Affiliation and Entry in First-Price Auctions with Heterogeneous Bidders*

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Abstract

In this paper we study the timber sales auctions in Oregon. We propsoe an entry and bidding model within the affiliated private value (APV) framework and with heterogeneous bidders, and establish existence of the entry equilibrium and existence and uniqueness of the bidding equilibrium with the joint distribution of private values belonging to the class of Archimedean copulas. We then estimate the resulting structural model, and find that the hauling distance plays a significant role in bidders' entry and bidding decisions. We quantify the extent to which the potential bidders' private values and entry costs are affiliated. The structural estimates are then used to conduct counterfactual analyses to address policy related issues. In particular, we quantify the effects of reserve price, affiliation, and merger on the end auction outcomes.

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1 Introduction

Auctions have long been used as a means for price determination under a competitive setting and an incomplete information environment. Auction theory developed within the game-theoretic framework with incomplete information (Harsanyi (1967/1968)) not only helps us understand how auctions work, but also offers insight in analyzing many other economic problems. A celebrated result in auction theory is Vickrey's (1961) revenue equivalence theorem, which postulates that all the four auction formats (first-price sealed-bid, second-price sealed bid, English, and Dutch auctions) generate the same average revenue for the seller with symmetric, independent, and risk-neutral bidders.

The revenue equivalence theorem is a powerful result that offers insight into how auction mechanisms work, and also raises important questions as to how this powerful result can be affected when the standard assumptions are relaxed. A large part of the auction theory has focused on answering these questions. Milgrom and Weber (1982) give revenue ranking with symmetric and affiliated bidders in which the English auction generates highest revenue among the four formats and the second-price auction ranks next; they also establish that with symmetric, affiliated, and risk-averse bidders who have constant absolute risk aversion, the English auction can generate at least as high revenue as the second-price auction. Myerson (1981) derives the optimal auctions with asymmetric bidders, and Maskin and Riley (1984) consider the case with risk-averse bidders. Levin and Smith (1994) extend the revenue equivalence and ranking results from Vickrey (1961) and Milgrom and Weber (1982) to the case with symmetric bidders (independent or affiliated) using mixed entry strategies.

Using timber sale auctions organized by the Oregon Department of Forestry (ODF), this paper attempts to address a set of questions that include with heterogeneous bidders and when entry is taken into account, how the seller's revenue could change with the extent to which the bidders' private values are affiliated, and whether the reserve price currently set by the ODF is optimal with respect to maximizing the seller's revenue/profit. Moreover, merger and bidder coalition have been an important issue to economists interested in competition policy, yet no empirical work has studied this issue taking into account endogenous participation from potential bidders. That we consider heterogeneous bidders is motivated by the evidence from the previous work studying the timber auctions in Oregon (e.g. Brannman and Froeb (2000) using data consisting of oral auctions, and Li and Zhang (2008) using the same data used in this paper comprising first-price sealed-bid auctions), that hauling distance plays an important role in bidders' bidding (Brannman and Froeb (2000)) and entry (Li and Zhang (2008)) decisions. This means that bidders are asymmetric and heterogeneous. Furthermore, Li and Zhang (2008) find a small but strongly significant level of affiliation among potential bidders' private information (either private values or entry costs). Lastly, recent empirical work in auctions in general and in timber auctions in particular (e.g. Athey, Levin and Seira (2004), Bajari and Hortacsu (2003), Kransnokutskaya and Seim (2006), Li and Zheng (2007, 2009)) has demonstrated that bidders' participation and entry decision is an integrated part of the decision making process that has to be taken into account when studying auctions. In view of these, in this paper we attempt to study the timber auctions organized by the ODF within a general framework in which potential bidders are affiliated and heterogeneous, and they make endogenous entry decisions before submitting bids.

Auction theory offers little guidance in answering these questions for auctions with entry and asymmetric potential bidders with affiliated private values. On the other hand, to gain insight on these questions from an empirical perspective, one needs to observe two states of world, such as pre and post the change of the affiliation level, or pre and post merger. Usually in auction data, as is the case in our data, however, one cannot observe these two states of world. Therefore we adopt the structural approach in our empirical analysis.

We develop an entry and bidding model for asymmetric bidders within the APV paradigm. Establishing existence and uniqueness of the entry and bidding equilbria within the APV model with entry and with heterogeneous bidders is a challenging problem. We extend the results by Lebrun (1999, 2006) for the IPV case with asymmetric bidders and without entry to our case and establish the existence and uniqueness of the bidding equlibrium and existence of the entry equlibrium for a general class of joint (affiliated) distribution of private values. This can be viewed as a contribution of the paper to the literature.

Because of the general framework we adopt, the answers to the aforementioned questions of our interest depend on the interactions of affiliation, entry, and asymmetry, as well as competition. As is well known, the optimal reserve price in a symmetric independent private value (IPV) model without entry does not depend on the number of potential bidders. This result can change if entry is introduced (see, e.g., Levin and Smith (1994), Samuelson (1985), Li and Zheng (2007)), or if bidders have affiliated private values (Levin and Smith (1996), Li, Perrigne, and Vuong (2003)). In our case, on the other hand, assessing the optimal reserve price is complicated further by the APV framework with entry and asymmetric bidders. Therefore we can only address this issue through a counterfactual analysis using the structural estimates. Furthermore, while the effect of the number of potential bidders on winning bids and seller's revenue is clear in an IPV model with symmetric bidders and without entry, it becomes less clear in a more general setting, such as the IPV model with entry and symmetric bidders (Li and Zheng (2007, 2009)), and the APV model without entry (Pinkse and Tan (2005)). In particular, Li and Zheng (2007, 2009) show that in terms of the relationship between the number of potential bidders and the expected seller's revenue, in addition to the usual "competition effect," there is an opposite effect due to the entry which they term as the "entry effect." On the other hand, Pinkse and Tan (2005) postulate that in a conditionally independent private value model, a special case of the APV paradigm, in addition to the "competition effect," there is an opposite effect caused by affiliation they term as the "affiliation effect." Zhang (2008) shows that in the APV model with entry and symmetric bidders, these three effects, namely, the "competition effect," the "entry effect," and the "affiliation effect" are at work. While we expect these three effects to remain in the APV framework with entry and asymmetric bidders, it becomes challenging to pinpoint them with asymmetric bidders. Since the effect of merger is closely related to how the seller's revenue changes with the set of potential bidders, i.e, not only the number of potential bidders, but also the identity of potential bidders when they are heterogeneous, and at the same time, theory does not yield good predictions, we rely on the structural analysis to gain insight on this issue.

We then develop a structural framework to estimate the entry and bidding model we propose. We use the estimated structural parameters to conduct counterfactual analyses of our interest. We find that for a representative auction, the optimal reserve price should be much larger than the current one. In evaluating the merger effects we find that **(TO BE ADDED LATER)**

Asymmetry is an indispensable element of the model given the asymmetric feature of the data. The analysis of the model, however, is complicated from both theoretical and econometric viewpoints due to the introduction of asymmetry. Because of the complexity of the model, and in particular, because that there is no closed form solution for the bidding function, we have to rely on some numerical approximation procedure. Moreover, while the structural analysis of auctions with asymmetric bidders has focused on the case with two types of bidders (Athey, Levin, and Seira (2004), Campo, Perrigne and Vuong (2003), and Kransnokutskaya and Seim (2006)), our model allows for all potential bidders to be different from each other, motivated by the fact that in our data, asymmetry is driven by the difference among bidders' hauling distances.

This paper makes contribution to the growing literature of the structural analysis of auction data since Paarsch (1992). While the structural approach has been extended to the APV paradigm by Li, Perrigne and Vuong (2000, 2002), Campo, Perrigne and Vuong (2003), and Li, Paarsch and Hubbard (2007), this paper is the first one in estimating a structural model within the APV paradigm and taking into account entry. On the other hand, while the recent work has started to pay attention to the problem of endogenous participation and entry, all the work has focused on the IPV framework with Bajari and Hortaçsu (2003) being an exception as they consider a common value (CV) model. In contrast, this paper considers the entry problem within the APV paradigm, a more general framework.

