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Convergence or Polarization? 21st Century Interprovincial and Gender Based Distributional Variation in the Incomes, Ages and Education of Canadians.

By Gordon John Anderson

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Very Preliminary please do not quote.

Abstract:

Growing economic inequalities between a confederations' constituencies can be a catalyst for the deterioration of its cohesiveness. The underlying idea is that inequalities that are more equally shared amongst a collection of subgroups, the more easily are they borne by the collection as a society. In focussing on empirical and theoretical bases for average income processes trending towards multiple or singular poles of attraction, the growth and convergence literature has long concerned itself with such issues. However, focussing on averages can be problematic since it can mask important distributional differences that can only be revealed when distributions are compared in their entirety. Here tools for examining distributional differences, exceptionalities and similarities which surmount these problems are employed in an interprovincial / gender based study of the progress of Canadian personal incomes and proxies for its latent experience and embodied human capital drivers namely age and experience. While the joint distributions of the drivers appear to be diverging, income distributions appear to be converging. However, closer inspection reveals that, when viewed separately, female and male income distributions are each diverging across the provinces, but the divergence is masked by the overall convergence of male and female distributions.

Introduction.

The last half century has seen deterioration in the unity of many confederated states across the globe with intractable governance problems, irreconcilable ethnic differences and diverging economic fundamentals between constituencies all cited as sources of a lack of cohesion. From the Nova Scotian movement in the 1860's, to the Quebec separatists of the 19th and 20th centuries, to the Western separatism of the late 20th and early 21st century, Canada has had its own history of provincially based secessionist movements fuelled in part by perceived differences in economic wellbeing and the sense in which those differences are growing or diminishing (See for example Fortin and Lemieux 2015).

Milanovic (2011) forcefully makes the point that growing economic inequalities between constituencies that transcend overall inequality within a federation can be a catalyst for the deterioration of its cohesion. The idea is that when constituencies are equally unequal, with relatively similar income distributions, there is a commonality of situation which promotes cohesion, whereas a more divisive and alienated situation arises when constituent income distributions are not so equally shared and the differences are growing. Such between group economic inequality trends have been the focus of a growth and convergence literature which, under the banner of Beta and Sigma convergence, concerned itself with the sense in which average income processes are moving towards or away from each other (see for example Barro and Sala-i-Martin 1992, 1995, Galor 1996, Sala-i-Martin 1996, Quah 1997, Barro 1998, Higgins, Levy, and Young. 2006).

Whereas early neoclassical growth models predict that constituencies with common technological, population structure, growth, and savings rate circumstances converge to a unique steady-state equilibrium per capita income level regardless of their respective initial conditions, later models held out the possibility of multiple equilibria engendering distinct poles of attraction

for collections of constituent income distributions (Durlauf and Johnson 1995, Quah 1996) based on the diversity of these circumstances. Such diversity of circumstance results in a concomitant diversity of subgroup income distributions in the form of "Convergence Clubs" centered on multiple poles of attraction.

Modern unified growth theory (Galor 2011) suggests a multilateral nation convergence club typology with a slow or zero growth "Malthusian" club, a sustained steady growth club and faster growing club which is effectively transitioning from the Malthusian state. To the extent that the various capital, labour and technology factors become perfectly and freely mobile across nation boundaries, diversity of circumstances (and hence the diversity of income distributions and nation typologies) should shrink[1] suggesting that, in such a world, convergence clubs could be a transitory and temporary phenomenon with the diversity of incomes shrinking as in a sigma convergence paradigm.

While between group differences may be diminishing, there are good theoretical reasons for expecting increasing variation in income generation and consumption patterns within a given subgroup facing common circumstances. Following Modigliani and Brumberg (1954), classical economic models of income and consumption behaviour (Friedman 1957, Hall 1978) explicitly or implicitly employ stochastic process theory, usually a version of Gibrats law (Gibrat 1931), which generally predict increasingly unequal consumption and income distributions (Battistin, Blundell, and Lewbel 2009, Blundell and Preston 1998, Browning and Lusardi 1996, Durlauf 1996, Anderson 2012). The nature of the process determines the extent to which inequality progresses and, under standard economic growth assumptions, growing within group inequality is to be expected. However, when constituencies face different circumstances their processes

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[1] Indeed, common markets between nation states or provinces are set up to this end.

likely differ, potentially producing convergence or divergence between the subgroup distributions of such a collection. Thus overall increasing income diversity may be observed within the context of an increasingly convergent collection of distributions facilitating increasingly unequal yet more alike or increasingly unequal and less alike scenarios[2]. From a national cohesiveness perspective, the basic question to be answered is: Is the collection of distributions in question converging or diverging over time?

Both Quah (1993) and Friedman (1992) argue that variation of average incomes across groups is of interest since it speaks to whether the distribution of income across economies is becoming more or less equitable. Frequently in convergence/divergence literatures (see for example the seminal works of Barro 1998, Lee, Pesaran and Smith 1998), indeed in distributional difference literatures generally (see for example the equal opportunity – mobility literatures Peragine, Palmisano and Brunori 2014, Solon 1992, 2008), differences have been assessed by comparing conditional moments like means and variances. However, such an approach can be problematic since it casts a veil of ignorance over potentially important distributional differences that can only be revealed when distributions are compared in their entirety (Carneiro, Hansen and Heckman 2003, Durlauf and Quah 2002).

Within any given population, the primary drivers of an individuals position in the income distribution are their embodied human capital, experience and gender, and all have been used to explore differences in personal incomes. For example, age cohorts, a proxy for experience levels,

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[2] This notion has much in common with distinctions made in the Equal Opportunity/Social Justice literatures (Arneson 1989, Dworkin 2000, Roemer 1998, 2006), wherein outcome inequalities engendered by individual choice (such as variation in effort) are of no concern whereas outcome inequalities engendered by circumstances beyond an individuals control (such as inherited traits) are, so the policy goal is to equalize circumstance conditioned outcome distributions. In both cases the transcendentally optimal state is commonality of distribution across circumstance groupings, and in both cases this optimal state will rarely obtain (Atkinson 2012, Sen 2009) hence policy evaluation should concern itself with measuring progress (or the lack of it) toward the policy goal which is the equality of a collection of distributions.

were considered in Blundell and Preston (1998), embodied human capital has been considered in terms of educational attainment levels in so-called Mincer Equations (see Heckman et. al. 2006, Autor 2014 and Acemoglu and Autor 2012) and gender has been considered in Goldin (2014) and Anderson, Leo and Muelhaupt (2014). If personal income is a monotonically non decreasing concave function of human capital and experience, jointly trending inequalities across constituencies in these dimensions could also underlay, indeed promote inter-provincial differences. Assessing the extent to which the distributions of these drivers are converging or diverging across constituencies will contribute toward our understanding of the progress of interprovincial income distribution inequalities.

Here, new tools which surmount the problems highlighted in the Carniero et. al (2003), Durlauf and Quah (2002) critiques are employed to analyze the extent of differences in household income distributions within the collection of provinces which constitutes Canada. After some preliminary discussion of the theoretical background for distributional variation in Section 1, Section 2 outlines the multilateral comparison techniques to be employed in the analysis. Section 3 considers the income / age / education nexus, interprovincial gender based differences in the joint age, education distribution and the income distributions by gender are considered in Section 4 and conclusions are drawn in Section 5.

Section 1. Models of Subgroup Distributional Variation.

As a simplified model of an income process, Gibrat's law of proportionate effects (Gibrat 1931, Sutton 1997) posits that, within a collection of individuals with common characteristics h, y_t, the groups representative agent income in a given period t, is stochastically proportionate to its income in the previous period so that $y_{t+1} = \theta_t \, y_t$. The factor of proportionality is such that $\theta_t = 1 + \varepsilon_t$ where ε_t is a random variable with a mean $\delta(h)$ (the growth rate of y) that is small

in absolute value relative to 1 and a variance $\sigma^2(h)$. Gibrats' Law, essentially a central limit theorem, predicts that, given an initial value y_0, ultimately y_T, the representative agents income at some future time T for the collection of individuals, will be log normally distributed [3] with mean and variance that depends upon T and the mean of ε_t.

Generically, letting $lny_T = Y_T$, it will be the case that:

$$Y_T \sim N\left(Y_0 + T\left(\delta(h) + \frac{\sigma^2(h)}{2}\right), T\sigma^2(h)\right) \qquad [1]$$

so that $\bar{Y}_T$, the per capita or sample average log income in period t will be an integrated variable of order one whose path can be described by a co-integrated error correction model (Engle and Granger 1987) whose simplest form would be:

$$\bar{Y}_t - \bar{Y}_{t-1} = \beta(\alpha_t - \bar{Y}_{t-1}) + u_t \qquad [1a]$$

Where $0 < \beta < 1$ is the error correction parameter related to the rate of convergence, $\alpha_t = Y_0 + T(\delta(h) + \frac{\sigma^2(h)}{2})$ is the long run equilibrium value of Y and u_t is a stationary error process.

