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On Transiting to a Sustainable World Population: Lessons from an Overlapping Generations Model on the associated Problems, Prospects and Time Horizons.

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Summary.

Conventional wisdom suggests the planet's current population is roughly twice what it should be for sustainability purposes and, due to prevailing high fertility rates (the number of live births per adult female) it continues to grow. Transition to a stable sustainable population level requires lowering the fertility rate for an extended period of time, followed by a return to a rate which will stabilize the sustainable level. However, by changing its age distribution which in turn changes the configuration of its population's needs and the ability to service those needs, switching to a persistently low fertility rate presents an economy with some short run challenges. Here, to elucidate what those challenges could be, an overlapping generations model of the life cycle age distribution is developed and the time profiles of fertility rates over the last 60 years of over 200 nations are examined to see what the prospects for, and problems associated with, achieving a stable sustainable population could be. The good news is that fertility rates are on the right trajectory, the bad news is it will be a long time coming and the path is beset with short run fiscal policy challenges.

Keywords: Fertility, Sustainability, Fiscal Challenge, Overlapping Generations models

JEL codes E10;J11;J13;C53

Introduction.

Evidence and opinion from the sustainability literature (Daily et.al 1994, Dasgupta 2019 Lianos and Pseiridis 2015, Pimentel et. al. 1994, Tucker 2019) suggests that the current world population (approximately 8.1 billion) is not sustainable and needs to be at most half of what it is. Indeed, the United Nations has projected the world's population to grow to over 10 billion by the end of the century (United Nations 2024). Absent any infant mortality, stable populations which routinely replicate themselves require fertility rates (the number of live births per adult female) to be two[1], with more than two resulting in a growing and less than two resulting in a shrinking populations. Thus, the main route for transiting to sustainable lower population levels require fertility rates less than two be maintained for several generations, followed by a return to a fertility rate of 2. Here the prospects for such a situation are examined.

A nation's fertility rate is the net result of a combination of economic choices that its households make. Rather than a desire for a sustainable world population level, households are motivated by concerns regarding family size, income level, and labor force participation, all of which are conditioned by the nations prevailing cultural norms and government policies they confront. The evidence is that this nexus of influences is going through a sea change. Older Malthusian based theories of relationships between fertility and income or labour force participation that explain the historical data are being replaced with alternative theories, which explain more recent data, relating fertility to women's career and family choices in terms of factors like family policy, cooperative fathers, favourable social norms and more flexible labour markets (Becker 1965, Da Rocha and Fuster 2006, Doepke et. al. 2023, Goldin 2021, Mincer 1962, Mincer and Ofek 1982, Sear et.al. 2016). However, it is not the purpose here to examine the pertinence of fertility level drivers (though such an examination will be helpful in developing policies for maintaining fertility levels when the sustainable population level is reached), rather it is to understand the broader implications of persistently low fertility levels for the world economy at large.

When a society switches to a lower fertility rate, it is presented with many challenges, challenges which have much to do with the fact that such a switch changes its population structure, with older age groups increasing in size relative to younger age groups, engendering an ageing population

[1] Strictly speaking to account for early childhood mortality, conventionally it is around 2.1.

whose average age is increasing[2]. Such a sustained population imbalance results in greater demand for things that older age groups need with diminished demands for things that younger age groups need.

To exemplify, health outcomes are not constant across age groups or gender, they deteriorate with age (Deaton and Paxson, 1998; Kerkhofs and Lindeboom, 1997; Miller et al., 2019) and, in a somewhat paradoxical fashion, differ by gender in that, at comparable life stages, women typically experience inferior health outcomes to men, yet live longer (Case and Paxon, 2005; Nusselder et al., 2010; Oksuzyan et al., 2009; Van Oyen, 2013). With such disparities in health outcomes, it is clear that a low fertility switch engenders increased demands for health care by relatively more populous older age groups and more so for female specific healthcare, care that is supplied by relatively less populous younger age groups. Furthermore, demands for state and privately provided support for the growing elderly population in the form of old age security benefits and pensions will increase, support that is financed by taxes and contributions drawn from a shrinking younger tax paying population base, presenting a so-called fiscal challenge (Heer et.al. 2020) especially in pay as you go environments (Preston 2014). At the other end of the age spectrum, ceteris paribus, younger age groups will be diminishing in absolute and relative size so the demand for goods and services they need (education and training come to mind) will decline, educational institutions will experience declining enrolments and surplus human and physical capital stocks.

More generally, the totality of a society's product is consumed by the whole of that society (hence constructs such as per capita GNP) whereas it is produced by its working population and the fertility rate affects the balance between the working population and the total population with low fertility reducing the proportion of a society that is of working age[3]. At a national level, beyond improvements in technology which make the working age population more efficient, these shortages on either side of the supply or demand divide can and have been resolved by immigration policies, but that is inevitably a short run solution, since when all nations are in a low fertility mode, there will be no "surplus" younger populations to fill the shortages and in a world context net migration is zero.

[2] Contrary to popular perceptions, low fertility has more to do with aging populations than does increasing life expectancy, indeed increasing life expectancy is neither necessary nor sufficient for the average age in a population to increase.

[3] This should be qualified by noting that low fertility will also release some home carers into the workforce.

To elucidate the impact of fertility rates on population growth, population aging, dependency ratios and time horizons for achieving a sustainable population level, and highlight the challenges presented by low fertility rates, an extended overlapping generations model of a society's population structure is developed in Section 1. Using data drawn from the United Nations Department of Economic and Social Affairs, Population Division (United Nations 2024) Section 2 examines the time paths of these constructs over the period 1960 – 2022 and conclusions are drawn in Section 3. To anticipate the results, the good news is that fertility rates are on the right trajectory for achieving a sustainable population level, the bad news is it will take a while and the path is beset with short run challenges.

1) A Fertility based Overlapping Generations Model of Population Size and Structure.

Following Samuelson (1958), Diamond (1965) and Galor (1992) Overlapping Generations Models have typically been used in economic growth analysis to determine the progress of the size of the working population. In these models, per capita income is employed under the presumption that all adults work and the children that emerge in a period do so at the very end of the period so they would not be included in the working population. The human life cycle underlying such models usually consists of two stages, childhood and adulthood with the period t adults that disappear at the end of that period being replaced in period $t + 1$ by the children produced in period t (see for example Ashraf and Galor 2011). Thus, the adult population, which is the working population in a given period, is determined by the size of the working population in the previous period times the number of surviving children per adult (in effect a fertility rate that is endogenously determined by the Malthusian desire for children) in that period.

More recently, to emphasize the dependency of some groups in the population on other groups in the population, overlapping generations models have involved three life stages (see for example Preston 2014), a human capital acquisition stage, a work life stage and a retirement stage. Population growth in these models is usually exogenously given and not related to fertility choice. Here, in order to examine population growth, aging and age-group dependency issues relevant for achieving a stable sustainable population level, a model more aligned with a four-stage life cycle reality is required. The growth, ageing and age-group dependency constructs will be determined by a pre-ordained fertility rate in the context of an overlapping generations model similar to the foregoing 3 stage model, but which has a two-component work life stage with a working and procreating component preceding a working but no longer procreating component.

