University of Toronto
Department of Economics

Working Paper 629

Policy Distortions and Aggregate Productivity with Endogenous Establishment-Level Productivity

By Jose-Maria Da-Rocha, Diego Restuccia and Marina M. Tavares

February 07, 2019

Policy Distortions and Aggregate Productivity with Endogenous Establishment-Level Productivity*

José-María Da-Rocha
Universidade de Vigo†

Diego Restuccia
University of Toronto and NBER‡

Marina M. Tavares
International Monetary Fund§

February 2019

Abstract

What accounts for differences in output per capita and total factor productivity (TFP) across countries? Empirical evidence points to resource misallocation across heterogeneous production units as an important factor. We study misallocation in a general equilibrium model of establishment productivity where the distribution of productivity is characterized in closed form and responds to the same policy distortions that create misallocation. In this framework, policy distortions not only misallocate resources across a given set of productive units (static effect), but also create disincentives for productivity improvement thereby altering the productivity distribution and equilibrium prices (dynamic effect), further lowering aggregate output and TFP. The dynamic effect is substantial contributing to a doubling of the static misallocation effect. Reducing the dispersion in distortions by 25 percentage points to the level of the U.S. benchmark economy implies an increase in relative aggregate output of 123 percent, where 54 percent arises from factor misallocation (static effect).

Keywords: distortions, misallocation, investment, endogenous productivity, establishments.
JEL codes: O11, O3, O41, O43, O5, E0, E13, C02, C61.

*For helpful comments we thank Lukasz Drozd, Sampreet Goraya, Hugo Hopenhayn, Alex Monge-Naranjo, B. Ravikumar, Loris Rubini, and participants at ITAM/INEGI Workshop on Productivity, ITAM, St. Louis Fed, SED Meetings in Toronto, North American Summer Meeting of the Econometric Society in Minneapolis, National Bank of Poland, and Banco de Mexico. All remaining errors are our own. Restuccia gratefully acknowledges the financial support from the Canada Research Chairs program and SSHRC of Canada. Tavares gratefully acknowledges the financial support of DIFD. Da Rocha gratefully acknowledges the financial support of Xunta de Galicia (ref. GRC2015/014 and ECOBAS).

†ECOBAS, Facultad CC Económicas y Emp., Campus Universitario Lagoas-Marcosende, 36310-Vigo, Spain (e-mail: jmrocha@uvigo.es).

‡150 St. George Street, Toronto, ON M5S 3G7, Canada (e-mail: diego.restuccia@utoronto.ca).

§700 19th Street, N.W., Washington, D.C. 20431, USA (e-mail: marinamendestavares@gmail.com).

1 Introduction

A crucial question in economic growth and development is why some countries are rich and others poor. A consensus has emerged in the literature whereby the large differences in income per capita across countries are mostly accounted for by differences in labor productivity and in particular total factor productivity (TFP) (Klenow and Rodriguez-Clare, 1997; Prescott, 1998; Hall and Jones, 1999). Hence, a key question is: what accounts for differences in TFP across countries? An important channel that has been emphasized is the (mis)allocation of factors across heterogeneous production units.[1] We study factor misallocation in a model where establishment-level productivity is determined endogenously. In this framework, policy distortions not only misallocate resources across a given set of productive units (static effect), but also create disincentives for productivity improvement (dynamic effect) thereby affecting the productivity distribution across establishments and equilibrium prices in the economy, further contributing to lower aggregate output and TFP.

A recent branch of the literature has emphasized the dynamic implications of misallocation by considering variants of the growth model with establishment-level productivity dynamics.[2] We build on this literature by developing a general equilibrium model of establishment productivity where the distribution of productivity is characterized in closed form as a function of the economic environment which is affected by policy distortions. In our framework, not only there is a tight mapping between abstract policy distortions and the empirical counterparts of dispersion in revenue products and factor misallocation, but also compared to the model with an exogenous distribution of productivity, the effect on aggregate output of empirically-plausible policy distortions is substantial. For instance, in our framework, the output gain from reducing the standard deviation of log revenue productivity (TFPR) from 0.74 to 0.49 in the benchmark economy is 123%, substantially larger than

[1]See Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Guner et al. (2008), and Hsieh and Klenow (2009). See also the surveys of the literature in Restuccia and Rogerson (2013), Restuccia (2013a), Hopenhayn (2014), and Restuccia and Rogerson (2017).

[2]Some of the early contributions on the endogenous productivity distribution include Restuccia (2013b), Bello et al. (2011), Acemoglu et al. (2018), Ranasinghe (2014), Bhattacharya et al. (2013), Gabler and Poschke (2013), Rubini (2014), Hsieh and Klenow (2014), Bento and Restuccia (2017), Guner et al. (2018), Peters (2015), Buera and Fattal-Jaef (2018), among others. See also Restuccia and Rogerson (2017) for a discussion of this literature.

the 54% gain from reduced factor misallocation (static effect). These effects in the model compare to the magnitudes of factor misallocation reported for 1998 China in Hsieh and Klenow (2009) and the actual gap in productivity between China and the United States. Moreover, our analytical solution of the distribution of productivity and how is affected by distortions can potentially be useful in empirical applications of dynamic misallocation across countries using panel micro data, an essential issue in the misallocation literature (Restuccia and Rogerson, 2017).

We develop a general equilibrium framework with heterogeneous production units that builds on Hopenhayn (1992) and Restuccia and Rogerson (2008). The framework is a standard neoclassical growth model with production heterogeneity extended to incorporate the dynamic implications of distortions on the distribution of establishment-level productivity. We use this framework to study the impact of policy distortions on misallocation and aggregate output and measured TFP. The key elements of the model are on the production side. In each period, there is a single good produced in establishments. Establishments are heterogeneous with respect to total factor productivity and have access to a decreasing returns to scale technology with capital and labor as inputs. Establishments are subject to an exogenous exit rate but differently from the standard framework, the distribution of establishment-level productivity is not exogenous, rather it is determined by establishment's decisions. In other words, productivity of establishments is determined endogenously in the model by the properties of the economic environment such as policy distortions.

