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Finance, Managerial Inputs, and Misallocation

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Abstract

In standard macro-finance models, financial constraints mainly affect small or young firms but not large or old ones due to the self-financing mechanism, and the dispersion of marginal revenue product of capital (MRPK) of a firm cohort is less persistent than in the data. We extend a standard model by allowing firms to hire managers and large firms hire disproportionately more managers, consistent with data. In our model, financial constraints and the dispersion of MRPK persist, and even large firms are likely to be constrained. The productivity loss from financial frictions is also substantially amplified.

Keywords: Collateral Constraint, Managerial Inputs, Elasticity of Scale, Misallocation, Aggregate Productivity, China.

JEL classification: E13, G21, L16, L26, O16, O41.

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A recent literature argues that misallocation of production factors, especially the misallocation of capital, is a main reason for low total factor productivity (TFP) in developing countries (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009), which in turn, is the main source of per capita income differences (Klenow and Rodríguez-Clare, 1997; Caselli, 2005). While financial frictions are a natural source of capital misallocation, the literature assessing the quantitative importance of this channel on aggregate TFP finds small effects (Buera et al., 2011; Moll, 2014; Midrigan and Xu, 2014). The literature typically models financial frictions by assuming that firms face collateral constraints. In these models, the quantitative effects of collateral constraints are small due to the incentive of productive firms to undo them through self financing.[1] These models' predictions are also at odds with the imprint of misallocation across both the firm size distribution and the firm life cycle in developing countries. While firm-level evidence from developing countries suggests that large firms may face more severe distortions (Hsieh and Olken, 2014), these models predict that financial constraints distort mainly small or young firms. Furthermore, while data suggest that the dispersion in marginal revenue product of capital (MRPK) for a cohort of firms is highly persistent over time (Banerjee and Moll, 2010; David and Venkateswaran, 2019), these models predict a short dispersion half-life as self-financing quickly undoes financial frictions (Moll, 2014).

A common assumption used in financial constraint models is that firm-level TFP is exogenous and not affected by financial frictions. Recently, however, a growing literature emphasizes that firms can improve their TFP by investing in management practices or hiring professional managers (Bloom and Van Reenen, 2007, 2010; Guner et al., 2018). The literature has also offered evidence that managerial inputs are non-homothetic in firm size: large firms hire disproportionately more managers (Grobovsek, 2020; Akcigit et al., 2021). In this paper, we introduce such firm expenditures on managerial inputs to the standard collateral constraint model used by Midrigan and Xu (2014) and re-examine the quantitative effects of financial frictions on capital misallocation and aggregate TFP. We show that our model with non-homothetic managerial inputs can better match financial constraints faced by firms

[1] The literature does find larger effects of financial constraints at the extensive margin, on entry decisions, technology adoption, or sectoral choice of firms. Our focus in this paper, however, is on the misallocation of production factors among incumbent firms.

across both the firm size distribution and the firm life cycle in the data, and amplify the impact of financial frictions on aggregate productivity.

The logic of our argument is as follows: Consider a firm with a productive blueprint but little collateral in a country with weak financial development, and consequently, tight collateral requirements. Initially, the firm can only operate on a small scale due to its limited collateral, which yields an MRPK that is substantially higher than the interest rate. This high MRPK then incentivizes the firm to save towards relaxing the collateral constraint, and as it does so, its MRPK declines. This is the standard self-financing channel that mitigates the impact of collateral constraints on aggregate TFP, limits distortions to small firms and young firms, and generates quantitatively fast-resolving MRPK dispersion for a cohort of firms. We introduce the option to hire professional managers. Such an option has two implications: First, in our framework, as in the data, expenditures on hiring managers are non-homothetic, with large firms hiring disproportionately more managers. Therefore, as a firm becomes larger by accumulating collateral and physical capital, its expenditure share on managers increases, and its profit margin and hence the ability to self finance declines. Second, as a firm hires more managers, its elasticity of scale increases, which in turn increases capital demand and hence the MRPK. These new channels partially offset the speed at which the self-financing channel undoes collateral constraints. With non-homothetic managerial inputs, the MRPK dispersion of a firm cohort is more persistent, and large firms and old firms are more likely to be financially constrained.

To quantitatively assess the contribution of our novel channels, we compare our benchmark model with non-homothetic managerial inputs and an otherwise identical model but without managerial inputs. To ensure a fair comparison, we calibrate both models to match exactly the same set of moments commonly chosen in the literature using firm-level data from China. Particularly, we calibrate the collateral constraint and productivity shock process such that, in the steady state, both models generate the same debt-to-output ratio and output dispersion and autocorrelation—moments used in Moll (2014) and Midrigan and Xu (2014), among others. We calibrate the management parameters to match the distribution of manager-to-worker ratio in the firm-level data, i.e., large firms hire more managers per employee. Since our benchmark model has one more factor of input (managers), we make sure that both models have the same aggregate profit margin and capital share in the stationary equilibrium.

Comparing the results from the two calibrated models, we find that it takes twice as much time for a high-productivity but low-collateral entrepreneur to save up to the unconstrained level when we allow for non-homothetic managerial inputs. The dispersion in MRPK of a cohort of firms is also more persistent in our model: It takes roughly twice as many periods for the cohort to eliminate 90 percent of its initial dispersion in MRPK through self financing. Consequently, in the steady state, firms are more likely to face binding financial constraints in our benchmark model than in the model without managerial inputs, and as a result, the efficiency gain associated with eliminating the collateral constraint from our model (6.4 percent) is almost twice as large as that of the model without managerial inputs (3.7 percent).

We also highlight that the key to our results is that managerial inputs are non-homothetic in firm size, consistent with the empirical evidence. If instead the production function is homothetic and the equilibrium manager-to-worker ratio is constant among firms, then allowing for managerial inputs would have no impact on the effects of collateral constraints, provided that the models are calibrated to match the aggregate capital share in the data. Given the central role of the non-homotheticity, our results can be interpreted more broadly: Allowing for other inputs, such as skilled labor or innovation efforts, that enter the production function non-homothetically could amplify the impacts of collateral constraints in a similar way.

Our paper mainly contributes to the misallocation literature.[2] Several papers have also documented that endogenizing firm TFP or allowing for productivity-enhancing inputs amplifies aggregate productivity loss arising from policy distortions (e.g. Gabler and Poschke, 2013; Bhattacharya et al., 2013; Ranasinghe, 2014; Da-Rocha et al., 2019) and more specifically from collateral constraints (e.g. Lopez-Martin, 2016; Vereshchagina, 2022). We differ from these papers by highlighting that, in addition to effects on aggregate productivity, allowing for non-homothetic managerial inputs allows us to better match the persistence in

[2] See Restuccia and Rogerson (2008), Guner et al. (2008), Hsieh and Klenow (2009), Buera and Shin (2013), Moll (2014), Midrigan and Xu (2014), Adamopoulos and Restuccia (2014), Hsieh et al. (2019), and Chen et al. (2022), among others.

