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Are Eurozone Household Income Distributions Converging? Introducing MGT and DisGini, New Tools for Multilateral Distributional Comparisons.

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Abstract

The recent rise of economic nationalism together with the economic demise or potential departure of some of its constituent nations, has raised concerns regarding the coherence of the European Union. Formed to promote commonality of wellbeing among its constituents, there is interest in seeing whether its nation income distributions are converging. The usual practice of comparing per capita incomes has met with some criticism in the treatment effects and convergence literatures (Carniero et. al. 2003, Durlauf and Quah 2002) since such moment comparisons can mask important distributional differences relevant for analysis. Here, in response to these concerns, some indices and tests for quantifying the commonality or differentness in collections of distributions are proposed and implemented in the context of a sigma convergence study of 21st century Eurozone household income distributions. The results indicate that, when the Eurozone is considered as an entity with no nation boundaries, in a latent four class mixture distribution model, both weighted and unweighted versions of the statistics record convergence as the theory predicts for such a confederation. However when constituent nations are considered as separate entities within a confederation, unweighted versions of the tests record convergence, whereas weighted versions reveal divergence with increasing segmentation among the more populous nations in the Eurozone outweighing the convergent patterns exhibited by the smaller populated countries recently admitted to the Eurozone.

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1 Introduction

Milanovic (2011) was not the first to note that growing inequalities between constituencies within a federation can be a catalyst for the deterioration of its social cohesion and institutional support. The growth and convergence literature has a history of similar concerns. Both Quah (1993) and Friedman (1992) suggest that variation of average incomes across constituencies is of interest because it speaks directly to the question of whether the distribution of income across economies is becoming more or less equitable and a consequent cause for concern. Deterioration of cohesiveness has much to do with the extent to which economic wellbeing differs across constituencies and the sense in which such differences are perceived by agents within those constituencies. In this context, cohesiveness is more than just a matter of whether or not constituencies have similar average incomes, it is more a matter of whether or not they have common income distributions. When member nations are equally unequal with relatively similar income levels and distributions, there is a commonality of situation amongst member constituents which promotes cohesion, whereas a more divisive and alienated situation arises when such inequalities and income levels are not so equally shared in a more segmented society.

Empirical growth models predict that trans-nation variation in technologies, factor labour ratios and savings rates supported by nation boundary impediments will result in the formation of convergence clubs (Durlauf and Johnson 1995, Quah 1996, Galor 2011). The free flow of factors facilitated by the Eurozone's open borders should ameliorate the situation and promote convergence of those groups. The convergence literatures' approach to empirically assessing this issue, under the banner of sigma convergence (see for example Barro and Sala-i-Martin 1992, 1995; Galor 1996; Sala-i-Martin 1996a, 1996b; Quah 1997; Barro 1998; Higgins, Levy, and Young 2006), was to consider trends in the variation of subgroup average incomes (or logs thereof) that would be implied if there were negative covariation of growth and initial income levels. Carneiro, Hansen and Heckman (2002, 2003) highlight a problem with this approach in the context of fixed effects models, they demonstrate that employing such summary statistics to explore distributional variation over collections of populations can be misleading. The point is that just considering conditional means ignores important information about distributional differences which creates a "veil of ignorance" that can only be countervailed by comparing subgroup distributions across their complete range. This can be seen in a growth and convergence model context by noting that, if within subgroup variation grows sufficiently fast relative to average income variation, distributions will increasingly overlap and become more alike, effectively converging regardless of increasing subgroup mean variation (which records divergence). Durlauf and Quah (2002) make a similar point and argue for a means of comparing a collection of distributions throughout their range of variation. In truth seeking commonality in a collection of distributions is a somewhat stronger form of convergence than the sigma convergence alluded to above which only requires commonality of distributional locations.

It is also important to note that such conclusions are not independent of how the subgroups are weighted. Sometimes evaluation of the differences between constituent distributions without reference to their importance in the collection is required, other times relative constituent sizes will matter. If for example a collection of distributions is converging except for a particularly small subgroup, unweighted versions will exaggerate the lack of convergence whereas weighted versions will not. Curiously it has been the usual practice in convergence analyses to compare constituent mean deviations regardless of the relative size of those constituencies. Here, in answer to these concerns, new tools are introduced and employed to analyze the extent of differences in household income distributions within a substantial subset of the European Union, namely 18 of the 19 constituent nations of the Euro Area[1] (i.e. those nations who use the common Euro currency) in both population weighted and unweighted contexts over the period 2006-2015.

When comparing a collection of subgroups, there are good reasons for preferring (relative) average mean differences (Gini 1921) to the variance of a collection of objects (Yitzhaki 2003). Unlike the variance, squared coefficient of variation or Theil (1967) entropic measures, the Gini and Absolute Gini (Yitzhaki 1983, Chakravarty 1988) are not subgroup decomposable but that is an advantage in the present context since Gini can be decomposed to reflect the extent of latent commonality in the collection of distributions which the subgroup decomposable measures cannot. Here, another construct of Ginis, Distributional Transvariation (Gini 1916, 1959), is extended and employed to generate new Gini-like measures of relative distributional inequality in a collection of distributions in a multilateral comparison setting, in the form of weighted

[1]Technically, there are 19 nations in the Eurozone but Malta has been excluded due to limited data availability.

and unweighted Multilateral Transvariation (MGT) and Distributional Gini (DisGini) coefficients. In addition a radar chart is introduced (with a concomitant statistic) which provides a useful pictorial image of the convergence process. The measures are then applied in an analysis of the mutual segmentation/convergence of household income distributions in Eurozone nations.

It should be noted that a great variety of potential applications for these multilateral comparators are possible. Indeed, multilateral comparison of aspects of collections of distributions are ubiquitous. International ranking exercises of all kinds, be it health, wealth, education, income etc., in portfolio analysis finance returns of a collection of portfolios are compared, in mobility/equal opportunity, treatment effect, event and policy outcome literatures, circumstance conditioned outcome distributions are compared.[2] All essentially employ summary statistics recording distributional locations, conditional means, spreads and other features of interest relating to those variates over collections of populations, the analysis of which could be improved by employing multilateral distributional comparators.

The multilateral generalizations of Gini's Transvariation statistic and the Distributional Gini coefficient, each record the extent of distributional inequality of a collection of K potentially multivariate distributions. The collection may be considered in the context of individual distributions being components within a mixture $f(x) = \sum_{k=1}^{K} w_k f_k(x)$ where w_k are weights reflecting the importance of the component in the mixture. So, for example, $f(x)$ may refer to a societal income distribution with $f_k(x)$ being the income distribution of the k'th constituency and w_k its relative population size. The weights reflect the relative size of the nation, and hence its importance in the calculus. Alternatively, from a representative agent or treatment effect perspective, the distributions in the collection could be compared directly, without reference to their importance, in which case w_k would be set to $1/K$ for all k. Here, the degree of commonality between constituencies without reference to the relative size of alternative populations is of intrinsic interest as it would be for example, when an agent

[2]For examples in international comparisons Banerjee and Duflo (2008), UNDP (2016), for wellbeing measurement, Blackorby and Donaldson (1978) discuss wellbeing measures for ordering a collection of distributions based upon functions of summary statistics. In finance, following Markowitz (1952), portfolio returns distributions are compared on a combined mean - variance basis (e.g. Bali, Brown, and Demirtas 2013; Banz 1981; Basu 1983; Jegadeesh 1990). In the Equality of Opportunity and Mobility literatures (e.g. Arrow, Bowles and Durlauf 2000; Herrnstein and Murray 1994; Roemer 1998; Weymark 2003) ability distributions of various forms are compared. In treatment effect, event and matching study and policy evaluation literatures (Angrist and Krueger, 2001) assessment is based upon comparisons of conditional means of distributions across outcome states.

contemplates membership of any of the nations in the collection under comparison in a classic Harsanyi–Rawls veil of ignorance exercise (Harsanyi 1953, 1959, Rawls 1971, 2001). In this context the index would reflect the extent of multilateral commonality or difference such an agent confronts in her choice process.

In Europe, the 21st century rise of economic nationalism (Pazzanese 2017, Zettelmeyer 2019), the economic demise of some of its constituent nations, and the potential departure of others, has given cause for concern regarding the Unions coherence (Lindberg 2019, Krastev 2014, Webber 2018, Brandolini and Rosolia 2019) In response, a study of the evolution of household income inequality in the Eurozone is performed. What emerges is a collection of distributions that result in a Eurozone with an increasingly unequal overall income distribution comprised of an increasingly similar (i.e. convergent) collection of unweighted distributions that, when population weighted, become divergent as a collection. In the following, Section 2 develops two versions of the Multilateral Transvariation measure, the corresponding Distributional Gini coefficients are developed in Section 3 and some properties of the measures are explored in Section 4. Section 5 reports the main results of the analysis of the EuroArea income distribution and conclusions are drawn in Section 6.

2 Multilateral Transvariation: A Many Distribution Generalization of Gini's Transvariation Measure

In his transvariation measure GT, Gini (1916, 1959) provided a measure of the difference between two distributions[3] which, for two distributions $f_i(x), f_j(x)$ whose support[4] is confined to $\mathbb{R}^+$, can be also defined as (Anderson, Linton and Thomas 2017):

$$GT_{ij} = \frac{1}{2}\int_0^{\infty} |f_i(x) - f_j(x)|\, dx = \\ = \frac{1}{2}\int_0^{\infty} [max(f_i(x), f_j(x)) - min(f_i(x), f_j(x))]dx \quad (1)$$

[3]See also Pittau and Zelli (2017) for an overview of Gini's original concepts of transvariation.

[4]Since, as comprehensively discussed in Manero (2017), the Gini coefficient has problems with negative values of x, the present discussion is confined to distributions defined on $\mathbb{R}^+$, though the Multilateral Transvariation and distributional Gini Coefficients introduced later do not suffer the same difficulties and are defined for x on the real line $\mathbb{R}$.

Since $0 \leq \int_0^\infty |f_i(x) - f_j(x)|\, dx \leq 2$, pre-multiplying by 0.5 yields a statistic that will be 0 when the two distributions are identical and 1 when they have mutually exclusive support. Generalizing equation (1) to K distributions indexed $k = 1, \cdots, K$, suggests contemplating a Multilateral Gini Transvariation measure (MGT) where:

$$MGT = \frac{1}{K} \int_0^\infty \left[max\left(f_1(x), f_2(x), \cdots, f_K(x)\right) - min\left(f_1(x), f_2(x), \cdots, f_K(x)\right)\right] dx \tag{2}$$

When the distributions have mutually exclusive support $MGT = 1$, when the distributions are identical $MGT = 0$.

