University of Toronto
Department of Economics

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Robust Ordering of Canadian Provincial Human Resource Stocks: Measurement in the Absence of Cardinal Measure.

By Gordon John Anderson

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Summary.

The stock of human resources available to a society is integral to its economic growth and development. Unfortunately, as a composite of levels of embodied human capital and accumulated experience, its measurement and comparison across societies is hampered by its inherently latent nature. Usually, for the purpose of analysis, some form of Cantril scale is arbitrarily attached to the ordered categorical variable proxies of educational status and age group, but this is problematic since results can be ambiguous when using scale dependent summary statistics for comparison purposes. Here, new scale independent techniques for making inferences about the respective levels of, and differences between, human resource stocks across groups are proposed that are not subject to such concerns. Their effectiveness is exemplified in an application comparing Canadian Provincial Human Resource Stocks in the 21st century.

Introduction.

In many empirical literatures, ordered categorical variables are used as proxies for latent unobserved theoretical constructs, classic examples are Human Developmental Status and the Ability drivers of an individuals' position in income and wage distributions. Combinations of numerically scaled ordered categories of cognition, language and motor skills and adaptive behavior are the basis for measuring early childhood development. Ability, based upon a three dimensional Skill Set, is an amalgam of an individuals embodied human capital (EHC), accumulated experience (AE) and gender, and all have been used to explore and reflect levels and differences in human resources. With respect to ability, while gender is observed, EHC and AE are intrinsically latent variables requiring proxies. For example, Age Cohorts are used as proxies for experience levels in Blundell and Preston (1998), Educational Attainment categories are used as a proxy for embodied human capital in "Mincer Equations" (see Heckman et. al. 2006, Autor 2014 and Acemoglu and Autor 2012) and gender is considered directly in Goldin (2014) and Anderson, Leo and Muelhaupt (2014). Frequently, for analysis purposes, scales are devised for and attached to assessment categories (Bayley 1969, Cantril 1965, de Jong Gierveld and Kamphuis 1985, Russell 1996 are early examples) and summary statistics employed to capture differences between various groups, therein lays a problem. Most summary statistics are scale dependent and generally, beyond respecting the ordinal nature of the categories, the applied scales are arbitrary which can lead to ambiguously incoherent results based on different but none-the-less equally valid scaling choices.

Kahneman and Krueger (2006), in noting the increasing use of self reported wellness data in evaluating subjective wellbeing, reference a large literature that employs summary statistics applied to some variant of a Cantril scale attributed by an investigator to ordered self assessed categories of wellness (see for example Lachowska 2017). In stressing the importance of recognizing that self assessment represents an individuals' perception of experience rather than their actual utility level as conventionally conceived, they raise concerns regarding the results dependency upon an arbitrarily assigned scale[1]. The scale dependency inherent in such approaches presents challenges when making multilateral group comparisons since, as noted in Schroder and Yitzhaki (2017) and Bond and Lang (2019), results are only robust to the use of alternative, equally valid, scale choices when certain data conditions prevail. Unfortunately, those conditions, essentially stochastic dominance relationships, seldom prevail in practice.

Much like self reported happiness measures and the wellbeing levels they represent, age group and education categories are but ordinal measures of the corresponding, cardinally conceived, latent experience and embodied human capital levels they are being employed as proxies for. It makes no sense to think that a 50-year-old individual actually has twice the cumulated experience they had as a 25-year-old or that the 2nd level of education corresponds to half the embodied human capital of the 4th level of education. While cumulated human experience can be presumed to increase with age and embodied human capital increase with education level, the most that can be averred is that experience or embodied human capital increase with their respective category ordering in some monotonic non-decreasing fashion. Indeed, any use of an

[1] For example, there is no reason to believe the 2nd ordered category corresponds to half of the wellness enjoyed in the 4th ordered category which use of the scale implies.

arbitrary scale applied to ordered categorical status as a proxy for some continuous latent variable faces similar concerns[2]. Employing the integer accorded a level of education as a measure of latent embodied human capital or using age group category as a measure of latent cumulated lifetime experience renders summary statistics of such data similarly dependent upon an arbitrarily chosen scale. Furthermore, the problem is compounded if the variables are combined in a joint analysis with some equally arbitrarily chosen weighting scheme.

Summarizing statistics such as conditional means, medians, quantiles, variances, coefficients of variation, Gini coefficients, regression coefficients etc. and multidimensional variants thereof are all scale dependent instruments which can present challenges when making multilateral multidimensional group comparisons with ordered categorical data, since results will confront the same robustness issues as in wellbeing measurement using self reported data. Here some resolution to this dilemma is offered in the form of scale independent methods for the cardinal ordering of, and measuring variation in, subgroups in a population based upon possibly multidimensional, ordinal categorical data.

The exemplar application is a Canadian provincial and gender based study of the progress over time of levels and differences in human resource stocks in the guise of embodied human capital and experience. While early neoclassical growth and convergence models predict steady state convergence for constituencies with common technological and population resources, modern unified growth theory (Galor 2011) holds out the possibility of multiple equilibria engendering distinct poles of attraction when there are barriers to inter-provincial free flow of those resources. Promotion of the free flow of such mobile assets and the consequent commonality of income distributions is one of the objectives of institutionalised common markets such as the confederation of Provinces and territories that is Canada. If there are differences in these abilities across provinces and genders they could well provide the basis for similar differences in income distributions across those respective divides.

In the following, Section 1 outlines the basic ordering concept in a univariate comparison environment, extensions to multivariate environments are considered in section 2. Section 3 provides measures of between group inequalities and section 4 develops the multidimensional, multilateral ranking and ordering concept and some index properties are explored in section 5. Results of the application are reported in section 6. To anticipate the results, both genders have advanced in terms of human resource levels with gender specific provincial distributions converging, i.e. female provincial distributions are becoming more similar over time as are male provincial distributions. However, within this context the genders are themselves diverging with male – female transvariations increasing over time, with female distributions invariably dominating male distributions which does not accord with what is observed with income distributions over the same period (Anderson 2020).

[2] Multiple Treatment effect models where outcomes are recorded in ordered categories are a case in point.