In order to study the progress of income diversity in a collection of groups that variation in h would engender, the growth and convergence literature explored patterns in variation in the $\bar{Y}_t's$ across the collection under some assumptions about the homogeneity of [1]. The focus of attention was "Beta" and "Sigma" convergence where Beta convergence concerns itself with the

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[3] Modifications to the process, for example subjecting y_t to a lower or upper reflective boundary such as social security benefit or confining the longevity of the process to a random variable T governed by an exponential distribution (Gabaix 1999, Reed 2001), can be shown to generate closely related distributions, so that for some groups the underlying distribution may be something other than normal. The point is, different groups subjected to different circumstances "h" may be uniquely associated with different distributions, so that the overall income distribution at a particular point in time would be a mixture of types dependent upon h where the weights are population proportions of such groups.

negative covariance of growth rates and per capita income levels ($0<\beta<1$ in formulation [1]).

Assuming K homogenous societies indexed k=1,..,K and $\alpha_t = \alpha$ it was explored with

regressions of the form:

$$\bar{Y}_{tk} - \bar{Y}_{t-1k} = \alpha - \beta^* \bar{Y}_{t-1k} + u_{tk} \qquad [1a^*]$$

Where Beta convergence was defined as $0 < \beta^* < 1$. Sigma convergence concerns itself with

the group of societies becoming more alike in terms of a reduction in the cross sectional variation

of $\bar{Y}_t$ namely $\sigma_{tG}^2 = \frac{1}{K}\sum_{k=1}^{K}(\bar{Y}_{tk} - \bar{Y}_t^*)^2$ where $\bar{Y}_t^* = \frac{1}{K}\sum_{k=1}^{K}\bar{Y}_{tk}$. Assuming $V(u_{tk}) = \sigma_{ut}^2$ for all k

and noting that $\sigma_{tG}^2 \approx (1 - \beta^*)^2\sigma_{t-1G}^2 + \sigma_{ut}^2$, it may be seen that, when σ_{ut}^2 is constant over t,

Beta convergence is necessary for the stability of σ_{tG}^2 yielding a steady state variance $\sigma^{2*} =$

$\frac{\sigma_{ut}^2}{(1-(1-\beta)^2)}$. Note however that, in setting α_t to be a constant, $[1a^*]$ is at odds with models based

in Gibrat's Law since the right hand side of the equation is no longer a co-integrated relationship

and u_{tk} would then have to be a non-stationary integrated process of the same order as $\bar{Y}_{t-1k}$

(note the variance σ_{ut}^2 under the proportionate effects model grows with T).

Typical growth and convergence literature analyses of distributional variation usually employ

some measure of the variation of subgroup means, either an absolute measure based upon the

juxtaposition of conditional means or a relative measure relating the variation in means to an

overall average. Unfortunately, such measures present problems for convergence / divergence

analysis. In the context of a treatment effects setting, Carneiro, Hansen and Heckman (2002,

2003) demonstrate that, employing such summary statistics to explore distributional variation

over collections of populations can be misleading. Locational differences ignore important

information about distributional variation, creating a "veil of ignorance" which can only be

countervailed by comparing complete outcome distributions in their entirety.

An alternative interpretation of the Carniero, Hansen and Heckman (2003) critique is that it is really a robustness issue, the fact that the average outcome in group A is higher than average outcome in group B does not mean that outcomes in every strata of group A are better than outcomes in group B's corresponding strata, which would be the case if one distribution stochastically dominated the other. A Kolmagorov-Smirnov distributional difference test (Kolmagorov 1933, Smirnov 1946)[4] employed in a FOD context can resolve this issue[5]. The point is variation in means measures could record divergence even when the respective subgroup distributions are becoming more alike as would be the case when subgroup variances are growing faster than are subgroup means growing apart. Indeed, it is easy to contrive many counter examples, suppose subgroup variances diminish over time while subgroup mean differences remain constant or even shrink a little, then distributions would be diverging (measures of between group commonalities or overlaps would diminish) while difference in means measures would remain unchanged recording no divergence. Alternatively, it has been shown that groups can even converge or polarize over time with unchanging means and variances (all that is required with mean and variance preserving skewing of subgroup distributions in opposite directions, see Anderson 2004). This is pertinent in the present context (Durlauf and Quah 2002) and demands a means of comparing a collection of distributions

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[4]Indeed, when X is an ordered variable without cardinal measure (such as the ranked levels of education or age which will be employed here) no real meaning can be attached to "averages" since there is no established metric or unit of measure for comparison. However, First Order Dominance comparisons have meaning since they have measure, i.e. probability.

[5] Let $\hat{F}_a(x)$ and $\hat{F}_b(x)$ be the estimated cumulative distribution functions for group A and group B respectively. Compute $D(\hat{F}_a(x), \hat{F}_b(x)) = \sup_x \left(\hat{F}_a(x) - \hat{F}_b(x) \right)$ and $DR\left(\hat{F}_a(x), \hat{F}_b(x) \right) = \left| \inf_x \left(\hat{F}_a(x) - \hat{F}_b(x) \right) \right|$ and compare each to a critical value $c(n_a n_b \alpha) = \sqrt{-0.5\ln(\alpha)} \left(\frac{n_a+n_b}{n_a n_b} \right)$, where n_a and n_b are the respective sample sizes and α is the chosen size of the test. Rejection of one together with non-rejection of the other indicates a first order dominance relation.

throughout the overall range of variation. DisGini, a Gini coefficient measuring the extent of differentness in a collection of distributions (Anderson, Linton, Pittau, Whang and Zelli 2019) provides a solution.

Section 2. Measuring Differentness in Collections of Distributions.

The Gini coefficient (Gini 1921) is not generally subgroup decomposable (Bourguignon 1970), except when subgroup distributions are perfectly segmented with mutually exclusive, closed and bounded support (Mookherjee and Shorrocks 1982), however, as Yitzhaki (2003) observes, this can be an advantage. In the present context it will yield information on the extent to which distributions overlap (i.e. are not segmented) which subgroup decomposable measures such as the variance or coefficient of variation cannot supply. Without loss of generality[6] given a collection of K subgroups indexed k=1,..,K with respective income distributions $f_k(x)$ with corresponding means and population shares μ_k and w_k, an overall education / income distribution $f(x)$, with mean μ, and Gini coefficient G, may be written as:

$$f(x) = \sum_{k=1}^{K} w_k f_k(x)$$

$$\mu = \sum_{k=1}^{K} w_k \mu_k \qquad\qquad [2]$$

$$G = \sum_{k=1}^{K} w_k^2 \frac{\mu_k}{\mu} G_k + \frac{1}{\mu} \sum_{k=2}^{K} \sum_{j=1}^{k} w_k w_j \left| \mu_k - \mu_j \right| + \frac{2}{\mu} \sum_{k=2}^{K} \sum_{j=1}^{k-1} w_k w_j \int_0^\infty f_k(y) \int_y^\infty f_j(x)\,(x - y)\,dx\,dy$$

Thus, G can be seen to be a sum of three terms: (i) a weighted sum of subgroup Ginis' (WGINI), (ii) a term which is the equivalent of a between group Gini coefficient of subgroup means

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[6] Formulations for discrete cardinally ordered variables, with or without subgroup weighting are similarly derivable.

(BGINI)[7], and (iii) a term measuring the extent to which subgroups overlap or are not segmented (NSF). In essence it is a weighted sum of the expected value of the extent to which elements in subgroup j exceed elements in subgroup for all subgroup pairs k,j k≠j.

Knowledge of subgroup means, shares and Ginis' results in WGINI and BGINI being readily computed. Since G=WGINI+BGINI+NSF, this last term (NSF) can also be easily computed. Generally, all terms are bounded between 0 and 1, and the equation can be re-arranged to provide a convenient statistic measuring the extent to which distributions are similar or different i.e.:

$$SI = 1 - NSF/G \qquad\qquad [3]$$

Limitations of the Gini coefficient are its difficulty in handling negative values (Manero 2017b) and the fact that it falls foul of the Carneiro, Hansen and Heckman (2003), Durlauf and Quah (2002) "veil of ignorance" critiques in that it just compares subgroup means rather than entire subgroup distributions. However, the extent to which a collection of distributions differ, when they are multidimentional and potentially cover negative possibilities, can be measured by using a Distributional Gini coefficient (Anderson et. al. 2019), furthermore this measure does not fall foul of the veil of ignorance critique.

Gini (1916, 1959) provided a measure of the difference between two distributions in his Transvariation measure GT which, for two distributions $f_i(x)\ and\ f_j(x)$, is given by:

$$GT_{i,j} = \frac{1}{2}\int_0^\infty |f_i(x) - f_j(x)|dx = \frac{1}{2}\int_0^\infty [\max\left(f_i(x), f_j(x)\right) - min\left(f_i(x), f_j(x)\right)]dx \qquad [4]$$

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[7] It is an unweighted, unstandardized version of this term that is the focus of the sigma convergence literature and highlights what it is that those measures ignore i.e. the NSF term.

GT will be 0 when the two distributions are identical and 1when they have mutually exclusive

support. It can be readily shown that $GT_{i,j} = 1 - OV_{i,j}$ where $OV_{i,j}$ is the overlap measure

$\int \min\left(f_i(x), f_j(x)\right) dx$ measuring the degree to which the distributions have common values,

Anderson, Linton and Whang (2012) provides its asymptotic distribution. Anderson, Linton,

Pittau, Whang and Zelli (2019) employ this in developing a Distributional Gini coefficient

DISGINI (together with its asymptotic standard error):

$$DISGINI = \frac{1}{(1 - \sum_{k=1}^{K} w_k^2)} \sum_{i=1}^{K} \sum_{j=1}^{K} w_i w_j (1 - OV_{ij}) = \frac{2}{(1 - \sum_{k=1}^{K} w_k^2)} \sum_{i=2}^{K} \sum_{j=1}^{i-1} w_i w_j (GT_{ij}) \quad [5]$$

This statistic measures similarities and differences multilaterally, again it is an index between 0

and 1 measuring the lack of commonality over all K distributions.

