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Financing Innovation with Innovation

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Abstract

This paper documents that firms are increasingly financing innovation using their stock of innovation, measured as patents. We refer to this behavior as *financing innovation with innovation*. Drawing on patent collateral data from both the US and China, we first show that (1) in both countries, the total number and share of patents pledged as collateral have been rising steadily, (2) Chinese firms employ patents as collateral on a smaller scale and with a lower intensity than US firms, (3) firms increase their borrowing and innovation after they start to use patent collateral. We then construct a heterogeneous firm general equilibrium model featuring idiosyncratic productivity risk, innovation capital investment, and borrowing constrained by patent collateral. The model emphasizes two barriers that hinder the use of patent collateral: high inspection costs and low liquidation values of patent assets. We parameterize the model to firm-level panel data in the US and China and find that both barriers are significantly more severe in China than in the US. Finally, counterfactual analyses show that the gains in innovation, output, and welfare from reducing the inspection costs in China to the US level are substantial, moreso than enhancing the liquidation value of patent assets.

Keywords: Patent collateral; innovation investment; financial barriers; firm dynamics; *JEL Classification*: E22, G32, O31, O33

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1 Introduction

Financing innovation is especially challenging since the stock of innovation (patents) can hardly be used as collateral. Historically, financial institutions have not accepted patents as collateral because their value cannot be easily assessed (Hall and Lerner, 2010). However, modern firms are increasingly pledging patents as collateral to obtain debt financing, and this new pledgeability of patents has contributed to the financing of innovation (Mann, 2018). We refer to the phenomenon of obtaining external debt financing using patent collateral as "*financing innovation with innovation*". In this paper, we document novel cross-country stylized facts of this phenomenon and provide a quantitative assessment of the underlying barriers, mechanisms, and welfare implications.

The penetration of patents as collateral differs across countries. Drawing on patent collateral data from the world's two largest economies, the US and China, we document three stylized cross-country facts. First, both the number and share of patents pledged as collateral have been rising steadily in both countries. Second, Chinese firms employ patents as collateral on a smaller scale and with a lower intensity than US firms. In the US, patents have been used as collateral to support external financing since 1980 (since 2003 in China), with the patent pledge ratio exceeding 15% in 2014 (2% in China). Since the stock of patents has grown substantially over time, the growth in pledged shares represents a massive increase in the number of patents pledged as collateral. Finally, firm-level regressions show that firms increase their borrowing and innovation after they start to pledge patents as collateral.

To rationalize these three stylized facts and shed light on the underlying barriers, mechanisms, and welfare implications surrounding patent pledging, we develop a heterogeneous firm general equilibrium model incorporating idiosyncratic productivity shocks, innovation capital, and collateral constraints. The key novel feature of our model is that firms can borrow against innovation capital up to a certain ratio that equals the liquidation value of patents after paying a fixed inspection cost. These two barriers reflect the quality of financial institutions: (1) the inspection technology used to evaluate patents and (2) the ease of liquidating patents in the technological market. These two barriers jointly govern the prevalence of using patents as collateral.

We use the share of pledged patents and the participation rate of firms pledging to capture these two barriers. For institutions to support patent-backed loans, market participants need to be able to accurately assess the market value of patents and identify pledgeable patents (Kamiyama, Sheehan, and Martinez, 2006). When issuing loans based on patent collateral, a lender would consider the fixed inspection cost and the liquidation value of the patent on the resale market.¹

¹The ease of reselling patents after default depends on the transactional barriers in the technological market

The ease of doing so determines the equilibrium share of pledged patents and the equilibrium pledging participation rate in the economy. By comparing the model to the data using listed firms from the US and China, we find that both barriers are much more severe in China than in the US, with magnitudes of about three to hundreds of times.

We then demonstrate the roles of both barriers in shaping the prevalence of pledging patents as collateral (i.e., the share of pledged patents and the participation rate of firms) and characterize the resulting dynamics using firm-level data from both the US and China. Both barriers play essential roles in understanding *financing innovation with innovation*. Matching the dynamics of targeted moments shows that the inspection cost has been substantially reduced in China (though it remains much higher than in the US) over the last decade, but the liquidation value is roughly stable over time. As an external validation, we replicate our empirical results that firms increase their borrowing and innovation activities once they start to pledge patents as collateral using the same regressions with model-simulated firm-level data.

With such severe barriers, there is ample room for China (and other countries where patent pledging is rare) to stimulate innovation and economic development. To estimate the benefits of reducing barriers to using patents as collateral, we conduct several counterfactual studies in which China reduces its barriers to the US level. First, reducing inspection costs or increasing patent liquidation values generates more innovation, more output, and welfare gains. Second, reducing the inspection cost generates larger improvements. Third, if both barriers in China are reduced to the US level, we estimate China would create 9% more innovation capital and increase its social welfare by 1.3%.

Literature Review To the best of our knowledge, this paper provides the first cross-country quantitative study on the implications of patent collateral. Although the practice of using patent collateral to obtain debt financing in the US is well documented (see Amable, Chatelain, and Ralf (2010); Loumioti (2012); Hochberg, Serrano, and Ziedonis (2018); Mann (2018); Akcigit et al. (2014), among others), little is known about patent collateral in other countries. This limits our understanding of policy implications for countries with less developed financial markets. The stylized cross-country facts we document for both the US and China provide a global perspective of the increasing trend and future potential of promoting patent collateral. Our study implies that countries, such as China, that manage to improve the pledgeability of patents may enjoy substantial advancements in innovation and welfare.

⁽Akcigit, Celik, and Greenwood, 2016). The scale of patent transactions also differs greatly between the US and China. Zhang (2021) documents that, between 1998 and 2013, the percentage of all granted patents that were assigned were approximately 15% in the US but only 4.4% in China. This reflects higher barriers in the Chinese technology market than in the US.

This study also contributes to a vast literature of quantitative studies on the impact of financial development on innovation investment and welfare. Most of these frameworks do not tackle the role of patent collateral in debt financing (see Aghion et al. (2012); Midrigan and Xu (2014); Vereshchagina (2018); Caggese (2019); Altomonte et al. (2021); Chen (2022), among others). Further, the common stylized theoretical model, which features patent collateral entering the firm's borrowing constraint, does not match the actual data on patent collateral (see Amable, Chatelain, and Ralf (2010) for example) and correspondingly cannot provide realistic welfare and policy implications of improving the pledgeability of patents. We provide an initial step in filling this gap by constructing and calibrating a heterogeneous firm model in which patents can be pledged as collateral to obtain debt financing. By doing so, we quantify the impact of using patent collateral on innovation and welfare, which has strong policy implications for knowledge economies in years ahead.

Lastly, we contribute to a broader literature exploring the relationship between financial markets and innovation investment. Financial constraints have been found to have negative impacts on innovation activities (see Rajan and Zingales (1998); Cornaggia et al. (2015); Varela (2018); Duval, Hong, and Timmer (2020), among others)². We add to this literature by emphasizing the role of patent collateral in facilitating the external financing of innovative firms in the presence of financial barriers. Specifically, we first document the growing trend of using patent collateral in both the US and China, and then quantify the welfare implications of improving patent pledgeability in China to the US level.

Roadmap The remainder of this paper is organized as follows. In Section 2, we present stylized facts on the practice of using patent collateral in the US and China. In Section 3, we present the model. Section 4 provides the quantitative analysis, which includes model calibration and counterfactual analysis. We conclude the paper in Section 5.

2 Stylized Facts on Patent Pledging in the US and China

2.1 Institutions for Pledging Patents as Collateral

Institutions for pledging patents as collateral are similar in the US and China. Figure 1(a) shows a flow chart of the process of obtaining external funds by employing patents as collateral. There are three main participants: patent owners, valuation agents, and lenders. Financially constrained patent owners who wish to pledge their patents as collateral need first to obtain an evaluation re-

²See Kerr and Nanda (2015) for an earlier review of relevant empirical evidence.



Figure 1: Stylized Facts on Patent Pledges in the US and China

(d) Participation Rate (Firm-level)

(e) Share of Pledged Patents (Firm-level)

Notes: Plot (a) shows a flow chart characterizing the process of how firms obtain patent-backed loans. Plot (b) shows the total number of pledged utility patents in the US and pledged all types of patents (invention, utility model, and designs) in China. In China, the total number of pledged patents in China was zero before 2003. Plot (c) shows the share of pledged patents by granting year. We use invention patents to calculate the share of pledged patents in China and utility patents in the US. For each granting vintage, we calculate the share of pledged patents as the number of pledged patents within five years of granting to the total number of patents in the granting vintage. Plot (d) presents the firm participation rate in using patent collateral for both the US and China. The participation rate is calculated as the number of firms that use patent collateral divided by the total number of firms. The left vertical axis of the plot (d) is for the US, while the right vertical axis is for China. In plot (e), we show the patent pledging ratio using firm-level data. The share of pledged patents is the number of active patents.

port from the valuation agents.³ Lenders rely on the patent valuation report to decide whether to accept the patent as collateral. If the lender decides to take the patent as collateral, a security interest agreement is drafted and signed as an enforceable legal claim on the pledged patent, which gives the lender the right to repossess the patent rights if the borrower defaults.⁴ After receiving the loan, the borrower could use it for purposes of production, marketing, and innovation.