1. The Basic Idea.

Suppose K ordered education (or age) categories indexed k=1,..,K where each successive category corresponds to a higher level of latent embodied human capital (or experience) respectively and suppose within group g for groups $g = 1, . . , G$ a probability density function is defined where the probability of being in the k'th category is $p_{k,g}, for\ k = 1, . . , K$ such that $p_{k,g} \geq 0\ and\ \sum_{k=1}^{K} p_{k,g} = 1$. $F_{j,g}$, the cumulative distribution function (CDF), is obtained by compounding the probabilities so that $F_{j,g} = \sum_{i=1}^{j} p_{i,g}\ for\ j = 1, . . , K$. When g's CDF is everywhere less than or equal to h's CDF i.e.:

$$F_{j,g} \leq F_{j,h}\ for\ j = 1, . . , K, with\ strict\ inequality\ somewhere \quad [1]$$

g is said to stochastically dominate h at the first order (Yalonetzky 2013)[3]. In such a case, if the numerical category ordering had true cardinal value with respect to latent embodied human capital or experience, call it x, any Skill Value Function $SVF(x)$ with $\frac{dSVF(x)}{dx} > 0$ would have an expected value under g's distribution that is at least as great as its expected value under h's distribution. The ordering is unambiguous in the sense that it is robust to any specific SVF whose first derivative is positive. However, the luxury of cardinality (or a fully specified wellbeing value function which attaches a real number to each level of wellbeing) is not available. Non the less, a sense of the distance between the two groups could be gleaned from adding the differences in cumulated probability over the outcome categories to yield a measure of the extent to which g is preferable to h given by $CDif(g, h)$, where:

$$CDif(g, h) = \sum_{j=1}^{K} \left(F_{j,h} - F_{j,g}\right) \quad [2]$$

Recalling that 1-$F_{j,g}$ is the chance of a randomly selected person in g group having a better outcome than the j'th level, this index has the interpretation of measuring the extent to which group g has bigger chances of better outcomes at each level than group h with [2] being an unweighted sum of the differences. The important point to note is that this measure does not depend upon the value of x or any such scale, it would retain the same value whatever values of x were attributed to the various category levels, it is in effect a measure of distance in probability that abstracts from, or is independent of, scaling, but non-the-less reflects an ordering of Skill Values experienced by the two groups.

Note that $CDif(g, h) > 0$ does not necessarily reflect the unambiguous superiority of g groups' wellbeing over that of h group, however it would be true if each component of [2] were non negative and at least one were positive. This can be checked by seeing if, for all Cumulative Density Pairs that are not identical, the sum of the absolute values of the differences in [2] were equal to the absolute value of their sum. Effectively this corresponds to checking if the two distributions were separate, based upon the proximity of a first order surface separation index SS1 to 1 or -1 where:

[3] Note, reversing the inequality in [1] would establish 1st order dominance of h over g.

$$-1 < SS1(g,h) = \frac{CDif(g,h)}{\sum_{j=1}^{K} |(F_{j,h} - F_{j,g})|} \le 1$$

[3]

When SS1=1 (-1) there is separation between the two CDF's in the sense that a piecewise continuous separating hyperplane exists between the two groups. Furthermore, in such a case it can be asserted that g group outcomes are unambiguously better (worse) than h group outcomes.

When $|SS1(g,h)| < 1$, a First Order Dominance relationship between g and h groups does not prevail. In a cardinally measureable situation such as income wellbeing measurement, researchers usually seek clarity in exploring higher order dominance conditions[4] involving integrals of continuous CDF's (See for example Levy 1998, Whang 2019). These conditions reflect additional concavity constraints on the Skill Value Function which in turn reflect concerns for the diminishing marginal value of skill levels (in wellbeing literatures they reflect ethical judgements regarding inequality and poorness, i.e. differences in outcomes at the lower end of the spectrum). In essence successive integration levels attach increasing weight to outcomes at the lower end of the outcome spectrum.

However, integrating CDF's is not possible without the luxury of cardinal measure, none-the-less the approach can be mimicked by discretely compounding the CDF's across the ordered levels to develop a Compounded Cumulative Density Function (CCDF) which, in the present context, for group g, has a typical element $CF_{j,g}$ where:

$$CF_{j,g} = \sum_{j^*=1}^{j} F_{j^*,g} \quad j = 1,..,K$$

The intuition behind these second order ordinal comparators is that, like their cardinally measured counterparts, they attach more weight to, and thus focus more attention upon, probability differences at the lower end of the value spectrum. So, if the latent value function is deemed strongly concave or if discrepancies between groups with respect to the worst outcomes are of greater concern, in giving a sense of distance between distributions which emphasises differences at the lower end of ordered outcomes, these 2nd order measures may be the appropriate comparator. The corresponding surface separation index SS2 where:

$$-1 < SS2(g,h) = \frac{\mathrm{C}CDif(g,h)}{\sum_{j=1}^{K} |(CF_{j,h} - CF_{j,g})|} \le 1$$

follows quite naturally.

2. Joint Distributions.

An attractive feature of these measuring instruments is that they work equally well in multivariate environments and avoid arbitrary dimension weighting problems associated with multivariate measures (Klugman, Rodríguez & Choi 2011). In the present case the combined levels of experience and embodied human capital are to be evaluated in terms of the joint probability density function (JPDF) of their proxies and its joint cumulative distribution function counterpart (JCDF). With respect to the JPDF, $p_{i,k,g}$ is now the probability that an agent

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[4] A rationale for such an approach is to be found in Lemma 1 in Davidson and Duclos (2000). The lemma demonstrates that, if distribution A first order dominates distribution B over some limited range at the lower end of the outcome spectrum, then there will be some order of integration, say K, at which distribution A K'th order dominates distribution B over its complete range.

randomly selected from group g is in the $i'th$ age and $k'th$ education categories for $i = 1,..,I$ and $k = 1,..,K$ respectively, where $p_{i,k,g} \geq 0$ and $\sum_{i=1}^{I}\sum_{k=1}^{K} p_{i,k,g} = 1$. Its corresponding JCDF has a typical element $F_{i,k,g}$ where $0 \leq F_{i,k,g} = \sum_{i*=1}^{i}\sum_{k*=1}^{k} p_{i^*,k^*,g} \leq 1$, (essentially compounding probabilities across 2 dimensions) in this case $F_{i,k,g}$ corresponds to the probability that a randomly selected agent from group g has an embodied human capital outcome no greater than the $i'th$ category and an experience outcome no greater than the $k'th$ category. In this context, if embodied human capital and experience are temporarily endowed with the cardinality measures x and y respectively, and a Skill Value Function $SVF(x,y)$ with $\frac{dSVF(x,y)}{dx} \geq 0, \frac{dSVF(x,y)}{dy} \geq 0$ and $\frac{d^2SVF(x,y)}{dxdy} \leq 0$ is posited, a necessary and sufficient condition for $E(SVF(x,y)|g) \geq E(U(x,y)|h)$ is that Group g outcomes stochastically dominate Group h outcomes (Atkinson and Bourguignon 1982) which demands:

$$F_{i,k,g} \leq F_{i,k,h} \; for \; i = 1,..,I \; and \; k = 1,..,K \; with \; strict \; inequality \; somewhere.$$

Note that, in this case, Group g's outcomes would be unambiguously preferred to Group h's for any Skill Value Function in the specified class. In particular, since $U(x,y)$ is a function which implicitly weights dimensions x and y, this would hold for any weighting scheme[5] consistent with the specified class of SVF's, in particular it doesn't depend upon an arbitrary weighting of the two dimensions.

