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Experimental elicitation of ambiguity attitude using the random incentive system

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EXPERIMENTAL ELICITATION OF AMBIGUITY ATTITUDE USING THE RANDOM INCENTIVE SYSTEM

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Abstract. We demonstrate how the standard usage of the random incentive system in ambiguity experiments eliciting certainty and probability equivalents might not be incentive compatible if the decision-maker is ambiguity averse. We propose a slight modification of the procedure in which the randomization takes place before decisions are made and the state is realized, and prove that if subjects evaluate the experimental environment in that way (first - risk, second - uncertainty), incentive compatibility may be restored.

1. Introduction

The ambiguity literature has been motivated by thought experiments (Keynes, 1921; Ellsberg, 1961). However, when it comes to experimental implementation, theoretical difficulties arise. This paper concerns the theoretical validity of the most commonly used incentive mechanism - the random incentive system (RIS). This mechanism asks a subject to make a series of choices and then randomly implements only one for payment.

To demonstrate the challenge, consider the basic Ellsberg two-color environment in which a subject is asked to choose between a bet on an uncertain event (e.g., drawing a red ball from an urn with an unknown composition of red and black balls) and a bet on an event with a known probability of 0.5 (e.g., drawing a red ball from an urn with 50% red balls and 50% black balls). Facing such a choice, most subjects will choose the risky bet. This is not sufficient evidence for ambiguity aversion since we could rationalize the choice by a simple prior over the composition of the unknown urn. For example, the subject might be suspicious that the unknown urn is biased to his disadvantage. Alternatively, he may simply believe that there are more black balls than red balls in the unknown urn.

There are several ways to overcome these difficulties. One is to ask the subject to choose the color to bet on, and only then ask him to choose between the urns (e.g., Halevy, 2007). This may overcome the suspicion concerns, but several drawbacks remain.[1] Indeed, many experimentalists opted for a different solution in which the

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[1]First, a subject preferring the unknown urn may be ambiguity seeking or may have nonsymmetric beliefs over the colors. Second, even if the subject prefers a bet on the known urn, the experimenter

subject is asked to make two choices: between a bet on red in the urn whose composition is unknown and a bet on red in the urn with 50% red balls, and similarly between a bet on black in the unknown urn and a bet on black in the urn with 50% black balls. Then, only one of the choices is selected at random to determine the payment, usually using an objective randomization device (e.g., coin toss).

This usage of the random incentive system may be problematic for the following reason. Consider an ambiguity-averse subject whose underlying preferences are to bet on the two known rather than on the two ambiguous events. However, suppose he views the two choices as a single decision problem. In that case, he may conclude that by choosing bets on the two unknown events, he will win with a probability of 0.5, independent of the event that will materialize. Similarly, he would obtain an identical probability of winning by choosing the two risky bets. Therefore, he will be indifferent between truthfully reporting his preferences and reporting the opposite preferences! The underlying reason for this phenomenon is that the random incentive provides the subject with a randomization device that allows him to hedge the ambiguity. This point is closely related to Raiffa's (1961) argument against the normative appeal of Ellsbergian behavior, since the decision-maker can choose to flip a coin between the two uncertain bets. While the cognitive requirements to implement Raiffa's suggestion may be high, the random incentive system implements this strategy for the subject.

Similar observations have been made by Bade (2015, first version 2011), Kuzmics (2017, first version 2012) and Oechssler and Roomets (2014, first version 2013). Bade (2015) considers a general (possibly uncertain) randomization mechanism. She discusses alternative ways to observe ambiguity preferences but concludes that there is, in most cases, no clear solution. Kuzmics (2017) studies the implication of Wald complete class theorem for ambiguity averse subjects. He argues that an experimenter can never observe the true preferences of an ambiguity averse subject who is Waldian and hence satisfies Anscombe and Aumann's (1963) axiom of reversal of order (i.e., the subject is indifferent between a risky lottery over acts or the equivalent act yielding risky lotteries). Oechssler and Roomets (2014) make the point for Ellsberg three-color experiments and discussed the risk of misclassifying subjects as ambiguity neutral. Azrieli et al. (2018) study a general model of incentives in experiments and show that a condition of *statewise monotonicity*, which is close to Savage's P3,[2] is equivalent to incentive compatibility of a class of experiments that use the RIS. They note that in the domain of uncertainty, reversal of order (which they term *reduction*) together with statewise monotonicity implies ambiguity neutrality. It follows that if a subject is ambiguity averse and satisfies reversal of order, there exist experiments using the RIS that are not incentive compatible.

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can measure ambiguity only by invoking an identifying assumption that beliefs are symmetric. This is a major concern when symmetry is not a natural assumption (e.g., bets on the stock market or weather).

[2]It also relates to Machina and Schmeidler's (1995) substitution hypothesis, but in the latter, the order of lotteries and acts is reversed.

We begin by demonstrating (Proposition 5) that the problem of eliciting ambiguity preferences is pervasive in the most common elicitation mechanisms used by experimentalists to study ambiguity attitudes – experiments in which probability or certainty equivalents are elicited. If the experimentalist is using RIS in which the randomization occurs after the realization of the event, an ambiguity averse subject with maxmin expected utility preferences (Gilboa and Schmeidler, 1989) will exhibit a behavior that can be rationalized by subjective expected utility preferences.[3]

We then turn to answer whether there exists an implementation of the RIS that will be incentive compatible in eliciting ambiguity preferences. We note that the above result depends on the randomization performed by the RIS occurring *after* the uncertainty is resolved. This induces an Anscombe and Aumann (1963) act which assigns to every state an objective lottery and where uncertainty can be hedged. If, however, the randomization occurs *before* the resolution of uncertainty, and the subject evaluates the experiment *in this order*, incentive compatibility may be restored. We prove this result assuming expected utility under risk and show that it extends to some non-expected utility preferences.

Our theoretical approach to solving the practical problem is inspired by strong experimental evidence suggesting that ambiguity aversion is tightly associated with violation of reduction of compound lotteries (Halevy, 2007; Chew et al., 2017; Abdellaoui et al., 2015; Gillen et al., 2019; Dean and Ortoleva, 2019). As shown by Seo (2009), the relaxation of the reversal of order assumption is required to provide an axiomatic foundation to the smooth model that accommodates the violation of reduction of compound lotteries (see also Saito 2015; Ke and Zhang 2020). We, therefore, believe that relaxing the reversal of order axiom is a reasonable theoretical approach. It follows that the elicitation of ambiguity preferences using the RIS is incentive-compatible if the latter *precedes* the ambiguity studied in the experiment. Note that we assume the same level of sophistication as we have assumed so far (i.e., the subject is able to see the whole experiment as a single decision problem). The experimenter credibly informs him that the RIS is performed before the resolution of uncertainty, and his perception of the ordering of the various stages changes accordingly. This order matters since his preferences do not satisfy reversal of order, and therefore incentive compatibility is maintained. In the terminology introduced by Azrieli et al. (2018): once reduction is relaxed, state monotonicity is equivalent to incentive compatibility of RIS if the ordering of randomizations is "correct."

Our theoretical results lead to a very practical recommendation for experiments using RIS in ambiguous environments: the experimenter can make this order (risk

[3]This is very different from the existing critiques of the RIS, which rely on non-expected utility under risk (Holt, 1986; Karni and Safra, 1987b; Segal, 1988; Freeman et al., 2019), since our result is obtained even if the subject satisfies expected utility under risk. In that literature, violation of the mixture independence axiom results in nonseparability of preferences that invalidates the incentive compatibility of the RIS in general. Following the tradition in models of choice under uncertainty, we will maintain the assumption of expected utility under risk and concentrate on the additional complications introduced by violations of Savage's (1954) Sure Thing Principle.

before uncertainty) more salient by explicitly performing the RIS randomization before choices are made and the uncertainty is resolved. In this case, the experimental design is internally consistent with the model under investigation.

2. Description of typical experiments under ambiguity

Let $S = \{s_1, \cdots, s_n\}$ be a finite set *of states of nature. Events* are subsets of S and are typically denoted by E. *Acts* map states into $[0, 1]$, the *outcome set*, with generic elements denoted by f, g. A *bet on* E is the binary act 1_E0, paying 1 if event E obtains and 0 otherwise. *A constant act*, which assigns the same outcome k to all states of the world, is denoted by its unique outcome.

The first type of experiments we study are those eliciting certainty equivalents. The objective of the experimenter ("she") is to find the sure amount of money that a subject ("he") values as much as an act f. If only one act is studied, the experimenter can use a Becker-DeGroot-Marschak (1964, henceforth, BDM) procedure: she asks the subject to report a number k, then she randomly draws a number $0 \leq \ell \leq 1$, and the subject receives ℓ if $\ell > k$ and f otherwise. Alternatively, she can use a series of choices (e.g., a choice list) and RIS: the subject is asked to choose between a sure amount ℓ and f for many possible realizations of ℓ, and one of the choices is randomly selected for payment.[4] If the subject's preferences satisfy the expected utility assumptions, both procedures (BDM or sequence of choices with RIS) are theoretically equivalent here[5] and would not pose any difficulty if the experimenter considers only one act. Of course, as shown by Holt (1986) and Karni and Safra (1987b), these procedures would be problematic should we not assume expected utility under risk. Here, we focus on a different challenge caused by the fact that the experimenter uses several acts in the experiment, which creates hedging opportunities for the subject. As explained in the Introduction, experiments studying ambiguity attitudes have to elicit preferences for bets on more than a single event, and therefore deal with more than a single act. This implies that the experimenter must randomly select which act will be considered for payment (first randomization), and implement a second randomization for this act (the BDM procedure or the randomization over the choices the subject made for this act).