MRPK dispersion among firm cohorts and to explain how large and productive firms may be constrained as well. We also highlight theoretically that the key to these results is that large firms spend disproportionately more on managerial inputs, a pattern consistent with data. In this way, our paper is also related to the recent macroeconomic literature on the firm size distribution, firm management, and their relationship to economic development.[3] Finally, our paper studies frictions in the Chinese context, and hence is also related to the literature on misallocation in China.[4]

1 Evidence on Firm Size and Managerial Inputs

Literature has provided evidence that managerial inputs are non-homothetic in firm size. Akcigit et al. (2021) use firm-level data from the U.S. and document that firms with more than 100 employees have higher than 10 percent managerial employment shares on average, while firms with fewer than five employees have virtually no managers. Similarly, Grobovsek (2020) studies French firm-level data and documents two important facts: First, the share of managerial employees increases with firm size. Second, the share of managerial compensation is higher for larger firms and hence the profit share declines with firm size.

We also observe this non-homotheticity of managerial inputs by firm size in the Chinese firm-level data, the Annual Surveys of Industrial Production for the period of 1998-2007 from the National Bureau of Statistics of China. The data include information on firms' capital, labor, intermediate input, and output. Additionally the 2004 sample also includes information on worker composition.[5] We define managers as workers with senior titles but are not technicians, sort firms by size (number of employees), and then calculate the percentage of employees working as managers in each size group as the total headcount of managers divided by the total headcount of employees. The results are in Table 1. Among all firms, around 4.1 percent of workers are managers. This number is clearly increasing in firm size: For instance, among the largest five percent firms, this number is 4.9 percent, which is

[3]See Garicano and Rossi-Hansberg (2006), Bloom and Van Reenen (2010), Haltiwanger et al. (2013), Hsieh and Klenow (2014), Grobovsek (2020), and Akcigit et al. (2021), among others.

[4]See Brandt et al. (2013), Hsieh and Song (2015), Bai et al. (2018), Tombe and Zhu (2019), Gai et al. (2021), König et al. (2022), Adamopoulos et al. (2022a), and Adamopoulos et al. (2022b), among others.

[5]Appendix A provides a detailed description of data.

Table 1: Manager-Worker Ratio among Firms of Different Size

	Employment Threshold	Managers as Percentage of Total Employees
All Firms	–	4.1 (0.1)
Largest 25 Percent Firms	200	4.2 (0.1)
Largest 10 Percent Firms	435	4.5 (0.2)
Largest 5 Percent Firms	733	4.9 (0.3)
Largest 1 Percent Firms	2,200	5.8 (0.8)

Note: This table lists the percentage of total employees working as managers across different size of firms. For each group, we separately calculate the total headcount of managers and divide it by the total headcount of employees to obtain our percentage. Standard errors obtained from bootstrap repetitions are in brackets. Data are the Annual Surveys of Industrial Production from the National Bureau of Statistics of China.

significantly higher than that of the entire sample (4.1 percent). It further increases to 5.8 percent among the largest one percent firms and it is significantly different from that of the largest five percent firms.[6] Clearly, larger firms hire disproportionately more managers compared to smaller firms.[7]

Motivated by the evidence, we next introduce non-homothetic managerial inputs into an otherwise standard collateral constraint model and re-examine the role of financial frictions in capital misallocation.

2 Model

2.1 Preferences and Endowments

The economy consists of two types of infinitely-lived individuals: workers and entrepreneurs. There is a measure N_w of infinitely-lived workers. In each period, each worker has one unit of time that is supplied inelastically to the labor market and earns a flat wage income. Workers' labor supply is used as production labor or transformed to managerial inputs. Workers do

[6] Note that the elasticity of manager share with respect to firm size is smaller than those found in Grobovsek (2020) and Akcigit et al. (2021), who use French and U.S. data. This is likely because of more severe contracting frictions in China between entrepreneurs and managers, which hinder productive firms from hiring outside managers and expanding, as discussed in Grobovsek (2020) and Akcigit et al. (2021).

[7] While our focus in this paper is on managerial inputs, the non-homotheticity also applies more generally to skilled labor inputs. For instance, 8.3 percent of employees have some college education in our sample. If we restrict to the largest five percent firms, this number increases to 9.6 percent, and it further increases to 10.7 percent among the largest one percent firms. Details are in Table 1 in Appendix B.

not save and live hand-to-mouth. In addition, there is a measure N_e of infinitely-lived entrepreneurs, who differ in exogenous entrepreneurial ability z. Entrepreneurs operate firms to produce the single output good, which is treated as the numeraire and can be used for consumption or capital formation. Note that we abstract from the occupational choice problem between entrepreneurs and workers to focus on the misallocation among incumbent firms rather than selection.

Entrepreneurs' preferences are described by the following utility function:

$$U(\mathbf{c}) = \mathbb{E}_z\Big[\sum_{t=0}^{\infty} \beta^t u(c_t)\Big], \quad \text{where} \quad u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}.$$

Here, β is the discount factor and σ is the coefficient of relative risk aversion. The expectation is taken over the realization of ability z, which varies over time according to a stochastic process known to entrepreneurs. Worker preferences are similar except that they are not subject to the uncertainty arising from entrepreneurial ability.

2.2 The Entrepreneur's Problem

An entrepreneur with ability z can operate a firm with endogenous productivity $A(z, m)$. Here, m is the number of hired managers. We assume that $A(z, m)$ is increasing in z and m. The production function is

$$y = A(z, m)^{1-\gamma} \left(k^\alpha n^{1-\alpha}\right)^\gamma,$$

where k and n are capital and production labor input, respectively, and α and γ determine the factor shares and the span of control.

We follow Evans and Jovanovic (1989) and Moll (2014) by assuming that the collateral constraint takes the form of $k \leqslant \phi a$. Here a is the entrepreneur's asset holdings used as collateral, and parameter ϕ hence governs the stringency of the collateral constraint, where a smaller ϕ indicates a tighter constraint. This parameter can be easily micro-founded by the degree of contract enforcement in an economy, as in Buera et al. (2011) and Midrigan and Xu (2014).

