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Measuring â•œGoodâ•š and â•œBadâ•š Inequality: A Cautionary Tale.

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Abstract.

While income inequality is generally thought to have a deleterious effect on societal wellbeing, some inequality, necessary for optimal resource allocation, is beneficial. Inequality as usually measured is an amalgam of both, but from both policy and wellbeing measurement perspectives, distinguishing between beneficial and less beneficial inequality typologies makes sense. Here the distinction is explored in considering the progress of personal incomes in 21st Century Canada. Using standard inequality measures, techniques for identifying and measuring “Good” and “Bad” inequality are proposed and applied in analysing Human Resource, Gender and Immigrant status-based income differences. Analysis categorizing Human Resource-based differences as efficiency promoting “Good” inequalities and Gender and Immigrant status-based differences as discriminatory and “Bad” inequalities, reveals that under all proposed measures, whilst Overall and “Good” inequalities grew over the sample period, “Bad” inequality components diminished, emphasising the point that inequality measures need to be fit for purpose lest they be misleading.

Key words: Inequality Measurement,

Jel codes C10, D31, D63

Introduction.

Increasing income inequality, generally considered detrimental to societal wellbeing, has been the focus of much public and academic discourse in recent times (see for example Autor 2014, Blundell et. al. 2018, Chandra 2003, Goldin 2014, Piketty 2014). It is invariably measured, either in terms of the standardized totality of multilateral differences (as in Gini type coefficients), or as an aggregation of all differences from some focus point like a mean or median centrality measure (as in the Atkinson (1970) equivalized income and Thiel (1967) information theoretic inequality measure families)[1] or as in upper – lower quantile differences in a polarization style measure (Blundell et. al. 2018, Piketty 2014). Yet not all inequality is unequivocally normatively bad. Krueger (2003) posed and debated the question "When is inequality too much of a good thing?" the inference being that in some contexts it is demonstrably a "Good" thing and in others it is "Bad" (Autor (2014) makes a similar point in arguing that some wage differentials are necessary for resource allocation efficiency reasons). If this is indeed the case, it behoves analysts and policy makers to make the distinction and, rather than use overall inequality, measure the extent of "Good" and "Bad" inequalities and specifically address them differentially in wellbeing measurement or as a policy evaluation metric. When both types of inequality are trending in the same direction this is less of an issue, but when the two are progressing in different directions, the distinction is crucial for effective analysis.

This raises the question: "Is the proposed inequality measure fit for purpose?", the answer, of course, depends upon its purpose and focus point. For example, suppose a societies concern is that all constituents should have the best possible health outcome and an inequality measure is to be used to evaluate policy progress regarding the diversity of experiences. In essence the society has a twin policy objective, namely equalizing and levelling up health outcomes, standard inequality measures that quantify differences from a focus point in the middle of the health outcome range could be misleading in this regard. Minimising average relative distance from the best health outcome is the joint policy goal. A fortiori, a larger average distance from the worst health outcome is a "good thing" to be measured as a "Good Inequality" and maximised,

[1] Sen (1976) advocated measuring a societies aggregate wellbeing by discounting National Income by some decreasing function of the Gini coefficient, an aggregation of all possible bilateral differences whereas UNDP (2019) use an Atkinson (1970) inequality measure based upon the ratio of the geometric mean of incomes to the average income, an aggregation of all differences from the average.

whereas a larger average distance from the best health outcome is a bad thing and to be minimised. Furthermore, in securing commonality at some outcome level anywhere other than at the top of the health outcome range, for example measuring inequality as the average relative distance from the average health outcome, could be misleading in the short run.