Following the literature, the economy faces policy distortions which, for simplicity, take the form of output taxes on individual producers. That is, each producer faces an idiosyncratic tax and it is the properties of policy distortions that generate misallocation in the model. Revenues collected from these taxes are rebated back to the households as a lump-sum transfer. We emphasize that the output distortions we consider are abstract representations (catch all) of the myriad of implicit and explicit distortions faced by individual producers. While the literature has made substantial progress in identifying the specific policies and institutions that create misallocation—as discussed in Restuccia and Rogerson (2017)—the emphasis in our paper is on the dynamic consequences of misallocation created by all the distortions on the productivity distribution in the economy. As a result, our paper represents a general quantitative assessment of the broader consequences of misallocation.

We provide an analytical solution of this model in continuous time. In particular, we solve in closed form for the stationary distribution of establishments which is an endogenous object that varies across economies. We show the equilibrium productivity distribution is a Pareto distribution with tail index that depends on policy distortions and on the response of incumbent establishments to distortions when selecting the growth rate of productivity. This allows us to characterize the behavior of aggregate output and TFP across distortionary policy configurations as well as the size and productivity growth rate of establishments, the size distribution of establishments, among other statistics of interest. The analytical solution of the productivity distribution also allows us to decompose analytically the static and dynamic effects of misallocation.

To explore the quantitative properties of the model relative to the existing literature, we calibrate the model and provide a set of relevant quantitative experiments. We consider a benchmark economy with distortions that is calibrated to data for the United States. The key calibrated parameters in our analysis are the curvature in the cost function for productivity growth, the variance in the distribution of productivity, and the growth rate of establishment size, which are targeted to data on the aggregate growth rate of TFP, the average employment growth of establishments, and the right tail index of the share of employment distribution in the U.S. data. We then perform quantitative analysis by exploring the implications of increased distortions for aggregate output and TFP.

Our main result is that policy distortions generate substantial negative effects on aggregate output and TFP, roughly doubling the effect of static misallocation—that is doubling the effect of distortions in a model with an exogenous distribution of productivity. In particular, reducing distortions in an economy by 25 percentage points to the dispersion in the U.S. benchmark economy, implies an increase in aggregate output of 123%, where about one half of this increase is due to improved factor allocation and the remaining half due to an improved productivity distribution. In this economy, the growth rate of establishment productivity and employment is less than half of that in the benchmark economy. Hence, reducing distortions to the benchmark roughly doubles the growth rate of establishments in this economy. These effects are broadly consistent with some evidence that in more distorted economies the productivity and employment growth of establishments are lower (e.g., Hsieh and Klenow, 2014) and with some evidence that when distortions are removed, the growth rate of establishments increases (e.g., Pavcnik, 2002; Bustos, 2011).

Our paper is related to a large literature on misallocation and productivity discussed earlier. The literature has emphasized various separate channels such as life-cycle investment of plants, human capital accumulation of managers, experimentation, step-by-step innovation, selection, among many others; and different contexts such as trade and labor policies, financial frictions, and specific sectors. We complement this literature by developing a general model of establishment growth featuring a distribution of establishment productivity that can be characterized in closed form. This theoretical characterization is important for our quantitative analysis of policy distortions because it allows us to tractably separate the role of changes in the productivity distribution and equilibrium prices (dynamic effect) from factor misallocation (static effect) in accounting for aggregate output and TFP differences. More importantly, our theoretical characterization can be useful in developing methods to estimate the role of dynamic misallocation using panel data of firms, and hence we hope our analysis can facilitate more empirical applications.

Two closely related papers to ours are Hsieh and Klenow (2014) and Bento and Restuccia (2017). Hsieh and Klenow (2014) consider the model of establishment innovation in Atkeson and Burstein (2010) to emphasize the life-cycle growth of establishments and its response to distortions, whereas Bento and Restuccia (2017) emphasize both entry productivity and life-cycle growth. We emphasize two key distinctions with our work. First, in these papers entering establishments draw their productivity from an exogenous and constant distribution across countries, whereas the entire productivity distribution is a key equilibrium object in our framework that responds to policy distortions. Second, we differ in the tools used to characterize the economy, in particular, we solve analytically for the entire distribution of productivity using continuous time and Brownian motion processes. These tools are increasingly popular in the growth literature allowing both a tighter theoretical characterization and more efficient computation (e.g., Lucas and Moll, 2014; Benhabib et al., 2014; Buera and Oberfield, 2014). More closely linked, these tools were prominently used in the seminal work of Luttmer (2007) to study the size distribution of establishments in the United States (see also Da-Rocha and Pujolas, 2011; Fattal, 2014; Gourio and Roys, 2014).

The paper proceeds as follows. In the next section we present the details of the model and Section 3 characterizes the equilibrium solution. In Section 4, we characterize aggregate output and measured TFP. Section 5 calibrates a benchmark economy with distortions to U.S. data. In Section 6, we perform a series of quantitative experiments to assess the impact of increased policy distortions on aggregate output, TFP, and other relevant statistics. We conclude in Section 7.

2 Economic Environment

We consider a standard version of the neoclassical growth model with producer heterogeneity as in Restuccia and Rogerson (2008). We extend this framework to allow establishments to invest in their productivity. As as a result, with on-going entry and exit of establishments, the framework generates an invariant distribution of productivity across establishments associated with the economic environment that may differ across countries. Time is continuous and the horizon is infinite. Establishments have access to a decreasing return to scale technology, pay a one-time fixed cost of entry, and die at an exogenous rate. Establishments hire labor and rent capital services in competitive markets. New entrants enter with a level of productivity z_e which is endogenous. We focus on a stationary equilibrium of this model and study the effects of idiosyncratic policy distortions on the allocation of factors across establishments. In this framework, policy distortions also affect establishment's decisions regarding their productivity growth, and hence the stationary distribution of productivity across establishments, aggregate output, and measured TFP. In what follows we describe the economic environment in more detail.

2.1 Baseline Model

There is an infinity-lived representative household with preferences over consumption goods described by the utility function,

$$\max \int_0^{+\infty} e^{-\rho t} u(c) dt,$$

where c is consumption and ρ is the discount rate. The household is endowed with one unit of productive time at each instant and $k_0 > 0$ units of the capital stock at date 0.