2.2 Aggregate-level Stylized Facts

Data Sources We first obtain data on patent collateral in the United States from the Patent Assignment Dataset. The Patent Assignment Dataset contains information on patent transactions recorded by the United States Patent and Trademark Office (USPTO), including security interest agreements that reflect patents being pledged as collateral from 1970 to 2019. The data for all patent applications and granted patents are from the Historical Patent Data Files. This contains annual counts of patent applications, patent grants, and patents in force from 1840 to 2014. The term "patents" throughout refers to regular utility patents, which exclude applications and patents for designs, statutory invention registrations, plants, reissues, and defensive publications. We merge the two databases using the WRDS US Patents Compustat Link.

We then obtain the data on Chinese patent collateral from the China National Intellectual Property Administration (CNIPA). CNIPA records the patent identification number, the pledgor and pledgee, the application date and authorization date, and the length of the pledge for each patent used as collateral. We focus on invention patents to make them comparable to the US patent collateral data.⁵ We then obtain annual counts of applications and granted patents from China's Statistical Yearbook. The use of patents as collateral was not possible in China before 2003. To facilitate the comparison between China and the US, we draw on these records between 2003 and 2014. Finally, for detailed firm-level information, we obtain variables from CSMAR (China's version of Compustat) and merge them with the patent data. See details in Appendix A.

The Number of Pledged Patents Figure 1(b) shows the total number of pledged patents between 1980 and 2019. Using patents as collateral to borrow has been a practice in the US since 1980 and has been growing steadily over the past three decades. However, it is relatively novel for Chinese firms to borrow against patents. The earliest record of patent pledges in China was

³The patent valuation services range from basic valuation to comprehensive valuation tailored to the product's market valuation. See *Transactions IP* (https://transactionsip.com/patent-valuation-services) and *Ji Hui* (http://zcpg.bjjihui.com/a/pxkc/lm1/1.html) for examples from the US and China, respectively.

⁴Recently, Ma, Tong, and Wang (2021) document that secured creditors exercise their control rights on collateralized patents when the innovative debtor firm goes bankrupt.

⁵In Appendix A.1.3, we show patent pledge ratios for different types of patents (invention, utility, and design) in China, verifying that the invention patent is the most frequently used in patent collateral.

in 2003, but the number of pledged Chinese patents has been growing exponentially since 2010. This rapid growth suggests a significant unrealized potential for firms to use patent collateral to support innovation investment.

The share of pledged patents Figure 1(c) shows the share of pledged patents (frequencyadjusted) in the aggregate patent vintage data in both the US and China from 1980 to 2014. Following Mann (2018), we normalize the number of pledged patents using the total number of patents by year of granting. For each granting vintage, we calculate the share of pledged patents as the number of pledged patents within five years of granting to the total number of patents in the granting vintage.⁶ The figure shows that US inventors pledge patents as collateral much more often than Chinese inventors. In the US, the share of pledged patents rose from below 1% to over 16% from 1980 to 2014, implying an increased propensity to use patent pledges to obtain external funding. In sharp contrast, in China, the pledge ratio climbed from close to zero in 2002 to only around 2% in 2014. The relative lack of patent pledging in China likely indicates significant financial market barriers, the relaxing of which could spur substantial innovation investment and yield national welfare gains. We estimate these gains using a quantitative model in Section 4.

2.3 Firm-level Evidence

To investigate the effect of using patent collateral on firm behavior, we employ firm-level data to investigate both the extensive margin (participation rate) and the intensive margin (share of pledged patents), as well as whether patent collateral increases firm borrowing or innovation investment. The firm-level data also allows us to extend the time series to 2019.

The Extensive and Intensive Margins of Patent Collateral We show the participation rate in Figure 1(d) and the share of pledged patents in Figure 1(e) using our firm-level data to measure the use of patent collateral at both the extensive and intensive margins. The participation rate is calculated as the percentage of firms using patent collateral, which potentially captures participation barriers such as the inspection cost of the patent evaluation. The share of pledged patents is calculated as the number of newly pledged patents over the total number of active patents.

Both margins matter for using patents as collateral. At the extensive margin, more and more firms are using patents as collateral in both the US and China. In particular, Chinese firms employ patent collateral at a much smaller but faster-growing intensity. The participation rate in patent pledging was 0.1% in 2009 but rose to above 1.7% in 2019. At the intensive margin, between 2011

⁶The aggregate measures are truncated in 2014 due to the measure of patent vintages. The calculation methods for both the frequency-adjusted and non-frequency-adjusted ratios, along with additional figures presenting results for different periods, can be found in Appendix Appendix A.1.

and 2019, the share of pledged patents for US firms was between 11% and 16%, while this ratio was below 1.2% in China. Recently, a smaller share of US patents has been pledged, possibly due to the substantial increase in the total number of active patents held by US firms.⁷ In China, despite strong growth in patenting, the share of patents pledged as collateral is still rising.

Responses of Leverage and R&D to Patent Collateral Finally, we examine the association between debt borrowing, R&D investment, and the use of patent collateral. We run the following two-way fixed effects specification:

$$Y_{it} = \alpha + \beta P C_{it} + \gamma \mathbf{Z}'_{it} + \lambda_i + \lambda_t + \xi_{it}, \qquad (1)$$

where Y_{it} is the outcome variable and PC_{it} is an indicator of using the patent as collateral. We set PC_{it} to be one starting from the time the firm first uses pledge patents as collateral and afterward. Otherwise, PC_{it} is equal to zero if the firm has never yet used patent collateral. We use the firm's leverage ratio (total debt/total assets) to measure the firm's borrowing responses to patent collateral. To examine the innovation response, we use the firm's R&D expenditures. Z_{it} is a vector of control variables. Our control variables include firms' ROE, ROA, Tobin's q, and total assets. These variables are standard in the macro-finance and corporate finance literature. To account for unobserved firm-level factors that may lead to endogeneity issues, we control for the firm-level fixed effects λ_i . λ_t contains several dummies that account for the influence of macro factors such as monetary and fiscal policy adjustments. The error term is ξ_{it} . The parameter of interest is β . The estimates of β capture the impact of employing patent collateral on the outcomes. We run the regression using the US data and Chinese data separately.

Table 1(a) reports the estimation results for the regressions based on equation (1) using US firm-level data. US firms pledging patents as collateral increase their leverage by 0.3%-0.8% and their R&D expenditures by 2.5%-6.8%, depending on the empirical specifications. Table 1(b) reports the estimation results for regressions based on equation (1) using Chinese firm-level data. Firms that employ patents as collateral increase their leverage by 2%-3.1%. We also find growth in R&D expenditures (9.8%-16.8%), suggesting that firms use patent collateral to finance their innovation.

2.4 Remarks on the Stylized Facts

In this section, we have documented three stylized facts. First, firms are increasingly using patent collateral to borrow against their stock of innovation, measured as patents. Second, US patent

⁷This decreasing trend disappears when we plot the share of pledged patents by granting year as in Figure 1(c).

