Appendix to "International Portfolio Diversification and the Structure of Global Production" (for online publication only)

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Appendix A: Detailed theoretical analysis

This section of the appendix contains additional details about the theoretical analysis.

To solve for equilibrium portfolios in the theoretical model, I follow the same approach as HP: find portfolio weights that decentralize the optimal allocation. I also follow HP's approach to deriving intuition for the solution from the risk-sharing condition.

A.1 Competitive equilibrium first-order conditions

The first-order conditions of the representative production firm in country i are:

$$w_i(s^t) = v(1-\alpha)p_{i,i}(s^t)\left(\frac{y_i(s^t)}{\ell_i(s^t)}\right), \tag{1}$$

$$p_{i,i}(s^t) = (1-v)\mu p_{i,i}(s^t)\left(\frac{y_i(s^t)}{m_{i,i}(s^t)}\right), \tag{2}$$

$$p_{i,j}(s^t) = (1-v)\left(\frac{1-\mu}{I-1}\right)p_{i,i}(s^t)\left(\frac{y_i(s^t)}{m_{i,j}(s^t)}\right),\ j \neq i, \tag{3}$$

$$Q_i(s^t) = \sum_{s_{t+1}\in S} Q_i(s^t, s_{t+1})\left[v\alpha p_{i,i}(s^t, s_{t+1})\left(\frac{y_i(s^t, s_{t+1})}{k_i(s^t, s_{t+1})}\right) + 1 - \delta\right]. \tag{4}$$

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The first-order conditions of the representative retailer are

$$p_{i,i}(s^t) = \omega \left(\frac{g_i(s^t)}{g_{i,i}(s^t)} \right), \tag{5}$$

$$p_{i,j}(s^t) = \left(\frac{1-\omega}{I-1} \right) \left(\frac{g_i(s^t)}{g_{i,j}(s^t)} \right). \tag{6}$$

The household's first-order conditions are

$$u_{i,c}(s^t) w_i(s^t) + u_{i,\ell}(s^t) = 0, \tag{7}$$

$$u_{i,c}(s^t) e_{i,j}(s^t) q_j(s^t) = \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s^t) u_{i,c}(s^t, s_{t+1}) e_{i,j}(s^t, s_{t+1}) (q_j(s^t, s_{t+1}) + d_j(s^t, s_{t+1})), \tag{8}$$

where $u_{i,c}(s^t) = u_c(c_i(s^t))$ and $u_{i,\ell}(s^t) = u_\ell(\ell_i(s^t))$. The household's budget constraint and all market clearing conditions must also be satisfied.

A.2 Planner's problem

The optimal allocation is the solution to the following equal-weighted social planner's problem:

$$\max_{\left(c_i(s^t), \ell_i(s^t), x_i(s^t), k_i(s^t), \left(g_{i,j}(s^t), m_{i,j}(s^t)\right)_{j \in I}\right)_{i \in I}} \frac{1}{I} \sum_{i \in I} \sum_{t=0} \sum_{s^t \in S^t} \pi(s^t) \beta u(c_i(s^t), \ell_i(s^t)) \tag{9}$$

subject to

$$y_i(s^t) = \left[z_i(s^t) k_i(s^{t-1})^\alpha \ell_i(s^t)^{1-\alpha} \right]^v \left[m_{i,i}(s^t)^\mu \left(\prod_{j \neq i} m_{i,j}(s^t)^{\frac{1-\mu}{I-1}} \right) \right]^{1-v}, \tag{10}$$

$$g_i(s^t) = g_{i,i}(s^t)^\omega \left(\prod_{j \neq i} g_{i,j}(s^t)^{\frac{1-\omega}{1-I}} \right), \tag{11}$$

$$y_i(s^t) = \sum_{j=1}^{I} \left(m_{j,i}(s^t) + g_{j,i}(s^t) \right), \tag{12}$$

$$g_i(s^t) = c_i(s^t) + x_i(s^t), \tag{13}$$

$$k_i(s^t) = (1-\delta) k_i(s^{t-1}) + x_i(s^t). \tag{14}$$

We can write the first-order conditions of this problem for each country i as

$$u_{i,c}(s^t)\left[v(1-\alpha)\frac{y_i(s^t)}{\ell_i(s^t)}\right]\left[\omega\frac{g_i(s^t)}{g_{i,i}(s^t)}\right]+u_{i,\ell}(s^t)=0, \tag{15}$$

$$u_{i,c}(s^t)\left[\omega\frac{g_i(s^t)}{g_{i,i}(s^t)}\right]=u_{j,c}(s^t)\left[\frac{1-\omega}{I-1}\frac{g_j(s^t)}{g_{j,i}(s^t)}\right], \tag{16}$$

$$m_{i,i}(s^t)=(1-v)\mu y_i(s^t), \tag{17}$$

$$m_{i,j}(s^t)=\left[(1-v)\frac{1-\mu}{1-I}y_i(s^t)\right]\left[\frac{\omega}{(1-\omega)/(I-1)}\frac{g_{i,j}(s^t)}{g_{i,i}(s^t)}\right], \tag{18}$$

$$u_{i,c}(s^t)=\beta\sum_{s_{t+1}\in S}u_{i,c}(s^t,s_{t+1})\left[\left(\omega\frac{g_i(s^t,s_{t+1})}{g_{i,i}(s^t,s_{t+1})}\right)\left(\alpha v\frac{y_i(s^t,s_{t+1})}{k_i(s^t)}\right)+1-\delta\right]. \tag{19}$$

These equations, together with the production functions (10)–(11) and the resource constraints (12) and (13), characterize the solution to the planner's problem.

A.3 Proof of proposition 1

We will show that if equilibrium portfolios are given by

$$1-\lambda=\frac{(I-1)(1-D\omega-F(1-\omega))}{I-1+\alpha\left[D+(I-1)F-I(D\omega+F(1-\omega))\right]}, \tag{20}$$

where the constants D and F are defined as

$$D=\frac{1-\mu-\mu v-(I-2)v}{I\mu+v-I\mu v-I}, \tag{21}$$

$$F=\frac{(1-v)(\mu-1)}{I\mu+v-I\mu v-I}, \tag{22}$$

then there exists a set prices at which the planner's solution satisfies all of the competitive equilibrium first-order conditions. The candidate prices are as follows: the prices of gross output relative to domestic final goods, $p_{i,i}(s^t)$ and $p_{i,j}(s^t)$, are given by equations (5) and (6); wages be given by (1); the real exchange rates are given by the law of one price,

$$e_{i,j}(s^t)p_{j,j}(s^t)=p_{i,j}(s^t); \tag{23}$$

and stock prices are given by $q_i(s^t)=k_i(s^t)$.