Our empirical analysis of the timber auctions and the resulting findings offer new insight on timber sale auctions and policy related issues. While most of the empirical analysis of timber sale auctions is based on the IPV model without entry (e.g. Paarsch (1997), Baldwin, Marshall and Richard (1997), Haile (2001), Haile and Tamer (2003), Li and Perrigne (2003)) or the IPV model with entry (Athey, Levin and Seira (2004), Li and Zheng (2007)), ours is based on the APV model with entry and heterogeneous bidders. As a result, our findings can be more robust, and also can be more useful for addressing the policy-related issues as our analysis takes into account the affiliation effect, the entry effect, and the asymmetry effect. Moreover and probably more interestingly, we study the merger effect within the asymmetric APV framework with entry, and offer new insight into how merger as well as other issues related to competition policy can be affected by complications arising from affiliation, entry, and asymmetry, and how they can be addressed within a unified framework as adopted in this paper.[1]

[1] It is worth noting that to the best of our knowledge, Brannman and Froeb (2000), considering oral timber auctions within an IPV paradigm without entry, is the only paper assessing the merger effect in auctions using the structural approach.

This paper is organized as follows. Section 2 describes the data we analyze in the paper. In Section 3 we propose the asymmetric APV model with entry. Section 4 is devoted to the structural analysis of the data, and Section 5 conducts a set of counterfactual analyses studying the effects of reserve prices, affiliation levels, and mergers. Section 6 concludes.

2 Data

The data we study in this paper are from the timber auctions organized by the ODF between January 2002 and June 2007. Before an auction is advertised, the ODF "cruises" the selected tract of timber and obtains information of the tract, such as the composition of the species, the quality grade of the timber and so on. Based on the information it obtains, the ODF sets its appraised price for the tract, which serves also as the reserve price. After the "cruise," a detailed bid notice is usually released 4-6 weeks prior to the sale date, which provides information about the auction, including the date and location of the sale, species volume, quality grade of the timber, appraised price as well as other related information. Potential bidders acquire their own information or private values through different ways and decide whether and how much to bid. Bids are submitted in sealed envelopes that are opened at a bid opening session at the ODF district office offering the sale. The sale is awarded to the bidder with the highest bid. All the sales are therefore first price sealed bid scale auctions.

The original data contain 415 sales in total. Among them, some sales have more than one bid species, which are deleted from our sample because of the "skewed bidding" issue discussed in Athey and Levin (2001). We focus on the sales in which Douglas-fir is the only bid species and drop the sales with other than Douglas-fir as bid species, because Douglas-fir is a majority species. Considering the time that our estimation program takes, we focus on the auctions with at most 8 potential bidders. The resulting final sample has 203 sales and 1074 observed bids.

For each sale, we directly observe some sale-specific variables including the location and the region of the sale, appraised price, appraised volume measured in thousand board feet or MBF, length of the contract, and diameter at breast height (DBH) as well. Noting that the bid species is often a combination of a mixture of several grades of quality, we use number 1, 2, $\cdots$, up to 18 to denote the letter-grades used by ODF so that the final grade of a sale is the weighted average of grades with volumes of grades as the weight. In addition to sale-specific variables, as shown in Brannman and Froeb (2000) and Li and Zhang (2008), respectively, hauling distance is an important bidder-specific variable that affects bidders' bidding and entry decisions. However, hauling distance is not observed directly. We use the hauling distance variable constructed in Li and Zhang (2008) who transfer the location of a tract into latitude and longitude through the Oregon Latitude and Longitude Locator[2] and find the distances between the tract and the mills of firms by using Google Map.

The key information related to endogenous entry is the identities of potential bidders, which

[2] It is available at http://salemgis.odf.state.or.us/scripts/esrimap.dll?name=locate&cmd=start

are not observed. Unlike some procurement auctions, where information on bidders who have requested bidding proposal is available and can be used as a proxy for potential bidders (Li and Zheng (2009)), we do not have such information in our case, as is usual for timber sale auctions. Therefore we follow Athey, Levin, and Seira (2004) and Li and Zheng (2007) to construct potential bidders. Specifically, we first divide all sales in the original data set into 146 groups each of which contains all sales held in the same district in the same quarter of the same year. The potential bidders of a sale are then all bidders who submit at least one bid in the sales of the group that the sale belongs to. In other words, all auctions in the same group have the same set of potential bidders. Note that in constructing the potential bidders we use the original data set including all auctions removed from our final sample. Summary statistics of the data are given in Table 1. Notably, the entry proportion, which is calculated as the ratio of the number of actual bidders and the number of potential bidders, is about 0.66 on average, meaning that while there is strong evidence of entry pattern from the potential bidders, on average more than half of the potential bidders would participate in the auction.

3 The Model

In this section we propose a theoretical two-stage model to characterize the timber sales, extending the models in Athey, Levin, and Seira (2004) and Krasnokutskaya and Seim (2006) with two groups of bidders within an IPV paradigm to the APV paradigm that allows potential bidders to be different from each other. Specifically, motivated by the finding of Brannman and Froeb (2000) that the hauling distance plays a significant role in bidders' bidding decision in oral timber auctions in Oregon, and the finding of Li and Zhang (2008) using the same data studied in this paper that the hauling distance is important in potential bidders' entry decision and potential bidders are affiliated through their private information (either private values or entry costs), we consider a first-price sealed-bid auction within the APV paradigm with a public reserve price, endogenous entry, and asymmetric bidders.

In the model, a single object is auctioned off to N heterogenous and risk-neutral potential bidders, who are affiliated in their private information. Bidder i has a private entry cost k_i, including the cost of obtaining private information and bid preparation, and does not obtain his private value v_i until he participates in the auction. We allow both private values and entry costs to be affiliated across bidders, that is $V_1, \ldots, V_N$ and $K_1, \ldots, K_N$ jointly follow a distribution $F(\cdot, \ldots, \cdot)$ with support $[\underline{v} = r, \overline{v}]^N$, and a distribution $G(\cdot, \ldots, \cdot)$ with support $\left[\underline{k}, \overline{k}\right]^N$, respectively, where r is the public reserve price of the auction. Affiliation is a terminology describing the positive dependence among random variables, which was first introduced into the study of auctions by Milgrom and Weber (1982). It is equivalent to the concept called multivariate total positivity of order 2 (MTP_2) in the multivariate statistics literature. Following Milgrom and Weber (1982), affiliation has the following formal definition.

Definition. *Let z and z' be any two values of a vector of random variables $Z \subseteq \mathbb{R}^n$ with a density*

$f(\cdot)$. It is said that all elements of Z are affiliated if $f(z \vee z') f(z \wedge z') \geq f(z) f(z')$, where $z \vee z'$ denotes the component-wise maximum of z and z', and $z \wedge z'$ denotes the component-wise minimum of z and z'.

Intuitively, affiliation means that large values for some of the components in Z make other components more likely to be large than small. We also denote the marginal distribution and density of bidder i's private value by $F_i(\cdot)$ and $f_i(\cdot)$ and marginal distribution and density of bidder i's entry cost by $G_i(\cdot)$ and $g_i(\cdot)$, respectively, and assume that $f_i(\cdot)$ is continuously differentiable and bounded away from zero on $[\underline{v} = r, \overline{v}]$. The subscript of distribution function implies that all potential bidders are of different types. This assumption is motivated by the fact that heterogeneity among bidders arises from different hauling distances in our data.