Panel (a) US Data								
		lev	erage			log((R&D)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PC	0.008***	0.006***	0.003**	0.003**	0.068***	0.028***	0.025***	0.025***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.008)	(0.007)	(0.008)	(0.008)
L.log(asset)		0.033***	0.046***	0.046***		0.575***	0.589***	0.602***
		(0.001)	(0.001)	(0.001)		(0.005)	(0.007)	(0.008)
L.Tobin's Q			0.526***	0.521***			0.028	-0.013
			(0.003)	(0.003)			(0.021)	(0.021)
L.ROE			-0.052***				-0.211***	
			(0.003)				(0.025)	
L.ROA				-0.105***				-0.843***
				(0.010)				(0.070)
N	102093	92128	48821	48822	46953	41648	20678	20679
adj. R^2	0.754	0.778	0.890	0.889	0.944	0.960	0.970	0.970
			Panel (b) Chinese	Data			
		lev	erage		log(R&D)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PC	0.031***	0.022**	0.021**	0.020**	0.168***	0.105**	0.100*	0.098*
	(0.010)	(0.009)	(0.009)	(0.009)	(0.056)	(0.053)	(0.053)	(0.053)
L.log(asset)		0.052***	0.072^{***}	0.072^{***}		0.610***	0.633***	0.634***
		(0.003)	(0.003)	(0.003)		(0.022)	(0.023)	(0.023)
L.Tobin's Q			0.016***	0.018***			0.044***	0.042***
			(0.001)	(0.001)			(0.009)	(0.009)
L.ROE			-0.237***				1.017^{***}	
			(0.017)				(0.129)	
L.ROA				-0.777***				2.605***
				(0.030)				(0.222)
N	24000	20971	20325	20327	21901	19204	18651	18653
adj. R^2	0.725	0.752	0.763	0.774	0.808	0.840	0.844	0.845

Table 1: Responses of Leverage and R&D to Patent Collateral

Notes: All regressions include firm- and year-fixed effects. Standard errors are in parentheses. * p < 10%, ** p < 5%, *** p < 1%. The regression specification is $Y_{it} = \alpha + \beta PC_{it} + \gamma Z'_{it} + \lambda_i + \lambda_t + \xi_{it}$ where Y_{it} is the outcome variable and PC_{it} is an indicator of using the patent as collateral. We set PC_{it} to be one starting from the time the firm first uses pledge patents as collateral and afterward. Otherwise, PC_{it} is equal to zero if the firm has never yet used patent collateral. We use the firm's leverage ratio (total debt/total assets) to measure the firm's borrowing responses to patent collateral. To examine the innovation response, we use the firm's R&D expenditures. Z_{it} is a vector of control variables. Our control variables include firms' ROE, ROA, Tobin's q, and total assets. These variables are standard in the macro-finance and corporate finance literature. To account for unobserved firm-level factors that may lead to endogeneity issues, we control for the firm-level fixed effects λ_i . λ_t contains several dummies that account for the influence of macro factors such as monetary and fiscal policy adjustments. The error term is ξ_{it} . The parameter of interest is β . The estimates of β capture the impact of employing patent collateral on the outcomes. We run these regressions using the US data and Chinese data separately.

holders employ patent collateral on a greater scale and with a higher intensity than Chinese patent owners. Finally, firms increase their borrowing and innovation once they begin to use patent collateral. Below, we construct a quantitative model to rationalize these three stylized facts, shed light on the underlying barriers moderating the use of patent collateral, and evaluate the innovation and welfare implications of relaxing these financial frictions.

3 The Model

We consider an economy with heterogeneous firms which undertake innovation investment subject to financial constraints. Time *t* is discrete and infinite, t = 1, 2, ..., N Each innovative firm i = 1, ..., N is subject to idiosyncratic productivity shocks.

3.1 Innovative Firms

Each innovative firm *i* produces with productivity that consists of an idiosyncratic stochastic component z_{it} , and an accumulated stock of innovation capital a_{it} , measured as patents, capital k_{it} , and labor l_{it} using the following production function:

$$y_{it} = (z_{it}a_{it}^{\gamma})k_{it}^{\alpha}l_{it}^{\nu}, \quad \gamma + \alpha + \nu < 1$$

where z_{it} is the stochastic idiosyncratic component of productivity for firm *i*, which follows an exogenous Markov process $\log(z_{it}) = \rho_z \log(z_{it-1}) + \sigma_z \varepsilon_{it}$, where ε_{it} follows a standard normal random process. a_{it} is the endogenous component of productivity. γ , α and ν are the income shares of innovation capital, physical capital and labor, respectively. We require that $\gamma + \alpha + \nu < 1$ so that the production technology features decreasing returns to scale.

Firms rent physical capital and labor from the market with market prices r_t^k and w_t . Their only intertemporal investment is innovation investment. We can directly calculate firm profits after paying wages and capital rentals $\{y_{it} - w_t l_{it} - (r_t^k)k_{it}\}$. The optimal choices of labor and capital are given by: $l_{it}^* = \left[\left(\frac{v}{w_t}\right)^{1-\alpha} \left(\frac{\alpha}{r_t^k}\right)^{\alpha} z_{it} a_{it}^{\gamma}\right]^{\frac{1}{1-\alpha-\nu}}$ and $k_{it}^* = \left[\left(\frac{v}{w_{it}}\right)^{\nu} \left(\frac{\alpha}{r_t^k}\right)^{1-\nu} z_{it} a_{it}^{\gamma}\right]^{\frac{1}{1-\alpha-\nu}}$. Thus, the firm's production revenue after paying wages and capital rentals is

$$f(z_{it}, a_{it}) = \max_{k,l} \left\{ y_{it} - w_t l_{it} - r_t^k k_{it} \right\} = \left(\frac{\nu}{w_t}\right)^{\frac{\nu}{1-\alpha-\nu}} \left(\frac{\alpha}{r_t^k}\right)^{\frac{\alpha}{1-\alpha-\nu}} \left(z_{it} a_{it}^{\gamma}\right)^{\frac{1}{1-\alpha-\nu}}$$
(2)

3.2 Financing Innovation

Firm *i* can issue one-period bond b_{it} to finance its innovation investment $[a_{it} - (1 - \delta_a)a_{it-1}]$, where δ_a is innovation capital depreciation rate. Since firms rent capital and labor, which they could always repay within-period, the only purpose of debt in the model is to finance its innovation investment in the model. Financial barriers occur due to imperfect information and uncertainty in returns. Lenders require some collateral to back up their debt holdings in case of bad return shocks. Consistent with the stylized facts, we allow innovation capital to be used as collateral with two conditions. First, debt holders discount the value of innovation collateral because of the associated uncertainty of innovation returns, so the liquidation value of patents is low. Second, firms need to hire a professional agent to evaluate the collateral value of their innovation capital, which incurs a fixed inspection cost. We assume the inspection cost is a uniformly distributed random variable $\xi \in [0, \overline{\xi}]$ paid in units of labor.⁸ The independent draw of fixed inspection costs generates endogenous size-dependent barriers. Mapping to reality, it is easier for larger firms to overcome such a fixed cost.

Let $F_{it} = \{A, N\}$ indicate whether the firm decides to pay the fixed inspection cost. When $F_{it} = A$, the firm pays the inspection cost, and when $F_{it} = N$, it does not pay the inspection costs and can only fund innovation investment using internal funds. For simplicity, we do not allow firms to finance innovation with equity issuance, so we constrain the dividend $d_{it} \ge 0$. In sum, firms face a collateral constraint as follows:

$$b_{it}(1+r_t) \leq \begin{cases} \chi(1-\delta_a)a_{it} & \text{if } F_{it} = A\\ 0 & \text{if } F_{it} = N \end{cases}$$

where debt that needs to be repaid next period is always less than the innovation capital stock at t+1. Since the firm always has the choice to reduce innovation capital and repay its debt, the non-negative dividend condition can always be satisfied, and default never happens in equilibrium.

3.3 Recursive Problem for an Innovative Firm

We write the firm's optimization recursively as in Benhima et al. (2022). Firm decisions are divided into two sub-periods. In the first sub-period, firms maximize their total net revenue given their productivity and starting net worth. Given the innovation capital price q_t^a , the firm decides how much innovation capital $q_t^a a_{it}$ to invest in, whether to use patent collateral F_{it} , and how much debt

⁸This random fixed cost setup is widely used in the lumpy investment literature (see Khan and Thomas (2008), Fang (2020), and Fang (2022)). This assumption helps address the fact that firms are not perfectly sorted by their states of productivity and net worth, which matches the data.

 b_{it} to hold if borrowing. The individual state variables of a firm are its idiosyncratic productivity z_{it} and its starting net worth entering the period n_{it-1} . Given the presence of the collateral constraint, the firm maximizes its end-of-period total net revenue:

$$\pi^*(z_{it}, n_{it-1}, F_{it}) = \max_{a_{it}, b_{it}} \left\{ f(z_{it}, a_{it}) + (1 - \delta^a) q_t^a a_{it} - (1 + r_t) b_{it} \right\},\tag{3}$$

subject to both constraints

$$q_t^a a_{it} = n_{it-1} + b_{it}, (4)$$

$$b_{it}(1+r_t) \le F_{it} \cdot \chi(1-\delta_a)a_{it}.$$
(5)

where $F_{it} = A$ denotes that the firm uses patent collateral, $F_{it} = N$ denotes that the firm opts out, and χ stands for the liquidation value of patents.