The firm, operated by an entrepreneur with ability z and asset holdings a, has the following profit maximization problem:

$$\pi(a,z) = \max_{m,k,n} \left\{ A(z,m)^{1-\gamma} \left(k^{\alpha} n^{1-\alpha}\right)^{\gamma} - Rk - wn - p_m m \right\}, \quad s.t. \quad k \leqslant \phi a, \tag{1}$$

where R and w are the interest rate and wage rate, respectively, and p_m is the unit cost of managerial service. Denote the demand for capital, labor, and managerial input as $k^d(a,z)$, $n^d(a,z)$, and $m^d(a,z)$, respectively, and the output as $y(a,z)$.

An entrepreneur begins a period with asset holdings a and ability z. Her consumption-savings problem can be written in recursive form:

$$V(a,z) = \max_{a' \geqslant 0} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_{z'}[V(a',z')|z] \right\},$$
$$\text{s.t.} \quad c + a' \leqslant (1+r)a + \pi(a,z),$$

where we use x' to denote the value of x in the next period.

2.3 Market Arrangement

A representative financial intermediary owns capital and rents it to entrepreneurs at interest rate R. This financial intermediary finances its capital through issuing a one-period risk-free bond (a) that is held by entrepreneurs, which is in turn used as their collateral. The interest rate of this bond is r. The financial intermediary makes zero profit, and hence we have $R = r + \delta$ in equilibrium, where δ is the depreciation rate of capital.

It costs κ units of labor to produce a manager, where $\kappa > 1$ represents for instance costs for training managers. The unit cost of m is hence $p_m = w\kappa$. As such, aggregate labor demand consists of two components: production labor and labor used to train managers.

2.4 Aggregation and Equilibrium

Let $G(a,z)$ be the joint distribution of entrepreneurs over the asset holdings and ability. Aggregate demand for managers m is given by

$$M^d = N_e \int_{a,z} m^d(a,z) G(da, dz),$$

where $m^d(a,z)$ represents the demand for managers of a firm with asset (bond) holdings a and entrepreneurial productivity z. To produce M^d managers, $M^d\kappa$ units of labor is used. Aggregate capital, labor, and output demands are hence given by

$$K^d = N_e \int_{a,z} k^d(a,z)G(da,dz),$$
$$N^d = N_e \int_{a,z} n^d(a,z)G(da,dz) + M^d\kappa,$$
$$Y^d = \delta K^d + N_e \int_{a,z} c(a,z)G(da,dz) + N_w c_w,$$

where $c(a,z)$ is the consumption of entrepreneurs of type (a,z) and c_w is worker consumption. Aggregate capital, labor, and output supplies are

$$K^s = N_e \int_{a,z} aG(da,dz),$$
$$N^s = N_w,$$
$$Y^s = N_e \int_{a,z} y(a,z)G(da,dz).$$

The formal definition of the stationary competitive equilibrium is in Appendix C.

2.5 Non-homothetic Productivity and Financial Constraints

Before moving on to quantitative exercises, We first discuss a key property of our model, non-homotheticity of managerial inputs, and how it interacts with firms' financial constraints.

The firm optimization problem (1) can be written in two steps. First, given the capital stock, the firm chooses the number of managers m and labor input n to maximize the operating profit:

$$\tilde{\pi}(z,k) = \max_{m,n} \left\{ A(z,m)^{1-\gamma} \left(k^\alpha n^{1-\alpha}\right)^\gamma - wn - p_m m \right\}. \tag{2}$$

Then, the optimization problem (1) can be rewritten as

$$\pi(a,z) = \max_k \left\{ \tilde{\pi}(z,k) - Rk \right\}, \quad s.t. \quad k \leqslant \phi a.$$

The first order conditions of the first step optimization problem (2) are

$$\gamma(1-\alpha)A(z,m)^{1-\gamma}k^{\alpha\gamma}n^{(1-\alpha)\gamma-1} = w, \quad (1-\gamma)A(z,m)^{-\gamma}\frac{\partial A(z,m)}{\partial m}k^{\alpha\gamma}n^{(1-\alpha)\gamma} = p_m.$$

It can be easily shown that the optimal m and n are both increasing in k. Let

$$\varepsilon_{A,m} = \frac{\partial \ln A(z,m)}{\partial \ln m}$$

be the elasticity of productivity $A(z,m)$ with respect to the number of managers m. Then, from the first order conditions above we have

$$\frac{m}{n} = \frac{w}{p_m}\frac{1-\gamma}{\gamma}\frac{1}{1-\alpha}\varepsilon_{A,m}. \tag{3}$$

That is, the optimal manager-to-production-worker ratio of a firm is proportional to the elasticity $\varepsilon_{A,m}$. As we documented in Section 1, this ratio is increasing with firm size and hence the number of mamagers m in the data. To be consistent with the empirical fact, we make the following assumption about the productivity function:

Assumption 1. *The productivity function $A(z,m)$ is such that $\varepsilon_{A,m}$ increases in m.*

This empirically motivated assumption has three important implications. First, it implies that the productivity function is non-homothetic with respect to the number of managers. Second, it implies that the share of operating profit is decreasing in firm size, which can be seen clearly from the following equation:

$$\frac{\tilde{\pi}(z,k)}{y(z,k)} = 1-\gamma(1-\alpha)-(1-\gamma)\varepsilon_{A,m}.$$

Thus, as an entrepreneur accumulates assets, capital $k = \lambda a$ increases, m increases, and then the profit margin decreases, consistent with evidence in Grobovsek (2020). As a result, as a firm grows, entrepreneur's ability to accumulate assets does not increase proportionately due to lower profit margin earned by the entrepreneur.

Finally, this assumption implies that a firm's elasticity of scale also increases with firm size. Specifically, the elasticity of scale, which characterizes what happens to output if we scale up all input by some small amount ψ, is given by

$$e(m,k,n) = \frac{d\ln y(\psi m, \psi k, \psi n)}{d\ln \psi}\Big|_{\psi=1} = (1-\gamma)\frac{d\ln A(z,\psi m)}{d\ln\psi}\Big|_{\psi=1} + \gamma = (1-\gamma)\varepsilon_{A,m} + \gamma.$$

If $\varepsilon_{A,m} \in (0,1)$ and $\varepsilon_{A,m}$ increases in m, then the technology has decreasing returns to scale, but the elasticity of scale increases in m and hence in firm size. This property that hiring managers helps increase the elasticity of scale is consistent with evidence in Grobovsek (2020) and Akcigit et al. (2021), and our framework can be viewed as a reduced-form approach of the hierarchy models such as Garicano and Rossi-Hansberg (2006) and Grobovsek (2020).

The increasing elasticity of scale tends to partially offset the self financing mechanism: As an entrepreneur with binding collateral constraint accumulates assets, she is able to borrow more; k increases and hence the marginal product of capital (MRPK) declines, which is the standard self-financing mechanism. In our model, however, as k increases, the optimal m increases. With increasing $\varepsilon_{A,m}$, the elasticity of scale increases, which in turn increases capital demand and raises the MRPK. This mechanism partially undoes the standard self-financing mechanism. As a result, firm MRPK declines more slowly along with self finance, making the MRPK dispersion among firm cohorts more persistent.