First, note that all market clearing conditions for goods are satisfied directly — they are constraints on the planner's problem — and that the candidate portfolio solution satisfies the market clearing condition for equities. It remains to show that the first-order conditions and the household's budget constraint are satisfied.

The first-order conditions (5), (6), (1), and (23) are trivially satisfied. The first-order condition for domestic intermediates, equation (2), is the same as the planner's optimality condition (17), so the former is satisfied. Many of the other first-order conditions are yielded directly by substiting the candidate prices. Using the candidates for $p_{i,i}(s^t)$ and $p_{i,j}(s^t)$ in the planner's condition (18) yields the first-order condition for imported interrmediates, equation (3), so that is satisfied as well. Using the candidates for $w_i(s^t)$ and $p_{i,i}(s^t)$ in the planner's condition (15) yields the household's intratemporal first-order condition (7), so that is also satisfied. Using the candidate for $p_{i,i}(s^t)$ in the planner's condition (19) yields the first-order condition for investment (4), so that is satisfied.

To show that the portfolio-choice first-order conditions (8) are satisfied, first note that using the candidate prices in the planner's optimality condition (16) yields the standard risk-sharing condition

$$u_{i,c}(s^t)e_{i,j}(s^t) = u_{j,c}(s^t). \tag{24}$$

Also note that we can express dividends as

$$d_i(s^t) = \alpha v p_{i,i}(s^t)y_i(s^t) - x_i(s^t) = \left(\omega \frac{g_i(s^t)}{g_{i,i}(s^t)}\right) \alpha v y_i(s^t) - k_i(s^{t+1}) + (1-\delta)k_i(s^t) \tag{25}$$

Multiply both sides of the planner's choice for country-i investment (19) by $k_i(s^t)$:

$$u_{i,c}(s^t)k_i(s^t) = \beta \sum_{s_{t+1} \in S} u_{i,c}(s^t, s_{t+1}) \left[\left(\omega \frac{g_i(s^t, s_{t+1})}{g_{i,i}(s^t, s_{t+1})}\right) \alpha v y_i(s^t, s_{t+1}) + (1-\delta)k_i(s^t) \right]$$

If we use the candidate stock price and the dividend expression above we get the first-order condition for the choice of domestic stock. If we use the version of this equation for country j and use the risk-sharing equation (24) we get country i's first-order condition for the choice of country j's stock.

All that remains is to show that the budget constraints are satisfied. Since portfolios are assumed constant and symmetric, budget constraints reduce to (suppressing state dependent notation for brevity)

$$c_i = w_i \ell_i + \lambda d_i + \left(\frac{1-\lambda}{I-1}\right) \sum_{j \neq i} e_{i,j} d_j.$$

where λ is the portfolio weight on domestic stock. Use the candidate wage function and the dividend expression (25):

$$c_i = v(1-\alpha)p_{i,i}y_i + \lambda\left(v\alpha p_{i,i}y_i - x_i\right) + \left(\frac{1-\lambda}{I-1}\right) \sum_{j \neq i} e_{i,j}\left(v\alpha p_{j,j}y_j - x_j\right)$$

Use the candidate real exchange rate and rearrange:

$$c_i = [v(1-\alpha) + \lambda v\alpha]\, p_{i,i}y_i + \left(\frac{1-\lambda}{I-1}\right) \sum_{j \neq i} v\alpha e_{i,j} p_{j,j} y_j - \lambda x_i - \left(\frac{1-\lambda}{I-1}\right) \sum_{j \neq i} e_{i,j} x_j$$

From here, I complete the proof for two separate cases. In the simpler one, where $v = 1$ so that there are no intermediate inputs, we can take the same approach as in HP. Use market clearing for gross output and the law of one price:

$$\begin{aligned}
c_i = {} & [1-\alpha+\lambda\alpha] \left[p_{i,i} g_{i,i} + \sum_{k \neq i} e_{i,j} p_{j,i} g_{j,i} \right] \\
& + \alpha \left(\frac{1-\lambda}{I-1}\right) \sum_{j \neq i} \left[p_{i,j} g_{i,j} + e_{i,j} p_{j,j} g_{j,j} + \sum_{k \neq i,j} e_{i,k} p_{k,j} g_{k,j} \right] \\
& - \lambda x_i - \left(\frac{1-\lambda}{I-1}\right) \sum_{j \neq i} e_{i,j} x_j
\end{aligned}$$

Use the candidate gross output prices and rearrange:

$$\begin{aligned}
c_i = {} & [1-\alpha+\lambda\alpha] \left[\omega g_i + \sum_{k \neq i} e_{i,k} \left(\frac{1-\omega}{I-1}\right) g_k \right] \\
& + \left(\frac{1-\lambda}{I-1}\right) \alpha \sum_{j \neq i} \left[\left(\frac{1-\omega}{I-1}\right) g_i + e_{i,j} \omega g_j + \sum_{k \neq i,j} e_{i,k} \left(\frac{1-\omega}{I-1}\right) g_k \right] \\
& - \lambda x_i - \left(\frac{1-\lambda}{I-1}\right) \sum_{j \neq i} e_{i,j} x_j
\end{aligned}$$

Collect like terms to get

$$\begin{aligned}
c_i = {} & \left[\omega + \left(\frac{1-\lambda}{I-1}\right) \alpha \left(1 - I\omega\right) \right] g_i + \left(\frac{1}{I-1}\right) \sum_{j \neq i} e_{i,j} \left[1 - \omega - \left(\frac{1-\lambda}{I-1}\right) \alpha (1 - I\omega) \right] g_j \\
& - \lambda x_i - \left(\frac{1-\lambda}{I-1}\right) \sum_{j \neq i} e_{i,j} x_j.
\end{aligned}$$

Rearrange and use market clearing for final goods:

$$\begin{aligned}
g_i = {} & \left[\omega + \left(\frac{1-\lambda}{I-1}\right) \alpha \left(1 - I\omega\right) \right] g_i + \left(\frac{1}{I-1}\right) \sum_{j \neq i} e_{i,j} \left[1 - \omega - \left(\frac{1-\lambda}{I-1}\right) \alpha (1 - I\omega) \right] g_j \\
& + (1-\lambda)(g_i - c_i) - \left(\frac{1-\lambda}{I-1}\right) \sum_{j \neq i} e_{i,j} (g_j - c_j).
\end{aligned}$$