To weight or not to weight, that is the question?

The foregoing has cast the difference in distributions question in terms of the importance of the

various constituencies in the population, thus if a very small constituency is egregiously different

from the rest it will be of no great account whereas if it were a larger constituency it would have

somewhat more import. Both Beta and Sigma convergence literatures generally do not employ

population weighting, treating each population subgroup as equally important. It is possible to

compare the distributions directly with DISGINI without reference to their relative size in the

distribution. Simply setting $w_i = {}^1/_K$ $for\ all\ i$ yields an unweighted "representative agent"

version of the foregoing statistics which is more appropriate when subgroups need to be

compared directly without regard to their importance in the population.

On Exceptionality.

Gini's Transvariation idea can be exploited in examining the differentness or exceptionality of a particular group (or collection of subgroups) from the rest of the subgroups in the collection. For example, a given subgroup distribution can be compared with a weighted (or unweighted) average of the other distributions in the collection. Suppose distribution $f_j(x)$, $j \in 1,..K$ is to be compared to a distribution which is not the j'th, call it $f_{\setminus j}(x)$ (which may be an amalgam of the rest of the distributions in the collection), for example consider the "average" distribution of the rest to be $f_{\setminus j}(x) = \frac{1}{(1-w_j)}\sum_{\substack{k=1 \\ k \neq j}}^{K} w_k f_k(x)$ and contemplate:

$$GT_{\setminus j,j} = \frac{1}{2}\int_0^\infty |f_{\setminus j}(x) - f_j(x)| dx = 1 - OV_{\setminus J,j} \qquad [6]$$

The weights could be the mixture weights w_k $k = 1,..,K$ or, if direct comparison of distributions is preferred, $w_k = \frac{1}{K} for\ all\ k$. $GT_{\setminus j,j}$ will equal 1 when $f_j(x)$ is perfectly segmented from all other distributions and will be 0 when $f_j(x)$ is equal to the average (weighted or otherwise) of all other distributions in the collection for all x. If the subgroups are independently randomly sampled the sampling distribution of [6] follows Anderson, Linton and Whang (2012).

Alternatively, GT can be extended to examine distributional differences in higher order integrals of distributions. Letting $F^i(x) = \int_0^x F^{i-1}(z)dz$ with $F^0(x) = f(x)$, $EI(f_j(x)\ j = 1,..,K; i)$, Leshno and Levi (2002) introduced the idea of "almost dominance" of distribution $f_k(x)$ over distribution $f_j(x)$ by contemplating θ, which represents the size of the dominance contradiction region where $0 \leq \theta = \int_0^\infty \left(F_k^1(x) - F_j^1(x)\right) I\left(\left(F_k^1(x) - F_j^1(x)\right) > 0\right) dx$ with $I(.)$ being an

indicator function recording 1 when its argument is true and 0 otherwise. For $f_k(x)$ to First

Order dominate $f_j(x)$, $F_k^1(x)$ should be everywhere not above $F_j^1(x)$ and somewhere below it (in

essence $\theta = 0$), so that, if they do cross "just a little bit" (θ close to zero), almost dominance is

declared. This exploits an important condition for stochastic dominance, namely that function

lines (or surfaces in the case of multivariate distributions) should not cross. In essence the

functions should be such that there exists a line (or surface) such that one function is everywhere

above or equal to it while the other function is everywhere below or equal to it, a surface

separator function much like a separating hyperplane.

An i'th order Surface Separation or Exceptionality Index can be contemplated for $i = 1, ...$ of the

form:

$$EI\big(f_j(x), f_j(x); i\big) = \frac{\int_0^\infty \big(F_k^i(x) - F_j^i(x)\big)\, dx}{\int_0^\infty \big|F_k^i(x) - F_j^i(x)\big|\, dx} \qquad [7]$$

This index reflects the i'th order superiority of the j'th distribution with respect to the k'th

distribution (alternatively it can be considered a measure of surface separation of the two

distributions) at the i'th order of integration. Since $0 \le \left|\int_0^\infty \big(F_k^i(x) - F_j^i(x)\big)\, dx\right| \le$

$\int_0^\infty \big|F_k^i(x) - F_j^i(x)\big|\, dx$, it will be the case that $-1 \le EI\big(f_j(x), f_j(x), i\big) \le 1$, approaching 0

when $F_k^i(x) = F_j^i(x)$ or have the same i'th integral, -1 when $f_k(x)$ stochastically dominates

$f_j(x)$ at the i'th order and +1 when $f_j^i(x)$ dominates $f_k^i(x)$ at the i'th order. In this case, the

standard errors can be obtained from Anderson, Post and Whang (2019). The index can be

normalized to the [0, 1] interval with the transformation $0.5\big(1 + EI\big(f_j(x); i\big)\big)$ In essence it

reflects the degree of difference in the distributions at the i'th order of integration.

It is also possible to check the exceptionality of a collection of subgroups, suppose the set of subscripts for the subgroup collection of interest is J, where the elements of J come from the set K* (k=1,..,K) and let J$^+$ be the complement of J in K* then form

$$F_{\backslash J}^i(x) = \frac{1}{\left(1 - \sum_{\substack{k=1 \\ k \in J}}^{K} w_k\right)} \sum_{\substack{k=1 \\ k \in J^+}}^{K} w_k F_k^i(x)$$

and

$$F_J^i(x) = \frac{1}{\left(1 - \sum_{\substack{k=1 \\ k \in J^+}}^{K} w_k\right)} \sum_{\substack{k=1 \\ k \in J}}^{K} w_k F_k^i(x)$$

And insert in [7] above.

On the other hand the notion of extreme exceptionality could be contemplated wherein $F_{\backslash J}^i(x)$ is redefined as:

$$FMAX_{\backslash J}^i(x) = \max_{\substack{k=1,K \\ k \neq j}} \{F_k^i(x)\} \qquad \text{or} \qquad FMIN_{\backslash J}^i(x) = \min_{\substack{k=1,K \\ k \neq j}} \{F_k^i(x)\}$$

In essence $FMAX_{\backslash J}^i(x)$ is the upper envelope of the i'th order integrals of the collection of distributions k=1,2,..,K k≠j and $FMIN_{\backslash J}^i(x)$ is the lower envelope of the collection. These correspond respectively to the unambiguously worst/best scenarios that could be contrived at the i'th order of integration by amalgamating all the distributions in the collection (see Anderson Leo 2014, Anderson Post and Whang 2019). If one could establish (via [7]) that $F_J^i(x)$ was either everywhere above $FMAX_{\backslash J}^i(x)$ or everywhere below $FMIN_{\backslash J}^i(x)$, then $f_j(x)$ would unambiguously correspond to the unambiguous extremely poor distribution or unambiguous extremely rich distribution respectively at the i'th level of integration. This idea can naturally be extended to collections of groups. Suppose J is a proper subset of the distributions $f_k(x)$ for k=1,..,K and the whole collection is partitioned into two groups $k \in J$ and its complement

denoted J^C and for the set of integers H, let $FMAX_H^i(x) = \max\limits_{k \in H}\{F_k^i(x)\}$ and $FMIN_H^i(x) =$

$\min\limits_{k \in H}\{F_k^i(x)\}$ be the corresponding upper and lower envelopes of the set at the i'th level of

integration. Suppose that $FMIN_J^i(x) \geq FMAX_{J^C}^i(x)$ can be established with strict inequality

somewhere, then all distributions in J are unambiguously poorer than all distributions in J^C.

Section 3. The Results.

The data were drawn from the Canadian Census Individual Public Use data files for the years

2001, 2006, 2011 and 2016. Incomes before taxes, educational status, age group and gender for

all persons over the age of 20 with nominal annual before tax incomes greater than 0 and less

than 1000000$C were selected (since the only comparisons to be made were relative income

comparisons within year, intertemporal income deflation is not necessary). Probability density

function values for continuous variates were estimated using a standard normal kernel.

The income, age, education, gender relationship is first explored via a simple regression model.

Supposing the characteristics "h" to be embodied human capital, experience and gender, the

mean of the stochastic process ($\delta(h)$) can be construed as being positively related to educational

attainment (an observable proxy for embodied human capital) and age group (an observable

proxy for experience) each interacted with each other and modified by a gender indicator for

particular collections of individuals. Then [1] can be thought of as the income distribution of a

given age cohort with a given level of embodied human capital. The relationship between

income and its human capital, experience and gender drivers can be explored via an extended

Mincer equation (Heckman, Lochner and Todd 2006). To facilitate diminishing marginal effects,

quadratic forms are contemplated, resulting in a workhorse regression of the form:

$$\ln(x_i) = \alpha_0 + \alpha_1 D + \beta_1 y_i + \beta_2 y_i^2 + \beta_3 y_i D + \beta_4 y_i^2 D + \gamma_1 z_i + \gamma_2 z_i^2 + \gamma_3 z_i D + \gamma_4 z_i^2 D$$

$$+ \vartheta_1 (y * z)_i + \vartheta_2 (y * z)_i D + \varepsilon_i$$

Here, x is nominal before tax income, y corresponds to an age group index and z to an educational attainment index with D, a dummy variable equal to 0 if the agent is male and 1 if the agent is female. The results of this simple exploratory regression are reported in Table 1.

