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Policy Distortions and Aggregate Productivity with Endogenous Establishment-Level Productivity

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ABSTRACT

What accounts for income per capita and total factor productivity (TFP) differences across countries? We study resource misallocation across heterogeneous production units in a general equilibrium model where establishment productivity and size are affected by policy distortions. We solve the model in closed form and show that the effect of policy distortions on aggregate productivity is substantially magnified when the distribution of (relative) establishment sizes is constant across economies as supported by some empirical evidence. In this case, more distorted economies feature higher establishment lifespan, amplifying the negative effect of distortions on establishment productivity growth. Policy distortions in this environment substantially reduce aggregate productivity, an effect that is 2.8-fold larger than the model with unrestricted relative size distribution.

Keywords: distortions, misallocation, investment, productivity, establishment size.
JEL codes: O11, O3, O41, O43, O5, E0, E13, C02, C61.

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1 Introduction

A crucial question in economic growth and development is why some countries are rich and others poor. A consensus has emerged in the literature whereby the large differences in income per capita across countries are mostly accounted for by differences in labor productivity and in particular total factor productivity (TFP) (Klenow and Rodriguez-Clare, 1997; Prescott, 1998; Hall and Jones, 1999). A key question then is: what accounts for differences in TFP across countries? An important channel that has been emphasized is the (mis)allocation of factors across heterogeneous production units.[1] We study misallocation in a model where establishment-level productivity is determined endogenously by the investment decisions of firms. In particular, we focus on economies with a constant distribution of (relative) establishment sizes and show that policy distortions substantially reduce aggregate productivity, by several fold factors compared to the model when the size distribution is unrestricted.

A recent branch of the literature has emphasized the dynamic implications of misallocation by considering variants of the growth model with establishment-level productivity dynamics.[2] We build on this literature by developing a general equilibrium model of establishments where the distributions of establishment productivity and size are characterized in closed form as a function of the economic environment which is affected by policy distortions.

A key insight of our work arises from exploiting two well-known properties of distorted economies, that establishment size is proportional to distortions and productivity (Restuccia and Rogerson, 2008) and that the distribution of (relative) sizes across establishments is fairly similar across countries (Hopenhayn, 2014b). It is also closely connected to evidence that the distribution of firm size approximates Zipf's law (Axtell, 2001; Gabaix, 2009; Luttmer, 2007), which in the limit

[1]See Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Guner et al. (2008), and Hsieh and Klenow (2009). See also the surveys of the literature in Restuccia and Rogerson (2013), Restuccia (2013a), Hopenhayn (2014a), and Restuccia and Rogerson (2017).

[2]Some of the early contributions on the endogenous productivity distribution include Restuccia (2013b), Bello et al. (2011), Acemoglu et al. (2018), Ranasinghe (2014), Bhattacharya et al. (2013), Gabler and Poschke (2013), Rubini (2014), Hsieh and Klenow (2014), Bento and Restuccia (2017), Guner et al. (2018), Peters (2020), Buera and Fattal Jaef (2018), among others. See also Restuccia and Rogerson (2017) for a discussion of this literature.

makes the distribution of relative sizes constant across economies. The implication of these features is that variations in policy distortions across economies are mostly reflected in differences in the productivity distribution. We find that this effect is quantitatively important as distortions in our framework reduce aggregate TFP by 57% compared with only 21% in the same economy that does not restrict the size distribution of establishments (a 2.8-fold larger impact). Moreover, our analytical solution of the distribution of productivity and how it is affected by distortions can potentially be useful in empirical applications of dynamic misallocation across countries using panel micro data, an essential issue in the misallocation literature (Restuccia and Rogerson, 2017).

We develop a general equilibrium framework with heterogeneous production units that builds on Hopenhayn (1992) and Restuccia and Rogerson (2008). The framework is a standard neoclassical growth model with production heterogeneity extended to incorporate the dynamic effect of distortions on productivity investment and hence the distribution of establishment-level productivity. The key elements of the model are on the production side. In each period, there is a single good produced in establishments. Establishments are heterogeneous with respect to total factor productivity and have access to a decreasing returns to scale technology with capital and labor as inputs. Establishments are subject to an exogenous exit rate but differently from the standard framework, the distribution of establishment-level productivity is not exogenous, rather it is determined by establishment's investment decisions. In other words, the productivity of establishments is determined endogenously in the model by the properties of the economic environment such as policy distortions. Nevertheless, despite growth in establishment productivity, with exogenous exit, the economy features a stationary distribution of establishments across productivity levels and hence an invariant level of aggregate TFP.

Following the literature, the economy faces policy distortions which, for simplicity, take the form of output taxes on individual producers. That is, each producer faces an idiosyncratic tax and it is the properties of policy distortions that generate misallocation in the model. Revenues collected from these taxes are rebated back to the households as a lump-sum transfer. We emphasize that the output distortions we consider are abstract representations, a catch-all for the myriad of implicit and explicit distortions faced by individual producers. While the literature has made substantial progress in identifying the specific policies and institutions that create misallocation, as discussed in Restuccia and Rogerson (2017), the emphasis in our paper is on effect of distortions on the productivity distribution in the economy. As a result, our paper represents a general assessment of the broader consequences of misallocation.

We provide an analytical solution of this model in continuous time. In particular, we solve in closed form for the stationary distributions of establishment sizes and productivities which are endogenous objects that may vary across economies. We show that the equilibrium productivity distributions are Double Pareto distributions with tail index that depends on policy distortions and on the response of incumbent establishments to distortions when selecting the growth rate of productivity. This allows us to characterize the behavior of aggregate output and TFP across distortionary policy configurations as well as the size and productivity growth rate of establishments, the inequality of the distributions of size and establishment-level productivity, among other statistics of interest.

To explore the quantitative properties of the model, we calibrate the model and provide a set of relevant quantitative experiments. We consider a benchmark economy with distortions that is calibrated to data for the United States. We then perform quantitative analysis by exploring the implications of increasing distortions for aggregate output and TFP under a variety of configurations. Our main result is that in economies featuring constant (relative) size distributions, policy distortions substantially reduce not only aggregate TFP, but also the average size of establishments, establishment entry productivity, and establishment productivity growth; effects which are broadly consistent with evidence on average establishment size (Bento and Restuccia, 2017), with evidence that in more distorted economies the productivity and employment growth of establishments are lower (Hsieh and Klenow, 2014), and with evidence that when distortions are lowered in economic reforms, the productivity growth rate of establishments increases (Pavcnik, 2002; Bustos, 2011). These effects are quantitatively much stronger compared with the model where the relative size distribution is unrestricted.

Our paper is related to a large literature on misallocation and productivity discussed earlier. The literature has emphasized various separate channels such as life-cycle investment of plants, human capital accumulation of managers, experimentation, step-by-step innovation, selection, among many others; and different contexts such as trade and labor policies, financial frictions, and specific sectors. We complement this literature by developing a general model of establishment growth featuring a distribution of establishment productivity that can be characterized in closed form. More importantly, our theoretical characterization can be useful in developing methods to estimate the role of dynamic misallocation using panel data of firms, and hence we hope our analysis can facilitate more empirical applications.

Two closely related papers to ours are Hsieh and Klenow (2014) and Bento and Restuccia (2017). Hsieh and Klenow (2014) consider the model of establishment innovation in Atkeson and Burstein (2010) to emphasize the life-cycle growth of establishments and its response to distortions, whereas Bento and Restuccia (2017) emphasize both entry productivity and life-cycle growth. We emphasize two key distinctions with our work. First, in these papers entering establishments draw their productivity from an exogenous and constant distribution across countries, whereas the entire productivity distribution is a key equilibrium object in our framework that responds to policy distortions. Second, we differ in the tools used to characterize the economy, in particular, we solve analytically for the entire distribution of productivity using continuous time and Brownian motion processes. These tools are increasingly popular in the growth literature allowing both a tighter theoretical characterization and more efficient computation (e.g., Lucas and Moll, 2014; Benhabib et al., 2014; Buera and Oberfield, 2014). More closely linked, these tools were prominently used in the seminal work of Luttmer (2007) to study the size distribution of establishments in the United States (see also Da-Rocha and Pujolas, 2011; Fattal Jaef, 2018; Gourio and Roys, 2014; Da-Rocha et al., 2019).

The paper proceeds as follows. In the next section, we present the details of the model and section 3 characterizes the equilibrium solution. In section 4, we characterize aggregate output and TFP and provide a closed-form solution of the model. Section 5 provides a quantitative assessment of the impact of policy distortions on aggregate output, TFP, and other relevant statistics. We conclude in Section 6.

2 Economic Environment

We consider a standard version of the neoclassical growth model with producer heterogeneity as in Restuccia and Rogerson (2008). We extend this framework to allow establishments to invest in their productivity. As as a result, with on-going entry and exit of establishments, the framework generates an invariant distribution of productivity across establishments associated with the economic environment that may differ across countries. Time is continuous and the horizon is infinite. Establishments have access to a decreasing return to scale technology, pay a one-time fixed cost of entry, and exit at an exogenous rate. Establishments hire labor and rent capital services in competitive markets. New entrants enter with a level of productivity which is endogenous. We focus on a stationary equilibrium of this model and study the effects of idiosyncratic policy distortions on the allocation of factors across establishments. In what follows, we provide more details of the economic environment.