Use the risk-sharing equation (24):

$$g_i = \left[\omega + \left(\frac{1-\lambda}{I-1}\right)(I-1+\alpha(1-I\omega))\right] g_i + \left(\frac{1}{I-1}\right)\sum_{j\neq i} e_{i,j}\left[1-\omega-\left(\frac{1-\lambda}{I-1}\right)(I-1+\alpha(1-I\omega))\right] g_j.$$

This equation is satisfied if and only if the proposition is true, i.e.,

$$1-\lambda = \frac{(I-1)(1-\omega)}{I-1+\alpha(1-I\omega)}.$$

If $v < 1$, we must use exploit the nature of roundabout production to complete the proof. Following Johnson and Noguera (2012), we can write the system of gross output market clearing conditions in expenditure form as

$$\begin{bmatrix} p_{1,1}y_1 \\ p_{2,2}y_2 \\ \vdots \\ p_{I,I}y_I \end{bmatrix} = \begin{bmatrix} p_{1,1}m_{1,1} + e_{1,2}p_{2,1}m_{21} + \ldots + e_{1,I}p_{I,1}m_{I,1} \\ e_{2,1}p_{1,2}m_{1,2} + p_{2,2}m_{2,2} + \ldots + e_{2,I}p_{I,2}m_{I,2} \\ \vdots \\ e_{I,1}p_{1,I}m_{1,I} + e_{I,2}p_{2,I}m_{2,I} + \ldots + p_{I,I}m_{I,I} \end{bmatrix} + \begin{bmatrix} p_{1,1}g_{1,1} + e_{1,2}p_{2,1}g_{21} + \ldots + e_{1,I}p_{I,1}g_{I,1} \\ e_{2,1}p_{1,2}g_{1,2} + p_{2,2}g_{2,2} + \ldots + e_{2,I}p_{I,2}g_{I,2} \\ \vdots \\ e_{I,1}p_{1,I}g_{1,I} + e_{I,2}p_{2,I}g_{2,I} + \ldots + p_{I,I}g_{I,I} \end{bmatrix}$$

This is equivalent to

$$\begin{bmatrix} p_{1,1}y_1 \\ p_{2,2}y_2 \\ \vdots \\ p_{I,I}y_I \end{bmatrix} = A \begin{bmatrix} p_{1,1}y_1 \\ p_{2,2}y_2 \\ \vdots \\ p_{I,I}y_I \end{bmatrix} + \begin{bmatrix} p_{1,1}g_{1,1} + e_{1,2}p_{2,1}g_{21} + \ldots + e_{1,I}p_{I,1}g_{I,1} \\ e_{2,1}p_{1,2}g_{1,2} + p_{2,2}g_{2,2} + \ldots + e_{2,I}p_{I,2}g_{I,2} \\ \vdots \\ e_{I,1}p_{1,I}g_{1,I} + e_{I,2}p_{2,I}g_{2,I} + \ldots + p_{I,I}g_{I,I} \end{bmatrix}$$

where

$$A = (1-v)\begin{bmatrix} \mu & \frac{1-\mu}{I-1} & \cdots & \frac{1-\mu}{I-1} \\ \frac{1-\mu}{I-1} & \mu & \cdots & \frac{1-\mu}{I-1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1-\mu}{I-1} & \frac{1-\mu}{I-1} & \cdots & \mu \end{bmatrix}$$

This, in turn, is equivalent to

$$\begin{bmatrix} p_{1,1}y_1 \\ p_{2,2}y_2 \\ \vdots \\ p_{I,I}y_I \end{bmatrix} = (I-A)^{-1} \begin{bmatrix} p_{1,1}g_{1,1} + e_{1,2}p_{2,1}g_{21} + \ldots + e_{1,I}p_{I,1}g_{I,1} \\ e_{2,1}p_{1,2}g_{1,2} + p_{2,2}g_{2,2} + \ldots + e_{2,I}p_{I,2}g_{I,2} \\ \vdots \\ e_{I,1}p_{1,I}g_{1,I} + e_{I,2}p_{2,I}g_{2,I} + \ldots + p_{I,I}g_{I,I} \end{bmatrix}$$

The Leontief inverse matrix, $(I-A)^{-1}$, is equal to

$$[v\,(I\mu+v-I\mu v-I)]^{-1}\begin{bmatrix}(1-\mu-\mu v-(I-2)v) & (1-v)(\mu-1) & \dots & (1-v)(\mu-1)\\ (1-v)(\mu-1) & (1-\mu-\mu v-(I-2)v) & \dots & (1-v)(\mu-1)\\ \vdots & \vdots & \ddots & \vdots\\ (1-v)(\mu-1) & (1-v)(\mu-1) & \dots & (1-\mu-\mu v-(I-2)v)\end{bmatrix}$$

Substitute this into the budget constraint to obtain

$$\begin{aligned}c_i &= [1-\alpha+\lambda\alpha]\left\{D\left[p_{i,i}g_{i,i}+\sum_{k\neq i}e_{i,k}p_{k,i}g_{k,i}\right]+\sum_{j\neq i}F\left[p_{i,j}g_{i,j}+e_{i,j}p_{j,j}g_{j,j}+\sum_{k\neq j}e_{i,k}p_{k,j}g_{k,j}\right]\right\}\\ &+\alpha\left(\frac{1-\lambda}{I-1}\right)\sum_{j\neq i}\left\{D\left[e_{i,j}p_{j,j}g_{j,j}+p_{i,j}g_{i,j}+\sum_{k\neq j}e_{i,k}p_{k,j}g_{k,j}\right]+F\sum_{k\neq j}\left[e_{i,j}p_{j,k}g_{j,k}+p_{i,k}g_{i,k}+\sum_{l\neq k}e_{i,l}p_{l,k}g_{l,k}\right]\right\}\\ &-\lambda x_i-\left(\frac{1-\lambda}{I-1}\right)\sum_{j\neq i}e_{i,j}x_j\end{aligned}$$

where

$$D=\frac{1-\mu-\mu v-(I-2)v}{I\mu+v-I\mu v-I},$$
$$F=\frac{(1-v)(\mu-1)}{I\mu+v-I\mu v-I}.$$