Table 1. Income, age, education and gender regressions.

Dependent Variable: ln(income) (Standard errors in brackets).	2001	2006	2011	2016
Constant	5.54771	6.38819	6.26647	5.87101
	(0.03481)	(0.02869)	(0.02940)	(0.04286)
Female dummy (1|female, 0 else)	1.60389	0.98416	0.82649	0.89938
	(0.05030)	(0.04151)	(0.04292)	(0.06144)
Age (an Age Group Index)	0.58986	0.50980	0.52136	0.63337
	(0.00502)	(0.00408)	(0.00419)	(0.00616)
Age squared	-0.02008	-0.01760	-0.01772	-0.02189
	(0.00018)	(0.00014)	(0.00015)	(0.00022)
Education (an educational attainment Index)	0.20361	0.14966	0.17301	0.15722
	(0.00520)	(0.00437)	(0.00443)	(0.00622)
Education squared	-0.01316	-0.00917	-0.01068	-0.00685
	(0.00037)	(0.00030)	(0.00031)	(0.00045)
Age-Education interacted	0.00197	0.00289	0.00260	0.00073
	(0.00032)	(0.00025)	(0.00025)	(0.00034)
Age*Female dummy	-0.34550	-0.23470	-0.18746	-0.15983
	(0.00705)	(0.00573)	(0.00593)	(0.00856)
Age squared*Female dummy	0.01290	0.00876	0.00685	0.00544
	(0.00025)	(0.00020)	(0.00021)	(0.00030)
Education* Female dummy	0.04870	0.04766	0.02957	-0.02393
	(0.00782)	(0.00650)	(0.00657)	(0.00912)
Education squared*female dummy	0.00132	0.00028	0.00136	0.00454
	(0.00055)	(0.00045)	(0.00045)	(0.00063)
Age-Education interacted*female dummy	-0.00226	-0.00195	-0.00179	-0.00088
	(0.00046)	(0.00037)	(0.00036)	(0.00048)
sigma squared	1.17787	0.87355	0.99594	2.05052
R squared	0.13922	0.14902	0.13298	0.07009
F test of no Gender effects.	5273.639	5173.583	3134.226	835.378
Sample Size	591026	633529	672042	711389

All coefficients are significant by the usual criteria in each year, Income appears to be a monotonic non-decreasing concave function of both proxy variables (Age Group and Educational Attainment) with gender having a significant differentiating effect. Age (the experience proxy) and Education (the embodied human capital proxy) appear to reinforce each other, though to a lesser degree for females reflecting a world in which embodied human capital

and experience have positive, diminishing but mutually reinforcing marginal effects. Being

female dilutes embodied human capital effects (perhaps a reflection of the motherhood wage

penalty Yu and Kuo 2017?), enhances the experience effect and mutes their mutual

reinforcement.

Table 1a. Male and Female Incremental Returns to experience and human capital at sample median experience and education levels.

$$\partial y_{male}/\partial AGE_{2001} = 0.5899 - 0.0402AGE + 0.0020ED \qquad (0.3950)$$

$$\partial y_{male}/\partial AGE_{2006} = 0.5098 - 0.0352AGE + 0.0029ED \qquad (0.3425)$$

$$\partial y_{male}/\partial AGE_{2011} = 0.5214 - 0.0354AGE + 0.0026ED \qquad (0.3520)$$

$$\partial y_{male}/\partial AGE_{2016} = 0.6334 - 0.0438AGE + 0.0007ED \qquad (0.4167)$$

$$\partial y_{female}/\partial AGE_{2001} = 0.2444 - 0.0144AGE - 0.0003ED \qquad (0.1717)$$

$$\partial y_{female}/\partial AGE_{2006} = 0.2751 - 0.0177AGE - 0.0009ED \qquad (0.1895)$$

$$\partial y_{female}/\partial AGE_{2011} = 0.3339 - 0.0217AGE + 0.0008ED \qquad (0.2276)$$

$$\partial y_{female}/\partial AGE_{2016} = 0.4735 - 0.0329AGE - 0.0002ED \qquad (0.3086)$$

$$\partial y_{male}/\partial ED_{2001} = 0.2036 - 0.0263ED + 0.0020AGE \qquad (0.1345)$$

$$\partial y_{male}/\partial ED_{2006} = 0.1497 - 0.0183ED + 0.0029AGE \qquad (0.1091)$$

$$\partial y_{male}/\partial ED_{2011} = 0.1730 - 0.0214ED + 0.0026AGE \qquad (0.1219)$$

$$\partial y_{male}/\partial ED_{2016} = 0.1572 - 0.0137ED + 0.0007AGE \qquad (0.1198)$$

$$\partial y_{female}/\partial ED_{2001} = 0.2523 - 0.0237ED - 0.0003AGE \qquad (0.1798)$$

$$\partial y_{female}/\partial ED_{2006} = 0.1973 - 0.0178ED - 0.0009AGE \qquad (0.1487)$$

$$\partial y_{female}/\partial ED_{2011} = 0.2026 - 0.0186ED - 0.0008AGE \qquad (0.1507)$$

$$\partial y_{female}/\partial ED_{2016} = 0.1333 - 0.0046ED - 0.0002AGE \qquad (0.1187)$$

Differences in Relative median returns to experience and embodied human capital.

	2001	2006	2011	2016
$\dfrac{r_{age}^{male} - r_{age}^{female}}{(r_{age}^{male} + r_{age}^{female})/2}$	0.78807	0.57519	0.42926	0.09056
$\dfrac{r_{edu}^{male} - r_{edu}^{female}}{(r_{edu}^{male} + r_{edu}^{female})/2}$	-0.28826	-0.30721	-0.21130	0.00922

To get an idea of impact differentials, gender based incremental returns to experience and human capital are reported in Table 1a for each of the observation years computed at sample median experience and education levels. It is interesting to note that male returns to experience exceed that of female returns whereas the male returns to education are exceeded by that of female returns, however in both cases the differential is diminishing over time. Experience and embodied human capital are complementary for males but appear to be substitutes for females though the effect is small in this case.

3.1. Distributional Differences, The Income Drivers.

To get some background for interprovincial differences in income distributions, interprovincial differences in the proxy variables for the drivers of those distributions are first examined. With respect to the experience driver, Table 2 reports $f(X|Age\ range) = P(x \in Age\ range)$, the proportion of the sample population (people over the age of 20) in a given age cohort, together with average ages in the sample and the male – female transvariation. Historically, life expectancy, and hence age distributions, differ by gender with females enjoying greater longevity than males in the modern era. However, Table 2 shows that the Transvariation measures of the extent to which Canadian gender based age distributions differ are diminishing significantly over time (The asymptotic standard error for the difference of any two coefficients is at most 0.001). Essentially the genders are becoming more alike in their age structure which is

to say the respective male and female age distributions are converging. Females are losing their

longevity advantage with relative average age differentials of 0.0246, 0.0220, 0.0201 and 0.0192

respectively over the 2001-2016 quinquennial observation years.

Table 2. Age cohort distributions by Gender

	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85 ≤	Ave Age	Gender Transv
2001																
Overall	0.0878	0.0868	0.0959	0.1154	0.1180	0.1064	0.0926	0.0703	0.0573	0.0521	0.0457	0.0367	0.0214	0.0134	46.973	
Female	0.0852	0.0865	0.0953	0.1140	0.1168	0.1048	0.0884	0.0665	0.0560	0.0526	0.0488	0.0415	0.0259	0.0175	47.547	0.0692
Male	0.0905	0.0871	0.0964	0.1169	0.1193	0.1081	0.0970	0.0743	0.0587	0.0516	0.0425	0.0316	0.0168	0.0092	46.391	
2006																
Overall	0.0878	0.0845	0.0864	0.0941	0.1114	0.1100	0.0984	0.0859	0.0668	0.0526	0.0445	0.0368	0.0249	0.0159	47.950	
Female	0.0853	0.0834	0.0858	0.0941	0.1099	0.1086	0.0965	0.0824	0.0653	0.0529	0.0465	0.0398	0.0290	0.0205	48.457	0.0575
Male	0.0904	0.0857	0.0871	0.0941	0.1129	0.1116	0.1004	0.0895	0.0684	0.0522	0.0423	0.0336	0.0205	0.0112	47.407	
2011																
Overall	0.0878	0.0866	0.0860	0.0867	0.0929	0.1060	0.1044	0.0905	0.0806	0.0597	0.0440	0.0344	0.0239	0.0168	48.436	
Female	0.0839	0.0845	0.0866	0.0875	0.0927	0.1051	0.1021	0.0877	0.0798	0.0602	0.0452	0.0365	0.0271	0.0212	48.890	0.0527
Male	0.0918	0.0887	0.0854	0.0858	0.0930	0.1069	0.1068	0.0934	0.0814	0.0591	0.0426	0.0322	0.0205	0.0121	47.916	
2016																
Overall	0.0843	0.0851	0.0872	0.0852	0.0837	0.0871	0.0996	0.0967	0.0842	0.0735	0.0522	0.0369	0.0249	0.0195	49.378	
Female	0.0803	0.0824	0.0865	0.0854	0.0841	0.0865	0.0987	0.0952	0.0843	0.0745	0.0529	0.0390	0.0267	0.0234	49.831	0.0426
male	0.0886	0.0879	0.0879	0.0849	0.0833	0.0877	0.1005	0.0983	0.0841	0.0724	0.0514	0.0347	0.0230	0.0153	48.881	

Table 3. Kolmogorov Smirnov 1st order dominance tests combined male and female*.