3. Assessing Distributional Differences.

When ranking and ordering groups, it is important to have a sense of the extent to which they differ which can be assessed by considering differences in the various subgroup distributions. GT, Gini's Transvariation (Gini 1916) provides a useful starting point where:

$$0 \leq GT_{g,h} = 0.5\sum_{k=1}^{K}|p_{k,g} - p_{k,h}| = 1 - OV_{g,h} \leq 1$$

Where $OV_{g,h}\{= \sum_{k=1}^{K} min(p_{k,g}, p_{k,h})\}$ corresponds to the extent of overlap of two distributions, Anderson, Linton and Whang (2012) provided the asymptotic distribution for this construct in a multivariate continuous distribution setting. When the distributions are segmented, so that agents in one group have nothing in common with agents in the other group, the statistic will record a value 1. When the distributions are identical, the statistic will record a value of 0. Anderson et. al. (2020) extended this to a multilateral setting where, with G groups MGT, the Multilateral Transvariation is given by:

$$MGT_{g=1,..,G} = \frac{1}{G}\sum_{k=1}^{K}\Big(max(p_{k,g}: g = 1,..,G) - min(p_{k,g}: g = 1,..,G)\Big)$$

$MGT_{g=1,..,G}$ will give a sense of the differences in the collection of distributions but it is very much in the nature of a range statistic measuring differences in the abscissa extremes in the collection and ignoring between group differences in the interior. An alternative which solves

[5] The choice of which has been the source of some controversy in wellbeing measurement, see for example Klugman et. al. (2011) and references therein.

this problem and is somewhat more informative is DisGin, the Distributional Gini coefficient (Anderson et. al. 2020) a distributional analogue of the standard Gini coefficient[6] where:

$$DisGin = \frac{2}{G(G-1)} \sum_{g=2}^{G} \sum_{h=1}^{g-1} GT_{g,h}$$

Which in essence is the average between group transvariation across all possible group pairings. Again when all the distributions are completely segmented, these statistics will record a value 1, when they are all identical they will take on a value 0.

4. Ranking and Ordering Groups.

However, exploiting this idea of distance further, let $F_{UENV,i}$ be the Upper Envelope of the collection of CDF's under comparison, which is given by:

$$F_k^{UENV} = \max_g \left(F_{k,g}\right) \; for \;\; k = 1,..,K$$

F_k^{UENV} is also a CDF and corresponds to the synthetic worst outcome distribution that could be contrived by combining the worst aspects of all the groups (If there were a group that was uniquely dominated by all other groups in the collection, its CDF would coincide with $F_{i,UENV}$ and it would unambiguously correspond to the "worst" group). As such, it is first order stochastically dominated by all other groups, and a measure of the merit of any group j relative to this synthetic worst case scenario is given by:

$$\mathrm{CDif(UENV, j)} = \sum_i \left(F_i^{UENV} - F_i^j\right)$$

As an index, its lower bound is 0 (which will arise when a particular group CDF coincides with the upper envelope) and in theory its upper bound is the number of assessment categories less 1, so if a standardized index confined to the unit interval is required, one could divide by the number of outcome categories less 1[7]. However, this turns out to be a little extreme, in effect it assumes a worst case scenario group where all members are in the lowest assessment category. A more palatable alternative would be to consider the lower envelope $F_{i,LENV}$ where:

$$F_k^{LENV} = \min_g \left(F_{k,g}\right) \; for \;\; k = 1,..,K$$

This is also a CDF which corresponds to the synthetic best outcome distribution that could be contrived by combining the best aspects of all the groups (If there were a group that uniquely dominated all other groups in the collection, its CDF would coincide with F_k^{LENV} and it would unambiguously be the "best" group. Then $\mathrm{CDif(g, LENV)}$ where:

[6] Whereas the conventional Gini coefficient is based upon aggregated differences in the cardinal measure of the variate, the Distributional Gini is based upon aggregated differences in the cardinal measure of its ordinates. Subgroup importance weighted versions of these statistics are also readily available.

[7] At the extreme if all members of group j were in the highest category and all members of any other group were in the lowest category the maximum value of CDif(UENV,j) would be 4 in a 5 category case for example.

$$\text{CDif}(\text{g}, \text{LENV}) = \sum_k \left(F_{k,g} - F_k^{LENV}\right)$$

would correspond to the distance of group g from the synthetic best case scenario, in essence a measure of how "bad" group g was relative to the synthetic best case scenario.

Then $UD1$(j), a first order relative wellbeing index could be contemplated where:

$$0 \leq \text{UD1}(\text{g}) = \frac{\sum_k \left(F_k^{UENV} - F_{k,g}\right)}{\sum_k (F_k^{UENV} - F_k^{LENV})} \leq 1$$

UD1 is quite naturally extended to higher order comparisons which reflect inequality and poorness concerns. For example, consider UD2, a second order index constructed in terms of the compounded cumulative densities of the form:

$$0 \leq \text{UD2}(\text{g}) = \frac{\sum_k \left(CF_k^{UENV} - CF_{k,g}\right)}{\sum_k (CF_k^{UENV} - CF_k^{LENV})} \leq 1$$

These indices, the discrete ordered categorical distribution analogues of the family of Utopia-Dystopia index (Anderson, Post and Whang 2019, Anderson and Leo 2020) developed for continuous distributions, has many advantages. It can be shown to satisfy many of the axioms required of wellbeing indices (Sen 1987, 1995), it is continuous (at least piecewise), independent of scale and functional form and consistent (if a distribution is more preferred the statistic yields a larger value), it is normalized and has an independence of irrelevant alternatives property (Anderson and Leo 2020). In addition, unlike most other normalized statistics, it attains the value 0 (1) only when a particular group has unambiguously the worst (best) outcomes. Furthermore, higher order measures can be seen as a response to the veil of ignorance problem associated with comparing conditional means in treatment effects models (Carneiro, Hansen and Heckman 2003). Its asymptotic distribution is readily obtained (See Appendix) which facilitates inference and furthermore, it is readily extended to multidimensional, multilateral comparisons obviating many of the weighting issues associated with a multidimensional environment.

A sense of the extent to which the ordering is definitive or unambiguous can be gleaned from considering AA, an Absence of Ambiguity Index which is one minus the average value of the surface separation index at the appropriate order so, for example, the first order index would be:

$$0 \leq AA1 = 1 - \tfrac{2}{K(K-1)} \textstyle\sum_{g=2}^{K} \sum_{h=1}^{g-1} |SS1(g,h)| \leq 1$$

When $AA1$ is 0 there is complete absence of ambiguity in the collection of distributions at the first order comparison level, when it is 1 all distributions are identical and there is complete ambiguity in the ordering. Ambiguity at the Compounded Cumulative Distribution level can be assessed using $AA2$ where:

$$0 \leq AA2 = 1 - \tfrac{2}{K(K-1)} \textstyle\sum_{g=2}^{K} \sum_{h=1}^{g-1} |SS2(g,h)| \leq 1$$

Generally, since compounding reflects a more restrictive class of Skill Value Functions it will reduce the potential for ambiguity so that $AA2 \leq AA1$.