More generally, in an idealized, meritocratic, incentive driven, fully informed society with an uninhibited free labour market, where individual income is the sole reward for expending effort in the productive application of its human resources, income variation across individuals simply reflects their varying contributions to the societal good which in turn reflects the variety of uninhibited choices regarding personal efforts and human resource stocks. When all individuals could freely choose another's path but didn't, each will be in their preferred place in the income distribution, any resultant inequality will have been necessary in securing the efficient allocation of resources which would be optimal in terms of aggregate societal wellbeing. In such circumstances it is difficult to construe the inequality as "Bad" for that society and it should probably not count as a negative factor in societal wellbeing computations. Indeed, any forced equalization of incomes would result in individuals being in less preferred positions than the ones they would have freely chosen and the consequent reduction in inequality would be associated with a reduction in overall societal wellbeing. It follows that, concomitant with the "optimal" income distribution, there may be an optimal level of income inequality (call it "Good" inequality) in such a society which, if departed from in either direction, would result in reduced societal wellbeing[2]. In this context the extent of distortion of the optimal income distribution would be the "Bad" Inequality that needs to be identified and addressed by policy.

Clearly, such fully informed and uninhibited labour markets rarely exist in practice and overall measured income inequality is an amalgam of both "Good" and "Bad" inequalities. Equally clearly the two types could progress in different directions, so from both policy and wellbeing measurement perspectives, in assessing the deleterious effects of inequality on a society, it behoves researchers and practitioners to make the typology distinction and address the "Bad" and "Good" inequalities differentially. However, if inequality modulated wellbeing measurement

[2] The "Good" and "Bad" distinction is also relevant in growth theory (Galor 2011) where "Good" inequalities could be considered growth promoting and "Bad" inequalities growth impeding and, since the seminal work of Kuznets (1955), empirical growth models that relate inequality and growth have employed standard measures of inequality which make no distinction between "Good" and "Bad" inequality typologies.

is anything to go by over the past 100 years (see for example Dalton 1920, Atkinson 1970, Sen 1976, Foster 1994, UNDP 2019), all income inequality is generally deemed uniformly detrimental to the social good and thus normatively a "Bad" thing[3]. Indeed, beyond the approaches taken in the equal opportunity literature (see for example references in Ferreira and Peragine 2015), little has been done to distinguish between "Good" and "Bad" inequalities, regardless of it making sense to do so.

The challenge is to identify the normatively "Good" and normatively "Bad" inequality components in these measures and weigh them accordingly for an appropriate inequality modulated wellbeing measure for use in growth models or policy intervention purposes. Section 2 considers some possibilities for measuring the relative magnitudes of "good" and "bad" inequalities in terms of commonly employed inequality measures and a similar exercise is pursued in Section 3 in terms of distributional differences. Assuming income differences predicated on differences in human resources (embodied human capital, experience and efforts) are not wellbeing diminishing whereas income differences based upon gender and immigrant status are, an illustrative example using 21st century Canadian Income distributions is provided in Section 4 and some conclusions drawn in section 5.

Section 2. Measuring Good and Bad Inequality with the Gini coefficient and Coefficient of Variation.

Some insight into the relative magnitudes of good and bad inequality can be gleaned from a subgroup decomposition of the Gini coefficient, a mean standardized unit free average of all multilateral differences in the sampled population. In considering K subgroups indexed $k = 1, .., K$ with respective means, subgroup Ginis and population weights μ_k , $GINI_k$ and w_k, Anderson and Thomas (2019) decomposed the GINI coefficient as:

$$GINI = \sum_{k=1}^{K} w_k^2 \frac{\mu_k}{\mu} GINI_k + \frac{1}{\mu} \sum_{k=2}^{K} \sum_{j=1}^{k} w_k w_j \left|\mu_j - \mu_k\right| + NSF \qquad [1]$$

$$= WGINI + BGINI + NSF$$

[3] Most objective inequality measures used in analysis (e.g. mean absolute deviations, coefficients of variation, information theoretic Theil measures and Gini coefficients, second and higher order dominance comparisons) are straightforward aggregations of all absolute bilateral differences or individual differences from some locational measure, as though all such differences, or monotonic transformations of them, have a negative impact on overall societal wellbeing or growth.

