Brexit and the Macroeconomic Impact of Trade Policy Uncertainty

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Abstract

The United Kingdom has voted to leave the European Union but the trade policies that will replace E.U. membership are uncertain, and speculation abounds that this uncertainty will cause immediate harm to the U.K. economy. To assess the impact of uncertainty about post-Brexit trade policies, I study a dynamic general equilibrium model with endogenous export participation and uncertainty about whether future U.K.-E.U. trade costs will be high or low. I find that the total consumption-equivalent welfare cost of Brexit for U.K. households is between 0.4 and 1.1 percent, while the cost of Brexit uncertainty is about 1/1,000 of a percent.

1 Introduction

The United Kingdom voted to leave the European Union on June 23, 2016, but the law that authorized the vote was silent about the trade policies that will replace E.U. membership. The Brexit vote was followed by widespread speculation that uncertainty about future U.K.-E.U. trade policies would cause immediate harm to the U.K. economy, although recent national income accounting data suggest that this harm has yet to materialize. In this paper, I analyze the effects of Brexit on U.K. macroeconomic dynamics and provide the first estimate of the cost of Brexit uncertainty.

I use a dynamic, stochastic, general equilibrium model of the United Kingdom, the European Union, and the rest of the world to address two quantitative questions about the consequences of Brexit. First, how will departure from the European Union affect the U.K. economy in the short and long run? Second, how will uncertainty about the trade policies that will replace E.U. membership in the future affect the U.K. economy in the present?

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The model features three countries, an input-output production structure, heterogeneous firms, and, most importantly, uncertainty about trade costs. There are two channels in the model through which this uncertainty affects international trade dynamics. First, as in Alessandria and Choi (2007, 2016) and Alessandria et al. (2015, 2016), firms' export participation decisions are forward-looking because the fixed cost of starting to export is larger than the fixed cost to continue. Second, following Handley and Limão (2013), firms make these decisions before uncertainty about trade costs is resolved. Uncertainty can also affect households' decisions about saving and investment. To compute the model's equilibrium I use a novel global method that provides an exact solution, which allows for accurate analysis of welfare and the effects of uncertainty.

In my quantitative analysis, I calibrate the model's parameters so that its steady state matches an input-output matrix from 2011, when the possibility of Brexit had not yet entered the global consciousness. To assess the overall impact of Brexit, I compare this no-Brexit steady state to an equilibrium in which trade costs follow a stochastic process that captures uncertainty about the outcome of the Brexit referendum and about post-Brexit changes in trade policy. To assess the impact of this uncertainty, I compare the stochastic equilibrium to a set of deterministic equilibria in which model agents have perfect foresight about these events. I find that overall welfare losses from Brexit will be substantial. Consumption-equivalent welfare losses are between 0.4 and 1.1 percent depending on how much post-Brexit trade costs rise; the present value of these losses amounts to £7,000–18,000 per person. The welfare cost of uncertainty about Brexit is small, however: U.K. households would pay no more than £50 per person to avoid this uncertainty.

In order to assess the impact of Brexit I must specify the set of possible trade policies that could replace E.U. membership in my model. Following Dhingra et al. (2016b,c), I take a parsimonious approach with two possible scenarios. If soft Brexit occurs, the United Kingdom retains tariff-free trade with the European single market through either continued membership in the European Economic Area or bilateral negotiation.[1] If, on the other hand, hard Brexit occurs, the United Kingdom loses single-market access and trades with the European Union according to World Trade Organization rules. In addition to formal tariffs, I incorporate non-tariff barriers to trade which I model as iceberg transportation costs. The literature on trade costs has found that non-tariff barriers are often larger than tariffs (Anderson and van Wincoop, 2004; Allen, 2014; Lim, 2016), particularly in the services sector where tariffs are essentially nonexistent. One of the major concerns about Brexit, in fact, centers around whether or not financial services firms will retain passporting rights that enable them to operate in the European Union. I use the estimates of Francois et al. (2013) for non-tariff barriers in E.U. trade with the United States as an upper bound for post-Brexit non-tariff barriers in E.U. trade with the United Kingdom.

In order to analyze the effects of uncertainty about Brexit, I must also model the timing of the Brexit process and the likelihood of each scenario. The first real indication that Brexit might be a possibility arose

[1] Recently, Prime Minister Theresa May has indicated that continued European Economic Area membership is off the table but that membership in a customs union is not. See `https://www.nytimes.com/2017/01/17/world/europe/brexit-theresa-may-uk-eu.html`.

in January of 2013, when Prime Minister David Cameron promised that he would hold a referendum on European Union membership if his Conservative party was reelected in May of 2015. The Conservatives won reelection and the European Union Referendum Act 2015, which authorized a popular vote on E.U. membership, was introduced to the House of Commons shortly thereafter. The bill passed the House of Commons the next month and was approved by the House of Lords in December of 2015. The referendum date was formally announced in February of 2016 and the vote itself took place in June of 2016. Since then, the British government has converged on a March, 2019 target for completion of negotiations with the European Union about post-Brexit policies. I capture this timeline in my model as follows. The economy begins in the no-Brexit steady state in which agents believe trade costs with the European Union will remain at their 2011 levels forever. In 2015 there is an unanticipated shock that initiates a stochastic process for trade costs with the European Union. This process, depicted in figure 1, involves two uncertain events. The first is the Brexit referendum which occurs in 2016, one year after the unanticipated shock. The referendum fails with probability Π_{vote} and passes with probability $1 - \Pi_{vote}$. If the referendum fails, trade costs stay at 2011 levels forever. If the referendum passes, Brexit will occur in 2019 but model agents do not learn which Brexit scenario they will face until this time. The probability of soft Brexit is Π_{brexit} and the probability of hard Brexit is $1 - \Pi_{brexit}$. After the unanticipated shock in 2015, agents have rational expectations about this process. I set Π_{vote}, the probability that the referendum fails, to 75 percent based on prediction market price data. I set Π_{brexit}, the probability of soft Brexit conditional on the referendum's success, to 50 percent. None of my results are sensitive to these transition probabilities, however.

In the long run, Brexit will have a large impact on the U.K. macroeconomy. Depending on which scenario occurs, trade with the remainder of the European Union will fall by 11–49 percent, real GDP will fall by 0.5–1.4 percent, and consumption will fall by 0.4–1.2 percent. In the short run, most macroeconomic variables remain close to their no-Brexit steady state values until Brexit occurs in 2019; the announcement of the referendum and the outcome of the vote have little impact on U.K. macroeconomic dynamics. This prediction is consistent with the recent national income accounting data listed in table 1. Once Brexit occurs, though, export participation, trade flows, and macroeconomic variables begin to decline towards their long-run levels. I propose two methods of measuring welfare losses that take these transition dynamics into account. The backward-looking method, which conditions on the outcome of Brexit, asks U.K. households in each scenario what fraction of their annual consumption they would give up to have remained in the no-Brexit steady state instead. Backward-looking welfare losses are 0.4 percent and 1.1 percent for soft and hard Brexit, respectively; the present values of these figures are equivalent to about £7,000 and £18,000 per person. The forward-looking method asks U.K. households how much they would pay to avoid Brexit before learning which outcome they face. Prior to learning the outcome of the referendum, households would give up 0.2 percent of annual consumpion to remain in the no-Brexit steady state, and once the referendum has succeeded they would give up 0.8 percent.

Uncertainty about Brexit in the short run will have little impact on both U.K. macroeconomic dynamics and welfare. I demonstrate this by comparing the stochastic equilibrium described in figure 1 with two perfect-foresight equilibria, one for each possible Brexit scenario, in which households learn immediately after the referendum which scenario will occur. In both the long and short run, macroeconomic dynamics and trade flows in the stochastic equilibrium are virtually identical to their perfect-foresight counterparts. The consumption-equivalent welfare differences between the baseline model and perfect-foresight models are about 1/1,000 of a percent; the present values of these welfare losses are less than £50 per person.

I have conducted a wide variety of sensitivity analyses and have found that all of my results are robust. I have studied the role of exporter dynamics in alternative models with less costly export partcipation, experimented with different assumptions about financial markets, analyzed a multi-sector environment with a more detailed input-output structure, and varied assigned parameters like transition probabilities and elasticites. None of these sensitivity analyses affected my results significantly. In particular, the welfare cost of uncertainty about Brexit is small in all versions of the model.

This paper contributes to several strands of the international trade and macroeconomics literatures. First, it contributes to the literature on the economic consequences of Brexit. A number of recent studies use static models and reduced-form estimations to analyze the impact of Brexit, from increased trade costs and other factors, on U.K. welfare and trade with the European Union (Dhingra et al., 2016b,c; Ebell et al., 2016; Baker et al., 2016). My paper is the first to use a dynamic general equilibrium model to assess the impact of Brexit on the U.K. economy in both the short and long run, and the first to quantify the cost of short-run uncertainty about Brexit. My study is limited, though, to the economic impact of increased post-Brexit trade costs. The United Kingdom stands to benefit from reduced fiscal transfers to the European Union after Brexit, and changes in immigration policy may also affect U.K. households' welfare. Further, leaving the European Union could lower U.K. productivity due to lower foreign direct investment (Dhingra et al., 2016a; Pain and Young, 2004).

More broadly, a number of recent studies analyze the welfare impact of trade reforms in models with capital accumulation and other dynamic adjustment margins (Baldwin, 1992; Bajona and Kehoe, 2010; Dix-Carneiro, 2014; Alessandria et al., 2015; Brooks and Pujolas, 2016). My model features both capital and intertemporal trade. My results indicate, though, that these features play minor roles in determining U.K. welfare losses from Brexit. My paper also contributes to the related literature on trade dynamics with heterogeneous firms and endogenous export participation (Alessandria and Choi, 2007; Ruhl, 2008; Alessandria et al., 2013b, 2015, 2016; Alessandria and Choi, 2016; Ramanarayanan, 2016). My model builds on these studies by extending the exporter dynamics framework of Alessandria and Choi (2007, 2016) and Alessandria et al. (2016) to a multi-country environment.

Finally, my paper contributes to the emerging literature on trade policy uncertainty. In contrast to my finding that the effects of uncertainty about Brexit are small, several studies in this literature have found large effects in other contexts. Pierce and Schott (2016) and Handley and Limão (2013), for example, argue that before China joined the World Trade Organization in 2001, uncertainty about U.S. trade policy towards Chinese goods significantly affected U.S. imports from China and lowered U.S. households' welfare. I use my model to conduct a series of hypothetical trade reform exercises to analyze whether differences between the circumstances of Brexit and those of pre-W.T.O. China could account for this discrepancy. My analysis indicates that uncertainty about U.S. trade policy towards Chinese goods actually had little macroeconomic impact, highlighting the need for further quantitative research in this area.

2 Model

I now develop a dynamic, stochastic, general equilibrium model with three countries: the United Kingdom, the European Union, and the rest of the world. Each country is populated by a representative household and a unit measure of heterogeneous firms. Households work, consume, invest, and save. Firms produce differentiated goods and endogenously enter and exit the export market in response to idiosyncratic productivity shocks and changes —or anticipation of possible future changes — in bilateral trade costs.

2.1 Aggregate uncertainty and trade costs

In each period t the model economy experiences an aggregate shock, Z_t, which is drawn from a finite set $\mathcal{Z}_t$. The vector $Z^t = (Z_0, Z_1, \dots, Z_t)$ denotes a history of aggregate shocks. $\Pi(Z^t)$ is the probability of a given history Z^t. There are two kinds of trade costs, both of which depend on the realization of the aggregate shock: import tariffs, which are rebated lump-sum to households; and non-tariff iceberg trade costs. $\tau_{i,j}(Z^t)$ is the import tariff on goods produced in country j and sold in country i, and $\xi_{i,j}(Z^t)$ is the iceberg cost of shipping those goods. The process for Z_t is assumed to be non-stationary: the set of possible shocks and the associated probabilities depend on the period as well as the previous shock. This is necessary to capture the nature of the uncertainty about Brexit.

2.2 Households

The representative household in each country $i \in I = \{uk, eu, rw\}$ chooses consumption, $C_i(Z^t)$, investment, $X_i(Z^t)$, and bonds, $B_i(Z^t)$ to maximize lifetime utility,

$$\sum_{t=0}^{\infty} \sum_{Z^t} \beta^t \Pi(Z^t) \frac{C_i(Z_t)^{1-\gamma}}{1-\gamma}, \tag{1}$$

subject to a sequence of budget constraints,

$$P_i(Z^t)(C_i(Z^t) + X_i(Z^t)) + Q(Z^t)B_i(Z^t) = W_i(Z^t)\bar{L}_i + R_i(Z^t)K_i(Z^{t-1}) + B_i(Z^{t-1}) + T_i(Z^t) + D_i(Z^t), \quad (2)$$

a law of motion for capital,

$$K_i(Z^t) = \frac{1}{\varphi}\left[\delta^{1-\varphi}\left(\frac{X_i(Z^t)}{K_i(Z^{t-1})}\right)^{\varphi} - (1-\varphi)\delta\right]K_i(Z^{t-1}) + (1-\delta)K_i(Z^{t-1}), \quad (3)$$

and initial conditions for capital and bonds, $B_i(Z_0)$ and $K_i(Z_0)$. Labor is supplied inelastically. $T_i(Z^t)$ is the lump-sum transfer of tariff revenue from the government and $D_i(Z^t)$ is the aggregate dividend payment from firms in the household's home country. International financial markets are exogenously incomplete.[2]. Bonds are denominated in units of the British consumer price index which is normalized to one without loss of generality. The parameter φ governs the cost of adjusting the capital stock. When $\varphi < 1$, large investments are less effective in augmenting the capital stock as in Eaton et al. (2011) and Lucas and Prescott (1971).

2.3 Aggregation technologies

In each country there is a large number of distributors who combine domestic and imported varieties to produce a nontradable aggregate good that is used for consumption, investment, and intermediate inputs. The aggregation technology has a nested CES structure. The top level takes the standard Armington form,

$$Y_i(Z^t) = \left[\sum_{j \in I} (\mu_{i,j})^{\frac{1}{\zeta}}\, Y_{i,j}(Z^t)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}, \quad (4)$$

where $Y_i(Z^t)$ is the aggregate good and $Y_{i,j}(Z^t)$ is a bundle of goods purchased from source country j. ζ is the elasticity of substitution between goods from different countries, commonly referred to as the Armington elasticity. At the bottom level, the source-specific bundles $Y_{i,j}(Z^t)$ are produced by aggregating over sets of differentiated varieties:

$$Y_{i,j}(Z^t) = \left[\int_{\nu \in N_{i,j}(Z^t)} y_{i,j}(Z^t, \nu)^{\frac{\theta-1}{\theta}}\, \mathrm{d}\nu\right]^{\frac{\theta}{\theta-1}}. \quad (5)$$

$N_{i,j}(Z^t)$, the set of varieties produced in j and sold in i, is endogenous. $y_{i,j}(Z^t, \nu)$ is the quantity of variety ν purchased from country j. The parameter θ governs the elasticity of substitution between varieties from the same source country.

Aggregators are competitive and choose inputs of each available variety to maximize profits taking

[2] All results reported in this paper, including the welfare losses associated with uncertainty about Brexit, are robust to alternative assumptions about international financial markets. In section 5 I study a version of the model with financial autarky.

prices as given:

$$\max_{y_{i,j}(Z^t,\nu)} \left\{ P_i(Z^t)Y_i(Z^t) - \sum_{j\in I} \int_{\nu\in N_{i,j}(Z^t)} (1+\tau_{i,j}(Z^t))p_{i,j}(Z^t,\nu)y_{i,j}(Z^t,\nu)\,\mathrm{d}\nu \right\} \tag{6}$$

subject to equations (4) and (5). The price of the aggregate good is then

$$P_i(Z^t) = \left[\sum_{j\in I} \mu_{i,j} P_{i,j}(Z^t)^{1-\zeta} \right]^{\frac{1}{1-\zeta}}, \tag{7}$$

where the prices of each source-specific bundle are given by

$$P_{i,j}(Z^t) = \left[\int_{\nu\in N_{i,j}(Z^t)} \left((1+\tau_{i,j}(Z^t))p_{i,j}(Z^t,\nu)\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \mathrm{d}\nu. \tag{8}$$

Demand for source country j's variety ν can be written as a function of its price as:

$$y_{i,j}(Z^t,p) = \left[(1+\tau_{i,j}(Z^t))^{-\theta} P_{i,j}(Z^t)^{\theta} Y_{i,j}(Z^t) \right] p^{-\theta}. \tag{9}$$

2.4 Firms

Each country has a continuum of monopolistically competitive firms that produce differentiated varieties. Each firm is identified with a particular variety ν. A firm's gross output is a Leontief[3] combination of value added and intermediate inputs:

$$y_i(Z^t,\nu) = e^{a_i(Z^t,\nu)} \min\left\{ \frac{k_i(Z^t,\nu)^{\alpha}\ell_i(Z^t,\nu)^{1-\alpha}}{\eta_i}, \frac{m_i(Z^t,\nu)}{1-\eta_i} \right\}. \tag{10}$$

$m_i(Z^t,\nu)$ denotes the firm's intermediate inputs, and its value added is a Cobb-Douglas combination of its capital, $k_i(Z^t,\nu)$, and labor, $\ell_i(Z^t,\nu)$. Firms are heterogeneous in gross output productivity, $a_i(Z^t,\nu)$, which is i.i.d. over time according to a distribution $F_i(a)$.[4] The parameter η_i governs the share of value added in gross output. I assume that firms can serve at most one export destination $d \in I \setminus \{i\}$ and that each firm's destination is assigned exogenously: a fraction $\omega_{i,d}$ of the firms in country i can serve each destination d.[5] A

[3]The literature indicates that value added and intermediates are almost perfectly complementary (Kehoe et al., 2013; Atalay, 2014). This assumption does not affect significantly affect welfare results and delivers more reasonable investment dynamics in the leadup to and immediate aftermath of Brexit.