3.1 Bidding Strategy

Because the entry decision is based on the pre-entry expected profit, which depends on the bidding strategy of bidder i, we first describe the bidding strategy of bidder i. We assume that bidder i knows the number of the actual competitors in the bidding stage,[3] and thus bidder i's bidding strategy is determined by the first order condition of the following maximization problem,

$$\max (v_i - b_i) \Pr (B_j < b_i | v_i; a_{-i}),$$

where B_j denotes the maximum bid among other actual bidders and

$$a_{-i} \in A_{-i} = \{(a_1, \ldots, a_N) | a_j = 0 \text{ or } 1, j = 1, \ldots N, j \neq i\}$$

is one possibility of the 2^{N-1} combinations of entry behaviors of $N-1$ other potential bidders. Denote the number of actual bidders of the combination a_{-i} by $n_{a_{-i}}$. As usual we consider a continuously differentiable and strictly increasing bidding strategy, $b_i = s_i(v_i)$, therefore the first order condition is

$$-F_{V_{-i}|v_i}\left(s_j^{-1}(b_i), j \neq i | v_i\right) + (v_i - b_i) \sum_{j \neq i}^{n_{a_{-i}}} \frac{\partial F_{V_{-i}|v_i}\left(s_j^{-1}(b_i), j \neq i | v_i\right)}{\partial v_j} \frac{\partial s_j^{-1}(b_i)}{\partial b_i} = 0, \qquad (1)$$

where $F_{V_{-i}|v_i}$ denotes the joint distribution of $V_j, j \neq i$ conditional on $V_i = v_i$ and $s_i^{-1}(\cdot)$ is the inverse function of the bidding function of bidder i. A set of equation (1) for $i = 1, \ldots, n$ form a system of differential equations characterizing the equilibrium bids for all n actual bidders. We denote the post-entry profit of bidder i by $\pi_i(v_i | a_{-i})$.

[3] When the lower support of private value is below the reserve price, bidder i only knows the active bidders who participate in the auction but not actual bidders who submit bids. In our case, the number of active bidders is equal to the number of actual bidders, since the lower support of private value is assumed to be just the reserve price.

3.2 Entry Decision

In the initial participation stage, each potential bidder i only knows his own entry cost, joint distributions of entry costs and private values. Therefore the entry decision of bidder i is determined by his pre-entry expected profit from participation, Π_i. Specifically, he participates in the auction only if his entry cost is less then Π_i. Let p_i denote the entry probability of bidder i, respectively. The ex ante expected profit Π_i is given by

$$\Pi_i = \sum_{a_{-i} \in A_{-i}} \int_{\underline{v}}^{\overline{v}} \pi_i \left(v_i | a_{-i} \right) dF_i \left(v_i \right) \Pr \left(a_{-i} | a_i = 1 \right), \tag{2}$$

where $\Pr \left(a_{-i} | a_i = 1 \right)$ is a function of $p_i, i = 1, \ldots, N$, which can be denoted by $\Pr \left(a_{-i}; p_1, \ldots, p_N | a_i = 1 \right).$ As a result, the pre-entry expected profit is the sum of 2^{N-1} products of the post-entry profits and corresponding probabilities with the unknown private value integrated out. On the other hand, the ex ante probability of entry is given by $p_i = \Pr \left(K_i < \Pi_i \right) = G_i \left(\Pi_i \right).$

Note that although the number of potential bidders does not directly affect the bidding strategy in the bidding stage, it affects the number and the identities of actual bidders, which in turn have impact on the bidding strategy.

3.3 Characterization of the Equilibrium

Existence and uniqueness of the Bayesian Nash equilibrium with asymmetric bidders has been a challenging problem studied in the recent auction theory literature. See, e.g. Lebrun (1999, 2006) and Maskin and Riley (2000, 2003) within the IPV framework, Lizzeri and Persico (2000) within the APV framework and two types of bidders. The analysis of our model is further complicated by the introduction of affiliation and entry, as well as that we allow all potential bidders to be different from each other. To address the issue of existence and uniqueness in our case, we look at the case where the joint distribution of bidders' private values is characterized by the family of Archimedean copulas. For the copula concept and the characterization of the Archimedean copulas, see Nelsen (1999). Copula can provide a flexible way of modeling joint dependence of multivariate variables using the marginal distributions.

Specifically, by Sklar's theorem (Sklar (1973)), for a joint distribution $F \left(x_1, \ldots, x_N \right)$, there is a unique copula C, such that $C \left(F_1 \left(x_1 \right), \ldots, F_N \left(x_N \right) \right) = F \left(x_1, \ldots, x_N \right)$. For the Archimedean copulas, the copula C can be expressed as $C \left(u_1, \ldots, u_n \right) = \phi^{[-1]} \left(\phi \left(u_1 \right) + \cdots + \phi \left(u_n \right) \right)$, where ϕ is a generator of the copula and is a decreasing and convex function, and $\phi^{[-1]}$ denotes the pseudo-inverse of ϕ[4]. The family of Archimedean copulas include a wide range of copulas. For example, the generators $\phi \left(u \right) = \frac{1}{q} \left(u^{-q} - 1 \right)$, $\phi \left(u \right) = \left(-\ln \left(u \right) \right)^q$, and $\phi \left(u \right) = \ln \left(\frac{\exp(qu)-1}{\exp(q)-1} \right)$ correspond to the

[4] ϕ is a decreasing convex function from $[0,1]$ to $(0,\infty]$ with $\phi(1) = 0$. $\phi^{[-1]}$ is defined as

$$\phi^{[-1]} \left(u \right) = \begin{cases} \phi^{-1} \left(u \right), & 0 \leq u \leq \phi \left(0 \right), \\ 0, & \phi \left(0 \right) \leq u \leq \infty. \end{cases}$$

widely used Clayton copula, Gumbel copula, and Frank copula, respectively. Since we consider a differentiable bidding strategy, we have to confine ourself to the strict generator, that is $\phi^{[-1]} = \phi^{-1}$. Since $C_i\left(F_1\left(x_1\right), \ldots, F_N\left(x_N\right)\right) = F_{X_{-i}|x_i}\left(x_1, \ldots, x_N\right)$ (e.g. Li, Paarsch, and Hubbard (2007)), the first order condition (1) determining the equilibrium bids can be written as follows

$$\frac{ds_i^{-1}(b)}{db} = \frac{\phi^{-1\prime}\left(\sum_k \phi\left(F_k\left(s_k^{-1}(b)\right)\right)\right)}{\left(n_{a_{-i}}-1\right)\phi'\left(F_i\left(s_i^{-1}(b)\right)\right) f_i\left(s_i^{-1}(b)\right)\phi^{-1\prime\prime}\left(\sum_k \phi\left(F_k\left(s_k^{-1}(b)\right)\right)\right)}\left[\sum_{k\neq i}\frac{1}{s_k^{-1}(b)-b} - \frac{n_{a_{-i}}-2}{s_i^{-1}(b)-b}\right]. \tag{3}$$

Note that with the copula specification for the joint entry cost distribution, the entry probabilities in (2) can be expressed in terms of the joint entry cost distribution. For example, $\Pr\left(a_{-i}; p_1, \ldots, p_N | a_i = 1\right)$, for the case that given the participation of bidder i, bidder 1 up to bidder $i-1$ participate in the auction while bidder $i+1$ up to bidder N do not, can be expressed as

$$\begin{aligned} & \Pr\left(a_1 = \cdots a_{i-1} = 1, a_{i+1} = \cdots a_N = 0 | a_i = 1\right) \\ = & \frac{\Pr\left(a_1 = \cdots a_i = 1, a_{i+1} = \cdots a_N = 0\right)}{\Pr\left(a_i = 1\right)} \end{aligned} \tag{4}$$

where

$$\begin{aligned} & \Pr\left(a_1 = \cdots a_i = 1, a_{i+1} = \cdots a_N = 0\right) \\ = & C\left(p_1, \ldots, p_i, 1, \ldots, 1; q_k\right) - \sum_{i+1 \leq j \leq N} C\left(p_1, \ldots, p_i, p_j, 1, \ldots, 1; q_k\right) \\ & \cdots + (-1)^{N-i} C\left(p_1, \ldots, p_N; q_k\right), \end{aligned}$$

and $\Pr\left(a_i = 1\right) = C\left(1, \ldots, 1, p_i, 1, \ldots, 1; q_k\right)$.