Assume no migration into or out of a population which is made up of households which are single genderless entities which self-replicate[4]. Households have four, equal length[5], life stages or generations indexed $j = 1,..,4$. At stage $j = 1$, the "junior stage" households acquire skills, learn and prepare for future adulthood, but do not yet produce goods and services or procreate. At the "young adult stage" stage $j = 2$, households produce goods and services and procreate. At the "middle age stage" $j = 3$,households cease procreation but continue producing goods and services. At stage $j = 4$, the "old age stage" households retire from productive work and die at the end of the stage. Passage of time is demarcated in epochs indexed $t = 0, 1, 2,..$ where an epoch has a length in years of a life stage which is the time it takes in years to move to the next stage. Households born at the beginning of epoch t live through 4 epochs $t + i$ for $i = 0,..,3$.

Letting $K_{j,t}$ represent the current population size (the number of households) of generation j in epoc t, the respective generations in epoch t will have population sizes $K_{j,t}$ for $j = 1,..,4$, with the overall population size in epoch t given by: $K_{O,t} = \sum_{j=1}^{4} K_{j,t}$. All households live the same amount of time, dying at the end of stage 4 and procreating at stage 2 with a replication or fertility rate of p, with their offspring being the new stage 1 population in the next epoch i.e. $K_{1,t+1} = pK_{2,t}$ which replaces the $K_{4,t}$ households in the overall population that died in the previous epoch. Generically, at the end of an epoch all generations move on to the next life stage so that epoch t's generation j becomes epoch $t + 1$'s generation $j + 1$ for $j = 1,.,3$ i.e. $K_{j+1,t+1} = K_{j,t}$ in a stable population $p = 1$ so that $K_{j,t} = 0.25K_{O,t}\ \forall\ j = 1,..,4$.

In this world without immigration or emigration[6], the fertility rate is the driver and determinant of all things, epoch-based population growth, population aging, the dependency ratio, the number of epochs it takes to reduce overall population size to a sustainable level and indeed the number of epochs to doomsday itself if low fertility remains unchecked, all of which are properties of this

[4] Conventional replication/fertility rate measures, which count the number of births per adult female, should be divided by 2 (or 2.1 if infant mortality is taken into account) to relate them to this model's rates.
[5] To relate to a society with a life expectancy of 90 years, the length of a life stage would be 22.5 years, retirement would be at 67.5 years and lived for 22.5 years.
[6] Clearly, the absence of any form of migration is an heroic assumption, since there is ample evidence that immigration has been used as a solution to worker shortages for example. However, in the context of achieving a sustainable world population, and noting that one nations immigrant is another nations emigrant, it greatly simplifies the analysis.

model which may be described as follows. Examples of potential values for these various epochal constructs over a range of fertility/replication rates follow in Table 1.

Epochal Population Growth.

To understand the nature of population growth, consider a succession of 4 epochs $t = 2,..,5$, in $t = 2$, $K_{O,2} = pK_{2,1} + K_{2,2} + K_{3,2} + K_{4,2}$, in $t = 3$, $K_{O,3} = p^2K_{2,1} + pK_{2,1} + K_{3,3} + K_{4,3}$, in $t = 4$, $K_{O,4} = p^3K_{2,1} + p^2K_{2,1} + pK_{2,1} + K_{4,4}$ and in epoch $t = 5$, $K_{O,5} = p^4K_{2,1} + p^3K_{2,1} + p^2K_{2,1} + pK_{2,1} = (p^4 + p^3 + p^2 + p)K_{2,1}$. Thus in each successive epoch $t = 5 + i$ the overall population will be $K_{O,5+i} = p^iK_{O,5}$ so that, given a sustained fertility rate of p, g, population growth in epoch $t = 5 + i$ is given by $g = \frac{K_{O,5+i} - K_{O,5+i-1}}{K_{O,5+i-1}} = \frac{(p^i - p^{i-1})}{p^{i-1}} = \frac{p^i}{p^{i-1}} - 1 = p - 1$ which is negative when $p < 1$, positive when $p > 1$ and zero when $p = 1$.

Epochal Population Ageing.

To associate the life stage indicator j with the average age of the population at that life-stage, let the epoch length in years be el, then average age of the population at life stage j is $el * (j - 1) + \frac{el}{2}$, and the average age of the overall population in any given epoch $A_{O,t}$ will be a population weighted sum of the life stage average ages i.e. $A_{O,t} = \sum_{j=1}^{4}\left(el * (j - 1) + \frac{el}{2}\right) K_{j,t}/K_{O,t} = el\sum_{j=1}^{4} jK_{j,t}/K_{O,t} - \frac{el}{2}$ which may be written as: $el\frac{p^3 + 2p^2 + 3p^1 + 4}{p^3 + p^2 + p + 1} - \frac{el}{2}$. Noting that epoch length el is related to life expectancy le ($el = 0.25le$), and $\frac{p^3 + 2p^2 + 3p^1 + 4}{p^3 + p^2 + p + 1} = 1 + \frac{p^2 + 2p + 3}{p^3 + p^2 + p + 1}$, observe the following:

$$\frac{\partial A_{O,t}}{\partial le} = 0.25\left(\frac{7}{8} + \frac{p^2 + 2p + 3}{p^3 + p^2 + p + 1}\right) > 0 \; \forall \, p > 0$$

And:

$$\frac{\partial A_{O,t}}{\partial p} = \frac{le}{4}\left(\frac{-(p^4 + 3p^3 + 7p^2 + 4p + 1)}{(p^3 + p^2 + p + 1)^2}\right) < 0 \; \forall \, p > 0\,.$$

While average age is an increasing function of life expectancy, it will be a decreasing function of fertility so that a diminishing fertility/replication rate increases the average age in the population for a given life expectancy. In essence decreasing the fertility/replication rate alters the age distribution, increasing the proportion of older people in the population and reducing the proportion of younger people, bringing about an increase in the average age without any increase in life expectancy (examples of the effect of different fertility rates on the age distribution are reported in Appendix 1). Given that in this model the young and the old depend upon the middle aged for support, the ratios of those respective populations will be an important feature.

The Epochal Dependency Ratio

In this model, the stage 1 and stage 4 populations are non-productive and depend upon the efforts of the population at stages 2 and 3 for their wellbeing, an important epoch parameter in the analysis is the dependency ratio $d_t = \frac{(K_{1,t}+K_{4,t})}{(K_{2,t}+K_{3,t})} = \frac{(p^4+p^1)}{(p^3+p^2)} = \frac{K_{O,t}-(p^3+p^2)}{(p^3+p^2)}$, which reflects the size of the "dependent population" relative to the size of the "depended upon population". In effect d_t is an index of the dependency burden underlying the fiscal challenge which in a stable population (when $p = 1$) will equal 1. Note that in this case:

$\frac{\partial d_t}{\partial p} = \frac{(4p^3+1)(p^3+p^2)-(3p^2+2p)(p^4+p^1)}{(p^3+p^2)^2} = 0$ and $\frac{\partial^2 d_t}{\partial p^2} = \frac{36}{16} > 0 \rightarrow d_t$ is at a minimum when $p = 1$.