In summary, in our model with a productivity function that is non-homothetic in managerial inputs, as a firm accumulates collateral and physical capital, its profit margin declines, which slows down asset accumulation, and its elasticity of scale increases, which reduces the negative effect of asset accumulation on MRPK. Both of these effects lengthen the time when the firm faces financial constraints. These effects would be absent, however, if the technology does not allow for managerial inputs or if the productivity elasticity with respect to the number of managers is constant.

3 Quantitative Results

We now quantify the role of financial constraints in our model that is calibrated to Chinese data. In particular, we compare the predictions of our model to that of an otherwise identical model without managerial inputs, which is similar to Buera et al. (2011) or Midrigan and Xu

(2014) without firm entry and exit. We also use our quantitative results to further highlight the importance of non-homothetic productivity function with respect to managerial inputs.

3.1 Calibration

Given that our goal is to compare the quantitative predictions of two models—with and without non-homothetic managerial inputs—it is crucial that we calibrate them to match the same set of data moments. Equally important is that we target a set of moments that are typically chosen in the literature to help the comparison. We calculate the data moments using information from the Annual Surveys of Industrial Production and the 2005 Chinese Population Census. Note that matching the same set of moments does not imply the same parameter values between the two models.

3.1.1 Parameterization

For our quantitative analysis, we assume the functional form of firm productivity to be

$$A(z,m) = e^z\left(T^{\frac{\theta-1}{\theta}} + \lambda m^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta\eta}{\theta-1}},$$

where T is the entrepreneur's own time spent in management, which we normalize to unity without loss of generality; m is the measure of hired professional managers; $\lambda < 1$ governs the contribution of the outside managers to firm productivity; $\theta > 1$ governs the elasticity of substitution between entrepreneur's own time and that of outside managers; and η determines the maximum elasticity of scale. Our setup nests Akcigit et al. (2021) as a special case when we set θ to infinity and $\eta = 1$. λ can be interpreted as supervision efficiency, given that the entrepreneur needs to expend effort to supervise the outside managers, with a lower λ indicating a lower supervision efficiency (Akcigit et al., 2021). One can verify that

$$\varepsilon_{A,m} = \eta\frac{\lambda m^{\frac{\theta-1}{\theta}}}{T^{\frac{\theta-1}{\theta}} + \lambda m^{\frac{\theta-1}{\theta}}},$$

and hence it is increasing in m as long as $\theta > 1$. Note that $\varepsilon_{A,m} \leqslant \eta$ and hence the maximum elasticity of scale is $\gamma + (1-\gamma)\eta$.

We follow Midrigan and Xu (2014) in assuming that entrepreneurial ability z has a permanent component $\bar{z}$ and a transitory component $\tilde{z}$: $z = \bar{z} + \tilde{z}$. The permanent component $\bar{z}$ follows a Gaussian distribution with standard deviation $\sigma_{\bar{z}}$. The transitory component $\tilde{z}$ follows an AR(1) process with Gaussian disturbances:

$$\tilde{z}' = (1 - \rho)\tilde{z} + \varepsilon_{\tilde{z}},$$

where ρ determines the persistence of the transitory component, and $\varepsilon_{\tilde{z}}$ is the disturbance term with a standard deviation $\sigma_{\tilde{z}}$. This AR(1) process is then approximated using Rouwenhorst method in our quantitative analysis.

3.1.2 Determining Parameter Values

Demographics and Preferences.—N_w and N_e govern the population share of workers and entrepreneurs. We normalize $N_e = 1$ and choose $N_w = 5.43$ such that 15.5 percent of individuals are entrepreneurs, as in the 2005 Chinese Population Census. We choose the coefficient of relative risk aversion σ to be 2. The discount factor β is chosen in both models to match an overall capital-to-output ratio of 3 as we observed in the Penn World Table (Feenstra et al., 2015).[8]

Entrepreneurial Ability Distribution.—We follow Midrigan and Xu (2014) and choose the persistence parameter ρ and dispersion parameters $\sigma_{\bar{z}}$ and $\sigma_{\tilde{z}}$ to jointly match three moments from the data: the one-year and five-year autocorrelation of log output of 0.88 and 0.77, respectively, and the standard deviation of log output of 1.26.

Technologies.—The elasticity of capital input α is chosen to match capital share of 0.33.[9] In our benchmark model γ is not the typical span-of-control parameter since the managerial input enters $A(z, m)$. Because the span-of-control is closely related to the profit margin, we choose γ such that the aggregate profit margin is identical between the two models at 0.3,

[8] The capital-to-output ratio for China varies substantially in different versions of the Penn World Table. For the year 2004, the capital-output ratio is around 3.4 in version 9.1 while it changes to 2.6 in version 10.0. We hence choose a value of 3 which falls roughly in the middle.

[9] Note that, with collateral constraints, a capital share of 0.33 does not necessarily imply $\alpha\gamma = 0.33$, since MRPK does not necessarily equal the interest rate in our case.

which is crucial in comparison (Vereshchagina, 2022).[10] The rate of depreciation δ is set to 0.06.

Collateral Constraint.—We follow the common practice in the literature of choosing ϕ to match the debt-to-output ratio in the Chinese data of 0.64.

Management.—The management parameters only apply to our benchmark model with non-homothetic managerial inputs. Recall that each unit of managerial inputs is produced with κ units of labor. We choose $\kappa = 1.96$ such that the wage premium of managers relative to workers is 1.96 as in the 2005 Chinese Population Census. We choose $\eta = 0.63$ such that the elasticity of scale is bounded from above by $\gamma + (1 - \gamma)\eta = 0.9$, or the largest firm in the limit has a profit share of 10 percent. The efficiency of supervision λ and the elasticity between entrepreneur's own time and that of managers θ are chosen to jointly match two moments: In aggregate, 4.1 percent of the worker population works as managers; and the manager-to-worker ratio of the largest one percent firms is 19.9 percent higher than that of the largest five percent firms. The second moment exploits the key prediction of our model that m/n increases in firm size.

[10] The targeted profit margin varies in the literature: For instance, Restuccia and Rogerson (2008) and Midrigan and Xu (2014) choose 0.15 while Yang (2021) chooses 0.5. Our choice of 0.3 falls in the ballpark. Note that a lower profit margin and hence a higher γ implies larger misallocation (Hopenhayn, 2014).