Equilibrium of the model consists of two parts, entry equilibrium and bidding equilibrium. Based on the choice of Archimedean copulas for the joint distribution of private values, the existence of the equilibrium is guaranteed. Moreover, with some additional conditions, the bidding equilibrium is unique. The next proposition describes the equilibrium formally.

Proposition (Characterization of Equilibrium)**.** *Assume* (*a*) *the marginal distribution of entry cost of bidder i,* G_i *is continuous over* $\left[\underline{k}, \overline{k}\right]$ *for all i;* (*b*) *marginal distribution of private value of bidder i is differentiable over* $(\underline{v}, \overline{v}]$ *with a derivative* f_i *locally bounded away from zero over this interval for all i;* (*c*) *joint distribution of private values follows an Archimedean Copula.*

i. Bidding Equilibrium *In the bidding equilibrium, bidder i adopts a continuously differentiable and strictly increasing bidding function* $b_i = s_i(v)$ *over* $(\underline{v}, \overline{v}]$*. The inverse functions of* s_i *for all i,* $s_1^{-1}, \ldots, s_n^{-1}$ *are the solution of the system of differential equations* (3) *with boundary* *conditions* (5) *and* (6) :

$$s_i^{-1}(\underline{v}) = \underline{v} \tag{5}$$

$$s_i^{-1}(\eta) = \overline{v}. \tag{6}$$

for some η.

ii. Uniqueness of Bidding Equilibrium *Moreover, if* $F_i(\underline{v}) > 0$ *and* $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ *is decreasing in* u*, then the bidding equilibrium is unique.*

iii. Entry Equilibrium *In the entry equilibrium, bidder* i *chooses to participate in the auction if his entry cost is less than the threshold* $\Pi_i(p)$ *and stay out otherwise, where* $p = (p_1, \ldots p_N)$ *and* p_i *is the entry probability of bidder* i *and is determined by*

$$p_i = G_i(\Pi_i(p)). \tag{7}$$

As is seen here, the existence of the entry equilibrium is equivalent to the existence of the entry probability p_i, given by the equation (7). Since Π_i is continuous in p_i and thus G_i is continuous over $[0,1]$, there exists a solution p_i of equation (7), according to Kakutani's fixed point theorem (Kakutani (1941)). To show the uniqueness of the bidding equilibrium is to show that there is a unique η such that $s_i^{-1}(\eta) = \overline{v}$. Then starting from η, according to Lipschitz uniqueness theorem, s_i^{-1} is unique over $(\underline{v}, \eta]$. Note that a Clayton copula satisfies the condition for uniqueness that $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u. The formal proofs are provided in Appendix A.

4 The Structural Analysis

We estimate the model proposed in the last section using the timber sales data. Our objective is to recover the underlying joint distributions of private values and entry costs using observed bids and the number of actual bidders. The structural inference in our case is complicated because of the generality of our model that accounts for affiliation, asymmetry, and entry. Our approach circumvents the complications arising from the estimation of our model and makes the structural inference tractable. First, to model the affiliation in a flexible way, we adopt the copula approach in modeling the joint distribution of private values and the joint distribution of entry costs.[5] Second, since we allow bidders to be asymmetric, the system of differential equations consisting of equation (3) that characterizes bidders' Bayesian Nash equilibrium strategies does not yield closed-form solutions. To address this problem we adopt a numerical method based on Marshall, Meurer, Richard, and Stromquist (1994) and Gayle (2004). Third, because of the various covariates we try to control for and the relatively small size of the data set, the nonparametric method does not work well here. Therefore, we adopt a fully parametric approach.

[5] Li, Paarsch, and Hubbard (2007) use the copula approach to model affiliation within the symmetric APV framework without entry and propose a semiparametric estimation method.

4.1 Specifications

We adopt the Clayton copula to model the joint distributions of both private values and entry costs. With the generator of Clayton copula given above, the joint distribution of private value is specified as $F(v_1, \ldots v_n) = \left(\sum_i F_i(v_1)^{-q_v} - n + 1\right)^{-1/q_v}$, and the joint distrition of entry costs is specified as $G(k_1, \ldots, k_n) = \left(\sum_i G_i(v_1)^{-q_k} - n + 1\right)^{-1/q_k}$, where q_v and q_k are dependence parameters and F_i and G_i are the marginal distributions of private value and entry cost, which are specified as truncated exponential distributions given as follows, $F_{V_{\ell i}}(v|\mathbf{x}_{\ell i};\beta) = \frac{\frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\underline{v}\right) - \frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}v\right)}{\frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\underline{v}\right) - \frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\overline{v}\right)}$, $G_{K_{\ell i}}(k|\mathbf{x}_{\ell i};\beta) = \frac{\frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}\underline{k}\right) - \frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}k\right)}{\frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}\underline{k}\right) - \frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}\overline{k}\right)}$ for bidder i of the ℓ-th auction, $\ell = 1, \ldots, L$, where L is the number of auctions, $\lambda_{v_{\ell i}}$ and $\lambda_{k_{\ell i}}$ are the private value and entry cost means and equal $\exp(\beta\mathbf{x}_{\ell i})$ and $\exp(\alpha\mathbf{x}_{\ell i})$, respectively, and $\mathbf{x}_{\ell i}$ is a vector of covariates that are auction specific or bidder specific, and in our case includes variables such as hauling distance, volume, duration, grade, and DBH.[6] In practice, $\underline{v}$ is equal to the reserve price of ℓ-th auction, $\overline{v}$ is equal to \$1500/MBF, the lower bound of entry cost is equal to zero and the upper bound $\overline{k}$ is \$940/MBF, an arbitrarily large number.[7] We then model the joint distributions of private values and entry costs in auction ℓ as Clayton copula with different dependence parameters q_v and q_k. The use of the Clayton copula offers several advantages. First, it guarantees the existence and uniqueness of the equilibrium as discussed in Section 3.3. Second, it preserves the same dependence structure when the number of potential bidders changes. Third, it is relatively easy to draw dependent data from the Clayton copula, as it has a closed form that can be used to draw data recursively. Lastly, since q is the only parameter that measures the dependence, we can easily evaluate the impact of the dependence level on the end outcomes of an auction by changing the value of q.

Note that in these specifications, the asymmetry across potential bidders is captured by the inclusion of the hauling distance variable in $\mathbf{x}_{\ell i}$, while both α and β are kept constant across different bidders. This enables us to estimate a relatively parsimonious structural model and at the same time control for the asymmetry.

4.2 Estimation Method

Because of the complexity of our structural model, we employ the indirect inference method to estimate the model. Initially proposed in the nonlinear time series context by Smith (1993) and developed further by Gourieroux, Monfort, and Renault (1993) and Gallant and Tauchen (1996), the indirect inference method is simulation based and obtains the estimates of parameters by minimizing a measure of distance between the estimates for the auxiliary parameters of an auxiliary model using the original data and simulated data. More specifically, let θ denote the vector of parameters of

[6] Here we do not introduce unobserved auction heterogeneity into the model, as Li and Zhang (2008) show that it does not have a significant effect in bidders' entry behaviors.