2.1 Baseline Model

There is an infinity-lived representative household with preferences over consumption goods described by the utility function,

$$\max \int_0^{+\infty} e^{-\varrho t} u(c) dt,$$

where c is consumption and ϱ is the discount rate. The household is endowed with one unit of productive time at each instant and $k_0 > 0$ units of the capital stock at date 0.

The unit of production in the economy is the establishment. Each establishment is described by a production function $f(z, k, n)$ that combines establishment productivity z, capital services k, and labor services n to produce output. The function f is assumed to exhibit decreasing returns to scale in capital and labor jointly and to satisfy the usual Inada conditions. The production function is given by:

$$y = z^{\theta(1-\alpha-\gamma)} k^{\alpha} n^{\gamma}, \quad \alpha, \gamma \in (0,1), \quad 0 < \gamma + \alpha < 1, \quad \theta > 1, \tag{1}$$

where establishment productivity z is stochastic but establishments can invest in upgrading their productivity at a cost and θ is a scaling parameter for establishment TFP. Note that establishment TFP is $z^{\theta(1-\alpha-\gamma)}$ and hence θ influences the units in which establishment productivity z is measured. Scaling productivity by θ is convenient for algebraic manipulations of the model as this parameter also represents the curvature in the cost function of establishment productivity growth, but other than the economics of productivity investment, this scaling of productivity is innocuous for the analysis. Establishments also face an exogenous probability of exit λ.

New establishments can also be created. Entrants must pay an entry cost c_e measured in units of output and as in the literature the expected value of entry satisfies the zero profit condition in equilibrium. Feasibility in the model requires:

$$C + I + Q = Y - E,$$

where C is aggregate consumption, I is aggregate investment in physical capital, Q is aggregate cost of investing in establishment productivity, E is the aggregate cost of entry, and Y is aggregate output.

2.2 Policy Distortions

We introduce policies that create idiosyncratic distortions to establishment-level decisions as in Restuccia and Rogerson (2008). We model these distortions as idiosyncratic output taxes but none of our results are critically dependent on the particular source of distortions. While the policies we consider are hypothetical, there is a large empirical literature documenting the extent of idiosyncratic distortions across countries and our framework allows for a simple mapping between distortions and empirical observations (Hsieh and Klenow, 2009; Bartelsman et al., 2013; Restuccia and Rogerson, 2017).

In our framework, distortions not only affect the allocation of resources across existing production units, but also the growth rate of establishment productivity, thereby affecting the distribution of productive units in the economy. Specifically, we assume that each establishment faces its own policy distortion (idiosyncratic distortions) reflected as an output tax rate τ_y. In what follows, for convenience we rewrite distortions as $\tau = (1-\tau_y)^{\frac{1}{\theta(1-\alpha-\gamma)}}$. Note that this transformation implies that an establishment with no distortions $\tau_y = 0$ faces $\tau = 1$, whereas a positive output tax $\tau_y > 0$ implies $\tau < 1$ and an output subsidy $\tau_y < 0$ implies $\tau > 1$.

In order to generate dispersion in distortions across productive units, we assume that τ follows a standard stochastic process, a Geometric Brownian motion,

$$d\tau = \mu_\tau \tau dt + \sigma_\tau \tau dw_\tau,$$

where μ_τ is the drift, σ_τ is the standard deviation and dw_τ is the standard Wiener process of the Brownian motion. In this specification σ_τ controls the dispersion of distortions across producers and hence the dispersion in marginal revenue products.

Establishment's productivity z follows a Geometric Brownian motion and establishments can invest in upgrading their productivity by choosing the drift of the Brownian motion μ_z, establishment productivity follows:

$$dz = \mu_z z dt + \sigma_z z dw_z,$$

where σ_z is the standard deviation and dw_z is the standard Wiener process of the Brownian motion. We assume that the output tax and productivity can be correlated, that is $E(dw_\tau, dw_z) = \rho \in (-1, 0]$. Note that a negative value of ρ, corresponds to the notion of correlated distortions in Restuccia and Rogerson (2008), whereby distortions impact more heavily on more productive establishments. This feature of the environment has been shown to be important in accounting for misallocation (e.g., David and Venkateswaran, 2019).

At the time of entry, the entry distortion τ_e is known and establishments enter with a productivity z_e that is determined in equilibrium and implies a value of entrants that satisfies the zero profit condition. In this economy, the relevant information for establishment's decisions is the joint distribution over productivity and distortions. We denote this joint distribution by $g(z, \tau)$.

A given distribution of establishment-level distortion and productivity may not lead to a balanced budget for the government. As a result, we assume that budget balance is achieved by either lump-sum taxation or redistribution to the representative household, denoting the lump-sum tax by T.

3 Equilibrium

We focus on a stationary equilibrium of this economy. The stationary equilibrium is characterized by an invariant distribution of establishments $g(z, \tau)$ over productivity z and distortion τ, a constant entry productivity z_e, and constant allocation functions. In the stationary equilibrium, the rental price for labor and capital services are also constant and we denote them by w and r. Before defining the stationary equilibrium formally, it is useful to consider the decision problems faced by incumbents, entrants, and consumers. We describe these problems in turn.

3.1 Incumbent establishments

Incumbent establishments maximize the present value of profits by making static and dynamic decisions. The static problem is to choose the amount of capital and labor services, whereas the dynamic problem involves solving for the establishment productivity drift. We now describe these problems in detail.

Static problem At any instant of time an establishment chooses how much capital to rent k and how much labor to hire n. These decisions are static and depend on the establishment's productivity z, the establishment's distortion τ, the rental rate of capital r, and the wage rate w. Formally, the instant profit function $\pi(z,\tau)$ is defined by:

$$\pi(z,\tau) = \max_{k,n} (\tau z)^{\theta(1-\alpha-\gamma)} k^{\alpha} n^{\gamma} - wn - rk, \tag{2}$$

from which we obtain the optimal demand for labor and capital:

$$n(z,\tau) = \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\gamma}{w}\right)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\gamma}} z^{\theta} \tau^{\theta}, \tag{3}$$

$$k(z,\tau) = \left[\left(\frac{\alpha}{r}\right)^{1-\gamma} \left(\frac{\gamma}{w}\right)^{\gamma} \right]^{\frac{1}{1-\alpha-\gamma}} z^{\theta} \tau^{\theta}. \tag{4}$$

For future reference, we redefine instant profits as a function of the optimal demand for factors:

$$\pi(z,\tau) = m(w,r) z^{\theta} \tau^{\theta}, \tag{5}$$

where $m(w,r) = (1-\alpha-\gamma)\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}}$ is a constant across establishments that depends on equilibrium prices. Note that since factor demands are linear in $(z\tau)^{\theta}$, we find it convenient to define size s as $s \equiv (z\tau)^{\theta}$ so that factor demands are proportional to size s. A key insight of the misallocation literature is that the relationship between size and productivity is fundamentally affected by distortions. In this setting, distortions put a wedge between factor demands (and revenue) and physical output. Unlike factor demands (as well as revenue and profits), establishment output $y = z^{\theta(1-\alpha-\gamma)} k^{\alpha} n^{\gamma}$ is not a linear function of size. We explicitly derive the relationship between output, distortions, and productivity below.

Dynamic problem Incumbent establishments choose the drift of their productivity μ_z. The cost of investing in productivity is expressed in units of output, described by a cost function $q(\mu_z)$ that is increasing and convex in the productivity drift. We follow a large literature in innovation and growth by specifying the cost function in units that are proportional to firm's relative revenue (Atkeson and Burstein, 2010; Buera and Fattal Jaef, 2018), but in our framework, revenue is proportional to productivity and distortions which is natural in this context. Specifically, we assume $q(\mu_z) = \left(\frac{c_\mu \mu_z^\theta}{\theta}\right)\frac{z^\theta \tau^\theta}{\bar{s}}$, where θ controls the convexity of the cost function and c_μ is a common scale parameter. The cost function is proportional to (relative) establishment size and revenue by a factor $\left(\frac{c_\mu \mu_z^\theta}{\theta}\right)$.

The optimal decision of productivity improvement is characterized by maximizing the present value of profits subject to the Brownian motion governing the evolution of productivity and the Brownian motion governing the evolution of distortions. Formally, incumbent establishments solve the following dynamic problem:

$$\begin{aligned} W(z,\tau) &= \max_{\mu_z}\left\{m(w,r)z^\theta\tau^\theta - q(\mu_z) + \frac{1}{1+(\lambda+R)dt}E_{z,\tau}W(z+dz,\tau+d\tau)\right\}, \\ s.t. \quad dz &= \mu_z z dt + \sigma_z z dw_z, \\ d\tau &= \mu_\tau \tau dt + \sigma_\tau \tau dw_{\tau,} \end{aligned} \tag{6}$$

where λ is the exogenous exit probability of establishments and R is the stationary equilibrium real interest rate. Next, we define the Hamilton-Jacobi-Bellman of the stationary solution,

$$\begin{aligned} (\lambda+R)W(z,\tau) \;=\; \max_{\mu_z}\Big\{ & m(w,r)z^\theta\tau^\theta - \frac{c_\mu}{\bar{s}}\frac{\mu_z^\theta}{\theta}z^\theta\tau^\theta \\ & +\mu_z z W_z' + \frac{\sigma_z^2}{2}z^2 W_{zz}'' + \mu_\tau \tau W_\tau' + \frac{\sigma_\tau^2}{2}\tau^2 W_{\tau\tau}'' + \sigma_z\sigma_\tau\rho z\tau W_{z\tau}''\Big\}. \end{aligned} \tag{7}$$

In the following Lemma 1 we characterize formally the endogenous productivity drift.