The unit of production in the economy is the establishment. Each establishment is described by a production function $f(z, k, n)$ that combines capital services k and labor services n to produce output. The function f is assumed to exhibit decreasing returns to scale in capital and labor jointly and to satisfy the usual Inada conditions. The production function is given by:

$$y = z^{\theta(1-\alpha-\gamma)} k^{\alpha} n^{\gamma}, \quad \alpha, \gamma \in (0,1), \quad 0 < \gamma + \alpha < 1, \quad \theta > 1, \tag{1}$$

where establishment productivity z is stochastic but establishments can invest in upgrading their productivity at a cost and θ is a scaling parameter for TFP. Note that establishment TFP is $z^{\theta(1-\alpha-\gamma)}$ and hence θ affects the units in which establishment productivity z is measured. Scaling productivity by θ is convenient for algebraic manipulations below as this parameter also represents the curvature in the cost function of establishment productivity growth. Establishments also face an exogenous probability of death λ.

New establishments can also be created. Entrants must pay an entry cost c_e measured in units of output and as in the literature the expected value of entry satisfies the zero profit condition in equilibrium. Feasibility in the model requires:

$$C + I + Q = Y - E,$$

where C is aggregate consumption, I is aggregate investment in physical capital, Q is aggregate cost of investing in establishment productivity, E is the aggregate cost of entry, and Y is aggregate output.

2.2 Policy Distortions

We introduce policies that create idiosyncratic distortions to establishment-level decisions as in Restuccia and Rogerson (2008). We model these distortions as idiosyncratic output taxes but none of our results are critically dependent on the particular source of distortions. While the policies we consider are hypothetical, there is a large empirical literature documenting the extent of idiosyncratic distortions across countries and our framework allows for a tight mapping between the distortions and empirical observations (e.g., Hsieh and Klenow, 2009; Bartelsman et al., 2013; Restuccia and Rogerson, 2017).

In our framework, distortions not only affect the allocation of resources across existing production units, but also the growth rate of establishment productivity, thereby affecting the distribution of productive units in the economy. Specifically, we assume that each establishment faces its own policy distortion (idiosyncratic distortions) reflected as an output tax rate τ_y. In what follows, for simplicity in our algebraic expressions we rewrite distortions as $\tau = (1 - \tau_y)^{\frac{1}{\theta(1-\alpha-\gamma)}}$. Note that this transformation implies that an establishment with no distortions $\tau_y = 0$ faces $\tau = 1$, whereas a positive output tax $\tau_y > 0$ implies $\tau < 1$ and an output subsidy $\tau_y < 0$ implies $\tau > 1$.

In order to generate dispersion in distortions across productive units, we assume that τ follows a standard stochastic process, a Geometric Brownian motion,

$$d\tau = \mu_\tau \tau dt + \sigma_\tau \tau dw_\tau,$$

where μ_τ is the drift, σ_τ is the standard deviation and dw_τ is the standard Wiener process of the Brownian motion. In this specification σ_τ controls the dispersion of distortions across producers and hence the dispersion in marginal revenue products that is restricted to data. As noted earlier, our specification of distortions is reduced form and abstract, simply standing in for the myriad of policies and institutions that effectively create dispersion in individual producer prices (e.g., Restuccia and Rogerson, 2008; Buera et al., 2013).

Establishment's productivity z follows a Geometric Brownian motion and establishments can invest in upgrading their productivity by choosing the drift of the Brownian motion μ_z that is determined in equilibrium, establishment productivity follows:

$$dz = \mu_z z dt + \sigma_z z dw_z,$$

where σ_z is the standard deviation and dw_z is the standard Wiener process of the Brownian motion. We assume that the output tax and productivity can be correlated, that is $E(dw_\tau, dw_z) = \rho_{\tau,z} \in (-1, 0]$. Note that a negative value of $\rho_{\tau,z}$, corresponds to correlated distortions, as in Restuccia and Rogerson (2008), whereby distortions impact more heavily on more productive establishments.

At the time of entry, the establishment-entry distortion τ_e is known and establishments enter with a productivity z_e that is determined in equilibrium and implies an expected value of entrants that satisfies the zero profit condition. In this economy, the relevant information for establishment's decisions is the joint distribution over productivity and distortions. We denote this joint distribution by $g(z, \tau)$.

A given distribution of establishment-level distortion and productivity may not lead to a balanced budget for the government. As a result, we assume that budget balance is achieved by either lump-sum taxation or redistribution to the representative household. We denote the lump-sum tax by T.

3 Equilibrium

We focus on a stationary equilibrium of this economy. The stationary equilibrium is characterized by an invariant distribution of establishments $g(z, \tau)$ over productivity z and distortion τ and an entry productivity z_e. In the stationary equilibrium, the rental price for labor and capital services are constant and we denote them by w and r. Before defining the stationary equilibrium formally, it is useful to consider the decision problems faced by incumbents, entrants, and consumers. We describe these problems in turn.

3.1 Incumbent establishments

Incumbent establishments maximize the present value of profits by making static and dynamic decisions. The static problem is to choose the amount of capital and labor services, whereas the dynamic problem involves solving for the drift in establishment productivity. We now describe these problems in detail.

Static problem At any instant of time an establishment chooses how much capital to rent k and how much labor to hire n. These decisions are static and depend on the establishment's productivity z, the establishment's distortion τ, the rental rate of capital r, and the wage rate w. Formally, the instant profit function $\pi(z,\tau)$ is defined by:

$$\pi(z,\tau) = \max_{k,n} (\tau z)^{\theta(1-\alpha-\gamma)} k^{\alpha} n^{\gamma} - wn - rk,$$

from which we obtain the optimal demand for labor and capital:

$$n(z,\tau) = \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\gamma}{w}\right)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\gamma}} z^{\theta} \tau^{\theta}, \tag{2}$$

$$k(z,\tau) = \left[\left(\frac{\alpha}{r}\right)^{1-\gamma} \left(\frac{\gamma}{w}\right)^{\gamma} \right]^{\frac{1}{1-\alpha-\gamma}} z^{\theta} \tau^{\theta}. \tag{3}$$

For future reference, we redefine instant profits as a function of the optimal demand for factors:

$$\pi(z,\tau) = m(w,r) z^{\theta} \tau^{\theta}, \tag{4}$$

where $m(w,r) = (1-\alpha-\gamma)\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}}$ is a constant across establishments that depends on equilibrium prices. Note that since factor demands are linear in $(z\tau)^{\theta}$, we find it convenient to define size s as $s \equiv (z\tau)^{\theta}$ so that factor demands are proportional to size s. A key insight of the misallocation literature is that the relationship between size and productivity is fundamentally affected by distortions.