5. Index Properties.

To facilitate interpretation and comparability, the Economic Wellbeing Literature develops indices on an axiomatic foundation (Sen 1995, Gravel Magdalou and Moyes 2020 are examples). Wellbeing analysts seek usually indices that obey certain axioms like Continuity, Scale Independence, Coherency, Normalization, Monotonicity and Inequality Sensitivity. Continuity of the index is usually required with respect to the variable x (usually income or consumption) which in the present case is not continuously measured, however these indices are piecewise continuous in the variable p. Scale Independence is required to secure independence from monotonic translations of the wellbeing function, note that scaling measures do not appear in any formulae here (the indices are scale independent by construction) so independence is secured. Coherency (if group g's distribution is preferred to group h's distribution then indices should reflect that i.e. $UD(g) > UD(h)$) will be satisfied if the indices can be shown to be monotonic increasing in category ordering. Normalization requires that indices values are bounded on the unit interval which is the case with respect to the normalized intervals. Monotonicity requires that indices are sensitive to category ordering so that ceteris paribus, if an agent enjoys a better outcome the index should increase. This is obviously related to the coherency property, but has special import in the case of ordered categories. Finally, in wellbeing measurement Inequality Sensitivity is an ethically grounded property that requires measures be sensitive to dispersion of wellbeing so that for a given level of aggregate wellbeing, the society that has it more equally shared is the society that is preferred, here it reflects the notion that the Skill Value Function is strongly concave. In the current situation adherence to the last two properties needs to be demonstrated.

To fix ideas suppose there exist S ordered categories indexed s=1,..,S in line with their ordering so that category s is preferred to category t when s > t. The probability density function (PDF) defines the chance that a randomly selected agent resides in a particular category so the categories have associated probabilities $p_s \geq 0 \; where \; \sum_{s=1}^{S} p_s = 1$ and the corresponding associated cumulative distribution function (CDF) is given by:

$$F_j = \sum_{i=1}^{j} p_i \; for \; j = 1,..,S$$

The Cumulative CDF is given by:

$$CF_j = \sum_{i=1}^{j} F_i = \sum_{i=1}^{j} \sum_{k=1}^{i} p_k \; for \; j = 1,..,S$$

Suppose G groups indexed $g = 1,..,G$, their respective PDF's, CDF's and Cumulative CDF's are identified with a second subscript g (so that $p_{s,g}$ is the probability that a randomly selected individual in group g is in category s). Define the upper envelope of the collection of group CDF's as F_i^{UENV}

$$F_s^{UENV} = \max_g \left(F_{s,g}\right) \; for \;\; s = 1,..,S. \;\; g = 1,..,G$$

And the upper envelope of the group cumulative CDF's

$$CF_s^{UENV} = \max_g\left(CF_{s,g}\right) \, for \;\; s = 1,..,S. \;\, g = 1,..,G$$

The un-normalized first and second order Utopia-Dystopia indices[8] for the g'th group is given by:

$$\mathrm{UD1(UENV, g)} = \sum_s \left(F_s^{UENV} - F_{s,g}\right)$$

$$\mathrm{UD2(UENV, j)} = \sum_s \left(CF_s^{UENV} - CF_{s,g}\right)$$

Monotonicity property – if an agent transfers to a higher category the index should increase. Working with nUD1 where n is the population of the $g'th$ group and assume that group is not part of the upper envelope (if it were things just net out). $np_{s,g}$ corresponds to the number of people in the $g'th$ group in the $s'th$ category and $nF_{s,g}$ corresponds to the number of people in the $s'th$ category or lower. Suppose that an agent moves from category s to category s+k and let superscript A denote the "after the move" index. Note that:

$np_{t,g}^A = np_{t,g} \; for \; all \; t < s, t = s + l, l = 1,.,k \; and \; t > s + k;$

$and \; np_{s,g}^A = np_{s,g} - 1; \, np_{s+k,g}^A = np_{s+k,g} + 1.$

It follows that:

$nF_{t,g}^A = nF_{t,g} \; for \; all \; t < s; \; nF_{s+l,g}^A = nF_{s+l,g} - 1 \; for \; l = 0,..,k-1 \; and \; nF_{t,g}^A = nF_{t,g}$ $for \; all \; t > s + k.$

So that $n\mathrm{UD1(UENV, g)}^A = n\mathrm{UD1(UENV, g)} + k \Rightarrow \mathrm{UD1(UENV, g)}^A > \mathrm{UD1(UENV, g)}$

For any $s < S - k$: Furthermore:

$nCF_{t,g}^A = nCF_{t,g} \; for \; all \; t < s; \; nCF_{s+i,g}^A = nCF_{s+i,g} - i; \; for \; all \; i = 0,.,k \; and \; nCF_{s+k+i,g}^A = nCF_{s+k+i,g} - k \; for \; i = 0,..,S - k - s$

So that $n\mathrm{UD2(UENV, g)}^A = n\mathrm{UD2(UENV, g)} + k! + (S - s - k)k \Rightarrow \mathrm{UD2(UENV, g)}^A > \mathrm{UD2(UENV, g)}$

As for distribution sensitivity, the requirement is that movement to an adjacent improved category for someone in the lower end of the category spectrum should result in a greater increase in the index than a similar movement for someone in the upper end of the category spectrum. Whilst this is not the case for UD1 it is the case for UD2 since $n\mathrm{UD2(UENV, g)}^A - n\mathrm{UD2(UENV, g)}$ diminishes with increasing s so that a transfer in the upper end of the spectrum results in a smaller increase than a similar transfer at the lower end of the spectrum.

[8] To satisfy normalization axioms UD1 and UD2 are divided by $\sum_s (F_s^{UENV} - F_s^{LENV})$ and $\sum_s (CF_s^{UENV} - CF_s^{LENV})$ respectively

Results.

The data were drawn from the Canadian Census Individual Public Use data files for the years 2001, 2006, 2011 and 2016. Educational status, age group, gender and province of domicile for all persons over the age of 20 with nominal annual before tax incomes greater than 0 and less than 1000000$C were selected. 22 subgroups, defined by gender and province of domicile, make up the confederation which is Canada and are analysed as entities. Age groupings were set at six decadal categories 20-29, 30-39, .. etc (the final category was "70 and over") and six educational categories were set at A) No degree, certificate or diploma. B) High school graduation certificate. C) Trades certificate/diploma, College certificate/diploma, University certificate/diploma lower than a bachelors degree. D) University degree: Bachelors level. E) Post bachelor level University degree: certificate, medical degree, Masters degree. F) Post Masters university degree: Earned doctorate.

Table 1 reports statistics recording the evolution of provincial and gender based distributional differences in ordinal educational status and age distributions when they are seen as proxies for embodied human capital (EHC) and experience. Underlying much distributional similarity across provincial and gender divides (with Multilateral Transvariation and Distributional Gini Coefficients generally close to, but significantly different from, zero), is a steady intertemporal increase in distributional inequality recorded in both statistics signalling distributional divergence as opposed convergence in the classical sense. Since both statistics are asymptotically normal (Anderson et. al. 2020) note that whilst increases in Multilateral Transvariation are seldom statistically significant at usual levels of significance, the Distributional Gini Coefficients are in terms of increases well beyond two standard errors. All in all, Canadian Provinces and Genders appear to be diverging over the first 15 years of the 21st Century.

Table 1 here.