We now turn to formally describe such an incentive scheme. Let $\mathfrak{f} = (f_1, ..., f_m)$ be a list of acts, $\tilde{i}$ be a random distribution governing the randomization of which act will determine the subject's payment with realization i and support $\{1, ..., m\}$.

[4]The highest ℓ for which the act is chosen and the lowest ℓ chosen over the act give an interval for the k elicited in the BDM. See Bardsley et al. (2009) pp.266-267 and 271 for variants of the RIS and the BDM.

[5]We assume both procedures are understood by the subject, while empirically the BDM is probably more demanding.

Definition 1. Let $\mathfrak{k} = (k_1, \cdots, k_m) \in [0,1]^m$ be a vector of reports and ℓ be a random number following distribution $\tilde{\ell}$ with support $[0,1]$.[6]

$$I(\mathfrak{f}, \mathfrak{k}, i, \ell) = \begin{cases} f_i & \text{if } \ell \leq k_i, \\ \ell & \text{otherwise.} \end{cases}$$

A *Certainty Equivalent Random Incentive Scheme (CERIS)* is defined by $I\left(\mathfrak{f}, \mathfrak{k}, \tilde{i}, \tilde{\ell}\right)$.

A second type of experiment involves probability equivalents. Let $\Delta([0,1])$ be the *set of simple lotteries* over $[0,1]$, and 1_p0 be a *binary lottery*, which pays 1 with probability p and 0 otherwise. If the experimenter elicits the binary lottery 1_p0 that the subject finds as good as an act f, we call p the *probability equivalent* of f. If f is a bet on E, we can think of p as the subjective belief of E. This procedure can be used to detect departures from additivity of belief and study ambiguity attitudes. Probability equivalents p can be elicited with a BDM procedure (as suggested by Grether, 1981; Holt, 2007; Karni, 2009) or with a series of choices and RIS. As discussed above, difficulties arise when considering experiments eliciting probability equivalents for more than a single act, which necessitates determining which act will be considered for payment. We now define such an incentive scheme for probability equivalents:

Definition 2. Let $\mathfrak{p} = (p_1, \cdots, p_m) \in [0,1]^m$ be a vector of reports and $q \in [0,1]$ be a random number following distribution $\tilde{q}$ with support $[0,1]$

$$J(\mathfrak{f}, \mathfrak{p}, i, q) = \begin{cases} f_i & \text{if } q \leq p_i, \\ 1_q0 & \text{otherwise.} \end{cases}$$

A *Probability Equivalent Random Incentive Scheme (PERIS)* is defined by $J\left(\mathfrak{f}, \mathfrak{p}, \tilde{i}, \tilde{\ell}\right)$.

An *incentive scheme* (CERIS or PERIS) designed by the experimenter is characterized by a vector of acts and a description of the random processes $\tilde{i}$, and $\tilde{\ell}$ or $\tilde{q}$ underlying i, and ℓ or q. For simplicity, we will assume that these random variables follow uniform distributions (discrete or continuous) over their respective (full) support. With everything clearly defined, we may suppress all arguments and use only I and J to denote the incentive scheme.

We assume that the incentive scheme is fully described to the subject, who is asked to report $\mathfrak{k}$ (for CERIS) or $\mathfrak{p}$ (for PERIS). We call the incentive scheme given the subject's report *an experiment* (when all uncertainties have not been resolved). Suppose that the subject's preferences over experiments are represented by the utility function $v(\cdot)$. The subject chooses a report that maximizes the utility from the experiments given by v. The subject employs the same utility function to evaluate

[6]Implementations of the BDM mostly employ discrete distributions (for instance, ℓ can only take values expressed in cents). For mathematical simplicity, in what follows, we assume a continuous uniform distribution of ℓ.

acts and experiments.[7] In what follows, v will always be expressed in terms of a utility index u over outcomes (normalized by $u(0) = 0$ and $u(1) = 1$).

For example, suppose that v is subjective expected utility[8] (Savage, 1954). There exists an additive belief μ over S such that the subject evaluates each act by the expected utility of the lottery that μ induces through the act. Imagine PERIS where $f = (1_E 0)$.[9] The subject evaluates the act f by $v(1_E 0) = \mu(E)$, i.e., it is worth him as much as $1_{\mu(E)}0$, and therefore his probability equivalent should be $p = \mu(E)$. The subject's evaluation of the experiment is more subtle: for each report p he receives the bet $1_E 0$ with probability p, and otherwise a lottery $1_q 0$, where q is in $[p, 1]$. The utility of the experiment is $v(J((1_E 0), (p), (1, \tilde{q}))) = p\mu(E) + \int_p^1 q dq$. Therefore, according to the first-order condition, the subject's optimal strategy is to report $p^* = \mu(E)$. Under the assumption that the subject's preferences are represented by subjective expected utility, the incentive scheme does not distort his report: he reports his valuation of the act if he optimally responds to the experimental environment. Such an incentive scheme will be referred to as incentive compatible.

We now define the optimal report strategy of a subject in an incentive scheme and then use it to define incentive compatibility for any evaluation function v.

Definition 3. A subject's *optimal report strategy* is a vector of reports $\mathfrak{k}^* = (k_1^*, ..., k_m^*)$ or $\mathfrak{p}^* = (p_1^*, ..., p_m^*)$ that maximizes his evaluation of the experiment induced by his report through the incentive scheme:

$$\mathfrak{k}^* \in argmax_{\mathfrak{k}} v\left(I\left((f_1, \cdots, f_m), (k_1, \cdots, k_m), \left(\tilde{i}, \tilde{\ell}\right)\right)\right)$$

$$\mathfrak{p}^* \in argmax_{\mathfrak{p}} v\left(J\left((f_1, \cdots, f_m), (p_1, \cdots, p_m), \left(\tilde{i}, \tilde{q}\right)\right)\right)$$

We now define incentive compatibility:

Definition 4. CERIS is *incentive compatible* (IC) with respect to v if $v(k_i^*) = v(f_i)$ for all k_i^*. Similarly, PERIS is IC with respect to v if $v\left(1_{p_i^*}0\right) = v(f_i)$ for all p_i^*.

[7]The domain of v is an extension of the domain used by Seo (2009). A typical element of it has the first two stages of events with given probabilities (risk), followed by one stage of events without given probabilities (ambiguity), and then two stages of risk (Seo had only one stage of risk before and after the ambiguity stage). In the domain of v, acts can be modeled as an element with all risk stages degenerate. In section 3 (4), we will consider only elements whose first (last) two stages are degenerate.

[8]For events with objectively given probabilities, their subjective probabilities are equated to the objectively given ones.

[9]The subject knows E is out of the experimenter's control (e.g. E is related to the weather) and since his evaluation function is subjective expected utility, he is not ambiguity averse. Hence, the explanation, which required to use more than a single act, does not apply here.

3. Random Incentive after Uncertainty Resolves

We now show that if the randomization used in the incentive scheme occurs after uncertainty is resolved, CERIS and PERIS may not be incentive compatible. We consider a subject whose preferences are represented by maxmin expected utility (MEU, Gilboa and Schmeidler, 1989), and a simple experiment whose goal is to evaluate the subject's behavior under ambiguity. We prove that the subject's optimal response under CERIS and PERIS is to act as if he had a unique probability measure over the state space and not a set of priors as the model assumes. As a consequence, the experimenter would underestimate the extent of deviation from ambiguity neutrality in her experiment.

Consider an incentive scheme based on two acts $(1_{E_1}0, 1_{E_2}0)$, such that the events E_1 and E_2 partition the state space. For example, the events may correspond to drawing a red/black ball in the Ellsberg ambiguous urn. Assume that the subject has MEU preferences, and evaluates such acts by the lowest expected utility it might generate over his set of priors C, i.e. $v\left(1_{E_i}0\right) = \min_{\mu\in C}\left[\mu(E_i)u(1) + \left(1-\mu(E_i)\right)u(0)\right]$. Let $[a,b]$ denote all the values $\mu(E_1)$ can take in C, we obtain $v\left(1_{E_1}0\right) = a$ and $v\left(1_{E_2}0\right) = 1-b$. The experimenter would like to observe these values to obtain the boundaries of the subject's set of priors. By definition of IC, CERIS is IC only if k_i^* satisfies $v(k_i^*) = v(1_{E_i}0)$ for $i = 1,2$, which implies $k_1^* = u^{-1}(a)$ and $k_2^* = u^{-1}(1-b)$. Similarly, noting that $v(1_{p_i}0) = p_i$,[10] PERIS is IC only if $p_1^* = a$ and $p_2^* = 1-b$.