Lemma 1. *Given distortion τ, productivity z, and operating profits $m(w,r)$, the value function that solves the establishment dynamic problem is given by $W(z,\tau) = A(w,r)\tau^\theta z^\theta$ where*

$$A(w,r) = \frac{m(w,r)}{\lambda + R + \mu_z - \left[\theta(\mu_\tau+\mu_z) + \theta^2\sigma_z\sigma_\tau\rho + \frac{\theta(\theta-1)}{2}(\sigma_z^2+\sigma_\tau^2)\right]},$$

and the productivity drift

$$\mu_z = \left[\frac{\theta A(w,r)\bar{s}}{c_\mu}\right]^{\frac{1}{\theta-1}}.$$

Proof See Appendix A.1.

The implication of Lemma 1 is that the growth rate of productivity of individual establishments is constant and common across establishments, that is, it does not depend on the intrinsic characteristics of the establishment such as productivity z or distortion τ, as a result Gibrat's law holds. We recognize that there is some debate as to whether Gibrat's law holds empirically in developed or less developed countries, however, we note that this implication of the model is more neutral with regards to amplification, that is it implies a more muted negative effect of distortions on output, making our quantitative results conservative in this context. Atkeson and Burstein (2010) develop a model of firm-level innovation with the same property, whereas in Bhattacharya et al. (2013) and Hsieh and Klenow (2014) it holds for undistorted economies but not for distorted economies. While Lemma 1 implies that the endogenous productivity drift is constant across establishments, the drift can differ across economies with different policy distortions when distortions affect equilibrium wages, and this is a key element in our quantitative analysis.

Employment, which is proportional to size $n \propto s = (z\tau)^\theta$, also follows Gibrat's law, and the resulting Brownian motion of size implies the following drift:

$$\mu_s = \theta\left(\mu_z + \mu_\tau + \theta\sigma_z\sigma_\tau\rho\right) + \theta(\theta-1)(\sigma_z^2 + \sigma_\tau^2)/2. \tag{8}$$

In this environment, policy distortions affect productivity growth and employment growth differently. Employment is impacted by distortions directly through the dispersion of distortions σ_τ and its correlation with productivity ρ, and indirectly through changes in productivity growth $\mu_z(w,r)$. Not only size is not the same as productivity in distorted economies in this framework, but also growth in size is not the same as growth in productivity.

3.2 Entering establishments

Potential entering establishments face an entry cost c_e in units of output and make their entry decision knowing the entry distortion τ_e. For tractability, we assume that entrants start with the same level of productivity, denoted by z_e. The initial level of productivity is such that the value of entering establishments satisfies the zero profit condition:

$$W_e = A(w,r)(\tau_e z_e)^\theta - c_e = A(w,r)s_e - c_e = 0. \tag{9}$$

Note that such a value of productivity z_e exists and is unique which follows from the fact that the value of entry W_e inherits the properties of the value of incumbent establishments which is increasing in productivity z_e. In addition, in the special case where the model is deterministic, the value of entry is the same as in Restuccia and Rogerson (2008), which is the discounted value of establishments' profit.

For simplicity, we assume that the mass of establishments is exogenous and normalized to one. We have extended the model to allow for an endogenous mass of establishments in Appendix B and show that the qualitative and quantitative implications are robust to this extension, in particular, an endogenous mass of establishments further amplifies the impact of policy distortions on aggregate TFP.

3.3 Stationary distribution of establishments

Given the optimal decisions of incumbents and entering establishments, we are now ready to characterize the stationary distribution $g(z,\tau)$ over productivity z and distortion τ. The first step to characterize this distribution is to rewrite the Brownian motions of productivity z and distortion τ as a function of size s. In order to characterize the stationary distribution over size, it is useful to rewrite the model in logarithms. Let x denote the logarithm of relative size, that is $x = \log(s/s_e)$, where $s_e = (\tau_e z_e)^\theta$ is the size in which establishments enter. Now we can rewrite the Geometric

Brownian motion of x as

$$dx = \mu_x dt + \sigma_x dw_x,$$

where

$$\mu_x = \theta(\mu_z + \mu_\tau) + \frac{\theta(\theta-1)}{2}(\sigma_z^2 + \sigma_\tau^2) - \frac{\theta^2}{2}(\sigma_z^2 + \sigma_z^2),$$

and

$$\sigma_x^2 = \theta^2 \left(\sigma_z^2 + \sigma_\tau^2 + 2\sigma_z\sigma_\tau\rho\right). \tag{10}$$

We use the Kolmogorov-Fokker-Planck (KFP) equation to characterize the stationary distribution of x:

$$\frac{\partial f(x,t)}{\partial t} = -\mu_x \frac{\partial f(x,t)}{\partial x} + \frac{\sigma_x^2}{2}\frac{\partial^2 f(x,t)}{\partial x^2} - \lambda f(x,t) + b(0,t), \tag{11}$$

where λ is the exogenous exit rate of establishments, $b(0,t)$ is the measure of establishments that enter at time t and have size 0, after the normalization. The solution of this problem is discussed in Gabaix (2009). We are interested in solving for the steady state where $f(x,t) = f(x)$ and $b(0,t) = b(0)$. Therefore, we can rewrite the KFP equation (11) as:

$$f'(x) = -\mu_x f'(x) + \frac{\sigma_x^2}{2} f''(x) - \lambda f(x) + b\delta(x-0) = 0, \tag{12}$$

and we assume four boundary conditions:

$$\lim_{x\to-\infty} f(x) = 0, \qquad \lim_{x\to+\infty} f(x) = 0, \tag{13}$$

$$\lim_{x\to-\infty} f'(x) = 0, \qquad \lim_{x\to+\infty} f'(x) = 0, \tag{14}$$

and that $f(\cdot)$ is a *p.d.f*, i.e. $f(x) \geq 0$ and $\int_{-\infty}^{+\infty} f(x)dx = 1$. The first two boundary conditions (13) guarantee that the stationary distribution is bounded, whereas the next two boundary conditions (14) imply that b is equal to λ, that is, the stationary equilibrium entry rate is equal to the exogenous exit rate.[3] We can now characterize the stationary (log) size distribution, which is a double Pareto. Formally, Lemma 2 characterizes the stationary distribution.

Lemma 2. *Given wages w, rental rate of capital r, and a policy $(\tau_e, \mu_\tau, \sigma_\tau, \rho)$ the stationary size distribution is a double Pareto:*

$$g(s) = \begin{cases} C\left(\frac{s}{s_e}\right)^{-(\xi_-+1)} & \text{for} \quad s < s_e. \\ \\ C\left(\frac{s}{s_e}\right)^{-(\xi_++1)} & \text{for} \quad s \geq s_e. \end{cases} \tag{15}$$

where the tail index ξ_+ is the positive root and the tail index ξ_- is the negative root that solves the characteristic equation

$$\frac{\sigma_x^2}{2}\xi^2 + \left(\mu_s - \frac{\sigma_x^2}{2}\right)\xi - \lambda = 0 \tag{16}$$

and $C = \frac{-\xi_-\xi_+}{s_e(\xi_+-\xi_-)}$. Moreover, average establishments size $\bar{s}$ is given by:

$$\bar{s} = \left(\frac{\lambda}{\lambda - \mu_s}\right) s_e.$$

Proof See Appendix A.2.

We leave the poof of Lemma 2 to the Appendix. Lemma 2 characterizes the endogenous distribution as a function of establishments' size drift μ_s and entry size s_e, which in turn are affected by distortions. Using the same methodology as in Lemma 2, we can solve for the distributions of productivity and distortions (the equilibrium values of ξ_z and ξ_τ). The boundary conditions on

[3] Integrating (12) we obtain:

$$\int_{\infty}^{+\infty} f'(x)dx = \left(-\mu_x f(x) + \frac{\sigma_x^2}{2}f'(x)\right)\Big|_{-\infty}^{+\infty} - \lambda\int_{-\infty}^{+\infty} f(x)dx + \int_{-\infty}^{+\infty} b\delta(x-0)dx = 0,$$

and applying the boundary conditions and using the Dirac delta function, $\int_{-\infty}^{+\infty}\delta(x-0)dx = 1$, results in $b = \lambda$.

these distributions prevent establishments with negative values of productivity z and distortions τ.

3.4 Household's problem

The household problem essentially help us pin down the stationary real interest rate R. As such, the process for capital accumulation in this model follows the standard neoclassical growth model. The stand-in household seeks to maximize lifetime utility subject to the law of motion of wealth given by:

$$(RK + w + T + \Pi - bc_e - c)\,dt,$$

where w is the wage rate, R is the interest rate which in equilibrium is the rental price of capital minus capital depreciation ($R = r - \delta_k$), T is the lump-sum tax levied by the government, Π is the total profit from the operation of all establishments, bc_e is the total entry cost of establishments and c is consumption.

We assume that households have log utility, $u(c) = \log(c)$. The solution of this problem is standard and implies that $R = \varrho$. In the stationary equilibrium, aggregate consumption C and physical capital K are constants.

