University of Toronto
Department of Economics

Working Paper 694

Is there a Grand Gender Convergence in Canada? â•fi The Jury is Still Out.

By Gordon John Anderson

April 24, 2021

Is there a Grand Gender Convergence in Canada? – The Jury is Still Out.

Gordon Anderson

University of Toronto

April 2021

Abstract.

The increasing similarity of male and female roles in the labor market over the last 50 years has been dubbed “The Grand Gender Convergence”, though there is concern that the process has stalled. In the absence of gender discrimination in the labour market and assuming similar preferences for work and human resource acquisition across the gender divide, females and males with similar human resource characteristics should have similar income distributions in equilibrium, in effect there would be equality of opportunity across the gender divide. If that equilibrium is stable, convergence to the equilibrium state should see increasingly similar gender based income distributions accompanied by increasingly similar gender based human resource distributions. Viewed through the lens of an equal opportunity imperative, income convergence is a necessary, but not sufficient condition for a “Grand Gender Convergence” since similarities in income distributions could be achieved with gender based differences in human resources and efforts given a discriminatory rewards structure. Here, using new tools for empirically examining distributional convergence processes, the existence of a “Grand Gender Convergence” in 21st century Canada is examined in the context of such an Equal Opportunity paradigm. While income convergence is almost universally apparent, the same is not true for human resource stocks which appear to be diverging, raising questions about the existence of a Canadian Grand Gender convergence.

Keywords. Gender, Convergence, Distributional Differences, Human Resources.

J3, J16, J22, J24, J31, J33, N3

Introduction.

Pursuit of the “gender equity” imperative has been a long and arduous journey for those in search of economic and social justice for women. Since the mid nineteenth century “Equal Pay for Equal Work” has been an oft cited argument for female unionisation and hoped for consequence of equality at the ballot box across the gender divide[1]. However, not all inequality is bad, in meritocratic, incentive driven economies some inequalities are socially justifiable for the promotion of efficient resource allocation (Autor 2014), the trick is to separate the justified from the unjustified and seek reduction in the unjustified inequalities. In dubbing the last half centuries increasing similarity of male and female roles in the labor market “The Grand Gender Convergence”, Goldin (2014) expressed concern that the process, particularly with respect to earnings, had stalled. O’Neill (2003) had a similar theme, arguing that male and female roles in the home need to become more closely aligned for convergence to be achieved. Goldin and O’Neill wrote in the context of gender differences in roles, opportunities and outcomes, addressing normative issues of fairness and the extent to which unjustifiable differences were being redressed in the convergence process which raises questions as to what are the unjustifiable differences and how their continued reduction may be evaluated.

Denoting the amalgam of an individuals’ education, skill and experience as their human resource stock, earnings are the reward for an individuals’ efforts (length and intensity of work spells) in deploying their human resources in productive activity. Workplace gender equity or equal pay for equal work requires similarity of male and female earnings when producing similar goods. To the extent that efforts and human resources are substitutable in the workplace, this can be achieved by applying more or less effort to less or more human resources dependent upon the preferences of the individual, so that one gender could achieve the same productive outcome as the other by combining less human resources with more effort. However, demanding similarity in efforts in addition to similarity in rewards, requires similarity in human resources. In the context of complete similarity in all dimensions, gender designation becomes immaterial, it should not affect the earnings, hours and intensity of work and human resource stock

[1] “Join the union girls, and together say Equal Pay for Equal Work” Susan B. Anthony, The Revolution, 8 October 1869. "When women are given the ballot, there will be equal pay for equal work." Carrie Ashton Johnson, The Chicago Tribune Quote, 1895.

relationship, in effect males and females become perfect substitutes in the labour market and completely exchangeable in the workplace. Seeking gender parity in the earnings derived from given occupations together with parity in both the efforts and human resources that individuals bring, requires gender convergence of both the earnings distributions and the human resource distributions associated with those occupations which has implications for how such convergence is evaluated.

Equality of Opportunity, a Social Justice imperative placing primary importance on equal chances for all, provides a useful lens through which to view the gender equity issue. Founded upon notions of personal responsibility (Arneson 1989, Dworkin 2002 and Roemer 1998) it avers that, inequalities emanating from individual choice and voluntary action are of lesser import than inequalities resulting from constraining circumstances beyond individual control. The fundamental precept being that different choices voluntarily made by otherwise identical neighbours should not render them unequal since each had the opportunity to choose the others path. There are two basic opinions as to how inequality measurement should be approached, one concerns itself with circumstance types (Checchi and Peragine 2010, Ferreira and Gignoux 2011), the other concerns itself with effort types so that groups of the same effort type should face the same outcome distribution[2] (Checchi and Peragine 2010, Aaberge et al 2011, Ferreira and Peragine 2015). In reality, distinguishing between voluntary and involuntary choice is problematic[3], as is determining exactly what an "effort type" is. However, having sorted individuals into distinctly common effort or circumstance types, the equality of opportunity principle asserts that the reward patterns or distributions of those different groups should be similar, reflecting an equality of chances that a level playing field implies. Atkinson (2012) and Sen (2009) argued that the ultimate level playing field is seldom attainable and that policy objective should be the encouragement of convergence toward the optimal state which raises the issue of how one could measure the extent of convergence in this context.

Modelling the convergence process has been a feature of the economic growth literature concerning whether or not a collection of nations or regions is tending toward similar poles of attraction with respect to incomes (Barro and Sala-i-Martin 1996, Barro 1998, Galor 1996, 2011,

[2] Fleurbaey and Peragine (2013) show the two positions to be incompatible.

[3] For example, determining whether an individuals' choice of hours and intensity of work has been unconstrained or impeded.

Sala-i-Martin 1996). It is usually explored by comparing the progress of conditional means over time in those entities. In a similar vein, Goldin (2014), O'Neill (2003) and others[4] looked to examining Gender Convergence by employing regression techniques, usually Blinder-Oaxaca type decompositions (Oaxaca and Ransom. 1994, 1999), which examine the mutual proximity of conditional mean earnings. However, generically the conditional mean comparison approach has met with criticism (Carniero, Hansen and Heckman 2003, Durlauf and Quah 2002) since basic comparison of distributional locations can ignore other potentially substantive distributional differences unrelated to location. If gender equity is really about equal chances for comparable male and female groups, then the issue is more about the convergence of a collection of distributions rather than the convergence of their respective means and essentially requires measurement of the extent to which two probability distributions differ or overlap.

Given the exchangeability assumption, if the genders can be sorted into common human resource stock types, in an equal opportunity world, male and female income distributions should be similar in each human resource stock type, and gender based human resource stock distributions should be similar at any given income level and progressive similarity in both should be sought when inequality of opportunity prevails. Practically, measures of rewards and their distributions are not difficult to establish but the semi latent nature of embodied human resources makes its distribution somewhat more difficult to develop and many aspects of effort such as intensity of work are fundamentally unobservable. Here after the implications of the exchangeability assumption for distributional comparisons and distributional convergence analysis is outlined in Section 1, Section 2 discusses a means of dealing with the latent nature of some aspects of human resources in order to estimate embodied human capital distributions. Section 3 outlines means for examining convergence in distributions. The results of applying these ideas to data drawn from the Census of Canada Individual Files for the years 2001, 2006, 2011 and 2016 are reported in Section 4 and conclusions drawn in section 5. To anticipate the results, whilst convergence in gender based income distributions is almost universally observed over the period, convergence in their respective human resource stock distributions across the gender divide is not, calling in to question the notion of a Grand Gender Convergence in 21st Century Canada.

[4] See for example Chetty, Hendren, Jones and Porter 2020, Meara, Pastore & Webster (2020), Boudarbat and Connolly (2013), Fortin (2019), Jehn, Walters and Howells (2019), Schirle (2015)

1. The Income – Human Resource Distributional Convergence Issue.

Imagine a working population is comprised of two groups, G and B which are identical in every respect regarding their distributions of preferences for work and acquiring human capital, their only distinguishing features are their group designation and possible relative numerical size. With no gender discrimination in the labour market and full exchangeability, similar males and females will employ similar efforts in applying similar human resource stocks so that earnings y, are generated by efforts x and human resources z through a common reward function $y = r(x, z, e)$ where e is an unobserved random variable (luck), which, in the absence of gender discrimination, is distributed independently of x, z and the designations B and G. In labour market equilibrium, the respective joint distributions of x, z and y, $f_G(y, x, z)$ and $f_B(y, x, z)$ should be such that $f_G(y, x, z) = f_B(y, x, z) = f(y, x, z)$ for all x, z, y where the working population joint distribution $f(y, x, z) = w_G f_G(y, x, z) + w_B f_B(y, x, z)$ with w_G and w_B $(= 1 - w_G)$ representing the respective proportions of B and G in the working population.

In such an equilibrium, the marginal distributions of efforts $(f(x) = \int\int f(y, x, z)dydz)$, human resources $(f(z) = \int\int f(y, x, z)dydx)$ and earnings $(f(y) = \int\int f(y, x, z)dxdz)$ will be such that $f_G(x) = f_B(x)$, $f_G(z) = f_B(z)$, and $f_G(y) = f_B(y)$. In a similar fashion all conditional distributions would be identical across genders. The gender groups become exchangeable in the sense that the joint, marginal and conditional G distributions are perfectly substitutable for the corresponding B distributions in these relationships. Furthermore, if the equilibrium is stable, adjustment or tatonnement processes will exist such that when $f_G^*(y, x, z) \neq f_B^*(y, x, z)$, $f_G^*(y, x, z) \rightarrow f(y, x, z)$ and $f_B^*(y, x, z) \rightarrow f(y, x, z)$ in the long run (Arrow and Hahn 1971). Note also that male and female marginal and conditional distributions will similarly converge to common distributions. Indeed, establishing a "Grand Gender Convergence" in this context, requires examining the simultaneous convergence of the earnings, human resource and effort distributions. Note that commonality of means, conditional means, is only a necessary but not a sufficient condition for establishing equilibrium. In this sense the empirical problem is more like that of the generational dependency/equal opportunity literatures (Arrow, Bowles and Durlauf 2000, Morgan, Grusky and Fields 2006) requiring convergence in and similarity of distributions rather than convergence in and similarity of conditional means.

2. Challenges in Human Resource Stock Measurement.

Beyond their capacity for hours and intensity of work, the resources at an individuals' disposal for income generation are a complex combination of the net effect of received education and training, in essence their embodied human capital (EHC), augmented by their accumulated lifetime experiences (AE). Thus an individuals' human resource stock at a given point in time may be generically assumed to be an increasing, possibly concave, function of the level of its acquired embodied human capital and the longevity of its post acquisition work life. EHC and AE are inherently latent concepts, each individually hard to quantify. Moreover, when they are viewed as jointly determining an individuals' human resource stock, their effects have to be aggregated somehow making measurement particularly challenging.

The various levels or categories of education and training each have a different import for the amount of human capital that is embodied, and the rate at which experience is acquired clearly differs across stages of the lifecycle. However, provided that, at each successive stage of the acquisition process, the previously acquired stock has not deteriorated to an extent that exceeds the gains made at the current stage, then it can be reasonably assumed that successive stages in each constitute continued accretion, so that the human resource stock would be monotonic non decreasing across ordered education and age categories. In this case, ordered categories make viable proxies for the latent EHC and AE theoretical constructs.