Comparison	$\underset{x}{sup}\left(\hat{F}_{year\ a}(x) - \hat{F}_{year\ a+}(x)\right)$	$\left\|\underset{x}{Inf}\left(\hat{F}_{year\ a}(x) - \hat{F}_{year\ a+}(x)\right)\right\|$
2001-2006	0.03980	0.00000
2001-2011	0.06450	0.00000
2001-2016	0.09780	0.00000
2006-2011	0.02830	0.00310
2006-2016	0.06160	0.00000
2011-2016	0.03820	0.00000

* The comparator. $D(\hat{F}_a(x), \hat{F}_b(x)) = \underset{x}{sup}|\hat{F}_a(x) - \hat{F}_b(x)|$ is compared to a critical value $c(n_a n_b \alpha) = \sqrt{-0.5\ln(\alpha)}\left(\frac{n_a + n_b}{n_a n_b}\right)$, where n_a and n_b are the respective sample sizes and α is the chosen size of the test, for the present analysis for α=0.01 this has a value of smaller order than 0.000006. The null hypothesis of commonality is rejected if $D > c$. Stochastic dominance tests can be contrived using $D(\hat{F}_a(x), \hat{F}_b(x)) = \underset{x}{sup}\left(\hat{F}_a(x) - \hat{F}_b(x)\right)$ and $D\left(\hat{F}_a(x), \hat{F}_b(x)\right) = \underset{x}{Inf}\left(\hat{F}_a(x) - \hat{F}_b(x)\right)$. Rejection of one together with non-rejection of the other indicates a first order dominance relation.

Overall, the first order dominance comparisons reported in Table 3 indicate that the Canadas

population is aging significantly. With the exception of the 2006-2011 comparison, age cohort

distributions of successive observation years stochastically dominate preceding years so that

generically younger generations have proportionately fewer members and older generations have proportionately more members as time progresses. Aside from implying that Canadas age distribution is not stable or converging over time, it represents a much stronger result than simply observing a significant increase in the average age.

In terms of the provinces, Table 4, which reports the Distributional Gini coefficient for the provincial age cohort distributions, indicates that provinces are becoming more unalike in their age cohort structures with significantly increasing values of the coefficients over time. Perusal of the individual distributions in Appendix 1 reveals that maritime provinces are becoming richer in older cohorts and poorer in younger cohorts relative to other provinces in the confederation.

Table 4. Provincial Age Cohort Distributional Gini coefficients (approximate maximal standard error for a difference 0.0022)

	2001	2006	2011	2016
Distributional Gini	0.02564	0.02726	0.03275	0.03769

To sum up, the age distribution does not appear to be stable, overall Canadian population is ageing significantly, with the promise of proportionately larger elderly population cohorts being dependent upon proportionately smaller younger cohorts. Within this context, female and male age distributions, long seen as different, appear to be converging. However, at the provincial level these trends are happening at different rates across provinces with the effect that provincial age distributions are diverging with the maritime provinces suffering a greater depletion of their younger cohorts. All in all, in terms of experience, while the genders are converging the provinces are diverging in their respective "experience" distributions.

With respect to the Embodied human capital factor, Table 5 reports the summary statistics on educational attainment measured on a ten-point ordinal scale (see key to Table 5). It reveals a solid improvement in the first decade in the level of attainment which tails off in 2016 with the

female average attainment surpassing the male average attainment. Variation in attainment appears to be diminishing throughout the period in both absolute (standard deviation) and relative to the mean (coefficient of variation and Gini) terms. However, in Table 6 decomposition of the standard Gini coefficient reveals a different story in which the reduction in variation is accompanied by increased segmentation or separation of the subgroups in the first decade followed by a precipitous convergence in 2016 to levels of segmentation lower than in 2001. This is mirrored by the weighted and unweighted Distributional Gini coefficients which compare distributions over their whole support.

Table 5. National Educational Summary Statistics.

		Mean	Median	Standard Deviation	Coefficient of Variation	Coefficient of Skewness	Gini	N
2001	Overall	2.43537	2	1.22773	0.50412	1.06384	0.27836	591026
	Female	2.40841	2	1.20394	0.49989	1.01768	0.27643	305064
	Male	2.46413	2	1.25198	0.50808	1.11215	0.28017	285962
2006	Overall	2.66519	3	1.19360	0.44785	-0.84151	0.24866	633529
	Female	2.64780	3	1.18260	0.44663	-0.89345	0.24856	327722
	Male	2.68382	3	1.20499	0.44898	-0.78718	0.24859	305807
2011	Overall	2.80991	3	1.19681	0.42592	-0.47649	0.23680	672042
	Female	2.80854	3	1.18938	0.42349	-0.48292	0.23606	346993
	Male	2.81137	3	1.20470	0.42851	-0.46973	0.23745	325049
2016	Overall	2.79814	3	1.17611	0.42032	-0.51490	0.23321	711389
	Female	2.81914	3	1.17088	0.41533	-0.46340	0.23109	365342
	Male	2.77598	3	1.18121	0.42551	-0.56896	0.23524	346047

Key:1 No degree, certificate or diploma, 2 High school graduation certificate, 3 Trades certificate/diploma, 4 College certificate/diploma, 5 University certificate/diploma <bachelor level, 6 University degree: Bachelors degree, 7 University degree: certificate >bachelor level, 8 University degree: Medical degree, 9 University degree: Masters degree, 10 University degree: Earned doctorate.

Table 6. Gini Analysis.

	Gini Decomposition				Distributional Gini	
	Within	Between	Non Seg	Segmentation	Unweighted	Weighted
2001	0.04404	0.02746	0.29759	0.19371	0.11336	0.08655
2006	0.04101	0.02617	0.27361	0.19713	0.11287	0.09562
2011	0.04005	0.02571	0.26458	0.19906	0.12602	0.10226
2016	0.03913	0.02215	0.26391	0.18845	0.10614	0.08947

The reversal of the male-female mean education level ordering noted in Table 5 is striking, however, as Carniero, Hansen and Heckman (2003) point out, just comparing means can mask substantial differences, in essence mean comparisons are not necessarily robust. The issue can be seen more clearly by comparing cumulative density functions reported in Tables A2 and A3 in the appendix (to simplify matters the ordinal ten point scale is condensed into an ordinal six point scale[8]). Note first from Table A3, the progressive improvement of educational attainment by each gender, with the exception of the 2011-2016 comparison, each year first order dominating the preceding year implying a robust unequivocal improvement for both genders. Indeed, although there was some fallback in 2016, that year remained an improvement over 2001 and 2006.

As for educational differences between the genders, in 2001 males first order dominate females with better outcomes in all categories, in 2006 and 2011 males are no longer first order dominant, by 2016 Females dominate males up to degree level but are still outperformed at degree level and above. However, if the ordinal 6-point scale is artificially endowed with cardinality, a second order dominance comparison can be made and females second order dominate males in 2016, which is to say female outcomes would be preferred to male outcomes for all monotonic non-decreasing concave educational achievement value functions. In essence there has been somewhat more than an equalization of embodied human capital across the genders consistent with the average education level of females being greater than that of males for the first time in the 2016 data.

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[8] The Trades certificate/diploma, College certificate/diploma, University certificate/diploma lower than bachelors degree categories are individually very small, amalgamating them into one category lends some clarity.

To summarize there has been an equalization across the provinces, as well as between the genders, of the distribution of embodied human capital. So that provincially, while provincial embodied human capital distributions are converging, experience distributions appear to be diverging. Ultimately, with respect to the impact of the experience and embodied human capital drivers on the income generation processes, it is the progress of the variation in their joint distribution that matters and the Distributional Gini coefficient reported in Table 7 is adept at quantifying multivariate distributional variation.

Table 7. Distributional Gini for The Joint Distribution of Experience and Embodied Human Capital, *.

	Overall		Female		Male	
	Weighted	Unweighted	Weighted	Unweighted	Weighted	Unweighted
2001	0.1118	0,1272	0.0952	0.1098	0.1032	0.1066
	(0.0028)	(0.0029)	(0.0025)	(0.0028)	(0.0026)	(0.0027)
2006	0.1146	0.1290	0.0938	0.1057	0.1089	0.1123
	(0.0027)	(0.0028)	(0.0024)	(0.0026)	(0.0026)	(0.0027)
2011	0.1179	0.1402	0.1004	0.1196	0.1131	0.1199
	(0.0026)	(0.0028)	(0.0024)	(0.0027)	(0.0026)	(0.0027)
2016	0,1259	0.1442	0.1084	0.1194	0.1233	0.1256
	(0.0026)	(0.0028)	(0.0024)	(0.0026)	(0.0026)	(0.0027)

*Approximate standard errors in brackets.

As may be seen from Table 7 increases of the order of 13% in the overall distributional Gini with the largest increases for male distributions (19%, 18%) and somewhat smaller increases for female distributions (14% 9%). Since it is only age distributions that are diverging provincially it must be that source which is driving the divergence of the joint distributions.