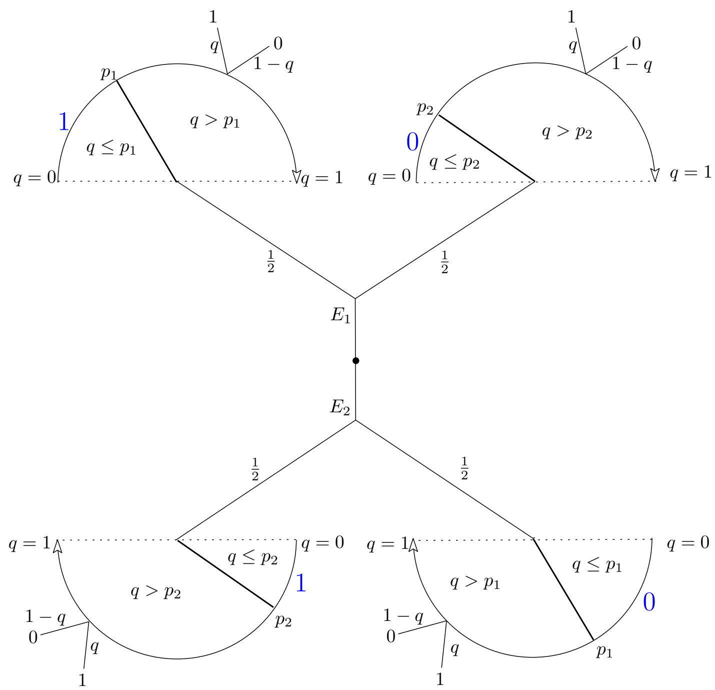
Consider PERIS $J\left(\left(1_{E_1}0, 1_{E_2}0\right), (p_1,p_2), \left(\tilde{i},\tilde{q}\right)\right)$. If realization of the events is perceived as occurring before that of the random processes used in the incentive scheme, the subject might perceive the experiment as an *Anscombe-Aumann act*, i.e. a mapping h from S to $\Delta([0,1])$. For such act h, MEU is defined as $v(h) = \min_{\mu\in C}\sum_S\left[\mu(s)Eu\left(h\left(s\right)\right)\right]$, where C is the subject's (convex) set of priors and Eu is the expected utility functional.

The subject's perception of the experiment induced by his report through the incentive schemes is depicted in Figure 3.1a.

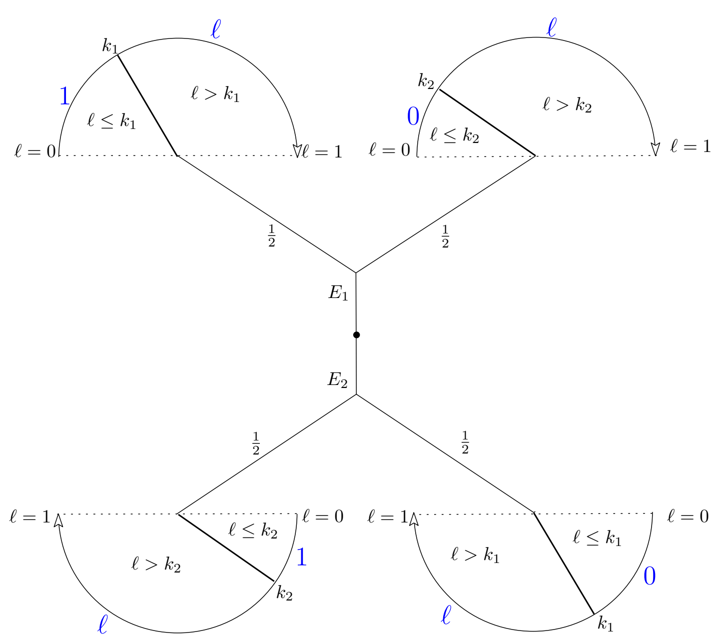
He understands that if E_1 happens, he has a 50% probability to be paid based on the report p_1 for $1_{E_1}0$, where he gets 1 with probability p_1,[11] and otherwise faces a lottery 1_q0 such that $q\in[p_1,1]$; he also has a 50% probability of being paid based on the report p_2 for $1_{E_2}0$ and would receive 0 with probability p_2 and otherwise a lottery 1_q0 whose winning probability is in $[p_2,1]$. Following the same reasoning for the case when E_1 does not happen, his utility of the experiment induced by his report (p_1,p_2)through the PERIS is:

[10] Formally, define an event with an objective probability of p_i and require that every prior in C assigns to this event a probability of p_i.

[11] If $q < p_1$ (hence with probability p_1 knowing that $\tilde{q}$ is uniform over $[0,1]$), he gets 1_E0. Since E occurred, he gets 1.



(A) PERIS



(B) CERIS

FIGURE 3.1. Experiments with RIS after resolution of uncertainty

$$\begin{aligned}&v\left(J\left(\left(1_{E_1}0,1_{E_2}0\right),\left(p_1,p_2\right),\left(\tilde{i},\tilde{q}\right)\right)\right)\\&=\min_{\mu\in C}\left[\tfrac{1}{2}\left[\left(\mu(E_1)\times p_1+(1-\mu(E_1))\times p_2\right)\right]+\tfrac{1}{2}\left(\int_{p_1}^{1}qdq+\int_{p_2}^{1}qdq\right)\right]\end{aligned} \tag{3.1}$$

A similar analysis can be applied to a CERIS experiment, described in Figure 3.1b. The experiment induced by his report of (k_1,k_2) through CERIS is $I\left(\left(1_{E_1}0,1_{E_2}0\right),\left(k_1,k_2\right),\left(\tilde{i},\tilde{\ell}\right)\right)$. Hence, he understands that if E_1 obtains, he has a 50% probability to be paid based on the report k_1 he made for $1_{E_1}0$. If so, he gets 1 with probability k_1,[12] and $\ell\in[k_1,1]$ otherwise; he also has a 50% probability to be paid based on the report k_2 he made for $1_{E_2}0$, and would receive 0 with probability k_2; otherwise, $\ell\in[k_2,1]$. Following the same reasoning for the case when E_1 does not happen, he obtains the evaluation for the experiment :

$$\begin{aligned}&v\left(I\left(\left(1_{E_1}0,1_{E_2}0\right),\left(k_1,k_2\right),\left(\tilde{i},\tilde{\ell}\right)\right)\right)\\&=\min_{\mu\in C}\left[\tfrac{1}{2}\left[\left(\mu(E_1)\times k_1+(1-\mu(E_1))\times k_2\right)\right]+\tfrac{1}{2}\left(\int_{k_1}^{1}u(\ell)d\ell+\int_{k_2}^{1}u(\ell)d\ell\right)\right]\end{aligned} \tag{3.2}$$

As a maxmin EU decision maker, the subject chooses p_1^* and p_2^* (k_1^* and k_2^*) so as to maximize Eq. 3.1 (Eq. 3.2). The following proposition establishes that PERIS and CERIS are not IC under the assumptions we have made in this section.

Proposition 5. *Consider an incentive scheme with the acts $f=(1_{E_1}0,1_{E_2}0)$ defined over complementary events E_1 and E_2, and assume maxmin expected utility with a set of priors such that $\mu(E_1)\in[a,b]$. With CERIS, the subject reports*

$$(k_1^*,k_2^*)=\begin{cases}\left(u^{-1}(b),u^{-1}(1-b)\right) & a<b<0.5\\ \left(u^{-1}(0.5),u^{-1}(0.5)\right) & a\le 0.5\le b\\ \left(u^{-1}(a),u^{-1}(1-a)\right) & 0.5<a<b\end{cases}$$

With PERIS, the subject reports

$$(p_1^*,p_2^*)=\begin{cases}(b,1-b) & a<b<0.5\\ (0.5,0.5) & a\le 0.5\le b\\ (a,1-a) & 0.5<a<b\end{cases}$$

Hence, CERIS and PERIS are not IC.

Proof. See Appendix A. □

The Appendix considers all possible cases in detail. Here, we provide the intuition for the simplest case with PERIS. Consider a MEU subject whose set of priors $[a,b]$ for event E_1 contains $\frac{1}{2}$, and $b>a$ as the decision maker thinks other probabilities are also

[12]If $\ell<k_1$ (hence with probability k_1 knowing that $\tilde{\ell}$ is uniform over $[0,1]$), he gets $1_{E_1}0$ and since E_1 occurred, he gets 1.

possible. However, by reporting $p_1 = p_2$, Eq. 3.1 becomes $\left[\frac{1}{2}p_1 + \int_{p_1}^{1} q dq\right]$, which does not depend on μ . Hence, the Anscombe-Aumann act generated by the experiment as a whole is no longer ambiguous if the decision maker reports $p_1 = p_2$. Maximizing $\frac{1}{2}p_1 + \int_{p_1}^{1} q dq = \frac{1}{2}(p_1 + 1 - p_1^2)$ results in the optimal strategy $p_1^* = p_2^* = \frac{1}{2}$, which differs from the valuations $v(1_{E_1}0) = a$ and $v(1_{E_2}0) = 1 - b$ that the experimenter should have observed had the experiment been IC.

When $[a, b]$ does not include $\frac{1}{2}$, we show in the Appendix that no matter which prior the decision-maker relies on, the optimal strategy always imposes $p_1^* = 1 - p_2^*$. This arises from both acts being evaluated simultaneously, i.e., with the same prior. Once a particular prior is fixed (by the minimization operator), the best strategy for the whole experiment consists of reporting it for both acts. Hence, in these cases again, the experimenter observes one prior only and wrongfully concludes that the decision-maker is ambiguity neutral. The arguments for CERIS are very similar. The only change is the additional impact of utility.