Where $WGINI$ reflects within group inequality, $BGINI$ is a Gini like coefficient based upon subgroup means reflecting between group differences in terms of their means and NSF is a non-segmentation factor reflecting the extent of commonality between the subgroups. In effect NSF measures the degree of overlap amongst the groups. When the subgroups are identical $NSF = 2\sum_{k=2}^{K}\sum_{j=1}^{k} w_k w_j GINI$ (its maximal value) when the subgroups have mutually exclusive outcomes, i.e. no outcomes in common, $NSF = 0$[4]. It follows that $SFR = (1 - NSF)/GINI$ provides a good measure of the extent to which the groups are segmented and don't overlap whereas $NSFR = NSF/GINI$ provides a measure of the extent of commonality between the groups. An alternative statistic, the Coefficient of Variation, is the ratio of the standard deviation of incomes to the mean, as a mean standardised average proximity of incomes to a mean focus in comparison to the mean, it captures a different aspect of inequality. Indeed, by considering deviation around some other centrality focus, it can be adapted to consider more nuanced measures of inequality.

In the following it is assumed that an individuals' human resource is a monotonic non-decreasing potentially concave function of cardinally calibrated but latent human capital (education and training) levels HC and experience EX, each respectively proxied for by K ordered categorical education and training level categories indexed $k = 1,..,K$ and J age level categories indexed $j = 1,..,J$ resulting in I= KxJ human resource categories. Let the proportion of the population in the k,j'th human resource category be $p_{k,j}$, and the average income in that category be $x_{k,j}$. Note that $\sum_{k=1}^{K}\sum_{j=1}^{J} p_{k,j} = 1$ and $\overline{X}$, average income in society is given by:

$$\overline{X} = \sum_{k=1}^{K}\sum_{j=1}^{J} p_{k,j}x_{k,j}$$

Suppose there are G distinguishable groups in society indexed $g = 1,..,G$ with the proportions of the overall population in group g being p_g. Let the proportion of the overall population in the k,j'th human resource category and the g'th group be $p_{k,j,g}$, and the average income of the g'th group in that category be $x_{k,j,g}$. Noting that $\sum_{g=1}^{G}\sum_{k=1}^{K}\sum_{j=1}^{J} p_{k,j,g} = 1$ and $\sum_{k=1}^{K}\sum_{j=1}^{J} p_{k,j,g} = p_g$, $\overline{X}_g$, the average income in group g is given by:

[4] In this case the Gini coefficient is subgroup decomposable (Mookherjee and Shorrocks 1982, Shorrocks 1984).

$$\overline{X}_g = \frac{1}{p_g} \sum_{k=1}^{K} \sum_{j=1}^{J} p_{k,j,g} x_{k,j,g}$$

$BGINI$, the Gini Coefficient and CV, the Coefficient of Variation between the groups are respectively:

$$BGINI = \frac{1}{2\overline{X}} \sum_{g=1}^{G} \sum_{g'=1}^{G} p_g p_{g'} \left|\overline{X}_g - \overline{X}_{g'}\right| : \quad CV = \frac{\sqrt{\sum_{g=1}^{G} p_g \left(\overline{X}_g - \overline{X}\right)^2}}{\overline{X}} \qquad [2]$$

However, these inequality measures do not take account of variation in incomes across human resource classes. To simplify matters, organise the population proportions $p_{k,j,g}$ in a 1xKxJxG vector $\underline{p}$ with typical element $p_i \ i = 1,..,(K.J.G)$ and a correspondingly organise the average incomes $x_{k,j,g}$ into a vector $\underline{x}$ with typical element $x_i \ i = 1,..,I \ (where \ I = K.J.G)$. When income variation across human resource categories is accounted for:

$$GIT = \frac{1}{2\overline{X}} \sum_{i=1}^{I} \sum_{i'=1}^{I} p_i p_{i'} \left|\overline{X}_i - \overline{X}_{i'}\right| : \quad CVT = \frac{\sqrt{\sum_{i=1}^{I} p_i \left(\overline{X}_i - \overline{X}\right)^2}}{\overline{X}} \qquad [2]$$