3.5 Stationary equilibrium

Definition Given a policy $\{\mu_\tau, \sigma_\tau, \rho, \tau_e\}$, a stationary equilibrium is an invariant size distribution $g(s)$, value and policy functions of incumbent establishments $W(s)$, $k(s)$, $n(s)$, establishment productivity growth μ_z, value of entrants W_e, entrants' productivity z_e, entry rate b, prices (r, w), transfer T, profits Π, capital K, and consumption C, such that:

i) Consumer optimization implies that $R = r - \delta_k = \varrho$ and aggregates C and K.

ii) Given prices, the incumbents' policy functions $\{k(s), n(s)\}$ solve the incumbents' static problem (2).

iii) Given prices, the incumbents' value function $W(s)$ solves the incumbents' dynamic problem (6), μ_z is optimal from this problem.

iv) Given prices, the value of entrants satisfy the zero profit condition in (9) and entry productivity z_e is determined by this condition.

v) The stationary distribution $g(s)$ and entry rate of establishments b solve the Kolmogorov-Fokker-Planck equation (11).

vi) Markets for capital and labor clear (market clearing in the goods market is satisfied by Walras' law):

   a) capital: $K = \int_0^{+\infty} k(s)g(s)ds$,

   b) labor: $1 = \int_0^{+\infty} n(s)g(s)ds$.

vii) Transfers T guarantee that the government's budget constraint is satisfied.

4 Aggregate Output and Productivity

Aggregate output Y is obtained by integrating over the distribution of establishment's output. From the establishment production function in equation (1), we substitute the demand for labor and capital in equations (3) and (4), and use the labor market clearing condition to substitute for the wage to obtain:

$$y(z,\tau) = \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1}{\bar{s}}\right)^{\frac{\gamma}{1-\alpha}} \tau^{\theta(\alpha+\gamma)} z^{\theta}. \tag{17}$$

Given that $y \propto \tau^{\theta(\alpha+\gamma)} z^{\theta}$ is a Brownian motion, the output drift is equal to

$$\mu_y = \mu_{\tau^{\theta(\alpha+\gamma)} z^{\theta}} = \theta\mu_z + \theta(\theta-1)\sigma_z^2/2 + (\alpha+\gamma)\left\{\theta\mu_\tau + \theta^2\rho\sigma_z\sigma_\tau + \left[\theta^2(\alpha+\gamma)-\theta\right]\sigma_\tau^2/2\right\}.$$

After integrating, using the same methodology as in Lemma 2, and rearranging terms we write aggregate output Y and total factor productivity Y/K^{α} as:[4]

$$
\begin{aligned}
Y &= \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\lambda-\mu_s}{\lambda-\mu_y}\right) \bar{s}^{\frac{1-\alpha-\gamma}{1-\alpha}} \left(\frac{1}{\tau_e^{\theta(1-\alpha-\gamma)}}\right), \\
\text{TFP} &= \left(\frac{\lambda-\mu_s}{\lambda-\mu_y}\right) \bar{s}^{1-\alpha-\gamma} \left(\frac{1}{\tau_e^{\theta(1-\alpha-\gamma)}}\right).
\end{aligned}
$$

Recalling that in undistorted economies the productivity, size and output drifts are all the same, these equations highlight the relevance of distortions in driving a wedge between the output and size drifts in distorted economies which lowers aggregate TFP. In particular, note that in this framework aggregate output and productivity may be lower with policy distortions not only through their effects on average establishment size $\bar{s}$, but also by the wedge between size and output drifts. These drifts determine average size and output relative to entry and their underlined distributions.

Model solution. We can easily solve for the stationary equilibrium. First, the stationary rental rate of capital r is pin down by ϱ and δ_k from the household problem in steady state. Second, we solve a non-linear system of five equations in five unknowns $\{z_e, \mu_z, \mu_s, \bar{s}, A\}$ as follows:

(1) The free entry condition:

$$A(\tau_e z_e)^{\theta} = c_e. \tag{18}$$

(2) The productivity growth rate μ_z:

$$\mu_z = \left[\frac{\theta A \bar{s}}{c_{\mu}}\right]^{\dfrac{1}{\theta-1}}. \tag{19}$$

[4]Aggregate capital is given by integrating equation (4) over size,

$$K = \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \bar{s}^{\frac{1-\alpha-\gamma}{1-\alpha}} = \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} \left[\left(\frac{\lambda}{\lambda-\mu_s}\right)\left(\tau_e^{\theta} z_e^{\theta}\right)\right]^{\frac{1-\alpha-\gamma}{1-\alpha}}.$$

(3) The size growth rate,

$$\mu_s = \theta\mu_z + \theta(\theta-1)\sigma_z^2/2 + \theta\mu_\tau + \theta(\theta-1)\sigma_\tau^2/2 + \theta^2\rho\sigma_z\sigma_\tau. \tag{20}$$

(4) The relative incumbent to entry size:

$$\frac{\bar{s}}{(\tau_e z_e)^\theta} = \left(\frac{\lambda}{\lambda-\mu_s}\right), \tag{21}$$

which given the labor market clearing condition implies the wage rate $w = \gamma\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \bar{s}^{\frac{1-\alpha-\gamma}{1-\alpha}}$.

(5) The establishment's value to size ratio A is:

$$A = \frac{(1-\alpha-\gamma)}{(\lambda+R+\mu_z-\mu_s)}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{(1-\alpha)}}\left(\frac{1}{\bar{s}}\right)^{\frac{\gamma}{1-\alpha}}. \tag{22}$$

Note that this system has a closed-form solution. From the free entry condition in equation (18), we substitute A in equation (19) and use the relative incumbent to entry size condition in equation (21) to substitute for $\bar{s}/(\tau_e z_e)^\theta$ and obtain the equilibrium condition between productivity and size growth rates:

$$\mu_z = \left[\frac{\theta c_e}{c_\mu}\left(\frac{\lambda}{\lambda-\mu_s}\right)\right]^{\dfrac{1}{\theta-1}}. \tag{23}$$

Using μ_s from equation (20), we obtain the solution for the productivity growth rate:

$$\theta\mu_z^\theta - \mu_z^{\theta-1}\left[\lambda - \theta(\theta-1)\sigma_z^2/2 - \theta\mu_\tau - \theta(\theta-1)\sigma_\tau^2/2 - \theta^2\rho\sigma_z\sigma_\tau\right] + \lambda\theta\frac{c_e}{c_\mu} = 0. \tag{24}$$

Given the solution for μ_z we can solve sequentially for all the other equilibrium variables as follows:

$$\begin{aligned} \mu_s &= \theta\mu_z + \theta(\theta-1)\sigma_z^2/2 + \theta\mu_\tau + \theta(\theta-1)\sigma_\tau^2/2 + \theta^2\rho\sigma_z\sigma_\tau & (25) \\ \bar{s}^{\frac{1-\alpha-\gamma}{1-\alpha}} &= \left(\frac{\lambda c_e}{\bar{m}(r)}\right)\left[\frac{\lambda + R - \mu_s + \mu_z}{\lambda - \mu_s}\right], & (26) \\ s_e &= \left(1 - \frac{\mu_s}{\lambda}\right)\bar{s}, & (27) \\ z_e &= s_e^{1/\theta}/\tau_e. & (28) \end{aligned}$$

We emphasize that the equilibrium values of μ_s and s_e can be used to solve for the size distribution in the economy using Lemma 2, in particular, the tail index ξ_+ is the positive root solution from the characteristic equation (16).

Restricting the (relative) size distribution. Our objective is to characterize the impact of distortions on the productivity distribution and hence aggregate TFP. To illustrate the effects, we exploit our analytical characterization under some simplifying assumptions. Nevertheless, in the next section, we quantify the impact of distortions under more general conditions. Following Axtell (2001) and Gabaix (2009), we characterize analytically the impact of distortions on the productivity drift by focusing on economies satisfying Zipf's law (for a more general discussion of power laws in economics, see Gabaix, 2016). Recall that the distribution of relative sizes (equation 15), satisfies the characteristic equation (16). Using equations (8) and (10), we rewrite the characteristic equation (16) as:

$$\lambda = \theta\xi\left\{\mu_z + \mu_\tau + (\xi\theta - 1)(\sigma_z^2 + \sigma_\tau^2)/2 + \theta\xi\rho\sigma_z\sigma_\tau\right\}. \tag{29}$$

Given μ_z in equation (24), we use equation (29) to substitute for λ, and taking the limit $\xi_+ \to 1$ (Zipf's law) we have $\theta\dfrac{c_e}{c_\mu} \to 0$. In this context, Zipf's law implies $\lambda \to 0$ (see also Gabaix, 2016) and the associated productivity drift is given by:

$$\mu_z = -[\mu_\tau + (\theta - 1)\sigma_\tau^2/2] - [\theta\rho\sigma_\tau\sigma_z + (\theta - 1)\sigma_z^2/2].$$

We use this expression to derive the change in the productivity drift μ_z from changes in distortions (σ_τ, ρ):

$$\frac{d\mu_z}{d\sigma_\tau} = -[(\theta - 1)\sigma_\tau + \theta\rho\sigma_z]. \tag{30}$$

Given that $\theta > 1$, the main insight from this expression is that productivity growth is lower with higher distortions (higher σ_τ). We also note that with negative correlation ($\rho < 1$), dispersion in τ and z have opposite effects on productivity growth. An implication of this result is that $\rho = -1$ provides the lowest impact of distortions on productivity growth. This is relevant in the context of the misallocation literature since assuming that $\tau = z^{-\nu}$ (e.g., Hsieh and Klenow, 2014; Bento and Restuccia, 2017; Buera and Fattal Jaef, 2018), implies in our model that $\rho = -1$ and that the dispersion in distortions is affected by changes in ν ($\sigma_\tau^2 = \nu^2\sigma_z^2$).