To sum up, Proposition 5 shows that neither PERIS nor CERIS is IC when uncertainty is resolved before the implementation of the random incentive. The results for CERIS and PERIS highlight that the experimenter will not be able to observe the set of priors $[a, b]$. She will conclude that the subject has a degenerate set of priors ($\{b\}$,$\left\{\frac{1}{2}\right\}$, or $\{a\}$), compatible with subjective expected utility or probabilistic sophistication, i.e., ambiguity neutrality. As a consequence, using CERIS or PERIS when the resolution of uncertainty precedes the random incentives lead to underestimating ambiguity aversion if the subject integrates the whole experiment into a single decision problem.

Proposition 5 considers one model only. Such a counterexample is enough to prove that PERIS and CERIS with RIS after uncertainty resolution is generally not IC for any class of preferences that includes MEU preferences, such as uncertainty averse preferences of Cerreia-Vioglio et al. (2011), the most general class of ambiguity averse preferences in the literature or α-MEU, which allows for ambiguity-seeking preferences.

Our result demonstrates how hedging affects the measurements of certainty equivalents and probability equivalents. It is not surprising in the sense that it could be predicted by the results of Bade (2015). It does highlight, though, that a MEU decision-maker, even if he mimics the optimal strategy of an ambiguity neutral agent, may still bear some ambiguity (if $\frac{1}{2}$ is not in the set of priors). The decision-maker faces a tradeoff between hedging and taking advantage that one event is more likely than the other *for all priors.*

In what follows, we relax the reversal of the order axiom and show that the elicitation of ambiguity preferences using the RIS is incentive-compatible if the latter precedes the ambiguity studied in the experiment. Note that we assume the same level of sophistication as we have assumed so far (i.e., that the subject is able to see the whole experiment as one decision). The experimenter will credibly inform him that the RIS is performed before the resolution of uncertainty and his perception of the ordering of the various stages changes accordingly.

4. Random Incentive Before Uncertainty Resolves

In this section, we show that if the random incentive system is employed before the resolution of uncertainty, and the subject evaluates the experiment in this order, CERIS and PERIS *are* IC if we assume expected utility under risk. We further study the robustness of this result by relaxing the expected utility assumption. All results below are derived for an arbitrary ambiguity model expressed in u terms, with u being the utility index that the subject uses under risk. In other words, v assigns utility $u(k)$ to outcome k. Virtually all ambiguity models satisfy this property. See for instance the general families of models proposed by Ghirardato and Marinacci (2001), Cerreia-Vioglio et al. (2011), and Grant and Polak (2013).[13]

Consider an experiment on $\mathfrak{f} = (f_1, \cdots, f_m)$ with CERIS and random incentives preceding subjective uncertainty. The experiment can be represented as in Figure 4.1b (for $m = 2$). With probability $\frac{1}{m}$, the subject's payoff depends on his report of k_i (and not on any $k_{j\neq i}$). In this case, he receives f_i (that he values at $v(f_i)$) with probability k_i, and $\ell \in [k_i, 1]$ otherwise. Figure 4.1a illustrates PERIS when the incentive system precedes subjective uncertainty (for $m = 2$). With probability $\frac{1}{m}$, the subject's payoff depends on his report of p_i (and not on any $p_{j\neq i}$). In this case, he receives f_i (that he values at $v(f_i)$) with probability p_i, and the lottery $(1_q 0)$ where $q \in [p_i, 1]$ otherwise.

4.1. Expected utility under risk.

Assuming expected utility,[14] the subject's optimal report of $\mathfrak{k}^*$ in CERIS maximizes

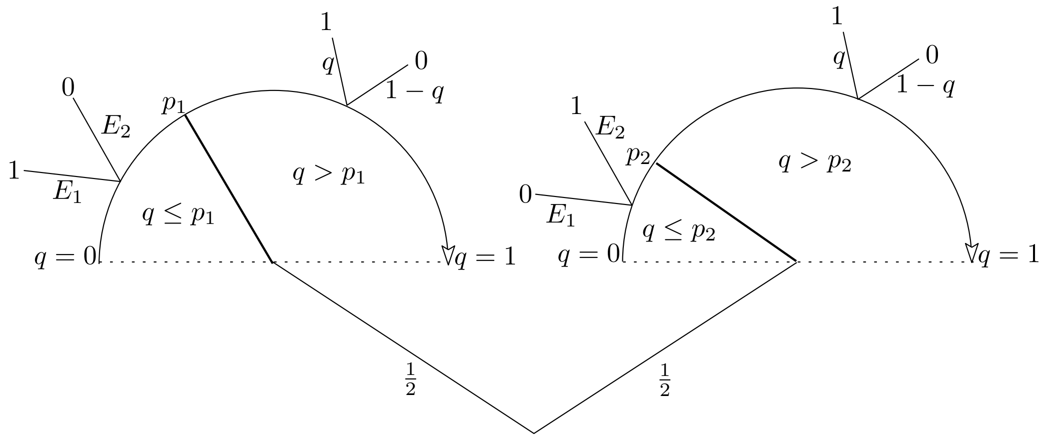
$$\sum_{i=1}^{m} \left(\frac{k_i}{m} v(f_i) + \frac{1}{m} \int_{k_i}^{1} u(\ell) d\ell \right). \tag{4.1}$$

We show that for all i reporting k_i^* such that $u(k_i^*) = v(f_i)$ is the subject's optimal strategy. First, reporting k_i such that $u(k_i) > v(f_i)$ is a dominated strategy since the experiment induced by reporting $k_i > u^{-1}(v(f_i))$ through CERIS can be obtained from that induced by reporting $k_i = u^{-1}(v(f_i))$ by transferring positive probability mass from higher utility values to $v(f_i)$. Conversely, reporting k_i such that $u(k_i) < v(f_i)$ would also be suboptimal, as the subject would receive outcomes with utility less than $v(f_i)$ instead of f_i. Therefore CERIS is IC, under the assumption of expected utility under risk.

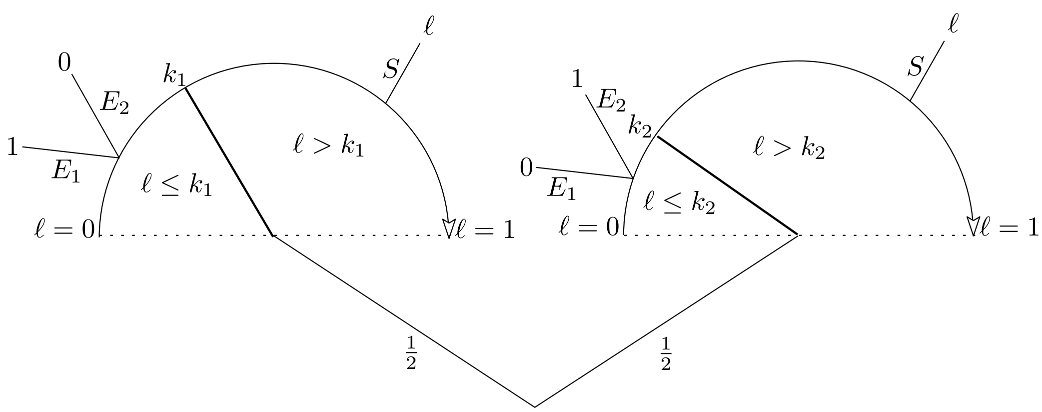
Similarly, when determining optimal reports $\mathfrak{p}^*$ for PERIS, assuming expected utility under risk, the subject maximizes

[13] We do not consider models with more than one utility index, such as Cerreia-Vioglio et al. (2015).

[14] We assume here the canonical expected utility model, satisfying reduction of compound lotteries. A generalization of expected utility with no reduction of compound lotteries was first introduced by Kreps and Porteus (1978).



(A) PERIS



(B) CERIS

FIGURE 4.1. Experiments with RIS before uncertainty ($m = 2$)

$$\sum_{i=1}^{m}\left(\frac{p_i}{m}v\left(f_i\right)+\frac{1}{m}\int_{p_i}^{1} qdq\right). \tag{4.2}$$

by reporting $p_i^* = v\left(f_i\right)$. The proof follows identical dominance argument as used above for CERIS.

Proposition 6. *Assume expected utility under risk with utility index u and an ambiguity model expressed in u terms. If the random incentive system (RIS) precedes the resolution of uncertainty, then CERIS and PERIS are incentive compatible (IC).*

4.2. **Nonexpected utility under risk with compound independence.** So far, we allowed for deviations from (subjective) expected utility for uncertain acts, but not for risky lotteries. We now relax the assumption of expected utility for risk, as nonexpected utility models have been shown to pose difficulties with random incentives. We consider all non-expected utility models for risk with utility index u that satisfies weak-ordering and stochastic dominance. If an incentive scheme is incentive compatible, the subject should report, under the assumption of weak-ordering and stochastic dominance: $k_i^* = u^{-1}(v(f_i))$ in CERIS, and p_i^* such that $v\left(1_{p_i^*}0\right) = v(f_i)$ in PERIS for all $i \in \{1, \cdots, m\}$.