Dynamic problem Incumbent establishments choose the drift of their productivity μ_z. The cost of investing in productivity is expressed in units of output, described by a cost function $q(\cdot)$ that is increasing and convex in the productivity drift, specifically we assume $q(\mu_z) = c_{\mu}(z\tau)^{\theta}\frac{\mu_z^{\theta}}{\theta}$, where θ controls the convexity of the cost function and c_{μ} is a common scale parameter. The cost function is also scaled by establishment size s which implies that all establishments choose the same productivity drift as we discuss below. The optimal decision of productivity improvement is characterized by maximizing the present value of profits subject to the Brownian motion governing the evolution of productivity and the Brownian motion governing the evolution of distortions. Formally, incumbent establishments solve the following dynamic problem:

$$W(z,\tau) = \max_{\mu_z}\left\{m(w,r)z^{\theta}\tau^{\theta} - q(\mu_z) + \frac{1}{1+(\lambda+R)dt}E_{z,\tau}W(z+dz,\tau+d\tau)\right\},$$

$$\begin{aligned} s.t. \quad dz &= \mu_z z dt + \sigma_z z dw_z, \\ d\tau &= \mu_{\tau}\tau dt + \sigma_{\tau}\tau dw_{\tau,} \end{aligned}$$

where λ is the exogenous exit probability of establishments and R is the stationary equilibrium real interest rate. Next, we define the Hamilton-Jacobi-Bellman of the stationary solution,

$$\begin{aligned} (\lambda+R)W(z,\tau) \quad = \quad & \max_{\mu_z}\left\{m(w,r)(z\tau)^{\theta} - c_{\mu}(z\tau)^{\theta}\frac{\mu_z^{\theta}}{\theta}\right. \\ & \left. +\mu_z z W_z' + \frac{\sigma_z^2}{2}z^2 W_{zz}'' + \mu_{\tau}\tau W_{\tau}' + \frac{\sigma_{\tau}^2}{2}\tau^2 W_{\tau\tau}'' + \sigma_z\sigma_{\tau}\rho_{z,\tau}z\tau W_{z\tau}''\right\}. \end{aligned} \quad (5)$$

In the following Lemma 1 we characterize formally the endogenous productivity drift.

Lemma 1. *Given a distortion τ, a productivity level z, and operating profits $m(w,r)$, the value function that solves the establishment dynamic problem is given by $W(z,\tau) = A(w,r)\tau^{\theta}z^{\theta}$, and the expected growth rate of establishment's productivity z follows Gibrat's law:*

$$\frac{dz}{z} = \left[\frac{\theta A(w,r)}{c_{\mu}}\right]^{\frac{1}{\theta - 1}} dt + \sigma_z dw_z,$$

where $A(w,r)$ is the solution of the polynomial in equation (A.1) and $\mu_z(w,r) = \left[\frac{\theta A(w,r)}{c_{\mu}}\right]^{\frac{1}{\theta-1}}$.

Proof See Appendix A.1.

The implication of Lemma 1 is that the growth rate of productivity of individual establishments does not depend on the intrinsic characteristics of the establishment, that is, it does not depend on the establishment productivity z or the distortion τ so Gibrat's law holds. We recognize that there is some debate as to whether Gibrat's law holds empirically, however, we note that this implication of the model implies a more muted negative effect of distortions on output, making our quantitative results conservative in this context. Atkeson and Burstein (2010) develop a model of firm-level innovation with the same property, whereas Bhattacharya et al. (2013) and Hsieh and Klenow (2014) consider environments where distortions create idiosyncratic effects across establishments. While Lemma 1 implies that the endogenous productivity drift is constant across establishments, the drift can differ across economies with different policy distortions if distortions affect equilibrium wages, and this is a key element in our quantitative analysis.

Employment, which is proportional to distortions and productivity $n \propto (z\tau)^{\theta}$, also follows Gibrat's law, and the resulting Brownian motion of $s = (z\tau)^{\theta}$ is given by:

$$\frac{ds}{s} = \left[\theta\left(\mu_z(w,r) + \mu_{\tau} + \theta\sigma_z\sigma_{\tau}\rho_{\tau,z}\right) + \frac{\theta(\theta-1)}{2}(\sigma_z^2 + \sigma_{\tau}^2)\right] dt + \theta(\sigma_z + \sigma_{\tau})dw_s, \tag{6}$$

where the drift μ_s is equal $\mu_s = \theta\left(\mu_z(w,r) + \mu_{\tau} + \theta\sigma_z\sigma_{\tau}\rho_{\tau,z}\right) + \frac{\theta(\theta-1)}{2}(\sigma_z^2 + \sigma_{\tau}^2)$. In this environment, policy distortions affect productivity growth and employment growth differently. Employment is impacted by distortions directly through the dispersion of distortions σ_τ and its correlation with productivity $\rho_{\tau,z}$, and indirectly through changes in productivity growth $\mu_z(w, r)$.

3.2 Entering establishments

Potential entering establishments face an entry cost c_e in units of output and make their entry decision knowing the output entering tax level τ_e. For tractability, we assume that entrants enter with the same level of expected productivity, denoted by z_e. The initial level of productivity is such that the value of entering establishments satisfies the usual zero profit condition:

$$W_e = W(z_e, \tau_e) - c_e.$$

Note that such a value of productivity z_e exists and is unique which follows from the fact that the value of entry W_e inherits the properties of the value of incumbent establishments which is increasing in productivity z. In addition, in the special case where the model is deterministic, the value of entering is the same as in Restuccia and Rogerson (2008), which is the establishments' expected profit.

3.3 Stationary distribution of establishments

Given the optimal decisions of incumbents and entering establishments, we are now ready to characterize the stationary distribution $g(z, \tau)$ over productivity z and distortion τ. The first step to characterize this distribution is to rewrite the Brownian motions of productivity z and distortion τ as a function of s as in equation (6). In order to characterize the stationary distribution over size s, it is useful to rewrite the model in logarithms. Let x denote the logarithm of s, that is $x = \log(s/s_e)$, where s_e is the size in which establishments enter. Now we can rewrite the Geometric Brownian motion in equation (6) as a Brownian motion in the logarithm of s,

$$dx = \mu_x dt + \sigma_x dw_x,$$

where $\mu_x = \mu_s - \frac{1}{2}\sigma_x^2$ and $\sigma_x^2 = \theta^2\left(\sigma_z^2 + \sigma_\tau^2 + 2\sigma_z\sigma_\tau\rho_{z,\tau}\right)$. Let $M(x,t)$ denote the number density function of establishments, i.e. the mass of size x establishments at time t. At time t, the total number of establishments is equal to $M(t) = \int_{-\infty}^{+\infty} M(x,t)dx$.