Turning to Embodied Human Capital differences, Table 2 reports the 1st and 2nd Order Utopia-Dystopia indices and their corresponding ranks for the educational status distributions. Note first that the rankings are relatively stable over time and across comparison orders with Maritimes, Manitoba, Northern Canada and Saskatchewan generally at the lower end of the spectrum and Alberta, British Columbia, Ontario and Quebec at the upper end. Equally notable is the fact that in the former group of provinces females generally rank higher than their male counterparts whereas in the latter group, males rank higher than their female counterparts although there has been a reversal in this respect in 2016. While no group is unambiguously worst or best under the first order comparison there is some clarity highlighted in the second order comparison with a distinct reduction of ambiguity in the comparison process.

Table 2 here.

In the assessment of experience, Table 3 reports a very different situation with respect to age. Now, with one or two exceptions, the Maritimes reside in the upper end of the scale with the rest of Canada in the lower part of the spectrum. Again within provinces females tend to be more experienced than their male counterparts (which is not surprising given their greater longevity). Again there is no unambiguously most or least experienced group in the first order comparison though there is in the second order comparison with Northern Canada Females and New Brunswick females highlighted, again ambiguity is reduced in the second order comparison. Unlike embodied human capital there are some significant trends in the experience variable. Generally, the Maritimes are gaining experience (aging) relative to their counterparts elsewhere in Canada with Manitoban and Saskatchewan females getting decidedly younger relatively speaking in the "Rest of Canada" group.

Table 3 here.

What really matters for the income generation process is the joint impact of an individuals' experience and embodied human capital so their joint distribution in the guise of ordered age and educational attainment categories is the appropriate instrument of analysis in the present context. Table 4 reports the First and Second Order Utopia-Dystopia indices for the joint education and age distributions. Again Second Order comparisons, in reflecting a more restrictive view of the Skill Value function reduce the ambiguity in the comparisons. Perhaps the most striking result is that, with just one exception (Northern Canada), females rank more highly than their male counterparts in every province. This probably reflects the importance of age in the joint distribution and the fact that women generally enjoy greater longevity. The prairie provinces of Alberta, Manitoba and Saskatchewan and Northern Canada dominate the bottom half of the Utopia-Dystopia spectrum with Maritime Provinces, Ontario, Quebec and British Columbia nearer the top. These very striking systematic gender differences province by province suggest an underlying relationship between the genders across Canada which is explored in the following trans Canada gender based analysis. Exploring the distributional relationships of the ordered educational and age categories via stochastic dominance comparisons facilitates the determination of improvement or regression without recourse to arbitrarily assigned scaling factors which could adversely influence the analysis.

Table 4 here.

Recalling that Transvariation measures the extent to which distributions differ on the unit interval (0 implies identical distributions 1 implies perfect segmentation) and a Surface Separation index of 1 (-1) is an indication of stochastic dominance at the corresponding order, Tables 5 and 6 chart the year on year progress of the respective genders in the acquisition of human capital and experience. The Transvariation estimate is asymptotically normally distributed (Anderson, Linton and Whang 2012) and a generous estimate of its standard error in these calculations is 0.0008, so the transvariations appear to be diminishing significantly over time which means the gender specific distributions are converging over time. None the less the surface separation indices are invariably equal 1 so the year by year improvements are palpable in the sense that outcomes in successive years stochastically dominate preceding years which corresponds to an unambiguous improvement. The exceptions are males in the educational status comparison 2011-2016, where it would appear that males have regressed (though not unambiguously so and males in the 2006-2011 age group comparison where there is evidence of progress though again, not unambiguously so.

Tables 5 and 6 here.

As for the female vs. male comparison, Tables 7 and 8 indicate that females dominate males in experience acquisition in all years. Perhaps the most interesting result is with respect to the acquisition of embodied human capital. Here males 1st Order dominated Females in 2001 and 2nd Order Dominated Females in 2006, whereas there was no dominance relationship in 2011 and by 2016 Females 2nd Order Dominated Males, clear evidence that Females have overtaken Males in human capital acquisition.

Tables 7 and 8 here.

Turning to the results for the joint distribution, Table 9 indicates progress being made by both genders over time with 1st or 2nd order dominance of succeeding over preceding years for all but males in the 2011-2016 comparison. Table 10 indicates that distributional differences between the genders -appear to be diverging over time (witness the increasing Transvariations in successive years) and, while 1st order dominance does not generally prevail, females are close to dominating males in 2011 and indeed 2nd order dominate males 2016.

Tables 9 and 10 here.

Conclusions.

Cantril type scales applied to ordered categorical data have been employed in a variety of empirical wellness and wellbeing environments. When standard summary statistics are applied to such numbers arbitrarily accorded to categories, the scale dependency of the statistics presents a problem (Kahneman and Krueger 2006, Schroder and Yitzhaki 2017, Bond and Lang 2019). The application of scales accorded ordered categorical data to reflect levels of latent continuous variates faces similar concerns. Using age groupings as categories representing acquired experience or the numerical value attributed to a level of education as a proxy for the level of embodied human capital is in effect applying some version of a Cantril Scale to the problem. Yet there is no reason to think that the 4th level of education represents twice the embodied human capital value that the 2nd level represents or that a fifty-year old has cumulated twice the amount of experience that a twenty five-year old has acquired. Here, scale independent techniques for ordering and ranking groups and examining the extent of their variation in multivariate, multilateral environments that circumvent these problems are proposed. They are implemented in a 21st Century Canadian interprovincial gender based analysis of age and education levels which are interpreted as proxies for latent acquired experience and embodied human capital variables.

Applying the 1st order and 2nd order Utopia-Dystopia indices (Anderson Post and Whang 2019) to multivariate multinomial categorical variates provides a scale independent means of ranking and ordering a collection of groups and Multilaterlal Transvariation and Distributional Gini coefficients provide scale independent means of analysing inequalities amongst the groups. These were applied to age group and educational attainment category data for males and females over the age of 20 in the collection of 11 provinces and territories that is Canada which yields 22 comparison groups for the years 2001, 2006, 2011 and 2016. Multilateral Transvariation and Distributional Gini coefficients indicate a steady and systematic increase in distributional inequality over time signalling some distributional divergence. Uni-dimensional Orderings with respect to Embodied Human Capital are relatively stable over time and across comparison orders with Maritimes, Manitoba, Northern Canada and Saskatchewan generally at the lower end of the spectrum and Alberta, British Columbia, Ontario and Quebec at the upper end. On the other hand, the Maritimes reside in the upper end of the ordering with the rest of Canada in the lower part of the spectrum with respect to experience (age) based orderings. In both cases in many provinces females outranked their male counterparts.

Ultimately it is the joint distribution of experience and embodied human capital that is of consequence in generating consumption wellbeing. In this case the Maritime Provinces, Ontario, Quebec and British Columbia feature near the top of the orderings with Alberta, Manitoba and Saskatchewan and Northern Canada occupying the bottom half of the Utopia-Dystopia spectrum. Again in most provinces females outrank their male counterparts which prompted a Trans-Canada gender based study. Both genders have advanced with gender specific distributions converging, i.e. becoming more similar over time. Within this context the genders are diverging with male – female transvariations increasing over time, with female distributions invariably dominating male distributions.

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Appendix.