Consider CERIS on $f = (f_1, \cdots, f_m)$. Figure 4.1b demonstrates that a report of k induces a three-stage experiment through CERIS. The first stage corresponds to the resolution of $\tilde{i}$, the random selection of the report k_i on which the subject's payoff depends. The second stage corresponds to the resolution of $\tilde{\ell}$, and the third stage is either the act f_i or a sure amount ℓ. Since expected utility is linear in probabilities, reducing the first two stages into a single stage does not impact the valuation of the experiment. However, when probabilities are weighted, this does not hold anymore. Therefore, the optimal report depends on the subject's evaluation of compound objective lotteries.

In this subsection, we consider a subject who does not reduce multi-stage lotteries but whose behavior satisfies the compound independence axiom (recursivity) instead (Segal, 1990).

Let $(X_1, r_1; ...; X_n, r_n)$ be a (possibly multi-stage) lottery yielding X_i with probability r_i, where X_i may be a (possibly multi-stage) lottery or an act. The experiment I induced by a report of $\mathfrak{k}$ can be written as $I = \left(B_1, \frac{1}{m}; \cdots; B_m, \frac{1}{m}\right)$ where B_i denotes the i^{th} second-stage branch, implied by i being drawn in the first stage and in which the subject's payoff depends on the report k_i. *Compound independence (recursivity)* implies that the evaluation function satisfies:

$$v(I) = v(u^{-1}(v(B_1)), \frac{1}{m}; \cdots; u^{-1}(v(B_m)), \frac{1}{m})$$

That is, the subject substitutes all second-stage branches by their certainty equivalents and then evaluate the implied one-stage lottery according to his risk preferences. Compound independence is, at the level of the evaluation function, what Azrieli et al.'s (2018) statewise monotonicity is for preferences.

Since there is no interaction between different branches, maximizing each $v(B_i)$ is sufficient to maximize $v(I)$. With the subject's risk preference satisfying stochastic dominance, following identical reasoning to the one employed for EU, the optimal report to maximize $v(B_i)$ is: $k_i^* = u^{-1}(v(f_i))$. Indeed, the experiment induced by reporting $k_i > u^{-1}(v(f_i))$ (or $k_i < u^{-1}(v(f_i))$) transfers positive probability masses from higher utility values (or $v(f_i)$) to $v(f_i)$ (or lower utility values).

In the case of PERIS, the experiment induced by a report of $\mathfrak{p}$ can be written as $J = \left(B'_1, \frac{1}{m}; \cdots; B'_m, \frac{1}{m}\right)$ where B'_i denotes the i^{th} second-stage branch, implied by i being drawn in the first stage and in which the subject's payoff depends on the report p_i. Assuming weak ordering, and stochastic dominance, compound independence also implies that maximizing each valuation $v(B'_i)$ is sufficient to maximize $v(J)$. It follows that the optimal report is p_i^* such that $v\left(1_{p_i^*}0\right) = v(f_i)$. By stochastic dominance, $v(1_{p_i}0) > v(1_{p_{i^*}}0)$ implies $p_i > p_i^*$. Hence, overreporting p_i ($p_i > p_i^*$) transfers positive probability masses from lotteries with higher utility values (e.g. $v(1_{p_i}0)$) to $v(f_i)$. With similar arguments, underreporting $p_i < p_i^*$ is also suboptimal.

Proposition 7. *Assume weak ordering, stochastic dominance, and compound independence under risk with utility index u and an ambiguity model expressed in u terms. If the random incentive system precedes the resolution of uncertainty, then CERIS and PERIS are IC.*

Our result is in line with the result of Azrieli et al. (2018), showing that RIS is IC if statewise monotonicity holds. The next section will show that statewise monotonicity is unnecessary and that CERIS is still IC even if subjects reduce compound lotteries.

4.3. **Nonexpected utility under risk with reduction of compound lotteries.** In this subsection, we no longer assume that the subject's behavior satisfies compound independence. Instead, we analyze the situation in which reduction of compound lotteries (ROCL) applies. Under this assumption, CERIS remains IC. Reducing the first and second stages simply means multiplying each second stage probability (or density) by $\frac{1}{m}$. Over-reporting k_i still implies a transfer of probability mass from outcomes with higher utility values to $v(f_i)$, and underreporting k_i implies a transfer of probability mass from $v(f_i)$ to outcomes with lower utility values.

However, in Appendix B we provide a counterexample that demonstrates that PERIS may not be incentive compatible when ROCL is assumed in conjunction with RDU (which satisfies weak-ordering and stochastic dominance). This result originates in the subject possibly being paid a lottery 1_q0. This lottery is compounded with the RIS stages. Therefore, under RDU and ROCL, the RIS stages influence the utility derived from the lottery and affect the subject's optimal report. By contrast, the utility of the outcome ℓ is independent of the RIS stages in CERIS, which retains incentive compatibility.

Proposition 8. *Assume weak ordering, stochastic dominance, and ROCL under risk with utility index u and an ambiguity model expressed in u terms. If the random incentive system precedes the resolution of uncertainty, then CERIS is IC, but PERIS may not be.*

For CERIS and PERIS to be incentive-compatible, the order of uncertainty resolution and compound independence are the essential conditions to be satisfied. They allow subjects to carry out backward induction. For CERIS, ROCL happens to induce the same decision situation as compound independence, whereas for PERIS, ROCL implies violation of compound independence, leading to incentive incompatibility in the same way as highlighted by Karni and Safra (1987a) for experiments under risk.

5. Empirical evidence from the literature

We reviewed the implementation of RIS in the experimental literature to analyze whether it influences how much ambiguity aversion was found. Taking our results at face value, we expect ambiguity aversion to be stronger when RIS is implemented before the resolution of uncertainty and even more so before the subjects complete the tasks. We would also expect more ambiguity aversion with CERIS than with PERIS. These predictions, however, rely on two strong assumptions: (i) enough (if not all) subjects should perceive the whole experiment as a single decision problem, and (ii) other methodological differences should remain minor. These two assumptions are far from being warranted, but we focused on studies based on Ellsberg's tasks to keep the results as comparable as possible.

We included papers reviewed by Trautmann and van de Kuilen (2015) and added papers that were published after 2015. We excluded papers that did not use PERIS or CERIS. We also excluded studies that were hypothetical or which were not incentive compatible even under expected utility. Table 1 presents a summary of the remaining papers. As Trautmann and van de Kuilen (2015) do in their Table 3.4, we report the average ambiguity premium of each study, defined as either the "the difference between the valuation of the risky act and valuation of the ambiguous act, divided by the expected value of the risky act" for CERIS or "the difference between the ambiguity-neutral matching probability and the actual matching probability for the ambiguous urn, divided by the ambiguity-neutral matching probability" for PERIS.

The rightmost column describes the order of implementation between the task (the decision phase), CERIS or PERIS, and the resolution of uncertainty. In all these studies, ambiguity was resolved last. This shows the relevance of our Section 4, studying incentive compatibility when the RIS is implemented before the uncertainty. Theoretical arguments made by Oechssler and Roomets (2014) and Bade (2015) assumed uncertainty to be resolved first. We did find two studies for which uncertainty was resolved first. Baillon and Bleichrodt (2015) and Baillon et al. (2018), not listed in Table 1 because they did not use an Ellsberg task but used the variation of a stock index during the experiment as the source of ambiguity. In the way these studies were conducted, the uncertainty had already been resolved when the RIS was implemented. These papers found some evidence for ambiguity seeking, but it could be due to their specific source of uncertainty. By contrast, a few recent papers implemented the RIS even before the decision task, making it salient that uncertainty was resolved last. Epstein and Halevy (2019) found ambiguity aversion with CERIS first and no evidence for hedging. Baillon and Placido (2019) and Li et al. (2020) also found some