A weighted version of MGT, $MGTW$ is also possible, and has the form:

$$MGTW = \int_0^\infty \left[max\left(w_1 f_1(x), w_2 f_2(x), ., w_K f_K(x)\right) - \right. \\ \left. - \, min\left(w_1 f_1(x), w_2 f_2(x), ., w_K f_K(x)\right)\right] dx \tag{3}$$

where w_k are the proportions associated to the distributions f_k, $k = 1, \cdots, K$. One problem with the multilateral transvariation measure is its maximum-minimum nature, like the range statistic for a collection of numbers which does not reflect differences in objects in the mid range, it does not reflect the many bi-lateral functional differences and similarities camouflaged by just considering the extremes. Indeed, it is in essence the distributional analogue of the relative range measure of a collection of numbers wherein the relative locations of interior and low weight members have little or no impact on its value. An alternative which does reflect the relative locations of "interior" distributions is the distributional equivalent of the Gini coefficient or what will be referred to as DisGini.

3 DisGini: The "Distributional" Gini Coefficient

To give some context, an off cited criticism of Gini's relative mean difference coefficient (Gini 1921) is that it is not subgroup decomposable (Bourguignon 1979), although when subgroups have mutually exclusive, closed and bounded support, Mookherjee and Shorrocks (1982) demonstrate that it is. Given a collection of K subgroups indexed $k = 1, \cdots, K$ with respective income distributions $f_k(x)$ with associated cumulative densities $F_k(x)$ and corresponding means and population shares μ_k and w_k, the overall income distribution $f(x)$, mean income μ, and Gini coefficient, may be written as:

$$f\left(x\right) = \sum_{k=1}^{K} w_k f_k(x), \sum_{k=1}^{K} w_k = 1 \text{ and } w_k \geq 0 \text{ for all } k \tag{4}$$

$$\mu = \sum_{k=1}^{K} w_k \mu_k \tag{5}$$

$$\begin{aligned} \text{Gini} = \sum_{k=1}^{K} w_k^2 \frac{\mu_k}{\mu} G_k + \frac{1}{\mu} \sum_{k=2}^{K} \sum_{j=1}^{k} w_k w_j |\mu_k - \mu_j| + \\ \frac{2}{\mu} \sum_{k=2}^{K} \sum_{j=1}^{k-1} w_k w_j \int_0^\infty f_k(y) \int_y^\infty f_j(x)(x-y) dx dy = \\ = \text{WGini} + \text{BGini} + \text{NSF}. \end{aligned} \tag{6}$$

Thus, Gini is comprised of three components, WGini, a weighted sum of subgroup Ginis', BGini, a term which is the equivalent of a between group Gini coefficient of subgroup means measuring the relative inequality of distributional locations which is analogous to a population weighted variance of subgroup means employed in sigma convergence studies, and NSF, a non-segmentation factor reflecting the extent to which subgroup distributions overlap or are not segmented.[5] This term can be used to calculate a Gini based "segmentation index" SI which reflects the extent of segmentation in the collection of distributions where SI=1-NSF/Gini (Anderson et al. 2018).

Since $\mu = \int x f(x) dx = \int (1 - F(x))\, dx$, the middle term may be written:

$$\text{BGini} = \frac{\sum_{k=2}^{K} \sum_{j=1}^{k} w_k w_j |\int (F_k(x) - F_j(x))\, dx|}{\int (1 - F(x))\, dx}$$

Generally, $\mu_j - \mu_i = \int_0^\infty (F_i(x) - F_j(x))\, dx \leq \int_0^\infty |F_i(x) - F_j(x)|\, dx$, however when distribution j First Order Dominates distribution i, equality prevails and, when the relationship prevails for all pairs whose indices reflect the ordering, observe that:[6]

$$\text{BGini} = \frac{\sum_{k=2}^{K} \sum_{j=1}^{k} w_k w_j \int |F_k(x) - F_j(x)|\, dx}{\int (1 - F(x))\, dx} = \frac{2 \sum_{k=2}^{K} \sum_{j=1}^{k} w_k w_j \text{GTF}_{kj}}{\int (1 - F(x))\, dx},$$

[5]Note that when sub-distributions have mutually exclusive closed and bounded support, this last term disappears, hence the Mookherjee and Shorrocks (1982) result. On the geometric interpretation of NSF see Lambert and Aronson (1993).

[6]For example suppose one intersection point at $0 \leq a \leq \infty$, where $F_i(x) \geq F_j(x)$ for $x < a$ and $F_i(x) < F_j(x)$ for $x > a$, then $\int_0^\infty |F_i(x) - F_j(x)|\, dx = \int_0^a (F_i(x) - F_j(x))\, dx - \int_a^\infty (F_i(x) - F_j(x))\, dx = \int_0^\infty (F_i(x) - F_j(x))\, dx - 2\int_a^\infty (F_i(x) - F_j(x))\, dx > \int_0^\infty (F_i(x) - F_j(x))\, dx$ since $2\int_a^\infty (F_i(x) - F_j(x))\, dx$ is negative.

where:

$$\mathrm{GTF}_{kj} = 0.5 \int |F_k\,(x) - F_j\,(x)\,|dx.$$

BGini can be adapted to reflect differences in other aspects of subgroup distributions, here DisGini, a Gini–like coefficient for the inequality of a collection of probability distribution functions is developed. Following the above expressions of the middle term in (6) above, consider:

$$\mathrm{DisGini} = \frac{1}{\varphi}\sum_{i=1}^{K}\sum_{j=1}^{K} 0.5 \int_0^\infty w_i w_j |f_i(x) - f_j(x)|\,dx = \frac{1}{\varphi}\sum_{i=1}^{K}\sum_{j=1}^{K} w_i w_j \mathrm{GT}_{ij} \qquad (7)$$

Where φ may be considered a scaling parameter. Note the term "$\int_0^\infty w_i w_j |f_i(x) - f_j(x)|\,dx$" may be written as "$w_i w_j 2\mathrm{GT}_{i,j}$" is twice Gini's Transvariation of sub distributions $f_i(x)$ and $f_j(x)$, multiplied by the product of the respective population shares. $\mathrm{GT}_{\mathrm{i,j}}$ is related to $\mathrm{OV}_{\mathrm{i,j}}$ the extent to which distributions $f_i(x)$ and $f_j(x)$ overlap (Anderson, Linton and Whang 2012):

$$\mathrm{OV}_{ij} = \int_0^\infty \min(f_i\,(x)\,, f_j\,(x))\,dx.$$

Essentially $\mathrm{GT}_{\mathrm{i,j}}$ = 1-$\mathrm{OV}_{\mathrm{i,j}}$ so that (7) may be written:

$$\mathrm{DisGini} = \frac{1}{\varphi}\sum_{i=1}^{K}\sum_{j=1}^{K} w_i w_j (1 - \mathrm{OV}_{ij})$$

Which, letting c be a K element column vector of ones, may be written as:

$$\frac{1}{\varphi}c' \begin{bmatrix} \int_0^\infty w_1 w_1(1 - OV_{11})dx & \int_0^\infty w_1 w_2(1 - OV_{12})dx & \dots & \int_0^\infty w_1 w_K(1 - OV_{1K})dx \\ \int_0^\infty w_2 w_1(1 - OV_{21})dx & \int_0^\infty w_2 w_2(1 - OV_{22})dx & \dots & \int_0^\infty w_2 w_K(1 - OV_{2K})dx \\ \vdots & \vdots & \ddots & \vdots \\ \int_0^\infty w_K w_1(1 - OV_{K1})dx & \int_0^\infty w_K w_2(1 - OV_{K2})dx & \dots & \int_0^\infty w_K w_K(1 - OV_{KK})dx \end{bmatrix} c \qquad (8)$$

Consider a typical element $\int_0^\infty w_i w_j (1 - \mathrm{OV}_{ij})dx$, when $i = j$ the element will be zero, also when $f_i(x) = f_j(x)$ for all x (i.e. subgroups i and j have identical distributions), the term will be 0. It follows that when all subgroups have identical distributions, expression (8) will be 0 since all of the elements are non-negative this will constitute a lower bound for DisGini.

Now consider the situation where all of the respective subgroup income distributions have mutually exclusive support, i.e. the subgroups are completely segmented so that for all $i \neq j$ and a given x, $f_i(x) \geq 0 \Rightarrow f_j(x) = 0$ and $f_j(x) \geq 0 \Rightarrow f_i(x) = 0$. This corresponds to the income mixture distribution situation where there is no commonality in incomes or distributional overlap between any constituency pairing, twice Gini's Transvariation would be at a maximum value of 2. It follows that all the elements of the matrix have their respective maximal values. Then:

$$\int_0^{\infty} w_i w_j |f_i(x) - f_j(x)| dx = \begin{cases} 2w_i w_j & \text{when i} \neq \text{j} \\ 0 & \text{when i=j} \end{cases}$$

In this case (7) may be written:

$$\frac{1}{\varphi} c' \begin{bmatrix} 0 & w_1 w_2 & \dots & w_1 w_K \\ w_2 w_1 & 0 & \dots & w_2 w_K \\ \vdots & \vdots & \ddots & \vdots \\ w_K w_1 & w_K w_2 & \dots & 0 \end{bmatrix} c = \frac{1}{\varphi} \sum_{k=1}^{K} w_k (1 - w_k) = \frac{1 - \sum_{k=1}^{K} w_k^2}{\varphi}$$

If the scaling parameter φ is set to $(1 - \sum_{k=1}^{K} w_k^2)$ then DisGini will always fall in the interval [0,1] and be equal to 1 when there is complete distributional inequality in terms of complete segmentation of the constituency distributions. It follows that DisGini may also be written as:

$$\text{DisGini} = \frac{1}{(1 - \sum_{k=1}^{K} w_k^2)} \sum_{i=1}^{K} \sum_{j=1}^{K} w_i w_j (1 - \text{OV}_{ij}) = \frac{2}{(1 - \sum_{k=1}^{K} w_k^2)} \sum_{i=2}^{K} \sum_{j=1}^{i-1} w_i w_j \text{GT}_{ij}. \tag{9}$$

If comparison of the distributions without subgroup weighting is desired, as in the aforementioned representative agent type scenarios, simply set $w_i = \frac{1}{K}$ for all $i = 1, \cdots, K$.