3.2 Interprovincial Inequalities in Income Distributions.

Turning to an analysis of the distribution of personal incomes, the impact of the all around (i.e. gender and provincial) convergence of embodied human capital is revealed in the overall convergence of provincial income distributions. Table 8 presents the gender based income distribution Transvariations together with the Distributional Gini measures of overall

distributional differences across provinces and genders. The Distributional Transvariations indicate systemic convergence in male and female income distributions throughout the observation period and, after an initial increase in 2006, the overall reduction in distributional inequality post 2006 is striking. However, as was observed in the previous section, while there was convergence in age and education distributions across genders, there was divergence across provinces in the joint densities of age and education. When, in Table 9, distributional disparities across provinces within the respective genders are examined, evidence of divergent income processes emerges. With the exception of the unweighted version of the Distributional Gini for females, measures of distributional disparity are significantly greater in 2016 than they were in 2001.

Table 8. Income Results. Overall Province & Gender

	Male-Female Transvariation	Weighted Distributional Gini	Unweighted Distributional Gini
2001	0.23106635 (0.00001)	0.16044833 (7.415e-08)	0.14558652 (2.470e-08)
2006	0.21623825 (0.00001)	0.16254843 (7.356e-08)	0.14919990 (2.398e-08)
2011	0.18120446 (0.00001)	0.14506129 (6.904e-08)	0.13688579 (2.184e-08)
2016	0.13930907 (0.00001)	0.11419822 (6.818e-08)	0.11820293 (2.509e-08)

Table 9. Within Gender across province.

	Females		Males	
	Weighted DisGini	Unweighted DisGini	Weighted DisGini	Unweighted DisGini
2001	0.076265368 9.5068867e-08	0.089120755 4.7199265e-08	0.066132697 9.2070786e-08	0.097457026 5.0644297e-08
2006	0.094737350 1.0355411e-07	0.095175480 4.6927373e-08	0.078054801 9.8784021e-08	0.11146011 5.0109521e-08
2011	0.10927432 1.0676425e-07	0.094633929 4.2282500e-08	0.079466315 1.0015831e-07	0.10632531 4.4706306e-08
2016	0.10553868 1.3250339e-07	0.087064557 5.4482802e-08	0.072141057 9.0141580e-08	0.11334808 4.8361658e-08

To summarise gender convergence of income distributions and the respective driver distributions appears to be masking underlying within gender divergence of income distributions across the provinces.

Specific Provincial-Gender Based Rankings and Exceptionalities.

Following Anderson and Leo (2014), Anderson, Post and Whang (2019) developed a family of dominance based Utopia-Dystopia indices which facilitate ordering a collection of groups. Based upon the construction of a synthetic best possible "Utopian" outcome distribution (the lower envelope of the collection of i'th order subgroup density integrals) and a synthetic worst possible "Dystopian" distribution (the upper envelope of the collection of i'th order integrals), the j'th distributions distance from the Dystopian outcome, measured by the area between the two curves, yields an index that is in the family of i'th order value functions. Dividing by the area between the Utopian and Dystopian distributions, yields an index confined to the 0-1 interval. While satisfying the usual axiomatic requirements of wellbeing indices (Sen 1995), it has some attractions over other ordering instruments (Anderson and Leo 2020). It possesses an independence of irrelevant alternatives property and always reveals when an outcome is unambiguously the worst (the index will be 0) or best (the index will equal 1) in a collection of outcomes at the i'th comparison level, which other standardized indices will not do.

Table 10 reports the gender based provincial 2nd order (income level – inequality sensitive) Utopia-Dystopia indices for the 4 observation years. Generally female distributions appear in the lower half of the table with male distributions in the upper half. The orderings are quite stable over the first 3 observation years though there appears to be some significant changes in ranks in 2016 with Ontario Males falling back and Northern Canada females and Alberta females advancing substantially (i.e. moving at least 3 places). With a conservative standard error

estimate of 0.004 Newfoundland and Labrador female income distributions appear to be

Dystopian in 2001 and 2006, no income distributions appear to be Utopian. Though Alberta

males have systematically been number one their index fell precipitously in 2016.

Table 10. Utopia-Dystopia Indices

	2001		2006		2011		2016	
	Index	Rank	Index	Rank	Index	Rank	Index	Rank
NewLab F	0.0025	22	0.0008	22	0.0230	22	0.0828	21
NewLab M	0.4421	12	0.4028	11	0.4396	8	0.6293	6
PEI F	0.1078	19	0.0898	19	0.0674	20	0.0858	19
PEI M	0.4423	11	0.3941	12	0.3702	12	0.3266	13
NovSco F	0.0735	20	0.0819	20	0.0722	19	0.0852	20
NovSco M	0.5742	8	0.4853	9	0.4374	10	0.4986	9
NewBru F	0.0437	21	0.0651	21	0.0257	21	0.0184	22
NewBru M	0.5361	10	0.4277	10	0.4144	11	0.3344	12
Quebec F	0.1392	17	0.1499	17	0.0825	18	0.0963	18
Quebec M	0.6375	7	0.5429	7	0.4376	9	0.4282	11
Ontario F	0.3061	13	0.2812	14	0.2101	14	0.2613	15
Ontario M	0.9150	3	0.7909	3	0.6400	4	0.5465	8
Manito F	0.1457	16	0.1461	18	0.0981	17	0.1849	16
Manito M	0.6442	5	0.5809	5	0.5109	6	0.5699	7
Saskat F	0.1196	18	0.1512	16	0.1861	15	0.2649	14
Saskat M	0.6390	6	0.5727	6	0.6615	3	0.7138	3
Alberta F	0.2275	15	0.2954	13	0.3263	13	0.4316	10
Alberta M	0.9538	1	0.9134	1	0.9567	1	0.8874	1
B.C. F	0.2359	14	0.1976	15	0.1286	16	0.1682	17
B.C. M	0.7739	4	0.7188	4	0.6348	5	0.6392	5
NorCan F	0.5415	9	0.4895	8	0.4870	7	0.7041	4
NorCan M	0.9352	2	0.8612	2	0.7919	2	0.7191	2

Question arise as to the extent to which these results are exceptional. With respect to overall

male and female incomes, Table 11 reports first order surface separation of overall gender based

incomes in 2001-2011 inclusive (males first order dominate females) which breaks down in 2016

with the relative rise in female incomes. However, there is no surface separation between the

female Utopian distribution and the male Dystopian distributions in those years with the Female

Utopian distribution sometimes appearing above the male dystopian distribution. However, there

is some surface separation in 2016 when for the first time, the female utopian distribution is

everywhere not above and sometimes below the Male dystopian distribution but this does not

indicate exceptionality where all female distributions dominate all male distributions which

would require the Female Dystopian distribution to be everywhere below the Male Utopian

distribution (i.e. the females worst to be better than the males best).

Table 11. Income Exceptionality indices females - males.

	Kolmogorov-Smirnov Test			Surface Separation
	$Max(F_f(x)-F_m(x))$ $Min(F_f(x)-F_m(x))$ CV		5%	
2001 CDF Difference	0.229260	0.000000	0.000008	1.000000
FemUtopia-MalDystopia envelopes	0.009834	-0.113048	0.000048	0.866497
2006 CDF Difference	0.211743	0.000000	0.000008	1.000000
FemUtopia-MalDystopia envelopes	0.014185	-0.109583	0.000043	0.931605
2011 CDF Difference	0.176840	0.000000	0.000007	1.000000
FemUtopia-MalDystopia envelopes	0.010654	-0.131269	0.000040	0.973480
2016 CDF Difference	0.121840	-0.011598	0.000007	0.986890
FemUtopia-MalDystopia envelopes	0.000000	-0.202795	0.000036	1.000000

Table 12 reports the exceptionality of the Newfoundland-Labrador female distribution and

Albertan male distribution. Neither are exceptional, though the Newfoundland-Labrador female

distribution is close to being everywhere below the female dystopian distribution in 2006 and

2016.

Table 12. Income Exceptionality Indices Newfoundland Females Alberta Males

	Newfoundland Females				Alberta Males			
	$(F_f(x)-F_m(x))$ Max	$(F_f(x)-F_m(x))$ Min	5% CV	Surface Separation	$(F_f(x)-F_m(x))$ Max	$(F_f(x)-F_m(x))$ Min	5% CV	Surface Separation
2001	0.054170	-0.001950	0.000249	0.886427	0.038902	-0.016916	0.000048	0.388414
2006	0.070450	-0.003166	0.000237	0.972167	0.023759	-0.028411	0.000044	0.380267
2011	0.025377	-0.017220	0.000249	0.280222	0.019372	-0.029658	0.000040	0.280644
2016	0.006820	-0.036289	0.000225	0.977189	0.012594	-0.043376	0.000036	0.028669

What about the provincial secessionist movements, have they been fueled by a sense of

exceptionality or lack of commonality? The urgency of the Quebec Separatist Movement has

diminished somewhat in recent years whereas recent events have lent some impetus to the

Albertan cause. It is of interest to see whether this is reflected in their respective income

distributions, and indeed it is. Both male and female income distributions in Alberta have seen

increased separation from the rest of Canada as have those of Quebec though to a somewhat

lesser degree. This is perhaps best characterized by trends in the Province versus the Rest

Transvariations reported in Table 13.

Table 13. Alberta and Quebec vs their Complementary Provinces.