TABLE 1. Literature Review Table

Study	Country	Task	Prize	N	VM*	IM**	Ambiguity Premium (in %)	RIS implementation***
Abdellaoui et al. (2011)	France	2-color	€25 ($34)	66	CE	CL	2.5	Task-CERIS-Ambiguity
Abdellaoui et al. (2015)	France	2-color (2 balls)	€50 ($67)	94	CE	CL	5.8	Task-CERIS-Ambiguity
		2-color (12 balls)	€50 ($67)				17.7	
Akay et al. (2012)	Ethiopia	2-color	ETN20 ($.5)	93	CE	CL	12.9	Task-CERIS-Ambiguity
Baillon and Placido (2019)	Netherlands	3-color	€30 ($39)	78	PE	CL	4.5	PERIS-Task-Ambiguity
Borghans et al. (2009)	Netherlands	2-color	€2 ($3)	347	WTA	BDM	13.2	Task-CERIS-Ambiguity
Cettolin and Riedl (2010)	Netherlands	2-color	€15 ($20)	55	PE	CL	10.0	Task-PERIS-Ambiguity
Chew et al. (2017)	Singapore	2-color	S$40 ($30)	56	CE	CL	22.9	Task-CERIS-Ambiguity
Cohen et al. (1987)	France	2-color	FF1,000 ($150)	134	CE	CL	23.4	Task-CERIS-Ambiguity
Cubitt et al. (2018)	Netherlands	2-color	€16 ($21)	88	CE	CL	0.6	Task-CERIS-Ambiguity
Eisenberger and Weber (1995)	Germany	2-color	DM10 ($7)	54	WTP	BDM	18.8	Task-CERIS-Ambiguity
					WTA	BDM	9	
Fairley and Sanfey (2017)	Netherlands	2-color	€5 ($6)	172	CE	CL	6.9	Task-CERIS-Ambiguity
Fox and Tversky (1995)	USA	2-color	$20	52	CE	CL	12.4	Task-CERIS-Ambiguity
		3-color	$50	53	WTA	BDM	10.3	Task-CERIS-Ambiguity
Füllbrunn et al. (2014)	Germany	2-color	€15 ($19.5) or €6.2 ($8.5)	20	WTP	BDM	7.2	Task-CERIS-Ambiguity
	Netherlands	2-color	€15 ($19.5) or €6.2 ($8.5)	12	WTP	BDM	4.8	
Epstein and Halevy (2019)	Canada	2-color	$25	74	CE	CL	12.8	CERIS-Task-Ambiguity
Keck et al. (2014)	USA	2-color	$20	90	CE	CL	17.5	Task-CERIS-Ambiguity
König-Kersting and Trautmann (2016)	Germany	2-color	€10 ($12)	194	PE	C	12	Task-PERIS-Ambiguity
		10-color			PE	C	-12	
Li et al. (2020)	Netherlands	3-color	€15 ($20)	80	PE	C	7	PERIS-Task-Ambiguity
Maffioletti and Santoni (2005)	Italy	2-color	ITL100,000 ($51)	25	WTA	BDM	24.2	Task-RIS-BDM-Ambiguity
Ross et al. (2012)	Laos	2-color	LAK20,000 ($2.5)	66	PE	CL	1.8	Task-PERIS-Ambiguity
Sutter et al. (2013)	Austria	2-color	€10 ($13)	487	CE	CL	15.3	Task-CERIS-Ambiguity
Trautmann et al. (2011)	Netherlands	2-color	€50 ($67)	74	WTP	BDM	40.7	Task-CERIS-Ambiguity
		2-color	€50 ($67)	79	CE	CL	10.9	Task-CERIS-Ambiguity
Qiu and Weitzel (2011)	Netherlands	2-color	€10 ($13)	208	WTP	BDM	21.7	Task-CERIS-Ambiguity

[*] *VM means valuation methods, CE stands for certainty equivalent, WTA (WTP) stands for willingness to accept (to pay), PE stands for probability equivalent (also called matching probability).

[**] **VM means incentive methods, CL stands for the choice list, C stands for separate choices, and BDM stands for the Becker-deGroot-Marschak procedure.

[***] *** The RIS implementation was inferred from the methods section of the corresponding papers.

degree of ambiguity aversion with PERIS coming before the decision tasks. Both papers' average ambiguity premiums are slightly lower than that of Epstein and Halevy (2019), possibly due to the use of PERIS instead of CERIS.

All studies reported in Table 1 found evidence of ambiguity aversion. There are at least two possible explanations for this. A first interpretation is that these empirical results are consistent with our theoretical results that RIS before ambiguity is incentive compatible (if subjects do not reduce compound lotteries). Alternatively, and maybe even more likely, subjects might not have perceived the whole experiment as one decision, unlike in our model, but simply made each choice in isolation. This would be in line with the literature on narrow bracketing (Tversky and Kahneman, 1981; Kahneman and Lovallo, 1993; Barberis et al., 2006; Rabin and Weizsäcker, 2009). Note that for subjects exhibiting narrow bracketing, any RIS is IC, since subjects do not perceive the interdependence between their various choices.

We discuss other studies that did not use PERIS or CERIS but other types of tasks with RIS. Several found little evidence for ambiguity aversion. Charness et al. (2013) used a task varying the degree of ambiguity, where the RIS preceded the resolution of uncertainty. A large majority of their subjects were classified as ambiguity neutral and less than 10% as ambiguity averse. Binmore et al. (2012) and Voorhoeve et al. (2016), with a variation of PERIS in a three-color Ellsberg experiment, found little evidence of ambiguity aversion as well. Ahn et al. (2014) conducted a portfolio-choice experiment with numerous budget sets and argued that it was difficult for subjects to figure out how to hedge. About 60% of their participant were ambiguity neutral, but they found evidence of ambiguity aversion among the others. Stahl (2014) varied outcomes assigned to ambiguous events and observed a majority of ambiguity neutral subjects.

By contrast, two papers that avoided hedging possibilities and used certainty equivalents or probability equivalents provided unequivocal support for ambiguity aversion. Halevy (2007) and Cettolin and Riedl (2019) eliminated the possibility of hedging by letting subjects choose the winning color (and therefore relying on belief symmetry to identify ambiguity seeking). The former elicited certainty equivalents and the latter probability equivalents. They both found substantial ambiguity aversion. Table 1 suggests that introducing RIS before the resolution of uncertainty and measuring certainty or probability equivalents (hence using PERIS or CERIS) still leads to observing ambiguity aversion. Some tasks, such as those of the studies mentioned in the previous paragraph, might be more sensitive to hedging when combined with RIS than certainty and probability equivalents.

We are aware of two direct attempts to study whether people behave the same way if randomization / risk precedes vs. succeeds ambiguity. Oechssler et al. (2019) asked their subjects to choose between four objects combining risk and ambiguity, where two objects provided a hedge against ambiguity, while the remaining two did not. Oechssler et al. (2019) also varied the order of resolving risk and ambiguity. In a second study called "alternative specification", they also included a treatment in which the risk mechanism preceded the decision. They found that the choices did not seem to be influenced by the various orders they implemented. Another attempt, by Baillon et al. (2021) (the authors of the present paper), directly compared the level of ambiguity aversion measured in a simple Ellsberg task in the absence vs. in the presence of RIS. Subjects were asked to choose between a bet based on a risky bag and a bet based on an ambiguous bag, with the latter paying 20 cents more (to break indifference). In the benchmark treatment, in which subjects only bet on the color they chose (without RIS), half of them preferred to bet on the risky bag, displaying strict ambiguity aversion. The introduction of RIS reduced this proportion to about 30%. However, there was little to no difference when changing the order of timing when the bags were prepared, RIS was implemented, draws from the bag occurred, and subjects made their decisions.

6. Conclusion

This study demonstrated the theoretical challenges posed by using a random incentive system in ambiguity experiments. In particular, we showed that when RIS follows the resolution of uncertainty, incentive compatibility might be lost. The reason for this result is that the RIS is incompatible with the structure of the preferences investigated. There could be an intrinsic inconsistency between the experimental design and the goal of measuring ambiguity aversion for subjects who perceive the whole experiment as a single decision problem.

We further showed that consistency with the ambiguity models requires that the RIS precedes the resolution of uncertainty. In this case, incentive compatibility of CERIS and PERIS is restored if the subject satisfies expected utility under risk. We extended this result to more general models of choice under risk and showed that both PERIS and CERIS are incentive-compatible if compound independence (recursivity) is satisfied. However, if the subject reduces compound objective lotteries (without compound independence), then only the incentive compatibility of CERIS is guaranteed.

The consistency obtained by having the RIS preceding the uncertainty resolution can be viewed as a minimum requirement. It does not guarantee that the RIS will be IC for all ambiguity-averse subjects who perceive the whole experiment as a single decision problem. It guarantees that it will be IC for at least some of these subjects (besides those who exhibit narrow bracketing). This is analogous to experiments measuring risk aversion assuming expected utility. Researchers know that many subjects may deviate from the expected utility. Nevertheless, we believe an experimental design that fails incentive compatibility under expected utility does not satisfy the minimum requirement for measuring risk aversion in experimental economics.

Our theoretical results have practical implications for experimental design involving ambiguity. To satisfy the minimal consistency requirement between the experiment and the decision model under investigation, RIS should precede the resolution of uncertainty. The experimenter can facilitate the subject's perception of this ordering by implementing the RIS before the decision is made and choosing a source of uncertainty whose resolution occurs after the decision. In practice, implementing the RIS before the decision can be implemented easily by letting the subject randomly draw a sealed envelope (containing the choice problem to be implemented) from a pile of envelopes at the beginning of the experiment. At the end of the experiment, the envelope is opened, and the uncertainty relating to the respective choice problem is resolved to determine the subject's payoff. Such strategy was successfully implemented by Epstein and Halevy (2019), who also included in their protocol choice problems that allowed the experimenter to observe if a subject "reversed the order" and hedged the ambiguity. Johnson et al. (2021) discussed other advantages of similar implementations of this protocol. These advantages, unlike the arguments of the present paper, go beyond the measurement of ambiguity aversion.

Appendix A. Proof of Proposition 5

Proof. **PERIS:** the subject maximizes Eq. 3.1, which can be simplified as:

$$v = \min_{\mu(E_1)\in[a,b]} \frac{1}{2}(p_1\mu(E_1) + p_2(1-\mu(E_1)) + \frac{1-p_1^2}{2} + \frac{1-p_2^2}{2}) \tag{A.1}$$

The DM has three different kinds of strategies: report $p_1 < p_2$, $p_1 > p_2$, or $p_1 = p_2$. Depending on the DM's prior beliefs, he chooses the strategy that maximizes his utility. We first list all available strategies and then analyze his optimal strategy given different prior beliefs.