An individuals' human resource level could be gauged by attaching some sort of Cantril (1965) scale to each of the various ordered categorical levels of AHC and AE they have achieved and combining the indices in some fashion for an overall measure of the individuals' human resource stock. However, recently this practice has met with criticism (Schroder and Yitzhaki 2017, Bond and Lang 2019). Most summarising statistics and weighting mechanisms are scale dependent and generally, beyond respecting the ordinal nature of categories, any applied scale or weighting process is arbitrary, which can lead to contradictory orderings when comparisons based on different but none-the-less equally valid scale and weighting choices are explored. Furthermore, when general distributional differences between groups are of interest, differences in corresponding locational summary statistics provide a weak, inconsistent test with low or zero power at any sample size against distributional differences unrelated to location (Carneiro, Hansen and Heckman 2003) potentially disguising distributional differences. This concern is relevant whenever location measures are compared as proxies for more general distributional differences but is of particular concern when arbitrary scales are applied to ordinal categorical data.

Following Anderson, Post and Whang (2020) and based on developments in Gravel et. al. (2020), Anderson and Leo (2021) provide a scale independent, dominance ordering based means of discriminating between a collection of groups in these situations. Suppose two dimensions, say education level "e", and age group "a" with increasing ordered categories $e_i \; i = 1,..,I$ and $a_j \; j = 1,..,J$. Ordering instruments may be developed as follows:

Letting $p_{i,j} = P(e_i \cap a_j) \; for \; i = 1,..,I \; and \; j = 1,..,J$, $F_{ic,jc}$, the Joint Cumulative Distribution Function, the probability of being no higher than e_{ic} and $a_{jc} \; for \; ic = 1,..,I \; and \; jc = 1,..,J$, is given by $F_{ic,jc} = \sum_{i=1}^{ic}\sum_{j=1}^{jc} p_{i,j}$ and $CF_{ic,jc}$, the Cumulative CDF may be written as: $CF_{ic,jc} = \sum_{i=1}^{ic}\sum_{j=1}^{jc} F_{i,j}$. In ordering female and male human resources defined by their respective joint cumulative distributions of education and age status $F_{ic,jc,G}$ and $F_{ic,jc,B}$, stochastic dominance conditions seek separation between the distributions. If the level of human resources is considered a monotonic non decreasing function of the latent embodied human capital and experience variables the respective categories represent, a necessary condition for females to have superior human resource stocks to males is that:

$$F_{ic,jc,G} \leq F_{ic,jc,B} \; for \; all \; ic \; and \; jc, with \; strict \; inequality \; somewhere$$

Similarly, if the level of human resources is considered a monotonic non decreasing concave function of the latent variables then:

$$CF_{ic,jc,G} \leq CF_{ic,jc,B} \; for \; all \; ic \; and \; jc, with \; strict \; inequality \; somewhere$$

Is required. These conditions may be examined by considering in the first instance CDCDF, the cumulated differences in the CDF's and its related dominance index CDCDFI where:

$$CDCDF_{M,F} = \sum_{i=1}^{I}\sum_{j=1}^{J}(F_{i,j,M} - F_{i,j,F})$$ and

$$CDCDFI_{M,F} = \sum_{i=1}^{I}\sum_{j=1}^{J}(F_{i,j,M} - F_{i,j,F}) / \sum_{i=1}^{I}\sum_{j=1}^{J}|F_{i,j,M} - F_{i,j,F}|$$

And in the second case by considering CDCCDF and CDCCDF where:

$CDCCDF_{M,F} = \sum_{i=1}^{I}\sum_{j=1}^{J}\left(CF_{i,j,M} - CF_{i,j,F}\right)$ and

$CDCCDFI_{M,F} = \sum_{i=1}^{I}\sum_{j=1}^{J}\left(CF_{i,j,M} - CF_{i,j,F}\right) / \sum_{i=1}^{I}\sum_{j=1}^{J}\left|CF_{i,j,M} - CF_{i,j,F}\right|$

$|CDCDF|$ ($|CDCCDF|$) can be seen as measures of distance between female and male distributions, the further apart they are the greater the value will be. Note that $-1 \leq CDCDFI\ (CDCCDFI) \leq +1$, reaching -1 when all the elements in the respective sums $CDCDF\ (CDCCDF)$ are non positive and at least one is negative (in which case the Male distribution dominates the Female distribution at that appropriate order) and reaching +1 when all the elements are non-negative and at least one is positive (in which case the Female distribution dominates the Male distribution).

More generally Anderson and Leo (2021) propose ranking and ordering G groups of individuals indexed g = 1,..,G with distribution functions $F_{i,j,g}$ by constructing $FU_{i,j}$ ($CFU_{i,j}$) and $FL_{i,j}$ ($CFL_{i,j}$) the upper and lower boundaries of the collection of distributions where:

$FU_{i,j}\left(CFU_{i,j}\right) = \max_{g=1,..,G}(F_{i,j,g})\ \left(\max_{g=1,..,G}(CF_{i,j,g})\right)$ and

$FL_{i,j}\left(CFL_{i,j}\right) = \min_{g=1,..,G}(F_{i,j,g})\ \left(\min_{g=1,..,G}(CF_{i,j,g})\right)$

The upper boundaries may be construed as representing the CDF and CCDF of the fictitious worst or Dystopian distribution that could be contrived by selecting the worst aspects of the distributions in the collection. If a particular distribution were dominated by all others in the collection it would correspond to the Dystopian Distribution. Similarly, the lower boundaries may be construed as the CDF and CCDF of the fictitious best or Utopian distribution that could be contrived by selecting the best aspects of the distributions in the collection. Again if a particular distribution in the collection dominated all others it would be the Utopian distribution. Noting that $CDCDFI_{U,g}\ \left(CDCCDFI_{U,g}\right) = 1$ for all g by construction, $\frac{CDCDF_{U,g}}{CDCDF_{U,L}}, \left(\frac{CDCCDF_{U,g}}{CDCCDF_{U,L}}\right)\ g = 1, .., G$ provide standardized, scale independent, indices of the volume distance from the worst possible outcome for each of the groups[5].

[5] For inference purposes, asymptotic distributions for all of these constructs are available in Anderson and Leo (2021).

3. On the Empirical Assessment of Convergence.

Generally, in examining equality of opportunity, the literature has followed two paths. Regression/Treatment Effect and Conditional Mean comparison approaches (Mulligan 1997, Solon 1992, 2008, Peragine at al.2014) employ differences in conditional location statistics to measure closeness of distributions, alternatively Lefranc et al. (2008, 2009) proposed distributional difference approaches based upon stochastic dominance relations. However, with regard to the former, Carneiro et al. (2002, 2003) and Durlauf and Quah (2002) demonstrate that employment of such summary statistics in this fashion can be misleading since they overlook important information about distributional differences which can only be countervailed by comparing complete outcome distribution profiles in their entirety. Simply put, variation in conditional means reveals nothing about distributional differences engendered by variation in conditional variances, skewness or kurtoses. In this regard, Lefranc et al. (2008, 2009), proposed tests for equality of opportunity by exploring whether areas under respective Generalized Lorenz Curves are similar. The difficulty with this approach is that they are not really a test of equality in distribution, which requires equality of the respective Generalized Lorenz Curves rather than equality of the areas beneath them.

The extent to which distributions differ can be quantified using Gini's Transvariation measure GT (Gini 1916, 1959) which, for group pdf,s $f_i(x)$ and $f_j(x)$, is given by:

$$GT_{i,j} = \frac{1}{2}\int_0^\infty |f_i(x) - f_j(x)|dx = \int_0^\infty [\max\left(f_i(x), f_j(x)\right) - min\left(f_i(x), f_j(x)\right)]dx \quad [1]$$

Since $GT_{i,j} = 1 - OV_{i,j}$, where $OV_{i,j} = \int \min\left(f_i(x), f_j(x)\right) dx$ is the overlap measure of the extent to which two distributions have common mass location (Anderson, Linton and Whang 2012), GT will take the value 0 when the distributions are identical and 1when they have mutually exclusive support.

At issue here is the extent of variation between male and female income distributions at various levels of human resource status and the degree to which that variation is diminishing or increasing over time both for individual groups and across the population as a whole. Denoting the probability density function of the income x of gender i with education status j in age group k at time t as: $f_{i,t}(x|j,k)$ where i=m,f, t= 1,..,T, j=1,..,J and k=1,..,K this may be examined using as a basis Ginis distributional transvariation GT which for males and females at time t in education group j and age group k may be written as:

$$GT_{t,j,k} = \frac{1}{2}\int_0^\infty \left|f_{m,t}(x|j,k) - f_{f,t}(x|j,k)\right|dx = 1 - OV_{t,j,k}$$

In the case of discrete categorical income distributions where for gender i in period t, the probability of being in category c given the j'th education level and k'th age group is $f_{i,t}(c|j,k)$ for income categories c=1,..,C, GT and OV may be written respectively as:

$$GT_{t,j,k} = \frac{1}{2}\sum_{c=1}^{C}\left|f_{m,t}(x_c|j,k) - f_{f,t}(x_c|j,k)\right| \; ; OV_{t,j,k} = \frac{1}{2}\sum_{c=1}^{C} min\left(f_{m,t}(x_c|j,k), f_{f,t}(x_c|j,k)\right).$$

For inference purposes, given independent samples of size n where $\widetilde{OV}_{t,j,k}$ is a kernel based estimate of $OV_{t,j,k}$, $\sqrt{n}(\widetilde{OV}_{t,j,k} - OV_{t,j,k}) \sim_a N\left(0, OV_{t,j,k}(1 - OV_{t,j,k})\right)$ Anderson, Linton and Whang (2012).

With regard to measuring convergence in the population as a whole, Anderson et. al. (2021) developed a family of unit free measures of multilateral distributional differences, distributional analogues of the Range, Coefficient of Variation and Gini coefficient used for measuring differences in collections of numbers. The Distributional Coefficient of Variation (DCV) is a weighted aggregation of the respective group and overall distribution transvariations, Letting $OV_{ko} = \int \min\left(f_k(x), f(x)\right) dx$, DCV, the weighted average of the group and target distribution transvariations, is given by:

$$DCV = \frac{1}{(1-\sum_{=1}^{K} w_k^2)}\sum_{k=1}^{K} w_k GT_{ko} = \frac{1}{(1-\sum_{=1}^{K} w_k^2)}\sum_{k=1}^{K} w_k\,(1 - OV_{ko}) \qquad [2]$$

DCV only measures distributional distance from some target distribution and does not reflect the full extent of distributional differences between all population of groups which are only captured by $DisGini$, the Distributional Gini coefficient which is given by:

$$DisGini = \frac{1}{(1-\sum_{k=1}^{K} w_k^2)}\sum_{i=1}^{K}\sum_{j=1}^{K} w_i w_j (1 - OV_{ij}) \;=\; \frac{1}{(1-\sum_{k=1}^{K} w_k^2)}\sum_{i=1}^{K}\sum_{j=1}^{K} w_i w_j (GT_{ij})$$

Unweighted versions of both [5] and [6] can be obtained by setting $w_k = 1/K$ for all k. Though both are useful for measuring overall variation in distributions across groups, in the present context they are more than required since all that equality of opportunity requires is commonality of income distribution in each human resource category, so ATR, the Average Transvariation over all categories is required i.e.

$$ATR = \sum_{k=1}^{K} w_k GT_{mfk} = \sum_{k=1}^{K} w_k \left(1 - OV_{mfk}\right) \qquad [3]$$

Where OV_{mfk} is the overlap of male and female income distributions in the k'th human resource category or the overlap of male and female human resource distributions in the k'th income category.