As can be seen, replication/fertility rates less than 1 (which are needed if population sustainability targets are to be met) will generate dependency ratios greater than 1 so that more than half the population is supported by less than half the population. Furthermore, the lower is p is the greater will be the supporting burden which, if growth is measured in per capita income terms, will affect growth rates in a constant technology world. The proportion of the population that is productively working is given by $w = \frac{(p^3+p^2)}{(p^4+p^3+p^2+p)} =$ which is readily shown to have a maximum of 0.5 at $p = 1$. The effect of fertility rates and w on economic growth is outlined in Appendix 2.

Epochal Time to a Sustainable Population Level and Existential Doomsday

When $p < 1$ persists, the society fails to replicate itself in successive epochs which will shrink the population size and it is of interest to examine when that society will achieve a sustainable population level and indeed when it could cease to exist. Given an overall population of $K_{O,0}$ in the initial epoch, the population in n epochs hence will be $p^n K_{O,0}$ so, achieving a target population which is half the current population will take n epochs where n is the solution to $p^n K_{O,0} = 0.5K_{O,0}$ i.e. $n = \frac{\ln(0.5)}{\ln(p)}$. A society will cease existence when the number of households is down to 1 so the number of epochs it will take for that to happen is n where n is the solution to $p^n K_{O,0} = 1$, in this case $n = -\ln(K_{O,0})/\ln(p)$.

Table 1. Fertility rate dependent growth, average age, dependency ratio time to sustainability and workforce share constructs.

Fertility Rate**	Pop. Growth rate***	Average Age ($el = 22.5$)	Dependency ratio	10 million pop doomsday epoch*	# epochs to halve population*	w
p	$p-1$	$el\frac{p^3+2p^2+3p^1+4}{p^3+p^2+p+1} - \frac{el}{2}$	$\frac{(p^4+p^1)}{(p^3+p^2)}$	$\frac{-\ln(10{,}000{,}000)}{\ln(p)}$	$\frac{\ln(0.5)}{\ln(p)}$	
0.5000	-0.5000	62.2500	1.5000	23.2535	1.0000	0.4000
0.8000	-0.2000	51.1890	1.0500	72.2320	3.1063	0.4878
0.8500	-0.1500	49.5370	1.0265	99.1767	4.2650	0.4935
0.9000	-0.1000	47.9540	1.0111	152.9804	6.5788	0.4972
0.9500	-0.0500	46.4415	1.0026	314.2340	13.5134	0.4993
1.0000	0.0000	45.0000	1.0000			0.5000
1.0500	0.0500	43.6287	1.0024			0.4994
1.1000	0.1000	42.3263	1.0091			0.4977
1.1500	0.1500	41.0908	1.0196			0.4952
1.2000	0.2000	39.9199	1.0333			0.4918
1.5000	0.5000	34.0962	1.1667			0.4615

*To get a sense of time in years, these values should be multiplied by el, the epoch length. ** To relate to conventional fertility rate measures these values should be multiplied by 2.1.*** This is an epoch growth rate which should be divided by el to get an annualized growth rate.

2) Trends in Fertility/Replication Rates and Dependency Ratios.

Data on fertility rates and population sizes for over 200 nations from 1960 to 2022 were obtained from United Nations (2024). These were employed to construct time profiles of 2 types of nation specific fertility rate probability density functions, one using standard national rates, the other national rates weighted by the respective population sizes. Since fertility choices will be influenced by the social mores and customs of a nation, the former is of interest in reflecting nation differences, the latter is of greater importance when considering the situation confronting the world. Mean and modal[7] location values of these distributions over the years together with spread (i.e. standard deviation) and skewness factors[8] are tabulated in Appendix 3.

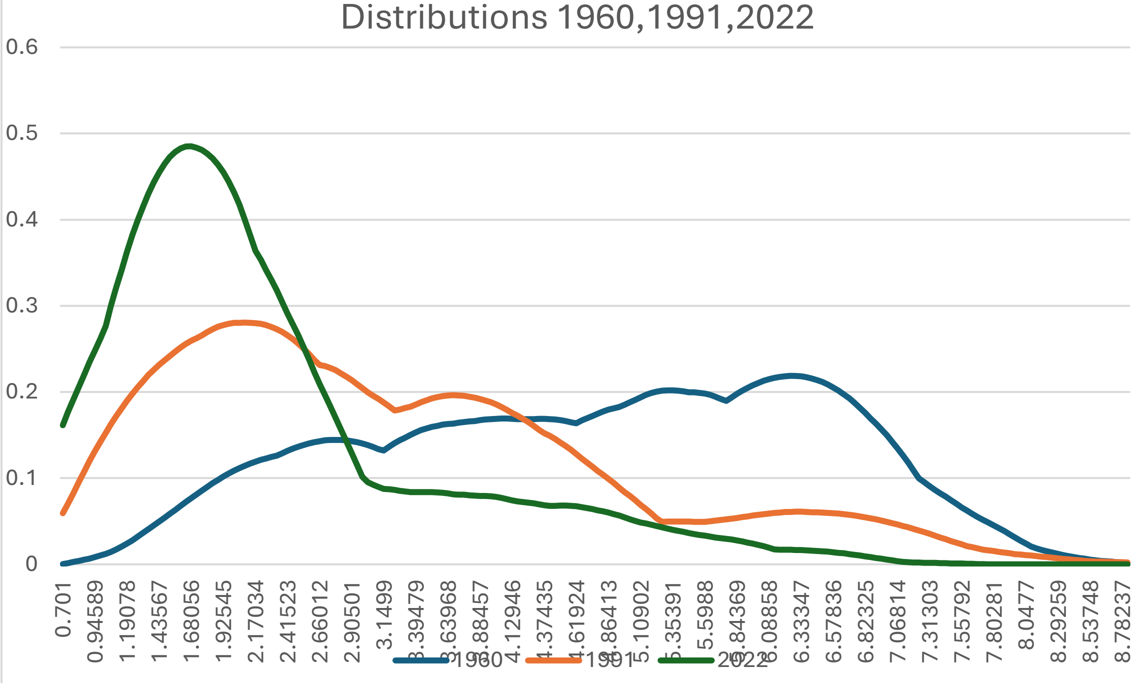
[7] There is a strong argument for using the modal rate since it is the most likely to be observed (the nation based modal rate was 3.012 times more likely to be observed than the mean rate in 1960 and 1.667 times more likely to be observed in 2022).