What is the quantitative impact of stochastic growth on aggregate productivity in this framework? We answer this question in the next section using a calibrated benchmark economy that does not rely on the stylized characterization under power laws.

5 Quantitative analysis

We assess the quantitative impact of policy distortions on aggregate output and productivity. An important feature of our results is to highlight the much larger amplification effect of policy distortions on aggregate productivity when the distribution of relative sizes is approximately constant across economies as suggested by the evidence discussed below.

Calibration. We calibrate a benchmark economy with distortions to U.S. data and then study the impact of alternative hypothetical policy distortions in the same spirit of Restuccia and Rogerson (2008). We start by selecting a set of parameters that are standard in the literature. These parameters have either well-known targets, which we match, or the values have been well discussed in the literature. Following the literature, we assume decreasing returns in the establishment-level production function and set $\alpha + \gamma = 0.85$ (Restuccia and Rogerson, 2008). Then we split it between α and γ by assigning 1/3 to capital and 2/3 to labor, implying $\alpha = 0.283$ and $\gamma = 0.567$. We set the discount rate ϱ to match a real interest rate of 4 percent and the depreciation rate of capital δ to 7 percent to match a capital to output ratio of 2.5. We normalize $\tau_e = 1$ ($\tau_{y,e} = 0$) and keep it constant across all economies. For the benchmark economy we set $\rho = -0.09$ based on the near zero elasticity between distortions and productivity in U.S. manufacturing (Hsieh and Klenow, 2009, 2014).

We then solve the stationary equilibrium and select the following 6 parameters to match 6 moments from the data. We select c_μ to match an establishment productivity growth rate of 4% ($\mu_z = 0.04$); θ to match an establishment output growth rate of 5% ($\mu_y = 0.05$); σ_z^2 and λ are selected to match an exit rate of 10% in line with estimates in the literature (Davis et al., 1998) and the positive root of the size distribution of 1.056 from Axtell (2001) ($\xi_+ = 1.056$); σ_τ^2 to match the standard deviation of the log of TFPR (see Appendix A.3) of 0.49 (Hsieh and Klenow, 2009); μ_τ to normalize mean distortions to 1 (mean $\tau^\theta = 1$); and c_e to match an average establishment size $\bar{s} = 21.85$ (Hsieh and Klenow, 2009). Parameter values and moments are summarized in Table 1.

Experiments. We study the impact of policy distortions on establishment-level productivity, aggregate output and TFP, and other relevant variables by comparing statistics in more distorted economies than the benchmark economy. We highlight the impact of policy distortions in our model with effects on establishment-level productivity contrasting the results in two alternative cases where the distribution of (relative) sizes ξ_+ is constant or not across distorted economies. To make the endogenous relative size distribution ξ_+ constant to the level calibrated in the benchmark

Table 1: Calibration of Benchmark Economy

Parameter	Value	Target	Value
c_μ	95.06	establishment productivity growth rate	0.04
θ	1.80	establishment output growth rate	0.05
σ_z^2	0.0390	Exit rate	0.10
λ	0.10	Positive root size distribution (ξ_+)	1.056
σ_τ^2	0.1492	SD log TFPR	0.49
μ_τ	-0.0597	Normalized, mean τ^θ	1.00
c_e	0.8937	Average size ($\bar{s}$)	21.85

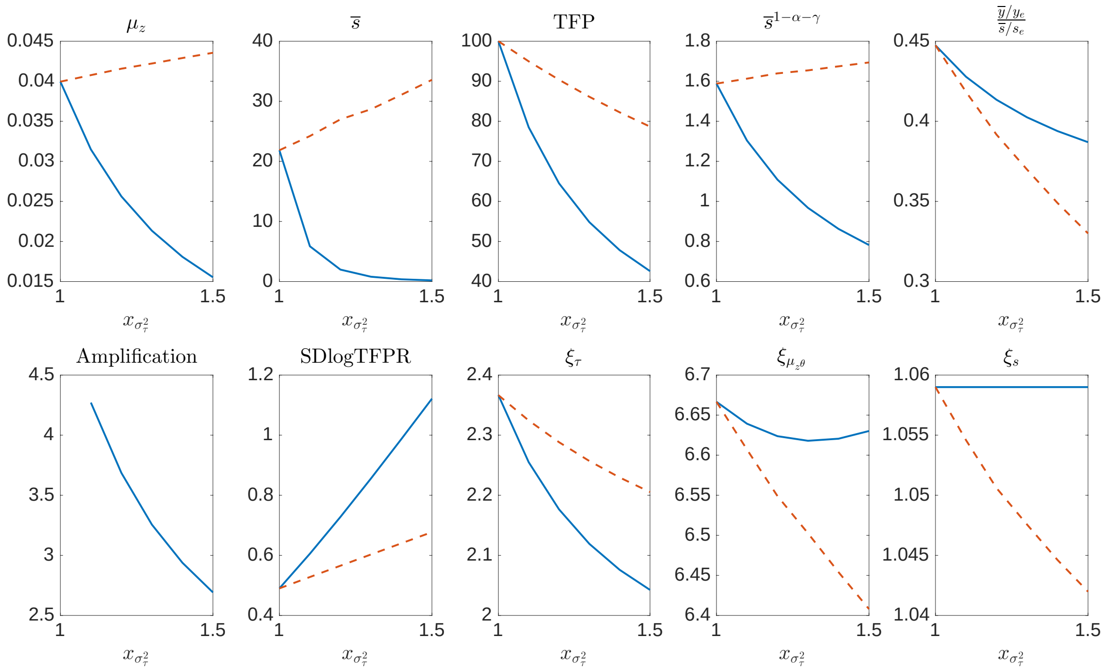
economy, we adjust the exit rate of establishments λ using the detailed characteristic equation (29). We find that a lower λ is required in more distorted economies.

We consider constant relative size distributions as motivated by two pieces of evidence. First, Hopenhayn (2014b) emphasizes that when conditioning on average size, there are no substantial differences in the size distributions of China and India relative to the United States. Second, there is some evidence that the death rate of firms is lower in developing countries as documented in McKenzie and Paffhausen (2019). Our two extreme scenarios bracket the possible outcomes in more distorted economies, with the evidence indicating that somewhat constant relative size distributions is the more likely case.

We study the impact of changes in policy distortions by considering economies that are relatively more distorted than the benchmark economy via changes in the dispersion of distortions σ_τ. Specifically, we increase σ_τ^2 while holding constant mean distortions. Note that mean $\tau^\theta = 1$ implies that $\theta\mu_\tau + \theta(\theta - 1)\sigma_\tau^2/2 = 0$. We consider variations in σ_τ within the range of estimates in the empirical literature (Hsieh and Klenow, 2009; Buera et al., 2013; Bento and Restuccia, 2017; Cirera et al., 2020; Restuccia and Rogerson, 2017). While we have witnessed a tremendous increase in firm-level data availability around the world, access to firm-level data and comparisons of samples across countries remain a challenge. For this reason, we find it useful to consider hypothetical economies to study the dynamic effects of misallocation. These experiments are not meant to represent any individual country but instead provide an assessment of the role of distortions on establishment-level productivity and aggregate output in our model relative to the existing literature.

We document our quantitative results in Figure 1 by reporting equilibrium values of the productivity drift μ_z, average size $\bar{s}$, entry size s_e, aggregate TFP, power laws relating to the distribution of distortions, TFP, and size; among other relevant statistics.

Figure 1: Effects of Changes in Policy Distortions σ_τ^2



Notes: The blue-solid line represents the benchmark economy with changes in σ_τ^2 and constant ξ_s (λ varies). The red-dashed line represents the same economies without restricting ξ_s (λ constant). In all economies, μ_τ is adjusted to normalize mean distortions to one, i.e., mean $\tau^\theta = 1$.

Figure 1, blue-solid line, reports the results of increases in distortions in the benchmark economy when ξ_+ is constant. Establishment productivity growth declines substantially as does average establishment size. The establishment size inequality (measured by $\xi_s = \xi_+$ in the figure) bounds the inequality in the establishment productivity distribution. The same (relative) establishment size distribution is compatible with a more unequal establishment productivity distribution if the establishment expects a higher lifespan (lower λ) that increases the disincentive to invest in productivity generated by the distortion. The effect of policy distortions on aggregate productivity is substantial when the relative size distribution is constant across economies, a reduction of 57 percent. We also note that the value for the left tail index of the relative size distribution in the benchmark economy is $\xi_- = -0.3341$ and the number of "small firms" (the density of $s < s_e$) increases in more distorted economies.