[4]The assumption that firms' productivities are i.i.d. over time reduces the dimensionality of the equilibrium by collapsing the joint productivity-export status distribution to a single aggregate: the export participation rate. Alessandria and Choi (2007) show that a more general AR1 process for firm productivities yields similar aggregate trade dynamics in an international business cycle model.

[5]In the data, most firms serve one export destination but the largest firms serve many destinations (Mayer and Ottaviano, 2008; Bernard et al., 2012). Allowing firms to choose to serve multiple destinations would complicate the analysis tremendously. Moreover, without some degree of exogenous heterogeneity in firms' ability to serve different export markets, one of the two destinations would be served only by very productive "mult-destination" exporters. The dynamics of destinations over exporters' life cycles is an interesting topic but it is beyond the scope of this paper.

firm's assigned destination, $d(\nu)$, is immutable. The firm's resource constraint is

$$y_i(Z^t,\nu) = y_{i,i}(Z^t,\nu) + (1+\xi_{d(\nu),i}(Z^t))s_i(Z^t,\nu)y_{d(\nu),i}(Z^t,\nu), \tag{11}$$

where $s_i(Z^t,\nu) \in \{0,1\}$ indicates the firm's status as an exporter. Conditional on export status, firms engage in monopolistic competition, choosing prices in each period to maximize intratemporal profits:

$$\pi_i(Z^t,\nu) = \max_{p_i,p_{d(\nu)},k,\ell,m} \left\{ p_i y_{i,i}(Z^t,p_i) + p_{d(\nu)} y_{d(\nu),i}(Z^t,p_{d(\nu)}) - W_i(Z^t)\ell - R_i(Z^t)k - P_i(Z^t)m \right\} \tag{12}$$

subject to (9), (10), and (11). The solution is characterized by the usual constant-markup pricing rules:

$$p_{i,i}(Z^t,\nu) = \left(\frac{\theta}{\theta-1}\right) C_i(Z^t)e^{-a_i(Z^t,\nu)}, \tag{13}$$

$$p_{d(\nu),i}(Z^t,\nu) = \left(\frac{\theta}{\theta-1}\right)(1+\xi_{d(\nu),i}(Z^t))C_i(Z^t)e^{-a_i(Z^t,\nu)}, \tag{14}$$

where

$$C_i(Z^t) = \eta_i \left[\left(\frac{R_i(Z^t)}{\alpha}\right)^{\alpha} \left(\frac{W_i(Z^t)}{1-\alpha}\right)^{1-\alpha} \right] + (1-\eta_i)P_i(Z^t). \tag{15}$$

$k_i(Z^t,\nu)$, $\ell_i(Z^t,\nu)$, and $m_i(Z^t,\nu)$ denote the firm's demand for factors and intermediate inputs.

Following Alessandria and Choi (2007, 2016) and Alessandria et al. (2016), in order to export to its assigned destination a firm must pay fixed costs, denominated in units of domestic labor, which depend on the firm's export status at the end of the previous period. The fixed cost for new exporters is $\kappa_{i,0}$, and the fixed cost for continuing exporters is $\kappa_{i,1}$. In the calibrated model, the startup cost is larger than the continuation cost which implies that the decision to begin exporting is forward-looking. This creates one channel through which uncertainty about future trade costs can affect export participation. Additionally, as in Handley (2014) and Handley and Limão (2013, 2015), I assume that firms must pay fixed exporting costs before learning the realization of the current aggregate shock. This creates a second channel through which uncertainty about trade costs can affect export participation. The timing in the firm's problem is as follows. First, the firm observes the current realization of its idiosyncratic productivity. Second, the firm chooses whether or not to begin or continue exporting. Third, it learns the current realization of the aggregate shock. Fourth, it chooses its price in each market it has chosen to serve, produces, and distributes profits to the domestic household.

All firms with the same assigned destination and state variables — productivity, a, and export status, s — will make the same choices in equilibrium, so we can index firms by destination and state instead of by their varieties. The firm's value function is

$$V_i(Z^{t-1}, a, s; d) = \max_{s' \in \{0,1\}} \left\{ \sum_{Z^t} \Pi(Z^t | Z^{t-1}) \left[\pi_i(Z^t, a, s'; d) - s' W_i(Z^t) \kappa_{i,s} + \tilde{V}_i(Z^t, s'; d) \right] \right\}, \tag{16}$$

where

$$\tilde{V}_i(Z^t, s; d) = Q(Z^t) \int_{a'} V_i(Z^t, a', s; d) \, \mathrm{d}a' \tag{17}$$

is the firm's discounted expected value at the beginning of the next period. The solution to the firm's dynamic problem is characterized by cutoff productivities:

$$\sum_{Z^t} \Pi(Z^t | Z^{t-1}) W_i(Z^t) \kappa_{i,0} = \sum_{Z^t} \Pi(Z^t | Z^{t-1}) \left[\Delta \pi_i(Z^t, a_i^+(Z^{t-1}; d), d) + \Delta \tilde{V}_i(Z^t; d) \right], \tag{18}$$

$$\sum_{Z^t} \Pi(Z^t | Z^{t-1}) W_i(Z^t) \kappa_{i,1} = \sum_{Z^t} \Pi(Z^t | Z^{t-1}) \left[\Delta \pi_i(Z^t, a_i^-(Z^{t-1}; d), d) + \Delta \tilde{V}_i(Z^t; d) \right], \tag{19}$$

where $\Delta \pi_i(Z^t, a; d)$ and $\Delta \tilde{V}_i(Z^t; d)$ are the differences between exporters' and non-exporters' current profits and continuation values, respectively. $a_i^+(Z^{t-1}; d)$ is the cutoff for new exporters and $a_i^-(Z^{t-1}; d)$ is the cutoff for continuing exporters. Note that, like the firm's value function, they depend on the previous period's aggregate state, not the current state, because of the timing of the firm's problem.

2.5 Export participation dynamics and market clearing

The export participation rate among firms that can export to destination d, $n_i(Z^t; d)$, evolves as follows:

$$n_i(Z^t; d) = n_i(Z^{t-1}; d) \left[1 - F_i(a_i^-(Z^{t-1}; d)) \right] + (1 - n_i(Z^{t-1}; d)) \left[1 - F_i(a_i^+(Z^{t-1}; d)) \right]. \tag{20}$$

The set of firms that export to destination d, $N_{d,i}(Z^t)$, has mass $\omega_{i,d} n_i(Z^t; d)$. The aggregate demand for capital by firms assigned to destination d is

$$\begin{aligned} K_i^D(Z^t; d) = \omega_{i,d} \Bigg\{ & n_i(Z^{t-1}; d) \left[\int_{-\infty}^{a_i^-(Z^{t-1}; d)} k_i(Z^t, a, 0; d) \, \mathrm{d}F(a) + \int_{a_i^-(Z^{t-1}; d)}^{\infty} k_i(Z^t, a, 1; d) \, \mathrm{d}F(a) \right] \\ & + (1 - n_i(Z^{t-1}; d)) \left[\int_{-\infty}^{a_i^+(Z^{t-1}; d)} k_i(Z^t, a, 0; d) \, \mathrm{d}F(a) + \int_{a_i^+(Z^{t-1}; d)}^{\infty} k_i(Z^t, a, 1; d) \, \mathrm{d}F(a) \right] \Bigg\}. \end{aligned} \tag{21}$$

Note that I've indexed the firm's policy function for capital by destination and state rather than by variety. Aggregate demand for productive labor, $L_i^D(Z^t; d)$, and intermediates, $M_i^D(Z^t; d)$, are calculated in a similar manner. Aggregate labor used for fixed exporting costs is given by

$$L_i^F(Z^t; d) = n_i(Z^{t-1}; d) \left[1 - F_i(a_i^-(Z^{t-1}; d)) \right] \kappa_{i,1} + (1 - n_i(Z^{t-1}; d)) \left[1 - F_i(a_i^+(Z^{t-1}; d)) \right] \kappa_{i,0} \tag{22}$$

There are four market clearing conditions that must be satisfied in equilibrium. First, each country's aggregate output $Y_{i,t}(Z^t)$ must be used for consumption, investment, or intermediate inputs:

$$Y_i(Z^t) = C_i(Z^t) + X_i(Z^t) + \sum_{d \in I \setminus \{i\}} M_i^D(Z^t; d). \tag{23}$$

Second and third, factor markets must also clear:

$$K_i(Z^{t-1}) = \sum_{d \in I \setminus \{i\}} K_i^D(Z^t; d) \tag{24}$$

$$\bar{L}_i = \sum_{d \in I \setminus \{i\}} \left[L_i^D(Z^t; d) + L_i^F(Z^t; d) \right]. \tag{25}$$

Finally, the bond market must clear:

$$\sum_{i \in I} B_i(Z^t) = 0 \tag{26}$$

2.6 Equilibrium and computation

An equilibrium is, for each country and all possible histories, a set of

- aggregate quantities, C_i, X_i, B_i, K_i, T_i, D_i, Y_i, $Y_{i,j}$,
- aggregate prices, W_i, R_i, P_i, $P_{i,j}$,
- firm allocations, y_i, k_i, ℓ_i, m_i, $y_{i,i}$, $y_{d,i}$, prices, $p_{i,i}$, $p_{d,i}$, and value functions,V_i,
- cutoff productivities, a_i^+ and a_i^-,
- and export participation rates, n_i,

that solve the household, distributor, and firm problems and satisfy market clearing conditions. If the aggregate shock Z_t is constant in the long run the model converges to a steady state in which the objects above are constant.

Most dynamic, stochastic, general equilibrium models in macroeconomics and international trade, including those that feature heterogeneous firms like Alessandria and Choi (2007, 2016) and Alessandria et al. (2016), use local methods to approximate the equilibrium near an invariant steady state. In my quantitative exercise, however, there are two steady states to which the equilibrium may converge: one associated with hard Brexit and another with soft Brexit.[6] Moreover, local approximation methods are ill-suited to the analysis of welfare and the effects of uncertainty, both of which take center stage in my study. Instead, I use a global method to solve for the exact equilibrium. The method is similar to that used in Kehoe et al. (2013),

[6]Strictly speaking, because I allow for unbalanced trade in the long run, steady states depend on net foreign assets, which are endogenous, as well as the trade policy regime (Kehoe et al., 2013). In truth, there is one set of possible steady states for soft Brexit, and another set of possible steady states for hard Brexit.

Alessandria et al. (2015), and others to solve for transition paths in deterministic models. The presence of uncertainty complicates matters but does not pose an insurmountable barrier as long as the number of possible histories is small, as is the case in my quantitative analysis which I describe in the next section. In brief, if one assumes that the equilibrium converges to a steady state after a finite number of periods, the equilibrium conditions for all possible histories, along with the corresponding equilibrium variables, can be represented by a single nonlinear system that can be solved using standard numerical methods. The appendix contains more details about my solution method.

3 Quantitative analysis

My quantitative analysis proceeds in three steps. The first is to construct a benchmark from which to measure the impact of Brexit. I construct this benchmark during my calibration procedure, in which I set the model's parameters so that its steady state matches macroeconomic and international trade data from 2011. This no-Brexit steady state is a counterfactual that represents the state of the world before Brexit entered the realm of possibility.

Second, I use external data on the costs of E.U. trade with non-U.K. trade partners to construct two possible post-Brexit trade policy regimes: soft Brexit, in which the United Kingdom retains access to the European single market by remaining in the European Economic Area or through bilateral negotiation; and hard Brexit, in which the United Kingdom loses single market access. Each scenario involves two exogenous effects: (i) changes in import tariffs; and (ii) changes in non-tariff iceberg trade costs. I also specify model agents' perceived probabilities that the Brexit referendum passes, and, conditional on that outcome, that Brexit will be hard or soft. The costs of trade with the rest of the world do not change in either scenario.[7]

Third, I solve for the equilibrium that arises following an unanticipated shock in 2015: Parliament authorizes a referendum on European Union membership in the following year. When the referendum is announced, model agents learn the probability that the referendum will pass and the details and likelihood (conditional on a "leave" vote) of each Brexit scenario. They must wait until 2016 to learn the outcome of the referendum and until 2019 to learn which Brexit scenario they will face. If Brexit occurs in 2019, the economy remains in either hard or soft Brexit forever. Figure 1 illustrates the timing in the Brexit equilibrium.

[7] E.U. regulations prevent the United Kingdom from negotiating free trade agreements with trade partners in the rest of the world while the United Kingdom remains an E.U. member, but once Brexit occurs the United Kingdom will be free to enter into such negotiations. Free-trade agreements with the rest of the world would increase trade with the rest of the world and offset some of the welfare losses caused by the reduction in trade with the European Union.

3.1 Calibrating the no-Brexit steady state

To calibrate the model, I first assign common parameters like the discount factor and elasticities of substitution to standard values. Given these assigned values, I calibrate the remaining parameters to that the model's steady state matches an input-output matrix from 2011 and several facts about exporter dynamics fron the literature. The calibrated parameter values are listed in table 2.

3.1.1 Input-output data

I use an input-output matrix from the World Input Output Database Timmer et al. (2015), henceforth abbreviated as WIOD, to specify production and trade relationships in the no-Brexit steady state. This dataset has been used widely in recent international trade studies including other analyses of Brexit like Dhingra et al. (2016b,c). I use the data from 2011, the last year available in the dataset and several years before Brexit was considered possible. I aggregate all industries into a single sector and aggregate countries according to the three-country scheme in the model. Panel (a) of table 3 shows the aggregated WIOD data. The first three columns list intermediate inputs, value added, and gross output for each country, while columns 4 through 6 list final demand. All data in the matrix have been normalized so that U.K. GDP is equal to 100.

Trade is unbalanced in the aggregated data, however; the United Kingdom and the rest of the world have trade deficits and the European Union has a trade surplus. In a steady state, in which current accounts are zero, trade imbalances represent interest payments on net foreign assets. A country that has a trade deficit has positive net foreign assets and vice versa. Consequently, treating the raw data as a steady state implies counterfactual net foreign asset positions, so I use the RAS procedure (Bacharach, 1965) to construct a similar input-output matrix in which each country's aggregate trade is balanced. This balanced matrix, which represents the no-Brexit steady state in my quantitative analysis, is shown in panel (b) of table 3. All differences between the balanced matrix and the raw data are minor.

3.1.2 Assigned parameters

I set the discount factor, β, so that the steady-state real interest rate is 2 percent per year. γ, which governs risk aversion and the elasticity of intertemporal substitution, is set to 2. The depreciation rate, δ, and the capital share, α, are set to 6 percent and one-third, respectively. I follow Alessandria and Choi (2016) and Alessandria et al. (2016) and set θ, the elasticity of substitution between varieties, to 5. φ, the parameter which governs capital adjustment costs, is set to 0.76 as in Steinberg (2016).

3.1.3 Calibrated parameters

The parameters that govern aggregate production and trade relationships are set using the balanced input-output matrix from section 3.1.1. I set the value added shares, η_i, and the Armington shares, $\mu_{i,j}$, so that the data in the matrix satisfy distributors' first-order conditions.[8] I set all bilateral trade costs to zero so that the Armington shares absorb trade costs as well as other sources of home bias. This is without loss of generality given the assumption that tariff revenues are rebated lump-sum to households. Each country's time endowment, $\bar{L}_i$, is set to a fraction $1-\alpha$ of its value added. For each country i and destination d, I set $\omega_{i,d}$, the fraction of firms that can export to that destination, equal to country d's share of country i's total exports.

The remaining parameters are set to match international trade facts from the literature. I set ζ, the Armington elasticity, so that the long-run trade elasticity is 5 (Costinot and Rodríguez-Clare, 2014). The calibrated value of 3.25 is lower than the target trade elasticity because the export participation rate changes in response to changes in trade costs. To set the dispersions of firms' productivities and the fixed exporting costs I follow Alessandria and Choi (2016) and Alessandria et al. (2016). σ_i, the standard deviations of firms' productivity distributions, are set so that the average exporter is 2.5 times larger than the average non-exporter in each country. The export entry costs, $\kappa_{i,0}$, are set to match an export participation rate of 25 percent. The continuation costs, $\kappa_{i,1}$, are set to match an export exit rate of 2 percent. As in the aforementioned studies, I find that the cost to begin exporting is significantly larger than the cost of continuing to export. Consequently, export participation decisions are forward-looking; continuation values, $\Delta\tilde{V}_i$, matter for export participation decisions as well as current profits, $\Delta\pi_i$.

3.2 Brexit scenarios

Having calibrated the model and constructed the no-Brexit steady state, I now describe the details of the two Brexit scenarios and the transition process for the aggregate shock. Table 4 provides a summary of this information.