The Universal Convergence Principle.

Effectively ATR is a weighted aggregation of the extent to which male and female distributions differ across subgroups which in turn suggests a new notion of Universal Convergence wherein all groups can be seen to be converging simultaneously. Denoting the i'th subgroup gender transvariation in period "t" as $GT_{i,t}\ for\ i = 1,..,K$ if $GT_{i,t} > GT_{i,t+c}\ for\ all\ i = 1,..,K$ then convergence is universal in that all subgroups have reduced variation over the period. This may be examined by testing the joint null hypothesis:

H_0: $GT_{i,t} - GT_{i,t+c} \geq 0\ for\ all\ i = 1,.,K$ against H_1: $GT_{i,t} - GT_{i,t+c} < 0\ for$ some $i = 1,.,K$

The test may be effected using Stoline and Ury (1979) maximum modulus distribution tables. Indeed, the terms $DIF_{i,t} = GT_{i,t} - GT_{i,t+c}$ can be assessed individually to see which groups are converging and which are diverging. Universality can also be simply established by checking if $\left|\sum_{i=1}^{K} DIF_{i,t}\right|/K = \sum_{i=1}^{K}\left|DIF_{i,t}\right|/K$.

In the following application achievement measures are multivariate. Unlike quantifying distributional differences using conditional moments, when x is a vector, multilateral comparisons of multivariate distributions is straightforward. Suppose the distributions are tri-variate $f(x, y, z)$, then simply write [1] as:

$$GT_{i,j} = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \frac{|f_i(x,y,z) - f_j(x,y,z)|}{2} dx\, dydz \left(= \sum_x \sum_y \sum_z \left|p_i(x,y,z) - p_j(x,y,z)\right| \right)$$

And substitute into [2] or [3] as appropriate, again with unweighted versions following as before. Comparing multivariate distributions in such a fashion provides a very natural way of combining the various outcome dimensions avoiding the invidious and arbitrary weighting problems involved in aggregation across dimensions.

3. Results.

To examine the income, human resource and effort nexus, data on the total income, age, gender and education status of individuals have been drawn from the Census of Canada: Individual File for the years 2001, 2006, 2011 and 2016. All agents over the age of 19 who received and income and reported age and educational status were included in the study and an agents' location in the income distribution was based upon its membership in one of the 20 income vingtiles.

An individuals' human resources were based upon their (ordered) education and training category and their age group membership, the six Education Categories were:

1. No certificate, diploma or degree.
2. Secondary (high) school diploma or equivalency certificate.
3. Trades certificate or diploma, Certificate of Apprenticeship or Certificate of Qualification.
4. Program of 3 months to 2 years (College, CEGEP and other non-university certificates or diplomas).
5. Program of more than 2 years (College, CEGEP and other non-university certificates or diplomas), University certificate or diploma below bachelor level or Bachelors degree.
6. University certificate or diploma above bachelor level, Degree in medicine, dentistry, veterinary medicine or optometry, Master's degree or Earned doctorate.

The six age groups were: 1) 20-29, 2) 30-39, 3) 40-49, 4) 50-59, 5) 60-69 and 6) 70 and over.

The unobservable in this relationship, apart from sheer luck, is a measure of an individuals' extent and intensity of effort which is based upon their preference for work. In the context of gender equity, as with luck, preferences for work are assumed to be similarly distributed across the genders. In such a situation, with no gender discrimination in the labour market, males and females should have the same joint income - human resource distributions so their overlap (transvariation), which is a measure of their proximity (differentness), should be 1 (0). Table 1 documents the changing proximity (differentness) of the female – male joint densities of the income, education, age group triple over the observation years based upon the extent of overlap of male and female distributions. There appears to be a reasonable amount of commonality between the distributions (of the order of 72% to 77%), though in all cases it is significantly different from 100%. Increasing overlaps (decreasing transvariations) over time are indicative of increasing proximity and thus convergence over the period which may be seen to be the case for all but the 2011-2016 period. Thus though convergence appears to be the prevailing process over the period, it is not uniform, with some evidence of divergence in the 2011-2016 period which prompts investigation.

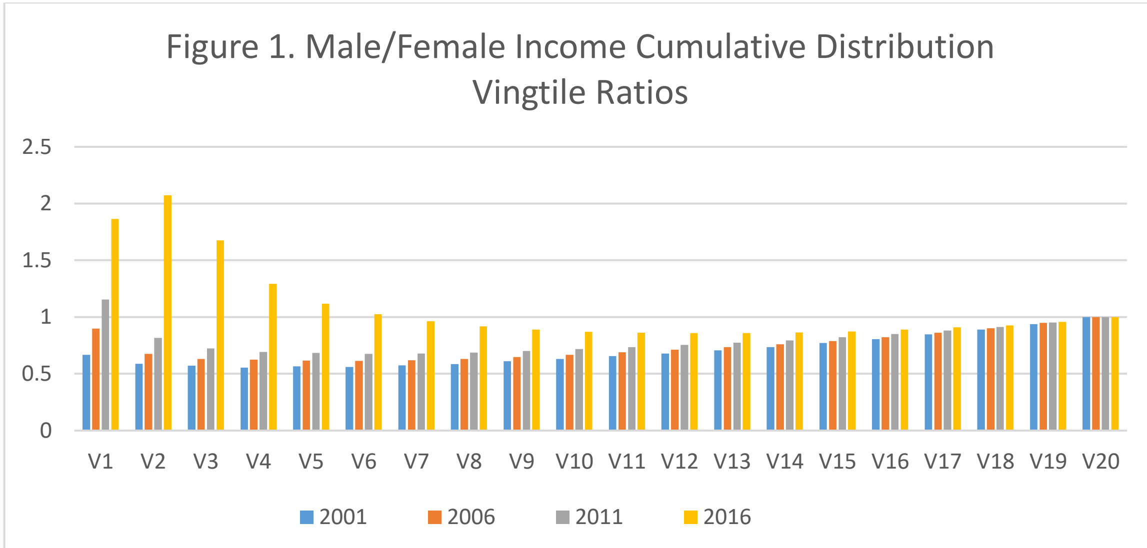
Table 1. Female –Male Income, Education, Age Joint Distributional Overlap

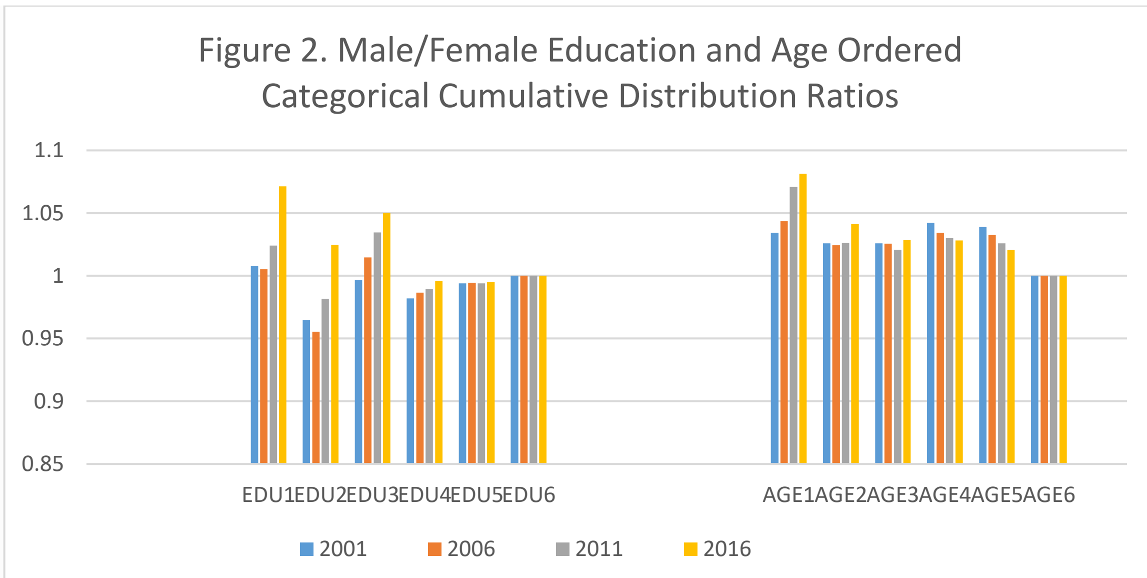
	2001	2006	2011	2016
Overlap	0.72664	0.74351	0.76881	0.75399
Transvariation	0.27336	0.25649	0.23229	0.24601
Standard Error	0.00176	0.00165	0.00153	0.00153

Convergence analysis. (OV(t+k)-OV(t) > 0 => Convergence)

Overlap Comparison	Difference	Standard Error	|Z score|
OV 2006 - OV 2001	0.01687	0.00240	7.03522
OV 2011 - OV 2001	0.04217	0.00232	18.20024
OV 2011 - OV 2006	0.02530	0.00225	11.24345
OV 2016 - OV 2001	0.02735	0.00232	11.80404
OV 2016 - OV 2006	0.01048	0.00225	4.65736
OV 2016 - OV 2011	-0.01482	0.00216	6.84923

Treating education and age group membership as joint determinants of an individuals latent embodied human resources generates 36 groups with common levels of educational and age status. In recording the male / female ratio of cumulative density functions for income, education status and age group, Figures 1 and 2 present a visual impression of the overall relative progress of female and male distributions over the observation period. Values less than or equal to 1 over the whole range of variation indicate first order distributional dominance of males over females, values greater than or equal to one over the whole range indicate dominance of females over males. Common distributions would yield a value of 1 in all vingtiles. The ratios ever increasing proximity to 1 over the period is indicative of convergence over the observation period. Clearly there are substantive differences in male and female income distributions, with the male distribution dominating the female distribution in 2001 and 2006, “almost dominating” in 2011 and, due to a preponderance of males in the lowest quartile, there is no dominance in 2016.





Dominance and convergence are less clear in the case of education although in 2016, a sequence of male/female cumulative cdf ratios $(\sum_{i=1}^{j} F_m(i)/\sum_{i=1}^{j} F_f(i)\ for\ j = 1,.,6))$ all above 1 in value {1.07135, 1.03683, 1.04425, 1.02367, 1.01471, 1.01119}indicates second order dominance of females over males in education. Females first order dominate males in the age category comparisons in all years though there is no clear convergence over the observation period.

However, these are overall illustrative comparisons, what matters for equality of opportunity and gender convergence, given identical proclivities for effort across the genders, is the increasing proximity of the income distributions of males and females with common levels of human resources, together with the increasing proximity of the education and age based bivariate human resource distributions for males and females at common levels of income.

3.1) Gender Convergence in Incomes.

Table 2. Differences in Density Functions Across Pooled Vingtiles.