The establishment productivity process can be modeled by a *modified* Kolmogorov-Fokker-Planck equation of the form:

$$\frac{\partial M(x,t)}{\partial t} = -\mu_x \frac{\partial M(x,t)}{\partial x} + \frac{\sigma_x^2}{2}\frac{\partial^2 M(x,t)}{\partial x^2} - \lambda M(x,t) + B(0,t), \quad (7)$$

where λ is the death rate of establishments and the function $B(0,t)$ are the new establishments that enter at time t and have size 0, after the renormalization. The solution of this problem is discussed in Gabaix (2009), and we solve by applying Laplace Transforms methods. Da-Rocha et al. (2019) show that the double Pareto is a particular solution in frameworks with inaction.

We are interested in a stationary distribution for the number density function, i.e. solutions that are separable in time t and are of the form $M(x,t) = M(t)f(x)$ and $B(0,t) = M(t)b\delta(x-0)$, where b is the establishment entry rate at point $x = 0$ and $\delta(\cdot)$ is a Dirac delta function which is equal to 1 at the entry, normalized to zero, and is equal to zero everywhere else. Mathematically, we can express this by using a Dirac delta function that is equal to infinity at the point on which new firms enter and zero otherwise. Let the function $b(0)$ be described by $\hat{b}(0) = b\delta(x-0)$, where δ denotes the Dirac delta function:

$$\delta(x) = \begin{cases} +\infty & \text{if} \quad x = 0, \\ 0 & \text{if} \quad x \neq 0. \end{cases}$$

Therefore, we can rewrite the *modified* Kolmogorov-Fokker-Planck equation equation (7) as:

$$\frac{M'(t)}{M(t)}f(x) = \eta f(x) = -\mu_x f'(x) + \frac{\sigma_x^2}{2}f''(x) - \lambda f(x) + b\delta(x-0), \tag{8}$$

where $\frac{M'(t)}{M(t)}$ is the growth rate of the mass of establishments denoted by η and $M(t) = e^{\eta t}M(0)$ in the balanced growth path. We normalize $M(0) = 1$. We assume four standard boundary conditions:

$$\lim_{x\to+\infty} f(x) = 0, \qquad \lim_{x\to+\infty} f'(x) = 0, \tag{9}$$

$$\lim_{x\to-\infty} f(x) = 0, \qquad \lim_{x\to-\infty} f'(x) = 0, \tag{10}$$

and

$$f(x) \geq 0, \qquad \int_{-\infty}^{+\infty} f(x)dx = 1. \tag{11}$$

The first four boundary conditions (9) and (10) guarantee that the stationary distribution is bounded, and equations (11) guarantee that f is a pdf. The boundary constraints restrict the growth rate of the mass of establishments η, by integrating (8) we find:

$$\eta \int_{\infty}^{+\infty} f(x)dx = \left(-\mu_x f(x) + \frac{\sigma_x^2}{2}f'(x)\right)\Bigg|_{-\infty}^{+\infty} - \lambda \int_{-\infty}^{+\infty} f(x)dx + \int_{-\infty}^{+\infty} b\delta(x-0)dx,$$

and applying the boundary conditions and using the Dirac delta function, we find that η is equal to:

$$\eta = b - \lambda,$$

which has a very intuitive interpretation, as it states that the growth rate of the mass of establishments η is equal to the net entry rate $(b - \lambda)$. After some algebraic manipulation from equation (8), we find that the stationary distribution must satisfy the following differential equation:

$$f''(x) - \frac{2\mu_x}{\sigma_x^2}f'(x) - \frac{2(\lambda+\eta)}{\sigma_x^2}f(x) = -\frac{2b}{\sigma_x^2}\delta(x-0), \tag{12}$$

subject to the boundary conditions and $f(\cdot)$ being a pdf. We can now characterize the stationary (log) size distribution, which is a double Pareto, with endogenous tail index, ξ, and endogenous net entry rate, $b - \lambda$ at $x = 0$. Formally, Lemma 2 characterizes the stationary distribution.

Lemma 2. *Given wages w, rental rate of capital r, and a policy (μ_τ, σ_τ) the stationary size distribution is a double Pareto:*

$$g(s) = \begin{cases} C\left(\frac{s}{s_e}\right)^{-(\xi_- + 1)} & \textit{for} \quad s < s_e. \\ \\ C\left(\frac{s}{s_e}\right)^{-(\xi_+ + 1)} & \textit{for} \quad s \geq s_e. \end{cases}$$

where the tail index ξ_+ is the positive root and the tail index ξ_- is the negative root that solves the characteristic equation $\frac{\sigma_x^2}{2}\xi^2 + \left(\mu_s - \frac{\sigma_x^2}{2}\right)\xi - (\lambda + \eta) = 0$ and $C = \frac{-\xi_- \xi_+}{s_e(\xi_+ - \xi_-)}$ where $\mu_x = \mu_s - \frac{1}{2}\sigma_x^2$ and $\sigma_x^2 = \theta^2\left(\sigma_z^2 + \sigma_\tau^2 + 2\sigma_z\sigma_\tau\rho_{z,\tau}\right)$. Moreover, the average size $\bar{s}$ of establishments is given by:

$$\frac{s_e}{\bar{s}} = 1 - \frac{\mu_s}{\eta + \lambda}.$$

Proof See Appendix A.2.

We leave the poof of Lemma 2 to the Appendix. Lemma 2 characterizes the endogenous distribution as a function of establishments' size drift μ_s and entry size s_e, which in turn are affected by distortions.