In the second sub-period, firms maximize their value function $v(z_{it}, n_{it-1}, F_{it})$ given their endof-period total net revenue $\pi^*(z_{it}, n_{it-1}, F_{it})$. We write the firm's optimization recursively. The expected equity value of a firm is given by $v(z_{it}, n_{it-1}) = \frac{\xi^*}{\xi}v(z_{it}, n_{it-1}, A) + (1 - \frac{\xi^*}{\xi})v(z_{it}, n_{it-1}, N)$. We denote the value function $v(z_{it}, n_{it-1}, F_{it})$ as:

$$v(z_{it}, n_{it-1}, F_{it}) = \max_{d_{it}} \left\{ d_{it}(z_{it}, n_{it-1}, F_{it}) + E[\Lambda_{t+1}v(z_{i,t+1}, n_{it})] \right\}$$
(6)

where the firm's dividend d_{it} is subject to the time *t* non-negative dividend constraint $d_{it} \ge 0$, and Λ_{t+1} is the firm's stochastic discount factor which is determined by household consumption. Net worth follows the accumulation rule:

$$n_{it}(z_{it}, n_{it-1}, F_{it}) = \pi^*(z_{it}, n_{it-1}, F_{it}) - d_{it}(z_{it}, n_{it-1}, F_{it}) - \xi_{it}$$

We then have a threshold value for the inspection costs:

$$\xi^*(z_{it}, n_{it-1}) = \frac{\pi^*(z_{it}, n_{it-1}, A) - \pi^*(z_{it}, n_{it-1}, N)}{w_t}.$$
(7)

Firms with state (z_{it}, n_{it-1}) who draw a fixed cost higher than $\xi^*(z_{it}, n_{it-1})$ will not pledge patents as collateral, otherwise, they pay the drawn fixed cost and borrow using patent collateral.

3.4 Other Firms, Households, and Equilibrium

Physical Capital Producer There is a representative physical capital producer who owns and produces new aggregate physical capital using the technology $\Phi(I_t^k/K_t)K_t$, where I_t^k are units

of the final good used to produce physical capital, $K_t = \int k_{it} di$ is the aggregate physical capital stock at the beginning of the period, $\Phi(I_t^k/K_t) = I_t^k + \frac{1}{2}\phi_k(I_t^k/K_t - \delta_k)^2K_t$, δ_k is the depreciation rate of physical capital, and ϕ_k reflects capital adjustment costs. Profit maximization pins down the rental price of physical capital as $r_{t+1}^k = \phi_k(\frac{I_t^k}{K_t} - \delta_k) + \delta_k$.

Innovation Capital Producer There is a representative innovation capital producer who produces new aggregate innovation capital using the technology $\Phi(I_t^a/A_t)A_t$, where I_t^a are units of the final good used to produce physical capital, $A_t = \int a_{it} di$ is the aggregate innovation capital stock at the beginning of the period, $\Phi(I_t^a/A_t) = \left(\frac{I_t^a/A_t}{\delta_a}\right)^{1/\phi_a}$, and δ_a is the steady-state innovation investment rate. Profit maximization pins down the relative price of innovation capital as $q_t^a = \frac{1}{\Phi'(I_t^a/A_t)} = \left(\frac{I_t^a/A_t}{\delta_a}\right)^{1/\phi_a}$.

Households There is a unit measure of continuous identical households with preferences over consumption C_t and labor supply L_t with utility $E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \psi L_t \right)$ subject to the budget constraint $C_t + \frac{1}{1+r_t}B_t \leq B_{t-1} + W_tL_t$ where E_0 is the expectation taken at the initial period 0, β is the discount factor of households, ψ is the disutility of working, r_t is the interest rate, B_t is one-period bonds and W_t is the nominal wage. Households choose consumption, labor, and bonds, which yields two Euler equations that determine both the real wage and the real interest rate (stochastic discount factor for firms as well): $W_t = -\frac{U_l(C_t,L_t)}{U_c(C_t,L_t)} = \psi C_t^{\eta}$ and $\Lambda_{t+1} = \frac{1}{1+r_t} = \beta \frac{U_c(C_{t+1},L_{t+1})}{U_c(C_t,L_t)} = \beta \left(\frac{C_t}{C_{t+1}} \right)^{\eta}$.

Equilibrium and Solution The equilibrium requires all firms optimizing, all capital producers optimizing, households optimizing, and market clearing in steady state and transition. The detailed equilibrium definition is given in Appendix B.1. We solve the model using global methods so the model can generate a rich cross-sectional distribution of firms and aggregate dynamics. The solution methods of the model are discussed in Appendix B.2.

4 Quantitative Analysis

We now quantitatively assess how patent collateral shapes firms' financing conditions and innovation. We first parameterize the model to both US and Chinese data using each country's average firm-level moments. The key parameters that capture barriers in using patent collateral are parameterized to match the financing patterns observed in our firm-level data. We then examine the ability of both barriers to replicate the time-series dynamics in Figure 1. We show that patents as collateral can quantitatively account for the observed patterns of innovation financing in our firm-level data. We finally conduct counterfactual exercises to quantify the innovation and welfare gains from expanding patent collateral in China to the US level.

4.1 Parameterization

There are two groups of parameters. The first group of parameters is common to the US and China, while those in the second group are chosen to match the average firm-level moments from each country. We provide the parameter values and the average firm-level moments in the data and model in Appendix B.3.

Fixed Parameters The model is calibrated at an annual frequency. We set the discount factor $\beta = 0.96$, a conventional value in an annual model. We choose logarithmic utility and hours of working equal to 1/3 so that $\eta = 1$ and $\psi = 2$. We choose a decreasing return to scale of 85% as in Ottonello and Winberry (2020). We then set the physical capital share to 25% and innovation capital share to 15%, following estimates by Corrado, Hulten, and Sichel (2009) and as in Perez-Orive (2016) and Lopez and Olivella (2018), so { α, γ, ν } = {0.20, 0.15, 0.50}. To match the corresponding 12% tangible investment to output ratio and 5% intangible investment to output ratio as in NIPA, we choose the physical capital depreciation rate $\delta_k = 10\%$ and the innovation capital depreciation rate $\delta_a = 20\%$.

Fitted Parameters The second group of parameters is chosen to match the following moments for each country: the average share of patent collateral, participation rate, and the standard deviation of patent assets relative to the mean. For the productivity process, we choose the persistence $\rho_z = 0.90$ and match the standard deviations $\sigma_z^{US} = 0.032$ and $\sigma_z^{CN} = 0.10$ to the standard deviation of patent assets relative to the mean for the US (56.6%) and China (121.7%), respectively. We then use the participation rate and the share of pledged patents (Figure 1(e) and (f)) to identify the fixed inspection costs parameters (ξ^{US} and ξ^{CN}) and the liquidation value parameters (χ^{US} and χ^{CN}). Since the fixed costs of inspection have a greater impact on the firm's decision to use patent collateral at the extensive margin, the participation rate will mostly identify the fixed cost parameters. Conditional on the inspection costs, the liquidation value of patents mostly affects the intensity of patent collateral use, so this moment pins down liquidation values.

We obtain fixed inspection costs $\xi^{US} = 0.0011$ and $\xi^{CN} = 1.21$, and the liquidation values $\chi^{US} = 0.32$ and $\chi^{CN} = 0.117$, using the average shares of pledged patents of both economies (US=13.91%, CN=0.47%) and the participation rates (US=55.84%, CN=1.06%) for the steady states. To intuitively understand the magnitudes of the inspection costs, consider that in the steady states, the average inspection cost is about 50% of an average firm's sales in China but is only about 0.05% in the US. These results demonstrate that Chinese firms face significantly higher inspection costs for innovation collateral and a much lower liquidation value for innovation capital. Only the largest and most productive Chinese firms are able to borrow against patents. We demonstrate how these barriers are identified in the section below. Both barriers jointly lead to a much lower share of

patent collateral and a much lower collateral participation rate.

4.2 The Roles and Dynamics of Patent Collateral Barriers

First, we show how the average shares of pledged patents and participation rates change with respect to variations of the parameters governing the barriers.

Figure 2(a) and (b) show how the average share of pledged patents and participation rates vary with inspection costs. The solid blue line stands for the US, and the red dashed line stands for China. The vertical blue line represents our equilibrium US calibration ($\xi_{US} = 0.0011$), but the baseline Chinese calibration ($\xi_{CN} = 1.21$) is off the chart. The first observation is that reducing the inspection cost significantly increases the shares of pledged patents and participation rates in both the US and China, though the increase is less pronounced in China. Second, the effects of reducing the inspection cost in the US and China vary in magnitude. When the inspection cost is large, the effects are stronger in China; otherwise, the effects are stronger in the US. This is because of very small Chinese liquidation values, so firms remain unwilling to use patent collateral even with minute inspection costs.