Assume independent subgroups indexed k=1,..,K with the i'th level self reported happiness level probability $p_{i,k}\ i = 1,..,m$ stacked in the m x 1 vector $\underline{p}_k$ which is multinomial with a variance

$$V\left(\underline{p}_k\right) = \begin{pmatrix} p_1 & 0 & . & 0 \\ 0 & p_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & p_m \end{pmatrix} - \begin{pmatrix} p_1^2 & p_1p_2 & . & p_1p_m \\ p_2p_1 & p_2^2 & . & p_2p_m \\ . & . & . & . \\ p_mp_1 & p_mp_2 & . & p_m^2 \end{pmatrix}$$

Given the m x m dimensioned integrating matrix D, where:

$$D = \begin{pmatrix} 1 & 0 & . & 0 \\ 1 & 1 & . & 0 \\ . & . & . & . \\ 1 & 1 & . & 1 \end{pmatrix}$$

$\underline{F}_k$, the CDF of the k'th group is such that $\underline{F}_k = D\underline{p}_k$ with variance $DV\left(\underline{p}_k\right)D'$

Generally, $\underline{F}_k - \underline{F}_l$ will have a variance $V(\underline{F}_k - \underline{F}_l) = D\left(V\left(\underline{p}_k\right) + V\left(\underline{p}_l\right) - 2COV\left(\underline{p}_k, \underline{p}_l\right)\right)D'$

and $D(\underline{F}_k - \underline{F}_l)$ will have a variance $DV(\underline{F}_k - \underline{F}_l)D'$. Note that $COV\left(\underline{p}_k, \underline{p}_l\right) = 0$ with subgroup independence, however when considering either $\underline{F}^{UENV} - \underline{F}_k$ or $\underline{F}_k - \underline{F}^{LENV}$ this will not necessarily be the case because the two vectors under comparison may have common elements (essentially when an element in $\underline{F}_k$ is a component of the corresponding frontier.

If the respective envelopes do not contain elements of $\underline{F}_k$, independence and zero covariance will prevail since the envelopes will be made up of elements from distributions that are independent of $\underline{F_k}$[9]. However, letting $\underline{p}^{ENV}$ be the PDF implied by the upper or lower envelope, if the envelopes and $\underline{F_k}$ have elements in common, the respective rows and columns of the variance-covariance matrix will be 0 since it will be the case that $D2COV\left(\underline{p_k}, \underline{p}^{ENV}\right)D' =$ $D\left(V\left(\underline{p_k}\right) + V\left(\underline{p}^{ENV}\right)\right)D'$ for those particular rows and columns and zero elsewhere and the Variance- covariance matrix in this case will thus be $D\left(V\left(\underline{p_k}\right) + V\left(\underline{p}^{ENV}\right)\right)D'$ with the corresponding rows and columns set to 0.

Typically, first and second order Utopia-Dystopia indices work with scaled sums of the differences which, letting d be the m dimensioned unit vector, are of the form:

$d'\left(\underline{F}^{UENV} - \underline{F_k}\right)$ and $d'D\left(\underline{F}^{UENV} - \underline{F_k}\right)$

With respective variances:

$d'V\left(\underline{F}^{UENV} - \underline{F_k}\right)d$ and $d'DV\left(\underline{F}^{UENV} - \underline{F_k}\right)D'd$.

For scaled indices where the scaling factors are $s_1 = d'\left(\underline{F}^{UENV} - \underline{F}^{LENV}\right)$ and $s_2 =$ $d'D\left(\underline{F}^{UENV} - \underline{F}^{LENV}\right)$ respectively the corresponding variances would be:

$d'V\left(\underline{F}^{UENV} - \underline{F_k}\right)d/s_1^2$ and $d'DV\left(\underline{F}^{UENV} - \underline{F_k}\right)D'd/s_2^2$.

When estimates of the underlying $p_i's$ are maximum likelihood estimates asymptotic normality of the sums and differences can be claimed (Rao 1973) so that, based upon a null hypothesis of no difference:

$$\sqrt{T}d'\left(\underline{F}^{UENV} - \underline{F_k}\right) \sim N\left(0, d'V\left(\underline{F}^{UENV} - \underline{F_k}\right)d\right)$$

And

$$\sqrt{T}d'D\left(\underline{F}^{UENV} - \underline{F_k}\right) \sim N\left(0, d'DV\left(\underline{F}^{UENV} - \underline{F_k}\right)D'd\right)$$

Where T is the appropriate sample size factor.

Note however, when examining the statistical difference between the n'th order Utopia-Dystopia indices of groups k and l, the statistic amounts to the comparison of the scaled sum of their respective n'th order cumulated differences since generally for given j = 0, 1, …:

$$\sqrt{T}d'D^i\left(\underline{F}^{UENV} - \underline{F_k}\right)/s_{i+1} - \sqrt{T}d'D^i\left(\underline{F}^{UENV} - \underline{F_l}\right)/s_{i+1} = \frac{\sqrt{T}d'D^i}{s_{i+1}}\left(\underline{F_l} - \underline{F_k}\right)$$

Which may effectively be examined by considering:

$$\sqrt{T}d'D^i\left(\underline{F_l} - \underline{F_k}\right)$$

[9] Similarly when $\underline{F}^{UENV}$ and $\underline{F}^{LENV}$ are being compared independence will prevail since by definition they will not have elements in common unless all distributions have a common element or elements which for most applications is unlikely.

Which has an easily established distribution as seen above.

Tables.

Table 1. Distributional Differences.

		EHC	Experience	Joint
2001	Multilateral Transvariation	0.02214	0.02006	0.04096
	Standard Error	0.00165	0.00160	0.00175
	Distributional Gini	0.08179	0.05766	0.13146
	Standard Error	0.00166	0.00140	0.00204
2006	Multilateral Transvariation	0.02356	0.02184	0.04075
	Standard Error	0.00173	0.00176	0.00176
	Distributional Gini	0.08714	0.06049	0.13529
	Standard Error	0.00165	0.00138	0.00200
2011	Multilateral Transvariation	0.02951	0.02317	0.04702
	Standard Error	0.00169	0.00170	0.00173
	Distributional Gini	0.09844	0.06601	0.14598
	Standard Error	0.00169	0.00140	0.00199
2016	Multilateral Transvariation	0.02929	0.02399	0.04655
	Standard Error	0.00166	0.00165	0.00168
	Distributional Gini	0.09804	0.07718	0.14834
	Standard Error	0.00165	0.00146	0.00196