Table 2: Calibration—Parameters and Values

Parameters	Value: Benchmark	Value: w/o Managers	Data Moments
N_e: measure of entrepreneurs	1	1	Normalization
N_w: measure of workers	5.433	5.433	Entrepreneur share of 15.5%
σ: coefficient of relative risk aversion	2	2	Literature
β: discount factor	0.928	0.932	Capital-output ratio of 3
ρ: autocorrelation of ability	0.489	0.486	1-year autocorrelation of output of 0.88
$\sigma_{\bar{z}}$: s.d. of permanent component	1.004	1.095	5-year autocorrelation of output of 0.77
$\sigma_{\tilde{z}}$: s.d. of i.i.d. disturbance	0.663	0.679	S.d. of log output of 1.26
γ: span of control	0.727	0.744	Identical profit share (0.3) between models
α: elasticity of capital ($\alpha\gamma$)	0.532	0.502	Capital share of 0.33
δ: depreciation rate	0.06	0.06	Literature
ϕ: collateral constraint	1.474	1.522	Debt-to-output ratio of 0.64
κ: labor used to produce management	1.960	–	Manager wage premium of 1.96
η: return to management	0.634	–	Highest return to scale of 0.9
λ: efficiency of supervision	0.256	–	4.1% of workers work as managers
θ: elasticity: entrepreneur and managers	1.675	–	Distribution of manager-worker ratio

Untargeted Moment	Benchmark	w/o Managers	Data
semi-elasticity of operating profit share with respect to size	−0.0072	0	−0.0142

Note: This table lists the calibrated parameter values and the model-implied elasticity of the operating profit share with respect to firm size in both our benchmark model and the model without managerial inputs, respectively, along with the corresponding data moments.

In summary, we have 15 parameters (11 for the model without managers) in total, with N_e, N_w, σ, κ, and δ taking directly assigned values, and η, γ, β, ρ, $\sigma_{\tilde{z}}$, $\sigma_{\bar{z}}$, α, ϕ, λ, and θ being jointly determined by comparing equilibrium model moments with those from the data. The value of these parameters are listed in Table 2.

A key implication of our model is that the operating profit share of value-added declines with firm size. Using the Chinese firm level panel data, we regress the operating profit share on firm size measured by log employment, controlling for firm fixed effects. We find a significantly negative coefficient on firm size of -0.0142, implying that a one hundred percent increase in firm size is related to a 1.42 percent decline in the operating profit share. Using the simulated data from our model, we find that a one hundred percent increase in firm size reduces the operating profit share by 0.72 percentage points. Hence, our model accounts for a large portion of the decline in the operating profit share with firm growth observed in the data.

3.2 Model Comparison

We now compare the quantitative predictions of our benchmark model with non-homothetic managerial inputs to those of the model without managers, both of which are calibrated to match the same sets of moments.

We begin by showing how allowing for non-homothetic managerial inputs quantitatively increases the persistence of collateral constraints for productive entrepreneurs. Consider in each model a peak-ability entrepreneur (with ability $\bar{z}$ and $\tilde{z}$ at the highest grid point) who has little collateral (with assets a at the 25th percentile of the stationary distribution), and for whom the collateral constraint initially binds in both models. In the model without managers, this entrepreneur, following her optimal policy function, undoes the collateral constraint in about 19 periods. In contrast, it takes the same entrepreneur 35 periods to self finance in our benchmark model, a significant increase in persistence.

Our model with non-homothetic managerial inputs also increases the persistence of MRPK dispersion of firm cohorts, improving the model's ability to match this feature of the data documented in, for instance, David and Venkateswaran (2019). Consider a firm cohort consisting of the peak-ability entrepreneurs, who have initial assets matching the equilibrium invariant marginal distribution for their type, $G(a, z|\bar{z} = \bar{z}^{max}, \tilde{z} = \tilde{z}^{max})$.[11] The collateral constraint is initially binding for most of them. We use their policy functions to calculate the evolution of their assets and to trace out the dispersion of MRPK within this cohort over time, keeping their ability invariant. The results are reported in the first four rows of Table 3. In the model without managers, the standard deviation of log MRPK falls to less than 10 percent of its initial level by the 15th period. In our benchmark model, however, the standard deviation falls to less than 10 percent of its initial level by the 29th period, which almost doubles the length of the model without managers.

Due to greater persistence in the collateral constraint, firms of all sizes are more likely to be financially constrained in our benchmark model than in the model without managerial inputs, as shown in the middle four rows of Table 3. After eliminating the collateral constraint, there is also more capital reallocation in our benchmark model than in the model without managers. The last four rows of Table 3 show that the amount of capital used by the top TFP quartile firms increases by 21.6 percent in the benchmark model, but only 16.7 percent in the model without managers. Consequently, allowing for non-homothetic managerial inputs substantially amplifies the effect of collateral constraints on aggregate TFP. Eliminating collateral constraints in the benchmark model increases aggregate TFP by 6.4 percent, in contrast to only 3.7 percent in the model without managerial inputs.

The larger aggregate TFP loss in our benchmark model are not driven by channels previously identified as important in the literature: higher equilibrium firm TFP dispersion or lower firm TFP persistence. First, the standard deviation of log of firm TFP $A(z, m)$ is similar between the two models (1.333 in the benchmark model and 1.336 in the model without managerial inputs), as we calibrate the ability distribution in both to match the same data moment, dispersion in firm output. Second, while the literature shows that less persistent ability processes increase TFP losses (e.g. Moll, 2014), this is not the driving reason in our case, as the calibrated values of ρ and the relative importance of $\bar{z}$ and $\tilde{z}$ are similar between the two models. Nevertheless, we conduct a robustness exercise in Appendix D by re-calibrating the model without managerial inputs that restricts the ability process z to be

[11]Note that, without entry or exit, firm age is not well defined in our model. We hence focus on a firm cohort constructed with assets matching its distribution in the stationary equilibrium.

Table 3: Comparison between Two Setups

	Benchmark Model	Model without Managers
Dispersion in MRPK of a constructed cohort:		
Initial (normalized, %)	100	100
14th period (%)	30.8	11.3
15th period (%)	29.3	2.0
28th period (%)	11.6	0.0
29th period (%)	7.6	0.0
% of firms with binding financial constraint, by firm size:		
Q1	30.9	24.2
Q2	42.3	35.3
Q3	47.3	43.1
Q4	59.3	52.2
Changes after eliminating financial constraint (%)		
Aggregate output	+6.4	+3.7
Firm capital usage, by productivity quartiles:		
Q1	−30.0	−24.7
Q2	−26.9	−21.0
Q3	−13.4	−15.2
Q4	+21.6	+16.7

Note: This table compares moments of interest computed at the stationary equilibrium for our benchmark model and the model without managerial inputs, both of which are calibrated to match the same data moments. MRPK dispersion is computed from a firm cohort consisting of entrepreneurs with ability z at the highest grid point and initial assets matching the equilibrium marginal distribution for their type, $G(a, z|\bar{z} = \bar{z}^{max}, \tilde{z} = \tilde{z}^{max})$.

identical to that of our benchmark model, while the remaining parameters are calibrated to match the same moments, and all predictions remain similar. More generally, Appendix D shows that our results hold even if we restrict the value of all parameters, except for the managerial ones, to be the same in both models. Hence, we conclude that our results are not driven by parameter value differences. Finally, the equilibrium dispersion in MRPK is slightly larger in the benchmark model than the model without managerial inputs. Our results hold if we re-calibrate the model without managerial inputs to match the same MRPK dispersion rather than the debt-to-output ratio.