	First Order Comparison $CDCDF_{F,M}$	$CDCDFI_{F,M}$	$\sum_{j=1}^{20} \left\| F_{j,F} - F_{j,M} \right\|$
2001	2.97228	1.00000	2.97228
2006	2.58900	1.00000	2.58900
2011	2.10953	0.99321	2.12395
2016	0.66177	0.55628	1.18963

As Figure 1 suggests, standard first order dominance comparisons reveal that the male income distribution first order dominates the corresponding female distribution in 2001 and 2006 and almost dominates in 2011 (Leshno and Levy 2002). However, in 2016 the male income distribution is far from dominating the female distribution, a reflection of the ever increasing proximity of the distributions indicated by the ever diminishing cumulated absolute CDF differences over vingtiles over time recorded in Table 2. Given that $\int_0^\infty (1 - F(x))dx = E(x)$, cumulated differences of Cdf's are much like comparing means and do not reflect similarities and differences in gender based outcomes as comprehensively as the overlap and transvariation measures which are reported in Table 3.

Overall transvariation falls significantly over the first decade but is attenuated in 2016 by a significant increase. However, these measures compare the income distribution of females from all human capital classes with that of males from all human capital classes whereas what matters from an equality of opportunity and convergence perspective is the overlaps/transvariations of the specific Human resource stock classes. In this case, the average transvariation over these classes reflects increasing similarity and ongoing convergence of female-male outcomes over the whole period.

The question as to whether or not the convergence was universal across all Human Resource Stock classes can be examined by comparing the transvariation of each class in subsequent years. If $GT_{t,j,k} \geq GT_{t+s,j,k}\ for\ s > 0\ and\ all\ j,k$, then universal convergence is ongoing over the period. Under the assumption that observations are independent across years, a test of the hypothesis: H[0]: $GT_{t,j,k} - GT_{t+s,j,k} \geq 0\ \forall\ j,k = 1,..6$ can be implemented using the Maximum

Modulus distribution Stoline and Ury (1979), the details of which are reported in Table 4. In the 2001-2006 comparison there was one significant rejection out of 36 comparisons (the lowest educated over 70's group), in 2006-2011 there were no rejections and there were 6 rejections in the 2011-2016 comparison (all age groups in education category 3), over the whole period 2001-2016 there were 3 rejections in 36 comparisons (the three youngest age groups in education category 3)[6]. So, convergence was close to universal over the first decade but there appears to be a slight reversal in the 2011-2016 period with respect to education category 3.

Table 3. Overall Transvariations and Overlaps.

	2001	2006	2011	2016
Male-Female Income Overlap	0.77187	0.79909	0.82580	0.82211
Male-Female Income Transvariation	0.22813	0.20091	0.17420	0.17789
Standard Error	(0.00110)	(0.00102)	(0.00094)	(0.00092)
Average Transvariation Across HR status	0.24749	0.22569	0.20036	0.19956
Standard Error	(0.00443)	(0.00318)	(0.00276)	(0.00316)
Female Sample Size	294194	317695	335512	353424
Male Sample Size	283591	302092	320040	337292

*Subgroups defined by education/age i.e. human resource stock status.

Casual perusal of Table 4 suggests some within year structure across the human capital classes which appears to be repeated in subsequent years. For information purposes this can be explored in a regression of human capital class transvariation on education (EI) and age (AI) class levels indexed by their respective category numbers. $OV_{t,j,k}$, the Female-Male Overlaps (income distribution similarities) are assumed to be a quadratic function of age and education group levels, time trend T and the overlap in the previous period $OV_{tl,j,k}$ represented in the following regression equation and reported in Table 5.

$$OV_{t,j,k} = \alpha + \beta_1 T_t + \beta_2 EI_{t,j} + \beta_3 EI_{t,j}^2 + \beta_4 AI_{t,k} + \beta_5 AI_{t,k}^2 + \beta_6 \left(EI_{t,j} AI_{t,k}\right) + \beta_7 OV_{tl,j,k} + e_{t,j,k}$$

The results record the extent of similarity of the genders with respect to incomes as a stationary process with an increasing concave function of the education level (the more highly educated are the parties, the more closely aligned are their incomes) and a decreasing concave function of age (income similarities diminish with increasing age) and a positive trend reflecting increasing similarity in male and female income distributions over time. At median education and age group

[6] The appendix reports a detailed comparison of mean differences.

Table 4. Period by Period Changes in Subgroup Income Transvariations

Edu	Age	2001-2006		2006-2011		2011-2016		2001-2016	
1	1	-0.00526	-0.54287	0.02555	2.43314	0.03151	2.99471	0.05180	5.33358
1	2	-0.00032	-0.03124	0.02489	2.01941	0.05775	4.67269	0.08232	8.00508
1	3	0.00583	0.69304	0.04646	4.74958	-0.00034	-0.03116	0.05195	5.35466
1	4	0.00168	0.19855	0.02835	3.08150	-0.00181	-0.19599	0.02822	3.32022
1	5	-0.02247	-2.69868	0.03865	4.18087	0.02317	2.49143	0.03935	4.69137
1	6	-0.02123	-3.29939	0.01919	2.76049	0.00879	1.27461	0.00675	1.05889
2	1	-0.00407	-0.70527	0.02156	3.91722	0.00622	1.20593	0.02371	4.35704
2	2	0.02223	2.78169	-0.00078	-0.09370	0.04661	5.78259	0.06806	8.82097
2	3	0.04572	6.48976	0.02298	3.34209	-0.00580	-0.79681	0.06290	8.45557
2	4	0.05177	6.16266	0.04213	6.15670	0.00684	1.09385	0.10074	12.70763
2	5	0.02456	2.06464	0.03022	3.19857	0.02841	3.64585	0.08319	7.82719
2	6	0.01810	1.54041	0.00982	1.00951	0.00778	0.88619	0.03570	3.25197
3	1	0.00796	1.24892	0.01268	2.09670	-0.08522	-13.35517	-0.06458	-9.65217
3	2	0.00261	0.40290	0.00777	1.18409	-0.04163	-6.30303	-0.03125	-4.79193
3	3	0.01770	2.89407	0.02167	3.72821	-0.05283	-8.55280	-0.01346	-2.08251
3	4	0.03403	4.60788	0.03805	6.07960	-0.06419	-10.89681	0.00789	1.11503
3	5	0.01287	1.27447	0.04763	5.82671	-0.05416	-7.58175	0.00634	0.68293
3	6	0.02004	1.92966	0.00165	0.18945	-0.14762	-16.89294	-0.12593	-12.09725
4	1	0.02634	4.38290	-0.00462	-0.90817	0.00609	1.27543	0.02781	4.83840
4	2	0.03001	3.95591	0.01082	1.59786	0.01171	1.83154	0.05254	7.24621
4	3	0.03132	3.88281	0.02422	3.52397	0.01147	1.79040	0.06701	8.73356
4	4	0.04068	4.37503	0.02545	3.39785	0.00161	0.23256	0.06774	7.65619
4	5	0.04853	3.26809	0.04274	4.13334	0.02292	2.77599	0.11419	8.46990
4	6	0.02641	1.58956	0.00026	0.01957	0.06408	5.70692	0.09075	6.04280
5	1	0.00698	0.50434	0.02596	2.31560	0.00740	0.79519	0.04034	3.26709
5	2	0.00491	0.41124	0.03415	3.30016	0.01798	1.97956	0.05704	5.25163
5	3	0.03562	3.08036	0.01995	2.02499	-0.00299	-0.32321	0.05258	4.75572
5	4	0.02428	1.82973	0.02993	2.67747	0.00520	0.49851	0.05941	4.69771
5	5	0.02203	1.07200	0.02338	1.57075	0.03411	2.91007	0.07952	4.32433
5	6	-0.00455	-0.16409	-0.00056	-0.02421	0.07604	4.11650	0.07093	2.95752
6	1	0.19282	1.42485	-0.04008	-0.39387	-0.09165	-0.77323	0.06109	0.41180
6	2	0.01539	0.36951	0.05539	1.60387	0.01708	0.63375	0.08786	2.46699
6	3	-0.01335	-0.37020	0.01800	0.54512	0.04669	1.61023	0.05134	1.58376
6	4	0.03036	0.68100	0.13111	3.83818	-0.06940	-2.47504	0.09207	2.29681
6	5	0.04742	0.77582	0.02104	0.45851	0.07945	2.31538	0.14791	2.79143
6	6	0.00767	0.08775	0.05623	0.87213	0.02746	0.51739	0.09136	1.15100

Stoline and Ury 5% Critical Value for 36 simultaneous comparisons is 3.19

levels of $\partial OV/\partial EI = 0.00733$ in the short run (0.02076 in the long run) $\partial OV/\partial AI = -0.00625$ short run (-0.01769 long run). Evidently there is a saddle point with the solution being the point where the respective education and age group levels are such that: ${}^{\partial OV}\!/_{\partial EI} = 0 \cap {}^{\partial OV}\!/_{\partial AI} = 0$, which is the solution to:

$$\begin{bmatrix} 0.00650 & 0.00061 \\ 0.00061 & -0.00564 \end{bmatrix} \begin{bmatrix} EI \\ AI \end{bmatrix} = \begin{bmatrix} 0.02866 \\ -0.02134 \end{bmatrix} => \begin{bmatrix} EI \\ AI \end{bmatrix} = \begin{bmatrix} 4.01341 \\ 4.21776 \end{bmatrix}$$

Suggesting the saddle point as being at the just below university entry – 50's age group nexus.

Table 5. Overlap Regression. (Dependent Variable Male-Female Income Distributional Overlap within education – age group.)

	Coefficient	Standard Error	"t" statistic.
constant	0.20573	0.05165	3.98330
trend	0.01484	0.00400	3.71360
education level	0.02866	0.01012	2.83178
education level squared	-0.00325	0.00129	-2.51886
age level	-0.02134	0.01020	-2.09294
age level squared	0.00282	0.00130	2.17730
education*age	-0.00061	0.00110	-0.55392
lagged overlap	0.66770	0.06011	11.10815

N=108, $\hat{\sigma}^2 =$ 0.00111; R^2 = 0.74762

3.2) Human Resource Stocks, A Gender Divergence?

As the primary driver of incomes the female / male similarities and diversities in the human resource stock distributions across genders is of interest. Recall that the latent measure of human resource stock is a bivariate function of education and training levels and experience (for which age is a proxy), the distribution of those two ordered categorical variates is bivariate. The Male-Female Human Resource Stock Transvariations (Standard Errors) of those bivariate distributions for the successive observation years are: 0.08541(0.00037), 0.09165(0.00037), 0.09457(0.00036) and 0.09541(0.00035) respectively so, contrary to the income distribution behaviour, the respective distributions appear to be diverging significantly.