Table 2. First Order Utopia – Dystopia Indices and Ranks Embodied Human Capital

Group	2001	2006	2011	2016
NewLab F	0.06512 22	0.06880 22	0.32262 20	0.35076 16
NewLab M	0.11695 21	0.12972 21	0.35366 19	0.29663 18
PEI F	0.51657 12	0.59254 10	0.64228 11	0.71660 9
PEI M	0.14538 20	0.15902 20	0.46814 16	0.25046 21
NovSco F	0.63695 9	0.60245 9	0.78160 7	0.74429 8
NovSco M	0.57589 10	0.52735 11	0.65807 10	0.52617 13
NewBru F	0.29085 17	0.27691 16	0.52154 14	0.45349 14
NewBru M	0.20450 18	0.21699 18	0.39156 17	0.25776 20
Quebec F	0.51574 13	0.62535 8	0.71810 9	0.77469 6
Quebec M	0.64478 8	0.73770 5	0.74030 8	0.71442 10
Ontario F	0.73849 5	0.80316 4	0.89700 4	0.88108 2
Ontario M	0.92915 2	0.91940 2	0.92950 1	0.83668 5
Manito F	0.29403 16	0.35591 13	0.57004 13	0.58125 12
Manito M	0.34468 14	0.33352 14	0.47984 15	0.37244 15
Saskat F	0.32012 15	0.36280 12	0.57208 12	0.60248 11
Saskat M	0.16111 19	0.17195 19	0.36608 18	0.30344 17
Alberta F	0.65651 7	0.68369 7	0.81029 5	0.84879 3
Alberta M	0.76478 4	0.72380 6	0.80569 6	0.75328 7
B.C. F	0.78733 3	0.86951 3	0.90542 3	0.90853 1
B.C. M	0.94872 1	0.95149 1	0.92657 2	0.84306 4
NorCan F	0.52058 11	0.28631 15	0.16477 21	0.28305 19
NorCan M	0.69470 6	0.27260 17	0.03531 22	0.02287 22
Ambiguity Index AA1	0.17828	0.16583	0.14604	0.19397

Second Order Utopia – Dystopia Indices and Ranks Embodied Human Capital.

Group	2001	2006	2011	2016
NewLab F	**0.00000 22**	**0.00000 22**	0.33349 20	0.39751 16
NewLab M	0.06114 21	0.07600 21	0.37345 19	0.34317 18
PEI F	0.54978 11	0.63982 8	0.72662 10	0.83182 8
PEI M	0.07821 20	0.11845 20	0.51517 15	0.29238 20
NovSco F	0.65811 8	0.61565 10	0.83978 7	0.83049 9
NovSco M	0.56304 10	0.52170 11	0.70277 11	0.58657 13
NewBru F	0.29025 16	0.25707 15	0.56481 14	0.52924 14
NewBru M	0.16359 18	0.18313 18	0.42635 17	0.29886 19
Quebec F	0.53401 13	0.63105 9	0.76798 9	0.84850 6
Quebec M	0.65769 9	0.75171 6	0.78947 8	0.78334 10
Ontario F	0.77643 5	0.82780 4	0.95607 4	0.96075 2
Ontario M	0.96006 2	0.94212 2	0.98162 2	0.90317 5
Manito F	0.26694 17	0.32951 13	0.60986 13	0.64984 12
Manito M	0.30092 15	0.28454 14	0.49582 16	0.40580 15
Saskat F	0.31635 14	0.36059 12	0.63503 12	0.69417 11
Saskat M	0.09930 19	0.12286 19	0.39046 18	0.35030 17
Alberta F	0.70479 7	0.72389 7	0.88367 5	0.95028 3
Alberta M	0.80808 4	0.75885 5	0.87662 6	0.84169 7
B.C. F	0.84496 3	0.92374 3	0.98084 3	**1.00000 1**
B.C. M	**1.00000 1**	**1.00000 1**	0.99437 1	0.92527 4
NorCan F	0.54199 12	0.22171 17	0.11654 21	0.27048 21
NorCan M	0.72019 6	0.22511 16	0.00591 22	0.00000 22
Ambiguity Index AA2	0.09202	0.07194	0.02788	0.01816

Table 3. First Order Utopia – Dystopia Indices and Ranks Age.

Group	2001	2006	2011	2016
NewLab F	0.77979 12	0.90474 5	0.94654 3	0.95817 3
NewLab M	0.74537 15	0.86480 9	0.87997 7	0.91015 5
PEI F	0.88427 5	0.90897 4	0.93967 4	0.93934 4
PEI M	0.78105 11	0.82321 11	0.83241 10	0.90025 6
NovSco F	0.93287 2	0.95341 1	0.99434 1	0.96651 2
NovSco M	0.78740 10	0.87994 8	0.92007 5	0.88020 8
NewBru F	0.88931 4	0.91118 3	0.96418 2	0.99460 1
NewBru M	0.75649 14	0.80194 12	0.83931 9	0.88906 7
Quebec F	0.87639 6	0.89524 6	0.88411 6	0.81415 9
Quebec M	0.68163 17	0.72214 16	0.75331 15	0.71798 13
Ontario F	0.80349 9	0.79175 14	0.80743 13	0.74565 11
Ontario M	0.67149 18	0.66982 18	0.71619 16	0.64348 15
Manito F	0.91076 3	0.85229 10	0.80002 14	0.63761 16
Manito M	0.71675 16	0.70036 17	0.66994 18	0.53288 17
Saskat F	0.97743 1	0.92414 2	0.81689 12	0.65771 14
Saskat M	0.81592 8	0.78776 15	0.68329 17	0.53276 18
Alberta F	0.55945 19	0.52024 19	0.51005 19	0.40960 19
Alberta M	0.44540 20	0.39902 20	0.42917 20	0.35076 20
B.C. F	0.86080 7	0.88077 7	0.87252 8	0.80687 10
B.C. M	0.77967 13	0.80031 13	0.82069 11	0.73410 12
NorCan F	0.00017 22	0.03388 22	0.00458 22	0.00010 22
NorCan M	0.09843 21	0.10430 21	0.09018 21	0.07891 21
Ambiguity Index AA1	0.13156	0.18058	0.11358	0.06614

Second Order Utopia – Dystopia Indices and Ranks Age.

Group	2001	2006	2011	2016
NewLab F	0.83526 12	0.96123 2	0.96158 3	0.97060 2
NewLab M	0.81656 14	0.94278 5	0.89938 6	0.92453 5
PEI F	0.91717 6	0.94722 3	0.95149 4	0.95506 4
PEI M	0.84283 9	0.86479 10	0.85601 10	0.91423 6
NovSco F	0.97330 2	0.98757 1	0.99819 1	0.95874 3
NovSco M	0.84580 8	0.93642 6	0.93459 5	0.87476 8
NewBru F	0.92599 3	0.94578 4	0.97592 2	**1.00000 1**
NewBru M	0.81142 15	0.85423 11	0.85915 9	0.89426 7
Quebec F	0.92493 4	0.92351 7	0.88139 7	0.80144 9
Quebec M	0.73939 17	0.75491 16	0.75940 15	0.71227 13
Ontario F	0.83599 11	0.80609 14	0.80106 12	0.73250 11
Ontario M	0.71463 18	0.68894 18	0.71199 16	0.62780 14
Manito F	0.92432 5	0.84700 12	0.77823 14	0.61278 16
Manito M	0.74797 16	0.70902 17	0.65481 17	0.51355 17
Saskat F	0.98462 1	0.91695 8	0.78391 13	0.62520 15
Saskat M	0.83977 10	0.79123 15	0.65468 18	0.50315 18
Alberta F	0.57861 19	0.50120 19	0.49436 19	0.39535 19
Alberta M	0.47103 20	0.37946 20	0.41390 20	0.33735 20
B.C. F	0.90118 7	0.91046 9	0.87186 8	0.79695 10
B.C. M	0.83289 13	0.82912 13	0.81487 11	0.71705 12
NorCan F	**0.00000 22**	0.00910 22	0.00119 22	**0.00000 22**
NorCan M	0.11498 21	0.07574 21	0.08624 21	0.06682 21
Ambiguity Index AA2	0.03858	0.03783	0.02175	0.04686

Table 4. First Order Utopia – Dystopia Indices and Ranks Joint Distribution.