3.4 Household's problem

The household problem is standard and essentially help us pin down the stationary real interest rate R. As such, the process for capital accumulation in this model follows the standard neoclassical growth model. The stand-in household seeks to maximize lifetime utility subject to the law of motion of wealth given by:

$$(RK + w + T + \Pi - bc_e - c)\,dt,$$

where w is the wage rate, R is the is the interest rate which in equilibrium is the rental price of capital minus capital depreciation ($R = r - \delta_k$), T is the lump-sum tax levied by the government, Π is the total profit from the operations of all establishment, bc_e is the entry cost and c is consumption.

We assume that households have log utility, $u(c) = \log(c)$, and we characterize the equilibrium interest rate by solving the household's problem. We define total wealth as:

$$a = K + \frac{w}{R} + \frac{T}{R} + \frac{\Pi}{R} - \frac{c_e b}{R},$$

and we rewrite the law of motion of wealth as $da = (Ra - c)dt$. The household solves the following Hamilton-Jacobi-Bellman equation:

$$\rho V(a) = \max_c \left\{ \log(c) + [Ra - c] V'(a) \right\}.$$

Lemma 3 establishes that in the stationary equilibrium the interest rate R is equal to the discount rate ρ.

Lemma 3. *In the stationary equilibrium the interest rate is equal to the discount rate $R = \rho$.*

3.5 Stationary equilibrium

Definition A stationary equilibrium is an invariant distribution $g(\cdot)$, a value function for incumbents $\{W(\cdot)\}$, a policy function for new entrants $\{z_e\}$, policy functions $k(\cdot)$, $n(\cdot)$, $\mu_z(\cdot)$, $c(\cdot)$, prices $\{r, w\}$, transfer $\{T\}$, and aggregate capital $\{K\}$, such that:

i) Given prices and transfer, the households' policy function $\{c(\cdot)\}$ solves the household dynamic problem.

ii) Given prices, the incumbents' policy functions $\{k(\cdot), n(\cdot)\}$ solve the incumbents' static problem.

iii) The incumbents' policy function $\{\mu_z(\cdot)\}$ together with the value function $\{W(\cdot)\}$ solve the incumbents' dynamic problem.

iv) The stationary distribution $\{g(\cdot)\}$ solve the Kolmogorov forward equation.

v) The entering establishments' policy function $\{z_e\}$ solves the free-entry condition.

vi) Market Clearing:

a) Capital: $K = \int_0^{+\infty} k(s, w, r)g(s)ds$,

b) Labor: $1 = \int_0^{+\infty} n(s, w, r)g(s)ds$.

vii) The government budget constraint is satisfied, $T = \int_0^{+\infty} \tau_y y(s)g(s)ds$.

The stationary equilibrium is a fixed point in measure and it is very simple to compute. From the household's problem, we solve for the stationary interest rate R and hence pin down the rental rate of capital r. From the incumbents' static problem, we solve the labor and capital demand as a function of prices $\{r, w\}$ and policies $\{\tau\}$. Given the solution of the static problem, incumbents solve the dynamic problem of choosing a productivity drift. The solution to this problem is a policy function $\{\mu_z(\cdot)\}$ that determines the Geometric Brownian motion for productivity of the entire economy. Given the Geometric Brownian motion for productivity, we solve for the stationary distribution $g(\cdot)$ that solves the Kolmogorov forward equation. After solving for the stationary distribution $g(\cdot)$, the entry rate at the entry size s_e must solve the free-entry condition, and the markets for capital and labor must clear. Formally, μ_z, A, μ_s, s_e, $\bar{s}$, and w are obtained by solving the following 6 equations:

(1) The productivity growth rate μ_z satisfies:

$$\mu_z = \left[\frac{\theta A(w, r)}{c_\mu}\right]^{\dfrac{1}{\theta - 1}}. \tag{13}$$

(2) The establishment's value function A satisfies:

$$A(w,r) = \frac{m(w,r)}{\lambda + R - (\theta-1)\mu_z(w,r) - \theta\left(\mu_\tau + \theta\sigma_z\sigma_\tau\rho_{\tau,z}\right) - \dfrac{\theta(\theta-1)}{2}(\sigma_z^2+\sigma_\tau^2)}, \tag{14}$$

where $m(w,r) = (1-\alpha-\gamma)\left[\left(\frac{\alpha}{r}\right)^\alpha \left(\frac{\gamma}{w}\right)^\gamma\right]^{\frac{1}{1-\alpha-\gamma}}$.

(3) The growth rate of establishment size μ_s is given by:

$$\mu_s = \theta\left(\mu_z(w,r) + \mu_\tau + \theta\sigma_z\sigma_\tau\rho_{\tau,z}\right) + \frac{\theta(\theta-1)}{2}(\sigma_z^2+\sigma_\tau^2). \tag{15}$$

(4) The entry establishment's size s_e is compatible with free entry:

$$c_e = A(w,r)s_e. \tag{16}$$

(5) Average size $\bar{s}$ is given by:

$$\frac{\bar{s}}{s_e} = \frac{\eta+\lambda}{\eta+\lambda-\mu_s}. \tag{17}$$

(6) The real wage rate w makes labor demand compatible with the inelastic one unit of labor supply:

$$\left[\left(\frac{\alpha}{r}\right)^\alpha \left(\frac{\gamma}{w}\right)^{(1-\alpha)}\right]^{\frac{1}{1-\alpha-\gamma}} \bar{s} = 1. \tag{18}$$

4 Aggregate Output and TFP

We characterize the impact of policy distortions on aggregate output and TFP using the well-known concept of revenue total factor productivity TFPR, which was disseminated in the context of the macro development literature by Hsieh and Klenow (2009). In our model, an establishment's TFPR is given by:

$$\text{TFPR} = \frac{y}{k^{\alpha/(\alpha+\gamma)}n^{\gamma/(\alpha+\gamma)}} \propto \frac{1}{\tau^{\theta(1-\alpha-\gamma)}} = \frac{1}{(1-\tau_y)},$$

which is equated across all establishments in the undistorted economy. In this context, misallocation arises from dispersion in TFPR across establishments.

Aggregate output Y is computed by integrating over the distribution of establishment's output. Using the establishment production function in equation (1) and substituting the demand for labor and capital in equations (2) and (3), we obtain:

$$y(z,\tau) = \left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}} \tau^{\theta(\alpha+\gamma)} z^{\theta}, \tag{19}$$

where the term in square brackets is just a constant across establishments that depends on factor prices and technology parameters. It is clear from this expression that misallocation implies that establishment size and in particular output are affected by distortions.