4 Further Considerations

4.1 Multivariate Distributions and Higher Order Integrals

An interesting feature of MGT and DisGini, is that they can handle multivariate distributions of discrete or continuous forms (or mixtures of both) which is a bit of a challenge for the standard Gini coefficient. Simply write (2) and (7) respectively as:

$$\text{MGT} = \frac{1}{K} \iint \int_0^{\infty} [\max(f_1(x,y,z), f_2(x,y,z), .., f_K(x,y,z)) - \min(f_1(x,y,z), f_2(x,y,z), .., f_K(x,y,z))] dxdydz$$

and

$$\text{DisGini} = \frac{1}{\varphi}\sum_{i=1}^{K}\sum_{j=1}^{K} w_i w_j \text{GT}_{ij} =$$

$$\frac{0.5}{\varphi}\sum_{i=1}^{K}\sum_{j=1}^{K}\iint\int_0^{\infty} w_i w_j |f_i(x,y,z) - f_j(x,y,z)|dxdydz$$

Formulae (3) and (9) then follow directly. In addition, by replacing the $f_i(x)$ with $F_i^h(x)$ where $F_i^h(x) = \int_0^x F_i^{(h-1)}(z)dz$ in (2) or (7) and adjusting the normalizing parameter accordingly, multilateral variation of higher order integrals of distribution functions could be contemplated reflecting the classic stochastic dominance criteria for more restrictive wellbeing structures (see Anderson, Post and Whang 2019). All of which is are matters for future research.

4.2 Axiomatic properties of inequality indices

These indices provide a complete ordering of collections of distributions with respect to their differentness, as such they satisfy some popular axioms in the inequality literature, anonymity, scale invariance (multiplying x by a constant does not influence the index), translation invariance (adding a constant to each x does not influence the index), and normalization axioms (the index is bounded between 0 and 1 with 0 corresponding to complete equality and 1 complete inequality) can be shown to be satisfied by MGT and DisGini.

Though, when sub-distributions are posited to be the atomistic equivalents of the sub-distributions employed in Duclos, Esteban and Ray (2004) and subjected to the same transformations, they comply with the polarization axioms posed therein, it should be noted that these indices do not generally comply with the principle of transfers (Dalton 1920) since counter examples are easy to contrive. It should also be noted that although the Gini coefficient has problems with negative incomes (see Manero 2017), it is not a problem for MGT and DisGini coefficients.

4.3 Inference and approximate standard errors

Since GT=1-OV and $\widehat{\text{OV}}$, the estimate of OV, has an asymptotically normal distribution $N\left(OV, V\left(\widehat{OV}\right)\right)$, where, to a close approximation, provided the contact set is empty and n_k is the sample size in the k'th population, $\widehat{V}\left(\widehat{OV}_{ij}\right) = \frac{n_i+n_j}{n_i n_j}\left(\widehat{OV}_{ij}(1-\widehat{OV}_{ij})\right)$

(Anderson, Linton and Whang 2012). These ideas are used in developing the distributions of MGT and DisGini in the Appendix.

5 Household income inequality in the Eurozone: 2006-2015

Since the late 20th century convergence debate (Galor 1996) many empirical growth models have held out the possibility of multiple equilibria engendering distinct poles of attraction for collections of national income distributions (Durlauf and Johnson 1995, Quah 1996). Whereas neoclassical growth models predict that nations with common technological, population structure and growth, and savings rate circumstances will converge to a unique steady-state equilibrium per capita income level regardless of their respective initial conditions, lack of commonality in any of these circumstances will lead to diverse poles of attraction based upon circumstance commonality within subgroups of the collection of nations. Such diversity of circumstance results in a concomitant diversity of subgroup income distributions in the form of "Convergence Clubs". Modern unified growth theory (Galor 2011) predicts a threefold nation convergence club typology with slow or zero growth Malthusian states, sustained steady growth states and faster growing states effectively transitioning from the Malthusian state. To the extent that the various capital, labour and technology factors become perfectly and freely mobile across nation boundaries, diversity of circumstances (and hence the diversity of income distributions and nation typologies) should shrink suggesting that convergence clubs could be a transitory and temporary phenomenon with the diversity of incomes shrinking as in a sigma convergence paradigm (Sala-i-Martin 1996a).

Viewed as an entity, the Eurozone household income distribution $f(x)$ is a mixture of the household income distributions $f_k(x)\, k = 1,..,K$ of its K constituent nations where the weights w_k correspond to relative population sizes so that:

$$f(x) = \sum_{k=1}^{K} w_k f_k(x) \quad k = 1, \cdots, K$$

Stochastic processes are frequently used to rationalize distributional structures and Gibrat's Law of Proportional Effects and some of its modifications (Gibrat 1931, Gabaix 1999, Reed 2001) have been foundational in providing a theoretical rationale for expecting increasing income inequality. The Law posits that household incomes in subgroup k follow a stochastic process which in its simplest form in period t, has the form:

$$x_{k,t} = (1 + \delta_{k,t})\, x_{k,t-1}$$

where $\delta_{k,t}$ is a random variable with mean δ_k (which is small relative to one in absolute value) and variance σ_k^2. The law predicts that, given a starting value x_0 and letting $X = \ln(x)$, after T periods X_{kT} will have a mean equal to $X_0 + T\,(\delta_k + 0.5\sigma_k^2)$ and variance equal to $T\sigma_k^2$, respectively i.e. log income variation that grows through time. Following Modigliani and Brumberg (1954), classical economic models of income (Friedman 1957, Hall 1978) use this idea to predict increasingly unequal income distributions (Battistin, Blundell, and Lewbel 2009, Blundell and Preston 1998, Browning and Lusardi 1996, Anderson 2012). When applied to the $k = 1, \cdots, K$ constituent societies in the Eurozone, clearly different configurations of pairs (δ_k, σ_k^2) for $k = 1, \cdots, K$ will yield collections of distributions that could be converging or diverging, segmenting or increasingly overlapping, becoming more or less equal in distribution.

The weighted and unweighted versions of the Multilateral Transvariation and Distributional Gini statistics can yield insights into the progress of such distributional inequalities over the era, tending toward 0 as distributions converge and tending toward 1 as they segment or diverge. The weighted versions give insight into distributional differences of the Eurozone as an entity, with small populations given low weight and large populations high weight. The unweighted versions can be construed as a representative agent model recording the juxtaposition of nation income distributions directly without respect to their relative importance in the overall Eurozone income distribution.

The data source is the European Union Survey on Income and Living Conditions (EU-SILC)[7], a harmonized household-level survey that is a collection of annual national surveys of socio-economic conditions of individuals and households in EU countries. Standard questionnaires and procedures for data processing have yielded ex-ante harmonized micro-data that allow homogeneous inter-country comparisons using a uniform protocol. To analyze the evolution of the Euro area income distribution over time, four temporally equi-spaced waves, 2006, 2009, 2012 and 2015 were chosen. Since data for Malta are only available from the 2008 wave, this country is excluded from analysis leaving 18 Euro zone countries. Income is the total household net disposable

[7] Version estatCROS 2019ki9, released in May 2019.

annual income (in thousands Euro) obtained by aggregation of all income sources from all household members net of direct taxes and social contributions.[8] All households are weighted by cross-sectional weights. Here we used population-weights to extrapolate from the sample of households to the total of the population. The assumption is that members of the the household equally share its income. Assuming consumption economies of scale in cohabitation, incomes are age and size-adjusted using the modified-OECD equivalence scale which assigns a value of 1 to the household head, of 0.5 to each additional adult member aged 14 and over and of 0.3 to each child aged under 14. Given significant disparities in the cost of living between countries, the PPP index for the household final consumption expenditure is used to adjust household incomes. For consistency with standard inequality comparisons (Cowell 2011, Cribb et al. 2013, San Martin et al. 2003), households whose income is less than zero were excluded from the sample (less than 0.5% of the sample units) though it should be noted, this is not necessary for the Multilateral Transvariation and Distributional Gini comparisons which can deal with negative income values.

The average equivalized disposable incomes in the EuroArea countries are reported in Table 1. As an entity, the Eurozone had overall household income Gini coefficients of 0.305, 0.313, 0.317, 0.335 for the years 2006, 2009, 2012 and 2015 respectively, suggesting ever increasing household income disparities in the area over the period. In the light of concerns regarding European disintegration, questions arise as to the extent to which such inequalities are equally shared across its various nations. Two approaches to the Eurozone household income distribution are taken, the first considers the constituent nations as the subgroups, the second considers a transnational decomposition based upon four latent household income classes that transcend nation boundaries treating the Eurozone as an entity which were determined by a semiparametric mixture distribution analysis (Anderson, Pittau, Zelli and Thomas 2018).

5.1 Nation subgroup analysis

To get some sense of the nation based distributional changes in the Eurozone, Table 2 shows the overall Gini coefficient and the decomposition results over the period 2006–2015. The BGini, the relative mean absolute difference of country means, being a function of the relative locations of distributions yields insight into their progress over

[8]The income reference period refers to the previous year, consequently analysis with EU-SILC files actually refers to 2005-2014.

Table 1: Mean household incomes in Eurozone countries

		Mean household income			
Countries		**2006**	**2009**	**2012**	**2015**
AT	Austria	19.17	21.23	22.13	23.59
BE	Belgium	17.83	18.92	19.49	21.68
CY	Cyprus	18.60	21.22	20.64	17.83
DE	Germany	16.97	20.68	21.40	23.14
EE	Estonia	7.64	10.51	10.61	13.59
EL	Greece	13.17	14.40	10.96	9.06
ES	Spain	12.63	16.28	15.05	13.05
FI	Finland	17.57	20.56	21.73	26.10
FR	France	19.27	23.37	23.33	28.70
IE	Ireland	18.77	19.65	18.58	19.90
IT	Italy	16.20	17.80	17.87	17.57
LT	Lithuania	6.33	9.87	8.80	11.44
LU	Luxembourg	27.83	28.99	27.12	29.22
LV	Latvia	6.40	9.27	8.33	10.60
NL	Netherlands	18.85	21.89	20.42	21.25
PT	Portugal	11.82	12.49	11.95	12.38
SI	Slovenia	13.74	15.88	15.48	16.40
SK	Slovakia	5.97	9.59	11.91	11.54

Note: Equivalized household disposable annual income ppp adjusted (000 €).

the period. Noting that, while the within group component WGini is growing steadily but relatively slowly over the period, consistent with Gibrat's law, there appears to be a substantial increase in between nation inequality over the period reflecting a sustained divergence of national average household incomes over the period in line with the overall Gini coefficient for the Eurozone which reflects potentially increasing inequality as perceived by Eurozone member nations. Notice that this is at odds with the variance of conditional means employed in sigma convergence studies which records a fall and then a rise in variation, however the weighted version, which is not typically employed in sigma convergence analysis, records a monotonic increase over the period. The segmentation index clearly indicates the monotonically increasing inequality in the Eurozone is largely a consequence of the apparent monotonically increasing segmentation of the Eurozone nation distributions.