In an idealized, meritocratic, incentive driven, fully informed society with an uninhibited free labour market, where individual income is the sole reward for expending effort in the productive application of its human resources, when all groups have identical effort distributions, all will be enjoying the average income level appropriate to their human resource status and level of effort. All income differences within and between human resource groups would be acceptable to all and the level of inequality recorded in [2] normatively acceptable and “good”. When there is discrimination between groups, while average income differences within groups across human resource categories would be acceptable, average income differences between groups within given human resource categories are normatively unacceptable and “Bad”. The JxK components of GIT that reflect this are:

$$GIB_{j,k} = \frac{1}{2\overline{X}} \sum_{g=1}^{G} \sum_{g'=1}^{G} p_{j,k,g} p_{j,k,g'} \left|x_{k,j,g} - x_{k,j,g'}\right| \ j = 1,..,J; k = 1,..,K \qquad [3]$$

and a sense of the extent of "Bad" inequality in overall inequality can be gleaned from:

$$\frac{\sum_{j=1}^{J}\sum_{k=1}^{K} GIB_{j,k}}{GIT} \qquad [4]$$

Regarding the Coefficient of Variation, the income differences that are bad are conveniently summarised in [1] and a sense of the extent of "Bad" inequality in overall inequality in this context can be gleaned from:

$$\frac{CV}{CVT} = \frac{\frac{\sqrt{\sum_{g=1}^{G} p_g\left(\overline{X}_g-\overline{X}\right)^2}}{\overline{X}}}{\frac{\sqrt{\sum_{i=1}^{I} p_i\left(\overline{X}_i-\overline{X}\right)^2}}{\overline{X}}} = \sqrt{\frac{\sum_{g=1}^{G} p_g\left(\overline{X}_g-\overline{X}\right)^2}{\sum_{i=1}^{I} p_i\left(\overline{X}_i-\overline{X}\right)^2}} \qquad [5]$$

For a more nuanced measure, consider replacing the mean in the variation element with some other target as a focus call it X^T so that [5] becomes:

$$\frac{CV}{CVT} = \frac{\frac{\sqrt{\sum_{g=1}^{G} p_g\left(\overline{X}_g-X^T\right)^2}}{\overline{X}}}{\frac{\sqrt{\sum_{i=1}^{I} p_i\left(\overline{X}_i-X^T\right)^2}}{\overline{X}}} = \sqrt{\frac{\sum_{g=1}^{G} p_g\left(\overline{X}_g-X^T\right)^2}{\sum_{i=1}^{I} p_i\left(\overline{X}_i-X^T\right)^2}} \qquad [5a]$$

Section 3. Distributional Equality.

The foregoing analysis has been pursued in terms of a distributional location measure, namely the mean which can potentially hide a great deal of inter group differences (Carniero Hansen and Heckman 2004). For example, if all groups at all human resource levels had common means but different higher moments, [1], [2] and [3] would reveal no inequality in a situation where differences between groups clearly existed. Basically, commonality of a summary statistic is only necessary but not sufficient for commonality of a distribution. This problem can be addressed by working with the Distributional Gini Coefficient (DGI) and Distributional Coefficients of Variation (DCV), (Anderson, Linton, Pittau, Whang and Zelli 2021) which compare distributions in their entirety[5]. These constructs are based upon TR, Gini's Transvariation measure (Gini 2016) which, for two probability density functions $f_g(x), f_{g'}(x)$ defined on a common support $[a, b]$, is given by:

[5] Though specific to probability density functions these distributional comparison instruments can easily be adapted and generalised to consider specific aspects of distributions by using Lorenz Curves, Generalized Lorenz Curves or $F^i(x)$, a higher order cumulant of the pdf, where: $F^i(x) = \int_{-\infty}^{x} F^{i-1}(x)dx \;\; i = 1, ..$ and $F^0(x) = f(x)$.