Figure 2(c) and (d) show how the shares of pledged patents and participation rates vary with the liquidation value. The solid blue line stands for the US, and the red dashed line stands for China. The vertical blue line stands for our equilibrium calibration of US ($\chi_{US} = 0.32$), and the vertical red line represents China ($\chi_{CN} = 0.117$). First, an increase in liquidation value significantly increases the share of pledged patents in both the US and China. Second, increasing the liquidation value will also significantly increase the intensity of using of patent collateral in both the US and China. Third, increasing liquidation values will also significantly increase participation rates in the US, but not in China, after the liquidation value rises above 50%. Even increasing the liquidation value to 100% cannot meaningfully increase the Chinese participation rate since the inspection cost is too high. Taken together, the four subplots (a) to (d) jointly show that both patent collateral barriers matter for financing innovation with innovation in the model. Reducing the severity of either barrier would significantly increase the use of patent collateral in the model on both the extensive and intensive margins. To achieve a high level of patent collateral, reducing both barriers jointly would be the most effective.

Second, we show how the dynamics of the liquidation value of patents and fixed inspection costs could explain the changes in the time series of the ratios of pledged patents and ratios of participation as shown in Figure 1 (e) and (f). The time series for each barrier in both countries are plotted in Figure 2 (e) and (f). First, inspection costs have been falling. From 2009 to 2019, Chinese firms experienced dramatic drops in inspection costs. This participation barrier parame-



Figure 2: The Roles and Dynamics of Patent Collateral Barriers

Notes: Sub-figures (a) to (d) plot the variations of the shares of pledged patents and the participation rates over the changes in the inspection cost and liquidation value for both the US and China. The solid blue line stands for the moments with respect to our calibrations of the inspection costs in the US, and the red dashed line stands for the moments with respect to our calibrations of the inspection costs in China. The dashed vertical reference lines indicate the parameter calibrations of the inspection cost in the US and China, respectively. Sub-figures (e) and (f) plot the estimated dynamics of the two barriers over the period of our firm-level sample.

ter was 18.5 in 2009, at the inception of Chinese patent collateral, and fell to 0.64 in 2019 after ten years of financial and legal development.⁹ During the same period, US peer firms experienced an inspection cost reduction from 0.002 to 0.0004, which is also a significant reduction. However, given that the cost was initially low, the falling cost did not translate into a spike in the participation rate. Second, liquidation values are quite stable compared to inspection costs. From 2009 to 2019, the liquidation value for Chinese firms fluctuated around 10%. There was also a slightly declining trend for US firms.

4.3 Financing Innovation with Innovation in the Model

We then replicate our empirical findings to show how patent collateral could boost innovation. We first simulate our steady state economies with 50,000 firms for 210 years and keep only the last ten years. We then run the same regression using equation (1) with essentially the same specifications with model-simulated firm-level data.¹⁰

The results are presented in Table 2. Panel (a) reports the estimation results for regressions based on equation (1) using the US model-simulated firm-level data. US firms pledging patents as collateral increase their leverage by around 3.3% (column (4)) and their R&D expenditures by around 5% (column (8)). These coefficients are significantly positive but slightly larger than our empirical findings. This is because, in the model, the only borrowing channel is *financing innovation with innovation*, which is particularly strong since other channels are not considered. Panel (b) reports the estimation results for regressions based on equation (1) using the Chinese model-simulated firm-level data. Firms pledging patents as collateral increase their leverage by around 2.6%. We also find a significant growth in R&D expenditures (around 10.6%, see column (8)), implying that firms probably use patents to finance their innovation. These coefficients are significant and quite close to our empirical findings. Overall, these results indicate that our model does a good job of fitting the non-targeted moments in the data, serving as a cross-validation of model calibration.

⁹These results are consistent with the fact that the Chinese government initiated a series of policies to stimulate the use of patent collateral by both firms and banks starting around 2009.

¹⁰The only two differences are that, first, in the model, there are no aggregate shocks, so we do not control timefixed effects; second, in the model, there is the resale of innovation capital, while in the data, we do not have negative R&D, so we replaced log(R&D) with log(1+R&D) to include most zero and negative values of changes in the stock of innovation capital. The results are robust if we exclude all zero and negative values.

Panel (a) US Model								
		leve	erage			log	(R&D)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PC	0.0674***	0.0588***	0.0426***	0.0336***	0.2012***	0.2502***	0.0498***	0.0516***
	(0.0004)	(0.0005)	(0.0005)	(0.0006)	(0.0020)	(0.0022)	(0.0015)	(0.0015)
L.log(asset)		0.0011***	-0.0005	0.0847***		-0.1240***	-0.1217***	0.0636***
		(0.0004)	(0.0004)	(0.0012)		(0.0018)	(0.0013)	(0.0042)
L.tobin's Q			0.2588***	0.1890***			3.6309***	3.4030***
			(0.0016)	(0.0019)			(0.0067)	(0.0076)
L.ROE			-1.3473***				-11.1536***	
			(0.0555)				(0.3489)	
L.ROA				-0.8417***				-1.9852***
				(0.0112)				(0.0401)
N	500000	450000	400000	400000	449931	449931	399938	399938
adj. R^2	0.073	0.043	0.123	0.138	0.016	0.030	0.594	0.591
			Panel	(b) Chinese	Model			
		leve	erage		log(R&D)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PC	0.0235***	0.0255***	0.0257***	0.0265***	0.1407***	0.0636***	0.1113***	0.1065***
	(0.0003)	(0.0004)	(0.0004)	(0.0004)	(0.0081)	(0.0073)	(0.0057)	(0.0056)
L.log(asset)		-0.0001***	0.0001***	-0.0008***		0.1281***	0.1901***	0.1893***
_		(0.0000)	(0.0000)	(0.0000)		(0.0015)	(0.0012)	(0.0013)
L.tobin's Q			0.0037***	0.0067***			1.3326***	1.3322***
			(0.0001)	(0.0002)			(0.0040)	(0.0049)
L.ROE			0.1238***				-0.1999***	
			(0.0043)				(0.0720)	
L.ROA				0.0139***				0.0073
				(0.0004)				(0.0078)
N	500000	450000	400000	400000	426936	426936	379459	379459
adj. R^2	0.127	0.134	0.180	0.185	0.002	0.053	0.445	0.445

Table 2: Responses of Leverage and R&D to Patent Collateral in the Model

Note: All regressions include firm and year-fixed effects. Standard errors are in parentheses. * p < 10%, *** p < 5%, *** p < 1%. The regression specification is $Y_{it} = \alpha + \beta PC_{it} + \gamma Z'_{it} + \lambda_i + \lambda_t + \xi_{it}$ where Y_{it} is the outcome variable. For firm *i* in year *t*, Y_{it} is the outcome variable of interest, and PC_{it} is an indicator which takes a value of 1 if the firm has ever used patents as collateral; otherwise, $PC_{it} = 0$. We use the firm's leverage ratio (total debt/total assets) to measure the firm's borrowing responses to patent collateral. To examine the innovation response, we use the firm's R&D expenditures. Z_{it} is a vector of control variables. Our control variables include firm ROE, ROA, Tobin's q, and total assets. These variables are standard in the macro-finance and corporate finance literature. To account for unobserved firm-level factors that may lead to endogeneity issues, we control for firm-level fixed effects λ_i . The variable λ_t is a group of complete-time dummies that account for the influence of macro factors such as monetary and fiscal policy adjustments. The error term is ξ_{it} . The parameter of interest is β . The estimates of β capture the impact of employing patent collateral on the outcomes. We run the regression using the US and Chinese data separately.

Benchmark	$\hat{\bar{\xi}}^{CN} = \bar{\xi}^{US}$	$\hat{\boldsymbol{\chi}}^{CN} = \boldsymbol{\chi}^{US}$	Both as US
0.47%	5.53%	4.30%	16.69%
1.09%	29.53%	3.09%	30.71%
-	1.50%	1.02%	4.67%
-	1.50%	1.00%	4.68%
-	2.88%	1.54%	8.97%
-	0.44%	0.63%	1.40%
-	0.42%	0.13%	1.27%
	Benchmark 0.47% 1.09%	Benchmark $\hat{\tilde{\xi}}^{CN} = \tilde{\xi}^{US}$ 0.47%5.53%1.09%29.53%-1.50%-2.88%-0.44%-0.42%	Benchmark $\hat{\xi}^{CN} = \bar{\xi}^{US}$ $\hat{\chi}^{CN} = \chi^{US}$ 0.47%5.53%4.30%1.09%29.53%3.09%-1.50%1.02%-1.50%1.00%-2.88%1.54%-0.44%0.63%-0.42%0.13%

Table 3: What if China had US-level barriers?

Notes: This table reports the counterfactual results of reducing the patent collateral barriers in China to the US level. In the three counterfactuals, we assume China has US-level barriers in terms of solely inspection cost, solely liquidation value, and both, respectively. We report on the prevalence of financing innovation and aggregate economic outcomes in each counterfactual.