[7] Here we need an upper bound for the private value to make the algorithem of finding the equilibrium bids possible. See Appendix B for the detail of the algorithem.

interest, γ be the parameters of the auxiliary model, $\widehat{\gamma}_T$ and $\widehat{\gamma}_{ST}^{(p)}(\theta)$ be the estimates of the auxiliary model using the original data and the p-th simulated data out of P sets of simulated data from the model given a specific θ, respectively. Then the estimator of θ, denoted by $\widehat{\theta}_{ST}$, is defined as

$$\widehat{\theta}_{ST} = \arg\min_{\theta} \left[\widehat{\gamma}_T - \frac{1}{P} \sum_{p=1}^{P} \widehat{\gamma}_{ST}^{(p)}(\theta) \right]' \Omega \left[\widehat{\gamma}_T - \frac{1}{P} \sum_{P=1}^{P} \widehat{\gamma}_{ST}^{(p)}(\theta) \right], \tag{8}$$

where Ω is a symmetric semi-positive definite matrix. Therefore to implement the indirect inference method, we have to draw data from the model for a given θ, which involves calculating the equilibrium bids and the thresholds of the entry costs. Basically we use numerical approximation method similar as the ones in Marshall, Meurer, Richard, and Stromquist (1994) and Gayle (2004) to find the equilibrium bids and iteration to find the equilibrium entry probabilities, both of which are illustrated in detail in Appendix B.

The auxiliary model, which is usually simpler than the original model and easier to estimate as well, plays an important role in the indirect inference method. In this paper, following the idea in Li (2005) we employ a relatively simple and easy-to-estimate auxiliary model to make the implementation tractable and the inference feasible. Specifically, since we use both bids and the number of actual bidders in the estimation of entry and bidding model, our auxiliary model includes two separate regressions: a linear regression of the observed bids and a Poisson regression of the number of actual bidders, which are described as follows

$$\begin{aligned} b_\ell &= \gamma_{10} + \sum_{h=1}^{H} X_{h\ell}\gamma_{11h} + \sum_{h=1}^{H} X_{h\ell}^2 \gamma_{12h} + \cdots + \sum_{h=1}^{H} X_{h\ell}^m \gamma_{1mh} + \varepsilon_1, \\ \Pr(n_\ell = k) &= \frac{\exp(-\lambda_\ell)\lambda_\ell^k}{k!}, \lambda_\ell = \gamma_{20} + \sum_{h=1}^{H} X_{h\ell}\gamma_{21h} + \sum_{h=1}^{H} X_{h\ell}^2\gamma_{22h} + \cdots + \sum_{h=1}^{H} X_{h\ell}^m \gamma_{2mh}, \end{aligned}$$

where b_ℓ is the average bid of auction ℓ, and $X_{h\ell}$, $h = 1, ...H$, denote the vector of auction-specific covariates of auction ℓ and the average of bidder-specific covariates, and H is the number of such covariates, which is 6 in our case. $m = 2$ makes our model over-identified.

An issue arising from the implementation of the indirect inference method is the discontinuity of the objective function of equation (8) because of the discrete dependent variable (the number of actual bidders) in the auxiliary model that makes gradient-based optimization algorithm invalid. We address this issue by using simplex, a nongradient-based algorithm. Alternatively, one can follow Keane and Smith (1993) to smooth the objective function using a logistic kernel.

4.3 Estimation Results

Table 2 reports the estimation results. For the (marginal) private value distribution, all the estimated parameters have the expected signs. Of particular interest is the parameter of the hauling distance variable, which is used to control for heterogeneity across bidders. Its estimate is negative, meaning that the longer the hauling distance is, the less is the private value mean. Furthermore, the average marginal effect of the hauling distance variable is about -0.09, meaning that one mile increase in the distance would reduce the private value mean by \$0.09/MBF while everything else is fixed. Another parameter of particular interest is the dependence parameter q_v in private values, which turns out to be relatively small ($q_v = 0.1014$) but significant. To get some idea of how large the dependence is with $q_v = 0.1014$, we use a measure called Kendall's τ (Nelsen (1999)), which is used to measure the concordance of two random variables. Concordance is not really the same concept as affiliation, but measures the positive dependence in a similar way. Kendall's τ is defined as the probability of concordance minus the probability of discordance:

$$\tau_{X,Y} = \Pr\left[(X_1 - X_2)(Y_1 - Y_2) > 0\right] - \Pr\left[(X_1 - X_2)(Y_1 - Y_2) < 0\right].$$

For the Clayton copula, $\tau = q/(q+2) = 0.047$. Therefore $q_v = 0.1014$ implies that the event of any two bidders' private values being concordant is about 4.7% more likely than the event of being discordant.

Two points in the estimates in the distribution of entry costs are worth noting. First, the hauling distance variable is positive in the entry cost distribution and its marginal effect is 1.34. Second, the dependence level among the entry costs is 0.5059, implying a Kendall's τ of 0.2 in the two bidders case. This indicates that the affiliation among the entry behaviors is mainly driven by the affiliation among the entry costs.

4.4 Model Fit

TO BE ADDED LATER

5 Counterfactual Analyses

With the estimated structural parameters we can now answer the questions put forward in the introduction section empirically. We focus on both end outcomes, namely, the number of actual bidders, and winning bids (or seller's unit revenue). We conduct counterfactual analyses on the 99th auction of our data. We use this auction as a representative auction, as the values of covariates of this auction are close to the average values of all auctions in our data set. In particular, the number of potential bidders in this auction is 7, about the same as the average number of potential bidders in the original data. In doing so we assume that the estimates derived from a subset of the data fit the whole data.

The seller's expected unit revenue is given as follows

$$\begin{aligned} E(w) &= E(w|w>0)\Pr(w>0) + E(w|w<0)\Pr(w<0) \\ &= E(w|w>0)\Pr(w>0) + v_0\Pr(w<0), \end{aligned}$$

where w denotes the winning bid and v_0 is the valuation of the timber to the seller, and the second equality follows the assumption that if the timber is not sold successfully then the seller gets his own value. In the following analyses we assume $v_0 = 0$, thus the expected revenue is equivalent to the expected profit.

5.1 Effects of Reserve Price and Dependence Level

Intuitively the effect of the reserve price can be seen from two aspects. On one hand, a higher reserve price is associated with a lower ex ante expected profit, i.e., a lower cut-off entry cost according to equation (2) as it narrows the integration range, and thus fewer participating bidders and lower probability of being sold, which may lower bids in our APV model with asymmetric bidders. On the other hand, a higher reserve price raises the lowest acceptable bids and of course makes bidders bid higher. Our counterfactuals shown in Figure 1 confirm such tradeoff. The three panels in Figure 1 show how the reserve price affects the number of actual bidders, the probability of being sold, and the seller's revenue. The number of actual bidders is decreasing in the reserve price as is shown in the first panel. The average number of participating bidders drops dramatically from 3.96 to 1.04 when the reserve price is raised from \$293.42/MBF to \$880/MBF. The probability of being sold is negatively related to the reserve price. The change in the winning bid is the final result of all effects associated with change in the reserve price. As is seen in the last panel, the optimal reserve price is around \$498/MBF, which is almost twice as large as the current reserve price. This implies that when the reserve price is below \$498/MBF the positive effect on the winning bid outweights the negative effect associated with the lower probability of being sold.

The APV model we estimated also enables us to quantify the effects of the dependence level among bidders. To this end, we change the values of the dependence parameters of both private values and entry costs while keeping other parameters fixed. We are able to conduct such analysis as the change of the dependence parameter does not affect the marginal distributions of private values and entry costs. As in the analysis of the effects of the reserve price, we are interested in three effects of the dependence parameters. Results are provided in Figure 2 and Figure 3. As is seen in the two figures, the dependence level does not affect the probability of being sold, which remains at about 98 percent. In Figure 2, it can be seen that the number of participating bidders, the probability of being sold and the seller's revenue are all slightly negatively related with the dependence level of private values. Since the dependence level of private values does not affect bidders' entry costs, it must be true that if bidders become more correlated with each other, their pre-entry expected profits become less, which leads to a decrease in the number of participating bidders. The drop in the probability of being sold is a direct result of the decrease in the number of participating bidders. Also, As can be seen in Figure 2, the seller's revenue drops as the dependence level of private values become higher.