Group	2001	2006	2011	2016
NewLab F	0.60837 14	0.67482 13	0.77899 10	0.82248 7
NewLab M	0.54893 19	0.62432 14	0.70731 14	0.73464 11
PEI F	0.80887 7	0.84893 5	0.86531 5	0.89123 2
PEI M	0.56381 18	0.59014 17	0.70512 15	0.70328 13
NovSco F	0.91385 1	0.88874 2	0.95378 1	0.93096 1
NovSco M	0.73202 11	0.75527 9	0.81126 8	0.76605 8
NewBru F	0.75884 10	0.75268 10	0.84616 6	0.87160 5
NewBru M	0.56649 17	0.60017 16	0.68629 16	0.69382 14
Quebec F	0.85227 4	0.88841 3	0.88944 4	0.87276 4
Quebec M	0.68762 12	0.73867 11	0.76536 11	0.73523 10
Ontario F	0.86088 3	0.86886 4	0.90180 3	0.85791 6
Ontario M	0.76005 9	0.76042 8	0.80127 9	0.71883 12
Manito F	0.76558 8	0.71313 12	0.74737 13	0.66237 16
Manito M	0.57433 16	0.54598 19	0.58899 18	0.47920 19
Saskat F	0.82868 5	0.77037 7	0.75494 12	0.68607 15
Saskat M	0.59556 15	0.56366 18	0.57130 20	0.47373 20
Alberta F	0.60910 13	0.60579 15	0.67244 17	0.61737 17
Alberta M	0.50385 20	0.48481 20	0.58138 19	0.49900 18
B.C. F	0.89308 2	0.92018 1	0.91684 2	0.88102 3
B.C. M	0.82178 6	0.82954 6	0.83938 7	0.76125 9
NorCan F	0.07077 22	0.08182 22	0.05742 21	0.11164 21
NorCan M	0.18347 21	0.09807 21	0.04065 22	0.02760 22
Ambiguity Index AA1	0.28039	0.27628	0.24837	0.25678

Second Order Utopia – Dystopia Indices and Ranks Joint Distribution.

Group	2001	2006	2011	2016
NewLab F	0.66954 13	0.73843 12	0.83682 8	0.90399 7
NewLab M	0.57972 15	0.66658 14	0.74814 14	0.80270 9
PEI F	0.85869 5	0.92921 5	0.92284 5	0.96963 2
PEI M	0.56774 17	0.61049 17	0.74782 15	0.76218 12
NovSco F	0.98995 1	0.95602 3	0.99556 1	0.99053 1
NovSco M	0.76874 11	0.79105 8	0.83217 9	0.80985 8
NewBru F	0.82747 8	0.81820 7	0.90073 6	0.95366 3
NewBru M	0.57588 16	0.63093 16	0.72967 16	0.74954 14
Quebec F	0.95464 3	0.97492 2	0.94658 4	0.94572 4
Quebec M	0.74260 12	0.78124 11	0.80082 11	0.78479 11
Ontario F	0.94033 4	0.93963 4	0.95138 3	0.92750 6
Ontario M	0.79237 9	0.78861 9	0.82194 10	0.75733 13
Manito F	0.77965 10	0.71579 13	0.76687 13	0.70817 16
Manito M	0.54857 19	0.51445 19	0.58474 19	0.50125 20
Saskat F	0.85236 6	0.78769 10	0.77477 12	0.73854 15
Saskat M	0.56227 18	0.53897 18	0.57290 20	0.50586 19
Alberta F	0.64645 14	0.63712 15	0.71982 17	0.69337 17
Alberta M	0.49467 20	0.47790 20	0.60848 18	0.54721 18
B.C. F	0.96193 2	0.98483 1	0.95686 2	0.93921 5
B.C. M	0.84871 7	0.85035 6	0.85007 7	0.79260 10
NorCan F	**0.00000 22**	0.01778 21	0.02542 21	0.12825 21
NorCan M	0.12261 21	0.01610 22	0.00270 22	**0.00000 22**
Ambiguity Index AA2	0.10233	0.10118	0.09133	0.10371

Table 5. Year on Year Progress in Embodied Human Capital (Educational Status)

$F_{year1} - F_{year2}$ Comparison	Transvariation	1st order Surface Separation (SS1)	2nd order Surface Separation (SS2)
2001-2006 Females	0.099985	1.000000	1.000000
2001-2006 Males	0.098815	1.000000	1.000000
2006-2011 Females	0.054490	1.000000	1.000000
2006-2011 Males	0.040700	1.000000	1.000000
2011-2016 Females	0.015395	0.649200	1.000000
2011-2016 Males	0.023135	-0.763089	-0.926233

Table 6. Year on Year Progress in Experience (Age)

$F_{year1} - F_{year2}$ Comparison	Transvariation	1st order Surface Separation (SS1)	2nd order Surface Separation (SS2)
2001-2006 Females	0.035790	1.000000	1.000000
2001-2006 Males	0.037700	1.000000	1.000000
2006-2011 Females	0.033080	0.714339	1.000000
2006-2011 Males	0.033740	0.860656	0.967055
2011-2016 Females	0.032795	1.000000	1.000000
2011-2016 Males	0.033555	1.000000	1.000000

Table 7. Gender Differences in Education Status

$F_{female-}F_{male}$ Comparison	Transvariation	1st order Surface Separation	2nd order Surface Separation
2001	0.036755	1.000000	1.000000
2006	0.046455	0.669330	1.000000
2011	0.045385	0.045287	-0.222323
2016	0.040475	-0.707550	-1.000000

Table 8. Gender Differences in Age Category

$F_{female-}F_{male}$ Comparison	Transvariation	1st order Surface Separation	2nd order Surface Separation
2001	0.029780	-1.000000	-1.000000
2006	0.026670	-1.000000	-1.000000
2011	0.021010	-1.000000	-1.000000
2016	0.018380	-1.000000	-1.000000

Table 9. Year on Year Progress, Joint Distribution

$F_{year1} - F_{year2}$ Comparison	Transvariation	1st order Surface Separation	2nd order Surface Separation
2001-2006 Females	0.12254	1.00000	1.00000
2001-2006 Males	0.11386	1.00000	1.00000
2006-2011 Females	0.07904	0.96358	1.00000
2006-2011 Males	0.06223	0.97661	1.00000
2011-2016 Females	0.04428	0.98209	1.00000
2011-2016 Males	0.04617	0.67002	0.62168

Table 10 Age-Education Joint Distribution Gender Comparison $(F_{female} - F_{male})$

$F_{female} - F_{male}$ Comparison	Transvariation	1st Order Separation	2nd Order Separation
2001	0.08248	-0.83744	-0.82182
2006	0.09116	-0.87032	-0.88107
2011	0.09438	-0.92672	-0.99018
2016	0.09553	-0.97848	-1.00000