Table 2: Gini overall inequality in the EuroArea and its decomposition

Year	**2006**	**2009**	**2012**	**2015**
Gini Total	30.48	31.30	31.74	33.52
WGini (Gini Within)	4.45	4.76	4.79	4.81
BGini (Gini Between)	9.62	9.65	10.90	16.07
NSF (Non Segmentation Factor)	16.41	16.05	15.84	12.64
SI (Segmentation Index)	46.19	46.04	49.43	62.29
Variance of conditional means	33.39	30.33	30.72	40.51
Variance of conditional means (weighted)	44.29	48.96	51.81	53.39

However, recalling the Durlauf and Quah (2002) and Carneiro, Hansen and Heckman (2002, 2003) veil of ignorance concerns, be aware that these results are based on summary statistic comparisons (essentially differences in means and progressions in variances) which veil overall distributional differences. Table 3 reports the unweighted and weighted Distributional Gini Coefficients (DisGini and DisGiniW) and Multilateral Gini Transvariation (MGT and MGTW). The income densities $f_k(x)$ are kernel estimated including the sample weights and using the Sheather and Jones (1991) bandwidth. Looking at the patterns of the indices a quite different stories emerge under population weighted and unweighted versions of the statistics. While the nation weighted versions of the indices, after a slight dip in 2009, show a significant increase, the unweighted versions record a decline over the whole period with respect to 2006. Thus a representative agent view of the Eurozone suggests increasing commonality of household disposable income distributions whereas the population weighted version suggests increasing segmentation. Thus nations with larger populations appear to be segmenting whereas nations with small populations are not.

Table 3: Unweighted and weighted Distributional Gini coefficients and Multilateral Gini Transvariation - nation group analysis

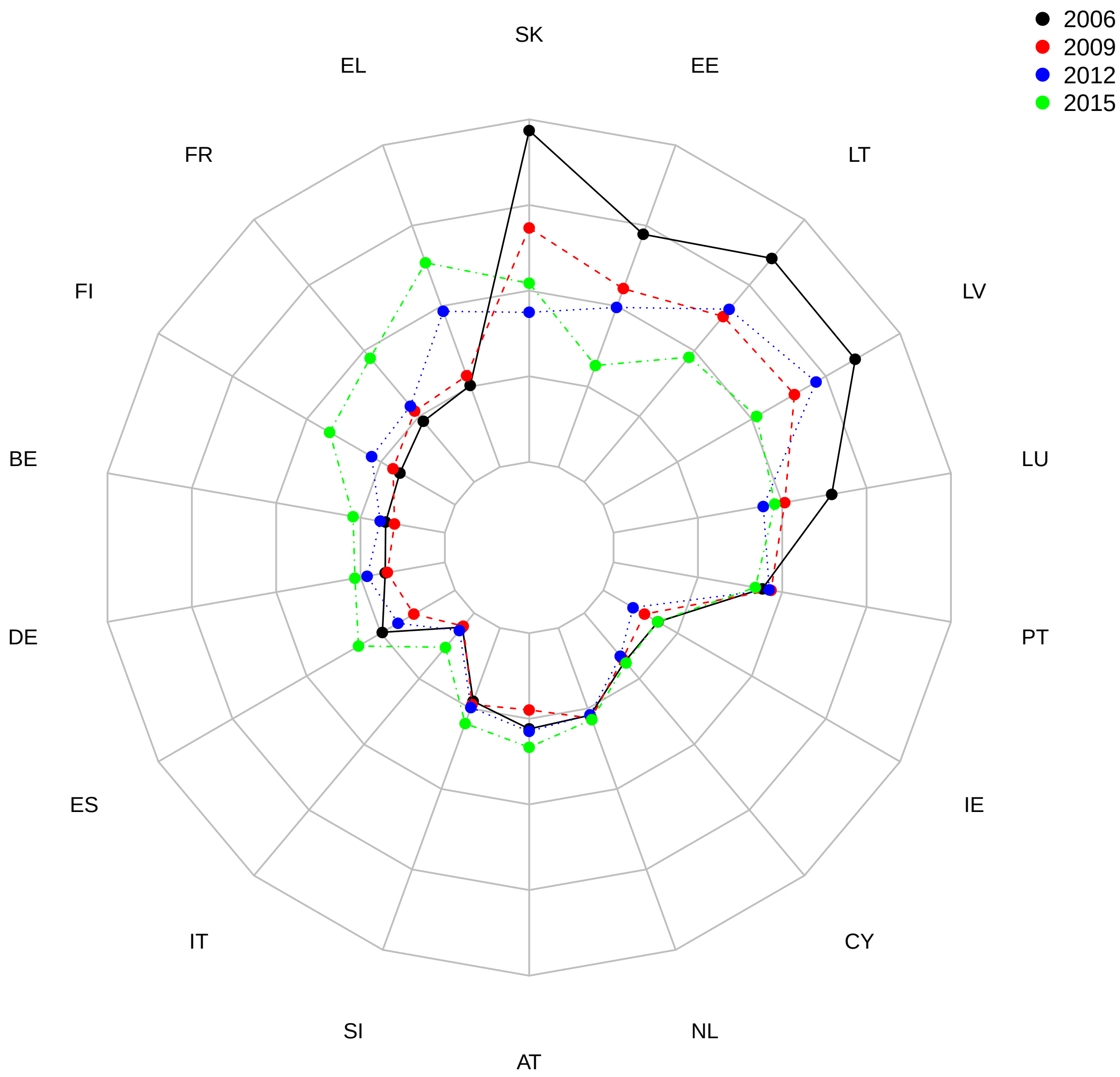
Year	**DisGini**	**DisGiniW**	**MGT**	**MGTW**
2006	0.386	0.237	0.135	0.291
	(0.003)	(0.003)	(0.004)	(0.005)
2009	0.341	0.222	0.111	0.279
	(0.003)	(0.003)	(0.004)	(0.005)
2012	0.326	0.282	0.106	0.323
	(0.003)	(0.003)	(0.003)	(0.005)
2015	0.341	0.361	0.107	0.349
	(0.003)	(0.003)	(0.004)	(0.005)

Note: Approximate standard errors are in brackets.

A further insight on the extent to which each country is converging or diverging to the Eurozone norm is given by the bilateral transvariations between each country and the whole Eurozone distribution. Remembering the measure GT equal to 1 means complete segmentation (two distribution are far apart), while GT equal to 0 means complete overlapping, Figure 1 reports a radar chart of the bilateral transvariation between each country and the Eurozone as a whole. The closer is a point on a nations spoke to the periphery, the higher is the transvariation of that nations income distribution with respect to Eurozone distribution. The closer to the center is a point the closer is that nations income distribution to convergence with the Eurozone income distribution. The points have been colour coded by year, so that intuitively nations with green dots nearer the centre than black dots are converging to the Eurozone distribution, whereas nations with green dots outside of the black dots are diverging from the Eurozone distribution over the observation period. The Radar Chart also suggests another convergence index, the value of the area enclosed by connecting the dots of a common colour. For Figure 1 this yields 2006 0.818, 2009 0.532, 2012 0.536 and 2015 0.571, reiterating the notion that the unweighted distributions are converging to the overall Eurozone distribution over the period.

The bilateral transvariations range from 0.03 for Italy to 0.68 for Slovakia in the year 2006. The pattern of this index shows a process of convergence toward the EuroArea distribution for Eastern European countries (notably low population countries)

Figure 1: Radar chart of bilateral transvariation of each country with respect to Eurozone. The center of the wheel corresponds to the minimum value of the measure or complete overlapping with respect to the Eurozone distribution. Moving to the periphery reflects divergence to, or decreasing commonality with, the Eurozone distribution. Countries are clockwise ordered starting with the largest positive difference between 2006 and 2015 (indicating convergence) and ending with the largest negative difference (indicating divergence).



and significant divergence from the Eurozone distribution for Spain, Finland, France and Greece. Figures 2 and 3 show the evolution of the income distributions of Slovakia, Estonia, Latvia, Lithuania and their overlapping with respect to the Eurozone distribution in 2006 and in 2015. Divergence occurs in Southern European countries, mainly Greece and Spain (notably more populated countries), due to a downward shift of their distributions. Divergence also occurs in France and Finland but in this case due to an upward shift of the distributions. Figures 4 and 5 show the evolution of the income distributions of Greece, Spain, France and Finland and their overlapping with respect to the Eurozone distribution in 2006 and in 2015. All the bilateral transvariation measures are reported in Appendix A.3.

Figure 2: Income distribution of Slovakia and Estonia and their overlapping with the Eurozone income distribution: years 2006 and 2015.