Gender Based Income Transvariations.

	Alberta	Quebec
2001 Female	0.0261 (0.0010)	0.0491 (0.0009)
Male	0.0595 (0.0015)	0.0711 (0.0011)
2006 Female	0.0573 (0.0014)	0.0652 (0.0010)
Male	0.1086 (0.0018)	0.0982 (0.0013)
2011 Female	0.0868 (0.0016)	0.0797 (0.0011)
Male	0.1520 (0.0021)	0.1078 (0.0013)
2016 Female	0.0946 (0.0016)	0.0653 (0.0010)
Male	0.1324 (0.0018)	0.1014 (0.0012)

All income distribution transvariations have grown significantly throughout the period with

Albertas' similarity with the rest of Canada diminishing to a greater degree than is the case in the

Quebec comparison. Questions arise as to the extent of differences in the drivers. Table 14

reports the corresponding comparisons for the experience and education drivers where it may be

seen that Alberta's age distribution is departing significantly from that of the rest of Canada

whereas Quebec's differential has not changed much at all. Both provinces education

distributions are distinguishing themselves from the rest of Canada in a significant fashion, in

this case Quebec more so than Alberta.

The location measures, average age and average education level, reported in Table 15, lend some

insight into the story. Alberta, typically younger than the rest of Canada, has not aged to the

extent that the rest of Canada has over the observation period, approximately 1.5 years as

opposed to the rest of Canada's 2.7 years. On the other hand, Quebec which is typically older

than the rest of Canada, has aged 2.6 years in line with the rest of Canada's 2.6 years. With

respect to education, both provinces have advanced more than the rest of Canada (0.38 versus

0.33 for Alberta and 0.41 versus 0.31 for Quebec).

Table 14. Gender Based Experience and Embodied Human Capital Driver Transvariations.

Age Transvariation

	Alberta	Quebec
2001 Female	0.1091 (0.0019)	0.0536 (0.0010)
Male	0.1117 (0.0020)	0.0489 (0.0009)
2006 Female	0.1219 (0.0019)	0.0519 (0.0009)
Male	0.1241 (0.0020)	0.0523 (0.0009)
2011 Female	0.1438 (0.0020)	0.0587 (0.0009)
Male	0.1453 (0.0020)	0.0520 (0.0009)
2016 Female	0.1716 (0.0020)	0.0590 (0.0009)
Male	0.1701 (0.0020)	0.0658 (0.0010)

Education Transvariation

	Alberta	Quebec
2001 Female	0.0703 (0.0016)	0.0959 (0.0013)
Male	0.0946 (0.0018)	0.0788 (0.0012)
2006 Female	0.0928 (0.0017)	0.1999 (0.0017)
Male	0.0616 (0.0014)	0.1880 (0.0017)
2011Female	0.0990 (0.0017)	0.2383 (0.0017)
Male	0.0779 (0.0015)	0.2309 (0.0017)
2016 Female	0.1274 (0.0018)	0.2818 (0.0018)
Male	0.1345 (0.0019)	0.2836 (0.0018)

Table 15. Mean Age and Education Levels.

	Alberta	vs The Rest	Quebec	vs The Rest
2001	45.0520	46.9003	47.1477	46.6907
2006	45.7065	48.0166	48.3084	47.7564
2011	45.8914	48.3642	48.8348	48.0698
2016	46.5642	49.5955	49.9465	49.2572
2001	2.4533	2.3513	2.3897	2.3577
2006	2.6429	2.5318	2.6316	2.5329
2011	2.8074	2.6689	2.7580	2.6738
2016	2.8277	2.6833	2.7931	2.6868

Conclusions.

It has been argued that growing economic inequalities between the constituencies of a confederation, which has been contrived to achieve some degree of commonality of experience and purpose, can be a catalyst for the deterioration of its cohesiveness. The success or otherwise of such arrangements is usually measured in terms of the coherence that exists amongst those separate entities in terms of summary statistics of distributional location and dispersion. Concerns have been raised over the extent to which such moment comparisons can conceal similarities and differences which only comparison over the entire range of distributions can reveal so, for example, while distributional locations are moving apart the collection of distributions may still exhibit more commonality if their respective spreads are growing sufficiently fast. Here new measures of differences between a collection of entities have been used to examine similarities and differences in Canadian Income distributions and their age and education drivers.

An exploratory pan Canada Mincer equation revealed that personal incomes are driven by monotonic non-decreasing concave functions of age (a proxy for experience) and education levels (a proxy for embodied human capital) with a degree of complementarity between the two. It indicated significant gender differences in returns to age and education where, at median income levels, returns to age for males exceeded that of females and returns to education for females exceeded that of males. In both cases the gender differentials in returns appear to be diminishing over time prompting a study of the juxtaposition of gender based provincial distributions.

Analysis of age and education drivers reveals considerable inter-provincial differences in embodied human capital (measured by educational attainment levels) and experience (measured

by age). Whilst the human capital component appears to be converging, or at least not diverging, there appears to be significant divergence in the experience factor with some provinces (mostly eastern) aging more rapidly than others.

Transvariation and Distributional Gini coefficients indicated overall provincial income distributional convergence, that is to say provincial income distributions appear to exhibit greater commonality over time. However, a more detailed gender based analysis suggests that the increased commonality of male and female income distributions, a result of the convergent returns to education and age, has masked some significant divergences between provinces. When male and female provincial income distributions are considered separately, the provinces are diverging significantly with respect to each gender. At the extremes Newfoundland and Labrador female distributions were the poorest and close to being first order stochastically dominated by all other male and female based income distributions, that is to say they were exceptionally low. At the other end of the spectrum Alberta male distributions were the richest but never exceptionally so, indeed there was some indication of them coming back to the rest of the pack in 2016.

With regard to gender differences, while, in aggregate, male distributions stochastically dominated female distributions in the first decade that ceased to be the case in 2016 and the differences were never exceptional in the sense that all provincial female income distributions were first order stochastically dominated by all male income distributions. A study of the relative position of the "secessionist" provinces Alberta and Quebec revealed upwardly trending differences from the rest of Canada in incomes for both provinces, (more so for Alberta than for Quebec) supported by increasing age and education distributional inequalities for Alberta and increasing education inequalities for Quebec.

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Appendix.