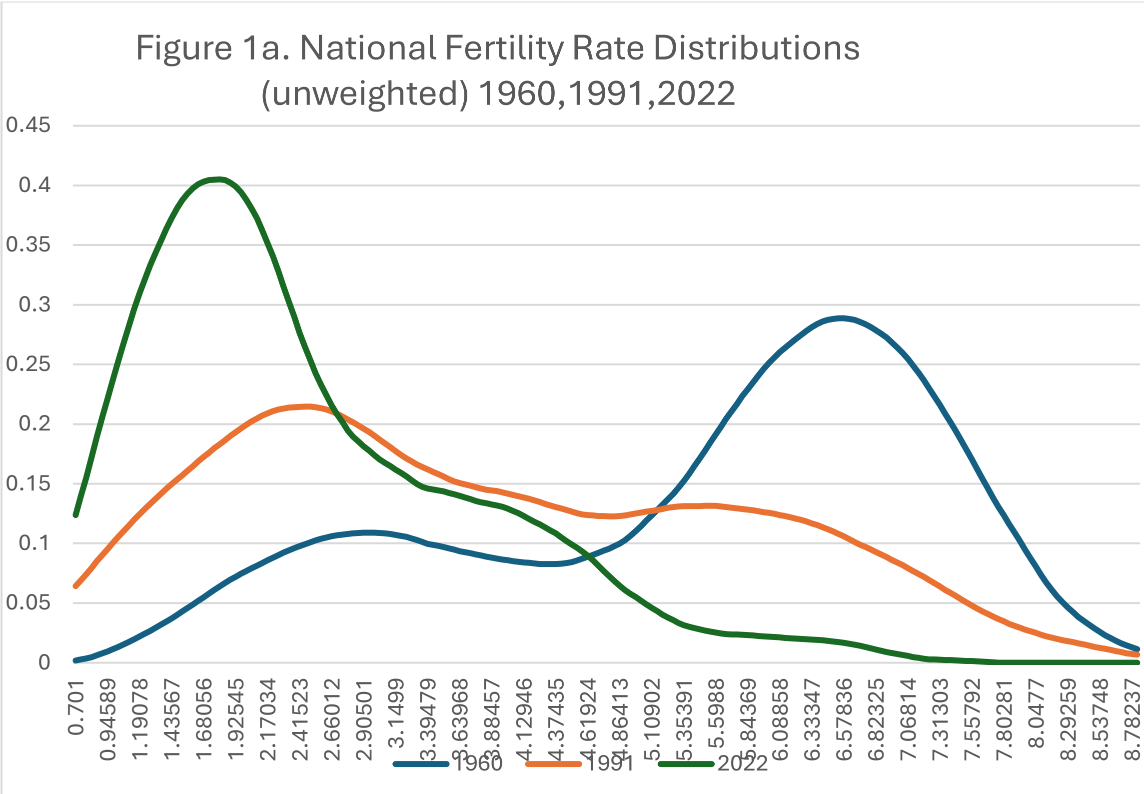
[8] The negative skewness factor was based upon $sf = -(P(p < Mean) - P(p < Mode))$. Skewness factors were all within the range of -0.3 to + 0.3 predominantly with absolute values around 0.25. The factors significance can be established by noting that under the hypothesis of no skewness its estimate is such that $n(\widehat{sf} - sf) \sim_a N(0, sf(1-sf))$ for a sample size n, which for a skewness factor of 0.25 yields a standard error of 0.0306.

Clearly fertility choices have undergone a sea-change over the period. While both population weighted and unweighted spreads characterized by the distributions standard deviation, have remained relatively constant, oscillating around 0.03, modal values diminished from an all-time high of 6.7 in the 1960's to a low of 1.6 in the 2020's effecting a reversal in the negative skewness factor of the distribution given by $(-\left(\int_0^{mean} f(p)dp - \int_0^{mode} f(p)\,dp\right))$. The prevalence of low fertility is reflected in the chance of observing low fertility ($\int_0^{2.1} f(p)dp$), i.e. a fertility rate < 2.1 which rose from 0.069 in 1960 to 0.553 in 2022. The sharp drop in the Most Likely fertility rates (both weighted and unweighted) in the late 1970's, which prompted the switch from negative skewness to positive skewness, coincided with the introduction of the one child policy in China during that period. Note that skewness reversal means that the modal rate will be higher than the average rate when the negative skew factor is positive, and the modal rate will be lower than the average rate when the negative skew factor is negative.

Perusal of Figures 1 and 1a clearly highlight the distributional changes that have taken place in what are fundamentally differing modality structures with a distinctly left (negatively) skewed multimodal 1960 distribution ending up as a right (positively) skewed unimodal distribution in 2022.

Figure 1. Population Weighted National Fertility Rate Distributions 1960,1991,2022





Based upon the most likely observed fertility rates in both weighted and unweighted versions of the distribution are in the region of 1.7-1.8 the most likely time to a sustainable population size would be in the region of 5 epochs, roughly 110 years given a life expectancy of 90 years.

The implications for achieving a sustainable world population and the challenges that will have to be confronted can be seen more clearly by picturing the time profiles of the average and most likely fertility rates of these distributions and the dependency ratios that they imply. These are illustrated in figures 2 and 2a where downward trends in both are apparent. Figure 3 pictures the progress of the proportion of the world's population or nations that have the low fertility rates required to achieve a sustainable population.

Most likely low fertility rates have clearly persisted in the modern era and do not appear to be a temporary phenomenon, the time profiles for the fertility rates are clearly trending in the right direction for all specifications of the fertility rate whether it be Most Likely or Expected and weighted or unweighted. Similarly, the proportion of the world's population or nations that have low fertility rates is consistently increasing. Simple time series regressions on a quadratic time trend will yield a sense of what the minimum fertility rate will be and how many years it will take to get there. A similar regression for the proportion of the population or nations with an appropriately low fertility rate, will reveal how long it will be before all the world's population or nations are in a low fertility state are reported in Tables 2 and 3.