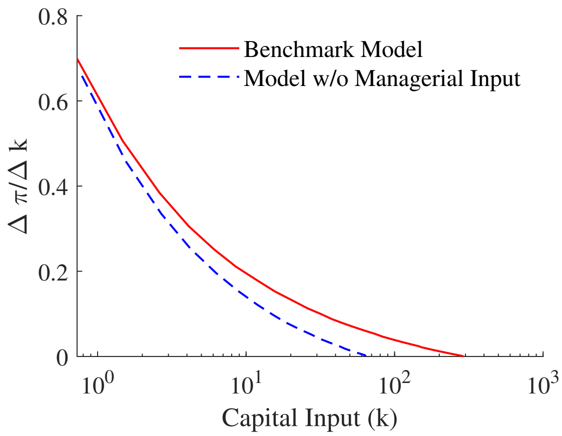
3.3 Discussion

3.3.1 Understanding the Mechanisms

To further illustrate how the mechanism of increasing the elasticity of scale matters, we plot in Figure 1 the return to capital, calculated as $\Delta\pi/\Delta k$, for the peak-ability entrepreneur for different levels of k, for both models. Note that we normalize capital employed by this entrepreneur by average capital per entrepreneur in each model to make sure it is unit free. One can clearly see that for all level of capital k, the return of additional capital is higher in the benchmark model than in the model without managerial inputs, highlighting the property that allowing for managerial inputs increases capital demand through increasing the elasticity of scale.

We present a decomposition exercise to illustrate the importance of the two mechanisms at play in our benchmark model—lower profit margins and increasing elasticity of scale. Specifically, we design a type-specific tax/subsidy to entrepreneurs, $\tau(a, z)$, such that the operating profit share, $\tilde{\pi}/y$, is constant, and this tax/subsidy is revenue neutral ($\int_{a,z} \tau(a, z)G(da, dz) = 0$), not involving any aggregate transfer between workers and entrepreneurs (Itskhoki and Moll, 2019). This tax affects the entrepreneur's consumption-savings decision by equalizing the operating profit share and hence shuts off the varying profit margin mechanism, but does not affect firm's profit maximization problem and the mechanism of varying the elasticity of scale. With this tax, it takes 23 periods to eliminate 90% of MRPK dispersion following an entrepreneur cohort with the highest grid point ability

Figure 1: Return to Capital



Note: This figure illustrates the return to capital, calculated as $\Delta\pi/\Delta k$, for the peak-ability entrepreneur (with ability $\bar{z}$ and $\tilde{z}$ at the highest grid point) for different levels of k, for the benchmark model and the model without managerial inputs, respectively.

and the equilibrium distribution of assets a, 6 periods shorter than in the benchmark model, but 8 periods longer than in the model without managerial inputs. Hence, both mechanisms contribute to greater persistence of MRPK dispersion.

Note that our calibration strategy implies that the average profit margin, and hence the average elasticity of scale, is identical between the two models. In our benchmark model, however, the profit share and the elasticity are heterogeneous. Specifically, low-ability entrepreneurs have smaller firms with higher operating profit share and lower elasticity of scale, and hence they self-finance faster than their counterparts in the model without managerial inputs; on the contrary, high-ability entrepreneurs have larger firms with lower operating profit share and higher elasticity of scale, and hence they self-finance slower. In the stationary equilibrium, high-ability entrepreneurs are more likely to have binding collateral constraints, and therefore on average it takes longer for entrepreneurs in our benchmark model to accumulate enough assets to become unconstrained.

3.3.2 The Importance of Varying $\varepsilon_{A,m}$

The assumption that $\varepsilon_{A,m}$ increases in m is key to our results. To see this, we consider an otherwise identical model with managerial inputs but we set $\theta = 1$ such that the elasticity of $A(z,m)$ with respect to m is constant. In this case, the production function is simply

$$y = (e^z)^{1-\gamma}\Omega m^{\frac{\lambda\eta(1-\gamma)}{1+\lambda}}(k^\alpha n^{1-\alpha})^\gamma,$$

where Ω is a collection of constants. Clearly, in this case, the manager-to-worker ratio m/n should be identical across firms. One can then redefine a composite labor input as

$$\tilde{n} = \left(m^{\frac{\lambda\eta(1-\gamma)}{1+\lambda}} n^{(1-\alpha)\gamma}\right)^{\frac{1}{(1-\alpha)\gamma}}$$

and then the production function can be written as

$$y = (e^z)^{1-\gamma}(k^\alpha \tilde{n}^{1-\alpha})^\gamma.$$

With a calibration strategy that targets the same profit share and capital/labor share, this production function is identical to the one in the model without managerial inputs and hence in this case allowing for managerial inputs does not directly affect the role of the collateral constraints. Intuitively, when m/n is identical across firms, by allowing for managerial inputs, we implicitly split the elasticity of labor into two components, that of raw labor and that of managers. This should not directly affect misallocation as long as there is no distortion between m and n.

4 Conclusion

The canonical model of collateral constraints typically predicts that they bind only for young firms and small firms, while older firms are unaffected due to their accumulated assets. This self-financing channel also leads these models to generate dispersion in MRPK that declines too rapidly within a firm cohort compared to the data. We argue that allowing firms to hire professional managers moves the predictions of models with collateral constraints closer to the data, by partly offsetting the effects of the self-financing channel. Particularly, along with self finance, firms grow larger and optimally spend disproportionately more on managerial inputs, reducing their profit margin and ability to self finance; in addition, hiring more managers helps increase the elasticity of scale, thus further increases capital demand.

We then calibrate our model to Chinese data. By comparing our benchmark model to a similarly calibrated model without managerial inputs, we find that, in our benchmark model, it takes twice as long for an entrepreneur with high productivity but low net worth to accumulate enough assets to become unconstrained, and the dispersion of MRPK within a firm cohort is also substantially more persistent. These properties imply that, with non-homothetic managerial inputs, high-productivity firms are more likely to be constrained and, as a result, the impact of collateral constraints on aggregate output is twice as large.