Use the candidate prices and collect like terms to get

$$\begin{aligned}c_i &= \left\{D\omega+F(1-\omega)+\left(\frac{1-\lambda}{I-1}\right)\alpha\,[D+(I-1)F-I(D\omega+F(1-\omega)]\right\}g_i\\ &+\left(\frac{1}{I-1}\right)\sum_{j\neq i}e_{i,j}\left\{1-D\omega-F(1-\omega)-\left(\frac{1-\lambda}{I-1}\right)\alpha\,[D+(I-1)F-I(D\omega+F(1-\omega)]\right\}g_j\\ &-\lambda x_i-\left(\frac{1-\lambda}{I-1}\right)\sum_{j\neq i}e_{i,j}x_j\end{aligned}$$

Use final goods market clearing and the risk-sharing equation:

$$\begin{aligned}g_i &= \left\{D\omega+F(1-\omega)+\left(\frac{1-\lambda}{I-1}\right)(I-1+\alpha\,[D+(I-1)F-I(D\omega+F(1-\omega))])\right\}g_i\\ &+\left(\frac{1}{I-1}\right)\sum_{j\neq i}e_{i,j}\left\{1-D\omega-F(1-\omega)-\left(\frac{1-\lambda}{I-1}\right)(I-1+\alpha\,[D+(I-1)F-I(D\omega+F(1-\omega)])\right\}g_j\end{aligned}$$

This equation is satisfied if and only if the proposition is true, i.e.,

$$1-\lambda=\frac{(I-1)(1-D\omega-F(1-\omega))}{I-1+\alpha\,[D+(I-1)F-I(D\omega+F(1-\omega))]}.$$

Differentiating this expression with respect to the key variables yields the relevant properties.

A.4 Risk-sharing intuition

First, note that with log utility, perfect risk sharing requires that

$$c_i(s^t) = e_{i,j}(s^t)c_j(s^t), \ \forall i, j.$$

Following HP, we can write this as $\Delta c_{i,j}(s^t) = 0$ for all i, j, where $\Delta c_{i,j}(s^t)$ is the exchange-rate adjusted difference between domestic and foreign consumption. Let $\Delta y_{i,j}(s^t)$ denote the exchange-rate adjusted difference in nominal gross output between country i and country j. Similarly, let $\Delta x_{i,j}(s^t)$ denote the exchange-rate adjusted difference in investment.

If portfolios are constant and symmetric, then country i's budget constraint implies

$$\begin{aligned} c_i(s^t) &= w_i(s^t)\ell_i(s^t) + \lambda d_i(s^t) + \frac{1-\lambda}{I-1}\sum_{j\neq i} e_{i,j}(s^t)d_j(s^t) \\ &= (1-\alpha)v p_{i,i}(s^t)y_i(s^t) + \lambda(\alpha v p_{i,i}(s^t)y_i(s^t) - x_i(s^t)) + \frac{1-\lambda}{I-1}\sum_{j\neq i} e_{i,j}(s^t)(\alpha v p_{j,j}(s^t)y_j(s^t) - x_j(s^t)). \end{aligned}$$

The second line follows from equation (25). Thus, we get

$$\Delta c_{i,j}(s^t) = v\left\{1 - \alpha\left[1 - \left(\frac{1-I\lambda}{I-1}\right)\right]\right\}\Delta y_{i,j}(s^t) + \left(\frac{1-I\lambda}{I-1}\right)\Delta x_{i,j}(s^t). \tag{26}$$

Note that the terms involving $y_k(s^t)$ and $x_k(s^t)$ for $k \neq i, j$ are cancelled out. When $I = 2$ and $v =$, this equation reduces to HP's equation (18).

I proceed from here in two separate cases again. If there are no intermediate inputs, then nominal gross output in country i is given by

$$p_{i,i}(s^t)y_i(s^t) = \sum_{j=1}^{I} e_{i,j}(s^t)p_{j,i}(s^t)g_{j,i}(s^t) = \omega g_i(s^t) + \left(\frac{1-\omega}{I-1}\right)\sum_{j\neq i} e_{i,j}(s^t)g_j(s^t).$$

Then $\Delta y_{i,j}(s^t)$ is

$$\Delta y_{i,j}(s^t) = \left(\frac{I\omega-1}{I-1}\right)(g_i(s^t) - e_{i,j}(s^t)g_j(s^t)) = \left(\frac{I\omega-1}{I-1}\right)(\Delta c_{i,j}(s^t) + \delta x_{i,j}(s^t))$$

Using this in (26), we get an I-country version of equation (22) in HP:

$$\Delta c_{i,j}(s^t) \propto \left(\frac{1-I\lambda}{I-1}\right)\Delta x_{i,j}(s^t) + \left\{1 - \alpha\left[1 - \left(\frac{1-I\lambda}{I-1}\right)\right]\right\}\left(\frac{I\omega-1}{I-1}\right)\Delta x_{i,j}(s^t)) - \Delta x_{i,j}(s^t).$$

With intermediate inputs, we have to use the Leontief inverse again. Nominal gross output in country i

is equal to

$$
\begin{aligned}
p_{i,i}(s^t)y_i(s^t) &= D\left[p_{i,i}g_{i,i} + \sum_{k\neq i} e_{i,k}p_{k,i}g_{k,i}\right] + \sum_{j\neq i} F\left[p_{i,j}g_{i,j} + e_{i,j}p_{j,j}g_{j,j} + \sum_{k\neq j} e_{i,k}p_{k,j}g_{k,j}\right] \\
&= \left[D\omega + F(1-\omega)\right] g_i(s^t) + \left[\frac{1 - D\omega - F(1-\omega)}{I-1}\right] \sum_{j\neq i} e_{i,j}(s^t)g_j(s^t)
\end{aligned}
$$

Thus, the gross output differential is now given by

$$
\Delta y_{i,j}(s^t) = \left[\frac{I(D\omega + F(1-\omega)) - 1}{I-1}\right](\Delta c_{i,j}(s^t) + \Delta x_{i,j}(s^t)).
$$

Using this in (26), we get

$$
\Delta c_{i,j}(s^t) \propto \left(\frac{1-I\lambda}{I-1}\right)\Delta x_{i,j}(s^t) + v\left\{1 - \alpha\left[1 - \left(\frac{I\lambda - 1}{I-1}\right)\right]\right\}\left[\frac{I(D\omega + F(1-\omega)) - 1}{I-1}\right]\Delta x_{i,j}(s^t)) - \Delta x_{i,j}(s^t).
$$

Note that when $v = 1$, $D = 1$ and $F = 0$ so this collapses to the expression for the first case.