Table 6 reports the distributional coefficients of variation across income vingtiles for education, age and their joint distributions. Female variations are generally larger than male variations, education variations are increasing through time whereas age variations are diminishing. Joint distribution transvariations are increasing through time highlighting the predominance of the

Table 6. Distributional Coefficients of Variation of Human Capital Factors Across Income vingtiles

	2001		2006		2011		2016	
Overall edu	0.14760	0.00049	0.14293	0.00046	0.15089	0.00045	0.22461	0.00049
Overall age	0.16027	0.00050	0.15343	0.00047	0.15122	0.00045	0.13813	0.00042
Overall joint	0.34449	0.00054	0.35789	0.00050	0.37682	0.00047	0.44894	0.00044
Female edu	0.16530	0.09091	0.16325	0.09035	0.17051	0.09233	0.22006	0.10490
Female age	0.15760	0.08877	0.15405	0.08777	0.15231	0.08727	0.14091	0.08394
Female Joint	0.34618	0.08877	0.36629	0.08777	0.38203	0.08727	0.44995	0.08394
Male edu	0.12801	0.08000	0.12069	0.07768	0.12749	0.07984	0.22892	0.10699
Male age	0.16530	0.09091	0.15497	0.08803	0.15132	0.08698	0.13661	0.08265
Male Joint	0.34327	0.09091	0.35228	0.08803	0.37345	0.08698	0.44761	0.08265

education as opposed to the age effect in the joint distribution. These can be explored in more detail by looking at the overlaps of the respective marginal education, age and joint education – age distributions by income vingtile in Tables 7, 8 and 9. Table 7, recording the similarity

Table 7. Education Distribution Gender Overlaps

Vingtile	2001		2006		2011		2016	
1	0.95950	0.00238	0.96494	0.00210	0.96177	0.00212	0.88414	0.00359
2	0.97001	0.00213	0.98632	0.00141	0.97060	0.00195	1.00000	0.00000
3	0.96543	0.00226	0.96966	0.00205	0.98154	0.00156	0.85112	0.00383
4	0.95832	0.00250	0.97746	0.00175	0.98069	0.00158	0.94075	0.00264
5	0.97426	0.00193	0.96097	0.00230	0.97314	0.00184	0.95908	0.00221
6	0.96521	0.00227	0.95201	0.00252	0.96324	0.00214	0.95658	0.00225
7	0.95470	0.00251	0.95827	0.00234	0.97644	0.00171	0.96337	0.00208
8	0.93799	0.00290	0.94005	0.00275	0.95309	0.00237	0.94221	0.00258
9	0.91240	0.00335	0.93192	0.00289	0.93695	0.00270	0.93045	0.00280
10	0.88604	0.00376	0.90970	0.00327	0.92593	0.00290	0.89178	0.00341
11	0.88492	0.00376	0.90502	0.00334	0.91031	0.00316	0.85592	0.00382
12	0.86517	0.00402	0.88982	0.00356	0.88962	0.00346	0.83903	0.00398
13	0.87040	0.00396	0.89653	0.00346	0.89109	0.00344	0.82021	0.00415
14	0.87301	0.00393	0.89160	0.00354	0.88400	0.00354	0.80678	0.00426
15	0.87716	0.00392	0.86905	0.00386	0.84977	0.00397	0.81222	0.00420
16	0.85196	0.00427	0.82340	0.00442	0.82123	0.00429	0.80938	0.00423
17	0.80685	0.00485	0.77626	0.00489	0.77474	0.00470	0.81350	0.00421
18	0.77557	0.00523	0.74304	0.00522	0.75999	0.00485	0.79446	0.00438
19	0.78655	0.00532	0.75867	0.00534	0.76895	0.00494	0.80155	0.00441
20	0.92086	0.00392	0.92079	0.00349	0.91160	0.00347	0.91514	0.00326

Table 8. Age Distribution Gender Overlaps.

Income Vingtile	2001		2006		2011		2016	
1	0.85730	0.00422	0.89344	0.00352	0.89850	0.00334	0.95211	0.00239
2	0.88481	0.00399	0.85007	0.00434	0.87710	0.00380	0.97075	0.00196
3	0.85053	0.00442	0.85594	0.00420	0.85290	0.00410	0.89843	0.00325
4	0.82931	0.00471	0.86911	0.00398	0.86882	0.00387	0.85720	0.00391
5	0.93798	0.00294	0.91493	0.00331	0.92086	0.00307	0.81915	0.00429
6	0.89031	0.00388	0.92536	0.00310	0.92854	0.00293	0.91214	0.00313
7	0.93774	0.00291	0.94200	0.00273	0.93987	0.00268	0.93654	0.00270
8	0.92271	0.00321	0.92875	0.00298	0.94230	0.00261	0.95042	0.00240
9	0.90574	0.00346	0.90377	0.00338	0.92545	0.00292	0.94568	0.00249
10	0.88605	0.00376	0.89837	0.00345	0.92164	0.00298	0.93759	0.00265
11	0.89616	0.00359	0.89672	0.00346	0.90544	0.00324	0.91771	0.00299
12	0.90057	0.00352	0.88217	0.00366	0.90776	0.00320	0.92934	0.00277
13	0.89902	0.00355	0.89515	0.00348	0.90688	0.00321	0.92138	0.00291
14	0.92710	0.00307	0.91034	0.00325	0.91159	0.00314	0.90717	0.00313
15	0.94634	0.00269	0.92459	0.00302	0.92786	0.00287	0.91845	0.00295
16	0.93984	0.00286	0.93045	0.00295	0.93738	0.00271	0.93603	0.00264
17	0.95361	0.00258	0.94403	0.00270	0.95222	0.00240	0.94604	0.00244
18	0.95749	0.00253	0.95397	0.00250	0.94837	0.00251	0.94654	0.00244
19	0.94850	0.00287	0.94668	0.00280	0.94079	0.00276	0.93554	0.00272
20	0.92988	0.00371	0.93672	0.00314	0.97142	0.00204	0.97469	0.00184

between female and male education distributions over the vingtiles, documents the diminishing similarity in educational status between the genders as incomes increase in each year whereas Table 8 documents the increasing similarity between the genders in age group status as incomes increase which is reflective of the diminishing life expectancy gap (Statistics Canada 2020) and Table 9 reports the diminishing similarity of joint education-age distribution reflecting the dominant contribution of education status to the joint analysis.

Table 9. Age - Education Joint Distribution Gender Overlaps.

Income Vingtile	2001		2006		2011		2016	
1	0.82643	0.00457	0.85451	0.00402	0.85692	0.00387	0.87563	0.00370
2	0.86094	0.00432	0.82986	0.00457	0.83590	0.00429	0.97075	0.00196
3	0.82642	0.00470	0.83379	0.00445	0.82029	0.00444	0.79512	0.00435
4	0.80063	0.00500	0.84710	0.00425	0.84494	0.00415	0.80583	0.00442
5	0.91272	0.00344	0.88097	0.00384	0.88519	0.00362	0.78590	0.00457
6	0.86205	0.00428	0.88190	0.00381	0.88249	0.00367	0.86552	0.00377
7	0.91071	0.00344	0.89591	0.00357	0.90144	0.00336	0.88284	0.00356
8	0.89020	0.00376	0.87637	0.00381	0.88746	0.00354	0.89789	0.00335
9	0.86128	0.00409	0.86059	0.00397	0.86945	0.00375	0.88257	0.00354
10	0.82957	0.00445	0.83943	0.00419	0.85825	0.00387	0.85356	0.00388
11	0.82787	0.00444	0.83619	0.00421	0.83280	0.00413	0.82571	0.00413
12	0.81310	0.00459	0.80480	0.00450	0.82585	0.00419	0.82265	0.00413
13	0.80334	0.00468	0.81197	0.00444	0.82321	0.00421	0.80111	0.00432
14	0.81030	0.00463	0.80492	0.00451	0.81569	0.00429	0.79308	0.00437
15	0.82232	0.00456	0.80443	0.00453	0.81181	0.00434	0.78955	0.00439
16	0.80332	0.00478	0.79439	0.00468	0.80540	0.00443	0.79264	0.00437
17	0.79808	0.00493	0.76610	0.00496	0.76410	0.00478	0.78592	0.00443
18	0.76218	0.00533	0.73047	0.00530	0.74821	0.00493	0.77449	0.00453
19	0.76662	0.00550	0.74790	0.00542	0.75724	0.00502	0.78688	0.00453
20	0.86617	0.00495	0.86886	0.00436	0.87356	0.00406	0.88112	0.00379

Table 10. Average Overlaps/transvariations of Education, Age, and Joint distributions across Income Vingtiles

	2001	2006	2011	2016
Education				
Overlap	0.89982	0.90127	0.90423	0.87938
Transvariation	0.10018	0.09873	0.09577	0.12062
Standard Error	0.00346	0.00322	0.00303	0.00331
Age				
Overlap	0.91005	0.91013	0.91928	0.92565
Transvariation	0.08995	0.08987	0.08072	0.07435
Standard Error	0.00342	0.00330	0.00302	0.00280
Joint Distribution				
Overlap	0.83271	0.82852	0.83501	0.83344
Transvariation	0.16729	0.17148	0.16499	0.16656
Standard Error	0.00452	0.00437	0.00415	0.00400

Table 10 reports average gender based similarities/differences over income vingtiles and records a significant increase in educational differences with a significant reduction in age differences with no significant change in the similarity or difference in the overall distribution.

Table 11. Joint Distribution First and Second Order Comparisons,

	First Order Comparison			Second Order Comparison	
	$CDCDF_{M,F}$	$CDCDFI_{M,F}$	$\sum_{i=1}^{6}\sum_{j=1}^{6}\lvert F_{i,j,F} - F_{i,j,M}\rvert$	$CDCCDF_{M,F}$	$CDCCDFI_{M,F}$
2001	0.60293	0.87193	0.69149	8.76497	1.00000
2006	0.60967	0.88730	0.68711	8.73739	1.00000
2011	0.70140	0.93815	0.74764	9.99909	1.00000
2016	0.84599	0.97938	0.86380	11.67396	1.00000

In contrast to their income distributions, the increasing cumulated absolute differences of Female and Male human resource CDFs over time reported in Table 11 are indicative of some distributional separation over the years. Indeed, Female distributions stochastically dominate Male distributions at the second order throughout the period so that, if human resources are a non-decreasing concave function of embodied human capital, females can be deemed to have superior human resources to males over the period, furthermore the gap is widening (in this situation the cumulated differences will be the same as the cumulated absolute differences, and they are growing over time).

However, equality of opportunity would dictate that, given similar proclivities for effort and application, males and females in a common income group should bring to bear similar human resource distributions in attaining that income level. To check this, the period on period changes in the human resource stock transvariations at each income vingtile are reported in Table 12 where negative values record divergence and positive values record convergence. Unlike the almost universally convergent income results of Table 3, the results here, either of convergence or divergence, are not universal. Over the 2001-2016 period there are 11 vingtiles recording divergence and 9 recording convergence, fairly evenly spread throughout the income spectrum. 2006-2011 was particularly divergent with only 4 convergent vingtiles.