3.2.1 Tariffs

There are no changes in import tariffs in the soft Brexit scenario because the United Kingdom retains single market access. In the hard Brexit scenario, tariffs are based on three sources of data: the European Union's most-favored-nation (MFN) tariff schedule for 6-digit HS goods industries published by the World Trade Organization (WTO); COMTRADE data on U.K. trade flows for these same industries; and the disaggregated

[8] As in Kehoe et al. (2013), I choose units so that all steady-state prices are one. This is without loss of generality. See the appendix for more details.

WIOD data from section 3.1.1. First, I use the WTO and COMTRADE data to calculate average MFN tariffs on U.K.-E.U. goods trade. The U.K. tariff on E.U. goods is computed as the average MFN tariff weighted by imports, while the E.U. tariff on U.K. goods is weighted by exports. Second, I multiply these goods-trade tariffs by the goods shares[9] of total U.K. imports from and exports to the European Union in the disaggregated WIOD data. This step adjusts tariffs downwards to reflect the fact that the United Kingdom and European Union trade services, on which tariffs are rarely levied, as well as goods.

3.2.2 Non-tariff barriers

To calculate changes in non-tariff barriers I use the same approach as Dhingra et al. (2016b,c), which is based on Francois et al. (2013)'s estimates of non-tariff barriers in trade between the United States and the European Union for a set of industries that approximately correspond to the 2-digit ISIC industries in the disaggregated WIOD data.[10] This study also reports the fraction of these barriers that could be reduced by policy action. I treat policy-reducible non-tariff barriers in E.U.-U.S.A. trade as worst-case upper bounds for post-Brexit non-tariff barriers in U.K.-E.U. trade. First, I compute average policy-reducible barriers using the WIOD data on U.K.-E.U. trade flows as weights as in section 3.2.1. Second, as in Dhingra et al. (2016b,c), I assume that that non-tariff barriers in the model increase by 25 percent and 75 percent of these averages following soft and hard Brexit, respectively.

3.2.3 Transition probabilities

The aggregate state in the equilibrium with Brexit follows a non-stationary Markov process. Let Z_{stay} denote the aggregate state associated with European Union membership, and let Z_{soft} and Z_{hard} denote the aggregate states associated with soft and hard and Brexit, respectively. We need a fourth pre-Brexit state, Z_{pb}, to which the economy enters after a "leave" vote in the referendum. Trade costs do not rise in the pre-Brexit state, but expectations about future trade costs change. The set of possible aggregate states in each period is given by

$$\mathcal{Z}_t = \begin{cases} \{Z_{stay}\} & t < 2016 \\ \{Z_{stay}, Z_{pb}\} & 2016 \leq t \leq 2018 \\ \{Z_{stay}, Z_{soft}, Z_{hard}\} & t \geq 2019 \end{cases} \tag{27}$$

Abusing notation slightly, let $\Pi_t(Z)$ denote the unconditional probability of aggregate state Z in period t, and let $\Pi_t(Z'|Z)$ denote the probability of transitioning from state Z in period $t-1$ to state Z' in period t. Both of these probability functions are time-varying.

[9] I define the goods sector as agriculture, resource extraction, and manufacturing.

[10] Several WIOD industries do not have counterparts in Francois et al. (2013). Many of these industries, such as the sale and maintenance of motor vehicles, are nontraded. See the appendix for more details.

Z_{stay} is the only possible state until 2016, the year of the referendum, so $\Pi_t(Z_{stay}) = 1$ for $t < 2016$. Let Π_{vote} denote the probability of a "stay" vote in the referendum. The unconditional probabilities for the aggregate state in 2016 are $\Pi_{2016}(Z_{stay}) = \Pi_{vote}$ and $\Pi_{2016}(Z_{pb}) = 1 - \Pi_{vote}$. If "stay" wins, the economy remains in this state forever: $\Pi_t(Z_{stay}|Z_{stay}) = 1$ for $t > 2016$. If "leave" wins, the economy remains in the pre-Brexit state, Z_{pb}, until Brexit occurs: $\Pi_t(Z_{pb}|Z_{pb}) = 1$ for $2016 < t < 2019$. In 2019, if the economy is in the pre-Brexit state, it switches to either the hard or soft Brexit scenario. Let Π_{brexit} denote the probability of soft Brexit. Then we have $\Pi_{2019}(Z_{soft}|Z_{pb}) = \Pi_{brexit}$ and $\Pi_{2019}(Z_{hard}|Z_{pb}) = 1 - \Pi_{brexit}$. Once this transition has occurred, the economy remains in hard or soft Brexit forever: $\Pi_t(Z_{soft}|Z_{soft}) = \Pi_t(Z_{hard}|P_{hard}) = 1$ for $t > 2019$. Figure 1 provides an illustration of this transition process.

There are two probabilities that we must assign: Π_{vote}, the probability that "stay" prevails in the 2016 referendum, and Π_{brexit}, the probability of soft Brexit conditional on a "leave" vote. Although "leave" won referendum, this outcome was viewed as unlikely by many until the votes began to be tallied. Prediction markets, in fact, reported a 75-percent probability that "stay" would win during the week before the referendum,[11] so I set Π_{vote} to 75 percent. Assigning the probability of soft Brexit, Π_{brexit}, is more problematic. There are no prediction markets that allow bettors to wager on the outcome of Brexit, and there is not yet sufficient post-referendum macroeconomic data to which one could calibrate this parameter using the model. Lacking a solid prior, I assume that hard and soft Brexit are equally likely. This choice maximizes the entropy of the Brexit outcome. However, as I show in section 5.3, none of my results are sensitive to this choice. Panel (c) of table 4 lists the assigned transition probabilities.

4 Impact of Brexit on the United Kingdom economy

I turn now to the analysis of the calibrated model's predictions about the consequences of Brexit. First, I discuss Brexit's effects on macroeconomic dynamics and trade. Second, I calculate the overall welfare cost of Brexit for U.K. households. Last, I discuss the macroeconomic impact of uncertainty about Brexit and calculate the welfare cost of this uncertainty.

4.1 Macroeconomic variables

Figure 3 depicts the impact of Brexit on U.K. macroeconomic variables. The solid blue lines (labeled "Pre-Brexit") depict the trajectories of these variables during 2015–2019, after the referendum is announced — and succeeds — but before Brexit actually takes place. In 2019, the equilibrium path forks. The dashed green and red lines (labeled "Soft" and "Hard") depict the trajectories of macroeconomic variables from 2019 onwards

[11] See, for example, http://www.slate.com/articles/news_and_politics/moneybox/2016/07/why_political_betting_markets_are_failing.html.

after soft and hard Brexit, respectively. The figures also show the long-rum effects of Brexit in each scenario using color-coded bars. I do not plot the counterfactual trajectory in which the referendum fails.

In the long run, real GDP, consumption, and investment fall permanently in both Brexit scenarios. The long-run drops in consumption — 0.44 percent and 1.19 percent for soft and hard Brexit, respectively — provide us with back-of-the-envelope measures of U.K. welfare losses from Brexit in each scenario. As we will soon see, these numbers are indeed close to the true welfare losses once transition dynamics into account. In the short run, the effects of Brexit on most macroeconomic variables are muted until Brexit actually occurs in 2019. GDP falls slightly during the pre-Brexit period, but does not drop substantially until Brexit takes place. Investment actually rises slightly in the pre-Brexit period in anticipation of higher future costs. The most pronounced pre-Brexit effects are seen in consumption, which begins to fall when the referendum is announced in 2015 and falls more dramatically in 2016, when the referendum succeeds, even though that trade costs do not rise for three more years. This is consistent with permanent income logic: when United Kingdom households learn that their expected long-run income has fallen they save to smooth their consumption over time. This behavior causes United Kingdom to run a trade surplus. Once Brexit occurs in 2019, U.K. households increase their consumption if soft Brexit occurs because their permanent income rises, and if hard Brexit occurs their permanent income, and thus consumption, fall further. The trade balance reverts towards zero as households have little further incentive to save. In the long run, trade surpluses turn to deficits as households use their accumulated savings to augment consumption.

The national accounts data that cover the period since the referendum act was introduced to Parliament are shown in table 1. These data indicate that, consistent with the model's results, anticipation of Brexit has had little impact on the U.K. macroeconomy. Real GDP, consumption, and trade flows have grown at approximately the same rates since the act's introduction as they did during the previous three years, and neither the investment rate nor the trade balance have changed significantly. The dynamics of U.K. macro variables during the period since since the Brexit vote, 2016Q3–2017Q1, are similar to their dynamics during the period between the act's introduction and the vote, 2015Q3–2016Q2, indicating that neither the announcement nor the outcome of the referendum appear to have had significant macroeconomic impact. The largest discrepancy between model and data is that the model predicts that consumption should have fallen when the referendum was announced and fallen again when "leave" won, while in the data consumption growth has been relatively strong. A version of the model with financial autarky, which prevents permanent-income-driven consumption smoothing, fits the pre-Brexit consumption data better and delivers similar results along all other dimensions.

4.2 Trade and exchange rates

Figure 4 illustrates the impact of Brexit on U.K. trade with the European Union. In the long run, imports from the European Union fall by 13 percent for soft Brexit and 49 percent for hard Brexit. These two numbers are directly implied by the calibration, which sets the Armington elasticity so that the long-run trade elasticity is five. Exports to the European Union fall less than imports in both scenarios, so the bilateral trade balance with the European Union improves. Permanent-income logic, as described above, helps explain this result, but there is a second mechanism at play. Trade costs on shipments from the United Kingdom to the European Union rise less than trade costs on shipments in the other direction, so imports fall more than exports. This is the same mechanism to which Barattieri (2014) attributes the decline in the U.S. trade balance in the 1990s. During this period, goods trade liberalized more quickly than services trade so the United States, which tends to export goods and import services, ran trade deficits. In both scenarios, U.K.-E.U. trade does not change significantly during the pre-Brexit period, but falls quickly once Brexit occurs despite the fact that export participation takes several years to adjust; the increase in trade costs triggers immediate adjustment on the intensive margin.

As figure 5 shows, Brexit will also cause trade with the rest of the world to rise. Imports from the rest of the world rise after Brexit because these goods are substitutes for those produced in the European Union, and exports to the rest of the world rise due to general equilibrium effects. Trade with the rest of the world rises more gradually than trade with the European Union falls because export participation rates take longer to adjust. This is because increasing export participation requires firms to pay large fixed entry costs, while reducing export participation simply requires firms to stop paying small continuation costs. If trade with the rest of the world did not adjust, it is likely that the welfare losses of U.K. households from Brexit would be larger. If the United Kingdom successfully negotiates free trade deals with countries in the rest of the world after exiting the European Union, trade with the rest of the world may increase more than the model predicts and welfare losses may be lower.

Data on bilateral trade during the pre-Brexit period are not yet available, but we can compare the model's predictions for aggregate trade with the national accounts. The model predicts that neither exports nor imports will respond significantly during the pre-Brexit period. As table 1 shows, real exports and imports grew at similar rates during this period compared to the previous three years. In 2016Q3, exports fell by almost 10 percent, while imports grew by 6 percent. This is not consistent with the model's predictions for aggregate trade in the period following the referendum, but we cannot draw much inference from one quarter's worth of data, particular for trade variables which are substantially more volatile than other macroeconomic aggregates (Alessandria et al., 2013a).

U.K. real exchange rates with both the European Union and the rest of the world depreciate during the pre-Brexit period in the model. This follows from Marshall-Lerner logic: permanent income motives drive up the U.K. trade balance so its real exchange depreciates in equilibrium to compensate. This prediction is qualitatively, but not quantitatively, consistent with the data. Figure 2 shows that U.K. real exchange rates with the European Union and other trade partners (I have used the United States as an example) have depreciated by more than 20 percent since the referendum act was introducted to Parliament, far more than the model predicts. It is widely known that international macroeconomic models have trouble generating as much real exchange rate volatility as we see in the data; the seminal study by Obstfeld and Rogoff (2001) identifies this as one of the major puzzles in the field. A version of the model with multiple sectors and import adjustment frictions fares better in generating pre-Brexit depreciation, and sticky wages and exogenous productivity losses further improve the model's performance on this dimension. Delayed nominal exchange rate passthrough may also account for the depreciation of the United Kingdom's real exchange rate in recent quarters; there is growing concern that the pound's recent weakness will soon cause inflation to rise which could reverse some of the recent real depreciation.

4.3 Welfare

I propose two ways to measure U.K. households' welfare losses from Brexit. The forward-looking method compares lifetime utility in the no-Brexit steady state to expected lifetime utility in the stochastic Brexit equilibrium before uncertainty about the referendum and the outcome of Brexit has been resolved. Formally. the forward-looking welfare loss following a history Z^t, $\mathcal{W}_i^f(Z^t)$, is measured in consumption-equivalent terms as

$$\frac{U\left((1-\mathcal{W}_i^f(Z^t))C_i^*\right)}{1-\beta} = \sum_{r=0}^{\infty} \sum_{Z^{t+r}} \Pi(Z^{t+r}|Z^t)\beta^r U(C_i(Z^{t+r})), \tag{28}$$

where C_i^* is consumption in the no-Brexit steady state. I present two versions of forward-looking welfare calculations: one for 2015, in which neither the outcome of the referendum nor Brexit have been revealed; and another for 2016, in which "leave" has won but the outcome of Brexit is still uncertain.

The backward-looking method compares welfare in the no-Brexit steady state to welfare from 2015 onwards in the history that leads to a particular long-run aggregate state $Z_{lr} \in \{Z_{soft}, Z_{hard}\}$.[12] The backward-looking welfare loss for soft-Brexit, $\mathcal{W}_i^{b,soft}$, for example, is given by

$$\frac{U\left((1-\mathcal{W}_i^{b,soft})C_i^*\right)}{1-\beta} = \sum_{t=0}^{\infty} \beta^t U(C_i(Z_0, Z_1, \dots, Z_{soft})), \tag{29}$$

where $(Z_0, Z_1, \dots, Z_{soft})$ is the unique history leading to soft Brexit. I present two versions of backward-looking welfare calculations as well: one for soft Brexit, and another for hard Brexit. These measures approx-

[12]Period $t = 0$ corresponds to the year 2015 in the model. I do not report differences in welfare between the no-Brexit steady state and the equilibrium in which the referendum occurs but does not pass.

imate lower and upper bounds on U.K. welfare losses. Panel (a) of table 5 lists the results.[13]

The backward-looking welfare losses for both Brexit scenarios are close to the long-run decreases in consumption: 0.42 percent for soft Brexit, and 1.09 percent for hard Brexit. These losses are large compared to estimates in the literature of the welfare effects of past trade reforms. Caliendo and Parro (2015), for example, find that U.S. welfare gains from NAFTA were only 0.08 percent, while di Giovanni et al. (2014) find that average country's welfare gain from trade with China is 0.42 percent. On the other hand, Dhingra et al. (2016b,c) predict even larger welfare losses from Brexit than I do, due in part to their assumption that the United Kingdom will miss out on future reductions in intra-E.U. trade costs. The present value of backward-looking losses range from £7,000 to £18,000 per person, or, equivalently, 18–46 percent of 2015 U.K. GDP.[14] The forward-looking welfare losses are approximately equal to weighted averages of the backward-looking losses. The forward-looking welfare loss from 2016 onward, conditional on the "leave" vote, is about equal to the Π_{brexit}-weighted average of the backward-looking welfare losses. The forward-looking measure for 2015 is about $1 - \Pi_{vote}$ times this figure. This indicates that losses driven by risk aversion are insignificant compared to overall welfare losses. Uncertainty may affect welfare losses through other channels, however. I discuss this further in the next section.

4.4 Uncertainty about Brexit

To assess the impact of uncertainty about Brexit, I compare the stochastic Brexit equilibrium depicted in figure 1 with two perfect-foresight equilibria. In the first one, model agents learn immediately in 2015 that soft Brexit will occur in 2019, and in the second they learn that hard Brexit will occur instead. The trajectories of trade costs in the perfect-foresight equilibria mirror the realized trade cost trajectories in the soft- and hard-Brexit histories in the stochastic equilibrium, so all differences in outcomes between the stochastic and perfect-foresight versions of the two Brexit scenarios are due solely to uncertainty. The trajectories of the variables of interest in the perfect-foresight equilibria are depicted as dotted lines in teal and orange (labeled "Soft (perf. foresight)" and "Hard (perf. foresight)") in figures 3–5.

The dynamics of macroeconomic variables, trade flows, and real exchange rates in the perfect-foresight equilibria are all similar to t4he stochastic equilibrium's dynamics. As figures 4–5 show, the largest differences are apparent in export participation rates. During the pre-Brexit period, export participation rates in the stochastic equilibrium are about halfway between their counterparts in the perfect-foresight versions of soft and hard Brexit, and after Brexit occurs in 2019 it takes several years for the stochastic and perfect-foresight export participation trajectories to converge. The trade policy uncertainty literature suggests that

[13]The table reports consumption-equivalent welfare losses in basis points because the welfare losses associated with uncertainty about Brexit are tiny.

[14]To compute the present value of consumption-equivalent welfare losses, I first compute the cost, in units of the 2015 U.K. CPI in the model, of purchasing no-Brexit steady-state consumption forever. I then multiply this figure by the ratio of 2015 consumption in the data (£2.17 trillion) and 2015 consumption in the model. From here, the conversion to per-capita or percent-GDP costs is straightforward.

these differences are due to the real option value of waiting until uncertainty about Brexit is resolved before paying sunk costs associated with changes in export participation status (Handley and Limão, 2013, 2015; Handley, 2014). In section 5 I study the role of this mechanism in more detail. The macroeconomic results, however, suggest that this mechanism does not impose large welfare costs.