Appendix B: Data

This section of the appendix contains additional details on data sources and data processing.

B.1 Measuring international portfolio diversification

I use the same measure of international portfolio diversification as HP: foreign assets and liabilities as a percent of total country assets and liabilities. I calculate portfolio diversification for a country (not region!) i as

$$
DIV_{it} = \frac{FA_{it} + FL_{it}}{2(K_{it} + FA_{it} - FL_{it})}, \tag{27}
$$

where FA_{it}, FL_{it}, and K_{it} are country i's gross foreign assets, gross foreign liabilities, and aggregate capital stock at time t (note here i indexes individual countries, not regions). I use the same source as Heathcote and Perri (2013) for the first two: the commonly-used dataset collected by Lane and Milesi-Ferretti (2007), which now covers the period 1970–2011. FA_{it} and FL_{it} capture long and short positions in any assets that represent a claim to country output: portfolio equity, FDI, debt, derivatives, and reserves. I express both as fractions of nominal GDP. I use a different data source to calculate K_{it}, however. HP look at OECD countries, so they naturally use the OECD Quarterly National Accounts as their source of national accounting data. The WIOD input-output matrices use a different set of countries, many of which are not in the OECD. Further none of the countries in the "rest of the world" are in the OECD. I therefore use the Penn World Tables I calculate

K_{it}, again as a fraction of GDP, as

$$K_{it} = \frac{CK_{i,t}}{CGDPO_{it}} \tag{28}$$

where CK_{it} is country i's capital stock at time t at current PPP's, and $CGDPO_{i,t}$ is output-side real GDP at current PPPs.

To calculate measures of portfolio diversification for each region, I take GDP-weighted averages. I remove outlier countries with extreme observations (portfolio diversification less than -1,000 percent or greater than 1,000 percent in any given year).[1] Using other weighting schemes (or taking the median) yields similar results for changes in regional diversification.

The python scripts `portfolios.py` and `plots_regions.py` in the "programs/python" folder in the online supplement perform these calculations.

B.2 Constructing region-level TFP using the Penn World Tables

I use the Penn World Tables version 8.0. I extract country-level real value added, capital stock, and labor input for each year in the data (1950–2011). I use the variable $RGDPO$ as my data analogue of real value added, v_{it}. For the capital stock, I use the variables CK and $CGDPO$ (capital and output at current PPPs) to calculate the capital-output ratio as above, and multiply this by $RGDPO$. For labor, I multiply EMP (persons engaged) by AVH (average hours per person engaged) and HC (human capital per person). Second, for each year in the data I sum value added, capital, and labor across countries by region. I assign all countries not in the WIOD dataset to the ROW region. This gives me value added, capital, and labor series for each of the four model regions. Third, I calculate region-level TFP (in logs) as

$$\log z_{it} = \log v_{it} - \alpha \log k_{it} - (1-\alpha)\log(n_{it}), \tag{29}$$

where v_{it}, k_{it}, and n_{it} are region-level real value added, capital, and labor respectively. Last, I remove a linear time trend before estimating the joint process

$$\begin{bmatrix} \log z_1(s^t) \\ \log z_2(s^t) \\ \log z_3(s^t) \\ \log z_4(s^t) \end{bmatrix} = P \begin{bmatrix} \log z_1(s^{t-1}) \\ \log z_2(s^{t-1}) \\ \log z_3(s^{t-1}) \\ \log z_4(s^{t-1}) \end{bmatrix} + \begin{bmatrix} \epsilon_1(s^t) \\ \epsilon_2(s^t) \\ \epsilon_3(s^t) \\ \epsilon_4(s^t) \end{bmatrix}. \tag{30}$$

The python script `tfp.py` in the "programs/python" folder in the online supplement performs these calculations.

[1] The outlier countries are: Panama, Syria, Vietnam, Yemen, Angola, Antigua and Barbuda, Bahrain, Belize, Chad, Congo, Ivory Coast, Dominica, Equatorial Guinea, Ethiopia, Grenada, Iceland, Ireland, Laos, Lebanon, Liberia, Luxembourg, Madagascar, Mali, Mozambique, Nigeria, Sao Tome and Principe, Sierra Leone, St. Lucia, St. Vincent, Sudan, Trinidad and Tobago, Zambia, Zimbabwe.

B.3 Input-output data

The input-output data come from the World Input Output Database (Timmer et al., 2015). This database contains a world input-output table for each year from 1995 through 2011. Each input-output table contains gross output, value added, intermediate inputs, consumption, and investment for 40 countries and 35 sectors. Final demand and intermediate use are listed by source and destination; the dataset distinguishes imports and exports by use as well as sector.

In addition to the 40 countries in the dataset, the WIOD data include a composite "rest of the world" which represents the group of developing countries that do not have good national input-output data. The rest of the world's gross output and value added are implied by world market clearing conditions; they are constructed by reconciling the national accounts of the 40 countries included in the database with world output and final demand in the UN National Accounts. The rest of the world's intermediate-input matrix is constructed by averaging the data for Brazil, China, India, Indonesia, Mexico, and Russia. Thus, we can think of the rest of the world as an additional composite emerging economy. To compute portfolio diversification and TFP for the rest of the world, I use all countries in the relevant dataset that are not in one of the first three regions.

Below, I describe the construction of each of the input-output tables used in the analysis. The spreadsheet "4country_alt_scenarios.xlsm" in the "excel" folder in the online supplement performs almost all of the steps.

B.3.1 Benchmark input-output tables

To construct the 1995 and 2011 benchmark tables, I start with the raw WIOD data for each year. First, I aggregate all industries into one sector and aggregate countries according to the regional aggregation in Table 1 in the main text. The raw underlying data are large, so I have not included them in the online supplement; they are available upon request. The python script `prepare_wiod_data.py` performs the aggregation and stores intermediate files in the folder "programs/python/wiod_data." These files are included in the online supplement. The python script `iomats.py` uses the intermediate files to construct unbalanced input-output tables and write them to .csv files which I manuall load into the excel spreadsheet. All steps in this section from here on out are contained therein.