Figure 2. World Fertility Rate Time Profiles

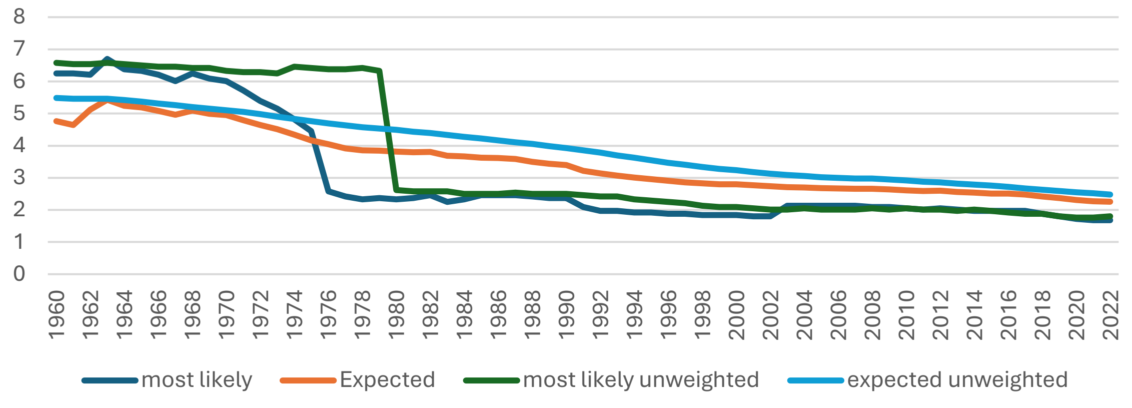


Figure 2a. World Dependency Ratio Time Profiles

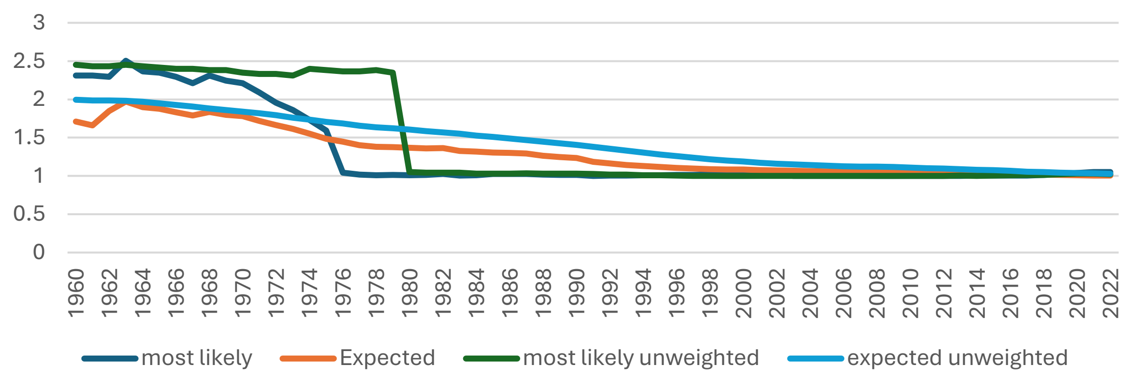


Figure 3. Proportion of the worlds populations and nations with low fertility rates.

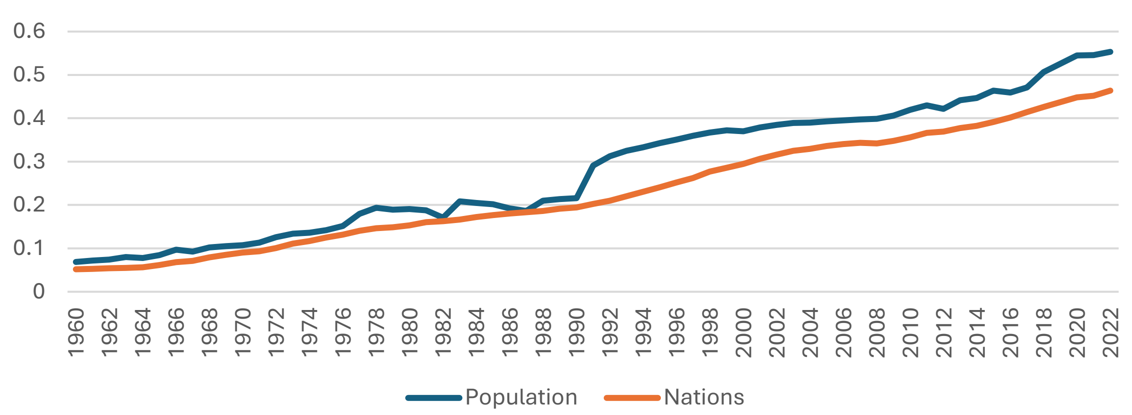


Table 2. ($y_t = \alpha + \beta_1 t + \beta_2 t^2 + \varepsilon_t$) where y_t is population proportion.

	Population Proportion			Nation Proportion		
	Coefficient	Std Err	P(T>|t|)	Coefficient	Std Err	P(T>|t|)
α	0.05102	0.00831	0.00000	0.04245	0.00317	0.00000
β_1	0.00637	0.00062	0.00000	0.00462	0.00024	0.00000
β_2	0.00003	0.00001	0.00563	0.00003	0.00000	0.00000
Standard Error	0.02268			0.00865		
R Squared	0.98587			0.99727		

*The time taken to secure all of the world's population or nations are in the low fertility state is the solution to $1 = \alpha + \beta_1 t + \beta_2 t^2$

If current trends continue, in about 100 years all the world's nations will be in a low fertility state and in about 70 years all the world's population will reside in nations with low fertility.

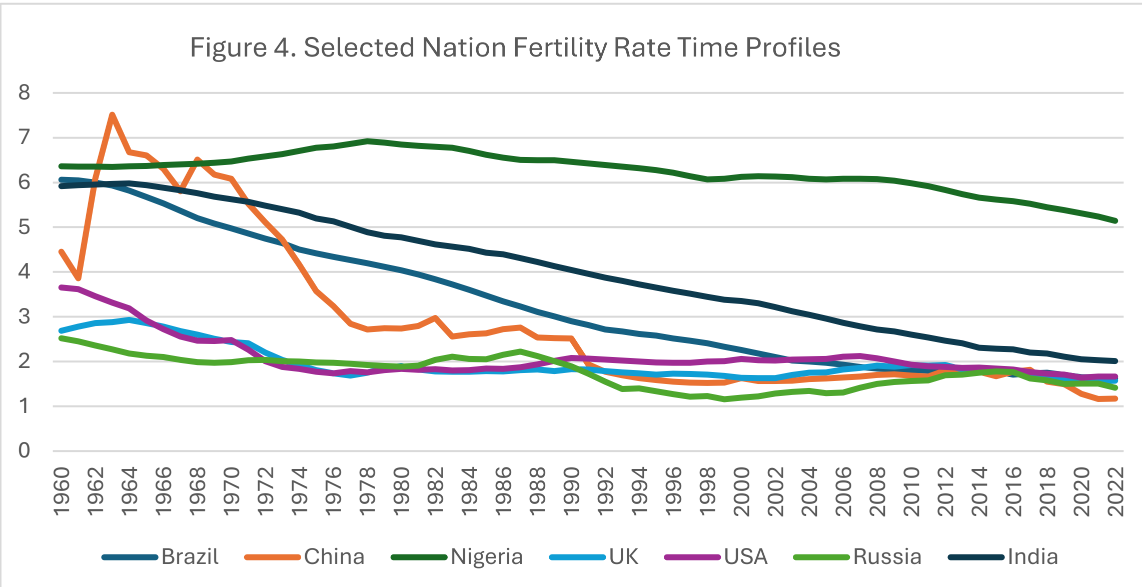
Table 3. ($y_t = \alpha + \beta_1 t + \beta_2 t^2 + \varepsilon_t$) where y_t is the fertility rate and t is time[9].