$$TR_{g,g'} = 0.5 \int_a^b \left| f_g(x) - f_{g'}(x) \right| dx$$

When the distributions are defined on discrete ordered support $h = 1,..,H$ where $f_{g,h}$, a typical component of the vector f_g is the probability of the h'th outcome under the g'th distribution:

$$TR_{g,g'} = 0.5 \sum_{h=1}^{H} \left| f_{g,h} - f_{g',h} \right|$$

TR is conveniently equal to $1 - OV$ where OV, the overlap of two distributions is of the form $\int_a^b min\left(f_g(x), f_{g'}(x)\right) dx$ and $\sum_{h=1}^{H} min\left(f_{g,h}, f_{g',h}\right)$ respectively for continuous and discrete paradigms[6] (in the application the discrete paradigm will be employed). Writing f_O, the Overall population distribution as a weighted sum of all subgroup distributions so that $f_O = \sum_{i=1}^{I} p_i f_i$, then the Overall Distributional Gini Coefficient and Distributional Coefficient of Variation may be written as:

$$DGIT = \frac{0.5}{\left(1 - \sum_{i=1}^{I} p_i^2\right)} \sum_{i=1}^{I} \sum_{i'=1}^{I} p_i p_{i'} TR_{i,i'} : \quad DCVT = \frac{1}{\left(1 - \sum_{i=1}^{I} p_i^2\right)} \sum_{i=1}^{I} p_i TR_{i,O} \qquad [6]$$

When there is discrimination between groups, while distributional differences within groups across human resource categories would be acceptable, distributional differences between groups within given human resource categories are normatively unacceptable and "bad". The JxK components of GIT that reflect this are:

$$DGIB_{j,k} = \frac{0.5}{\left(1 - \sum_{i=1}^{I} p_i^2\right)} \sum_{g=1}^{G} \sum_{g'=1}^{G} p_{j,k,g} p_{j,k,g'} TR_{(j,k,g),(j,k,g')} \quad j = 1,..,J;\, k = 1,..,K \qquad [7]$$

and a sense of the extent of "Bad" distributional inequality in overall inequality can be gleaned from:

$$\frac{\sum_{j=1}^{J} \sum_{k=1}^{K} DGIB_{j,k}}{DGIT} \qquad [8]$$

[6] This relationship is useful because, based upon random samples of a common size n, OV has been shown to have an asymptotically normal distribution such that $\sqrt{n}\left(\widehat{OV} - OV\right) \sim_a N\left(0, \left(OV(1 - OV)\right)\right)$

Group $g's$ distribution may be written as $f_g(x) = \frac{1}{p_g}\sum_{k=1}^{K}\sum_{j=1}^{J} p_{k,j,g} f_{k,j,g}$ and its Transvariation with the overall distribution denoted $TR_{g,O}$, and the "Bad" component of DCVT summarised as:

$$DCVB = \frac{1}{(1-\sum_{i=1}^{I} p_i^2)}\sum_{g=1}^{G} p_g TR_{g,O}$$

and a sense of the extent of "Bad" distributional inequality in overall inequality as measured by the Distributional Coefficient of Variation can be gleaned from:

$$\frac{DCVB}{DCVT} = \frac{\sum_{g=1}^{G} p_g TR_{g,O}}{\sum_{i=1}^{I} p_i TR_{i,O}} \qquad [9]$$

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Section 4. Good and Bad Inequalities in the Canadian Income Distribution.

To exemplify the effect of making the "good" versus "bad" inequality distinction an heroic assumption that at given levels of effort, ceteris paribus, the rewards structure across all human resource categories is socially acceptable. Thus, in long run equilibrium, all individuals in a given human resource category, exhibiting the same effort receive the same expected income and furthermore all are content with other individuals in other categories at other effort levels receiving different rewards. In such a world there will clearly be income inequality, but it is to be welcomed on efficient resource allocation grounds and is acceptable to all, it is in effect, a "Good" inequality. The existence of identifiable groups whose memberships, because of their group identity, do not enjoy the same level of income as others at a given human resource category and effort level, induces "Bad" inequalities into the mix, based upon income differences at those given human resource and effort levels. Here, for exemplification purposes and cognizant of discrimination concerns regarding gender (Goldin 2014) and immigrants (Aydemir, Chen and Corak 2013), such groups will be defined by immigrant/non-immigrant and gender status on the supposition that immigrants and females are potentially discriminated against in the labour market.