Second, I use the RAS algorithm outlined below to modify the table so that each region's aggregate trade balance is zero. I impose the following restrictions on row and column totals: (i) each region's GDP in the new table equals the its GDP in the raw data; (ii) each region's gross output in the new table equals its gross output in the raw data; (iii) each region's domestic absorption equals its GDP; and (iv) each region's investment rate in the new table is the same as in the raw data. These two tables are constructed in the worksheets "BenchBalanced-1995" and "BenchBalanced-2011" in the spreadsheet. The sheets "iomat-bal-bench-1995" and "iomat-bal-bench-2011," which are linked to the previous two, are used to save the CSV output used in the MATLAB code to calibrate the model.

B.3.2 Counterfactual 1: size only

To construct the first counterfactual input-output table, I use the RAS algorithm to modify the 1995 benchmark table obtained in the previous section so that is satisfies the following requirements: (i) each region's GDP in the new table matches its GDP in the 2011 benchmark table; (ii) each region's gross output/GDP ratio in the new table equals its gross output/GDP ratio in the 1995 benchmark; and (iii) each region's investment rate in the new table equals its investment rate in the 1995 benchmark. The counterfactual table is constructed in the sheet "Alt1-Balanced-Size" and the CSV output is contained in the sheet "iomat-bal-size-counter."

B.3.3 Counterfactual 2: trade openness only

To construct the second counterfactual input-output table, I use the RAS algorithm to modify the 1995 benchmark table obtained in the previous section so that is satisfies the following requirements: (i) the sum of each region's exports and imports as a fraction of world GDP in the new table is the same as in the 2011 benchmark; (ii) each region's net exports as a fraction of world GDP in the new table are the same as in the 1995 benchmark (i.e., zero); (iii) the shares of each region's imports that are used for intermediates, consumption, and investment in the new table are the same as in the 1995 benchmark; and (iv) each region's investment rate in the new table is the same as in the 1995 benchmark. The counterfactual table is constructed in the sheet "Alt2-Balanced-Trd" and the CSV output is contained in the sheet "iomat-bal-trd-counter."

B.3.4 Counterfactual 3: intermediate trade only

To construct the third counterfactual input-output table, I use the RAS algorithm to modify the 1995 benchmark table obtained in the previous section so that is satisfies the following requirements: (i) the sum of each region's intermediate trade (exports and imports) as a fraction of its total trade in the new table is the same as in the 2011 benchmark; (ii) the sum of each region's total trade as a fraction of world GDP in the new table is the same as in the 1995 benchmark; (iii) each region's intermediate and final trade balances as fractions of world GDP in the new table are the same as in the 1995 benchmark; (iv) the shares of each region's final imports that are used for consumption and investment in the new table are the same as in the 1995 benchmark; and (v) each region's consumption share of domestic absorption in the new table is the same as in the 1995 benchmark. The counterfactual table is constructed in the sheet "Alt1-Balanced-IO" and the CSV output is contained in the sheet "iomat-bal-io-counter."

B.3.5 Unbalanced-trade versions

The unbalanced-trade versions of the benchmark input-output tables use the RAS algorithm on the raw data just to ensure that all markets clear exactly; there are slight rounding errors in the raw data. These tables are l isted in the worksheets the worksheets "BenchmarkIOMatrix-1995" and "BenchmarkIOMatrix-2011" in the spreadsheet, and the CSV output is contained in the sheets "iomat-bal-bench-1995" and "iomat-bal-bench-2011."

The unbalanced-trade versions of the first three counterfactuals are constructed in exactly the same way as in the baseline analysis, except that I start with the unbalanced 1995 benchmark. The construction is in the sheets "Alt1-Size," "Alt2-Trd," and "Alt3-IO." The CSV output is contained in the sheets "iomat-size-counter," "iomat-trd-counter," and "iomat-io-counter."

To construct the fourth counterfactual input-output table, I start with the unbalanced 1995 benchmark table and use the RAS algorithm with the following restrictions: (i) each region's net exports as a fraction of world GDP in the new table is the same as in the 1995 benchmark; (ii) each region's total trade as a fraction of world GDP in the new table is the same as in the 1995 benchmark; and the same restrictions (iii) and (iv) imposed in the second counterfactual. The construction is performed in the sheet "Alt4-NX" and the CSV output is in the sheet "iomat-nx-counter."

B.3.6 Using RAS to construct counterfactual input-output matrices

The RAS algorithm (Bacharach, 1965) works as follows. Let $M_0 \in \mathbb{R}^{J\times K}$ denote the initial matrix which the user wishes to adjust (in this case M_0 is the 1995 benchmark matrix). Let $u \in \mathbb{R}^J$ denote the vector of user-supplied row sums which with the matrix should be made consistent. $v \in \mathbb{R}^K$ denotes the desired column sums. The goal of the procedure is to find a new matrix M' that is similar to M_0 and also satisfies $\sum_{k=1}^K M_{jk} = u_j$, $\forall j$ and $\sum_{j=1}^J M_{jk} = v_k$, $\forall k$.

First, define the operators $r : \mathbb{R}^{J\times K} \to \mathbb{R}^J$ and $s : \mathbb{R}^{J\times K} \to \mathbb{R}^K$ by

$$r_j(M) = \frac{u_j}{\sum_{k=1}^K M_{jk}},\ j = 1,\ldots,J \tag{31}$$

$$s_k(M) = \frac{v_k}{\sum_{j=1}^J M_{jk}},\ k = 1,\ldots,K. \tag{32}$$

Next, define the operators $T_1 : \mathbb{R}^{J\times K} \to \mathbb{R}^{J\times K}$ and $T_2 : \mathbb{R}^{J\times K} \to \mathbb{R}^{J\times K}$ by

$$T_1(M) = \mathrm{diag}(r(M))M \tag{33}$$

$$T_2(M) = M\,\mathrm{diag}(s(M)). \tag{34}$$

Last, define $T : \mathbb{R}^{J\times K} \to \mathbb{R}^{J\times K}$ by

$$T(M) = T_2(T_1(M)). \tag{35}$$

The desired matrix M' is obtained by iterating on T starting with M_0, i.e., $M' = \lim_{n\to\infty} M_n$, where $M_n = T(M_{n-1})$ for $n > 0$.

The spreadsheet described above contains an Excel VBA macro that implements the RAS algorithm and applies it to each of the input-output tables.

Appendix C: Solving for equilibrium portfolios numerically

This section of the appendix illustrates how to extend the method of Devereux and Sutherland (2011), henceforth DS, to my many-country, many-asset environment.