Table A1. Provincial Age Cohort Distributions

2001	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85 $\leq$
NewLab	0.0870	0.0802	0.0984	0.1118	0.1140	0.1159	0.1001	0.0734	0.0606	0.0506	0.0418	0.0311	0.0227	0.0124
PEI	0.0944	0.0780	0.0905	0.1027	0.1172	0.1085	0.1011	0.0709	0.0600	0.0545	0.0419	0.0353	0.0294	0.0157
NovSco	0.0834	0.0825	0.0908	0.1125	0.1141	0.1056	0.0949	0.0738	0.0602	0.0528	0.0476	0.0389	0.0248	0.0181
NewBru	0.0848	0.0854	0.0921	0.1121	0.1130	0.1084	0.0947	0.0720	0.0604	0.0513	0.0465	0.0378	0.0247	0.0165
Quebec	0.0895	0.0821	0.0890	0.1134	0.1200	0.1085	0.0936	0.0759	0.0603	0.0546	0.0467	0.0362	0.0191	0.0112
Ontario	0.0848	0.0881	0.1003	0.1193	0.1169	0.1038	0.0910	0.0681	0.0567	0.0528	0.0463	0.0375	0.0214	0.0130
Manito	0.0897	0.0898	0.0872	0.1135	0.1138	0.1020	0.0929	0.0692	0.0547	0.0491	0.0482	0.0427	0.0276	0.0197
Saskat	0.0940	0.0840	0.0844	0.1067	0.1114	0.1051	0.0837	0.0665	0.0574	0.0563	0.0508	0.0440	0.0325	0.0232
Alberta	0.1019	0.0995	0.1033	0.1195	0.1268	0.1092	0.0905	0.0635	0.0506	0.0431	0.0375	0.0276	0.0167	0.0103
B.C.	0.0817	0.0844	0.0966	0.1092	0.1161	0.1079	0.0968	0.0710	0.0581	0.0527	0.0472	0.0392	0.0232	0.0158
NorCan	0.1310	0.1052	0.1297	0.1442	0.1196	0.1102	0.0995	0.0598	0.0353	0.0258	0.0176	0.0101	0.0076	0.0044
2006	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85 $\leq$
NewLab	0.0774	0.0694	0.0858	0.0934	0.1062	0.1070	0.1080	0.1005	0.0776	0.0586	0.0463	0.0343	0.0212	0.0144
PEI	0.0880	0.0751	0.0794	0.0872	0.1105	0.1094	0.1000	0.0981	0.0681	0.0568	0.0467	0.0362	0.0284	0.0160
NovSco	0.0806	0.0748	0.0796	0.0872	0.1095	0.1107	0.0978	0.0931	0.0752	0.0583	0.0481	0.0387	0.0259	0.0205
NewBru	0.0815	0.0772	0.0853	0.0890	0.1083	0.1088	0.1036	0.0922	0.0715	0.0573	0.0451	0.0342	0.0244	0.0217
Quebec	0.0823	0.0861	0.0825	0.0891	0.1093	0.1115	0.1004	0.0882	0.0746	0.0548	0.0456	0.0379	0.0238	0.0138
Ontario	0.0884	0.0830	0.0878	0.0979	0.1153	0.1091	0.0966	0.0835	0.0636	0.0522	0.0451	0.0367	0.0255	0.0153
Manito	0.0931	0.0835	0.0887	0.0897	0.1079	0.1088	0.0911	0.0851	0.0639	0.0509	0.0457	0.0399	0.0295	0.0222
Saskat	0.0961	0.0853	0.0767	0.0821	0.1020	0.1080	0.1015	0.0845	0.0617	0.0522	0.0487	0.0439	0.0317	0.0255
Alberta	0.1042	0.1014	0.0983	0.1002	0.1127	0.1126	0.0985	0.0776	0.0554	0.0429	0.0359	0.0299	0.0183	0.0122
B.C.	0.0836	0.0780	0.0834	0.0936	0.1071	0.1094	0.0993	0.0904	0.0690	0.0550	0.0452	0.0389	0.0278	0.0193
NorCan	0.1068	0.1114	0.1275	0.1212	0.1252	0.1062	0.1027	0.0842	0.0462	0.0214	0.0202	0.0127	0.0087	0.0058
2011	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85 $\leq$
NewLab	0.0819	0.0726	0.0670	0.0894	0.0964	0.1080	0.1009	0.1021	0.0898	0.0681	0.0519	0.0370	0.0200	0.0151
PEI	0.0931	0.0654	0.0712	0.0819	0.0902	0.1096	0.1109	0.1096	0.0943	0.0525	0.0484	0.0397	0.0182	0.0149
NovSco	0.0819	0.0731	0.0760	0.0757	0.0883	0.1064	0.1062	0.0968	0.0949	0.0679	0.0488	0.0355	0.0271	0.0214
NewBru	0.0711	0.0798	0.0800	0.0855	0.0931	0.1062	0.1038	0.1018	0.0880	0.0643	0.0474	0.0360	0.0234	0.0197
Quebec	0.0829	0.0831	0.0891	0.0839	0.0878	0.1040	0.1059	0.0924	0.0850	0.0664	0.0459	0.0348	0.0239	0.0149
Ontario	0.0900	0.0848	0.0826	0.0879	0.0960	0.1093	0.1041	0.0876	0.0784	0.0578	0.0441	0.0350	0.0249	0.0175
Manito	0.0939	0.0910	0.0876	0.0852	0.0867	0.1014	0.1033	0.0908	0.0814	0.0574	0.0432	0.0349	0.0231	0.0202
Saskat	0.0965	0.0943	0.0889	0.0820	0.0848	0.0971	0.0995	0.0931	0.0750	0.0566	0.0442	0.0383	0.0279	0.0220
Alberta	0.0969	0.1068	0.1034	0.0992	0.0985	0.1052	0.1037	0.0848	0.0672	0.0454	0.0334	0.0263	0.0175	0.0116
B.C.	0.0835	0.0850	0.0804	0.0822	0.0922	0.1033	0.1041	0.0941	0.0851	0.0623	0.0464	0.0367	0.0257	0.0190
NorCan	0.1302	0.1161	0.1222	0.0891	0.1173	0.1241	0.1093	0.0817	0.0516	0.0369	0.0111	0.0074	0.0018	0.0012
2016	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85 $\leq$
NewLab	0.0684	0.0673	0.0706	0.0746	0.0798	0.0927	0.1048	0.1056	0.1026	0.0903	0.0629	0.0383	0.0249	0.0172
PEI	0.0780	0.0673	0.0662	0.0642	0.0796	0.0823	0.1178	0.1162	0.0953	0.0918	0.0610	0.0402	0.0252	0.0150
NovSco	0.0767	0.0734	0.0703	0.0711	0.0775	0.0827	0.1010	0.1078	0.0952	0.0879	0.0657	0.0437	0.0264	0.0207
NewBru	0.0712	0.0684	0.0663	0.0749	0.0854	0.0849	0.1054	0.1049	0.0960	0.0890	0.0611	0.0426	0.0289	0.0211
Quebec	0.0803	0.0790	0.0834	0.0879	0.0811	0.0826	0.1000	0.1005	0.0884	0.0780	0.0579	0.0387	0.0245	0.0178
Ontario	0.0883	0.0848	0.0847	0.0819	0.0845	0.0897	0.1017	0.0954	0.0807	0.0722	0.0505	0.0377	0.0262	0.0216
Manito	0.0948	0.0893	0.0935	0.0879	0.0844	0.0835	0.0969	0.0947	0.0816	0.0689	0.0482	0.0335	0.0221	0.0207
Saskat	0.0873	0.0988	0.0946	0.0900	0.0794	0.0793	0.0974	0.0927	0.0841	0.0647	0.0448	0.0376	0.0261	0.0232
Alberta	0.0869	0.1066	0.1107	0.1020	0.0927	0.0899	0.0937	0.0909	0.0751	0.0559	0.0389	0.0265	0.0178	0.0124
B.C.	0.0798	0.0822	0.0871	0.0800	0.0810	0.0881	0.0965	0.0951	0.0883	0.0792	0.0553	0.0385	0.0278	0.0212
NorCan	0.1118	0.1173	0.1221	0.1091	0.0809	0.1152	0.0981	0.0905	0.0706	0.0460	0.0213	0.0110	0.0021	0.0041

Table A2. Male – Female 6 Category Cumulative Densities

	<HSC	HSC	HSC>D	D(B)	PG>MA	MA+
2001 Female	0.30120	0.53705	0.80631	0.95014	0.99689	1.00000
2001 Male	0.29827	0.51297	0.80127	0.93243	0.99092	1.00000
2006 Female	0.20135	0.46463	0.75242	0.93766	0.99614	1.00000
2006 Male	0.19957	0.44036	0.76136	0.92437	0.99053	1.00000
2011 Female	0.16009	0.41108	0.70522	0.92024	0.99483	1.00000
2011 Male	0.16234	0.40042	0.72727	0.90986	0.98874	1.00000
2016 Female	0.14735	0.40964	0.70654	0.92259	0.99474	1.00000
2016 Male	0.15702	0.41835	0.74070	0.91823	0.98972	1.00000

Key: <HSC: No degree, certificate or diploma. HSC: High school graduation certificate. HSC>D Trades certificate/diploma, College certificate/diploma, University certificate/diploma lower than bachelors degree. D(B) University degree: Bachelors level. PG>MA Post bachelor level University degree: certificate, medical degree, Masters degree. MA+: Post Masters university degree: Earned doctorate

Table A3. Cumulative Density Differences (note the 1% critical value for these comparisons is approximately 0.0015).

Females Year on Year	<HSC	HSC	HSC>D	D(B)	PG>MA	MA+
$F_{2001}(x)$-$F_{2006}(x)$	0.09985	0.07242	0.05389	0.01248	0.00075	0.00000
$F_{2006}(x)$-$F_{2011}(x)$	0.04126	0.05355	0.04720	0.01742	0.00131	0.00000
$F_{2011}(x)$-$F_{2016}(x)$	0.01274	0.00144	-0.00132	-0.00235	0.00009	0.00000
$F_{2001}(x)$-$F_{2011}(x)$	0.14111	0.12597	0.10109	0.02990	0.00206	0.00000
$F_{2001}(x)$-$F_{2016}(x)$	0.15385	0.12741	0.09977	0.02755	0.00215	0.00000
$F_{2006}(x)$-$F_{2016}(x)$	0.05400	0.05499	0.04588	0.01507	0.00140	0.00000
Males Year on Year						
$F_{2001}(x)$-$F_{2006}(x)$	0.09870	0.07261	0.03991	0.00806	0.00039	0.00000
$F_{2006}(x)$-$F_{2011}(x)$	0.03723	0.03994	0.03409	0.01451	0.00179	0.00000
$F_{2011}(x)$-$F_{2016}(x)$	0.00532	-0.01793	-0.01343	-0.00837	-0.00098	0.00000
$F_{2001}(x)$-$F_{2011}(x)$	0.13593	0.11255	0.07400	0.02257	0.00218	0.00000
$F_{2001}(x)$-$F_{2016}(x)$	0.14125	0.09462	0.06057	0.01420	0.00120	0.00000
$F_{2006}(x)$-$F_{2016}(x)$	0.04255	0.02201	0.02066	0.00614	0.00081	0.00000
CDF Differences Female-Male						
2001	0.00293	0.02408	0.00504	0.01771	0.00597	0.00000
2006	0.00178	0.02427	-0.00894	0.01329	0.00561	0.00000
2011	-0.00225	0.01066	-0.02205	0.01038	0.00609	0.00000
2016	-0.00967	-0.00871	-0.03416	0.00436	0.00502	0.00000
Integrated CDF Differences Female-Male						
2001	0.00293	0.02701	0.03205	0.04976	0.05573	0.05573
2006	0.00178	0.02605	0.01711	0.03040	0.03601	0.03601
2011	-0.00225	0.00841	-0.01364	-0.00326	0.00283	0.00283
2016	-0.00967	-0.01838	-0.05254	-0.04818	-0.04316	-0.04316

Key: <HSC: No degree, certificate or diploma. HSC: High school graduation certificate. HSC>D Trades certificate/diploma, College certificate/diploma, University certificate/diploma lower than bachelors degree. D(B) University degree: Bachelors level. PG>MA Post bachelor level University degree: certificate, medical degree, Masters degree. MA+: Post Masters university degree: Earned doctorate