The effects of the dependence level of entry costs are different than the dependence level of private values. The first panel in Figure 3 presents a positive relationship between the dependence level of entry costs and the number of participating bidders at least for the dependence levels that are less than 0.20 (kendall's τ). It is intuitive because the entry process is mainly governed by the entry cost. When the entry costs become more affiliated, more bidders will choose the same entry decision, which is to participate in the auction in this case. This is also the idea of the affiliation test in Li and Zhang (2008). Note that the relationship between the dependence level of entry costs has the same pattern as the relationship between the dependence level of entry costs and the number of participating bidders, which can be seen from the comparison of the first panel and the last panel of Figure 3. This is consistent with what we found in Figure 2.

5.2 Effect of Bidding Coalition or Merger

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6 Conclusion

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Appendix A: Proof of Proposition

The proof of Proposition adapts Lebrun (1999, 2006). We first need the following lemma.

Lemma. *Consider a continuously differentiable and strictly increasing bidding strategy. Assume* $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ *is decreasing in* u*. If* $\widetilde{\eta} > \eta$ *and* $\widetilde{s}_i^{-1}(b)$ *and* $s_i^{-1}(b)$ *for all* i *are two solutions of the system of differential equations* (3) *with boundary condition* (6) *over* $(\widetilde{\gamma}, \widetilde{\eta}]$ *and* $(\gamma, \eta]$*, respectively, then the inverse bidding functions satisfy the following condition:* $\widetilde{s}_i^{-1}(b) < s_i^{-1}(b)$ *for all* b *in* $(\max(\gamma, \widetilde{\gamma}), \eta]$*, where* $\gamma > \underline{v}$*.*

Proof. Since we know that s_i^{-1} is strictly increasing over $(\gamma, \eta]$, we have $\widetilde{s}_i^{-1}(\eta) < s_i^{-1}(\eta) = \overline{v}$. Define g in $[\max(\gamma, \widetilde{\gamma}), \eta]$ as follows:

$$g = \inf\left\{b \in [\max(\gamma, \widetilde{\gamma}), \eta] \,\middle|\, \widetilde{s}_i^{-1}(b') < s_i^{-1}(b'),\ \text{for all } i \text{ and all } b' \in (b, \eta]\right\}.$$

We want to prove that $g = \max(\gamma, \widetilde{\gamma})$. According to the definition of g, $\eta > g$. Suppose that $g > \max(\gamma, \widetilde{\gamma})$. By continuity, there exists i such that $\widetilde{s}_i^{-1}(g) = s_i^{-1}(g)$. From the definition of g, we also have $\widetilde{s}_j^{-1}(g) \le s_j^{-1}(g)$ for all j. Moreover, there exists $j \neq i$ such that $\widetilde{s}_j^{-1}(g) < s_j^{-1}(g)$, because all the solutions conincide at the point g and therefore conincide in $(g, \eta]$ due to the fact that the right hand side of equation (3) is locally Lipschitz at $b = g$, which contradicts the fact that at point η $\widetilde{s}_i^{-1}(\eta) < s_i^{-1}(\eta)$.

From equation (3), we know $ds_i^{-1}(b)/db$ is a strictly decreasing function of $s_j^{-1}(b)$, for all $j \neq i$, since $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u. Consequently, $d\widetilde{s}_i^{-1}(g)/db > ds_i^{-1}(g)/db$. Therefore there exists $\delta > 0$ such that $\widetilde{s}_i^{-1}(b) > s_i^{-1}(b)$, for all b in $(g, g+\delta)$. This contradicts the definition of g. □

Proof of Proposition

Proof. First we prove the first part of the proposition by showing that there exist an η, such that $s_i^{-1}(\eta) = \overline{v}$.

(i) Bidding Equilibrium

Let $i, 1 \le i \le n$ denote bidders who have the highest bids, denoted by η', at the upper bound of private value $\overline{v}$ and $j, 1 \le j \le n$ denote bidders who has the second highest bid, denoted by η, at the upper bound of private value $\overline{v}$. So $\eta' \ge \eta$.

For bidder i, we know that

$$\left(\overline{v} - \eta'\right) \Pr\left(B_{-i} < \eta' | \overline{v}\right) \ge \left(\overline{v} - \eta\right) \Pr\left(B_{-i} < \eta | \overline{v}\right).$$

It is obvious that $\Pr(B_{-i} < \eta' | \overline{v}) = 1$

$\Pr(B_{-i} < \eta | \overline{v}) = \Pr(b_j < \eta, b_k < \eta, k \neq i, j | v_i = \overline{v}) = \Pr(b_k < \eta, k \neq i, j | v_i = \overline{v})$, since b_j is not larger than η.

$\Pr(B_{-j} < \eta | \overline{v}) = \Pr(b_i < \eta, b_k < \eta, k \neq i, j | v_j = \overline{v})$.

Since the joint distribution of private values follows Archimedean copulas, we have

$$\begin{aligned}\Pr\left(B_{-i}<\eta|\overline{v}\right) &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}(\eta)\right)\right)+\phi\left(F_j\left(s_j^{-1}(\eta)\right)\right)+\phi\left(F_i\left(\overline{v}\right)\right)\right)\phi'\left(F_i\left(\overline{v}\right)\right)\\ &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}(\eta)\right)\right)+\phi(1)+\phi(1)\right)\phi'(1)\end{aligned}$$

and

$$\begin{aligned}\Pr\left(B_{-j}<\eta|\overline{v}\right) &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}(\eta)\right)\right)+\phi\left(F_j\left(\overline{v}\right)\right)+\phi\left(F_i\left(s_i^{-1}(\eta)\right)\right)\right)\phi'\left(F_j\left(\overline{v}\right)\right)\\ &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}(\eta)\right)\right)+\phi(1)+\phi\left(F_i\left(s_i^{-1}(\eta)\right)\right)\right)\phi'(1)\end{aligned}$$

If $F_i\left(s_i^{-1}(\eta)\right)<1$, then $\phi\left(F_i\left(s_i^{-1}(\eta)\right)\right)>\phi(1)$ and $\Pr\left(B_{-i}<\eta|\overline{v}\right)>\Pr\left(B_{-j}<\eta|\overline{v}\right)$ since $\phi'(1)<0$ and $\phi^{-1\prime}(x)$ is increasing in x. Therefore

$$\left(\overline{v}-\eta'\right)\Pr\left(B_{-j}<\eta'|\overline{v}\right)>\left(\overline{v}-\eta\right)\Pr\left(B_{-j}<\eta|\overline{v}\right)$$

since $\Pr\left(B_{-j}<\eta'|\overline{v}\right)=1$. But this is impossible because the optimal bid of bidder j at $\overline{v}$ is η, therefore we have $F_i\left(s_i^{-1}(\eta)\right)=1$ and $\eta'=\eta$.

(ii) Uniqueness of Bidding Equilibrium

Suppose that there exist two equilibria and thus two different values η and $\widetilde{\eta}$ such that the respective solutions $s_i^{-1}(b)$ and $\widetilde{s}_i^{-1}(b)$ are also solutions of the system of differential equations for all i. Without loss of generality, we can assume that $\eta<\widetilde{\eta}$. The value of $\ln\left(\Pr\left(v_j<s_j^{-1}(b_i),j\neq i|v_i\right)\right)$ at $b_i=\eta$ is strictly larger than the value of $\ln\left(\Pr\left(v_j<\widetilde{s}_j^{-1}(b_i),j\neq i|v_i\right)\right)$ at the same point. We have shown that $\widetilde{s}_i^{-1}(b)<s_i^{-1}(b)$ for all b in $(\underline{v},\eta]$. When b converges to $\underline{v}$, $s_i^{-1}(\underline{v})=\underline{v}$.