Following that study's lead, let $\alpha_{i,j}(s^t) = q_j(s^t)\alpha_{i,j}(s^t)$ denote the value of country i's holding of country j's stock. I will solve for the steady state values of these variables rather than the portfolio weights, $\lambda_{i,j}(s^t)$, directly. We can write households' budget constraints as

$$p_{i,c}(s^t)c_i(s^t) + W_i(s^t) = w_i(s^t)\ell_i(s^t) + \sum_{j=1}^{I} \alpha_{i,j}(s^t)(R_j(s^t) - R_I(s^t)) + W_i(s^{t-1})R_I(s^t), \tag{36}$$

where $W_i(s^t)$ is the total value of country i's portfolio:

$$W_i(s^t) = \sum_{j=1}^{I} \alpha_{i,j}(s^t). \tag{37}$$

In other words, its net foreign assets.

For this purpose, I ignore the portfolio wedges τ_i. The key second-order approximation of the portfolio choice first order conditions, the portfolio choice equation from the main text,

$$\begin{aligned} 0 = \sum_{s_{t+1}\in S} \pi(s^t, s_{t+1}) \Bigg[&\hat{R}_j(s^t, s_{t+1}) - \hat{R}_I(s^t, s_{t+1}) + \frac{1}{2}\left(\hat{R}_j(s^t, s_{t+1})^2 - \hat{R}_I(s^t, s_{t+1})^2\right) \\ &+ \left(-\gamma\hat{c}_i(s^t, s_{t+1}) - \hat{p}_{i,c}(s^t, s_{t+1})\right)\left(\hat{R}_j(s^t, s_{t+1}) - \hat{R}_I(s^t, s_{t+1})\right) + e^{\tau_i \mathbb{1}_{\{i\neq j\}}} - e^{\tau_i \mathbb{1}_{\{i\neq I\}}} \Bigg], \ \forall j < I, \end{aligned} \tag{38}$$

simplifies to

$$\begin{aligned} \sum_{s_{t+1}\in S} \Bigg[&\hat{R}_{j,x}(s^t, s_{t+1}) + \frac{1}{2}\left(\hat{R}_j(s^t, s_{t+1})^2 - \hat{R}_I(s^t, s_{t+1})^2\right) \\ &- (\gamma\hat{c}_i(s^t, s_{t+1}) + \hat{p}_{i,c}(s^t, s_{t+1}))\hat{R}_{j,x}(s^t, s_{t+1}) \Bigg] = 0, \ \forall i, \ \forall j \neq I, \end{aligned} \tag{39}$$

where $\hat{R}_{x,j}(s^t)$ is the excess return on tree j:

$$\hat{R}_{x,j}(s^t) = \hat{R}_j(s^t) - \hat{R}_I(s^t). \tag{40}$$

There are $I-1$ of these equations for each country I — $I(I-1)$ equations in total. This ought to be enough to pin down $\alpha_{i,j}$ for all $i = 1, \ldots, I$ and all $j = 1, \ldots I-1$. We can reduce this a bit, recognizing that we only need to solve for $\alpha_{i,j}$ for the first $I-1$ countries (the last country's portfolio shares will be implied by share market clearing). Combine them to yield

$$\sum_{s_{t+1} \in S} \left[\hat{c}_{i,D}(s^t, s_{t+1}) \hat{R}_{j,x}(s^t, s_{t+1}) \right] = 0, \ \forall i \neq I, \ \forall j \neq I, \tag{41}$$

where

$$\hat{c}_{i,D}(s^t) = -\gamma \left(\hat{c}_i(s^t, s_{t+1} - \hat{c}_I(s^t, s_{t+1}) \right) - \left(\hat{p}_{i,c}(s^t, s_{t+1}) - \hat{p}_{c,I}(s^t, s_{t+1}) \right) \tag{42}$$

are marginal utility differentials as in DS. This is $(I-1)(I-1)$ distinct equations, which ought to be just what we need to pin down $\overline{\alpha}_{i,j}$ for each of the first $I-1$ countries and $I-1$ asssets.

DS point out that evaluating the products of the marginal utility differentials and excess returns in the equations above to second-order accuracy requires only first-order accurate solutions for the two components $\hat{c}_{i,D}(s^t)$ and $\hat{R}_{i,x}(s^t)$. We can therefore linearize the rest of the equilibrium system around the non-stochastic steady-state given some choice of $\overline{\alpha}_{i,j}$, ignoring the portfolio shares and portfolio choice optimality conditions, and solve to get recursive decision rules as in any linearized macro model. We then use these decision rules to check that (42) are satisfied.

One could solve for $\overline{\alpha}_{i,j}$ in an iterative manner, making a guess, checking to see if (42) are satisfied, and updating the guess if not. However, following DS there is a much simpler approach. The expected value after history s^t of $\hat{R}_{j,x}(s^t, s_{t+1})$ is zero, so we can define an auxilliary i.i.d. exogenous variable

$$\xi_i(s^t) = \frac{1}{\beta} \sum_{j=1}^{I-1} \overline{\alpha}_{i,j} \hat{R}_{i,x}(s^t), \tag{43}$$

or in matrix notation

$$\xi_i(s^t) = \frac{1}{\beta} \overline{\alpha}_i' \hat{R}_x(s^t) = \tilde{\alpha}_i' \hat{R}_x(s^t). \tag{44}$$

Note that we now need one for each country since the total excess return of each country's portfolio dpends on its vector of shares $\overline{\alpha}_i$. In DS, there is only one auxilliary variable.

Replace each country's total excess portfolio returns in its budget constraint with $\xi_i(s^t)$:

$$p_{i,c}(s^t) c_i(s^t) + W_i(s^t) = w_i(s^t) \ell_i(s^t) + \xi_i(s^t) + W_i(s^{t-1}) R_I(s^t). \tag{45}$$

Using net foreign assets, $W_i(s^t)$, as state variables in addition to capital stocks, we can linearize the non-portfolio equations and solve for recursive decision rules for non-portfolio variables using standard methods. These rules take the following form in DS notation:

$$\mathbf{s}_{t+1} = F_1\mathbf{x}_t + F_2\mathbf{s}_2 + F_3\boldsymbol{\xi}_t \tag{46}$$

$$\mathbf{c}_t = P_1\mathbf{x}_t + P_2\mathbf{s}_2 + P_3\boldsymbol{\xi}_t \tag{47}$$

$$\mathbf{x}_t = N\mathbf{x}_{t-1} + \Sigma\mathbf{e}_t, \tag{48}$$

where $\mathbf{x}_t$ is the vector of exogenous state variables, $\mathbf{s}_t$ is the vector of endogenous states, $\mathbf{c}_t$ is the vector of controls,

$$\boldsymbol{\xi}_t = [\xi_{1,t}, \ldots, \xi_{I-1,t}]', \tag{49}$$

and the last equation is the Markov process for the exogenous states. From here on out, I omit the history-dependent notation given that all model variables are recursively defined.