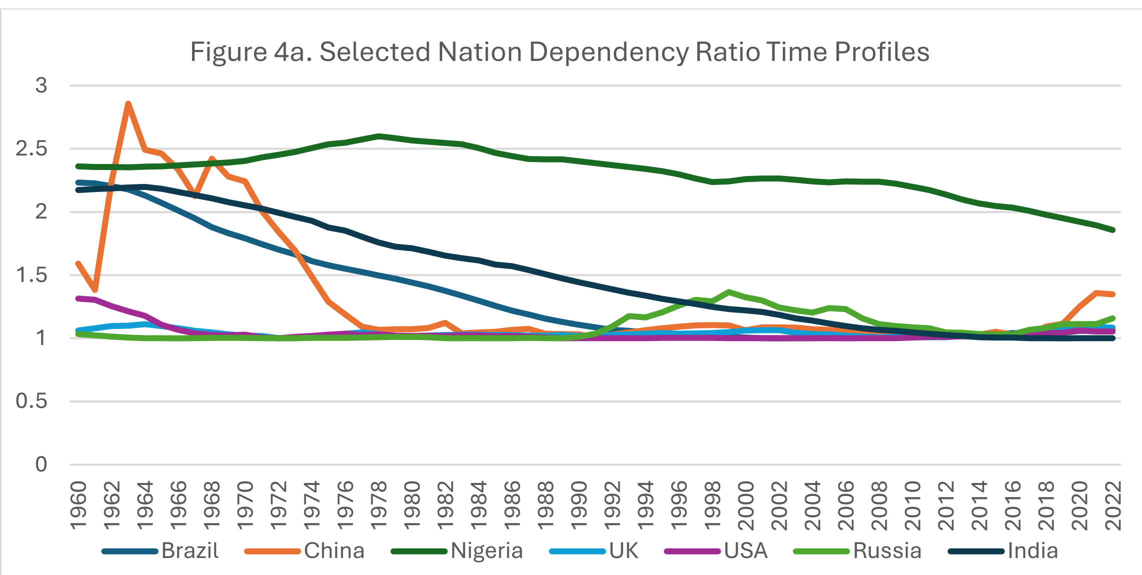
	Most Likely Fertility Rate (unweighted)			Expected Fertility Rate (unweighted)		
	Coefficient	Std Err	P(T>|t|)	Coefficient	Std Err	P(T>|t|)
α	7.81605	0.28367	0.00000	5.69941	0.22335	0.00000
β_1	-0.22809	0.02115	0.00000	-0.06865	0.01666	0.00006
β_2	0.00215	0.00033	0.00000	0.00026	0.00026	0.16473
Standard Error	0.77447			0.60978		
R Squared	0.90640			0.75752		
Minimum Value	1.76663			1.16785		
t to reach min	53 years			132 years		
t to reach 2	42 years			75 years		
	Most Likely Fertility Rate (weighted)			Expected Fertility Rate (weighted)		
	Coefficient	Std Err	P(T>|t|)	Coefficient	Std Err	P(T>|t|)
α	7.11464	0.20700	0.00000	5.42764	0.06406	0.00000
β_1	-0.24339	0.01544	0.00000	-0.08902	0.00478	0.00000
β_2	0.00270	0.00024	0.00000	0.00063	0.00007	0.00000
Standard Error	0.56514			0.17489		
R Squared	0.95657			0.98266		
Minimum Value	1.62959			2.28300		
t to reach min	45			71		
t to reach 2	33			Cannot be reached.		

With the exception of the weighted Expected Fertility Rate formulations these equations project a low fertility rate appropriate for achieving a sustainable population.

[9] All equations are concave functions of t with minima at time t^* where t^* is the solution to $\beta_1 + \beta_2 t^* = 0$ y, the value of the fertility rate at that point is given by: $y^* = \alpha + \beta_1 t^* + \beta_2 t^{*2}$ and the tears to reach a fertility rate of 2 is t^{**}, the solution to $0 = \alpha - 2 + \beta_1 t^{**} + \beta_2 t^{**2}$.

The foregoing treats the world as an entity or a representative nation Figure 4 reports a selection of nations (Brazil, China, Nigeria, UK USA, Russia and India) some of which, UK, USA and Russia, have experienced fertility rates less than 2.1 over the last 51 years (1972-2022). Interestingly enough, going back to 1960, all had fertility rates well above 2.1 (Brazil 6.061, China 4.451, Nigeria 6.364, United Kingdom 2.69, United States 3.643, Russia 2.62 and India 5.921). Chinas progress is noticeable[10] because of the One Child Policy.





[10] China announced a One Child Policy in 1978 which became effective in 1980 and brought the fertility rate down to less than 2.1 by the early 1990's recent attempts to relax the policy and raise the fertility rate have so far been unsuccessful

Figure 4a illustrates the corresponding dependency ratios where it may be observed that all but Nigeria and Russia have maintained low dependency ratios since the early 1990's though China's dependency ratio has taken an upswing in the 2020's largely due to its aging population.

3).Conclusions.

Halving the current world population to the level required for sustainability purposes requires lowering the fertility rate to a suitably low level for an extended period of time. This presents some challenges in the shorter run since it involves changing the age distribution in an economy, increasing its elderly portion of the population relative to its younger portion which in turn changes the configuration of the needs and the ability to service those needs in that economy's populace. To elucidate the problem an overlapping generations model was developed which allowed for four life stages: young, working and reproductive, working and not reproductive and elderly and not working. It facilitated an understanding of the influence on population growth and aging, the dependency ratio (the ratio of non-workers to workers) and the time it will take to reach a sustainable population of the fertility/replication rate, in the light of which current trends in world fertility rates were examined.

The distribution of fertility rates across nations has clearly undergone a sea change in the last 60 years from a heavily left skewed distribution to a heavily right skewed distribution, reflecting a solid downward trend in fertility. Based upon the latest distribution for 2022 the most likely national fertility rate is in the region of 1.7-1.8, making the time to achievement of a sustainable population for such a nation around 110 years. but not all nations are at the most likely fertility rate. Simple population share time-series regressions indicate that all of the world's population will be in a low fertility state in around 70 years. On the other hand, dependency ratios have not increased by much, largely because the fertility rates in aggregate have not gone too far below two. Finally, the progress of a selection of nations was studied progress toward a low fertility rate was found in all examples with some having a persistently low fertility rate for over 50 years.

The foregoing conclusions have been based upon a presumption of no migration since in the context of the world net migration will be zero. However, in a nation-based context that would certainly not be the case. immigration has been seen to be beneficial in redressing the imbalances brought about by low fertility in many nations (Dustmann and Frattini 2014, Preston 2014) with incoming younger groups filling the voids that emerge in an aging population and ameliorating the so-called fiscal challenge. However in the broader context this can only be a short-term solution which is only viable as long as there are nations whose fertility rates are greater than 2.1 and would not be an impediment to the ultimate objective of attaining a sustainable population.

Given the determinants of low fertility have little to do with the achievement of a Sustainable Population objective, the real challenge will arise when the Sustainable Population is arrived at and has to be maintained, since then the fertility / replication rate has to be increased to 2 after a long period of practice and custom of it being below 2. Recently some nations, notably China, Japan and Russia[11], have tried to increase their historically low fertility rates but to no avail. All of which leads to the conclusion that a sustainable world population appears to be achievable in the foreseeable future, unfortunately, the bad news is that there may be some issues with maintaining it.