Table 12. Period by Period Changes in Vingtile Human Resource Stock Transvariations

Vingtile	2001-2006		2006-2011		2011-2016		2001-2016	
1	-0.02808	-7.74560	-0.00241	-0.71088	-0.01871	-5.90266	-0.04920	-14.65438
2	0.03108	8.61250	-0.00604	-1.67998	-0.13485	-55.85166	-0.10981	-45.64021
3	-0.00737	-1.99193	0.01350	3.67876	0.02517	6.74361	0.03130	8.24851
4	-0.04647	-12.49639	0.00216	0.61907	0.03911	10.82285	-0.00520	-1.36154
5	0.03175	10.37191	-0.00422	-1.36307	0.09929	27.70419	0.12682	35.86798
6	-0.01985	-6.05777	-0.00059	-0.18955	0.01697	5.39212	-0.00347	-1.05309
7	0.01480	5.02078	-0.00553	-1.90458	0.01860	6.30586	0.02787	9.39252
8	0.01383	4.31511	-0.01109	-3.58104	-0.01043	-3.60805	-0.00769	-2.60822
9	0.00069	0.20198	-0.00886	-2.69639	-0.01312	-4.26466	-0.02129	-6.68666
10	-0.00986	-2.68756	-0.01882	-5.48497	0.00469	1.41726	-0.02399	-6.88727
11	-0.00832	-2.26030	0.00339	0.94326	0.00709	1.99882	0.00216	0.59303
12	0.00830	2.12910	-0.02105	-5.65730	0.00320	0.89314	-0.00955	-2.57925
13	-0.00863	-2.22519	-0.01124	-3.02492	0.02210	5.97795	0.00223	0.58172
14	0.00538	1.37939	-0.01077	-2.85399	0.02261	6.02338	0.01722	4.47244
15	0.01789	4.63241	-0.00738	-1.94490	0.02226	5.88990	0.03277	8.55288
16	0.00893	2.24380	-0.01101	-2.85863	0.01276	3.37427	0.01068	2.75872
17	0.03198	7.78136	0.00200	0.48823	-0.02182	-5.58889	0.01216	3.10656
18	0.03171	7.32847	-0.01774	-4.19909	-0.02628	-6.59846	-0.01231	-3.04076
19	0.01872	4.38810	-0.00934	-2.24549	-0.02964	-7.57268	-0.02026	-5.08016
20	-0.00269	-0.80795	-0.00470	-1.45629	-0.00756	-2.46112	-0.01495	-4.70914

Stoline and Ury 5% Critical Value for 36 simultaneous comparisons is 3.031

Tables A2 through A5 report the comparisons of male – female income distributions across human resource stocks and human resource distributions across the vingtiles using the Utopia-dystopia index rankings as well as the male female resource distribution stochastic dominance comparisons within an income vingtile. The results are striking, with regard to human resources, Female distributions outrank Male distributions at almost every vingtile in every year and correspondingly strictly second order dominate the corresponding male distribution. Notable exceptions are vingtile 10 in 2001 where there is no second order dominance, vingtile 20 in 2006 where there is only "Almost" second order dominance (Leshno and Levy 2002) and vingtiles 1 and 2 in 2016 where dominance of female distributions over male distributions is not established and rankings are reversed and dominance of males over females established in vingtile 2. The same is patently not true of incomes where generally male distributions outrank female distributions. The results are synthesized in Table 12 which records the average female-male rank differences and the corresponding average absolute rank differences where ranks are based upon Utopia – Dystopia indices of the first and second order. When negative (positive), the average difference implies male distributions outrank (are outranked by) female distributions on average, when its absolute value is equal to the mean absolute value the outranking is universal.

Table 13. Average Absolute Rank Differences.

	Income Distributions First Order Mean value	Income Distributions First Order Mean Abs Val	Income Distributions Second Order Mean Abs Val	Human Resource Distributions First Order Mean value	Human Resource Distributions First Order Mean Abs Val	Human Resource Distributions Second Order Mean Abs Val
2001	-16.38889	17.11111	17.55556	7.50000	7.50000	8.70000
Standard Error	0.31698	0.26573	0.27395	0.11192	0.11192	0.10533
2006	-15.44444	15.44444	16.00000	7.90000	7.90000	8.60000
Standard Error	0.24049	0.24049	0.25817	0.12108	0.12108	0.10510
2011	-12.44444	13.55556	13.88889	7.20000	7.20000	7.90000
Standard Error	0.27879	0.20510	0.21701	0.09646	0.09646	0.08498
2016	-7.00000	10.16667	9.88889	7.90000	8.20000	8.60000
Standard Error	0.32832	0.20174	0.18138	0.13401	0.12070	0.10741

So while generally male income distributions outrank female income distributions, though not universally, and differences in rankings are diminishing over time, female human resource distributions almost universally outrank male human resource distributions and there is little evidence of diminishing differences over time.

Conclusions.

The pursuit of Gender Equity in the workplace has seen slow and steady, but possibly questionable, progress (Goldin 2014). The increasing similarity of female and male labour market experiences (dubbed the Grand Gender Convergence) is usually established by observing increasingly similar gender conditional averages of hours of work, occupational choice and incomes. However, when viewed through an Equal Opportunity lens, assuming the distributions of the propensity to work and acquire human resources are similar and effectively exchangeable across the gender divide, these increasingly similarities need to be viewed in the context of increasingly similar individual human resource stocks in order to establish A Grand Gender Convergence. In such a world, in an equilibrium unconstrained by the circumstance of gender, males and females with similar human resource stocks should have similar income distributions and males and females at similar income levels should have similar distributions of human resource stocks. If the equilibrium is stable, when disrupted its return should see the convergence of the income distributions of males and females in common human resource groups matched by the convergence in the human resource stock distributions of males and females in groups with similar incomes. Furthermore, since even extensive comparison of location measures can conceal important distributional differences, the convergence needs to be examined in the context increasingly similar distributions rather than increasingly similar distributional locations.

To examine this issue in a Canadian context, the progress of female and male income distributions in 36 human resource stock classes and their corresponding human resource stock distributions in twenty income vingtiles over the years 2001, 2006, 2011 and 2016 were examined. It appears that, while male income distributions are generally superior to female distributions, the gaps are narrowing, on the other hand, while female human resource distributions are generally superior to the corresponding male distributions, the gaps are, if anything, widening. With respect to the latter, while age group patterns, the proxy for experience, are becoming increasingly similar across the genders (largely due to the diminishing life expectancy gap), male-female gaps in education and training, the proxy for human capital, appear to be widening, resulting in their joint age-education densities moving apart. Unless male and female proclivities for effort differ, this juxtaposition can only be reconciled by males being accorded a disproportionate return for their efforts or by females utilizing greater levels of human resources to achieve the same income level with the same effort. In either event, it is not a characteristic of a Grand Gender Convergence.

References.

Aaberge, Rolf, Magne Mogstad, and Vito Peragine (2011): "Measuring Long-term Inequality of Opportunity", Journal of Public Economics, 95 (3-4), 193-204

Anderson G., Linton, O., Whang, Y.-J. (2012). "Nonparametric Estimation and Inference about the Overlap of Two Distributions", Journal of Econometrics, 171(1), 1–23.

Anderson G., Linton, O., Pittau, M.G., Whang Y-J., R. Zelli (2021) "On Unit Free Assessment of Multilateral Distributional Variation" forthcoming The Econometrics Journal.

Anderson, G., Post, T., and Y-J Whang (2020) "Somewhere Between Utopia and Dystopia: Choosing From Multiple Incomparable Prospects" Journal of Business and Economic Statistics 38, 502-515

Anderson G. and T. W. Leo (2021) "In search of Complete and Unambiguous Orderings of Sets of Alternatives" mimeo University of Toronto.

Arrow K., Bowles S. and S. Durlauf (2000) eds. Meritocracy and Economic Inequality Princeton University Press. Princeton New Jersey.

Arrow K.J. and F.H. Hahn (1971) General Competitive Analysis. Oliver and Boyd Edinburgh.

Atkinson, A.B. (2012). "Public economics after the idea of justice". Journal of Human Development and Capabilities, 13(4), 521–536.

Autor, D. H. (2014) “Skills, Education, and the Rise of Earnings Inequality Among the ‘Other 99 Percent.’” Science 344, no. 6186: 843–851.

Barro RJ. 1998. Determinants of Economic Growth: A Cross Country Empirical Study. MIT Press: Cambridge, MA.

Barro, Robert J., and Xavier X. Sala-i-Martin. (1992) “Convergence.” Journal of Political Economy, 100, 223–51.

Bond T.N. and K. Lang (2019) "The Sad Truth about Happiness Scales," Journal of Political Economy 127, no. 4: 1629-1640.

Boudarbat, B. and M. Connolly (2013) “The gender wage gap among recent post-secondary graduates in Canada: a distributional approach” Canadian Journal of Economics 46, No. 3 1037-1065

Cantril, H. (1965). The pattern of human concerns. New Brunswick, NJ: Rutgers University Press.

Carneiro, P., Hansen, K. T., & Heckman, J. J. (2003). “Lawrence R. Klein Lecture Estimating Distributions of Treatment Effects with an Application to the Returns to Schooling and Measurement of the Effects of Uncertainty on College Choice”. International Economic Review, 44, 361-422.

Checchi, Daniele, and Vito Peragine (2010): “Inequality of Opportunity in Italy”, Journal of Economic Inequality, 8 (4), 429-450.

Raj Chetty, Nathaniel Hendren, Maggie R Jones, Sonya R Porter 2020 “Race and Economic Opportunity in the United States: an Intergenerational Perspective” The Quarterly Journal of Economics, Volume 135, pp 711–783,

Durlauf, S.N., Quah, D. (2002). The New Empirics of Economic Growth, Chapter 4 in Handbook of Macroeconomics, J.B. Taylor and M. Woodford (eds), North Holland.

Ferreira, Francisco H. G., and Jérémie Gignoux (2011): “The Measurement of Inequality of Opportunity: Theory and an Application to Latin America”, Review of Income and Wealth, 57 (4): 622-657.

Ferreira F.H.G and V. Peragine (2015) “Individual responsibility and Equality of Opportunity” Chapter 25 in the Oxford Handbook of Wellbeing and Public Policy edited by M.D. Adler and M. Fluerbaey

MARC FLEURBAEY and VITO PERAGINE (2013) Ex Ante Versus Ex Post Equality of Opportunity Economica (2013) 80, 118–130

Fortin N.M. (2019) “Increasing earnings inequality and the gender pay gap in Canada: Prospects for convergence” Canadian Journal of Economics 52, 407-440

Galor, O. (1996), "Convergence? Inference from Theoretical Models", Economic Journal, 106, 1056-1069.

Galor O. (2011) Unified Growth Theory Princeton University Press.

Goldin, C. (2014) “A Grand Gender Convergence: Its Last Chapter” American Economic Review 2014, 104(4): 1–30

Gravel, N., Magdalou, B., and P. Moyes (2020) “Ranking distributions of an ordinal variable” Economic Theory https://doi.org/10.1007/s00199-019-01241-4

Jehn, A., Walters, D. & Howells, S. (2019) “Employment and Wage Gaps Among Recent Canadian Male and Female Postsecondary Graduates.” Higher Education Policy. https://doi.org/10.1057/s41307-019-00162-0

Lefranc, A., Pistolesi, N., and Trannoy, A. (2008). Inequality of Opportunities vs. Inequality of Outcomes Review of Income and Wealth 54, 513-546

Lefranc, A., Pistolesi, N., and Trannoy, A. (2009). “Equality of Opportunity and Luck: Definitions and Testable Conditions, with an Application to Income in France". Journal of Public Economics, 93, 1189-1207.

Leshno, M. and Levy, H. (2002). Preferred by “ll" and Preferred by “Most" Decision Makers: Almost Stochastic Dominance. Management Science, 48(8), 1074-1085.

Katie Meara, Francesco Pastore & Allan Webster (2020) ”The gender pay gap in the USA: a matching study” Journal of Population Economics 33, pp 271–305

Morgan S.J., Grusky S.B. and G.S. Fields (2006) Mobility and Inequality: Frontiers of Research in Sociology and Economics.

Oaxaca, R. L., and M. R. Ransom. 1994. On discrimination and the decomposition of wage differentials. Journal of Econometrics 61: 5–21.

Oaxaca, R. L., and M. R. Ransom 1999. Identification in detailed wage decompositions. Review of Economics and Statistics 81: 154–157.

O'Neill, June. 2003. "The Gender Gap in Wages, circa 2000 ." American Economic Review, 93 (2): 309-314.