To measure the welfare costs of uncertainty about Brexit, I use both backward- and forward-looking methods as in section 4.3. For each Brexit scenario, the backward-looking method compares welfare in the perfect-foresight equilibrium with welfare along the history leading to that scenario in the stochastic equilibrium. This method asks U.K. households, once they learn which Brexit scenario they face in 2019, how much they would have paid to learn that outcome immediately in 2015 instead. The forward-looking method compares expected lifetime utility in the stochastic equilibrium to the lifetime utility of a deterministic consumption sequence given by the probability-weighted average of the perfect-foresight consumption sequences. This method is analogous to a risk compensation measure. As before, I compute forward-looking measures for 2015 and 2016. The former measures the cost of uncertainty about the outcomes of both the referendum and the outcome of Brexit, and the latter measures the cost of uncertainty about Brexit conditional on a "leave" vote.[15] Panel (b) of table 5 details these calculations. All measures of the welfare cost of uncertainty about Brexit are about 1/1,000 of one percent in consumption-equivalent terms, or equivalently, less than one percent of the overall welfare cost of Brexit. The present values of these measures are at most a few dozen pounds. Interestingly, my estimates of the cost of Brexit uncertainty have the same order of magnitude as estimates of the welfare cost of business cycles reported by Lucas (2003) and Imrohoroglu (2008).

5 Sensitivity analyses

I have conducted a wide range of sensitivity analyses using alternative calibrations and model structures. Table 5 lists the overall welfare cost of Brexit and the welfare cost of Brexit uncertainty in each of these alternatives. As the table shows, my findings about the welfare costs of Brexit are highly robust.

5.1 The role of exporter dynamics

The recent literature on trade policy uncertainty emphasizes the role of fixed export costs in creating a real option value of waiting to make export participation decisions until uncertainty is resolved. Handley and Limão (2013) and Pierce and Schott (2016), for example, highlight the role of trade policy uncertainty in delaying Chinese firms' decisions about exporting to the United States until China's 2001 accession to

[15] To compute the forward-looking measure for 2016 I construct two more perfect-foresight equilibria that begin in that year, one for soft Brexit and another for hard Brexit. The initial conditions for these equilibria are taken from the aggregate state variables in the stochastic equilibrium in 2016.

the W.T.O. Here, uncertainty about Brexit creates a real option value of delaying decisions about export participation in trade with both the European Union and the rest of the world. U.K. firms that can export to the rest of the world face similar circumstances to those faced by Chinese firms before W.T.O. accession. Although the cost of trade with the rest of the world remains unchanged regardless of the outcome of Brexit, demand for U.K. products in the rest of the world grows. Hard Brexit entails a larger increase in market size than soft Brexit, so U.K. firms that can export to the rest of the world have an incentive to wait until 2019, when they learn whether their markets will grow slightly or substantially, to pay fixed export entry costs. Firms in the rest of the world that can export to the United Kingdom have a similar incentive to delay their export participation decisions. U.K. firms that can export to the European Union also have an incentive to delay changes in their export status until uncertainty about post-Brexit trade policy is resolved; some incumbent exporters who would remain profitable after soft Brexit but not after hard Brexit continue to pay continuation costs to remain in the market in case soft Brexit occurs.

The results of my analysis indicate, however, that these forces are not important determinants of the welfare costs of Brexit for U.K. households. To support this conclusion, I compare the baseline model studied in the previous sections to two alternative models in which trade policies have less impact on export participation decisions. In the static export participation model, I set the costs of starting to export, $\kappa_{i,0}$, equal to the continuation costs, $\kappa_{i,1}$, so that future profits do not affect export participation decisions.[16] In the no extensive margin model, I set both costs to zero so that all firms export. In both alternatives, I recalibrate the Armington elasticity, ζ, so that the long-run trade elasticity is the same as in the baseline model. In the static export participation model I also recalibrate the costs of exporting so that export participation is the same as in the baseline model.

The static export participation model features sharper dynamics of trade flows and export participation than the baseline model. Here, although firms know that profits from exporting to the European Union will fall in the long run, export participation does not fall until Brexit occurs in 2019, at which point export profits fall and export participation rates fall almost immediately to their long run levels.[17] Consequently, trade flows with the European Union converge to their post-Brexit values more quickly, but they do not fall until after Brexit takes place. This postpones some of the pain that occurs during the pre-Brexit period in the baseline model. Export participation and trade with the rest of the world also move more sharply. Export participation rises swiftly after Brexit occurs, rather than gradually after the referendum as in the baseline model, and trade flows move in tandem. This delays the increase in trade with the rest of the world that offsets some of the United Kingdom's welfare losses during the pre-Brexit period in the baseline model. Export participation and trade with the rest of the world also increase less after Brexit in the static export participation model than in the baseline. Despite the differences in export participation and trade dynamics, consumption dynamics are almost exactly the same as in the baseline model. The overall welfare cost of

[16] Uncertainty about current-period profits still matters due to the timing of the firm's export participation decision.

[17] Because uncertainty affects firms' export decisions in 2019, it takes one more year for export participation rates to adjust completely.

Brexit is about five percent lower than in the baseline model, and the welfare cost of uncertainty about Brexit changes by no more than a few pounds.

In the no extensive margin model, trade with both partners does not change until after Brexit takes place. As in the static export participation model, trade with the rest of the world changes less than in the baseline. This time, though, exports to the rest of the world actually fall slightly in the long run instead of increasing. The differences in consumption dynamics and welfare losses are again minor. The overall welfare cost of Brexit is about five percent lower than in the baseline model, and while all measures of the cost of Brexit uncertainty are higher, the differences are small.

5.2 Multiple sectors

In the baseline model there is a single output sector. The literature indicates, however, that the welfare consequences of changes in trade policy may be sensitive to the level of aggregation; multi-sector models often predict larger welfare effects than their single-sector equivalents (Costinot and Rodríguez-Clare, 2014). In the case of Brexit, in particular, modeling trade in services and intermediate inputs could potentially be important because services and intermediates account for significant fractions of U.K.-E.U. trade.

My model's dynamic export participation framework is not tractable in a multi-sector setting, so in order to determine whether the overall cost of Brexit or the cost of Brexit uncertainty are sensitive to the level of aggregation I study a multi-sector version of the no extensive margin model from section 5.1. The multi-sector model features two output sectors — goods and services — and different aggregation technologies for consumption, investment, and intermediate inputs. Buillding on the work of Kehoe et al. (2013) and Eaton et al. (2011), the model features a rich input-output structure which distinguishes trade in intermediate inputs from trade in final purchases. I calibrate this structure to a two-sector version of the input-output matrix in table 3 constructed from the same WIOD source. I also study a version of the multi-sector model with convex trade adjustment frictions as in Krugman (1986) and Engel and Wang (2011) which exhibits gradual trade adjustment dynamics like the baseline model. The online appendix contains additional details about the multi-sector model, its calibration, and its results.

The overall welfare losses from Brexit are higher in both versions of the multi-sector model as compared to the one-sector baseline. In the frictionless version, the backward-looking losses are 2 percent and 18 percent higher for soft and hard Brexit, respectively, and the forward-looking losses are about 13 percent higher; these figures are similar for the version with import adjustment frictions. The welfare cost of uncertainty about Brexit is about the same in the frictionless multi-sector model as in the baseline, but in the version with adjustment frictions the backward-looking uncertainty cost measures are about ten times higher. Even these figures, though, amount to less than 1.5 percent of the overall welfare losses from Brexit.

5.3 Probability of hard vs. soft Brexit

Soft and hard Brexit are equally likely in the baseline calibration. This is an ad-hoc choice, so it is important to verify that it has little impact on the results. I consider two alternative calibrations: one in which hard Brexit is more likely ($\Pi_{brexit} = 0.25$), and another in which soft Brexit is more likely ($\Pi_{brexit} = 0.75$). The overall backward-looking welfare losses from Brexit are virtually identical to the losses in the baseline model. Forward-looking welfare losses are, as in the baseline model, approximately equal to probability-weighted averages of the backward-looking losses. The welfare losses from Brexit uncertainty are higher than in the baseline model when hard Brexit is more likely and lower when soft Brexit is more likely, but their magnitude is the same as in the baseline. As macroeconomic data for 2016 and beyond become available, it may be feasible to calibrate Π_{brexit} to match U.K. consumption dynamics. This parameter maps directly to the post-referendum dynamics of U.K. consumption because it governs the degree to which expected permanent income, and thus consumption, falls when the outcome of the referendum is announced. Regardless, it does not play an important role in determining the overall welfare cost of Brexit or the cost of Brexit uncertainty.

5.4 Financial autarky

Access to international financial markets allows U.K. households to smooth consumption in the baseline model. Most of the international trade literature, by contrast, including Handley and Limão (2013)'s study of U.S.-China trade policy uncertainty, assumes financial autarky. I have studied an alternative version of my model with financial autarky to determine whether the ability to run trade imbalances is a significant factor in determining the welfare cost of Brexit. The results indicate that it is not: both the overall welfare cost of Brexit and the welfare cost of Brexit uncertainty in the financial autarky model are similar to the baseline results.

5.5 Elasticities

The trade elasticity is a key parameter in calculations of the welfare effects of changes in trade policy because it governs the degree to which households can substitute foreign goods for domestic ones (Arkolakis et al., 2012). The baseline calibration targets a long-run trade elasticity of five, which is common in the international trade literature. The open-economy macro literature, which targets the volatilty of net exports and/or the real exchange rate, typically finds a lower elasticity. For example, Heathcote and Perri (2002) estimate an elasticity of 0.9. I have analyzed an alternative calibration in which I target a long-run trade elasticity of one. The overall welfare cost of Brexit is higher in this calibration, but not markedly so, as the decline in substitutability between domestic and foreign goods is offset by a smaller drop in trade flows. The trade elasticity has little impact on the welfare cost of Brexit uncertainty.

Finally, the baseline calibration sets γ, the coefficient of relative risk aversion, to the standard value of two. This parameter also governs the elasticity of intertemporal substitution. I have analyzed another alternative calibration in which I set γ to five to verify that increasing risk aversion does not substantially affect the results, particularly the cost of uncertainty about Brexit. The overall welfare cost of Brexit is similar in this alternative calibration to the baseline result. The welfare cost of Brexit uncertainty is higher in the alternative calibration, but is still less than one percent of the overall welfare cost.

6 Broader implications for trade policy uncertainty

In contrast to my finding that uncertainty about Brexit will have little macroeconomic impact, several studies have argued that trade policy uncertainty can have significant consequences. China's accession to the World Trade Organization in 2001, which eliminated uncertainty about future tariff increases on U.S. imports of Chinese goods, is a classic example. Pierce and Schott (2016) argue that China's W.T.O. accession significantly increased U.S. imports of Chinese goods and reduced U.S. manufacturing employment, and Handley and Limão (2013) estimate that pre-accession uncertainty about U.S.-China trade policy created large welfare losses for U.S. households. In this section I use my model to conduct a series of hypothetical trade reform exercises to investigate how the differences between Brexit and pre-W.T.O. China shape the macroeconomic consequences of trade policy uncertainty.

I offer four potential reasons that trade policy uncertainty could have had a larger impact in the case of China. First, Chinese exporters faced a larger potential increase in tariffs than exporters in my model. If China had lost its temporary MFN status, the average U.S. import tariff on Chinese goods would have risen by 27 percent (Handley and Limão, 2013). By contrast, in my model trade costs rise by less than 10 percent even in the hard Brexit scenario. Second, uncertainty about U.S.-China trade policy lasted for many years — China was granted MFN status in 1980 but this status was not made permanent until its accession to the WTO in 2001 — whereas uncertainty about Brexit lasts for only four years. Third, China's MFN status had to be renewed annually by the U.S. Congress while Brexit will occur only once. Last, Brexit involves uncertainty about how much trade costs with the European Union will rise, whereas Chinese exporters during the 1990s faced uncertainty about whether trade cost reductions were temporary or permanent. Uncertainty about the size of an increase in trade costs could be less costly than uncertainty about the permanance of a trade cost reduction because of the asymmetric costs of beginning and continuing to export. When trade costs fall, export participation rises, but entering the export market is expensive so potential new exporters have a strong incentive to delay entry decisions until learning about whether the reduction is permanent or temporary. Conversely, export participation falls when trade costs increase, but marginal exporters have little incentive to exit before the increase occurs, regardless of whether the size of the increase is uncertain or not, because continuing to export is cheap.

The first trade reform exercise asks whether uncertainty about reductions in trade costs is more costly than uncertainty about rising trade costs. The exercise begins in the hard-Brexit steady state from section 4. In 2015 a potential trade reform is announced: with probability Π_{reform}, U.K.-E.U. trade costs may fall to zero in the following year. If the reform occurs, the reform will be cancelled in 2019 with probability Π_{cancel} and trade costs will permanently revert to hard-Brexit levels. The second exercise highlights the role of the length of the uncertainty period. It is identical to the first exercise except that the period of uncertainty is twice as long: the reform may be cancelled in 2022 instead of 2019. The third exercise asks whether uncertain policy changes that may occur annually are more costly than uncertain policy changes that may occur only once. This exercise is identical to the first, except that the reform may be cancelled with probability Π_{cancel} each year during 2017–2019 instead of in 2019 only. The last exercise combines the second and third: the reform may be cancelled each year during 2017–2022.[18]

To facilitate comparison with the results of the Brexit analysis I use the calibration from table 2. In the first and second exercises I set $\Pi_{reform} = 1 - \Pi_{vote}$ and $\Pi_{cancel} = 1 - \Pi_{brexit}$. In the third and fourth, I set Π_{cancel} so that the unconditional probability of the history in which the reform is never cancelled is the same as in the first and second exercises. This ensures that U.K. households' permanent income does not depend on the number of periods in which the reform can be cancelled. To determine whether the size of the potential increase in tariffs affects the cost of trade policy uncertainty, I repeat these exercises under an alternate specification of hard Brexit trade costs in which bilateral trade costs in U.K.-E.U. trade rise by 27 percent.[19]

Panel (a) of table 6 lists the total welfare gains for U.K. households in the trade reform exercises relative to the hard Brexit steady state. The first two columns report the results for the baseline trade cost specification. I compute backward-looking welfare gains for the history in which the trade reform is permanent only. In all four exercises, these gains are close to the backward-looking welfare losses for hard Brexit reported in table 5. I report forward-looking welfare gains for 2015 only. All measures of the welfare gains from the trade reform are close to their counterparts from the Brexit analysis. The second two columns report the results for the specification with higher trade costs. As expected, the overall welfare gains from the trade reform are higher in this version of the analysis because trade costs fall more.

Panel (b) lists the welfare losses from uncertainty in the trade reform exercises. The backward-looking measure is the difference in welfare between the stochastic and perfect-foresight versions of the permanent-reform trajectory; this is the same measure of the welfare loss from trade policy uncertainty computed

[18]When the reform may be cancelled annually instead of only once, the model's uncertainty tree becomes substantially more complex. Pushing the end of the uncertainty window beyond 2022 makes the model computaitonally intractable. The results indicate, though, that the welfare cost of uncertainty is not significantly affected by the length of the uncertainty window, regardless of whether annual cancellation is possible.

[19]I use the same increase in non-tariff barriers as in the specification with baseline trade costs, and change the increase in tariffs so that the total cost of trade rises by 27 percent.

by Handley and Limão (2013). The forward-looking measure is, as in the Brexit analysis, the difference betweeen stochastic-equilibrium welfare and the lifetime utility associated with the probability-weighted average of perfect-foresight consumption sequences. The welfare losses from trade policy uncertainty are similar in all versions of the trade reform exercise. The welfare losses in the specification with baseline trade costs are slightly larger than in the Brexit analysis but they are still less than 1 percent of the overall change in welfare. The welfare losses in the specification with higher trade costs are slightly larger in levels but slightly smaller when measured relative to the overall chnage in welfare. These results are corroborated by the visual comparison of stochastic and perfect-foresight equilibrium dynamics shown in figures 6–8, which plot macro and trade variables for the history in which the trade reform is never cancelled in the specification with baseline trade costs. These figures are approximately mirror images (i.e. reflected around the x-axis) of their counterparts from the Brexit analysis.

The results from the trade reform exercises indicate that none of the four potential explanations hold water — welfare losses from trade policy uncertainty are not significantly affected by asymmetries in export participation reactions to falling versus rising trade costs, the length of the uncertainty period, whether trade policy changes can occur every year or just once, or the size of the potential increase in trade costs.

The results of these exercises indicate that trade policy uncertainty has little impact on macroeconomic dynamics and welfare in general, not just in the case of Brexit. What, then, might account for the findings of Handley and Limão (2013) and Pierce and Schott (2016) for the Chinese case? One possible explanation for the sharp increase in U.S. imports of Chinese goods following China's W.T.O. accession is that while U.S. tariffs on Chinese goods did not change, exchange rate depreciation made Chinese goods cheaper for U.S. importers; China's real exchange rate with the United States depreciated by almost 20 percent during the late 1990s and early 2000s and there is widespread perception that Chinese exchange rate manipulation is responsible for the widening U.S.-China trade imbalance during this period. Neither Handley and Limão (2013) or Pierce and Schott (2016) discuss this possibility, although the latter's empirical specification includes year fixed effects which should control for aggregate real exchange rate dynamics. Regardless, disentangling the roles of trade policy uncertainty and exchange rate dynamics is a promising direction for future research.