Following Lyuu (2002), we use equation (19) to write aggregate output Y as:

$$Y = \left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}} \int_0^{+\infty} z^{\theta} g_z(z)dz \int_0^{+\infty} \tau^{\theta(\alpha+\gamma)} g_{\tau}(\tau)d\tau,$$

where z and τ are Geometric Brownian motions with distributions that are double Pareto. Using the same methodology as in Lemma 2 and imposing market clearing in labor, we obtain the following simple expression for aggregate output Y:

$$Y \propto \left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{\tau^{\theta(\alpha+\gamma)}}}\right)\frac{1}{(1-\tau_{y,e})}\right]\left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{z^{\theta}}}\right)\frac{s_e}{\bar{s}^{\gamma/(1-\alpha)}}\right], \tag{20}$$

where we use the fact that $s = (z\tau)^{\theta}$ and $(1-\tau_y) = \tau^{\theta(1-\alpha-\gamma)}$. The first term in square brackets arises from the distribution of distortions whereas the second term in square brackets arises from the productivity distribution and changes in equilibrium prices. Appendix A.3 describes the explicit steps of derivation.

In equation (20), we associate the expression in the first square brackets with the static output gain from changes in misallocation, that is changes in TFPR dispersion across establishments (static effect), and we associate the expression in the second square brackets with the dynamic output gain from eliminating distortions via changes in the distribution of establishment productivity and changes in equilibrium prices (dynamic effect). In the quantitative analysis that follows, we emphasize the relative importance of these terms, static and dynamic effects, in accounting for income differences in the model. For completeness, we compute measured aggregate TFP following standard practice as aggregate output per unit of aggregate composite inputs:

$$\text{TFP} = \frac{Y}{K^{\alpha/(\alpha+\gamma)} N^{\gamma/(\alpha+\gamma)}}, \tag{21}$$

where aggregate labor N is equal to one by market clearing and aggregate capital is given by integrating the demand of capital of establishments in equation (3),

$$K = \left[\left(\frac{\alpha}{r} \right)^{1-\gamma} \left(\frac{\gamma}{w} \right)^{\gamma} \right]^{\frac{1}{1-\alpha-\gamma}} \bar{s},$$

which is a simple function of prices and average establishment size.

5 Calibration

We study the quantitative impact of policy distortions on aggregate TFP and GDP per capita in an economy that is relatively more distorted than the United States in the same spirit of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). For this reason, we calibrate a benchmark economy with distortions to U.S. data.

We start by selecting a set of parameters that are standard in the literature. These parameters have either well-known targets which we match or the values have been well discussed in the literature.

Following the literature, we assume decreasing returns in the establishment-level production function and set $\alpha + \gamma = 0.85$, e.g., Restuccia and Rogerson (2008). Then we split it between α and γ by assigning 1/3 to capital and 2/3 to labor, implying $\alpha = 0.283$ and $\gamma = 0.567$. We set the annual exit rate λ to be 10 percent, which is in line to the estimates in the literature, e.g., Davis et al. (1998). We set the discount rate ρ to match a real interest rate of 4 percent and the depreciation rate of capital δ to 7 percent to match a capital to output ratio of 2.5. To calibrate the exogenous growth rate of the mass of establishments η, we use the equilibrium implication of the model that the aggregate growth rate of TFP over time is proportional to the growth rate of the mass of establishments. Since the growth rate of TFP in the United States in the last 100 years is roughly equal to 2 percent, we set η equal to 0.02. We normalize $\tau_e = 1$ (or $\tau_{y,e} = 0$) for the benchmark economy.

Regarding distortions, we set the correlation between distortions and productivity across establishments to 0, $\rho_{\tau,z} = 0$ consistent with the near zero estimates of this correlation for the United States in Hsieh and Klenow (2014). Since policy distortions follow a Geometric Brownian motion, we can use the same methodology as in Lemma 2 to find the stationary distribution of τ_y and consequently the stationary distribution of TFPR (see appendix A.4) to compute the standard deviation of log TFPR.

We then calibrate the remaining parameters by solving the equilibrium of the model and making sure the equilibrium statistics match some targets. The remaining 6 parameters to calibrate are $\{\sigma_z^2, \mu_\tau, \sigma_\tau^2, c_e, c_\mu, \theta\}$. We construct the following 6 statistics in the model and match with the corresponding targets in the data:

(1) Standard deviation of log revenue total factor productivity (TFPR):

$$\text{SD log TFPR} = \sqrt{\frac{1}{\xi^2_{TFPR,-}} + \frac{1}{\xi^2_{TFPR,+}}}.$$

(2) Employment growth rate (with $\rho_{\tau,z} = 0$):

$$\mu_s = \left[\theta\left(\mu_z + \mu_\tau\right) + \frac{\theta(\theta-1)}{2}(\sigma_z^2 + \sigma_\tau^2)\right].$$

(3) Standard deviation of log employment (with $\rho_{\tau,z} = 0$):

$$\sigma_s^2 = \theta^2(\sigma_z^2 + \sigma_\tau^2).$$

(4) Productivity growth rate:

$$\mu_z = \left[\frac{\theta \quad c_e}{s_e \quad c_\mu}\right]^{\frac{1}{\theta-1}},$$

where we use the fact that the entry zero profit condition implies $A(w,r)s_e = c_e$.