Strategy 1: report $p_1 < p_2$. In this case, the valuation is decreasing in $\mu(E_1)$. Therefore a minimum is attained at $\mu(E_1) = b$:

$$v_1 = \frac{1}{2}\left(bp_1 + (1-b)\,p_2\right) + \frac{1}{2}\left(\frac{1-p_1^2}{2} + \frac{1-p_2^2}{2}\right)$$

Whenever the constraint $p_1 < p_2$ is satisfied, the optimal strategy, given by the first order condition, is to report $(p_1^*, p_2^*) = (b, 1-b)$ if it satisfies $p_1 < p_2$.

Strategy 2: report $p_1 > p_2$. In this case, $\mu(E_1) = a$ gives the minimal utility:

$$v_2 = \frac{1}{2}\left(ap_1 + (1-a)p_2\right) + \frac{1}{2}\left(\frac{1-p_1^2}{2} + \frac{1-p_2^2}{2}\right)$$

and a similar analysis shows that the optimal strategy is to report $(p_1^*, p_2^*) = (a, 1-a)$ if it satisfies $p_1 > p_2$.

Strategy 3: report $p_1 = p_2$. The term $\mu(E_1)$ drops out and the utility becomes:

$$v_3 = \frac{1}{2}(p_1 + 1 - p_1^2)$$

and the optimal strategy is to report $(p_1^*, p_2^*) = \left(\frac{1}{2}, \frac{1}{2}\right)$.

We next show how the DM chooses among the three strategies according to his prior beliefs.

Prior belief 1: $a < b < \frac{1}{2}$

For strategy 1, $p_1 < p_2$ is satisfied since $b < \frac{1}{2}$ implies $b < 1-b$. The attained maximum is: $v_1^* = \frac{1}{2}(\frac{3}{2} + b^2 - b)$, which is minimized at $\frac{1}{2}$. Given that $b < \frac{1}{2}$ then $v_1^* > \frac{5}{8}$,

For strategy 3, the maximum is $v_3^* = \frac{5}{8}$.

For strategy 2, $a < \frac{1}{2}$ implies $a < 1-a$ and therefore $p_1 > p_2$ is not satisfied. We show that when $a < \frac{1}{2}$ then strategy 2 (i.e., $p_1 > p_2$) implies $v_2^* < \frac{5}{8}$:The first inequality is obtained by replacing a by $\frac{1}{2}$ and the second by maximizing the quadratic function in p_1 and p_2. It is maximized at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Therefore $v_1^* > v_3^* > v_2^*$, implying that when the prior belief is $a < b < \frac{1}{2}$, reporting $(p_1^*, p_2^*) = (b, 1-b)$ is the optimal strategy.

Prior belief 2: $\frac{1}{2} < a < b$

In this case, when $b > \frac{1}{2}$ then strategy 1 (i.e., $p_1 < p_2$) implies $v_1^* < \frac{5}{8}$: the first inequality is obtained by replacing b by $\frac{1}{2}$ and the second by maximizing the quadratic function in p_1 and p_2. The maximum of $\frac{5}{8}$ is attained at $\left(\frac{1}{2}, \frac{1}{2}\right)$. As a consequence, $v_1^* < \frac{5}{8}$, $v_2^* = \frac{1}{2}(\frac{3}{2} + a^2 - a)$ (because $a > 1 - a$), and $v_3^* = \frac{5}{8}$. Therefore, $v_2^* > v_3^* > v_1^*$, implying that reporting $(p_1^*, p_2^*) = (a, 1 - a)$ is the optimal strategy.

Prior belief 3: $a \le \frac{1}{2} \le b$

In this case, $v_1^* < \frac{5}{8}$ and $v_2^* < \frac{5}{8}$. This can be proven as above, noting that the maximum $\frac{5}{8}$ of $\frac{1}{4}\left(2 - p_1^2 - p_2^2 + p_1 + p_2\right)$ is only reached if $p_1 = p_2 = \frac{1}{2}$ and therefore, not if $p_1 < p_2$ or $p_1 > p_2$. Furthermore, we know that $v_3^* = \frac{5}{8}$ and therefore, $v_3^* > v_1^*, v_3^* > v_2^*$. This implies that reporting $(p_1^*, p_2^*) = \left(\frac{1}{2}, \frac{1}{2}\right)$ is the optimal strategy.

-

CERIS

The subject maximizes his utility of the experiment as represented by

$$v\left(I\left(\left(f_1, f_2\right), \left(k_1, k_2\right), \left(\tilde{i}, \tilde{\ell}\right)\right)\right)$$
$$= \min_{\mu \in C}\left[\tfrac{1}{2}\left[\left(\mu(E_1) \times k_1 + (1 - \mu(E_1)) \times k_2\right)\right] + \tfrac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell + \int_{k_2}^1 u(\ell)d\ell\right)\right]$$

The DM can have three different strategies: reporting $k_1 < k_2$, $k_1 > k_2$ or $k_1 = k_2$. Depending on the prior beliefs of the DM, he chooses the strategy that maximizes his utility. We first list all available strategies and then analyze his optimal strategy given different prior beliefs.

Strategy 1: report $k_1 < k_2$. In this case, the valuation is decreasing in $\mu(E_1)$ therefore, $\mu(E_1) = b$ gives the minimal valuation:

$$v_1 = \frac{1}{2}\left[\left(bk_1 + (1 - b)\, k_2\right)\right] + \frac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell + \int_{k_2}^1 u(\ell)d\ell\right)$$

Whenever the constraint $k_1 < k_2$ is satisfied, the optimal strategy, given by the first-order condition, is to report $(k_1^*, k_2^*) = \left(u^{-1}(b), u^{-1}(1 - b)\right)$ if it satisfies $k_1 < k_2$.

Strategy 2: report $k_1 > k_2$. In this case, $\mu(E_1) = a$ gives the minimal valuation:

$$v_2 = \frac{1}{2}\left[\left(ak_1 + (1 - a)\, k_2\right)\right] + \frac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell + \int_{k_2}^1 u(\ell)d\ell\right)$$

and a similar analysis shows that the optimal strategy is to report $(k_1^*, k_2^*) = \left(u^{-1}(a), u^{-1}(1 - a)\right)$ if it satisfies $k_1 > k_2$.

Strategy 3: report $k_1 = k_2$. The $\mu(E_1)$ term drops out and valuation becomes:

$$v_3 = \frac{1}{2}k_1 + \int_{k_1}^1 u(\ell)d\ell$$

and the optimal strategy is to report $(k_1^*, k_2^*) = \left(u^{-1}\left(\frac{1}{2}\right), u^{-1}\left(\frac{1}{2}\right)\right)$.

Next, we analyze the subject's optimal strategy for three different cases of prior beliefs.

Prior belief 1: $a < b < \frac{1}{2}$

Since $b < \frac{1}{2}$, we have $1 - b > b$. Hence, $(k_1^*, k_2^*) = (u^{-1}(b), u^{-1}(1-b))$ satisfies $k_1 < k_2$ (u^{-1} is increasing) and therefore the maximum valuation under strategy 1 is attained at:

$$v_1^* = \frac{1}{2}\left[\left(bu^{-1}(b) + (1-b)\,u^{-1}(1-b)\right)\right] + \frac{1}{2}\left(\int_{u^{-1}(b)}^{1} u(\ell)d\ell + \int_{u^{-1}(1-b)}^{1} u(\ell)d\ell\right)$$

For strategy 3, the maximum is $v_3^* = \int_{u^{-1}(\frac{1}{2})}^{1} u(\ell)d\ell + \frac{1}{2}u^{-1}(\frac{1}{2}))$.
Next we show $v_1^* > v_3^*$. Consider the function:

$$f(x) = \frac{1}{2}(\int_{u^{-1}(x)}^{1} u(\ell)d\ell + \int_{u^{-1}(1-x)}^{1} u(\ell)d\ell + xu^{-1}(x) + (1-x)u^{-1}(1-x)). \tag{A.2}$$

The first order condition gives:

$$\begin{aligned} &-x\left(u^{-1}\right)'(x) - (1-x)\left(u^{-1}\right)'(1-x) + u^{-1}(x) + \\ &\qquad x\left(u^{-1}\right)'(x) + (1-x)\left(u^{-1}\right)'(1-x) - u^{-1}(1-x) = 0. \end{aligned} \tag{A.3}$$

Simplifying Eq. A.3 gives

$$\frac{1}{2}(u^{-1}(x) - u^{-1}(1-x)) = 0. \tag{A.4}$$

This implies that $u^{-1}(x) = u^{-1}(1-x)$ gives a stationary point. Since u^{-1} increases monotonically, it holds only for $x = 1 - x = \frac{1}{2}$. It is a minimum, since $f'(x) < 0$ for $x < \frac{1}{2}$ and $f'(x) > 0$ for $x > \frac{1}{2}$. Therefore, the minimum is attained at $x = \frac{1}{2}$.

Hence, $v_1^* = f(b) > f(\frac{1}{2}) = v_3^*$ as $b < \frac{1}{2}$.