Another possible explanation is that trade policy uncertainty affects the intensive margin of international trade as well as the extensive margin. Like Handley and Limão (2013) and others in this literature, I have focused on the effects of trade policy uncertainty on the extensive margin of trade. Marginal exporters account for only a small fraction of overall economic activity—only half of a percent of firms switch export status each period in my calibration—so it is perhaps not surprising that this channel has little aggregate impact. Channels that involve the intensive margin may have more potential. A preliminary study by Heise et al. (2017) analyzes the effects of trade policy uncertainty on the structure of supply chains. They find that elimination of U.S.-China trade policy uncertainty fostered Japanese-style, just-in-time intermediate input procurement practices, which increased the intensive margin of trade and raised U.S. households' welfare substantially.

7 Conclusion

In this paper I have used a model of the United Kingdom and its trade partners to assess the impact of the United Kingdom's impending departure from the European Union — and the impact of uncertainty about what form this departure will take — on trade flows, welfare, and macroeconomic dynamics. The model features two possible outcomes for Brexit: soft, in which the costs of trade with the European Union rise slightly; and hard, in which those costs rise substantially. Forward-looking model agents learn the details of these scenarios in 2015, when the Brexit referendum is announced, but do not know which scenario will occur until Brexit takes place in 2019.

The model predicts that Brexit will have a substantial impact on the U.K. economy, particularly in the long run. Compared to a counterfactual steady state in which Brexit never occurs, trade flows with the European Union will fall by 13–49 percent, consumption will fall by 0.4–1.2 percent, and the present value of U.K. households' welfare losses will amount to £7,000–£18,000 per person. The model also predicts, though, that uncertainty about Brexit will have little macroeconomic impact: perfect-foresight equilibria in which model agents learn immediately which Brexit scenario they will face are virtually identical to the baseline stochastic equilibrium. Consequently, the welfare cost of uncertainty about Brexit is tiny: U.K. households would pay no more than a few dozen pounds per person to avoid this uncertainty. I have shown that my findings are robust to a wide range of alternative assumptions about export participation, the level of sectoral aggregation, financial markets, the likelihood of each Brexit scenario, and assigned parameter values.

It is important to point out that this study is limited to an analysis of the increase in trade costs that will occur when the United Kingdom leaves the European Union. Brexit will likely cause other policies to change as well, particularly policies concerning immigration and fiscal benefit transfers. The United Kingdom is likely to benefit from cessation of fiscal benefits because it currently pays more into the benefit pool than it receives. Dhingra et al. (2016b,c) show, though, that the welfare gains from leaving the fiscal benefit system are likely to be small compared to the welfare losses associated with rising trade costs. Further work is needed to assess the impact of policies that restrict migration to and from the United Kingdom, which are likely to have differential effects across population segments. More broadly, the outcome of the Brexit referendum has caused substantial political turmoil which may affect a range of U.K. economic policies in the future.

I have also used my model to conduct a series of hypothetical trade reform exercises to analyze the role of differences between the circumstances of Brexit and other trade policy uncertainty episodes in driving my results. These exercises suggest that trade policy uncertainty has little impact on macroeconomic dynamics, trade flows, and welfare in a broad range of circumstances, highlighting the need for further quantitative analysis in this part of the literature.

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Table 1: Recent U.K. macroeconomic and trade dynamics

Variable	2012Q1–2015Q2	2015Q3–2016Q2	2016Q3–2017Q1
Real GDP growth (pct. per year)	2.28	1.72	1.79
Consumption growth (pct. per year)	1.67	2.43	1.95
Investment (pct. GDP)	16.31	16.80	16.67
Net exports (pct. GDP)	-2.09	-1.67	-1.93
Export growth (pct. per year)	1.73	3.50	2.15
Import growth (pct. per year)	3.38	3.41	3.96

Table 2: Calibrated parameter values

Parameter	Meaning	Value	Source or target
(a) Assigned parameters			
β	Discount factor	0.98	Long-run interest rate = 2%
γ	Risk aversion	2.00	Standard
δ	Depreciation rate	0.60	Standard
α	Capital share	0.33	Standard
θ	Elast. of subst. across varieties	5.00	Alessandria et al. (2016)
φ	Capital adjustment cost	0.76	Steinberg (2016)
(a) Calibrated parameters			
η_i	Value-added shares	$(0.46, 0.42, 0.40)$	Table 3
$\bar{L}_i$	Labor endowments	$(66.7, 194, 761)$	Table 3
$\mu_{uk,j}$	U.K. Armington shares	$(0.84, 0.07, 0.09)$	Table 3
$\mu_{eu,j}$	E.U. Armington shares	$(0.01, 0.89, 0.10)$	Table 3
$\mu_{rw,j}$	R.W. Armington shares	$(0.003, 0.24, 0.97)$	Table 3
$\omega_{uk,d}$	U.K. destination shares	$(0.49, 0.51)$	Table 3
$\omega_{eu,d}$	E.U. destination shares	$(0.11, 0.89)$	Table 3
$\omega_{rw,d}$	R.W. destination shares	$(0.13, 0.87)$	Table 3
ζ	Armington elasticity	3.25	Long-run trade elasticity = 5
σ_i	Productivity distributions	$(0.35, 0.40, 0.46)$	Exporter size/non-exporter size = 2.5
$\kappa_{i,0}$	Export entry costs	$(222, 1001, 1076)$	Export participation rate = 25%
$\kappa_{i,1}$	Export continuation cost	$(16.0, 57.9, 49.1)$	Export exit rate = 2%

Table 3: 2011 inter-country input-output matrix (UK GDP = 100)

	Intermediate inputs			Final demand			
	UK	EU	RW	UK	EU	RW	GO
(a) Raw WIOD matrix							
UK	71.4	10.0	10.3	87.8	4.2	5.5	189.2
EU	7.6	500.1	75.2	6.8	530.7	44.5	1,164.9
RW	10.2	72.5	2,346.7	6.0	31.6	2,248.7	4,715.8
VA	100.0	582.3	2,283.6	-	-	-	2,965.9
GO	189.2	1,164.9	4,715.8	100.6	566.6	2,298.7	
(b) Balanced-trade matrix							
UK	71.4	10.3	10.1	87.4	4.5	5.4	189.2
EU	7.3	495.7	70.7	6.4	543.3	41.5	1,164.9
RW	10.5	76.6	2,351.3	6.1	34.5	2,236.7	4,715.8
VA	100.0	582.3	2,283.6	-	-	-	2,965.9
GO	189.2	1,164.9	4,715.8	100.0	582.3	2,283.6	

Table 4: Brexit scenarios: transition probabilities and trade costs

Parameter	Meaning	Value	Source or target
(a) Soft Brexit trade costs			
$\tau_{uk,eu}$	Tariff on U.K. imports from E.U.	0.00%	Not applicable
$\tau_{eu,uk}$	Tariff on E.U. imports from U.K.	0.00%	Not applicable
$\xi_{uk,eu}$	NTB on U.K. imports from E.U.	2.18%	WIOD + Francois et al. (2013)
$\xi_{eu,uk}$	NTB on E.U. imports from U.K.	1.74%	WIOD + Francois et al. (2013)
(b) Hard Brexit trade costs			
$\tau_{uk,eu}$	Tariff on U.K. imports from E.U.	3.58%	COMTRADE + W.T.O.
$\tau_{eu,uk}$	Tariff on E.U. imports from U.K.	2.12%	COMTRADE + W.T.O.
$\xi_{uk,eu}$	NTB on U.K. imports from E.U.	6.53%	WIOD + Francois et al. (2013)
$\xi_{eu,uk}$	NTB on E.U. imports from U.K.	5.21%	WIOD + Francois et al. (2013)
(c) Transition probabilities			
Π_{vote}	Probability of "stay" vote	0.75	Prediction markets
Π_{brexit}	Probability of soft Brexit	0.5	Not applicable

Table 5: U.K. welfare losses from Brexit (consumption equivalent, 1/100 percent)

	Forward-looking		Backward-looking	
Model	2015	2016	Soft	Hard
(a) Total welfare loss				
Baseline	18.95	77.05	42.60	108.84
Static export participation	17.82	72.52	40.25	102.28
No extensive margin	17.97	73.14	40.12	103.68
Multi-sector	22.55	91.54	45.51	134.74
Multi-sector + frictions	22.49	91.02	46.39	132.57
More likely hard Brexit	23.01	93.81	42.77	108.58
More likely soft Brexit	14.79	60.09	42.44	108.95
Financial autarky	18.36	75.10	41.35	105.97
Lower trade elasticity	21.36	86.92	45.23	125.50
Higher risk aversion	18.81	76.02	41.66	107.55
(b) Loss from uncertainty				
Baseline	0.15	0.02	0.10	0.14
Static export participation	0.06	0.12	0.22	0.23
No extensive margin	0.05	0.16	0.23	0.37
Multi-sector	0.28	0.33	0.17	0.25
Multi-sector + frictions	0.18	0.34	0.97	1.70
More likely hard Brexit	0.13	0.22	0.26	0.40
More likely soft Brexit	0.07	0.01	0.06	0.03
Financial autarky	0.18	0.10	0.04	0.23
Lower trade elasticity	0.16	0.09	0.09	0.23
Higher risk aversion	0.15	0.48	0.66	0.79

Table 6: Welfare gains and losses in trade reform exercises (consumption equivalent, 1/100 percent)

Possible cancellation year(s)	Baseline trade costs		Higher trade costs	
	Forward-looking	Backward-looking	Forward-looking	Backward-looking
(a) Total welfare gain				
1. 2019 only	14.41	110.91	19.78	152.98
2. 2022 only	15.09	110.84	20.74	153.12
3. Annually, 2017–2019	13.96	110.96	20.31	153.03
4. Annually, 2017–2022	12.68	110.94	17.94	153.12
(b) Loss from uncertainty				
1. 2019 only	0.21	0.25	0.24	0.32
2. 2022 only	0.22	0.32	0.20	0.46
3. Annually, 2017–2019	0.38	0.20	0.50	0.37
4. Annually, 2017–2022	0.26	0.22	0.11	0.46

Figure 1: Model timing and uncertainty tree

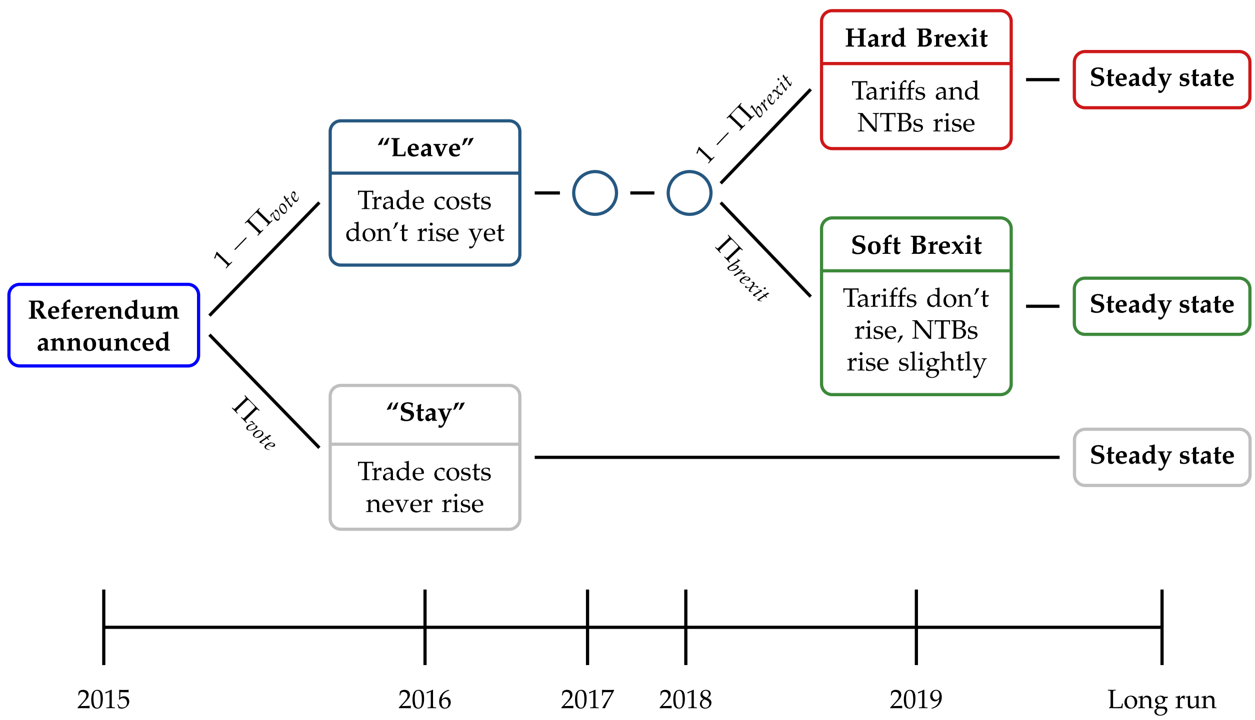


Figure 2: Recent dynamics of U.K. bilateral real exchange rates (2015Q2=100)

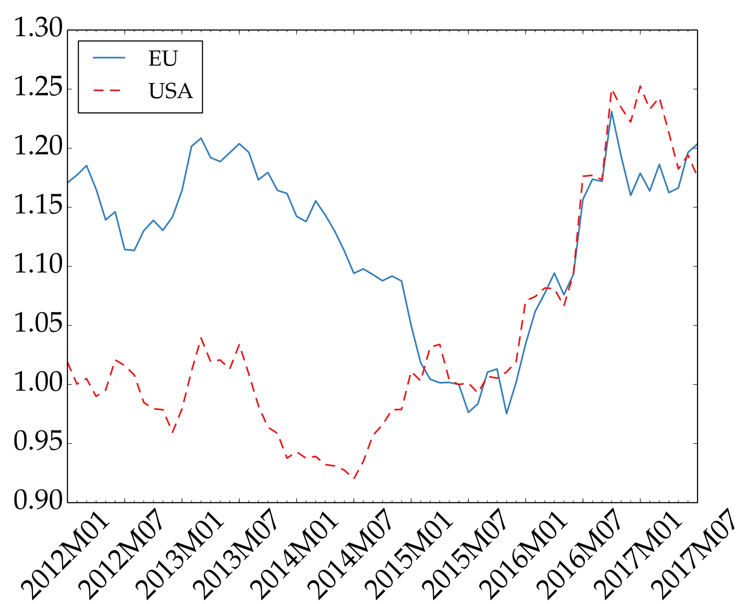


Figure 3: Impact of Brexit on U.K. macro variables (pct. change))