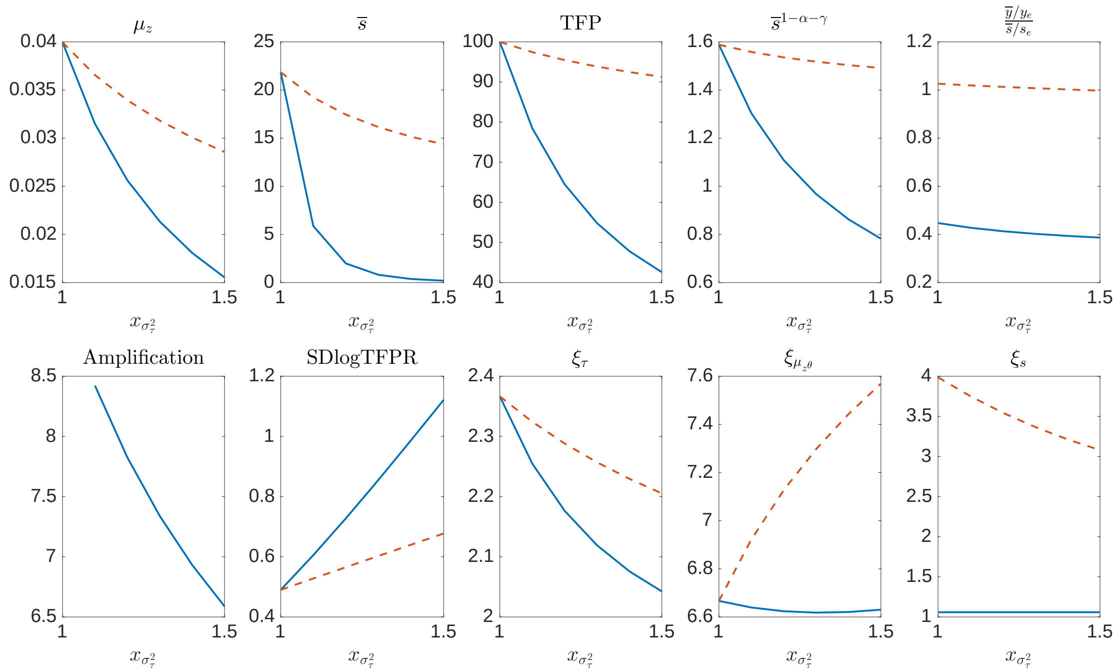
Figure 1, red-dashed line, reports the results of increases in distortions when λ is constant. An increase in policy distortions increases μ_z and generates a more unequal establishment distribution (lower ξ_s than in the benchmark economy). These effects arise from the fact that with correlated distortions, $\rho < 0$, an increase in policy distortions decreases stochastic growth, $\text{cov}(z^\theta, \tau^\theta) = \rho\theta^2\sigma_z\sigma_\tau$, and reduce (all other things equal) size growth $\mu_s = \mu_{z^\theta} + \text{cov}(z^\theta, \tau^\theta)$. Establishments react by investing in productivity.[5] And when $\rho \to 0$, the endogenous investment effect on productivity dominates the stochastic effect (note that when $\rho \to -1$ the opposite holds). Therefore, an increase in policy distortions increases μ_s and generates a more unequal establishment size distribution (lower $s_e/\bar{s}$). The effect of policy distortions on aggregate TFP is a reduction of 21 percent when the relative size distribution varies compared with 57 percent when the relative size distribution is constant (a 2.8-fold larger impact with constant relative size distribution).

Motivated by our analytical results, we now contrast our experiments with constant relative size distribution, the solid-blue line reproduced in Figure 2, with economies featuring $\rho = -1$ (red-dashed line) as in much of the existing literature. In this case, we recalibrate c_μ to match the establishment productivity growth rate of the benchmark economy. Note that in economies where $\rho \to -1$ the impact of misallocation on stochastic growth reaches its maximum (negative) value, whereby we expect a negative impact of policy distortions on μ_s.

Consistent with the literature, an increase in policy distortions decreases productivity growth, average size, entry productivity, and aggregate TFP. But the decline in aggregate TFP is substantially larger in our baseline case, of more than 57 percent, compared to the case when $\rho = -1$ where

[5]Note that $\theta\mu_z^\theta - \mu_z^{\theta-1}\left[\lambda - \theta(\theta-1)\sigma_z^2/2 - \text{cov}(z^\theta, \tau^\theta)\right] + \lambda\theta\frac{c_e}{c_\mu} = 0$. Given that $\mu_z > 0$ and $\frac{d\mu_z}{dc_{\mu_z}} < 0$, we have that $\frac{d\mu_z}{d\text{cov}(z^\theta,\tau^\theta)} > 0$.

Figure 2: The Role of Correlation between Distortions and Productivity



Notes: The blue-solid line represents the benchmark economy with changes in σ_τ^2 and constant ξ_s as in Figure 1. The red-dashed line represents alternative economies with $\rho = -1$ and without restricting ξ_s. In all economies, μ_τ is adjusted to normalize mean distortions to one, i.e., mean $\tau^\theta = 1$.

the decline in TFP is around 9 percent (a 6.9-fold larger impact). We emphasize that these results arise with only modest increases in the dispersion in revenue productivity, well within the ranges reported in the literature.

6 Conclusions

We develop a tractable dynamic model of heterogeneous producers to study the effect of distortions on the distribution of establishment-level productivity across economies. The model tractability allows us to obtain closed-form solutions that are useful in identifying the response of distortions on aggregate output. We show that policy distortions have substantial negative effects on aggregate output and TFP in this economy compared to the existing literature.

It would be interesting to explore specific policies and institutions—such as size-dependent policies, firing taxes, financial frictions—in the context of our framework with dynamic effects of distortions (see for example, Da-Rocha et al., 2019; Aghion et al., 2021). These explorations of specific policies in our framework may help reconcile the empirically large effects found in the literature. As a result, further progress aimed at broadening the empirical mapping of the model to the data may provide useful insights. Our analytical solution of the productivity distribution as a function of distortions is a critical first step in this mapping. But the mapping requires reliable and comparable panel data of producers across countries. While these data are increasingly available for some countries, comparability across countries remains an important limitation. We leave these interesting and important explorations for future work.

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A Appendix: Proofs

This appendix presents the proofs of Lemma 1 and Lemma 2, and the characterization of the distribution of TFPR.

A.1 Proof Lemma 1

From the first order condition for the productivity drift in equation (7), we can solve for the productivity drift μ_z as a function of the determinants of costs and benefits such as distortions τ, cost scale c_μ, and the marginal present value profits W'_z. In particular, equating the marginal cost and benefit from productivity growth implies,

$$\frac{c_\mu}{\bar{s}}(z\tau)^\theta \mu_z^{\theta-1} = zW'_z.$$

By guessing and verifying, we find that the optimal Hamilton-Jacobi-Bellman equation is given by $W(z,\tau) = A(w,r)z^\theta\tau^\theta$, where the constant $A(w,r)$ is the solution of the polynomial:

$$\left[\frac{(\lambda+R)}{(\theta-1)} - \frac{\theta\mu_\tau}{(\theta-1)} - \frac{\theta^2\sigma_z\sigma_\tau\rho}{(\theta-1)} - \frac{\theta(\sigma_z^2+\sigma_\tau^2)}{2}\right]A(w,r) - \left[\frac{\theta\bar{s}}{c_\mu}\right]^{\frac{1}{\theta-1}} A(w,r)^{\frac{\theta}{\theta-1}} = \frac{m(w,r)}{(\theta-1)}. \tag{A.1}$$

After this simplification, we rewrite equation (A.1) finding the following expression for $A(w,r)$:

$$A(w,r) = \frac{m(w,r)}{\lambda + R - \theta\mu_\tau - \theta^2\sigma_z\sigma_\tau\rho - \frac{\theta(\theta-1)}{2}(\sigma_z^2+\sigma_\tau^2) - (\theta-1)\mu_z}, \tag{A.2}$$

where μ_z depends on $A(w,r)$. Given the solution to this polynomial, the optimal productivity drift μ_z is independent of establishment characteristics τ and z:

$$\mu_z = \left[\frac{\theta A(w,r)\bar{s}}{c_\mu}\right]^{\frac{1}{\theta-1}}.$$

A.2 Proof Lemma 2

After some algebraic manipulation from equation (12), we find that the stationary distribution must satisfy the following differential equation:

$$f''(x) - \frac{2\mu_x}{\sigma_x^2} f'(x) - \frac{2\lambda}{\sigma_x^2} f(x) = -\frac{2b}{\sigma_x^2}\delta(x-0),$$

subject to the boundary conditions and $f(\cdot)$ being a pdf. Therefore, the stationary pdf is the solution of the boundary-value problem that consists of solving

$$\begin{array}{ll} f''(x) - \gamma_1 f'(x) - \gamma_2 f(x) = 0 & \text{if} \quad x \neq 0, \\ f''(x) - \gamma_1 f'(x) - \gamma_2 f(x) = -\gamma_3 \delta(x-0) & \text{if} \quad x = 0, \end{array}$$

where the constants γ_1, γ_2, and γ_3 are given by

$$\gamma_1 = \frac{2\mu_x}{\sigma_x^2} < 0, \quad \gamma_2 = \frac{2\lambda}{\sigma_x^2} > 0, \quad \gamma_3 = \frac{2\hat{b}}{\sigma_x^2} > 0.$$

We solve the boundary-value problem using Laplace transforms. Laplace transforms are given by

$$\begin{aligned} \mathscr{L}[f'(x)] &= s\mathscr{L}[f(x)] - f(0), \\ \mathscr{L}[f''(x)] &= s^2\mathscr{L}[f(x)] - sf(0) - f'(0). \end{aligned}$$