On the other hand, the first order condition can be written as follows

$$\frac{d\ln\left(\Pr\left(v_j<s_j^{-1}(b_i),j\neq i|v_i\right)\right)}{db}=\frac{1}{s_i^{-1}(b_i)-b_i}.$$

Therefore $\frac{d\ln\left(\Pr\left(v_j<s_j^{-1}(b),j\neq i|v_i\right)\right)}{db}<\frac{d\ln\left(\Pr\left(v_j<\widetilde{s}_j^{-1}(b),j\neq i|v_i\right)\right)}{db}$. Therefore, the difference between these two logarithms increases as b decreases towards $\underline{v}$. On the other hand, $\ln\left(\Pr\left(v_j<\underline{v},j\neq i|v_i\right)\right)$ is a finite value since $F_j(\underline{v})>0$. Therefore for two solutions, $\ln\left(\Pr\left(v_j<s_j^{-1}(b_i),j\neq i|v_i\right)\right)$ cannot both coverge to the same finite value as b decreases towards $\underline{v}$. Therefore η and $\widetilde{\eta}$ conincide and the equilibrium is unique.

(iii) Entry Equilibrium

The entry probability p_i is determined by equation (7). Let $p=(p_1,\ldots,p_n)\in[0,1]^n$ and

$G_p = (G_1 \circ \Pi_1(p), \ldots, G_n \circ \Pi_n(p))$. Since $s_i(v)$ and G_i is continuous, the pre-entry expected proft Π_i and $G_i \circ \Pi_i$ is continuous in p. So $G_p : [0,1]^n \to [0,1]^n$ and is continuous in p. A fixed point of p follows Kakutani's fixed point theorem (Kakutani (1941)). □

Appendix B: Solving Equilibrium Bids and Entry Probabilities

Equilibrium Bids

Note that with the choice of Clayton copula, the first order condition given in equation (3) can be written as follows,

$$(1+q)(v_i - b)\sum_{j\neq i}\frac{dF_j^{-q}\left(s_j^{-1}(b)\right)}{db} = -q\left(\sum_{k=1}^{n} F_k^{-q}\left(s_k^{-1}(b)\right) - n + 1\right).$$

Define $F_i^{-q}\left(s_i^{-1}(b)\right) = l_i(b)$, then $v_i = F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right)$, and F.O.C. becomes

$$(1+q)\left(F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) - b\right)\sum_{j\neq i} l_j'(b) = -q\left(\sum_{k=1}^{n} l_k(b) - n + 1\right)$$

Rewriting all terms in the equation as polynomials

$$\begin{aligned}
l_i(b) &= \sum_{j=0}^{\infty} a_{i,j}(b-b_0)^j, \\
l_i'(b) &= \sum_{j=0}^{\infty} (j+1)\, a_{i,j+1}(b-b_0)^j, \\
l_i^{-\frac{1}{q}}(b) &= \sum_{j=0}^{\infty} g_{i,j}(b-b_0)^j, \\
F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) &= \sum_{j=0}^{\infty} p_{i,j}(b-b_0)^j, \\
F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) - b &= \sum_{j=0}^{\infty} \widetilde{p}_{i,j}(b-b_0)^j, \\
F_i^{-1}(x) &= \sum_{j=0}^{\infty} d_{i,j}(x-x_0)^j, \\
x_i^{-\frac{1}{q}} &= \sum_{j=0}^{\infty} c_{i,j}(x-x_0)^j,
\end{aligned}$$

where $\widetilde{p}_{i,0} = p_{i,0} - b_0$, $\widetilde{p}_{i,1} = p_{i,1} - 1$, and $\widetilde{p}_{i,j} = p_{i,j}$ for $j > 1$.

Computation of $p_{i,j}, g_{i,j}$: following the lemma in Appendix C in Marshall, Meurer, Richard, and Stromquist (1994), we have

$$
\begin{aligned}
p_{i,J} &= \sum_{r=1}^{J} d_{i,r}\theta_{i,r,J} - z_J,\ p_{i,0} = F_i^{-1}\left(l_i^{-\frac{1}{q}}(b_0)\right) && \text{(10a)}\\
\theta_{i,r,J} &= \sum_{s=1}^{J-r+1} g_{i,s}\theta_{i,r-1,J-s},\ \theta_{i,0,0} = 1, && \text{(10b)}\\
g_{i,J} &= \sum_{r=1}^{J} c_{i,r}\varphi_{i,r,J}, && \text{(10c)}\\
\varphi_{i,r,J} &= \sum_{s=1}^{J-r+1} a_{i,s}\varphi_{i,r-1,J-s}, \varphi_{i,0,0} = 1. && \text{(10d)}
\end{aligned}
$$

Computation of $a_{i,j}$: from the FOC, we have

$$
\begin{aligned}
(1+q)\left(\sum_{j=0}^{\infty}\widetilde{p}_{i,j}(b-b_0)^j\right)\sum_{j\neq i}\sum_{s=0}^{\infty}(s+1)\,a_{j,s+1}(b-b_0)^s &= -q\left(\sum_{k=1}^{n}\sum_{s=0}^{\infty}a_{k,s}(b-b_0)^s - n + 1\right)\\
(1+q)\left(\sum_{j=0}^{\infty}\widetilde{p}_{i,j}(b-b_0)^j\right)\sum_{s=0}^{\infty}(s+1)\left(\sum_{j\neq i}a_{j,s+1}\right)(b-b_0)^s &= -q\left(\sum_{s=0}^{\infty}\left(\sum_{k=1}^{n}a_{k,s}\right)(b-b_0)^s - n + 1\right)\\
(1+q)\sum_{s=0}^{\infty}(s+1)\left(\sum_{r=0}^{s}\widetilde{p}_{i,s}\sum_{j\neq i}a_{j,s+1-r}\right)(b-b_0)^s &= -q\left(\sum_{s=0}^{\infty}\left(\sum_{k=1}^{n}a_{k,s}\right)(b-b_0)^s - n + 1\right)
\end{aligned}
$$

Comparing the coefficients of $(b-b_0)^s$ we have

$$
\begin{aligned}
(1+q)(s+1)\left(\sum_{r=0}^{s}\widetilde{p}_{i,s}\sum_{j\neq i}a_{j,s+1-r}\right) &= -q\left(\sum_{k=1}^{n}a_{k,s}\right),\ \text{for } s>0 && \text{(11a)}\\
(1+q)\,p_{i,0}\sum_{j\neq i}a_{j,1} &= -q\left(\sum_{k=1}^{n}a_{k,0} - n + 1\right),\ \text{for } s=0. && \text{(11b)}
\end{aligned}
$$

Algorithm:

1. $d_{i,j}, c_{i,j}$ for $j = 1, \ldots, J$, can be computed by Taylor expansion. In practice, $J = 3$.
2. Decide $a_{i,0}, \widetilde{p}_{i,0}, \theta_{i,0,0}$,and $\varphi_{i,0,0}$ by the boundary conditions.
3. Calculate $\widetilde{p}_{i,1}$ from equations (10) given $a_{i,0}, \widetilde{p}_{i,0}, \theta_{i,0,0}$,and $\varphi_{i,0,0}$.
4. Calculate $a_{i,1}$ from equations (11) given $\widetilde{p}_{i,1}$.
5. Repeat step 3 and 4 until $a_{i,j}, j = 1, \ldots, J$ are calculated.

Now we have found the coefficients of the Taylor expansion of the inverse bidding function up to the J-th order, so we are able to find the equilibrium bid for a given private value for bidder i through the obtained Taylor expansion at a appropriate point. One issue regarding the algorithm is the boundary conditions. From the Proposition we know that there are two boundary conditions associated with the equilibrium. Note that it cannot be used here although the boundary condition at the lower bound of bids is known to us, since it causes the problem of singularity. Therefore we have to use the condition at the upper bound, which is unfortunately unknown to us. To address this problem we follow the method described in Marshall, Meurer, Richard, and Stromquist (1994) and Gayle (2004) to find the common η first. Roughly, it is to find an η which generates the best equilibrium bids at point $\underline{v}$ according to the algorithm described above. Please refer to the two papers for details.