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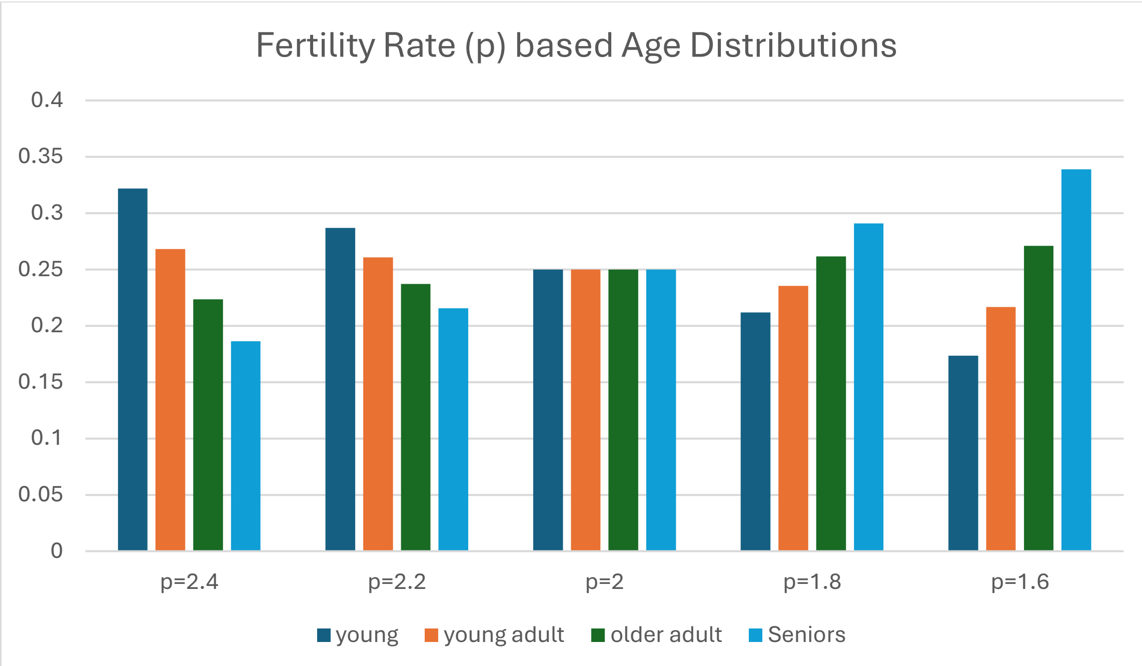
[11] China introduced a Three-Children Policy, alongside a package of financial and social benefits in 2021, Japan recently increased the child allowance and introduced financial aid for young couples and some regions of Russia have introduced monetary inducements of the order of US$1000 to young women who successfully carry a child to term, there is little evidence that these inducements have had an effect.

Appendix 1.



Appendix 2. Economic Growth with Fixed Technology and Capital Stock.

For comparison purposes economic growth is usually measured as the temporal rate of change in per capita output or per worker output (see for example Ashraf and Galor 2011). Epoch output Y_t is generated via a constant returns technology where L_t represents the epochs' working labor force ($L_t = \frac{(p^3+p^2)K_{O,t}}{(p^4+p^3+p^2+p)} = w_t K_{O,t}$) and X_t represents its stock of capital enhanced by technology level A. So output is given by $Y_t = (AX_t)^\alpha L_t{}^{1-\alpha} = (AX_t)^\alpha \left(w_t K_{O,t}\right)^{1-\alpha}$, output per capita[12] is given by $Y_t/K_{O,t} = \frac{(AX_t)^\alpha (w_t K_{O,t})^{1-\alpha}}{K_{O,t}} = (AX_t)^\alpha K_{O,t}{}^{-\alpha} w_t{}^{1-\alpha}$ and g_t, epoch growth given a constant capital stock and technology is given by:

$$g_{t+1} = \left(K_{O,t+1}{}^{-\alpha} w_{t+1}{}^{1-\alpha} - K_{O,t}{}^{-\alpha} w_t{}^{1-\alpha}\right)/K_{O,t}{}^{-\alpha} w_t{}^{1-\alpha}$$

When considering epoch $t+1$'s growth rate epoch t's per capita output is a give constant so g_{t+1} will be based upon changes in $K_{O,t+1}$ and w_{t+1} brought about by changes in p.

[12] Output per worker is given by $Y_t/wK_{O,t} = (AX_t)^\alpha \left(wK_{O,t}\right)^{-\alpha}$ which is somewhat larger than output per capita by a factor of $1/w$.

Note:

$\frac{\partial(Y_t/K_{O,t})}{\partial K_{O,t}} = -\alpha(AX_t)^{\alpha}K_{O,t}{}^{-(\alpha+1)}w_t{}^{1-\alpha} < 0 \; with \; \frac{\partial^2\left(\frac{Y_t}{K_{O,t}}\right)}{\partial K_{O,t}{}^2} > 0 \; \forall \; K_{O,t}$

So $\frac{Y_t}{K_{O,t}}$ is a decreasing convex function of $K_{O,t}$.

Note also:

$$\frac{\partial(Y_t/K_{O,t})}{\partial w_t} = (1-\alpha)(AX_t)^{\alpha}K_{O,t}{}^{-\alpha}w_t{}^{-\alpha} > 0 \; with \; \frac{\partial^2\left(\frac{Y_t}{K_{O,t}}\right)}{\partial w_t{}^2} < 0 \; \forall \; w_t$$

So $\frac{Y_t}{K_{O,t}}$ is a increasing concave function of w, furthermore:

$$\frac{\partial K_{O,t}}{\partial p} > 0$$

Since when p is below 1, increasing the fertility rate increases the proportion of the workforce in the population and when it is above 1 an increased fertility rate reduces that proportion i.e.:

$$\frac{\partial w}{\partial p} > 0 \; for \; p < 1 \; and < 0 \; for \; p > 1$$

$\frac{\partial(Y_t/K_{O,t})}{\partial p}dp$, the net effect of a change in the fertility rate on per capita income is then given by:

$$\left(\frac{\partial(Y_t/K_{O,t})}{\partial K_{O,t}}\frac{\partial K_{O,t}}{\partial p} + \frac{\partial(Y_t/K_{O,t})}{\partial w_t}\frac{\partial w_t}{\partial p}\right)dp$$

which is the sum of two terms, the former of which is always negative and the latter of which is negative when $p > 1$ and positive otherwise.

Assertion 1. When $p > 1$ increasing the fertility rate reduces per capita income while reducing the fertility rate increases per capita income.

On the other hand, when $p < 1$ it will depend upon which of $\left(\frac{\partial(Y_t/K_{O,t})}{\partial K_{O,t}}\frac{\partial K_{O,t}}{\partial p}\right)$ and $\left(\frac{\partial(Y_t/K_{O,t})}{\partial w_t}\frac{\partial w_t}{\partial p}\right)$ is larger in absolute value. If $\left|\frac{\partial(Y_t/K_{O,t})}{\partial K_{O,t}}\frac{\partial K_{O,t}}{\partial p}\right| < \left|\frac{\partial(Y_t/K_{O,t})}{\partial w_t}\frac{\partial w_t}{\partial p}\right|$, assertion 1 no longer holds and increasing the fertility rate increases per capita income and reducing the fertility rate reduces per capita income.