By applying Laplace transforms in equation (12), we obtain:

$$(s^2 - \gamma_1 s - \gamma_2)\mathscr{L}[f(x)] - (s-\gamma_1)f(0) - f'(0) = -\gamma_3 \mathscr{L}[\delta(x-0)].$$

Using the boundary condition $f(0) \geq 0$ and $\mathscr{L}[\delta(x-0)] = 1$ we find:

$$(s^2 - \gamma_1 s - \gamma_2)Y(s) = f'(0) + (s-\gamma_1)f(0) - \gamma_3,$$

where

$$Y(s) = \frac{f'(0) - \gamma_3 + (s - \gamma_1)f(0)}{(s^2 - \gamma_1 s - \gamma_2)}.$$

We obtain the solution by solving the Laplace inverses when $x \neq 0$ given by:

$$\begin{aligned}
\mathscr{L}^{-1}\left[\frac{1}{(s-r_1)(s-r_2)}\right] &= \frac{1}{(r_1-r_2)}\left(e^{r_1 x} - e^{r_2 x}\right), \\
\mathscr{L}^{-1}\left[\frac{(s-\gamma_1)}{(s-r_1)(s-r_2)}\right] &= \frac{1}{(r_1-r_2)}\left[(r_1-\gamma_1)e^{r_1 x} - (r_2-\gamma_1)e^{r_2 x}\right],
\end{aligned}$$

where the two roots (one positive and one negative) are given by $r = \dfrac{\gamma_1 \pm \sqrt{\gamma_1^2 + 4\gamma_2}}{2}$. We can rewrite the final solution for this case as:

$$\begin{aligned}
f(x) &= \tfrac{f'(0)}{(r_1-r_2)}\left(e^{r_1 x} - e^{r_2 x}\right) + \tfrac{f(0)}{(r_1-r_2)}\left[(r_1-\gamma_1)e^{r_1 x} - (r_2-\gamma_1)e^{r_2 x}\right] \quad \text{if} \quad x \neq 0, \\
f(x) &= \tfrac{f'(0)-\gamma_3}{(r_1-r_2)}\left(e^{r_1 x} - e^{r_2 x}\right) + \tfrac{f(0)}{(r_1-r_2)}\left[(r_1-\gamma_1)e^{r_1 x} - (r_2-\gamma_1)e^{r_2 x}\right] \quad \text{if} \quad x = 0.
\end{aligned}$$

When $x \neq 0$ (that is $\forall x \in (-\infty, 0) \cup (0, \infty)$), we have

$$f(x) = \begin{cases} C_1 e^{r_1 x} + C_2 e^{r_2 x}, & \text{if} \quad x < 0, \\ \\ C_1 e^{r_1 x} + C_2 e^{r_2 x}, & \text{if} \quad x > 0, \end{cases}$$

where

$$\begin{aligned}
C_1 &= \frac{1}{(r_1-r_2)}\left[f'(0) + f(0)(r_1-\gamma_1)\right], \\
C_2 &= \frac{-1}{(r_1-r_2)}\left[f'(0) + f(0)(r_2-\gamma_1)\right],
\end{aligned}$$

and $r_1 > 0$ and $r_2 < 0$. When $x > 0$ in order to $f(\cdot)$ be a pdf, it is necessary that $C_1 = 0$ and

$$f'(0) = -f(0)(r_1-\gamma_1) \Rightarrow C_2 = \frac{-1}{(r_1-r_2)}\left[f(0)(\gamma_1-r_1) + f(0)(r_2-\gamma_1)\right] = f(0).$$

Symmetrically when $x < 0$ we need $C_2 = 0$. Therefore,

$$f'(0) = -f(0)(r_2 - \gamma_1) \Rightarrow C_1 = \frac{1}{(r_1 - r_2)} \left[f(0)(\gamma_1 - r_2) + f(0)(r_1 - \gamma_1) \right] = f(0),$$

and

$$f(x) = \begin{cases} f(0)e^{r_1 x} & \text{if} \quad x < 0, \\ \\ f(0)e^{r_2 x} & \text{if} \quad x \geq 0, \end{cases}$$

where $f(0) = \left(\frac{r_1 r_2}{r_2 - r_1} \right)$. Finally we need to prove that: 1) for $x > 0$, $f'(0) = -f(0)(r_1 - \gamma_1)$ (i.e. $C_1 = 0$), and 2) for $x < 0$, $f'(0) = -f(0)(r_2 - \gamma_1)$ (i.e. $C_2 = 0$); Given that when $x > 0$ $f'(0) = r_2 f(0)$ (and when $x < 0$ $f'(0) = r_1 f(0)$) this is equivalent to show that

$$(r_2 + r_1) f(0) = \left(\frac{\gamma_1 - \sqrt{\gamma_1^2 + 4\gamma_2}}{2} + \frac{\gamma_1 + \sqrt{\gamma_1^2 + 4\gamma_2}}{2} \right) f(0) = f(0)\gamma_1.$$

When $x = 0$ we have

$$f(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x},$$

where

$$\begin{aligned} C_1 &= \frac{1}{(r_1 - r_2)} \left[f'(0) - \gamma_3 + f(0)(r_1 - \gamma_1) \right], \\ C_2 &= \frac{-1}{(r_1 - r_2)} \left[f'(0) - \gamma_3 + f(0)(r_2 - \gamma_1) \right]. \end{aligned}$$

Therefore

$$f(0) = C_1 + C_2 = \frac{1}{(r_1 - r_2)} \left[r_1 f(0) - r_2 f(0) \right] = f(0).$$

Using $s = s_e e^x$, we can recover the size distribution $g(s)$. That is

$$g(s) = \frac{1}{s} f(\ln(s/s_e)) = \begin{cases} f(0)\dfrac{s^{r_1-1}}{s_e^{r_1}} & \text{if} \quad s < s_e, \\ \\ f(0)\dfrac{s^{r_2-1}}{s_e^{r_2}} & \text{if} \quad s \geq s_e. \end{cases}$$

Note that this solution is equivalent to the guess and verify solution obtained by solving the characteristic equation $\frac{\sigma_x^2}{2}\xi^2 + \left(\mu_s - \frac{\sigma_x^2}{2}\right)\xi - \lambda = 0$ with $r_1 = -\xi_-$ and $r_2 = -\xi_+$.

Finally, average establishment size $\bar{s}$ is given by

$$\bar{s} = s_e \frac{-\xi_-\xi_+}{(\xi_+ - 1)(1 - \xi_-)} = s_e \left(\frac{\lambda}{\lambda - \mu_s}\right). \blacksquare$$

A.3 TFPR

In our model, an establishment's TFPR is given by:

$$\text{TFPR} \quad = \quad \frac{y}{k^{\alpha/(\alpha+\gamma)} n^{\gamma/(\alpha+\gamma)}} \propto \frac{1}{\tau^{\theta(1-\alpha-\gamma)}} = \frac{1}{(1-\tau_y)},$$

which is equated across all establishments in the undistorted economy. In this context, misallocation arises from dispersion in TFPR across establishments. In our environment, the distribution of distortions $g_\tau(\tau)$ is a Double Pareto. Therefore, log TFPR follows a Double Exponential with roots $\xi_{TFPR,-}$ and $\xi_{TFPR,+}$ that solve the characteristic equation:

$$\frac{\sigma_{TFPR}^2}{2}\xi^2 + \left(\mu_{TFPR} - \frac{\sigma_{TFPR}^2}{2}\right)\xi - \lambda = 0,$$

where $\mu_{TFPR} = -\theta(1-\alpha-\gamma)\mu_\tau - \theta(1-\alpha-\gamma)(-\theta(1-\alpha-\gamma)-1)\frac{\sigma_\tau^2}{2}$ and $\sigma_{TFPR}^2 = \theta^2(1-\alpha-\gamma)^2\sigma_\tau^2$.

Standard deviation of log revenue total factor productivity (TFPR):

$$\text{SD log TFPR} = \sqrt{\frac{1}{\xi_{TFPR,-}^2} + \frac{1}{\xi_{TFPR,+}^2}}.$$

B Appendix: Extension

We extend the model to allow for the mass of establishments to be determined in equilibrium and assess how our main results change in this case. We first highlight changes in the solution of the model with endogenous mass of establishments, and then we show quantitatively how the results change in this extended model.

B.1 Theory

We start by introducing the mass of entrants M in our model which is given by:

$$M = \int_{s_e}^{+\infty} g(s)ds$$

The mass depends on the distribution of firms $g(\cdot)$ and entrants' minimum size s_e that are endogenous. We assume, as in Bento and Restuccia (2017), that firms draw their initial productivity from the distribution $g(\cdot)$ and all firms with productivity above z_e enter. These firms enter with the minimum productivity requirement z_e.

B.1.1 Stationary Distribution

To find the stationary distribution, we proceed by following a similar methodology as in the main text. Given the optimal decisions of incumbents and entering establishments, we characterize the stationary distribution $g(z, \tau)$ over productivity z and distortion τ.