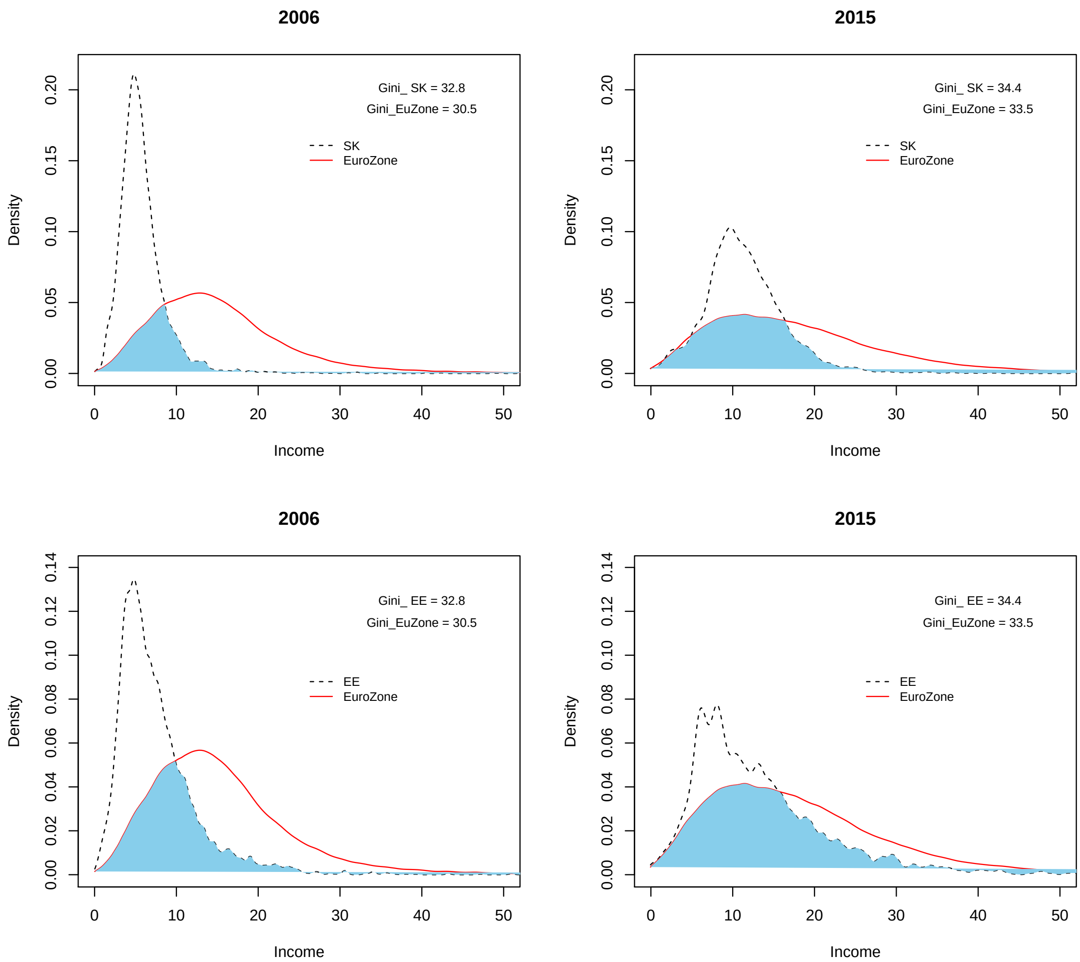


Figure 3: Income distribution of Latvia and Lithuania and their overlapping with the Eurozone income distribution: years 2006 and 2015.

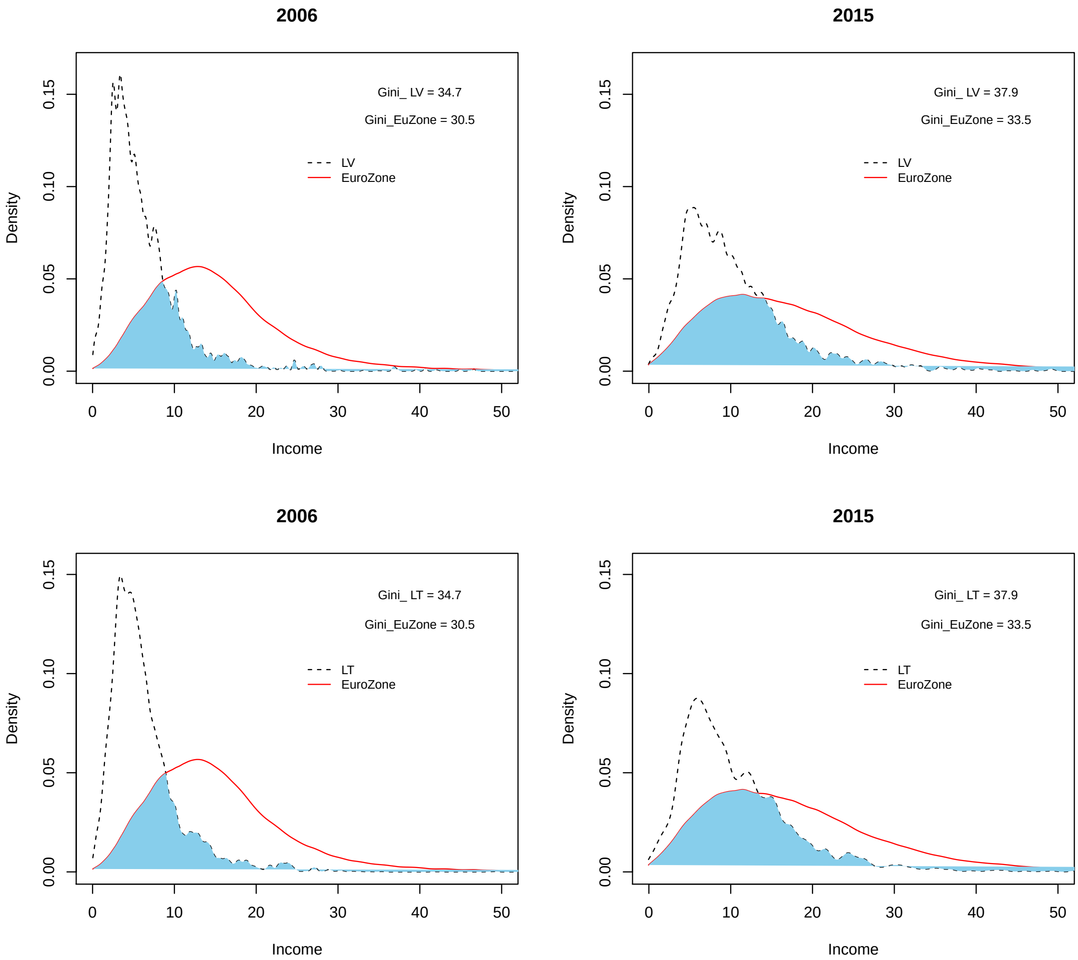


Figure 4: Income distribution of Greece and Spain and their overlapping with the Eurozone income distribution: years 2006 and 2015.

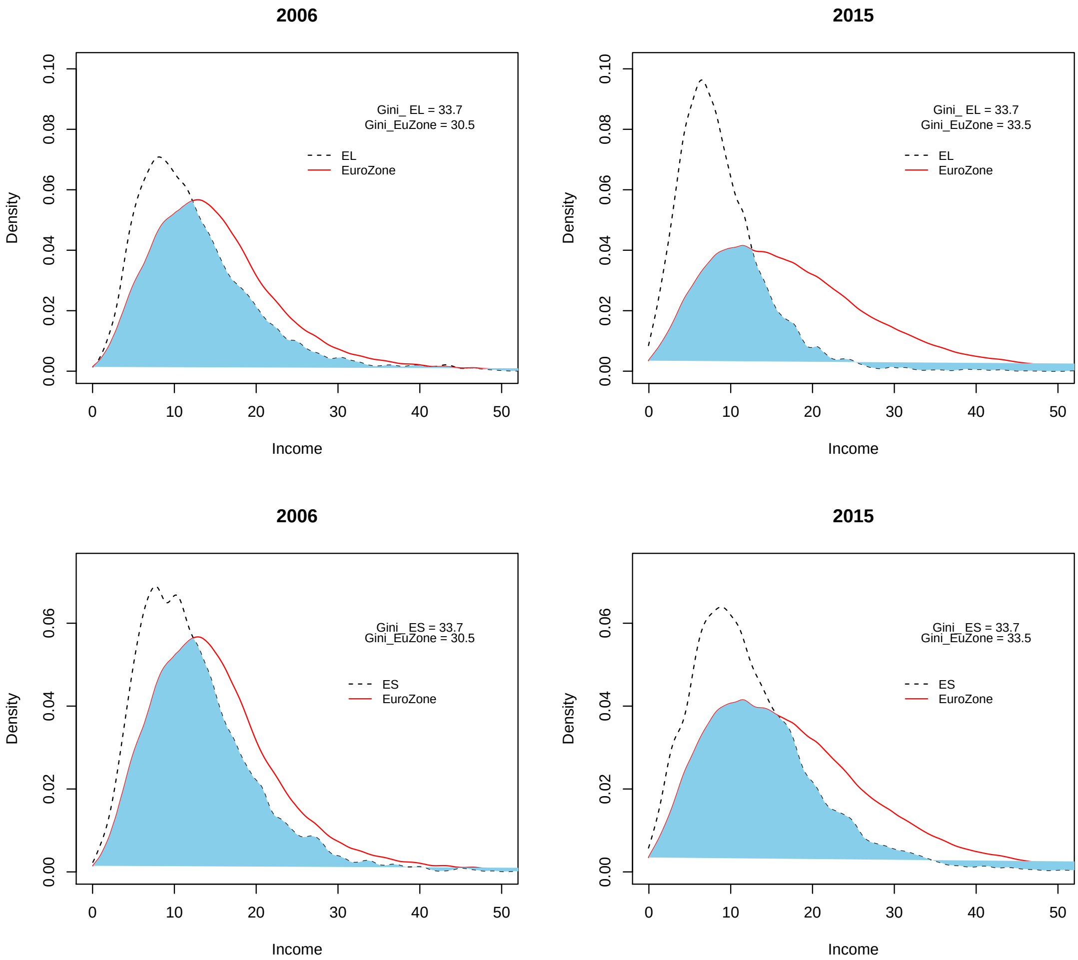
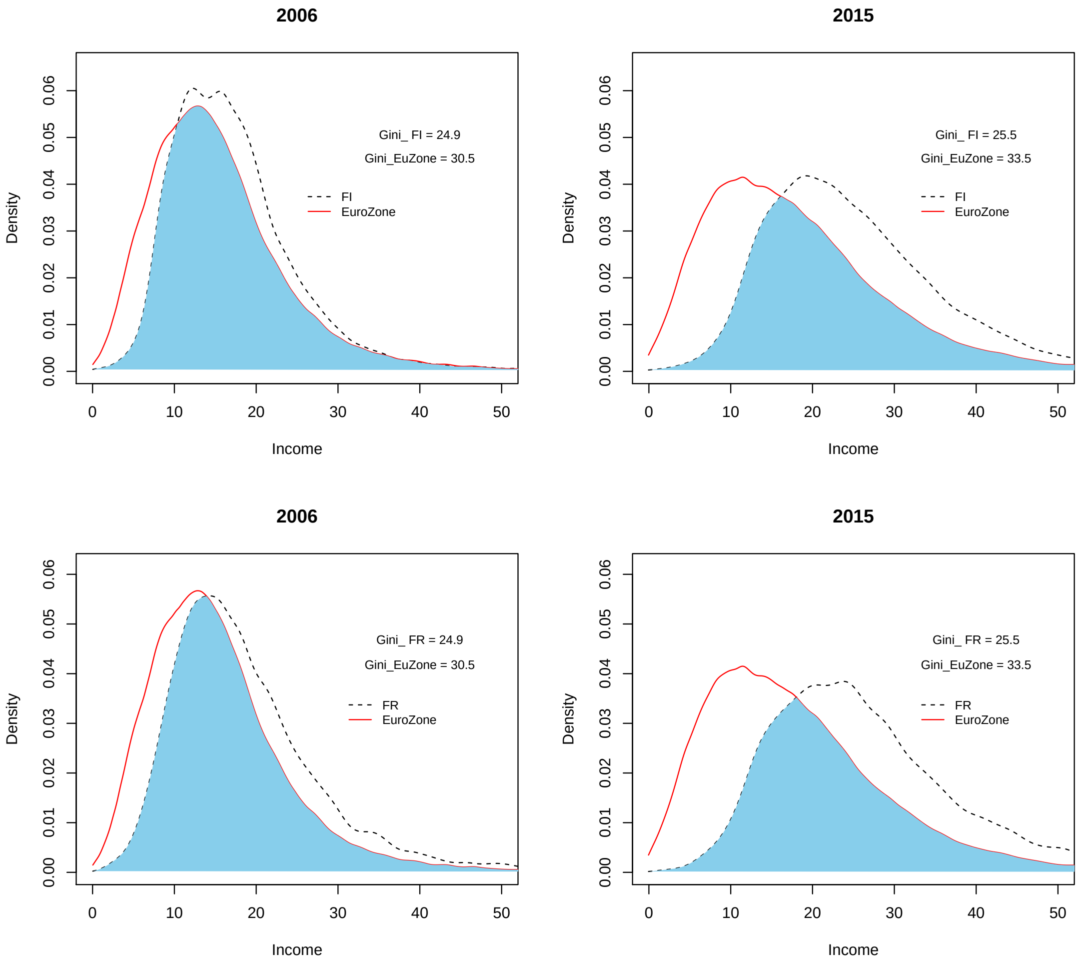