Sala-i-Martin, Xavier X. (1996) “Regional Cohesion: Evidence and Theories of Regional Growth and Convergence.” European Economic Review, 40, 1325–52.

Schirle T. (2015) “The Gender Wage Gap in the Canadian Provinces, 1997–2014” Canadian Public Policy 41 pp. 309-319

Schroder, C. and Yitzhaki, S. (2017). Revisiting the evidence for cardinal treatment of ordinal variables. European Economic Review, 92:337-358.

Sen, A.K. (2009). The Idea of Justice. Harvard University Press.

Statistics Canada. (2020) Table 13-10-0114-01 Life expectancy and other elements of the life table, Canada, https://doi.org/10.25318/1310011401-eng

Stoline M.R. and H.K. Ury (1979). “Tables of the Studentized Maximum Modulus Distribution and an Application to Multiple Comparisons among Means", Technometrics, 21, 87-93.

Appendix. Table A1. Female & Male Income and Human Resource Cumulative Densities.

Vingtile	2001 Female	2001 Male	2006 Female	2006 Male	2011 Female	2011 Male	2016 Female	2016 Male
1	0.05978	0.03986	0.05262	0.04724	0.04648	0.05369	0.03517	0.06554
2	0.12535	0.07370	0.11879	0.08024	0.10988	0.08964	0.06561	0.13603
3	0.18991	0.10860	0.18279	0.11551	0.17331	0.12556	0.11280	0.18898
4	0.25578	0.14213	0.24472	0.15297	0.23536	0.16293	0.17490	0.22630
5	0.31782	0.17964	0.30756	0.18946	0.29573	0.20206	0.23644	0.26420
6	0.38243	0.21448	0.36968	0.22673	0.35649	0.24078	0.29618	0.30400
7	0.44223	0.25432	0.42995	0.26592	0.41503	0.28183	0.35651	0.34318
8	0.50169	0.29451	0.48788	0.30758	0.47214	0.32437	0.41678	0.38241
9	0.55645	0.33957	0.54340	0.35177	0.52682	0.36947	0.47584	0.42293
10	0.61103	0.38481	0.59650	0.39851	0.57959	0.41656	0.53414	0.46422
11	0.66168	0.43414	0.64773	0.44722	0.63129	0.46477	0.59000	0.50808
12	0.71197	0.48384	0.69793	0.49701	0.68153	0.51453	0.64412	0.55377
13	0.75909	0.53683	0.74647	0.54854	0.73057	0.56553	0.69759	0.60013
14	0.80417	0.59193	0.79257	0.60264	0.77775	0.61849	0.74967	0.64795
15	0.84500	0.65145	0.83604	0.65952	0.82170	0.67484	0.79896	0.69870
16	0.88419	0.71266	0.87526	0.72085	0.86251	0.73446	0.84545	0.75237
17	0.91916	0.77826	0.91195	0.78485	0.90204	0.79545	0.88973	0.80836
18	0.95142	0.84665	0.94573	0.85191	0.93953	0.85856	0.93284	0.86559
19	0.97970	0.91919	0.97436	0.92438	0.97212	0.92681	0.97040	0.92863
20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1st Order Dominance	2.97228	1.00000	2.58909	1.00000	2.10953	0.99321	0.66177	0.55628
2nd Order Dominance	30.53074	1.00000	26.20981	1.00000	20.91752	0.99931	2.08763	0.36814

Age Group	Edication Group	2001 Female	2001 Male	2006 Female	2006 Male	2011 Female	2011 Male	2016 Female	2016 Male
1	1	0.02524	0.03628	0.01729	0.02543	0.01425	0.02198	0.01196	0.01880
1	2	0.06049	0.08109	0.03434	0.04831	0.02692	0.03864	0.02356	0.03615
1	3	0.10682	0.13709	0.06208	0.08474	0.04536	0.06343	0.03786	0.05520
1	4	0.15277	0.18762	0.09299	0.11955	0.07013	0.09498	0.06166	0.08711
1	5	0.20934	0.23855	0.13130	0.15391	0.10105	0.12311	0.09043	0.11565
1	6	0.29584	0.29813	0.19766	0.19867	0.15717	0.16095	0.14491	0.15525
2	1	0.07751	0.09785	0.06970	0.08910	0.06327	0.08539	0.06320	0.08613
2	2	0.16001	0.18892	0.12311	0.15319	0.10637	0.13894	0.10586	0.14397
2	3	0.26541	0.29297	0.20956	0.24105	0.16846	0.20629	0.15477	0.20326
2	4	0.34666	0.37512	0.29166	0.31957	0.24765	0.28429	0.23380	0.28400
2	5	0.42333	0.44125	0.36008	0.37641	0.31743	0.34214	0.31230	0.35135
2	6	0.53116	0.51248	0.45979	0.43926	0.40667	0.39927	0.40602	0.41597
3	1	0.13063	0.14747	0.12068	0.14149	0.11175	0.13933	0.10912	0.13664
3	2	0.28524	0.31010	0.23603	0.26763	0.21192	0.25321	0.20673	0.25459
3	3	0.45904	0.48836	0.39660	0.43578	0.34259	0.39150	0.31389	0.37248
3	4	0.58237	0.62014	0.53127	0.57724	0.48189	0.53934	0.45874	0.52368
3	5	0.67923	0.71296	0.62806	0.67069	0.58906	0.64108	0.58274	0.64203
3	6	0.80415	0.80155	0.75019	0.76126	0.70297	0.72724	0.70570	0.74102
4	1	0.16478	0.17239	0.15995	0.17018	0.15704	0.17223	0.15333	0.16923
4	2	0.35985	0.37010	0.32289	0.33483	0.30940	0.32507	0.30333	0.32458
4	3	0.56740	0.58000	0.52557	0.54071	0.48838	0.50420	0.45604	0.47839
4	4	0.71252	0.73507	0.69149	0.71381	0.66302	0.68603	0.63650	0.66171
4	5	0.81799	0.83727	0.80416	0.82339	0.79365	0.81147	0.78529	0.80492
4	6	0.94967	0.93259	0.93702	0.92443	0.91963	0.90993	0.92257	0.91855
5	1	0.17168	0.17748	0.16840	0.17572	0.16821	0.18012	0.16253	0.17573
5	2	0.38020	0.38901	0.34751	0.35531	0.34092	0.34955	0.33300	0.34642
5	3	0.60083	0.61407	0.56483	0.57762	0.53748	0.54704	0.50230	0.51562
5	4	0.75497	0.78294	0.74295	0.76544	0.72606	0.74501	0.69515	0.71261
5	5	0.86329	0.89160	0.86067	0.88422	0.86525	0.88308	0.85314	0.86743
5	6	0.99685	0.99092	0.99611	0.99052	0.99478	0.98875	0.99471	0.98983
6	1	0.17178	0.17766	0.16861	0.17596	0.16845	0.18039	0.16266	0.17590
6	2	0.38112	0.39098	0.34862	0.35713	0.34260	0.35158	0.33457	0.34838
6	3	0.60276	0.61831	0.56713	0.58168	0.54046	0.55170	0.50522	0.51960
6	4	0.75765	0.78958	0.74607	0.77168	0.73017	0.75207	0.69912	0.71889
6	5	0.86628	0.89988	0.86423	0.89236	0.87010	0.89258	0.85792	0.87557
6	6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1st Order Dominance		-0.60293	0.87193	-0.60967	0.88730	-0.70140	0.93815	-0.84599	0.97938
2nd Order Dominance		-8.76497	1.00000	-8.73739	1.00000	-9.99909	1.00000	-11.67396	1.00000

Table A2. Distribution Rankings and 1st and 2nd Order Female – Male dominance indices across Human Resource Groups 2001.

edu	age	Female 1st Order Rank	Male 1st Order Rank	Female 2nd Order Rank	Male 2nd Order Rank	$CDCDFI_{M,F}$	$CDCCDFI_{M,F}$
1	1	1	11	1	9	-1.00000	-1.00000
1	2	6	29	5	29	-1.00000	-1.00000
1	3	10	33	8	32	-1.00000	-1.00000
1	4	4	31	4	30	-1.00000	-1.00000
1	5	3	19	3	23	-1.00000	-1.00000
1	6	5	16	10	22	-1.00000	-1.00000
2	1	2	8	2	6	-1.00000	-1.00000
2	2	14	40	14	40	-1.00000	-1.00000
2	3	20	47	19	48	-1.00000	-1.00000
2	4	17	46	16	46	-1.00000	-1.00000
2	5	7	37	7	34	-1.00000	-1.00000
2	6	13	34	15	36	-0.99917	-0.99992
3	1	9	21	11	20	-1.00000	-1.00000
3	2	24	48	26	50	-1.00000	-1.00000
3	3	30	53	31	56	-1.00000	-1.00000
3	4	26	52	27	51	-1.00000	-1.00000
3	5	12	38	12	39	-1.00000	-1.00000
3	6	18	32	21	38	-1.00000	-1.00000
4	1	15	22	13	18	-0.98083	-0.99773
4	2	39	58	37	60	-1.00000	-1.00000
4	3	44	64	43	63	-1.00000	-1.00000
4	4	43	62	42	61	-1.00000	-1.00000
4	5	27	50	28	49	-1.00000	-1.00000
4	6	35	51	35	52	-0.99730	-0.99945
5	1	25	28	24	25	-0.71450	-0.46113
5	2	42	60	41	59	-1.00000	-1.00000
5	3	54	67	53	68	-1.00000	-1.00000
5	4	57	69	58	69	-1.00000	-1.00000
5	5	45	63	45	64	-1.00000	-1.00000
5	6	41	61	44	65	-1.00000	-1.00000
6	1	36	23	33	17	0.76786	1.00000
6	2	49	55	47	55	-0.99024	-0.99884
6	3	65	68	62	67	-0.99917	-0.99988
6	4	66	72	66	71	-1.00000	-1.00000
6	5	59	71	57	72	-1.00000	-1.00000
6	6	56	70	54	70	-1.00000	-1.00000

Distribution Rankings and 1st and 2nd Order Female – Male dominance indices across income vingtiles 2001.