Let $M(x, t)$ denote the number density function of establishments, i.e. the mass of size x establishments at time t. At time t, the total number of establishments is equal to $M(t) = \int_{-\infty}^{+\infty} M(x, t)dx$. The establishments relative size process can be modeled by a *modified* Kolmogorov-Fokker-Planck equation of the form:

$$\frac{\partial M(x,t)}{\partial t} = -\mu_x \frac{\partial M(x,t)}{\partial x} + \frac{\sigma_x^2}{2}\frac{\partial^2 M(x,t)}{\partial x^2} - \lambda M(x,t) + B(0,t), \tag{B.3}$$

where λ is the exit rate of establishments and the function $B(0, t)$ is new establishments that enter at t and have (normalized) size 0.

The solution of this problem is discussed in Gabaix (2009). We are interested in a stationary distribution for the number density function, i.e. solutions that are separable in time t and are of the form $M(x,t) = M(t)f(x)$ and $B(0,t) = M(t)b\delta(x-0)$, where b is the establishment entry rate at point $x = 0$ and $\delta(\cdot)$ is a Dirac delta function. Therefore, we can rewrite the *modified* Kolmogorov-Fokker-Planck equation equation (B.3) as:

$$\frac{M'(t)}{M(t)}f(x) = \eta f(x) = -\mu_x f'(x) + \frac{\sigma_x^2}{2}f''(x) - \lambda f(x) + b\delta(x-0), \tag{B.4}$$

where $\frac{M'(t)}{M(t)}$ is the mass growth rate denoted by η and $M(t) = e^{\eta t}M(0)$ in the balanced growth path. We normalize $M(0) = 1$. We assume four boundary conditions:

$$\lim_{x\to+\infty} f(x) = 0, \qquad \lim_{x\to+\infty} f'(x) = 0, \tag{B.5}$$

$$\lim_{x\to-\infty} f(x) = 0, \qquad \lim_{x\to-\infty} f'(x) = 0, \tag{B.6}$$

and

$$f(x) \geq 0, \qquad \int_{-\infty}^{+\infty} f(x)dx = 1. \tag{B.7}$$

The first four boundary conditions (B.5) and (B.6) guarantee that the stationary distribution is bounded, and equations (B.7) guarantee that f is a pdf. The boundary constraints restricts the growth rate η, by integrating (B.4) we find:

$$\eta \int_{\infty}^{+\infty} f(x)dx = \left(-\mu_x f(x) + \frac{\sigma_x^2}{2}f'(x)\right)\Bigg|_{-\infty}^{+\infty} - \lambda \int_{-\infty}^{+\infty} f(x)dx + \int_{-\infty}^{+\infty} b\delta(x-0)dx$$

and applying the boundary conditions and using the Dirac delta function, we find that the mass of establishments growth rate η is equal to:

$$\eta = b - \lambda.$$

The expression for η has a very intuitive interpretation, it states that the mass growth rate is equal to the net entry rate $(b-\lambda)$. After some algebraic manipulation from equation (B.4), we find that the stationary distribution must satisfy the following differential equation:

$$f''(x) - \frac{2\mu_x}{\sigma_x^2} f'(x) - \frac{2(\lambda+\eta)}{\sigma_x^2} f(x) = -\frac{2b}{\sigma_x^2}\delta(x-0), \tag{B.8}$$

subject to the boundary conditions and to $f(\cdot)$ be a pdf. We can now characterize the stationary (log) size distribution, which is a double Pareto, with endogenous tail index, ξ, and endogenous net entry rate, $b-\lambda$ at $x=0$. Formally, Lemma 2 characterizes the stationary distribution.

Lemma 3. *Given wages w and rental rate of capital r, the stationary size distribution associated with the output tax rate Geometric Brownian Motion is an double Pareto:*

$$g(s) = \begin{cases} C\left(\frac{s}{s_e}\right)^{-(\xi_- + 1)} & \textit{for} \quad s < s_e. \\ \\ C\left(\frac{s}{s_e}\right)^{-(\xi_+ + 1)} & \textit{for} \quad s \geq s_e. \end{cases}$$

where the tail indexes ξ_+ is the positive root and ξ_- is the negative root that solves the characteristic equation $\frac{\sigma_x^2}{2}\xi^2 + \left(\mu_s - \frac{\sigma_x^2}{2}\right)\xi - (\lambda+\eta) = 0$ and $C = \frac{-\xi_-\xi_+}{s_e(\xi_+ - \xi_-)}$. Moreover, the average size $\overline{s}$ is given by:

$$\frac{\overline{s}}{s_e} = \frac{\eta+\lambda}{\eta+\lambda-\mu_s}.$$

The poof of Lemma 3 is similar to the proof of Lemma 2 in the main text. The only difference between the endogenous distribution with and without the mass is the parameter η that governs the mass growth rate. Since we are looking for the stationary solution, we assume that $\eta = 0$, and consequently, the mass of firms is stationary. An important implication of this assumption is that the stationary distribution solution is the same when the mass is endogenous or exogenous. As a result, the only new variable we need to pin down to find the stationary equilibrium is the mass of firms M which adds a new equation to the stationary equilibrium defined below.

B.1.2 Stationary equilibrium

Definition Given a policy $\{\mu_\tau, \sigma_\tau, \rho, \tau_e\}$, a stationary equilibrium is an invariant size distribution $g(s)$, a mass of establishments M, value and policy functions of incumbent establishments $\{W(s),$ $k(s)$, $n(s)\}$, establishment productivity growth μ_z, value of entrants W_e, entrants' productivity z_e, entry rate b, prices $\{r, w\}$, transfer T, profits Π, capital K, and consumption C, such that:

i) Consumer optimization implies that $R = r - \delta_k = \varrho$ and aggregates C and K.

ii) Given prices, the incumbents' policy functions $\{k(s), n(s)\}$ solve the incumbents' static problem (2).

iii) Given prices, the incumbents' value function $W(s)$ solves the incumbents' dynamic problem (6), μ_z is optimal from this problem.

iv) Given prices, the value of entrants satisfy the zero profit condition in (9) and entry productivity z_e is determined by this condition.

v) The stationary distribution $g(s)$ and entry rate of establishments b solve the Kolmogorov-Fokker-Planck equation (B.3).

vi) The mass of establishments' M is given by

$$M = \int_{s_e}^{+\infty} g(s)ds$$

vii) Markets for capital and labor clear (market clearing in the goods market is satisfied by Walras' law):

a) capital: $K = M \int_0^{+\infty} k(s)g(s)ds,$

b) labor: $1 = M \int_0^{+\infty} n(s)g(s)ds.$

viii) Transfers T guarantee that the government's budget constraint is satisfied.

The inclusion of an endogenous mass of establishments has led to two changes to the stationary equilibrium. First, the mass enters the market clearing conditions for capital and labor. Second, a new equation characterizes the mass of establishments as a function of the stationary distribution and the productivity entry point.

B.1.3 Aggregate

We can follow the same methodology as in the main text to find the model main aggregates in closed form taking into consideration the mass of establishments. The key equation that changes is the labor market clearing condition that we use to pin down wages. Aggregate output Y and total factor productivity TFP explicitly depend on the mass of establishments M as shown in the equations below:

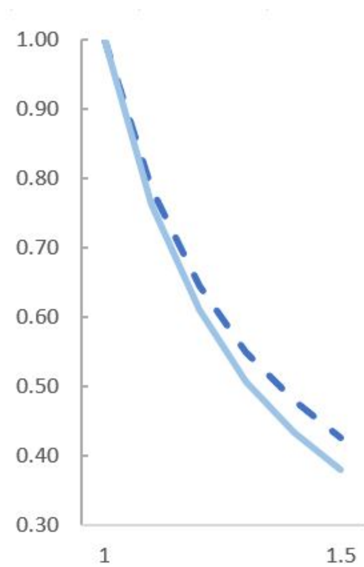
$$\begin{aligned}
Y &= \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\lambda-\mu_s}{\lambda-\mu_y}\right) (M\bar{s})^{\frac{1-\alpha-\gamma}{1-\alpha}} \left(\frac{1}{\tau_e^{\theta(1-\alpha-\gamma)}}\right), \\
\text{TFP} &= \left(\frac{\lambda-\mu_s}{\lambda-\mu_y}\right) (M\bar{s})^{1-\alpha-\gamma} \left(\frac{1}{\tau_e^{\theta(1-\alpha-\gamma)}}\right).
\end{aligned}$$

B.2 Quantitative

We can now assess quantitatively how the model predictions change when we introduce the mass of establishments endogenously. We depart from our calibrated economy, where the mass is exogenous and normalized to one, and compute the impact on TFP in the model with and without an endogenous mass of establishments. We perform this experiment using our benchmark economy when $\rho = -0.09$ and cases with constant relative size distribution (constant ξ_s).

We find that the mass introduction amplifies the impact of policy distortions on TFP and the impact is more significant when distortions are larger. In an economy where distortions are 50 percent larger than in the benchmark economy $\sigma_\tau^2 = 1.5$, TFP drops by 57 percentage points in an economy without an endogenous mass and 62 percentage points when the mass is endogenous. Our results point out that the mass can amplify the impact of policy distortions, particularly when distortions are more significant.

Figure 3: The Role of the Mass on TFP



Notes: The blue-dashed line represents the benchmark economy with changes in σ_τ^2 and constant ξ_s. The blue-solid line represents the same economies with an endogenous mass. In all economies, μ_τ is adjusted to normalize mean distortions to one, i.e., mean $\tau^\theta = 1$.