Figure 5: Income distribution of Finland and France and their overlapping with the Eurozone income distribution: years 2006 and 2015.



5.2 Latent subgroup analysis

Following the predictions of the unified growth theory (Galor 2011) and the enhanced factor mobility that the Eurozone offers, a transnational convergence club model seems appropriate. Using the same data set and invoking Gibrat's Law to define sub class distributions, Anderson, Pittau, Zelli and Thomas (2018) developed a latent class model for the Eurozone as an entity in itself with classes that transcend national borders determined by semi-parametric data – based machine learning methods. What emerged as the best fit solution, contrary to the modern growth theory prediction of a three class model (Galor 2011), was a four-class, increasingly unequal polarizing structure with income growth in all four classes, where the two middle income groups comprised more than 70% of the population and the poorest group increased in size from 14% to 19% of the population over the period. The means μ, standard deviations σ and population shares w of the normally distributed income distributions for the subgroups are reported in Table 4 together with the between subgroup Gini coefficients, BGini.

Table 4: Estimated parameters of the components of the mixtures

Year	2006			2009			2012			2015		
Class	μ	σ	w	μ	σ	w	μ	σ	w	μ	σ	w
Low (L)	6.23	2.54	0.14	8.48	3.27	0.18	8.34	3.60	0.18	8.00	3.64	0.19
Lower-Middle (LM)	12.79	4.02	0.48	14.76	4.56	0.38	14.66	4.76	0.38	15.40	5.15	0.39
Upper-Middle (UM)	20.40	5.86	0.33	21.97	6.46	0.35	22.70	6.90	0.37	25.17	7.57	0.36
High (H)	35.61	6.03	0.05	36.95	8.69	0.09	39.84	7.89	0.07	44.39	7.36	0.06
BGini	0.211			0.218			0.220			0.237		

Note: μ and σ are expressed in PPA-adjusted Euros. w are the mixing proportions.

Again, in concert with the overall Gini coefficient, the BGini coefficient suggests a sustained divergence of the subgroup income distributions over successive years that is not born out in the results for the Multilateral Transvariation and Distributional Gini coefficients reported in Table 5. In this scenario, the unweighted and weighted version of both the coefficients tell the same story of a significant fall in distributional variation over the 2006-2009 period (presumably a consequence of the convergence of the highest and lowest income classes due to the crisis in 2007-2008) followed by steady increases in distributional variation from post 2009 that fail to return to the levels of distributional variation in 2006. So that overall the weighted and unweighted versions of the four class Eurozone income distribution indicate convergence over the whole period.

Table 5: Weighted and unweighted Multilateral Transvariation and Distributional Gini coefficients - latent group analysis

Year	DisGini	DisGiniW	MGT	MGTW
2006	0.824	0.694	0.763	0.729
	(0.002)	(0.003)	(0.005)	(0.005)
2009	0.716	0.648	0.665	0.660
	(0.003)	(0.003)	(0.005)	(0.005)
2012	0.767	0.657	0.704	0.667
	(0.002)	(0.003)	(0.005)	(0.005)
2015	0.803	0.695	0.741	0.700
	(0.002)	(0.003)	(0.005)	(0.005)

Note: Approximate standard errors are in brackets.

6 Conclusions

The 21st century economic demise of some of its constituent nations, the potential departure of others, together with growing European Union household income inequality, could give cause for concern regarding the potential deterioration of its cohesiveness as a Union. This has much to do with the extent to which nation income distributions are segmenting or converging or whether differences are commonly felt across national boundaries. When constituent nations are equally unequal with relatively similar income levels, there is a commonality of situation which promotes cohesiveness, whereas, when such inequality and income levels are not equally shared, the situation is somewhat more divisive and alienating. Here, new measures of multilateral distributional variation, together with their standard errors, are introduced and employed to address these distinctions within a substantial subset of nations in the European Union, namely 18 of the constituent nations of the Eurozone over the 2006–2015 period.

In a representative agent view of the world, similar to that pursued in the Sigma convergence literature, the multilateral results present significant evidence of convergence in nation based household income distributions as well as in the household income class distributions. An agent, viewing the possibility of belonging to any one of the nations in the Eurozone, would perceive an increasingly similar collection of nations. All of which suggests that national household income distributions are becoming increasingly similar with growing transnational distributional inequalities being more equally shared, trends which pose less of a threat to the cohesion of the Eurozone. However, population weighted versions of the distributional inequality measures suggest that there is increasing distributional inequality in terms of increasingly segmented nations among the more populated nations in the Eurozone highlighting the need for careful consideration with regard to weighting schemes.

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A Appendix

A.1 Multilateral Transvariation

Suppose that $f_k(x)$, $k = 1,..,K$, are continuous density functions with closed and bounded support $[a,b]$, and suppose we have independent random samples from the k'th population X_{kh}, $h = 1,..,T_k$. We define the kernel estimates:

$$\widehat{f}_k(x) = \frac{1}{T_k}\sum_{h=1}^{T_k}\mathbb{K}_b(x - X_{k,h}), \tag{A.1}$$

where $\mathbb{K}$ is a (potentially d dimensioned multivariate) kernel with $\mathbb{K}_b(.) = \mathbb{K}(./b)/b^d$, where b is a positive bandwidth sequence. The K distribution unweighted transvariation index is of the form:

$$\begin{aligned}\widehat{\theta}_{KT} &= \frac{\left\{\int_a^b \max\left(\widehat{f_1}(x), \widehat{f_2}(x), \ldots, \widehat{f_K}(x)\right)dx - \int_a^b \min\left(\widehat{f_1}(x), \widehat{f_2}(x), \ldots, \widehat{f_K}(x)\right)dx\right\}}{g_{KT}(K)} \\ &:= \widehat{\theta}_{KTU} - \widehat{\theta}_{KTL},\end{aligned}$$

where

$$\begin{aligned}\widehat{\theta}_{KTU} &= \frac{\int_a^b \max\left(\widehat{f_1}(x), \widehat{f_2}(x), \ldots, \widehat{f_K}(x)\right)dx}{g_{KT}(K)}, \\ \widehat{\theta}_{KTL} &= \frac{\int_a^b \min\left(\widehat{f_1}(x), \widehat{f_2}(x), \ldots, \widehat{f_K}(x)\right)dx}{g_{KT}(K)},\end{aligned}$$

and $g_{KT}(K)$ is a known linear function of K, the number of distributions in question. Considering (A.1), assume for simplicity that the contact set (the set of values for which the densities are equal) is of Lebesgue measure zero. Define the sets $CK_{i,*}$ and $CK^{i,*}$:

$$\begin{aligned}CK_{i,*} &= \{x : f_i(x) < f_j(x)\, \forall j = 1, ..., K, j \neq i\} \quad \text{and} \\ CK^{i,*} &= \{x : f_i(x) > f_j(x)\, \forall j = 1, ..., K, j \neq i\}.\end{aligned}$$

Let $p_{kU} = \Pr\left(X_k \in CK^{i,*}\right)$ and $p_{kL} = \Pr\left(X_k \in CK_{i,*}\right)$, and note that $CK_{i,*} \cap CK^{i,*} = \emptyset$ so that $p_{kUL} = \Pr\left(X_k \in CK^{i,*} \cap CK_{i,*}\right) = 0$.

Then, under standard regularity conditions (see, e.g., Assumptions (A1) and (A3) of Anderson, Linton and Whang (2012)), we have

$$\widehat{\theta}_{KTU} - \theta_{KTU} = \frac{1}{g_{KT}(K)} \sum_{k=1}^{K} \int_{CK^{k,*}} \left(\widehat{f}_k(x) - E\left(\widehat{f}_k(x) \right) \right) dx + r_T, \tag{A.2}$$

where r_T is generic notation for a remainder term that is of smaller order in probability (r_T may be different from expression to expression).