4.4 What if China had the US level Barriers?

Finally, we demonstrate how reductions in the barriers could improve welfare by simulating counterfactuals of China with US-level barriers. The results are shown in Table 3. Compared to the benchmark, in all counterfactuals, Chinese firms increase financing innovation, and aggregate economic outcomes improve. However, the improvement in solely liquidation value yields smaller improvements in aggregate economic outcomes, resulting in increases of 1.02% in total output, 1.54% in total patents, and 0.13% in total welfare. On the other hand, reducing the inspection cost to the US level significantly stimulates innovation investment and improves aggregate economic outcomes, resulting in increases of 1.5% in total output, 2.88% in total patents, and 0.42% in total welfare. Reducing both barriers to the US level generates even more substantial gains in output, patenting, and welfare.

Our counterfactual analyses have strong real-world policy implications for lagged countries in terms of using patent collateral to promote economic growth. Given the stage of development of patent collateral in China, reducing fixed inspection costs (or, equivalently, adopting better evaluation technology) is much more effective than improving the liquidation value of patents in stimulating innovation and promoting welfare. Policies that include reducing barriers in technological transaction markets and improving the legal protection of intellectual property rights would be a first-order consideration in unleashing the potential of patent collateral.

5 Conclusion

This paper studies the emerging firm behavior of *financing innovation with innovation*. Using patent collateral data from both the US and China, we demonstrate that (1) both the number of pledged patents and the share of patents being used as collateral have been rising steadily in the US and China; (2) however, US patent holders employ patents as collateral on a greater scale and intensity than Chinese patent owners; and (3) firms that started using patent collateral increase their borrowing and innovation activities. We also rationalize these facts in a heterogeneous firm general equilibrium model with two barriers that hinder patent collateral. We show that both barriers – liquidation values and inspection costs - matter for the difference between the US and China, but given the early stage of the patent collateral market in China, the gains in output, innovation, and welfare from reducing Chinese inspection costs are more substantial.

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Online Appendix for "Financing Innovation with Innovation" by Zhiyuan Chen, Minjie Deng, and Min Fang (Not for Publication)

A Empirical Appendix

A.1 Patent Pledges

In this subsection, we provide a more detailed description of patent pledging in terms of the US and China datasets.

A.1.1 Frequency-Adjusted Pledge Ratios

A patent can be used as collateral multiple times during its lifetime. To reflect the intensity of using patent pledges more accurately, we adjust the number of patents using the frequency of patent collateral usage and calculate the ratios between pledged patents and total patents. We take advantage of this information and use it as a proxy for the liquidation value of the patent. In Appendix A.1.4, we document that around 56% (22%) of patents are used more than once as collateral in the US (China). That is, the patent being pledged more often is regarded as of higher liquidation value and can be resold more easily. Let $M_j(s)$ be the frequency of which that patent *j* is pledged as collateral within *s* years after its granting. We treat $M_j(s)$ as the proxy for the quality of this patent. $M_j(S)$) is used as a weight for patent *j*, reflecting its contribution when accounting for the usage of patent collateral. We then calculate the share of pledged patents in *s* years within granting vintage *t* as follows:

$$\gamma_t^q(s) = \sum_{j \in \mathcal{J}_t} M_j(s) / \sum_{j \in \mathcal{J}_t} (1 + M_j(s))$$
(8)

where \mathcal{J}_t is the index set of patents granted in year *t*. We calculate the share of pledged patents by choosing *s* = 1, 3, 5, 7 and set *s* = 5 in this benchmark.

A.1.2 Non Frequency-Adjusted Pledge Ratios

The non-quality weighted patent pledged ratio is calculated as follows:

$$\gamma_t^{nq}(s) = \frac{\sum_{j \in \mathcal{J}_t} \mathbb{I}_j(s)}{\|\mathcal{J}_t\|}$$
(9)

where $\mathbb{I}_j(s)$ is an indicator function equal to one if patent *j* is used as patent collateral during *s* periods within granting vintage *t*, and $\|\mathcal{J}_t\|$ is the total number of patents granted in year *t*.

Figure 3 Panel A shows the non-quality weighted patent pledge ratios with s = 1, 3, 5, 7. In Figure 3 Panel B, we display the corresponding quality-weighted patent pledge ratios. The dark solid line indicates the time horizon we choose as the benchmark (s = 5). As we increase s, the resulting pledge ratios increase because patents are more likely to be employed as patent collateral as they age. The gap in patent pledge ratios between the US and China is quite stable for the choice of different values for s, though the magnitude varies with different choices for s.

The quality-weighted patent pledge ratios are higher than the non-quality-weighted patent pledge ratios, as patents are usually pledged more than once. However, we only see a nuanced difference for Chinese data. This is because the frequencies of being pledged for each patent are lower than in the US. Lastly, we see steady growth in patent pledge ratios in China, indicating advancements in the patent market and improvements in the functioning of financial intermediaries.

A.1.3 Patent Pledges by Types

Figure 4 plots the fraction of patent pledges by the type of patent granted and the age of granting. Invention patents have the highest pledging ratio, utility model patents have a lower pledging ratio, and design patents have the lowest pledging fraction. This ranking is consistent with the usual conjecture on the ranking of the liquidation value for different patents (Chen and Zhang, 2019).¹¹ For invention patents, their pledging ratios have reached around 4 percent in recent years, with utility patents below 2 percent. Design patents, which have the least novelty, have pledging ratios close to zero.

¹¹In China's patent law, invention is referred to as a new technical solution proposed for the product, method, or related improvement; the utility model refers to a new technical solution suitable for practical use proposed for shape, construction, or combination. According to Article 22 of the Patent Law of the P.R.C.: any invention or utility model for which patent right may be granted must possess novelty, inventiveness, and practical applicability. In comparison, the requirement for the approval of design patents is in Article 24 of the Patent Law of the P.R.C as "... ... must not be identical with or similar to any design which, before the date of filing, has been publicly disclosed in publications in the country or abroad or has been publicly used in the country, and must not collide with any prior legal rights obtained by any other person."





Panel A: Non-Quality Weighted Patent Pledge Ratios





This figure compares the patent pledge ratios for US and China. Panel A shows the non-qualityweighted patent pledge ratios with s = 1, 3, 5, 7. Panel B displays the corresponding qualityweighted patent pledge ratios. The dark solid line indicates the time horizon we choose as the benchmark (s = 5). As we increase s, the resulting pledge ratios increase because patents are more likely to be employed as patent collateral as they age. The gap in patent pledge ratios between the US and China is quite stable to the choice of different values for s, though the magnitude varies with different choices for s.

Figure 4: Patent Pledges for Three Different Types of Patents in China



This figure plots the fraction of patent pledges by the type of patent granted and the age of granting. Invention patents have the highest pledging ratio, utility model patents have a lower pledging ratio, and design patents have the lowest pledging fraction.

A.1.4 Frequencies of Patent Pledges

Figure 5 shows the distribution of average repeated pledging times for all pledged patents in the US and China. Many patents are pledged only once. Around 44% (78%) of pledged patents in the US (China) are only pledged once. Pledged patents are more likely to be repeatedly used in the US than in China. Around 24% of pledged patents are repeatedly used twice in the US, but this number is only 15% in China. In the US, 15% of patents are pledged three times, and 6% of pledged patents are employed more than five times. In China, this fraction is zero.

A.2 Firm-level Data

A.2.1 The US Compustat and Patent Data

US Data We link firms with individual patents using the gvkey-patnum (gvkey identifies firm ID and patnum indicates the patent number) linkages provided by The WRDS US Patents Compustat Link. The Compustat contains rich firm-level information on publicly listed US firms, allowing us to explore the relevance of firm characteristics.¹² The WRDS US Patents Compustat Link covers patents granted between 2011 and 2019, and the matching is done with company names

¹²Although Compustat only includes public firms, it covers a big fraction of U.S. output.



Figure 5: Frequencies of Patent Pledges for US and China

This figure plots the distribution of average repeated pledging times for all pledged patents in the US and China. Many patents are pledged only once. Around 44% (78%) of pledged patents in the US (China) are only pledged once. Pledged patents are more likely to be repeatedly used in the US than in China. Around 24% of pledged patents are repeatedly used twice in the US, but this number is only 15% in China. In the US, 15% of patents are pledged three times, and 6% of pledged patents are employed more than five times. In China, this fraction is zero.

using fuzzy name-matching algorithms. The geographical information and corporate hierarchy information from the WRDS Subsidiary database have been used for fine-tuning the matching results. After matching individual patent information to its corresponding firms, we can compare the characteristics of firms that do or do not hold patents.