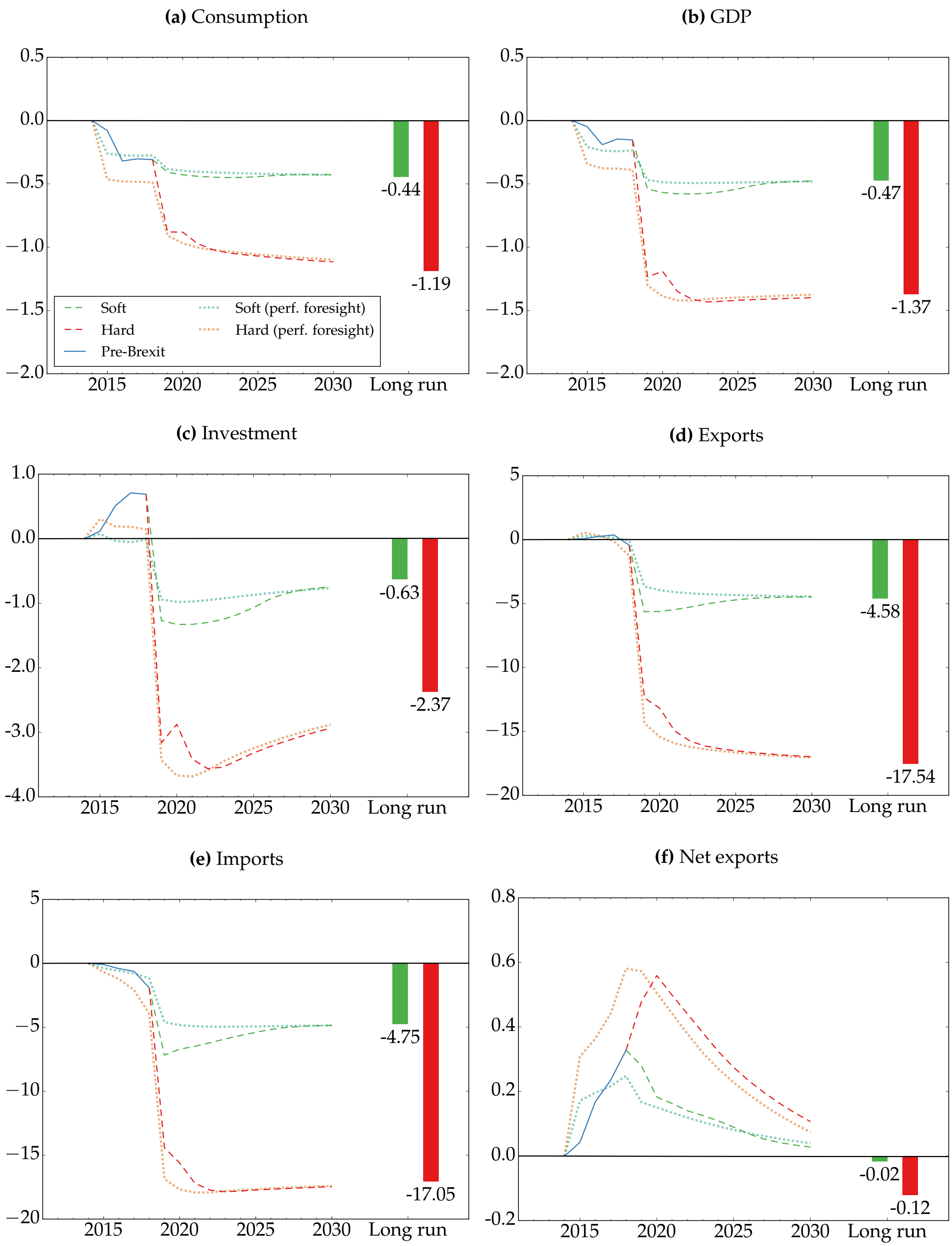
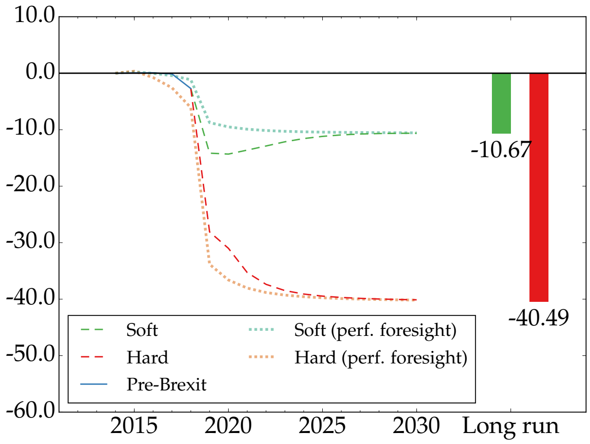
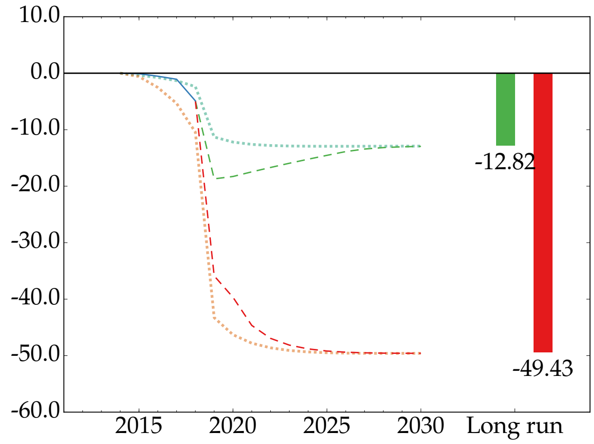


Figure 4: Impact of Brexit on U.K.-E.U. trade (pct. change)

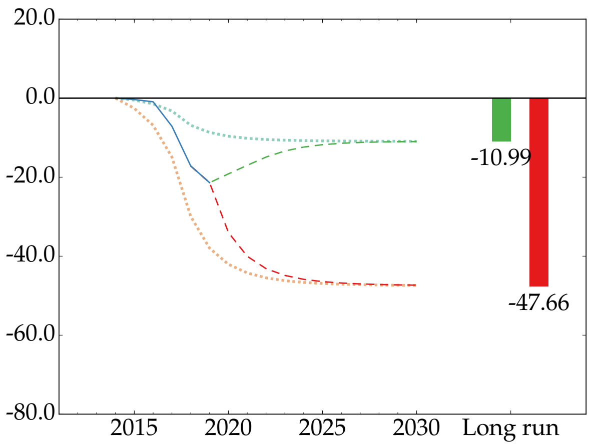
(a) Exports



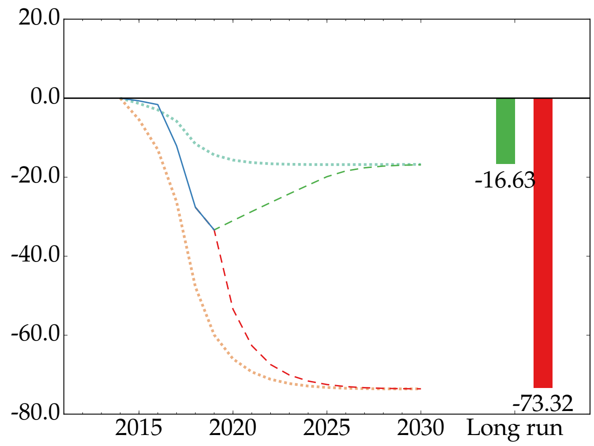
(b) Imports



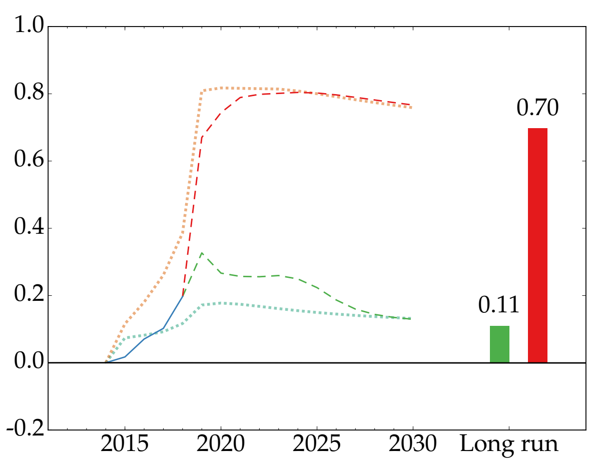
(c) U.K. export participation



(d) E.U. export participation



(e) Net exports



(f) Real exchange rate

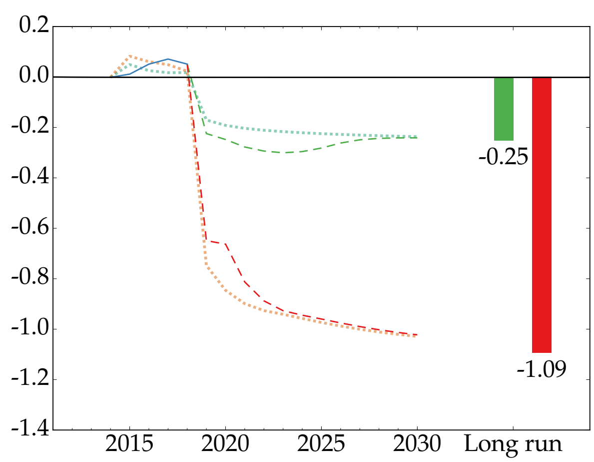


Figure 5: Impact of Brexit on U.K.-R.W. trade (pct. change)

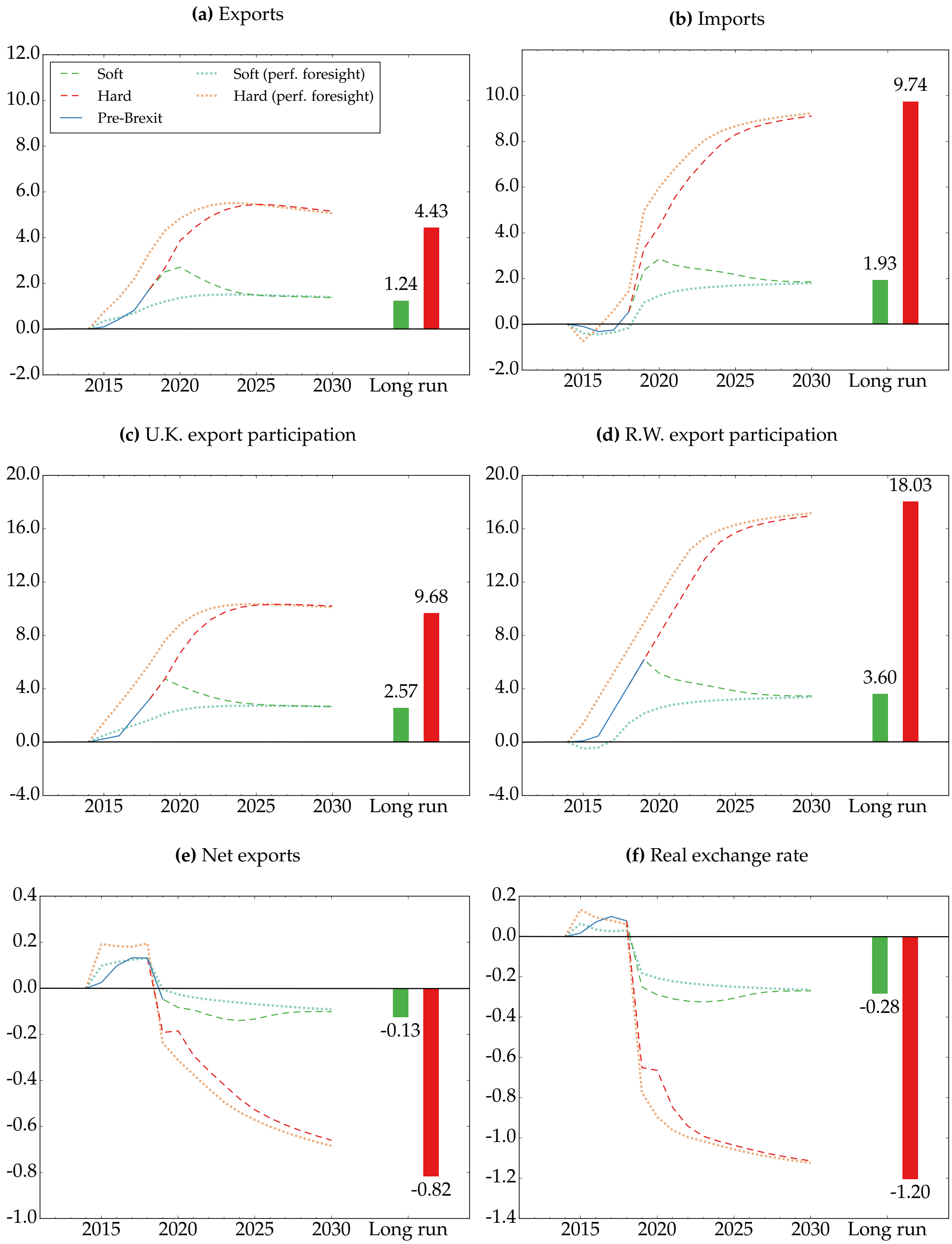
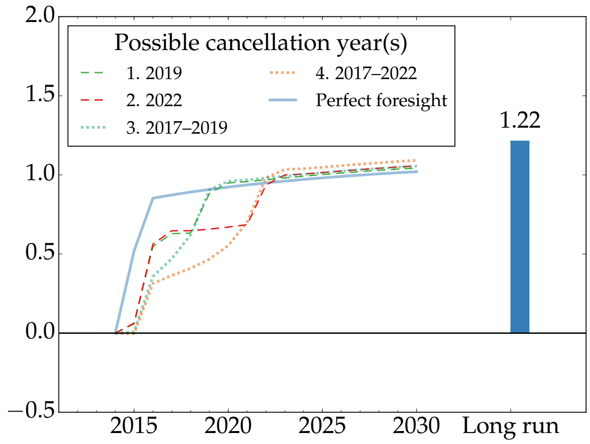
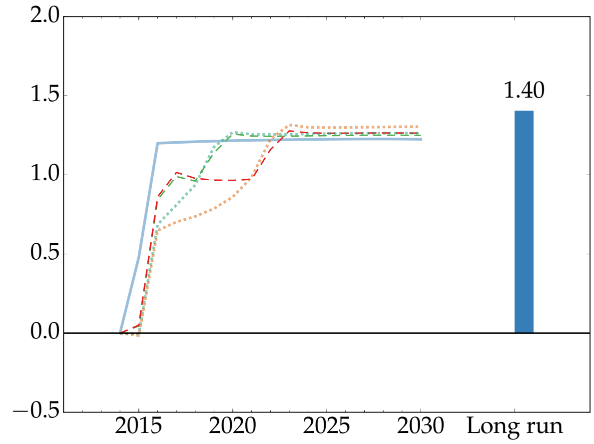


Figure 6: U.K. macro variables in trade reform exercises (pct. change))

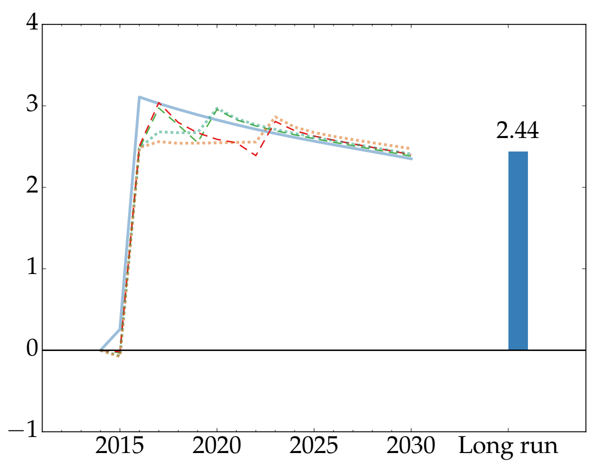
(a) Consumption



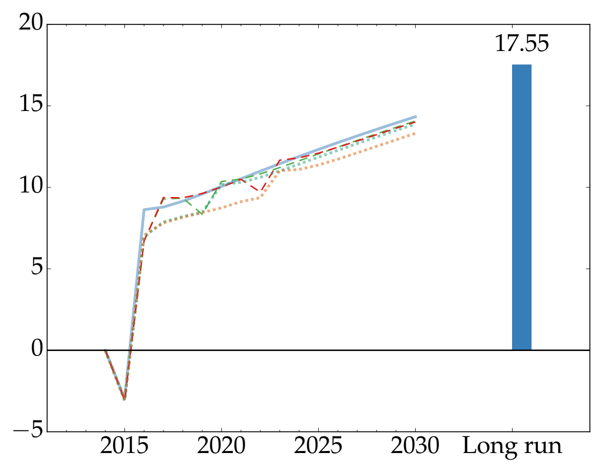
(b) GDP



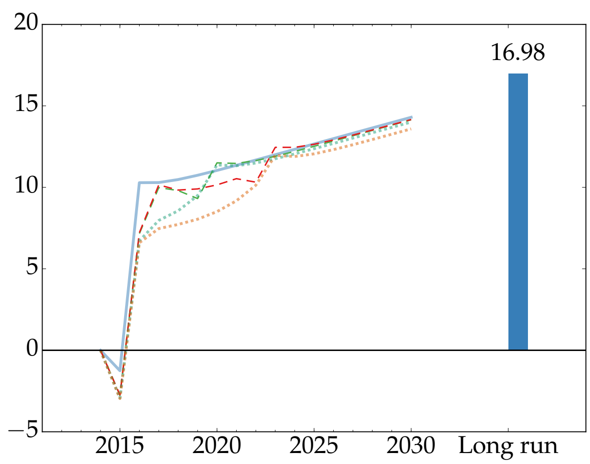
(c) Investment



(d) Exports



(e) Imports



(f) Net exports

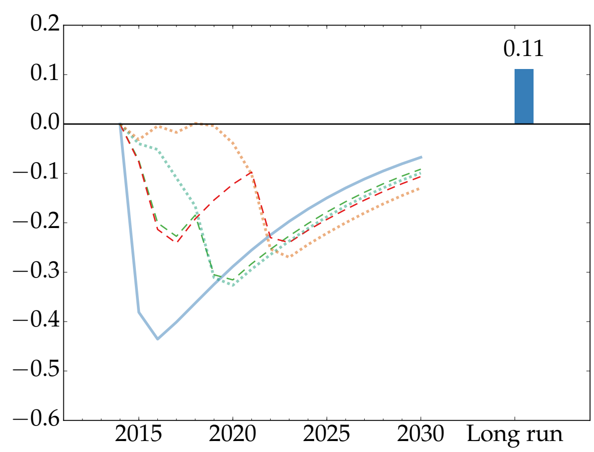
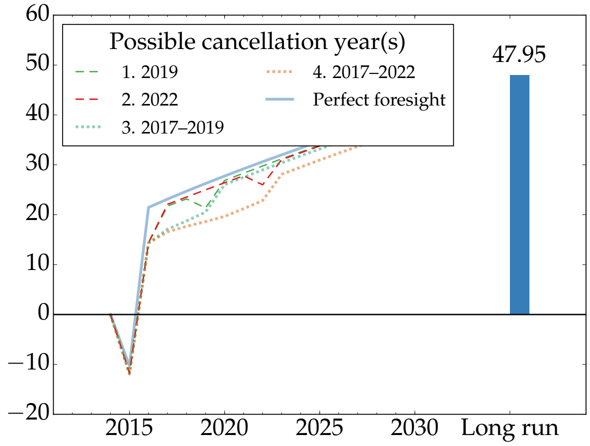
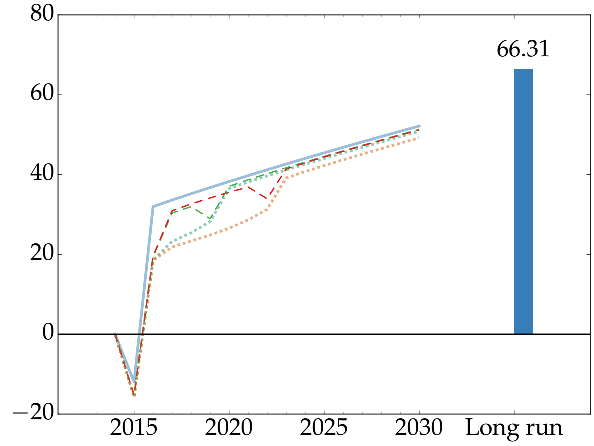