For notational convenience assume $T_k = T$ for all k and independent sampling over $k = 1, .., K$. Then, the asymptotic variance of $\widehat{\theta}_{KTU}$ is given by

$$AVAR\left(\widehat{\theta}_{KTU}\right) = \frac{1}{(g_{KT}(K))^2} \frac{1}{T} \sum_{k=1}^{K} p_{kU}\left(1 - p_{kU}\right), \tag{A.3}$$

see Lemma A.6 of Anderson, Linton and Whang (2012). (A.3) can be consistently estimated by

$$\frac{1}{(g_{KT}(K))^2} \frac{1}{T} \sum_{k=1}^{K} \widehat{p_{kU}}\left(1 - \widehat{p_{kU}}\right),$$

where

$$\widehat{p_{kU}} = \frac{1}{T} \sum_{h=1}^{T} 1\left(X_{kh} \in CK^{k,*}\right). \tag{A.4}$$

Similarly,

$$\widehat{\theta}_{KTL} - \theta_{KTL} = \frac{1}{g_{KT}(K)} \sum_{k=1}^{K} \int_{CK_{i,*}} \left(\widehat{f}_k(x) - E\left(\widehat{f}_k(x) \right) \right) dx + r_T. \tag{A.5}$$

The asymptotic variance of $\widehat{\theta}_{KTL}$ is given by

$$AVAR\left(\widehat{\theta}_{KTL}\right) = \frac{1}{(g_{KT}(K))^2} \frac{1}{T} \sum_{k=1}^{K} p_{kL}\left(1 - p_{kL}\right), \tag{A.6}$$

which can be consistently estimated by

$$\frac{1}{(g_{KT}(K))^2} \frac{1}{T} \sum_{k=1}^{K} \widehat{p_{kL}}\left(1 - \widehat{p_{kL}}\right),$$

where

$$\widehat{p_{kL}} = \frac{1}{T} \sum_{h=1}^{T} 1\left(X_{kh} \in CK_{k,*}\right). \tag{A.7}$$

Combining (A.2) and (A.5) toghether, we have

$$\widehat{\theta}_{KT} - \theta_{KT} = \frac{1}{g_{KT}(K)} \sum_{k=1}^{K} \int_{CK^{i,*}} \left(\widehat{f}_k(x) - E\left(\widehat{f}_k(x)\right)\right) dx$$
$$- \frac{1}{g_{KT}(K)} \sum_{k=1}^{K} \int_{CK_{i,*}} \left(\widehat{f}_k(x) - E\left(\widehat{f}_k(x)\right)\right) dx + r_T.$$

The asymptotic variance of $\widehat{\theta}_{KT}$ is given by

$$AVAR\left(\widehat{\theta}_{KT}\right) = \frac{1}{(g_{KT}(K))^2} \frac{1}{T} \sum_{k=1}^{K} \{p_{kU}\left(1 - p_{kU}\right) + p_{kL}\left(1 - p_{kL}\right) + 2p_{kU}p_{kL}\},$$

using the fact that $p_{kUL} = \Pr\left(X_k \in CK^{i,*} \cap CK_{i,*}\right) = 0$. It can be consistently estimated by

$$\frac{1}{(g_{KT}(K))^2} \frac{1}{T} \sum_{k=1}^{K} \{\widehat{p_{kU}}\left(1 - \widehat{p_{kU}}\right) + \widehat{p_{kL}}\left(1 - \widehat{p_{kL}}\right) + 2\widehat{p_{kU}}\widehat{p_{kL}}\}.$$

The distributional properties of KTRW can be derived as above by working with $w_k \widehat{f}_k(x)$ in place of $\widehat{f}_k(x)$ and modifying $g_{KT}(K)$ accordingly as in (3).

A.2 The Distributional Gini

The Distributional Gini Index (*DisGini* or *DG*) over K distributions is of the form:

$$\widehat{\theta}_{DG} = \frac{1}{g_{DG}(K)} \sum_{i=1}^{K} \sum_{j=1}^{K} w_i w_j \left\{2 - \int_a^b \min\left(\widehat{f}_i(x), \widehat{f}_j(x)\right) dx\right\}, \tag{A.8}$$

where $g_{DG}(K)$ is a known function of K and the $w_i^{\prime}$s are also assumed known. This may be written as

$$\widehat{\theta}_{DG} = \frac{1}{g_{DG}(K)} \left(2K^2 - \sum_{i=1}^{K} \sum_{j=1}^{K} w_i w_j \left\{\int_a^b \min\left(\widehat{f}_i(x), \widehat{f}_j(x)\right)\right\} dx\right).$$

So, for the distributional properties of $\widehat{\theta}_{DG}$ attention can be focussed upon:

$$\theta_{OV} = \sum_{i=1}^{K} \sum_{j=1}^{K} w_i w_j \left\{\int_a^b \min\left(\widehat{f}_i(x), \widehat{f}_j(x)\right) dx\right\} = \sum_{i=1}^{K} \sum_{j=1}^{K} w_i w_j \left\{\widehat{\theta}_{i,j}\right\}, \tag{A.9}$$

where $\widehat{f}_k(x)$ are defined as in (A.1).

Considering the $\widehat{\theta}_{i,j}$, for simplicity assume independent samples of T observations and that the contact sets are of measure 0. Define the sets $C_{i,j}$ $i,j=1,..,K$ $i\neq j$ as:

$$C_{i,j}=\{x:f_i(x)<f_j(x)\}.$$

Then

$$\widehat{\theta}_{i,j}-\theta_{i,j}=\int_{C_{i,j}}\left(\widehat{f}_i(x)-E\left(\widehat{f}_i(x)\right)\right)dx+\int_{C_{j.i}}\left(\widehat{f}_j(x)-E\left(\widehat{f}_j(x)\right)\right)dx+r_T$$

and thus

$$\widehat{\theta}_{OV}=\sum_{i=1}^{K}\sum_{j=1}^{K}w_iw_j\left(\widehat{\theta}_{i,j}-\theta_{i,j}\right)=$$

$$=\sum_{i=1}^{K}\sum_{j=1}^{K}w_iw_j\left(\int_{C_{i,j}}\left(\widehat{f}_i(x)-E\left(\widehat{f}_i(x)\right)\right)dx+\int_{C_{j,i}}\left(\widehat{f}_j(x)-E\left(\widehat{f}_j(x)\right)\right)dx\right)+r_T.$$

Turning to its asymptotic variation, define:

$$p_{i:ij}=P\left(X_i\in C_{i,j}\right)\text{ and }p_{ij}=\Pr\left(X_i\in C_{i,j}\cap X_j\in C_{j,i}\right).$$

Then generally,

$$AVAR\left(\widehat{\theta}_{i,j}\right)=\frac{1}{T}\left(p_{i:ij}\left(1-p_{i:ij}\right)+p_{j:ji}\left(1-p_{j:ji}\right)+2\left(p_{ij}-p_{i:ij}p_{j:ji}\right)\right),$$

which may simplify with independent sampling. However even if X_i and X_i are independent $\widehat{\theta}_{i,j}$ and $\widehat{\theta}_{k,l}$ will be dependent if they have one subscript in common so that $ACOV\left(\widehat{\theta}_{i,j},\widehat{\theta}_{k,l}\right)\neq 0$ when there is a commonality in subscripts. All such terms need to be considered so that a threefold summation is required involving probabilities of sets of the form:

$$C_{i,j}\cap C_{i,k}=\{x:f_i(x)<\min\left(f_j(x),f_k(x)\right)\}.$$

Ultimately:

$$AVAR\left(\widehat{\theta}_{OV}\right)=\frac{1}{T}\sum_{i=1}^{K}\sum_{j>i}w_i^2w_j^2\left(\Pr\left(X_i\in C_{ij}\right)-\Pr\left(X_i\in C_{ij}\right)^2\right)+$$

$$+\frac{2}{T}\sum_{i=1}^{K}\sum_{j>i}\sum_{k>j>i}w_i^2w_jw_k\left(\Pr\left(X_i\in C_{ij}\cap C_{ik}\right)-\Pr\left(X_i\in C_{ij}\right)\Pr\left(X_i\in C_{ik}\right)\right),$$

which may be consistently estimated by replacing the population quantities by their sample analogues.

A.3 Bilateral Transvariation Estimates

Table A.1: Transvariation measures, year 2006

	AT	BE	CY	DE	EE	EL	ES	FI	FR	IE	IT	LT	LU	LV	NL	PT	SI	SK
AT	0.000	0.084	0.089	0.128	0.677	0.363	0.088	0.056	0.368	0.140	0.194	0.749	0.315	0.746	0.050	0.493	0.304	0.823
BE	0.084	0.000	0.040	0.088	0.622	0.287	0.047	0.060	0.294	0.098	0.126	0.706	0.364	0.702	0.081	0.426	0.236	0.783
CY	0.089	0.040	0.000	0.105	0.622	0.296	0.072	0.047	0.301	0.082	0.125	0.703	0.341	0.699	0.087	0.430	0.247	0.780
DE	0.128	0.088	0.105	0.000	0.621	0.289	0.068	0.117	0.298	0.154	0.138	0.704	0.431	0.700	0.101	0.431	0.200	0.781
EE	0.677	0.622	0.622	0.621	0.000	0.362	0.633	0.653	0.358	0.583	0.506	0.136	0.793	0.185	0.687	0.249	0.520	0.214
EL	0.363	0.287	0.296	0.289	0.362	0.000	0.294	0.331	0.031	0.260	0.174	0.468	0.574	0.469	0.359	0.163	0.174	0.554
ES	0.088	0.047	0.072	0.068	0.633	0.294	0.000	0.082	0.302	0.123	0.139	0.720	0.391	0.716	0.081	0.435	0.226	0.798
FI	0.056	0.060	0.047	0.117	0.653	0.331	0.082	0.000	0.335	0.099	0.159	0.731	0.316	0.727	0.069	0.461	0.275	0.806
FR	0.368	0.294	0.301	0.298	0.358	0.031	0.302	0.335	0.000	0.263	0.179	0.468	0.567	0.467	0.365	0.147	0.188	0.553
IE	0.140	0.098	0.082	0.154	0.583	0.260	0.123	0.099	0.263	0.000	0.110	0.678	0.339	0.677	0.156	0.385	0.236	0.761
IT	0.194	0.126	0.125	0.138	0.506	0.174	0.139	0.159	0.179	0.110	0.000	0.600	0.437	0.599	0.191	0.309	0.158	0.682
LT	0.749	0.706	0.703	0.704	0.136	0.468	0.720	0.731	0.468	0.678	0.600	0.000	0.827	0.086	0.763	0.377	0.622	0.182
LU	0.315	0.364	0.341	0.431	0.793	0.574	0.391	0.316	0.567	0.339	0.437	0.827	0.000	0.834	0.335	0.641	0.570	0.892
LV	0.746	0.702	0.699	0.700	0.185	0.469	0.716	0.727	0.467	0.677	0.599	0.086	0.834	0.000	0.760	0.381	0.625	0.234
NL	0.050	0.081	0.087	0.101	0.687	0.359	0.081	0.069	0.365	0.156	0.191	0.763	0.335	0.760	0.000	0.495	0.279	0.837
PT	0.493	0.426	0.430	0.431	0.249	0.163	0.435	0.461	0.147	0.385	0.309	0.377	0.641	0.381	0.495	0.000	0.319	0.456
SI	0.304	0.236	0.247	0.200	0.520	0.174	0.226	0.275	0.188	0.236	0.158	0.622	0.570	0.625	0.279	0.319	0.000	0.707
SK	0.823	0.783	0.780	0.781	0.214	0.554	0.798	0.806	0.553	0.761	0.682	0.182	0.892	0.234	0.837	0.456	0.707	0.000