By extracting the appropriate coefficients from above, we can write

$$\underbrace{\hat{R}_{x,t+1}}_{(I-1)\times 1} = \underbrace{R_1}_{(I-1)\times(I-1)} \cdot \underbrace{\boldsymbol{\xi}_{t+1}}_{(I-1)\times 1} + \underbrace{R_2}_{(I-1)\times I} \cdot \underbrace{\mathbf{e}_{t+1}}_{I\times 1}, \tag{50}$$

where the matrices R_1 and R_2 are formed from the appropriate rows from the decision rule matrices above. This looks just like in DS, except that R_1 is an $(I-1)\times(I-1)$ matrix rather than a vector with length equal to the number of assets. This expression confirms that excess returns are indeed i.i.d., being functions of other i.i.d. variables only. Now, use the definition of $\xi_i(s^t)$,

$$\xi_{i,t+1} = \tilde{\alpha}_i'\hat{R}_{x,t+1}, \tag{51}$$

and stack to get

$$\boldsymbol{\xi}_{t+1} = \underbrace{\begin{bmatrix} \xi_{1,t+1} \\ \xi_{2,t+1} \\ \vdots \\ \xi_{I-1,t+1} \end{bmatrix}}_{(I-1)\times 1} = \underbrace{\begin{bmatrix} \tilde{\alpha}_1' \\ \tilde{\alpha}_2' \\ \vdots \\ \tilde{\alpha}_{I-1}' \end{bmatrix}}_{(I-1)\times(I-1)} \underbrace{\hat{R}_{x,t+1}}_{(I-1)\times 1} = \tilde{\alpha}'\hat{R}_{x,t+1} \tag{52}$$

Now plug this into (50) to get

$$\boldsymbol{\xi}_{t+1} = (\tilde{\alpha}'R_2)(\mathbb{I}_{(I-1)} - \tilde{\alpha}'R_1)^{-1}\mathbf{e}_{t+1} = \tilde{H}\mathbf{e}_{t+1} \tag{53}$$

$$\hat{R}_{x,t+1} = (R_1\tilde{H} + R_2)\mathbf{e}_{t+1} = \tilde{R}\mathbf{e}_{t+1}. \tag{54}$$

These are basically the same as (A.5) - (A.7) in DS, except that the identity matrix shows up in the inverse term instead of a unit scalar.

We perform a similar trick with the consumption differential, which can be written as

$$\begin{aligned} c_{i,D,t+1} &= D_{i,1}\boldsymbol{\xi}_{t+1} + D_{i,2}\mathbf{e}_{t+1} + D_{i,3}\begin{bmatrix}\mathbf{x}_t \\ \mathbf{s}_{t+1}\end{bmatrix} \\ &= (D_{i,1}\tilde{H} + D_{i,2})\mathbf{e}_{t+1} + D_{i,3}\begin{bmatrix}\mathbf{x}_t \\ \mathbf{s}_{t+1}\end{bmatrix} \\ &= \tilde{D}_i\mathbf{e}_{t+1} + D_{i,3}\begin{bmatrix}\mathbf{x}_t \\ \mathbf{s}_{t+1}\end{bmatrix}. \end{aligned} \tag{55}$$

Now multiply by the excess return:

$$\begin{aligned} 0 &= \mathbb{E}_t\left[\hat{c}_{i,D,t+1}\hat{R}_{x,t+1}\right] \\ &= \mathbb{E}_t\left[\left(\tilde{D}_i\mathbf{e}_{t+1} + D_{i,3}\begin{bmatrix}\mathbf{x}_t \\ \mathbf{s}_{t+1}\end{bmatrix}\right)\tilde{R}\mathbf{e}_{t+1}\right] \\ &= \mathbb{E}_t\left[\tilde{D}_i\mathbf{e}_{t+1}\tilde{R}\mathbf{e}_{t+1} + D_{i,3}\begin{bmatrix}\mathbf{x}_t \\ \mathbf{s}_{t+1}\end{bmatrix}\tilde{R}\mathbf{e}_{t+1}\right] \\ &= \mathbb{E}_t\left[\tilde{R}\mathbf{e}_{t+1}\mathbf{e}'_{t+1}\tilde{D}'_i\right] \\ &= \underbrace{\tilde{R}}_{(I-1)\times I}\underbrace{\Sigma}_{I\times I}\underbrace{\tilde{D}'_i}_{I\times 1}, \end{aligned} \tag{56}$$

where the fourth line follows from the fact that $\mathbf{e}_{t+1}$ is i.i.d., and the fifth from the fact that the expected value of $\mathbf{e}'_{t+1}\mathbf{e}_{t+1}$ is the variance-covariance matrix of the innovations.

The last step is to stack the resulting expressions above and solve for the portfolio shares. To be explicit, we solve for $\alpha_{i,j}$ such that

$$\begin{bmatrix}\tilde{R}\Sigma\tilde{D}'_1 \\ \tilde{R}\Sigma\tilde{D}'_2 \\ \vdots \\ \tilde{R}\Sigma\tilde{D}'_{I-1}\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ \vdots \\ 0\end{bmatrix}. \tag{57}$$

This is a system of $(I-1)(I-1)$ equations in the same number of variables.

The MATLAB script `lucas_trees.m` in the "programs/matlab/lucas_tree_model" folder in the online supplement illustrates how to apply this solution method to a simple Lucas tree model. Upon running the script, the user will find that the program matches the theoretical solution for portfolio diversification exactly for any number of countries.

The MATLAB script `quant_model.m` in the "programs/matlab/quantitative_model" folder in the online supplement performs all of the quantitative exercises in the paper. In each exercise, it calibrates the model to input-output data, linearizes the non-portfolio equations, and solves for equilibrium portfolios. When calibrating to the 1995 benchmark input-output table, it also calibrates the wedge parameters to match diversification data.

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