Figure 7: U.K.-E.U. trade in trade reform exercises (pct. change)

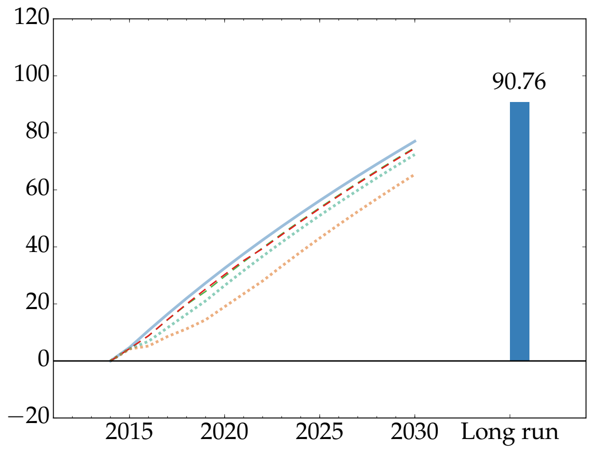
(a) Exports



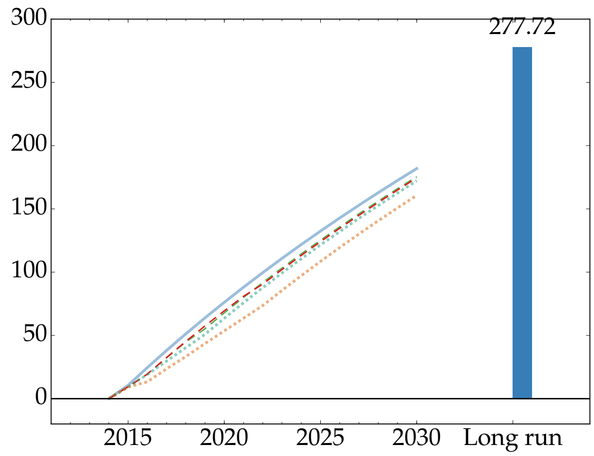
(b) Imports



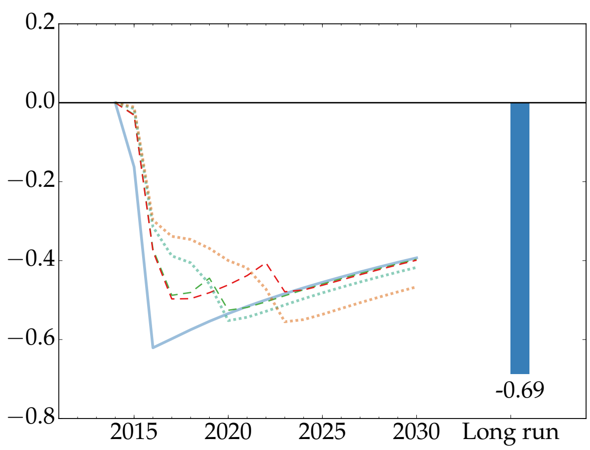
(c) U.K. export participation



(d) E.U. export participation



(e) Net exports



(f) Real exchange rate

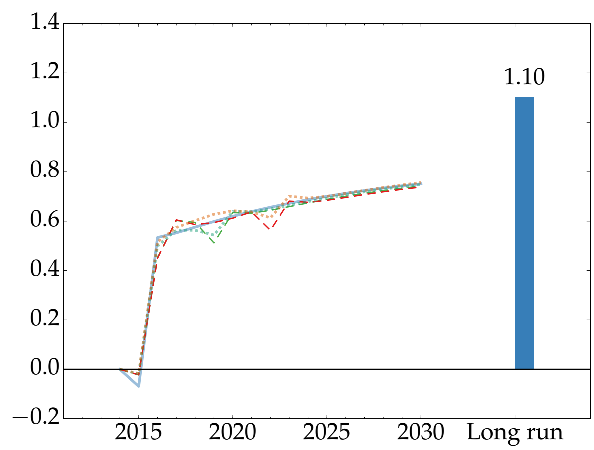
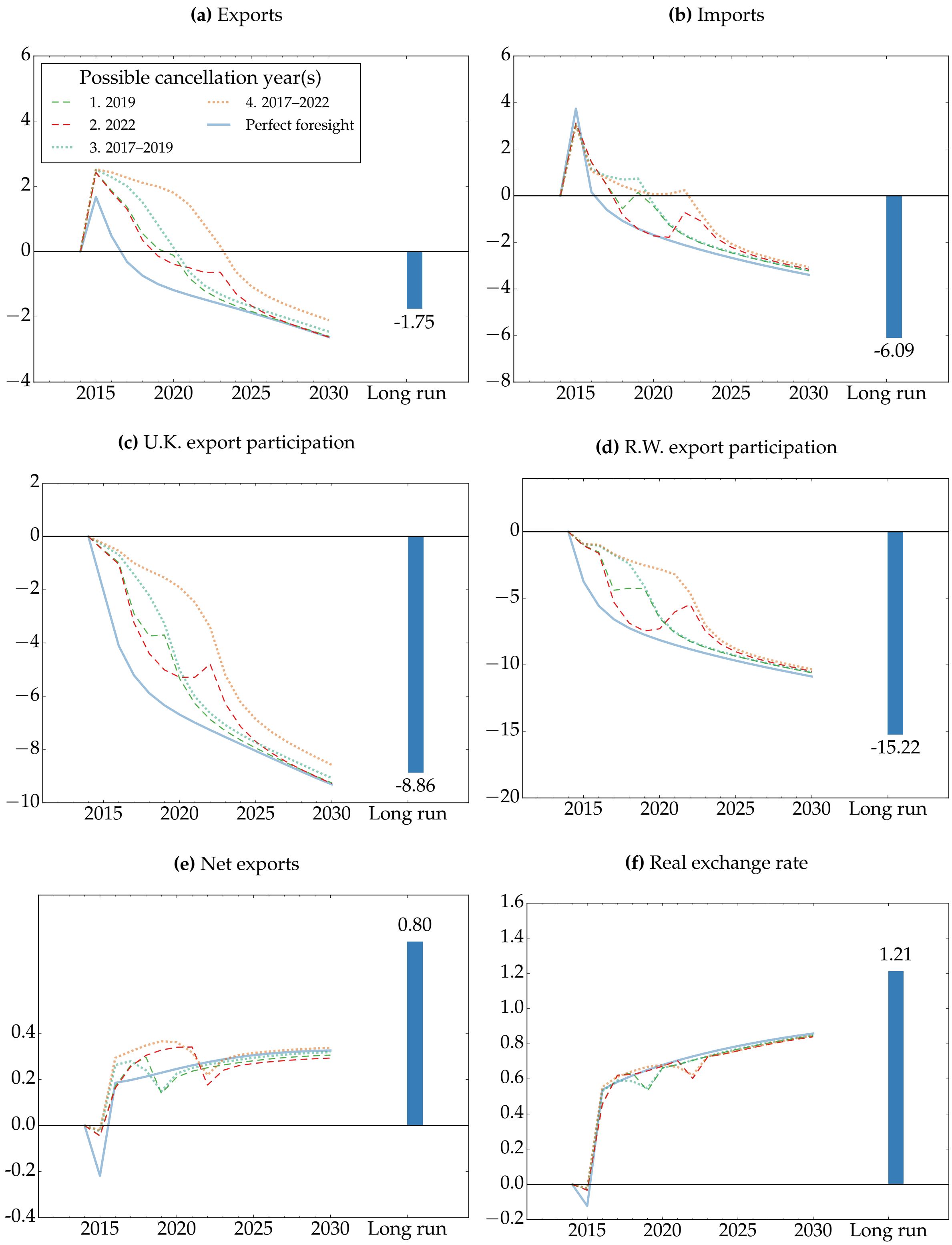


Figure 8: U.K.-R.W. trade in trade reform exercises (pct. change)



Appendix to "Brexit and the Macroeconomic Impact of Trade Policy Uncertainty" (for online publication only)

A Data

This section of the appendix provides additional details on data sources and data processing. Python scripts referenced below can be found in the folder "programs/python" in the online supplement.

A.1 Recent national accounts

All of the data reported in Table 1 were extracted from Eurostat. They have been seasonally and calendar adjusted. Filtering methods are well-known to give inaccurate results near the end of time series, so I simply compare the data that cover the period after the referendum act was introduced to Parliament to the data prior to the act's introduction. The first column of the table reports the average quarterly growth rates[20] of key macro variables between 2012Q1 and 2015Q2. I choose 2012Q1 as the starting point for the comparison period to eliminate the effects of recovery from the Great Recession; this is the first quarter in which consumption begins to rise after falling during recession. The second column lists average growth rates between 2015Q3 and 2016Q2, the period between the referendum's introduction to Parliament and the actual vote, and the third column lists the growth rates for 2016Q3–2017Q1, the three quarters for which we have data since the vote.

A.2 Input-output matrices

Here I describe the construction of the input-output matrices in Table 3. I begin with the 2011 world input-output matrix from the World Input Output Database (Timmer et al., 2015), which contains 40 countries and 35 industries. I aggregate all countries in the European Union (except for the United Kingdom) into a composite "E.U." country, and the remaining non-U.K. countries into a second "rest of the world" composite. I aggregate all industries into one sector. This yields the matrix in panel (a) of the table. To obtain the balanced matrix in panel (b), I use the RAS method (Bacharach, 1965) to find the most similar matrix in which each country's aggregate trade balance is zero. To apply the method, I add two additional rows for value added, so that each country's value added is in its own row. This allows me to ensure that these value added figures remain unchanged in the procedure. I then impose the restriction in the RAS algorithm that each country's final demand must equal its value added. This implies that net exports must be zero. The balanced matrix

[20] I report the average values of the investment rate and net exports/GDP instead of growth rates since these variables do not exhibit trend growth.

is shown in panel (b). Both of these steps are performed in the python script iomats.py.

A.3 Tariff and non-tariff barriers

A.3.1 Tariffs

To calculate the increases in U.K.-E.U. tariffs for hard Brexit, I use data from two sources:

- W.T.O. data on most-favored-nation tariff rates charged by the European Union on HS6-level products.
- COMTRADE data on HS6-level trade flows between the United Kingdom and the European Union for 2011.

I combine these two data sources to compute trade-flow-weighted average tariffs for each trade flow direction. The average tariff on U.K. imports from the E.U. is weighted by imports, while the average tariff on E.U. imports from the U.K. is weighted by exports. Then, I multiply these averages by the goods (agriculture, mining, and manufacturing) shares of total imports and exports, since there are no tariffs in services trade. These calculations are shown in the first two rows of Table 4, panel (c). These calculations are perfomed in the script tariffs.py.

A.3.2 Non-tariff barriers

The data sources for the non-tariff barrier increases are:

- Francois et al. (2013), who estimate the non-tariff barriers in trade between the European Union and the United States for a subset of ISIC Rev. 2 industries, as well as the fraction of these barriers that are policy-reducible.
- WIOD trade flows between the United Kingdom and the European Union at the ISIC Rev. 2 level (the data from A.2 before aggregating across industries).

First, I calculate the policy-reducible non-tariff barriers in E.U.-U.S.A. trade for each industry by multiplying total barriers by their policy-reducibility fractions. Second, I use a similar approach as in A.3.1 to calculating average barriers in U.K.-E.U. trade, here using the WIOD trade flow data as weights. This is complicated slightly by the fact that the Francois et al. (2013) data map closely, but not exactly, to the ISIC Rev. 2 industries. Table A1 lists the concordance that I use between ISIC and Francois et al. (2013) sectors. In several cases, one ISIC sector maps to multiple Francois et al. (2013) sectors (this is noted in the table with "+" signs). In this case, I calculate the non-tariff barrier for the ISIC sector by taking a simple average of the mapped Francois et al. (2013) sectors. In one case, three ISIC sectors maps to one Francois et al. (2013) sector. In this case, I use the same value for all three ISIC sectors. This calculation is performed in the script ntb.py.

B Details on calibration and equilibrium solution method

This section of the appendix provides additional details on the calibration procedure and the numerical method used to solve the model.

B.1 Calibration

Before choosing any parameters, I first add scaling factors, $\bar{Y}_i$ and $\bar{Y}_{i,j}$, to the aggregation technologies which I will calibrate so that all steady-state aggregate prices are one:

$$Y_i(Z^t) = \bar{Y}_i \left[\sum_{j \in I} \left(\mu_{i,j}\right)^{\frac{1}{\zeta}} \left(Y_{i,j}(Z^t)\right)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}$$

$$Y_{i,j}(Z^t) = \bar{Y}_{i,j} \left[\int_{\nu \in N_{i,j}(Z^t)} y_{i,j}(Z^t, \nu)^{\frac{\theta-1}{\theta}} \, \mathrm{d}\nu \right]^{\frac{\theta}{\theta-1}}$$

This is without loss of generality, but it facilitates the calibration procedure because the entries in the input-output matrix represent both real quantities and nominal expenditures (Kehoe et al., 2013).

B.1.1 Assigned parameter values

First, I assign the following parameter values:

- $\beta = 1/1.02$;
- $\delta = 0.06$;
- $\zeta = 3.25$ (this value, which yields a long-run trade elasticity of 5, was determined through manual experimentation);
- $\gamma = 2$;
- $\varphi = 0.8$;
- $\alpha = 1/3$;
- $\theta = 5$;
- $\omega_{uk,eu} = 0.489$;
- $\omega_{eu,uk} = 0.109$;
- and $\omega_{rw,uk} = 0.13$.

In addition, I set all trade costs to zero so that Armington shares will absorb both subjective home bias and trade costs as discussed in the main text.

B.1.2 Aggregation technology and household parameters

A number of parameters can be set directly from the input-output data. First, I set the value added shares, η_i, as follows:

$$\eta_i = \frac{1 - \left(\frac{\theta M_i^*}{(\theta-1)Y_i^*}\right)}{1 - \left(\frac{\theta M_i^*}{(\theta-1)Y_i^*}\right)\left(1 - \left(\frac{R_i^*}{\alpha}\right)^{\alpha}\left(\frac{W_i^*}{1-\alpha}\right)^{1-\alpha}\right)}$$

As in the main text, stars indicate steady-state equilibrium values, which are taken directly from the input-output matrix in panel (b) of table 3. Second, I calibrate the Armington share parameters, $\mu_{i,j}$, using marginal-product-pricing conditions:

$$\mu_{i,i} = \frac{1}{\sum_{j \in I}\left(Y_{i,j}^*/Y_{i,i}^*\right)};$$
$$\mu_{i,j} = \mu_{i,i}\left(Y_{i,j}^*/Y_{i,i}^*\right).$$

I normalize $\sum_{j \in I} \mu_{i,j} = 1$. Third, I set the top-level scaling factors, $\bar{Y}_i$:

$$\bar{Y}_i^* = \frac{Y_i^*}{\left[\sum_{j \in I}\left(\mu_{i,j}\right)^{\frac{1}{\zeta}}\left(Y_{i,j}^*\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}}}.$$

Fourth, I set labor endowments equal to steady state employment: $\bar{L}_i = L_i^*$.

B.1.3 Firm parameters

The remaining parameters are calibrated by solving a system of equations. These parameters are:

- productivity dispersion, σ_i, for $i \in I$;
- scaling factors, $\bar{Y}_{i,j}$, for $i, j \in I \times I$;
- entry costs, $\kappa_{i,0}$, for $i \in I$;
- and continuation costs, $\kappa_{i,1}$, for $i \in I$.

I solve for values of these parameters that jointly satisfy the following conditions:

- in each country i, the average exporter's revenues are 2.5 times those of the average non-exporter;
- for each country i, the export participation rate is 25 percent;

- for each country i, the export exit rate is 2 percent;
- for each pair i, j, the price of the bilateral aggregate, $Y^*_{i,j}$, is one.

The first three conditions are from Alessandria and Choi (2016) and Alessandria et al. (2016). While these parameter values must be solved jointly, each one loosely maps to one of the conditions. The dispersions, σ_i, control the relative size of exporters. The entry costs, $\kappa_{i,0}$, control export participation costs. The continuation costs, $\kappa_{i,1}$, control the exit rates. And the scaling factors, $\bar{Y}_{i,j}$, control bilateral aggregate prices.

B.2 Solution method

Typically, DSGE models are solved by linearizing the equilibrium conditions around an invariant, deterministic steady state (higher-order approximations are necessary for analyzing the effects of aggregate uncertainty and making welfare comparisons). The stochastic structure in my model is not amenable to this approach; the process for the aggregate state is non-stationary and there are three long-run steady states, each of which is selected endogenously.[21] Instead, I use a global method that provides an exact, not approximated, solution.