To examine these issues in the context of the income, human resource and effort nexus, data on the total income, age, gender, immigrant/non-immigrant and education status of individuals have been drawn from the Census of Canada: Individual Files for the years 2001, 2006, 2011 and 2016. All agents over the age of 19 who received and income and reported age and educational status were included in the study and an agents' location in the income distribution was based upon its membership in one of the 20 income vingtiles.

An individuals' human resources were based upon their (ordered) education and training category and their age group membership, the five Education and Training Categories were:

1. No certificate, diploma or degree.

2. Secondary (high) school diploma or equivalency certificate.

3. Trades certificate or diploma, Certificate of Apprenticeship or Certificate of Qualification. Program of 3 months to 2 years (College, CEGEP and other non-university certificates or diplomas).

4. Program of more than 2 years (College, CEGEP and other non-university certificates or diplomas), University certificate or diploma below bachelor level or Bachelors degree.

5. University certificate or diploma above bachelor level, Degree in medicine, dentistry, veterinary medicine or optometry, Master's degree or Earned doctorate.

The six age groups were: 1) 20-29, 2) 30-39, 3) 40-49, 4) 50-59, 5) 60-69 and 6) 70 and over.

The existence of 5 ordered categories of embodied human capital and 6 age groups proxying for experience yields 30 possible human resource categories. Four potential discrimination categories, based upon immigrant/non-immigrant and gender status were defined yielding 120 categories in all when gender and immigrant status are accounted for. When there is no discrimination in the labour market all females, males, immigrants and non-immigrants in a common human resource and effort category should receive the same income. Since Effort categories are not unobserved initially it will be assumed that all individuals exhibit the same effort.

Table 1. Subgroup Average Income and Category Shares.

	Male		Female	
	Non-Immigrant	Immigrant	Non-Immigrant	Immigrant
2001Average Income	37970.6379	35395.8301	24000.7892	22183.1930
Category Share.	0.3798	0.1111	0.3928	0.1164
2006 Average Income	50143.1828	45110.4938	30891.3513	27484.4777
Category Share.	0.3707	0.1160	0.3888	0.1246
2011 Average Income	56032.2567	49949.1617	37394.9443	32780.1094
Category Share.	0.3633	0.1246	0.3779	0.1342
2016 Average Income	66277.4820	55505.6979	43947.7075	36880.0036
Category Share.	0.3371	0.1276	0.3888	0.1464

Table 1 reports the average pre-tax incomes and relative category sample sizes over the 4 observation years for Male Non-Immigrants (MNI), Male Immigrants (MI), Female Non-Immigrants (FNI) and Female Immigrants (FI) respectively. Note that the groups have enjoyed differential income growth over the period with respective growth rates of 3.7% (MNI), 3.0% (MI), 4.0% (FNI) and 3.4% (FI) respectively.

Turning to the Gini coefficient, two decompositions are contemplated one with respect to all 120 groups defined by human resource, gender and immigration status categories and the other by just the 30 human resource categories reflecting the notion that there is no immigrant/gender discrimination in the labour market. Gini has risen steadily over the period with a hiatus in 2011, the aftermath of the 2008 recession which was seen to have an equalizing effect. Within group inequalities both broadly and narrowly defined have diminished somewhat over the period whereas between group inequalities have increased. Recall that NSF reflects the extent of commonality between groups which appears to have increased over the period.

Table 2. Gini Decomposition Analysis.