In addition to existing firm-level variables, such as assets and sales, we also construct firmlevel investment and leverage. Investment for a firm *j* at time *t* is defined as the ratio (×100%) of quarterly capital expenditures (*capxy*) to the lag of quarterly property, plant, and equipment (*ppentq*). Leverage is defined as the debt-to-assets ratio, which is the sum of debt maturing within one year and debt maturing in more than one year (*dlcq+dlttq*) over total assets (*atq*).

Sample Selection First, we keep observations with Current ISO Country Code - Headquarters (*loc*) as *USA*. Second, we disregard observations from financial sector firms (SICs 6000-6999), non-profit organizations, and governmental enterprises (SICs 8000s & 9000s), as well as utilities (SICs 4900-4999). Third, we drop firm-quarter observations with missing or negative sales and with missing or non-positive total assets. Lastly, we winsorize investment and leverage at the top and bottom 5% of the distribution.

Firm Distribution After merging the Compustat data with patent data using the WRDS US Patents Compustat Link, we have panel data containing 102,797 observations for 5,210 firms. In our sample, about 55% (2,848 out of 5,210) of the firms have at least one patent (see Table 4 Panel A). Firms with patents are generally bigger and invest more. Figure 6 compares the histograms of assets, sales, and investment for firms with at least one patent to the firms with no patents. The firms with patents are generally bigger and invest more.

We then match the above data with patent collateral information to compare the characteristics of firms that use their patents as collateral to firms whose patents are never used as collateral. Figure 7 compares the histograms of assets, sales, and investment for firms that have pledged their patents at least once to firms that have never pledged their patents. Firms that pledge patents are, in general, smaller but invest more. The comparison suggests that patent collateral provides a way for firms that are smaller but actively investing to obtain funding.

A.2.2 The Chinese CSMAR Data

Chinese Data We obtain rich firm-level financial and innovation variables on Chinese listed firms from CSMAR.¹³To explore the differences in firm characteristics in terms of patent pledging, we link the CSMAR data with the data on China's patent collateral transactions using firm names. To ensure the quality of the matching, we also perform fuzzy matching using the stem words in firm names and have manually checked its efficiency. In our sample during 2009 and 2019, 214 unique firms (347 firm-year observations) are matched to the patent collateral database. On average, each firm pledged patents as collateral 5.86 times, resulting in 2,035 patent-year counts and 1,629 unique patents.¹⁴

Because owning a patent is a prerequisite for using patents as collateral, we focus on innovative firms which hold at least one invention or utility patent in our sample. Table 4 Penal B characterizes firms by patent holding. Of 4,635 firms, 3,971 firms (around 85.7%) have at least one granted patent, of which only 214 firms (around 5.4%) have used patents as collateral.

Variables Construction We obtain detailed firm-level information from China Stock Market Accounting Research (CSMAR). We perform our empirical analysis using this yearly data. The variables of *logassets*, *logsales*, and *leverage* are defined identically to the Compustat data. We construct investment in two ways:

¹³CSMAR is usually viewed as the Compustat of China.

¹⁴Top three industries using patent collateral are computers & communication equipment, special equipment, and pharmaceutical manufacturing.



Figure 6: (U.S.) Firm characteristics with patents and without patents

(c) Investment

U.S. data. This figure compares the histograms that show the distribution of assets, sales, and investment for two groups of firms: those with at least one patent (referred to as "patenting" firms) and those with no patents (referred to as "non-patenting" firms). The data suggests that, on average, patenting firms are generally larger in terms of assets and sales and tend to invest more compared to non-patenting firms.



Figure 7: (U.S.) Firm characteristics with patents collateral and without patents collateral

U.S. data. This figure compares the histograms of assets, sales, and investment for two groups of firms: those that have pledged their patents at least once (referred to as "patent pledgors") and those that have never pledged their patents (referred to as "non-patent pledgors"). Both groups consist of firms that hold at least one patent. The data suggests that, on average, pledged firms are generally smaller in terms of assets and sales compared to non-pledged firms but tend to invest more. This suggests that patent collateral, or the use of a patent as collateral to obtain funding, provides a way for smaller firms that are actively investing to access funding.

Panel A: US			
	non patent pledgors	patent pledgors	Total
Non Innovator	2362	0	2362
Innovator	1306	1542	2848
Total	3668	1542	5210
Panel B: China			
	non patent pledgors	patent pledgors	Total
Non Innovator	664	0	664
Innovator	3,757	214	3,971
Total	4,421	214	4,635

Table 4: Number of Innovators and Patent Pledgors

This table reports the number of firms with patents (referred to as "innovators") and the number of firms that have pledged their patents at least once (referred to as "patent pledgors") in the US and China.

- Total investment in a year is calculated as the total cash payments for purchasing durable assets (including fixed assets, intangible assets, and other durable assets), subtracting cash earned from the disposal of durable assets. The investment rate is the ratio of total investment to the sum of lagged fixed assets and intangible assets.
- 2. Alternatively, total yearly investment is defined as the net increase in fixed assets, which is defined as current total fixed assets minus lagged total assets plus the depreciation of fixed assets, oil and gas assets, and biological assets. Then the investment rate is the ratio of total investment to lagged total fixed assets.

Sample Selection We merge the patent collateral database with the CSMAR data. We find that these listed firms started to pledge patents as collateral in 2010. The most recent year of patent collateral data is 2019. Thus, we include Chinese-listed firms between 2010 to 2019 that are contained in the CSMAR dataset. Our dataset includes mostly manufacturing firms but also firms operating in various service sectors. The final sample contains 29,537 observations and 4,305 firms. To avoid the influence of outliers, we also winsorize investment rate and leverage at the top and bottom 5% of the distribution.

Firm Distribution In the Chinese data, the difference between patenting firms and non-patenting firms is relatively more nuanced than we documented using the US data. Figure 8 compares the histograms of assets, sales, and investment for Chinese firms with at least one patent with those of firms without patents. We do not see much difference in firm sizes as measured by the log of as-

sets or sales. But in terms of investment, patenting firms invest more than non-patenting firms. Figure 9 compares the histograms of assets, sales, and investment for firms that have pledged their patents as collateral with those firms that have never pledged their patents. The firms who pledged their patents are generally slightly smaller but invest more.



Figure 8: (Chinese) Firm characteristics with patents and without patents

(c) Investment

Chinese data. This figure compares the histograms that show the distributions of assets, sales, and investment for two groups of firms: those with at least one patent (referred to as "patenting" firms) and those with no patents ("non-patenting" firms). The data suggest that patenting firms generally tend to invest more compared to non-patenting firms.



Figure 9: (Chinese) Firm characteristics with patents collateral and without patents collateral

(c) investment

Chinese data. This figure compares the histograms of assets, sales, and investment for two groups of firms: those that have pledged their patents at least once (referred to as "patent pledgors") and those that have never pledged their patents ("non-patent pledgors"). Both groups consist of firms that hold at least one patent. The data suggest that pledging firms are generally smaller in terms of assets and sales compared to non-pledging firms but tend to invest more. This suggests that patent collateral, or the use of a patent as collateral to obtain funding, provides a way for smaller firms that are actively investing to access funding.

B Theoretical Appendix

B.1 Equilibrium Definition

We now define the equilibrium of the model. We define $\mu(z, n, \xi)$ as the distribution of firms over their state vector (z, n, ξ) . The **Recursive Competitive Equilibrium** for this economy is defined by a set of value functions and policy functions { $v(z, n), v^A(z, n), v^N(z, n), \xi^*(z, n), a^{'A*}(z, n),$ $a^{'N*}(z, n)$ }, a set of quantities {C, L, K, Y, A}, a set of prices { w, Λ, r, r^k, q }, and a distribution $\mu(z, n, \xi)$ that solves the innovative firm's problem, other firms' problems, the household problem, and satisfies market clearing such that:

(i) [Firm Optimization] Taking the aggregate prices { w, r, r^k, q, Λ } as given, $v(z, n), v^A(z, n)$, $v^N(z, n)$, and $\xi^*(z, n)$ solve the innovative firms' optimization (3) – (7) with associated decision rules $a'^{A*}(z, n), a'^{N*}(z, n)$.

(ii) [Household Optimization] Taking the aggregate prices $\{w, r\}$ as given, $\{C, L, B\}$ and Λ solve the household's utility maximization.

(iii) [Other Firm Optimization] Both capital producers maximize profit which determines the physical capital rent r^k and innovation capital price q^a .

(iv) [Market Clearing] The labor market clears, the bond market clears, and the final good market clears $Y = C + I^k + I^a$.

B.2 Solution Method of the Model

Part I: Solving the Stationary Equilibrium for the Average Moments

We assume the economy is at its steady state given the aggregate moments. We search for an equilibrium wage to clear the labor market. The algorithm is as follows:

Step.1. Guess an equilibrium wage.