The basic approach is the same as in Kehoe et al. (2013) and Alessandria et al. (2015): assume that the equilibrium converges to a steady state after a finite number T of periods, then solve the resulting finite system of equilibrium variables and equations using the standard Newton-Raphson method. These papers study deterministic models, however. The presence of aggregate uncertainty in my paper complicates the application of this approach but does not prevent it because the number of possible histories is finite.

To reduce the size of the solution space, I solve for many of the equilibrium variables analytically as functions of "target" variables which I use in the solver. For example, consumption is implied by gross output, investment, and intermediate inputs through the aggregate market clearing condition (24). The equilibrium variables that are used in the solver are:

- wages, $\{W_i(Z^t)\}_{t=0}^T$;
- bilateral prices, $\{P_{i,j}(Z^t)\}_{t=0}^T$;
- bond prices, $\{Q(Z^t)\}_{t=0}^T$;
- rental rates, $\{R_i(Z^t)\}_{t=0}^T$;
- gross output, $\{Y_i(Z^t)\}_{t=0}^T$;
- and investment, $\{X_i(Z^t)\}_{t=0}^{T-1}$.

The equilibrium equations that are used in the solve are

[21] As mentioned in the main text, even holding trade costs fixed there is a continuum of possible steady states because I allow for unbalanced trade.

- numeraire normalization, $P_{uk}(Z^t) = 1$, for $t \leq T$;
- balance of payments for $i = uk, eu$ and $t \leq T$;
- labor market clearing for $i \in I$ and $t \leq T$;
- capital market clearing for $i \in I$ and $t \leq T$;
- Euler equations for $i \in I$ and $t < T$;
- and market clearing for bilateral trade, $Y_{i,j}(Z^t) = \int y_{i,j}(Z^t, \nu)$, for $i, j \in I \times I$ and $t \leq T$.

The program to solve the model is written in C. It can be found in the folder "programs/c" in the online supplement. Please note that I have compiled the program in Linux and linked to BLAS and LAPACK routines in the Intel MKL library. If you do not have acccess to this library, you can use alternative libraries instead (e.g. Atlas, GSL). If you are using Windows and need help, please contact me for assistance.

C Multi-sector model

In this section of the appendix, I describe the multi-sector version of the model briefly mentioned in the main text. I also explain the calibration of the multi-sector model.

Each country in the multi-sector model has two sectors, goods ($s = 1$) and services ($s = 2$). As in Armington, gross output is differentiated by source country and sector but homogeneous across firms within each country-sector pair.[22] International trade is conducted by intermediaries that aggregate purchases of domestic and foreign gross output into artificial composites, which are then sold to domestic households consumption and investment, and to firms for intermediate inputs. Households have the same preferences and solve almost the same maximization problem as in the one-sector model. The only difference is that households choose investment for each sector separately, and sectoral capital stocks follow a law of motion with adjustment costs as in equation (3) in the main text.

C.1 Production and demand system

C.1.1 International trade

In each country i and sector s, competitive distributors intermediate trade in intermediate inputs and final expenditures separately. The intermediate composite, $M_{i,s}(Z^t)$, is an Armington aggregate of sector-s

[22] The exporter dynamics framework of Alessandria and Choi (2007) used in the one-sector model is not tractable here. The Armington specification is numerically tractable and consistent with theoretical and quantitative findings in the literature regarding the macroeconomic effects of trade frictions. Adding static firm heterogeneity as in Eaton and Kortum (2002) and other "new" trade models would yield similar results (Arkolakis et al., 2012; Eaton et al., 2011).

intermediates from all source countries, $M_{i,s,j}(Z^t),\ j \in I$:

$$M_{i,s}(Z^t) = \left\{ \sum_{j \in I} \left(\mu_{i,s,j}\right)^{\frac{1}{\zeta_{i,s}}} \left(M_{i,s,j}(Z^t)\right)^{\frac{\zeta_{i,s}-1}{\zeta_{i,s}}} \right\}^{\frac{\zeta_{i,s}}{\zeta_{i,s}-1}}.$$

The final expenditure composite in country i, $F_{i,s}(Z^t)$, is given by

$$F_{i,s}(Z^t) = \left\{ \sum_{j \in I} \left(\theta_{i,s,j}\right)^{\frac{1}{\sigma_{i,s}}} \left(F_{i,s,j}(Z^t)\right)^{\frac{\sigma_{i,s}-1}{\sigma_{i,s}}} \right\}^{\frac{\sigma_{i,s}}{\sigma_{i,s}-1}}.$$

The elasticities of substitution vary across countries, sectors, and uses. Later, I will calibrate them to match WIOD trade flow data and elasticity estimates from Caliendo and Parro (2015).

As in the one-sector model, there are two kinds of trade costs: formal import tariffs and non-tariff iceberg trade costs. Each type of trade cost varies by destination country, sector, source country, use. $\tau^m_{i,s,j}(Z^t)$ and $\tau^f_{i,s,j}(Z^t)$ denote country i's tariffs on intermediate and final imports from country j's s-sector, respectively. All tariff revenues are rebated to households as lump-sum transfers. Similarly, $\xi^m_{i,s,j}(Z^t)$ and $\xi^f_{i,s,j}(Z^t)$ denote country i's non-tariff iceberg costs of intermediate and final imports from country j's s-sector.

C.1.2 Gross output

Gross output of country i's sector s, $Y_{i,s}(z^t)$, is produced by competitive firms using value added, $V_{i,s}(z^t)$, and intermediate inputs of goods and services purchased from distributors, $M^d_{i,s,1}(z^t)$ and $M^d_{i,s,2}(z^t)$, according to a Leontief technology:

$$Y_{i,s}(Z^t) = \min \left\{ \frac{V_{i,s}(Z^t)}{\eta_{i,s,v}}, \frac{M^d_{i,s,2}(Z^t)}{\eta_{i,s,1}}, \frac{M^d_{i,s,2}(Z^t)}{\eta_{i,s,2}} \right\},$$

Value added is produced using capital, $K^d_{i,s}(Z^t)$, and labor, $L^d_{i,s}(Z^t)$, according to the usual Cobb-Douglas function.

$$V_{i,s}(Z^t) = (K^d_{i,s}(Z^t))^{\alpha_{i,s}} (L^d_{i,s}(Z^t))^{1-\alpha_{i,s}}.$$

I use the superscript d to distinguish firms' demand for factors and intermediates from factor supply, which is chosen by households, and intermediate supply, which is chosen by distributors.

C.1.3 Final demand

Each country i's aggregate consumption basket is a CES aggregate of retail goods and services:

$$C_i(Z^t) = \left\{ (\epsilon_{i,1})^{\frac{1}{\rho}} \left(C_{i,1}(Z^t) \right)^{\frac{\rho-1}{\rho}} + (\epsilon_{i,2})^{\frac{1}{\rho}} \left(C_{i,2}(Z^t) \right)^{\frac{\rho-1}{\rho}} \right\}^{\frac{\rho}{\rho-1}}.$$

Following Bems (2008), aggregate investment in each country i is a Cobb-Douglas aggregate of inputs purchased from goods and services retailers:

$$X_i(Z^t) = (X_{i,1}(Z^t))^{\varepsilon_{i,1}} (X_{i,2}(Z^t))^{\varepsilon_{i,2}}$$

C.1.4 Market clearing

The market clearing conditions in the multi-sector model are:

$$\begin{aligned}
Y_{i,s}(Z^t) &= \sum_{j \in J} \left(M_{i,s,j}(Z^t) + F_{i,s,j}(Z^t) \right), \ \forall i \in I, \ \forall s \in S \\
M_{i,s}(Z^t) &= \sum_{r \in S} M^d_{i,r,s}(Z^t), \forall i \in I, \ \forall s \in S \\
F_{i,s}(Z^t) &= C_{i,s}(Z^t) + X_{i,s}(Z^t), \ \forall i \in I, \ \forall s \in S \\
X_i(Z^t) &= \sum_{s \in S} X^d_{i,s}(Z^t), \ \forall i \\
K_{i,s}(Z^t) &= K^d_{i,s}(Z^t), \ \forall i, s \\
\bar{L}_i(Z^t) &= \sum_{s \in S} L^d_{i,s}(Z^t), \ \forall i \\
0 &= \sum_{i \in I} B_i(Z^t)
\end{aligned}$$

C.2 Calibration

As in the one-sector model, I first assign elasticities of substitution and other common parameters, then calibrate remaining parameters so that the steady-state replicates an input-output matrix.

C.2.1 Assigned parameters

Assigned parameters, like the discount factor and the capital share, which have the same meaning in the multi-sector model as they do in the one-sector model, are set to the same values as in the main text.

There are several new elasticities, however. I follow Kehoe et al. (2013) and use Atalay (2014)'s estimate of 0.65 for the elasticity of substitution between goods and services in consumption, ρ. To set the Armington elasticities — which are also trade elasticities since this model has no extensive margin — I refer to Caliendo and Parro (2015), who estimate trade elasticities for the same 2-digit ISIC industries that comprise the goods sector in the input-output matrix described below. For each country i, I set the intermediate and final goods trade elasticities, $\zeta_{i,1}$ and $\sigma_{i,1}$, to averages of the Caliendo and Parro (2015) estimates, weighted by these industries shares' in country i's total intermediate goods imports and total final goods imports, respectively. Intermediate goods trade elasticities range from 6.6 to 7.6, while final goods trade elasticities range from 4.4 to 5.3. All countries have higher elasticities for intermediates than for final use. For the services sector, I follow Costinot and Rodríguez-Clare (2014) and set the intermediate and final services trade elasticities, ζ_{i2} and σ_{i2}, all to 5, the average of the Caliendo and Parro (2015) estimates. Note that the aggregate trade elasticities are close to 5 in the multi-sector model.

C.2.2 Parameters calibrated to input-output data

The remaining parameters are calibrated so that the no-Brexit steady state replicates a two-sector version of the input-output matrix constructed from the 2011 WIOD data. The goods sector includes agriculture, mining, and all manufacturing industries (2-digit ISIC codes 37 and lower). The services sector includes all other industries. The two-sector matrix is shown in table A3. With this matrix and the elasticities and other externally-calibrated parameters in hand, we can calibrate the remaining parameters using the model's equilibrium conditions as in the one-sector model.

C.3 Post-Brexit trade costs

To calculate the increases in tariffs and non-tariff barriers after Brexit in the multi-sector model I use the same approach as in the one-sector model, but perform the computations for each sector separately. For tariffs in hard Brexit, the goods-sector tariffs are the figures from A.3.1 prior to scaling by the goods shares of total trade flows; there are no tariffs for the services sector. For non-tariff barriers, I perform the computation separately by sector and use, since the WIOD data that are used as weights distinguish between intermediate and final trade. The results are in table **??**.

C.4 Version with import adjustment frictions

The baseline multi-sector model has been calibrated using trade elasticity estimates from Caliendo and Parro (2015), which are in line with other estimates in the literature of trade flows' sensitivity to price changes in the long run. As Ruhl (2008) points out, in the short run trade flows typically respond less to price changes than long-run elasticity estimates would dictate; Heathcote and Perri (2002) estimate an elasticity of 0.9 using

HP-filtered data. Recent theoretical work on trade dynamics and endogenous trade elasticities emphasizes fixed costs of importing or exporting as used in the one-sector model (Alessandria and Choi, 2007; Ruhl, 2008; Alessandria et al., 2013b, 2015; Ramanarayanan, 2016), source-specific durable goods (Engel and Wang, 2011), destination-specific marketing capital (Drozd and Nosal, 2012), and firm-to-firm relationship stickiness (Lim, 2016). Incorporating these sorts of features into the mult-sector model is not computationally feasible, so I take a similar approach to Engel and Wang (2011) and add convex costs of adjusting import quantities.

In this version of the model, distributors must pay costs to adjust the quantities of inputs they import from other countries. The total adjustment cost paid by sector-s distributors in country i for intermediate trade is

$$\sum_{j \in I \setminus i} \left[\frac{\varphi_m}{2} \left(\frac{M_{i,s,j}(Z^t)}{M_{i,s,j}(Z^{t-1})} - 1 \right)^2 \right].$$

The parameter φ_m governs the size of the adjustment costs, which are paid in units of labor. Decreasing inputs from one foreign country in favor of increasing inputs from another incurs two adjustment costs; substituting inputs from the rest of the world for inputs from the European Union after Brexit is particularly costly. There is a similar adjustment cost for trade in final expenditures. Distributors solve dynamic problems in this version of the model, choosing inputs to maximize the expected present value of dividends.

In contrast to the adjustment-cost model suggested by Krugman (1986) and studied in Drozd and Nosal (2012), in which producers pay costs to adjust export quantities, this model delivers time-varying trade elasticities. In this exercise, I calibrate the adjustment cost parameters φ_m and φ_f so that the average short-term trade elasticity for the United Kingdom, measured over the one-year period 2019–2020 immediately after Brexit implementation, is 1, the standard value in the international business cycle literature.

Table A1: Concordance between ISIC Rev 2. and Francois et al. (2013) sectors

ISIC code	ISIC sector name	Francois et al. (2013) sector name
AtB	Agriculture, Hunting, Forestry and Fishing	–
C	Mining and Quarrying	–
15t16	Food, Beverages and Tobacco	Food & beverages
17t18	Textiles and Textile Products	Textiles
19	Leather, Leather and Footwear	–
20	Wood and Products of Wood and Cork	Wood & paper products
21t22	Pulp, Paper, Paper , Printing and Publishing	Wood & paper products
23	Coke, Refined Petroleum and Nuclear Fuel	–
24	Chemicals and Chemical Products	Chemicals + Cosmetics + Pharmaceuticals
25	Rubber and Plastics	–
26	Other Non-Metallic Mineral	–
27t28	Basic Metals and Fabricated Metal	Metals
29	Machinery, Nec	Machinery
30t33	Electrical and Optical Equipment	Electronics + OICE + Biotech. + Medical equip.
34t35	Transport Equipment	Aerospace & Space + Automotive
36t37	Manufacturing, Nec; Recycling	–
E	Electricity, Gas and Water Supply	–
F	Construction	Construction
50	Sale, Maint. and Repair of Motor Vehicles	–
51	Wholesale Trade and Commission Trade	–
52	Retail Trade, Except Motor Vehicles	–
H	Hotels and Restaurants	Travel Services
60	Inland Transport	Transport Services
61	Water Transport	Transport Services
62	Air Transport	Transport Services
63	Other Transport Activities	–
64	Post and Telecommunications	Communication Services
J	Financial Intermediation	Financial Services + Insurance Services
70	Real Estate Activities	–
71t74	Renting of M&Eq, Other Business Activities	ICT + Other Business Services
L	Public Admin and Defence	–
M	Education	–
N	Health and Social Work	–
O	Other Community and Personal Services	Personal & Recreational Services
P	Private Households with Employed Persons	–

Table A2: Assigned parameters in multisector model

Parameter	Meaning	Value	Source or target
(a) Trade elasticities			Literature + WIOD
$\zeta_{uk,s}$	UK intermediates	(7.6,5.0)	
$\zeta_{eu,s}$	EU intermediates	(7.5,5.0)	
$\zeta_{rw,s}$	ROW intermediates	(6.6,5.0)	
$\sigma_{uk,s}$	UK final	(4.8,5.0)	
$\sigma_{eu,s}$	EU final	(4.4,5.0)	
$\sigma_{rw,s}$	ROW final	(5.3,5.0)	
(b) Other parameters			
ρ	Consumption elasticity	0.65	Atalay (2014)
β	Discount factor	0.98	2% long-run interest rate
γ	Risk aversion	2.0	Standard
α	capital share	0.33	Standard
δ	Depreciation rate	0.06	Standard
φ_k	Capital adjustment cost convexity	0.76	Steinberg (2016)

Table A3: 2011 world input-output table in multi-sector model

		Intermediate inputs						Final demand			
		UK		EU		ROW					
		Goods	Services	Goods	Services	Goods	Services	UK	EU	ROW	GO
UK	Goods	2.68	2.96	2.12	0.77	1.79	1.11	4.00	1.95	2.25	39.27
	Services	4.61	27.46	0.56	1.83	0.62	1.93	42.41	0.28	0.67	160.73
EU	Goods	1.77	1.41	62.49	29.10	16.11	8.38	3.24	56.30	19.68	396.98
	Services	0.10	0.74	48.01	124.72	5.11	10.13	0.33	224.21	3.84	834.37
ROW	Goods	1.87	1.61	18.16	6.71	474.06	211.32	2.58	14.24	288.49	2,038.10
	Services	0.29	1.64	4.63	8.81	163.50	391.38	0.60	2.48	900.01	2,946.68
VA		19.63	80.37	198.49	417.19	1,019.05	1,473.34	-	-	-	3,208.07
GO		39.27	160.73	396.98	834.37	2,038.10	2,946.68	106.34	598.92	2,429.88	9,551.27

Table A4: Brexit scenarios in multisector model

Exogenous change	Soft Brexit	Hard Brexit
(a) Tariffs on goods trade		
Imports from E.U.	0.00	4.23
Exports to E.U.	0.00	3.29
(b) Non-tariff barriers (imports from E.U.)		
Intermediate goods	1.76	5.23
Intermediate services	0.98	2.94
Final goods	3.08	9.24
Final services	0.38	1.13
(c) Non-tariff barriers (exports to E.U.)		
Intermediate goods	1.49	4.47
Intermediate services	1.44	4.32
Final goods	2.62	7.85
Final services	1.065	3.20