	Overall Gini	Between	Within	NSF
2001 All 120 Groups	0.4411	0.2081	0.0058	0.2271
2001 Just 30 HR Groups	0.4411	0.1658	0.0180	0.2572
2006 All 120 Groups	0.4792	0.2231	0.0057	0.2504
2006 Just 30 HR Groups	0.4792	0.1817	0.0177	0.2798
2011 All 120 Groups	0.4754	0.2164	0.0055	0.2535
2011 Just 30 HR Groups	0.4754	0.1834	0.0174	0.2746
2016 All 120 Groups	0.4913	0.2313	0.0052	0.2549
2016 Just 30 HR Groups	0.4913	0.1957	0.0171	0.2785

The decomposition using just Human Resource categories is appropriate if there were no discrimination by gender or immigration status, in the presence of discrimination, the decomposition focussed on all HR and MNI, MI, FNI and FI categories would be appropriate. If indeed there were no discrimination, so that all males and females, immigrants and non-immigrants in a common category would receive the same income at a given effort level, both decompositions would yield the same result so that differences between the two analyses yields information on the impact of discrimination. Making that comparison, increases in Between category inequalities increase while Within category inequalities diminish as does the extent of between category commonalities (NSF), all a result of the extension of categorization to include MNI, MN, FNI and FI distinctions and thus a reflection of the extent of "Bad" inequalities.

Working with just the category averages and sampling shares, BGINI, a Gini coefficient based upon subgroup means and population shares, can provide evidence of the extent of “Good” and “Bad” inequalities. Since the decomposition based upon all MNI, MI, FNI and FI based HR categories can be construed as an amalgam of “Good” and “Bad” mean differences whereas, under an assumption of no differences between MNI, MI, FNI and FI groupings, the decomposition based upon HR designations alone reflects only good differences, BGINI (All Categories) less BGINI (HR categories) provides a measure of the magnitude of “Bad” inequalities in the society.

Table 3 Good and Bad Inequality analysis.

	BGINI All categories	BGINI HR groups	Total-Good (Bad Inequality)	Good/Total	Bad/Total
2001	0.2081	0.1658	0.0423	0.7967	0.2033
2006	0.2231	0.1817	0.0414	0.8144	0.1856
2011	0.2164	0.1834	0.0330	0.8475	0.1525
2016	0.2313	0.1957	0.0356	0.8460	0.1540

Table 3 provides a breakdown indicating that, whereas overall inequality has increased, in counterpoint the share of “Bad” inequalities has diminished significantly over the period. If, as is usually the case, the progress of overall inequality was used to measure success in reducing “Bad” inequalities, policymakers would have been seriously misled.

In measuring aggregate proximity to some centrality parameter, the Coefficient of Variation, along with the Thiel family of inequality measures, offers a slightly different perspective on inequality as compared to the Gini coefficient which measures the average of all multilateral differences. Variants of the Coefficient of Variation also offer the opportunity of considering aggregate differences from some target other than a centrality parameter. Table 4 reports the coefficient of Variation overall and for MNI, MI, FNI and FI subgroups and, in accord with the Gini results, there appears to have been growth in inequality in all groups over the period with a hiatus in 2011.

Table 4. Coefficients of Variation

	Overall	Male		Female	
		Non-Immigrant	Immigrant	Non-Immigrant	Immigrant
2001	0.8918	0.8263	0.9051	0.8204	0.8816
2006	1.4668	1.4834	1.6948	1.0055	1.1237
2011	1.2893	1.3076	1.4901	0.9573	1.0814
2016	1.4915	1.5685	1.7080	1.0364	1.1641

Table 5 reports Coefficients of Variation amongst subgroup means across all groups and across human resource groups alone and the corresponding "Good" and "Bad" inequality differences. Two foci for the measures are entertained, average income, reflecting a policy aspiration of equalization toward the mean, and the highest subgroup average income, that of Male Non-Immigrants, reflecting a policy aspiration of upward equalization. In the mean focus case Bad inequality constituted a around 20% of total inequality, in the best focus case it constituted around 11% of total inequality. In both cases as a share of increasing overall inequality, the Bad inequality component is diminishing. Note that, when the best subgroup outcome is the focus, overall inequality and "Good" inequality is measurably greater while "Bad" inequality is measurably smaller emphasising the importance of choosing measurement instruments that are fit for purpose.