For strategy 2, $(k_1^*, k_2^*) = (u^{-1}(a), u^{-1}(1-a))$ does not satisfy $k_1 > k_2$ since $a < \frac{1}{2}$ and u^{-1} is increasing. Below we show that v_2 cannot exceed v_3^*:

$$\begin{aligned} v_2 &= \tfrac{1}{2}\left[(ak_1 + (1-a)\,k_2)\right] + \tfrac{1}{2}\left(\int_{k_1}^{1} u(\ell)d\ell + \int_{k_2}^{1} u(\ell)d\ell\right) \\ &< \tfrac{1}{4}(k_1 + k_2) + \tfrac{1}{2}\left(\int_{k_1}^{1} u(\ell)d\ell + \int_{k_2}^{1} u(\ell)d\ell\right) \\ &\leq \int_{u^{-1}(\frac{1}{2})}^{1} u(\ell)d\ell + \tfrac{1}{2}u^{-1}(\tfrac{1}{2})). \end{aligned}$$

The first inequality is obtained by replacing a with $\frac{1}{2}$ and the second by maximizing the function with respect to $k_1$and k_2. The maximum is reached at $\left(u^{-1}\left(\frac{1}{2}\right), u^{-1}\left(\frac{1}{2}\right)\right)$ and it is v_3^*. Therefore $v_1^* > v_3^* > v_2^*$, implying that reporting $(k_1^*, k_2^*) = (u^{-1}(b), u^{-1}(1-b))$ is the optimal strategy.

Prior belief 2: $\frac{1}{2} < a < b$

Since $a > \frac{1}{2}$, we have $a > 1 - a$. Hence, $(k_1^*, k_2^*) = (u^{-1}(a), u^{-1}(1-a))$ satisfies $k_1 > k_2$ (u^{-1} is increasing) and therefore the maximum valuation under strategy 2 is attained at:

$$v_2^* = \frac{1}{2}\left[\left(au^{-1}(a) + (1-a)\,u^{-1}(1-a)\right)\right] + \frac{1}{2}\left(\int_{u^{-1}(a)}^{1} u(\ell)d\ell + \int_{u^{-1}(1-a)}^{1} u(\ell)d\ell\right)$$

For strategy 3, the maximum is again $v_3^* = \int_{u^{-1}(\frac{1}{2})}^{1} u(\ell)d\ell + \frac{1}{2}u^{-1}(\frac{1}{2}))$.
Next we show $v_2^* > v_3^*$. Consider the function f as defined above and its first order condition Eq. A.3. With the same arguments as above, we obtain $v_2^* = f(a) > f(\frac{1}{2}) = v_3^*$ as $a > \frac{1}{2}$.
For strategy 3, $(k_1^*, k_2^*) = (u^{-1}(b), u^{-1}(1-b))$ does not satisfy $k_1 < k_2$ since $b > \frac{1}{2}$ and u^{-1} is increasing. Below we show that v_1 cannot exceed v_3^*:

$$\begin{aligned} v_1 &= \tfrac{1}{2}\left[(bk_1 + (1-b)\,k_2)\right] + \tfrac{1}{2}\left(\int_{k_1}^{1} u(\ell)d\ell + \int_{k_2}^{1} u(\ell)d\ell\right) \\ &< \tfrac{1}{4}(k_1 + k_2) + \tfrac{1}{2}\left(\int_{k_1}^{1} u(\ell)d\ell + \int_{k_2}^{1} u(\ell)d\ell\right) \\ &\leq \int_{u^{-1}(\frac{1}{2})}^{1} u(\ell)d\ell + \tfrac{1}{2}u^{-1}(\tfrac{1}{2})) \end{aligned}$$

The first inequality is obtained by replacing b with $\frac{1}{2}$and the second by maximizing the function with respect to k_1 and k_2. The maximum is reached at $\left(u^{-1}\left(\frac{1}{2}\right), u^{-1}\left(\frac{1}{2}\right)\right)$ and it is v_3^*.
Therefore $v_2^* > v_3^* > v_1^*$, implying that reporting $(k_1^*, k_2^*) = (u^{-1}(a), u^{-1}(1-a))$ is the optimal strategy.
Prior belief 3: $a \leq \frac{1}{2} \leq b$
In this case, $v_1^* < v_3^*$ and $v_2^* < v_3^*$. This can be proven as above, noting that the maximum v_3^* of $\frac{1}{4}(k_1 + k_2) + \frac{1}{2}\left(\int_{k_1}^{1} u(\ell)d\ell + \int_{k_2}^{1} u(\ell)d\ell\right)$ is only reached if $k_1 = k_2 = u^{-1}\left(\frac{1}{2}\right)$ and therefore, not if $k_1 < k_2$ or $k_1 > k_2$. This implies that reporting $(k_1^*, k_2^*) = \left(u^{-1}\left(\frac{1}{2}\right), u^{-1}\left(\frac{1}{2}\right)\right)$ is the optimal strategy. □

Appendix B. PERIS for RDU with ROCL may not be Incentive Compatible: An Example

To give an example where PERIS is not incentive-compatible, we consider a subject whose preference for risk is represented by *rank-dependent utility (RDU,* Quiggin, 1982), which also corresponds to *cumulative prospect theory* (Tversky and Kahneman, 1992) restricted to gains.

Let F denote a cumulative distribution function over the outcome set $[0, 1]$. The expected utility of F is $\int_0^1 u(x)dF(x)$ which is equivalent to $\int_0^1 (1 - F(x))du(x)$. Its RDU value is $\int_0^1 w(1 - F(x))du(x)$, where $w(\cdot)$ is a weighting function that is increasing and satisfies $w(0) = 0$ and $w(1) = 1$. RDU generalizes expected utility by allowing not only outcomes but also probabilities that are subjectively transformed by the subject. The RDU value of a binary lottery 1_p0 is $w(p)$. If an incentive scheme is incentive compatible, the subject should report, under the assumption of RDU: $p_i^* = w^{-1}(v(f_i))$ in PERIS for all $i \in \{1, \cdots, m\}$.

Consider the experiment $J\left(\left(f_1,f_2\right),\left(p_1,p_2\right),\left(\tilde{i},\tilde{q}\right)\right)$ induced by reporting (p_1,p_2) in PERIS. Assume RDU and ROCL under risk. We show that optimal reports (p_1^*,p_2^*) may deviate from $\left(w^{-1}\left(v(f_1)\right),w^{-1}\left(v(f_2)\right)\right)$. The reduction of J can be written as $(f_1,r_1;1,r_2;0,r_3;f_2,r_4)$, where $r_1=\frac{p_1}{2}$, $r_2=\frac{1-p_1^2}{4}+\frac{1-p_2^2}{4}$, $r_3=\frac{(1-p_1)^2}{4}+\frac{(1-p_2)^2}{4}$, and $r_4=\frac{p_2}{2}$.

To determine the RDU value of J, we need to know how the subject ranks the four acts f_1, 1, 0, and f_2. The constant acts 1 and 0 are obviously ranked highest and lowest, respectively. The ranks of the other two acts depend on their respective utility. Let's take for instance $v(f_1)=v(f_2)$. The subject's valuation of the experiment is:

$$v\left(J\right)=w(r_2)+\left(w(r_1+r_2+r_4)-w(r_2)\right)v(f_1) \tag{B.1}$$

The first order condition for p_1 implies:

$$w'(r_2)\frac{\partial r_2}{\partial p_1}+\left(w'(r_1+r_2+r_4)\frac{\partial(r_1+r_2+r_4)}{\partial p_1}-w'(r_2)\frac{\partial r_2}{\partial p_1}\right)v(f_1)=0. \tag{B.2}$$

Note that $\frac{\partial r_2}{\partial p_1}=\frac{-p_1}{2}$, $\frac{\partial r_1}{\partial p_1}=\frac{1}{2}$, and $\frac{\partial r_4}{\partial p_1}=0$ by definition. The first order condition then simplified to:

$$\frac{-p_1}{2}w'(r_2)+\left(\frac{1-p_1}{2}w'(r_1+r_2+r_4)-\frac{1-p_1}{2}w'(r_2)\right)v(f_1)=0. \tag{B.3}$$

Assume $w(r)=r^2$. The first order condition becomes

$$-p_1r_2+\left(1-p_1\right)\left(r_1+r_4\right)v(f_1)=0. \tag{B.4}$$

Suppose PERIS is IC and therefore $\left(p_i^*\right)^2=v(f_i)$ should satisfy the first order condition. Using $\left(p_i^*\right)^2=v(f_i)$ and also $r_1=\frac{p_1}{2}$, $r_2=\frac{1-p_1^2}{4}+\frac{1-p_2^2}{4}$, and $r_4=\frac{p_2}{2}$, the first order condition is now:

$$3p_1^3-2p_1^4-p_1=0. \tag{B.5}$$

We show that there exists $v\left(f_i\right)$ such that the first-order condition does not hold for p_i^* satisfying the IC condition $\left(p_i^*\right)^2=v(f_i)$. Assume for instance $v(f_i)=0.01$ and therefore p_i^* should be 0.1 for PERIS to be IC. However, Eq. B.5 does not hold in this case. This proves by contradiction that PERIS is not IC.

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