Although we interpret the productivity-enhancing input in our model as management practices, our findings are potentially more general. Other productivity-enhancing inputs, such as skilled labor or innovation efforts, could have similar effects provided that larger firms spend disproportionately more on these inputs.

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Finance, Managerial Inputs, and Misallocation
Online Appendix

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A Data

Our main data source is the NBS China's Annual Surveys of Industrial Production. This data set is widely used in the literature (e.g. Hsieh and Klenow, 2009). Our sample covers private manufacturing firms with sales above five million RMB (around eight hundred thousand USD) in the previous year and all state-owned enterprises regardless of sales. More than 1.6 million firm-year observations are recorded in our data, ranging from the year 1998 to 2007. We have information on firms' capital, labor, intermediate input, and output. Additionally, the 2004 wave also details information on worker composition, including information on hired managers.

We restrict our sample to manufacturing firms only, which is the common practice in the literature (Hsieh and Klenow, 2009). We follow Qian and Zhu (2012) and calculate firm value added as the sum of labor cost (wage and welfare expenditures), value-added tax, depreciation, and profit. We measure labor and capital input as the head count of employees and the value of net fixed asset, respectively. Given the 2004 wave's detailed employee composition, we count managers as employees with senior titles who are not technicians.

To control for the differences between state-owned firms and private firms, we explicitly regress log firm value added on ownership, industry, and year fixed effects to obtain the residual that is in turn used in calculating the dispersion and autocorrelation of output. We further trim our sample by one percent on each tail for value added when we calculate the dispersion.

We use the 2004 wave, which consists of information of employee composition, to calculate other moments used in calibration, such as the share of managers among all employees and debt-to-output ratio. The debt-to-output ratio is constructed as the ratio of interest expenditure and value added, both of which are observed in the data, divided by a nominal interest rate. The nominal interest rate is estimated as the sum of 5-year moving-average of inflation rate and a real interest rate, which is 5 percent per year as in the equilibrium of our baseline calibration.

We also use 2005 Chinese Population Census to calculate the share of entrepreneurs in the labor force and the wage premium of managers. In particular, we classify an individual as an entrepreneur if her reported status is either employer or self-employed and then calculate the share of entrepreneurs accordingly. We classify an individual as a manager if this person is considered a decision maker of an enterprise but serves as an employee rather than an employer. Note that we keep individuals who work in non-agriculture only. Within non-agriculture, the data do not clearly distinguish between manufacturing and service workers. We then calculate the average of log income among managers and among non-managers separately, the difference of which is our measure of the manager wage premium.

We obtain the capital-to-output ratio from the Penn World Table (Feenstra et al., 2015). We note that the capital-to-output ratio for China varies substantially in different versions of the Penn World Table (PWT). For the year 2004, the capital-output ratio is around 3.4 in PWT 9.1, while this number changes to 2.6 in PWT 10.0. We hence choose a value of 3 which falls roughly in the middle.

B Evidence on Skilled Labor

As we describe in Section 1, we find that the ratio of skilled worker increases with firm size. We define skilled workers as workers with college education. Our results also hold if we define skilled workers as workers with, for instance, high school education. We sort firms by size (number of employees), and then calculate the percentage of employees with college education

Table 1: Skilled-Labor Ratio among Firms of Different Size

	Employment Thresholds	Percentage of Employees with College Education
All Firms	–	8.3 (0.1)
Largest 25 Percent Firms	200	8.5 (0.2)
Largest 10 Percent Firms	435	9.1 (0.3)
Largest 5 Percent Firms	733	9.6 (0.4)
Largest 1 Percent Firms	2,200	10.7 (0.7)

Note: This table lists the percentage of total employees with college education across different size of firms. For each group, we separately calculate the total headcount of college graduates and divide it by the total headcount of employees to obtain our percentage. Standard errors obtained from bootstrap repetitions are in brackets. Data are the Annual Surveys of Industrial Production from the National Bureau of Statistics of China.

in each size group as the total headcount of college graduates divided by the total headcount of employees. The results are in Table 1. Among all firms, around 8.3 percent of workers are managers. This number is clearly increasing in firm size: For instance, among the largest five percentile firms, this number is 9.6 percent, which is significantly different from that of the entire sample (8.3 percent). It further increases to 10.7 percent among the largest one percentile firms and again it is significantly different from that of the entire sample. Clearly, larger firms hire disproportionately more skilled workers compared to smaller firms.

Another way to assess the data is to explore whether the share of managerial inputs or skilled labor rises with sales growth over time within firms. Unfortunately we only observe managerial input for one year. Nevertheless, we do observe that the average labor compensation per employee, which is highly related to the skill composition of a firm, increases with sales and the elasticity is 0.29.

C Definition of Competitive Equilibrium

We now define the stationary equilibrium as follows:

Definition 1 (Competitive Equilibrium). *A stationary competitive equilibrium for this economy consists of prices w and r; entrepreneur's optimal savings function $a'(a,z)$, optimal factor demands $k^d(a,z)$, $n^d(a,z)$, and $m^d(a,z)$, output $y(a,z)$, and profit $\pi(a,z)$; and a stationary distribution $G(a,z)$ over entrepreneur assets and ability that satisfies the following* *conditions:*

1. *Given prices and borrowing limits, $k^d(a,z)$, $n^d(a,z)$, $m^d(a,z)$, $y(a,z)$, and $\pi(a,z)$ solve the firm's problem.*

2. *Given prices, $a'(a,z)$ solves the entrepreneurs' consumption-savings problem.*

3. *Wage w and interest rate r clear the labor and capital markets, respectively: $N_w = N^d$, $K^s = K^d$.*

4. *The joint distribution of assets and productivity $G(a,z)$ is stationary.*

D Robustness on Parameter Values

As discussed in Section 4, we compare the quantitative predictions of two models after calibrating them to match the same set of moments. This strategy leads to different parameter values between models. We note that it is not appropriate to force the two models to have the same parameter values, as parameter values are only meaningful within a specific model setup. Nevertheless, in this section, we explore what happens if we restrict some parameters to be the same between two models, and highlight that our results are not driven by different parameter values between models.