Table 5. Good and Bad Inequality Analysis.

	Overall Inequality	Good Inequality	Bad Inequality	Bad Inequality Share
2001 Mean CV	0.3848	0.3025	0.0823	0.2139
"Best" CV	0.4861	0.4240	0.0621	0.1278
2006 Mean CV	0.4324	0.3410	0.0914	0.2114
"Best" CV	0.5490	0.4804	0.0686	0.1250
2011 Mean CV	0.4147	0.3416	0.0731	0.1763
"Best" CV	0.5153	0.4585	0.0568	0.1102
2016 Mean CV	0.4501	0.3630	0.0871	0.1935
"Best" CV	0.5688	0.5028	0.0660	0.1160

Cognizant of the fact that just employing subgroup means can cast a veil over a multitude of individual differences (Carniero, Hansen and Heckman 2003), the foregoing results van be checked by employing a distributional analysis using Distributional Gini and Distributional Coefficient of Variation measures (Anderson et. al. 2021). Table 6 reports the Distributional Gini results where it is observed that overall inequality has not increased over the period, however the share of "Good" Inequality has increased, and the share of "Bad" inequality has diminished.

Table 6. Distributional Gini Analysis.

	Overall Inequality	Good Inequality	Bad Inequality	Good Inequality Share	Bad Inequality Share
2001	0.1561	0.1299	0.0262	0.8322	0.1678
2006	0.1525	0.1263	0.0262	0.8282	0.1718
2011	0.1520	0.1309	0.0211	0.8612	0.1388
2016	0.1561	0.1342	0.0219	0.8597	0.1403

In Table 7, the Distributional Coefficient of Variation yields a slightly different perspective with overall average targeted inequality increasing marginally over the period, however the share of "Good" Inequality has increased and the share of "Bad" inequality has diminished. On the other hand, overall "best distribution targeted inequality has diminished over the period (groups getting closer to the target distribution) with the share of "Good" Inequality increasing and the share of "Bad" inequality has diminishing in accord with the other formulations.

Table 7. Distributional Coefficient of Variation Analysis.

	Overall Inequality	Good Inequality	Bad Inequality	Good Inequality Share	Bad Inequality Share
2001	0.2322	0.1953	0.0369	0.8411	0.1589
	0.2625	0.2251	0.0374	0.8575	0.1425
2006	0.2343	0.1966	0.0377	0.8391	0.1609
	0.2570	0.2189	0.0381	0.8518	0.1482
2011	0.2302	0.2029	0.0273	0.8814	0.1186
	0.2490	0.2188	0.0302	0.8787	0.1213
2016	0.2358	0.2058	0.0300	0.8728	0.1272
	0.2577	0.2294	0.0283	0.8902	0.1098

Section 5. Conclusions.

There is good reason to believe that not all inequality is detrimental to societal wellbeing and that overall inequality is an amalgam of "Good", wellbeing promoting and "Bad" wellbeing diminishing inequalities. From a policy perspective it makes sense to distinguish between the typologies especially if they have the potential for moving in different directions. Here possibilities for exploring the distinction are examined in the context of the progress of Canadian personal income distributions in the 21st Century. Making the heroic assumption that, in the absence of discrimination in the labour market, income variation across human resource categories was deemed to contribute positively to societal wellbeing and constitutes "Good" inequality, whereas discrimination within those categories with regard to gender and immigrant status was deemed to engender "Bad" inequalities. Measurement of those typologies using standard Gini Coefficient and Coefficient of Variation measures in terms of actual outcomes and distributions of outcomes was pursued. Under that categorization it was revealed that while, overall and "Good" inequalities was growing, the "Bad" inequality component, which was usually less than 25% of overall inequality, was diminishing over the period in all alternative measures. It should be stressed that while having some foundation in theory, the categorization was employed purely for illustrative purposes, the salutary point being that unequivocal employment of overall inequality measures could be misleading from policy guidance and wellbeing measurement perspectives.

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