Vingtile	Female 1st Order Rank	Male 1st Order Rank	Female 2nd Order Rank	Male 2nd Order Rank	$CDCDFI_{M,F}$	$CDCCDFI_{M,F}$
1	6	1	6	1	0.98373	1.00000
2	4	2	5	2	0.97445	1.00000
3	7	3	7	3	0.93068	1.00000
4	21	5	16	4	0.93020	1.00000
5	26	20	26	13	0.91132	1.00000
6	36	30	34	25	0.90560	1.00000
7	33	28	30	23	0.90890	1.00000
8	31	27	29	21	0.44035	1.00000
9	16	11	17	9	0.47777	1.00000
10	19	15	19	12	0.10324	0.97256
11	14	9	18	8	0.42959	1.00000
12	22	12	24	11	0.44227	1.00000
13	18	8	22	10	0.62491	1.00000
14	24	10	28	14	0.67725	1.00000
15	25	13	31	15	0.77263	1.00000
16	32	17	33	20	0.76092	1.00000
17	35	23	36	27	0.91142	1.00000
18	37	29	37	32	0.95548	1.00000
19	39	34	39	35	0.98537	1.00000
20	40	38	40	38	0.70949	1.00000

Table A3. Distribution Rankings and 1st and 2nd Order Female – Male dominance indices across Human Resource Groups 2006.

edu	age	Female 1st Order Rank	Male 1st Order Rank	Female 2nd Order Rank	Male 2nd Order Rank	$CDCDFI_{M,F}$	$CDCCDFI_{M,F}$
1	1	1	7	1	6	-0.97156	-0.99699
1	2	5	26	5	25	-0.98593	-0.99870
1	3	8	31	7	31	-1.00000	-1.00000
1	4	4	30	4	29	-1.00000	-1.00000
1	5	3	25	3	26	-1.00000	-1.00000
1	6	6	19	9	24	-1.00000	-1.00000
2	1	2	9	2	8	-0.99453	-0.99954
2	2	16	39	16	39	-0.99970	-0.99997
2	3	27	44	27	46	-1.00000	-1.00000
2	4	22	42	20	44	-1.00000	-1.00000
2	5	11	37	11	36	-1.00000	-1.00000
2	6	15	33	18	35	-0.99767	-0.99979
3	1	10	23	12	22	-0.99700	-0.99972
3	2	28	50	28	52	-1.00000	-1.00000
3	3	34	54	33	58	-1.00000	-1.00000
3	4	29	52	30	50	-1.00000	-1.00000
3	5	13	40	15	40	-1.00000	-1.00000
3	6	21	35	23	38	-0.99871	-1.00000
4	1	14	18	14	17	-0.91401	-0.96481
4	2	38	55	37	57	-1.00000	-1.00000
4	3	46	63	45	62	-1.00000	-1.00000
4	4	45	62	43	61	-1.00000	-1.00000
4	5	32	51	32	49	-1.00000	-1.00000
4	6	36	48	34	51	-0.99550	-0.99913
5	1	20	24	19	21	-0.98723	-0.99863
5	2	43	57	42	56	-0.98915	-0.99852
5	3	56	67	55	67	-1.00000	-1.00000
5	4	60	68	60	68	-0.99631	-0.99947
5	5	47	64	47	66	-0.99879	-0.99986
5	6	41	61	41	64	-1.00000	-1.00000
6	1	12	17	10	13	-0.87156	-0.99159
6	2	53	59	53	59	-0.91004	-0.93752
6	3	65	70	65	69	-0.99423	-0.99915
6	4	66	72	63	72	-1.00000	-1.00000
6	5	58	71	54	71	-1.00000	-1.00000
6	6	49	69	48	70	-0.99284	-0.99841

Distribution Rankings and 1st and 2nd Order Female – Male dominance indices across income vingtiles 2006.

Vingtile	Female 1st Order Rank	Male 1st Order Rank	Female 2nd Order Rank	Male 2nd Order Rank	$CDCDFI_{M,F}$	$CDCCDFI_{M,F}$
1	6	3	6	3	0.97776	1.00000
2	5	1	5	1	0.96450	1.00000
3	7	2	7	2	0.93035	1.00000
4	11	4	11	4	0.94831	1.00000
5	29	8	24	8	0.89362	1.00000
6	33	25	30	17	0.81402	1.00000
7	31	27	27	20	0.87188	1.00000
8	26	17	23	13	0.82428	1.00000
9	18	14	18	10	0.26014	1.00000
10	16	9	19	9	0.37840	1.00000
11	19	10	22	12	0.41780	1.00000
12	22	13	26	14	0.34450	1.00000
13	23	12	28	15	0.48210	1.00000
14	28	15	31	16	0.63901	1.00000
15	30	20	33	21	0.73126	1.00000
16	34	21	34	25	0.79183	1.00000
17	36	24	36	29	0.85790	1.00000
18	37	32	37	32	0.92595	1.00000
19	38	35	39	35	0.94340	1.00000
20	40	39	40	38	0.89885	0.99840

Table A4. Distribution Rankings and 1st and 2nd Order Female – Male dominance indices across Human Resource Groups 2011.

edu	age	Female 1st Order Rank	Male 1st Order Rank	Female 2nd Order Rank	Male 2nd Order Rank	$CDCDFI_{M,F}$	$CDCCDFI_{M,F}$
1	1	1	4	1	4	-0.90997	-0.97932
1	2	7	22	7	19	-0.95925	-0.99186
1	3	9	27	8	27	-0.99345	-0.99939
1	4	8	29	6	29	-1.00000	-1.00000
1	5	3	24	3	25	-1.00000	-1.00000
1	6	5	16	9	23	-1.00000	-1.00000
2	1	2	6	2	5	-0.96952	-0.99675
2	2	15	37	17	36	-0.98279	-0.99831
2	3	26	42	28	42	-0.99296	-0.99927
2	4	25	43	26	43	-1.00000	-1.00000
2	5	11	36	12	37	-1.00000	-1.00000
2	6	13	31	15	31	-0.99742	-0.99978
3	1	10	23	11	22	-0.98865	-0.99889
3	2	30	47	30	51	-0.99153	-0.99909
3	3	35	53	33	54	-0.99329	-0.99923
3	4	33	51	32	50	-1.00000	-1.00000
3	5	20	39	20	39	-1.00000	-1.00000
3	6	18	32	21	35	-1.00000	-1.00000
4	1	14	19	13	16	-0.84235	-0.91485
4	2	41	57	40	58	-0.99048	-0.99890
4	3	46	63	46	63	-0.99681	-0.99962
4	4	49	60	47	59	-1.00000	-1.00000
4	5	38	52	38	52	-0.99887	-0.99987
4	6	34	45	34	48	-0.98812	-0.99755
5	1	**21**	**17**	**18**	**14**	**0.68895**	**1.00000**
5	2	44	58	44	57	-0.98782	-0.99848
5	3	56	66	55	65	-1.00000	-1.00000
5	4	61	67	61	67	-0.98291	-0.99533
5	5	50	65	49	66	-0.99840	-0.99981
5	6	40	59	41	64	-1.00000	-1.00000
6	1	**28**	**12**	**24**	**10**	**0.99059**	**1.00000**
6	2	48	55	45	53	-0.98431	-0.99707
6	3	64	72	62	72	-1.00000	-1.00000
6	4	69	70	68	69	-0.97860	-0.99554
6	5	62	71	60	71	-1.00000	-1.00000
6	6	54	68	56	70	-0.99710	-0.99989

Distribution Rankings and 1st and 2nd Order Female – Male dominance indices across income vingtiles 2011.

Vingtile	Female 1st Order Rank	Male 1st Order Rank	Female 2nd Order Rank	Male 2nd Order Rank	$CDCDFI_{M,F}$	$CDCCDFI_{M,F}$
1	5	1	5	1	0.99777	1.00000
2	6	2	6	2	0.98057	1.00000
3	7	3	7	3	0.98037	1.00000
4	13	4	11	4	0.96855	1.00000
5	27	8	21	8	0.93127	1.00000
6	34	26	29	16	0.88564	1.00000
7	31	23	27	15	0.93523	1.00000
8	22	16	20	12	0.74916	1.00000
9	17	9	18	9	0.57963	1.00000
10	15	10	19	10	0.37653	1.00000
11	18	11	23	13	0.37769	1.00000
12	21	12	25	14	0.53321	1.00000
13	24	14	28	17	0.46239	1.00000
14	28	19	31	22	0.54984	1.00000
15	30	20	33	24	0.66628	1.00000
16	33	25	34	26	0.73158	1.00000
17	36	29	36	30	0.78931	1.00000
18	37	32	37	32	0.87229	1.00000
19	38	35	39	35	0.89250	1.00000
20	40	39	40	38	0.92700	1.00000

Table A5. Distribution Rankings and 1st and 2nd Order Female – Male dominance indices across Human Resource Groups 2016.

edu	age	Female 1st Order Rank	Male 1st Order Rank	Female 2nd Order Rank	Male 2nd Order Rank	$CDCDFI_{M,F}$	$CDCCDFI_{M,F}$
1	1	3	8	4	10	-0.86277	-0.94249
1	2	15	28	18	26	-0.95121	-0.98290
1	3	14	34	17	34	-0.99800	-0.99965
1	4	11	35	14	36	-1.00000	-1.00000
1	5	6	29	8	31	-1.00000	-1.00000
1	6	9	23	16	27	-1.00000	-1.00000
2	1	4	12	6	15	-0.94492	-0.98795
2	2	26	42	28	42	-0.98529	-0.99617
2	3	30	45	33	44	-1.00000	-1.00000
2	4	31	43	32	43	-1.00000	-1.00000
2	5	20	39	21	39	-1.00000	-1.00000
2	6	19	36	24	37	-0.99776	-0.99974
3	1	**7**	**2**	**7**	**2**	**0.59414**	**1.00000**
3	2	**21**	**17**	**19**	**9**	**0.34108**	**1.00000**
3	3	**27**	**18**	**22**	**12**	**0.41449**	**1.00000**
3	4	**24**	**13**	**20**	**5**	**0.63970**	**1.00000**
3	5	**16**	**5**	**13**	**3**	**0.76530**	**1.00000**
3	6	**10**	**1**	**11**	**1**	**0.95979**	**1.00000**
4	1	22	25	23	25	-0.63811	-0.55044
4	2	44	55	45	56	-0.98476	-0.99553
4	3	51	63	50	64	-0.99903	-0.99981
4	4	49	60	48	58	-1.00000	-1.00000
4	5	40	50	40	51	-1.00000	-1.00000
4	6	38	46	41	47	-0.97320	-0.99354
5	1	32	33	29	30	-0.69099	-0.69816
5	2	48	56	46	53	-0.99268	-0.99825
5	3	58	67	55	67	-0.99997	-0.99999
5	4	62	65	62	65	-0.99716	-0.99924
5	5	52	59	52	59	-0.99468	-0.99991
5	6	47	57	49	60	-0.99365	-0.99840
6	1	**41**	**37**	**38**	**35**	**0.63664**	**0.97263**
6	2	54	61	54	61	-0.96023	-0.99415
6	3	66	71	66	70	-1.00000	-1.00000
6	4	70	72	68	71	-0.98882	-1.00000
6	5	**68**	**64**	**69**	**63**	**0.69128**	**1.00000**
6	6	53	69	57	72	-0.98728	-0.99599

Distribution Rankings and 1st and 2nd Order Female – Male dominance indices across income vingtiles 2016.

Vingtile	Female 1st Order Rank	Male 1st Order Rank	Female 2nd Order Rank	Male 2nd Order Rank	$CDCDFI_{M,F}$	$CDCCDFI_{M,F}$
1	19	18	23	21	0.02502	0.60822
2	**28**	**31**	**30**	**32**	**-1.00000**	**-1.00000**
3	6	3	9	4	0.66463	1.00000
4	4	1	5	1	0.99588	1.00000
5	11	2	10	2	0.98453	1.00000
6	15	5	15	3	0.97388	1.00000
7	33	14	26	12	0.96454	1.00000
8	30	22	25	13	0.96798	1.00000
9	23	9	18	7	0.96936	1.00000
10	17	7	17	6	0.88584	1.00000
11	16	8	19	8	0.63603	1.00000
12	21	10	22	11	0.66008	1.00000
13	25	12	27	14	0.55292	1.00000
14	26	13	29	16	0.53785	1.00000
15	29	20	33	20	0.63511	1.00000
16	34	24	34	24	0.77048	1.00000
17	36	27	36	28	0.78808	1.00000
18	37	32	37	31	0.82701	1.00000
19	39	35	39	35	0.90362	1.00000
20	40	38	40	38	0.99989	1.00000