Appendix 3. Data

Year	Population Weighted				Unweighted				weighted
	Mode	Mean	Negative Skew Factor	Std Err	Mode	Mean	Negatve Skew factor	Std Err	Low FR prob
1960	6.25184	4.76644	0.29883	0.03205	6.57836	4.76644	0.27134	0.03113	0.06878
1961	6.25184	4.63864	0.26098	0.03075	6.53755	4.63864	0.26206	0.03079	0.07161
1962	6.21103	5.12322	0.29473	0.03192	6.53755	5.12322	0.26036	0.03072	0.07425
1963	6.70081	5.42957	0.26927	0.03106	6.57836	5.42957	0.26658	0.03096	0.08004
1964	6.37429	5.24129	0.27989	0.03143	6.53755	5.24129	0.26033	0.03072	0.07750
1965	6.33347	5.19293	0.28121	0.03148	6.49673	5.19293	0.25235	0.03041	0.08470
1966	6.21103	5.07954	0.29068	0.03179	6.45592	5.07954	0.24446	0.03009	0.09678
1967	6.00695	4.96753	0.28687	0.03167	6.45592	4.96753	0.25177	0.03039	0.09229
1968	6.25184	5.08757	0.28700	0.03167	6.41510	5.08757	0.24196	0.02998	0.10257
1969	6.08858	4.98976	0.28438	0.03158	6.41510	4.98976	0.24566	0.03014	0.10494
1970	6.00695	4.94897	0.27683	0.03133	6.33347	4.94897	0.23592	0.02973	0.10748
1971	5.72125	4.79324	0.25718	0.03060	6.29266	4.79324	0.22930	0.02943	0.11292
1972	5.39473	4.64778	0.20718	0.02838	6.29266	4.64778	0.23420	0.02965	0.12605
1973	5.14984	4.51436	0.17067	0.02634	6.25184	4.51436	0.23006	0.02947	0.13387
1974	4.82331	4.34885	0.11697	0.02250	6.45592	4.34885	0.26595	0.03093	0.13628
1975	4.45598	4.16026	0.06483	0.01724	6.41510	4.16026	0.26015	0.03072	0.14233
1976	2.57849	4.04799	-0.26588	0.03093	6.37429	4.04799	0.25599	0.03056	0.15142
1977	2.41523	3.92050	-0.26787	0.03101	6.37429	3.92050	0.25583	0.03055	0.17977
1978	2.33360	3.85566	-0.27765	0.03135	6.41510	3.85566	0.26770	0.03100	0.19399
1979	2.37441	3.84228	-0.26923	0.03106	6.33347	3.84228	0.25741	0.03061	0.18958
1980	2.33360	3.81920	-0.28296	0.03154	2.61930	3.81920	-0.25023	0.03033	0.19050
1981	2.37441	3.79451	-0.27410	0.03123	2.57849	3.79451	-0.25305	0.03044	0.18809
1982	2.45604	3.80939	-0.28081	0.03146	2.57849	3.80939	-0.25305	0.03044	0.17103
1983	2.25197	3.68660	-0.29425	0.03191	2.57849	3.68660	-0.25201	0.03040	0.20882
1984	2.33360	3.66400	-0.27636	0.03131	2.49686	3.66400	-0.25887	0.03067	0.20495
1985	2.45604	3.62873	-0.25131	0.03037	2.49686	3.62873	-0.25883	0.03067	0.20213
1986	2.45604	3.61489	-0.26522	0.03091	2.49686	3.61489	-0.25403	0.03048	0.19189
1987	2.45604	3.58303	-0.26983	0.03108	2.53767	3.58303	-0.24750	0.03022	0.18667
1988	2.41523	3.49248	-0.26103	0.03075	2.49686	3.49248	-0.25796	0.03063	0.20970
1989	2.37441	3.44106	-0.27137	0.03113	2.49686	3.44106	-0.25250	0.03042	0.21357
1990	2.37441	3.39491	-0.27178	0.03115	2.49686	3.39491	-0.25458	0.03050	0.21591
1991	2.08871	3.22096	-0.26590	0.03093	2.45604	3.22096	-0.25425	0.03049	0.29147
1992	1.96627	3.13437	-0.27576	0.03129	2.41523	3.13437	-0.25559	0.03054	0.31277
1993	1.96627	3.06491	-0.25802	0.03063	2.41523	3.06491	-0.24524	0.03012	0.32477
1994	1.92545	3.00993	-0.26532	0.03091	2.33360	3.00993	-0.25414	0.03048	0.33324
1995	1.92545	2.95464	-0.26112	0.03075	2.29278	2.95464	-0.25355	0.03046	0.34243
1996	1.88464	2.90614	-0.27009	0.03109	2.25197	2.90614	-0.25345	0.03046	0.35122
1997	1.88464	2.86319	-0.25639	0.03057	2.21116	2.86319	-0.26043	0.03073	0.35995
1998	1.84382	2.82711	-0.27571	0.03129	2.12953	2.82711	-0.26707	0.03098	0.36718
1999	1.84382	2.79721	-0.27286	0.03119	2.08871	2.79721	-0.27498	0.03126	0.37233
2000	1.84382	2.80225	-0.28864	0.03173	2.08871	2.80225	-0.27380	0.03122	0.37033
2001	1.80301	2.76467	-0.28940	0.03175	2.04790	2.76467	-0.26958	0.03107	0.37893
2002	1.80301	2.73636	-0.28815	0.03171	2.00708	2.73636	-0.27787	0.03136	0.38461
2003	2.12953	2.71107	-0.18641	0.02727	2.00708	2.71107	-0.27541	0.03128	0.38942
2004	2.12953	2.69961	-0.18397	0.02713	2.04790	2.69961	-0.26083	0.03074	0.38968
2005	2.12953	2.67867	-0.19121	0.02753	2.00708	2.67867	-0.26969	0.03107	0.39310
2006	2.12953	2.66786	-0.19954	0.02798	2.00708	2.66786	-0.27391	0.03122	0.39521
2007	2.12953	2.66282	-0.20689	0.02836	2.00708	2.66282	-0.27041	0.03110	0.39756
2008	2.08871	2.65780	-0.21938	0.02897	2.04790	2.65780	-0.26496	0.03090	0.39914
2009	2.08871	2.64020	-0.22270	0.02913	2.00708	2.64020	-0.28183	0.03150	0.40614
2010	2.04790	2.61239	-0.22739	0.02935	2.04790	2.61239	-0.25817	0.03064	0.41985
2011	2.00708	2.58766	-0.24753	0.03022	2.00708	2.58766	-0.26218	0.03079	0.42982
2012	2.04790	2.59663	-0.24274	0.03002	2.00708	2.59663	-0.25518	0.03052	0.42185
2013	2.00708	2.55726	-0.24192	0.02998	1.96627	2.55726	-0.25861	0.03066	0.44151
2014	1.96627	2.54333	-0.27218	0.03116	2.00708	2.54333	-0.24686	0.03019	0.44699
2015	1.96627	2.51018	-0.24999	0.03032	1.96627	2.51018	-0.25271	0.03043	0.46405
2016	1.96627	2.50872	-0.26071	0.03074	1.92545	2.50872	-0.25928	0.03068	0.45954
2017	1.96627	2.48248	-0.24557	0.03014	1.88464	2.48248	-0.26225	0.03080	0.47123
2018	1.88464	2.41470	-0.23463	0.02967	1.88464	2.41470	-0.24818	0.03024	0.50657
2019	1.80301	2.37256	-0.25422	0.03049	1.80301	2.37256	-0.26796	0.03101	0.52591
2020	1.72138	2.30506	-0.25970	0.03070	1.76219	2.30506	-0.27193	0.03115	0.54497
2021	1.68056	2.27309	-0.25630	0.03057	1.76219	2.27309	-0.26611	0.03094	0.54602
2022	1.68056	2.25586	-0.25650	0.03058	1.80301	2.25586	-0.23625	0.02974	0.55324

Modes were calculated using an Epanechnikov kernel estimate (Silverman 1986) of the probability density function and finding its maximum value. Skew factors were based upon the probability

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