To start with, as we described in Section 4.2, the ability process z matters for quantifying misallocation. We hence conduct the following robustness exercise: In the model without managerial inputs, we restrict the ability process z to be identical to that of the benchmark model, and re-calibrate all other parameters to match the same set of moments, except for the autocorrelations and dispersion of log output, which were used to pin down the ability process. The re-calibrated parameter values are in Table 2. Note that the model without managerial inputs now implies 1-year and 5-year autocorrelation to be lower than the data moments. With this alternative calibration, the model without managerial inputs implies that an entrepreneur with ability $\bar{z}$ and $\tilde{z}$ at the highest grid points but 25th percentile asset holdings only needs 18 periods of self-financing to attain the unconstrained level, similar to the same model in the baseline calibration, and is substantially shorter than the 35 periods

Table 2: Calibration—Robustness

Parameters	Value					
	Benchmark Model	Model without Managers				
	Baseline	Baseline	Same z processes	Same ϕ	Same α	Same MPK disp.
N_e	1	1	1	1	1	1
N_w	5.433	5.433	5.433	5.433	5.433	5.433
σ	2	2	2	2	2	2
β	0.928	0.932	0.933	0.930	0.931	0.928
ρ	0.489	0.486	0.489	0.487	0.491	0.492
$\sigma_{\bar{z}}$	1.004	1.095	1.004	1.094	1.093	1.089
$\sigma_{\tilde{z}}$	0.663	0.679	0.663	0.697	0.711	0.724
γ	0.727	0.744	0.742	0.749	0.752	0.756
α	0.532	0.502	0.501	0.506	0.532	0.511
δ	0.06	0.06	0.06	0.06	0.06	0.06
ϕ	1.474	1.522	1.529	1.474	1.450	1.421
κ_n	1.960	–	–	–	–	–
η	0.634	–	–	–	–	–
λ	0.256	–	–	–	–	–
θ	1.675	–	–	–	–	–

Note: This table lists the parameters and calibrated values associated with the alternative calibration exercises.

needed under the benchmark model with non-homothetic managerial inputs. Examining a firm cohort of entrepreneurs with highest grid point ability $\bar{z}$ and $\tilde{z}$ and the equilibrium distribution of assets a, the dispersion of MRPK reduces by 90 percent by the 14th period, which is again similar to that in the baseline calibration and is substantially shorter than the 29 periods of the benchmark model with non-homothetic managerial input. In addition, eliminating the collateral constraint increases aggregate output by 3.4 percent, which is still substantially smaller than the 6.4 percent of the benchmark model with non-homothetic managerial inputs. We summarize this comparison in Table 3.

In our baseline calibration, ϕ—the parameter governing the collateral constraint—turns out to be slightly lower in the benchmark model (1.474) than in the model without managerial inputs (1.522), in order to match the same debt-to-output ratio. Here, we explore the case where we restrict ϕ to be 1.474 in the model without managerial inputs as well, and re-calibrate all other parameters to match the same set of moments, except for the debt-to-output ratio, which was used to pin down ϕ. The re-calibrated parameter values are in Table 2. Note that the model without managerial inputs now implies a debt-to-output ratio of 0.616, which is lower than the data moment (0.638), and hence the collateral constraint is tighter. Even with this tighter collateral constraint, the model without managerial inputs implies a less persistent MRPK than the benchmark model. Particularly, an entrepreneur with ability $\bar{z}$ and $\tilde{z}$ at the highest grid points but 25th percentile asset only needs 20 periods of self-financing to attain the unconstrained level (compared to 35 periods needed under the benchmark model). Examining a firm cohort of entrepreneurs with highest grid point ability and the equilibrium distribution of assets, it takes 16 periods for the dispersion of MRPK to decline to less than 10 percent of its initial level, again substantially shorter than the 29 periods of the benchmark model. Efficiency gain of eliminating collateral constraint increases to 4.3 percent, still smaller than the 6.4 percent of the benchmark model. We summarize this comparison in Table 3.

In the baseline calibration, matching the same capital share of 0.33 implies a smaller α for the model without managerial inputs. We hence also experiment with restricting α to be the same here. Again, we re-calibrate the model without managerial inputs, restricting the value of α to be the same as that of the benchmark model, without targeting the capital income share.[1] In this case, the capital share in the model without managerial inputs is 0.348, which is higher than 0.33. While a larger capital share amplifies the role of the collateral constraint slightly, it is still considerably different from that of the benchmark model (see Table 3).

Lastly, we explore the case where we restrict all corresponding parameter values between the two models to be the same; i.e., for the model without managerial input, we directly use the parameter values from the benchmark model, except for setting λ to zero and θ being irrelevant. In this case we cannot match any calibration moments for the model without managerial inputs. Even in this extreme scenario the predicted role of the collateral constraint is still substantially smaller than that of the benchmark model (see Table 3).

We hence conclude that the different quantitative effects of the collateral constraint between the two models are not driven by differences in parameter values between models. More importantly, we argue again that the parameter values are only meaningful within a specific model setup, and hence a fair comparison between models should be in the baseline calibration, where we calibrate the two models to match the same set of moments, rather

[1] Note that the capital-to-output ratio is closely related to the capital income share. We hence do not target the capital-to-output ratio either but restrict β such that the equilibrium interest rate to be identical to that of the baseline calibration (0.05).

Table 3: Quantitative Predictions with Different Parameter Values

	Periods requiring self-financing	Periods to eliminate 90% of MRPK dispersion	Efficiency gain (+%)
Benchmark Model			
Baseline	35	29	6.4
Model without Managers			
Baseline	19	15	3.7
Same processes of z	18	14	3.4
Same ϕ	20	16	4.3
Same α	22	17	5.0
Same all parameters	18	15	3.9
Same MRPK dispersion	23	18	5.1

Note: This table lists the quantitative predictions associated with different parameter values. The first column reports the number of periods needed for an entrepreneur with highest grid point ability $\bar{z}$ and $\tilde{z}$ and 25th percentile asset a to self-finance her way out of her collateral constraint. The second column reports the number of periods required to eliminate 90 percent of firm MPK within a cohort of entrepreneurs with the highest grid point ability and the equilibrium distribution of assets a. The last column reports the efficiency gain of eliminating the collateral constraint.

than to restrict them to take on the same parameter values.

We also note that our results still hold if we calibrate the two models to match the same MRPK dispersion, rather than the same debt-to-output ratio. To do so, we calculate that the equilibrium MPK dispersion (standard deviation of the log MRPK) is 0.191 in the benchmark model. We then choose ϕ in the model without managerial inputs to match the same equilibrium MRPK dispersion. The associated parameter values are in the last column of Table 2. With this calibration approach, we again find that the predicted role of the collateral constraint is still substantially smaller than that of the benchmark model (see Table 3).

We argue that this alternative calibration strategy is less appropriate: Our theoretical and quantitative analysis shows that the model without managerial inputs is not able to generate the right MRPK distribution among firms, including the persistence and the dispersion, given the observed debt-to-output ratio. To obtain the same MRPK dispersion, we need to exacerbate the collateral constraint in the model without managerial inputs, implying a debt-to-output ratio of 0.590, which is much lower than the data moment (0.638).

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