Step.2. Solve the firm's problem using Value Function Iteration.

Step.3. Calculate aggregate variables from the firm distribution using Young (2010).

Step.4. Update the wage with a given weight and return to Step 2 until convergence.

After the convergence, we have the stationary equilibrium aggregate prices $\Omega^* = \{\Lambda^* = \beta, w^* = w^*\}$ since the economy is at its steady state, which yield the aggregate quantities $\{C^*, w^* = w^*\}$

 N^* , Y^* , K^* , A^* }, firm value functions { $V^*(n, z)$, $V^{A^*}(n, z)$, $V^{NA^*}(n, z)$, policy functions $\xi^{**}(k, z)$, $a^{**}(k, z)$, $b^{**}(k, z)$, and distribution $\mu(k, z)$ at the stationary equilibrium state.

Part II: Solving the Transitional Equilibrium for the Dynamics

With the stationary equilibrium solutions in hand, we now move to the solution of the transitional equilibrium using a shooting algorithm. The key assumption here is that the economy starts from one steady state, say *China-2009*, and evolves to another steady state, say *China-2019*. Along the path from *China-2009* to *China-2019*, we choose a path for the two barriers $\{\bar{\xi}_t, \chi_t\}_{t=2009}^{T=2019}$ to match the two key moments: the Participation Rate (%) and the share of pledged patents (%). The following steps outline the shooting algorithm:

Step.1. Fix two steady states SS_{t_0} and SS_T ;

Step.2. Guess or given a sequence of barriers $\{\bar{\xi}_t, \chi_t\}_{t_0}^T$ and aggregate prices $\{w_t, \Lambda_t\}_{t_0}^T$ such that the initial prices $\{w_{t_0} = w_{t_0}^*, \Lambda_{t_0} = \Lambda_{t_0}^*\}$ (just simply assuming all the prices stay at their steady state works well) and terminal prices $\{w_T = w_T^*, \Lambda_T = \Lambda_T^*\}$ which are from the two steady states. This implies a time series for the aggregate state $\{\Omega_t\}_{t=1}^T$. The aggregate state is just time *t*.

Step.3. We know that at time T, the economy is back to its steady state SS_T . We have the steady state value function $V(k, z; T) = V^*(k, z; T)$ in hand for time T. We solve for the firms' problem by **backward induction** given V(k, z; T), $\{\bar{\xi}_{T-1}, \chi_{T-1}\}$, and $\{w_{T-1}, \Lambda_{T-1}\}$. This yields the firm value function $V(k, z; \Omega_{T-1})$ and associated policy functions for capital a(k, z; T - 1) and debt $b(k, z; \Omega_{T-1})$. By iterating backward, we solve the whole series for both policy functions $\{a(k, z; \Omega_t)\}_{t=0}^T$ and $\{b(k, z; \Omega_t)\}_{t=0}^T$.

Step.4. Given the policy functions and the steady state distribution as the initial distribution $\mu(k, z; t_0)$, we use *forward simulation* with the non-stochastic simulation in Young (2010) to recover the whole path of $\{\mu(k, z; t)\}_{t=0}^{T}$.

Step.5. Using the distribution $\{\mu(k, z)\}_{1}^{T}$, we obtain all the *aggregate quantities*: aggregate output $\{Y\}_{t=0}^{T}$, aggregate investment $\{I\}_{t=0}^{T}$, aggregate labor demand $\{N\}_{t=0}^{T}$, and aggregate innovation $\{A\}_{t=1}^{T}$. We then use the goods market clearing condition to calculate aggregate consumption $\{C\}_{t=0}^{T}$. We then calculate the *Excessive Demand* $\{\Delta C\}_{t=0}^{T}$ by taking the differences between the currently iteration of $\{C\}_{t=0}^{T}$ and the previous iteration $\{C_{old}\}_{t=0}^{T}$.

Step.6. Given all the aggregate quantities in the previous step and the *Excessive Demand* $\{\Delta C\}_{t=1}^{T}$, we update all the *aggregate prices* and *both barriers*. We update all equilibrium prices with a line search: $X_t^{new} = speed \cdot f_X(\{\Delta C\}_{t=1}^{T}) + (1 - speed) \cdot X_t^{old}$.

Repeat Steps 2-6 until X_t^{new} and X_t^{old} are close enough. Updating all prices in all periods simultaneously reduces the computational burden dramatically. In all the experiments, we set a

step size of 0.1 to ensure convergence, with the distance between X_t^{new} and X_t^{old} very small. In practice, this method guarantees that the path is accurately capturing the dynamics of the moments. However, the convergence may be slow. Without loss of generality, we use the steady states of each period between the beginning and the ending as our initial guess of the path. This is because since the barriers are slowly adjusting, such an initial guess is closer to the final path.

B.3 Supplements to Parameterization

Parameter	Description	Value
β	Discount factor	0.96
η	Log utility	1
ψ	Leisure preference	2
α	Physical capital share	0.20
γ	Innovation capital share	0.15
ν	Labor share	0.50
δ_k	Physical capital depreciation rate	0.10
δ_a	Innovation capital depreciation rate	0.20

Table 5: Fixed Parameters

This table reports the values for the fixed parameters (assigned) in the model. A detailed description is in the calibration subsection 4.1.

Table 6: Fitted Parameters

Parameter	Description	U.S.	China
ξ	Inspection cost of innovation collateral	0.0011	1.21
χ	Innovation capital liquidation value	0.32	0.117
$ ho_z$	Productivity persistence (fixed)	0.90	0.90
σ_z	Productivity volatility	0.032	0.10

This table reports the values for the estimated parameters in the model to match the average firm-level moments in Table 7. A detailed description is in the calibration subsection 4.1.

	US		China	
	Data	Model	Data	Model
share of pledged patents (%)	13.91	14.20	0.47	0.47
Participation Rate (%)	55.84	54.75	1.06	1.09
Patent assets std/mean (%)	56.60	55.03	121.70	121.20

Table 7: Targeted Average Firm-level Moments

This table reports the moments that we target to estimate the parameters listed in Table 6. A detailed description is in the calibration subsection 4.1. A detailed description is in the calibration subsection 4.1. The moments are average annualized moments from 2009 to 2019 for China and from 2011 to 2019 for the U.S.. The share of pledged patents (%) is calculated as the ratio between the number of patents used as collateral and the total active number of patents. The participation rate (%) is the fraction of firms that have used their patents as collateral at least once during the year among the firms that have patents. These moments are averaged annual firm-level moments as in Figure 1. "Patent assets standard deviation and mean" calculates the standard deviation and mean for log(intangible assets), for which the units are millions of dollars (millions of RMBs for China).

	Ratio of (in %)		Ratio of (in %)		
	pledged patents		participation		
Year	US Data	China Data	US Data	China Data	
2009	-	0.102	-	0.119	
2010	-	0.053	-	0.282	
2011	16.047	0.073	48.663	0.301	
2012	15.682	0.250	51.860	0.438	
2013	15.221	0.568	54.527	0.952	
2014	14.959	0.324	56.000	0.773	
2015	14.682	0.335	56.612	0.892	
2016	13.785	0.433	57.349	0.935	
2017	12.895	0.551	57.993	1.656	
2018	12.138	1.111	58.009	1.596	
2019	11.349	0.524	57.988	1.762	

Table 8: Dynamics of Targeted Firm-level Moments

This table reports the moments that we directly used to plot Figure 1(e) and (f) as well as to match the dynamics of both barriers in Figure 2(e) and (f). The moments are annualized from 2009 to 2019 for China and from 2011 to 2019 for the U.S. The share of pledged patents (%) is calculated as the ratio between the number of patents used as collateral and the total active number of patents. The participation rate (%) is the fraction of firms that have used their patents as collateral at least once during the year among the firms that have patents.

	Inspec	ction cost	Liquidation value		
		ξ	χ		
Year	US Model	China Model	US Model	China Model	
2009	-	18.5	-	0.19	
2010	-	2.51	-	0.06	
2011	0.0020	2.52	0.38	0.07	
2012	0.0016	3.31	0.36	0.13	
2013	0.0013	1.82	0.34	0.15	
2014	0.0009	2.05	0.32	0.13	
2015	0.0008	1.41	0.32	0.11	
2016	0.0006	1.38	0.30	0.12	
2017	0.0004	0.67	0.27	0.10	
2018	0.0004	1.08	0.27	0.16	
2019	0.0004	0.64	0.25	0.09	

Table 9: Fitted Barrier Parameters

This table reports the fitted barrier parameters that we used to match the moments plotted in Figure 1(e) and (f). They are plotted in Figure 2(e) and (f).