Table A.2: Transvariation measures, year 2009

	AT	BE	CY	DE	EE	EL	ES	FI	FR	IE	IT	LT	LU	LV	NL	PT	SI	SK
AT	0.000	0.095	0.075	0.056	0.533	0.252	0.044	0.055	0.354	0.125	0.179	0.585	0.257	0.596	0.055	0.477	0.262	0.625
BE	0.095	0.000	0.070	0.071	0.473	0.185	0.069	0.127	0.279	0.082	0.117	0.529	0.331	0.538	0.121	0.412	0.172	0.562
CY	0.075	0.070	0.000	0.050	0.495	0.215	0.065	0.078	0.318	0.092	0.144	0.548	0.266	0.559	0.102	0.438	0.227	0.587
DE	0.056	0.071	0.050	0.000	0.501	0.215	0.062	0.083	0.316	0.098	0.141	0.554	0.277	0.564	0.082	0.443	0.218	0.593
EE	0.533	0.473	0.495	0.501	0.000	0.301	0.524	0.561	0.229	0.455	0.374	0.102	0.682	0.161	0.575	0.092	0.420	0.177
EL	0.252	0.185	0.215	0.215	0.301	0.000	0.240	0.283	0.110	0.164	0.077	0.360	0.431	0.367	0.292	0.239	0.186	0.391
ES	0.044	0.069	0.065	0.062	0.524	0.240	0.000	0.079	0.343	0.117	0.169	0.577	0.284	0.587	0.059	0.466	0.239	0.615
FI	0.055	0.127	0.078	0.083	0.561	0.283	0.079	0.000	0.385	0.143	0.212	0.612	0.210	0.623	0.067	0.504	0.297	0.653
FR	0.354	0.279	0.318	0.316	0.229	0.110	0.343	0.385	0.000	0.254	0.179	0.298	0.528	0.312	0.393	0.163	0.212	0.302
IE	0.125	0.082	0.092	0.098	0.455	0.164	0.117	0.143	0.254	0.000	0.106	0.515	0.309	0.522	0.159	0.391	0.173	0.543
IT	0.179	0.117	0.144	0.141	0.374	0.077	0.169	0.212	0.179	0.106	0.000	0.431	0.371	0.439	0.218	0.311	0.157	0.464
LT	0.585	0.529	0.548	0.554	0.102	0.360	0.577	0.612	0.298	0.515	0.431	0.000	0.715	0.108	0.628	0.166	0.483	0.205
LU	0.257	0.331	0.266	0.277	0.682	0.431	0.284	0.210	0.528	0.309	0.371	0.715	0.000	0.725	0.254	0.623	0.478	0.757
LV	0.596	0.538	0.559	0.564	0.161	0.367	0.587	0.623	0.312	0.522	0.439	0.108	0.725	0.000	0.638	0.196	0.489	0.253
NL	0.055	0.121	0.102	0.082	0.575	0.292	0.059	0.067	0.393	0.159	0.218	0.628	0.254	0.638	0.000	0.518	0.275	0.667
PT	0.477	0.412	0.438	0.443	0.092	0.239	0.466	0.504	0.163	0.391	0.311	0.166	0.623	0.196	0.518	0.000	0.361	0.179
SI	0.262	0.172	0.227	0.218	0.420	0.186	0.239	0.297	0.212	0.173	0.157	0.483	0.478	0.489	0.275	0.361	0.000	0.501
SK	0.625	0.562	0.587	0.593	0.177	0.391	0.615	0.653	0.302	0.543	0.464	0.205	0.757	0.253	0.667	0.179	0.501	0.000

Table A.3: Transvariation measures, year 2012

	AT	BE	CY	DE	EE	EL	ES	FI	FR	IE	IT	LT	LU	LV	NL	PT	SI	SK
AT	0.000	0.105	0.122	0.053	0.542	0.333	0.049	0.040	0.535	0.183	0.196	0.648	0.168	0.675	0.104	0.517	0.307	0.502
BE	0.105	0.000	0.080	0.076	0.469	0.252	0.090	0.104	0.457	0.107	0.119	0.581	0.251	0.623	0.082	0.442	0.214	0.409
CY	0.122	0.080	0.000	0.078	0.460	0.244	0.119	0.111	0.448	0.089	0.101	0.573	0.242	0.615	0.117	0.433	0.196	0.401
DE	0.053	0.076	0.078	0.000	0.504	0.294	0.059	0.050	0.498	0.143	0.157	0.614	0.184	0.644	0.101	0.481	0.266	0.462
EE	0.542	0.469	0.460	0.504	0.000	0.230	0.540	0.544	0.088	0.397	0.363	0.130	0.635	0.191	0.532	0.087	0.372	0.199
EL	0.333	0.252	0.244	0.294	0.230	0.000	0.330	0.335	0.208	0.173	0.148	0.338	0.430	0.392	0.318	0.199	0.192	0.201
ES	0.049	0.090	0.119	0.059	0.540	0.330	0.000	0.045	0.534	0.180	0.193	0.651	0.185	0.683	0.076	0.517	0.296	0.497
FI	0.040	0.104	0.111	0.050	0.544	0.335	0.045	0.000	0.538	0.183	0.197	0.652	0.165	0.682	0.090	0.519	0.304	0.505
FR	0.535	0.457	0.448	0.498	0.088	0.208	0.534	0.538	0.000	0.378	0.353	0.165	0.628	0.230	0.523	0.091	0.341	0.164
IE	0.183	0.107	0.089	0.143	0.397	0.173	0.180	0.183	0.378	0.000	0.059	0.506	0.298	0.553	0.160	0.364	0.147	0.324
IT	0.196	0.119	0.101	0.157	0.363	0.148	0.193	0.197	0.353	0.059	0.000	0.478	0.320	0.517	0.175	0.341	0.142	0.315
LT	0.648	0.581	0.573	0.614	0.130	0.338	0.651	0.652	0.165	0.506	0.478	0.000	0.729	0.091	0.646	0.190	0.478	0.306
LU	0.168	0.251	0.242	0.184	0.635	0.430	0.185	0.165	0.628	0.298	0.320	0.729	0.000	0.754	0.244	0.607	0.434	0.605
LV	0.675	0.623	0.615	0.644	0.191	0.392	0.683	0.682	0.230	0.553	0.517	0.091	0.754	0.000	0.681	0.256	0.533	0.374
NL	0.104	0.082	0.117	0.101	0.532	0.318	0.076	0.090	0.523	0.160	0.175	0.646	0.244	0.681	0.000	0.508	0.233	0.466
PT	0.517	0.442	0.433	0.481	0.087	0.199	0.517	0.519	0.091	0.364	0.341	0.190	0.607	0.256	0.508	0.000	0.339	0.153
SI	0.307	0.214	0.196	0.266	0.372	0.192	0.296	0.304	0.341	0.147	0.142	0.478	0.434	0.533	0.233	0.339	0.000	0.256
SK	0.502	0.409	0.401	0.462	0.199	0.201	0.497	0.505	0.164	0.324	0.315	0.306	0.605	0.374	0.466	0.153	0.256	0.000

Table A.4: Transvariation measures, year 2015

	AT	BE	CY	DE	EE	EL	ES	FI	FR	IE	IT	LT	LU	LV	NL	PT	SI	SK
AT	0.000	0.079	0.287	0.066	0.452	0.459	0.091	0.146	0.670	0.188	0.258	0.562	0.179	0.593	0.122	0.526	0.329	0.588
BE	0.079	0.000	0.220	0.061	0.387	0.395	0.147	0.204	0.624	0.131	0.188	0.509	0.231	0.536	0.079	0.462	0.257	0.518
CY	0.287	0.220	0.000	0.238	0.209	0.198	0.354	0.396	0.438	0.125	0.083	0.337	0.402	0.351	0.225	0.252	0.133	0.306
DE	0.066	0.061	0.238	0.000	0.405	0.413	0.125	0.177	0.627	0.141	0.208	0.518	0.198	0.548	0.109	0.479	0.281	0.540
EE	0.452	0.387	0.209	0.405	0.000	0.077	0.515	0.545	0.245	0.292	0.213	0.144	0.547	0.162	0.393	0.101	0.264	0.225
EL	0.459	0.395	0.198	0.413	0.077	0.000	0.523	0.556	0.241	0.297	0.218	0.146	0.558	0.158	0.402	0.085	0.258	0.221
ES	0.091	0.147	0.354	0.125	0.515	0.523	0.000	0.059	0.725	0.255	0.328	0.624	0.097	0.652	0.203	0.588	0.402	0.654
FI	0.146	0.204	0.396	0.177	0.545	0.556	0.059	0.000	0.745	0.303	0.373	0.650	0.059	0.676	0.259	0.616	0.452	0.687
FR	0.670	0.624	0.438	0.627	0.245	0.241	0.725	0.745	0.000	0.533	0.450	0.124	0.745	0.094	0.632	0.201	0.494	0.302
IE	0.188	0.131	0.125	0.141	0.292	0.297	0.255	0.303	0.533	0.000	0.099	0.422	0.312	0.447	0.131	0.359	0.156	0.405
IT	0.258	0.188	0.083	0.208	0.213	0.218	0.328	0.373	0.450	0.099	0.000	0.336	0.380	0.364	0.186	0.285	0.126	0.338
LT	0.562	0.509	0.337	0.518	0.144	0.146	0.624	0.650	0.124	0.422	0.336	0.000	0.651	0.066	0.517	0.120	0.387	0.267
LU	0.179	0.231	0.402	0.198	0.547	0.558	0.097	0.059	0.745	0.312	0.380	0.651	0.000	0.678	0.282	0.617	0.461	0.689
LV	0.593	0.536	0.351	0.548	0.162	0.158	0.652	0.676	0.094	0.447	0.364	0.066	0.678	0.000	0.544	0.123	0.409	0.265
NL	0.122	0.079	0.225	0.109	0.393	0.402	0.203	0.259	0.632	0.131	0.186	0.517	0.282	0.544	0.000	0.468	0.219	0.510
PT	0.526	0.462	0.252	0.479	0.101	0.085	0.588	0.616	0.201	0.359	0.285	0.120	0.617	0.123	0.468	0.000	0.319	0.181
SI	0.329	0.257	0.133	0.281	0.264	0.258	0.402	0.452	0.494	0.156	0.126	0.387	0.461	0.409	0.219	0.319	0.000	0.333
SK	0.588	0.518	0.306	0.540	0.225	0.221	0.654	0.687	0.302	0.405	0.338	0.267	0.689	0.265	0.510	0.181	0.333	0.000