Equilibrium Entry Probabilities

As is seen from the Proposition and equation (7) the equilibrium entry probabilities are determined by a fixed point problem. We solve it through iteration, described in detail as follows,

1. Given an initial guess of $p_i^{old}, i = 1, \ldots, N$, we calculate the probabilities of all possible entry behavior occurring according to equation (4).

2. Given the calculated $\int_{\underline{v}}^{\overline{v}} \pi_i (v_i | a_{-i}) \, dF_i (v_i)$ for all possible entry behavior in equation (2) and associated probabilities given in step 1, we calculate the post-entry expected profit $\Pi_i, i = 1, \ldots, N$.

3. Calculate new entry probabilities according to $p_i^{new} = G_i (\Pi_i), i = 1, \ldots, N$.

4. If the difference of p_i^{new} and p_i^{old} is smaller than a given small positive number, ε, then $p_i^{new}, i = 1, \ldots, N$ are the equilibrium entry probabilities and the iteration stops; otherwise, let $p_i^{old} = p_i^{new}$ and go to step 1.

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Table 1: Summary Statistics of Bidder- and Auction-specific Covariates

	Observation	**Mean**	**Std. Dev.**
Bid	1074	384.5844	103.7889
# of Potential Bidders	203	5.8276	1.5690
# of Actual Bidders	203	3.6946	1.7250
Entry Proportion	203	.6550	.2826
Appraised Price	203	331.291	94.322
Distance	1183	75.2779	45.6976
Volume	203	3318.468	2674.112
Duration	203	780.010	199.965
Grade	203	10.326	.461
DBH	203	16.722	4.812

Table 2: Estimation Results

	Private Value distribution		**Entry Cost distribution**	
	Coefficient	**Std. Error**	**Coefficient**	**Std. Error**
Hauling Distance	-.1391		1.3807	
Volume	.0599		.0691	
Duration	-.0795		.0294	
Grade	.9479		1.0781	
DBH	.1733		-.0762	
Dependence Parameter	.1025		0.5059	

Figure 1: Effect of Reserve Price

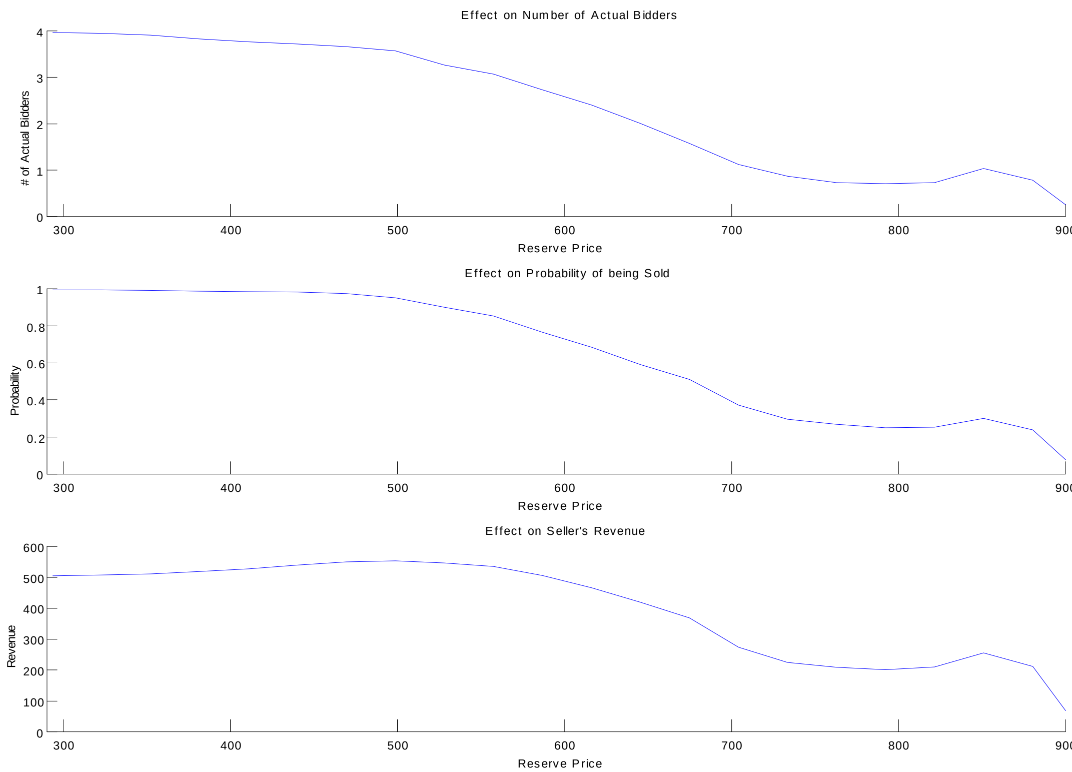

Figure 2: Effect of Dependence Level of Private Values

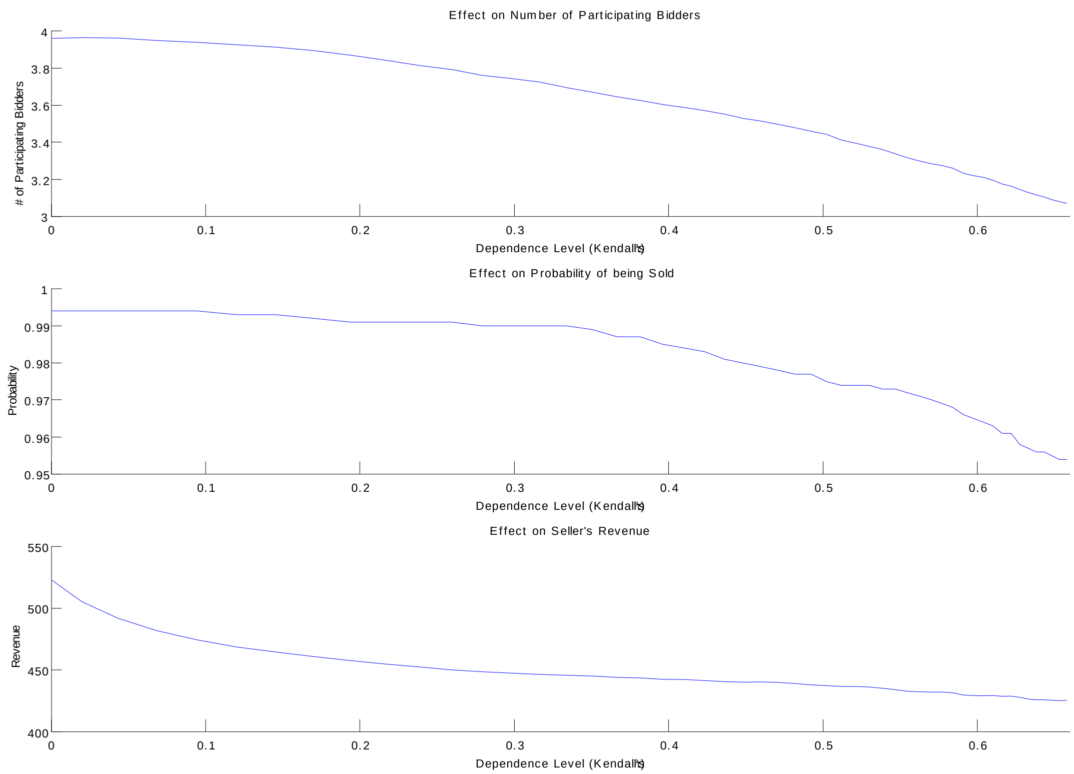

Figure 3: Effect of Dependence Level of Entry Costs