(5) Average establishment size:

$$\bar{s} = s_e \frac{\eta + \lambda}{\eta + \lambda - \mu_s}.$$

(6) Cumulative distribution function (CDF) of establishment size:

$$F(s \le \bar{s}) = 1 - \left(\frac{-\xi_-\xi_+}{\xi_+ - \xi_-}\right) e^{-(\xi_+)\log(\bar{s}/s_e)}.$$

The six parameters are selected simultaneously to match six targets for the above statistics, but since some parameters have a first-order impact on some targets we discuss them in turn. The policy distortions parameters σ_τ and μ_τ help matching the standard deviation of log of TFPR in the United States from Hsieh and Klenow (2009) and the employment growth over the life cycle of plants in the United States from Hsieh and Klenow (2014). The elasticity of the productivity investment cost function θ helps matching the annual measured productivity growth rate of establishments of 4.7 percent from Hsieh and Klenow (2014) for the United States. The standard deviation of productivity σ_z is calibrated to match the standard deviation of employment across establishments compatible with Zipf's law, so we set ξ_+ equal to 1.059 from Axtell (2001). The entry cost c_e

Table 1: Calibration to U.S. Data

Parameters	Values	Targets	
c_e	1.7919	Average establishment size	21.85
θ	1.1610	Productivity growth rate	4.7%
c_μ	0.2553	(CDF) % small establishments	92.5%
σ_z^2	0.4639	Zipf's law, ξ_+	1.059
σ_τ^2	0.3764	SD log TFPR	0.49
μ_τ	-0.0741	Employment growth rate	4.7%

and the level parameter of the productivity cost function c_μ are calibrated to match the average establishment size and the share of small establishments in the United States in 1997 from Hsieh and Klenow (2009). The implied parameters values from this procedure are summarized in Table 1. We next study the quantitative impact of increased policy distortions in this model.

6 Quantitative Experiments

We study the impact of policy distortions on establishment-level productivity, aggregate output, aggregate TFP, and other relevant variables by comparing these statistics in more distorted economies relative to the benchmark economy. We highlight the impact of policy distortions in our model with effects on the distribution of establishment-level productivity relative to a version of the model where the distribution of productivity is exogenous and hence invariant to changes in policy distortions as in Restuccia and Rogerson (2008). We show that empirically-plausible policy distortions generate substantial negative effects on aggregate output and TFP. These effects are larger than the static effect of misallocation estimated in the literature. In our framework, distortions generate dynamic effects on productivity via two channels affecting the invariant distribution of productivity in the economy. First, distortions reduce the growth of establishments in both employment and productivity. Second, the lower growth of establishments make entrants more like incumbent firms altering the invariant distribution of establishment-level productivity in the economy.

6.1 Changes in policy distortions

We study the impact of changes in policy distortions across economies via changes in three parameters: the dispersion of distortions σ_τ, the correlation between establishment-level distortions and productivity $\rho_{\tau,z}$, and the distortion of entrants τ_e which simply determines the level of taxes in the economy. All other parameters remain the same as in the benchmark economy. We choose these three parameters to generate economies that differ in distortions, in particular, we target economies that differ in: (a) the dispersion in distortions, measured by the standard deviation of the log in TFPR, (b) the static output gain from reallocation, and (c) the establishment growth rate of employment. We choose a poor economy that has a standard deviation of log TFPR of 1, that has a static output gain from reducing distortions to the level of the benchmark economy of a factor of around 2-fold, and that has a zero growth rate in establishment employment. For this poor economy, the calibrated values are $\sigma_\tau = 1.04$, $\rho_{\tau,z} = 0.10$, and $(1 - \tau_e) = 2.09$. While the particular values for calibration chosen for the poor economy are somewhat arbitrary, we note that they broadly characterize the empirical pattern of distortions found in the literature (e.g., Buera et al., 2013; Bento and Restuccia, 2017; Cirera et al., 2017; Restuccia and Rogerson, 2017).

We then linearly interpolate the values of policy distortion parameters between the benchmark economy and the poor economy to generate economies that feature different magnitudes of dispersion in log TFPR that are comparable in magnitude to those estimated in the empirical literature. For instance, a standard deviation of log TFPR of 0.67 for 1991 India and 0.74 for 1998 China from Hsieh and Klenow (2009), and of 0.85 from the evidence in very poor countries (Chen et al., 2017; Restuccia and Santaeulalia-Llopis, 2017).

We emphasize that the static gains from reallocation depend not only on the dispersion of log TFPR, but also on the dispersion of productivity in the economy (more generally, on the joint distribution of productivity and distortions). For this reason, for example, Hsieh and Klenow (2009) report the same dispersion in log TFPR for India in 1991 and 1994 of 0.67, yet the resulting static output gains from reallocation in each case are 41.4% in 1991 and 59.2% in 1994 relative to gains from reallocation in 1997 United States. Hence, we also don't expect the model to exactly reproduce the empirical static gains from reallocation.

Table 2 reports the results for economies that differ in policy distortions. The table reports aggregate output and measured TFP for each economy relative to that of the benchmark economy. Hence, the results reported are closely linked to the exercise in Hsieh and Klenow (2009) of calculating the aggregate output gains from reducing the dispersion in marginal revenue products in China and India to the level observed in the United States. We note that while Hsieh and Klenow (2009) report TFP gains, in their static setting with constant factors and number of firms, TFP and output gains are identical.

Table 2: Effects of Changes in Policy Distortions

	SD(logTFPR)				
	0.49	0.67	0.74	0.85	1.00
σ_τ	0.38	0.60	0.69	0.84	1.04
$\rho_{\tau,z}$	0.00	-0.03	-0.05	-0.07	-0.10
$(1-\tau_{y,e})$	1.00	1.37	1.52	1.76	2.09
Relative output Y	1.000	0.535	0.448	0.347	0.215
Relative TFP	1.000	0.540	0.454	0.352	0.214
Entry size relative to incumbents $(s_e/\bar{s})$	0.611	0.762	0.823	0.912	1.004
Establishment productivity growth	0.047	0.029	0.023	0.012	0.001
Establishment employment growth	0.047	0.029	0.021	0.011	0.000

Notes: Output Y and total factor productivity (TFP) are reported relative to the benchmark economy. Entry size $s_e/\bar{s}$ is defined as the size of entrants relative to the average incumbent. Establishment productivity growth is the growth rate of establishment productivity and is related to the endogenous drift μ_z. Establishment employment growth is the growth rate of establishment employment and is related to the endogenous drift in employment μ_s. Hsieh and Klenow (2009) report that the standard deviation of log TFPR is 0.49 for the United States in 1997, 0.67 for India in 1991, and 0.74 for China in 1998.

Our results are quite striking. For instance, the economies with dispersion in distortions of 0.67 and 0.74, as documented in Hsieh and Klenow (2009) for China and India, have aggregate output that is 53.5% and 44.8% of that in the benchmark economy. Economies with larger dispersion in distortions feature much lower relative output, 34.7% and 21.5% of the benchmark economy. We find similar quantitative effects for aggregate measured TFP. These results represent substantial decreases in output and TFP. In more distorted economies, establishment productivity and their employment size do not grow as much (low μ_z and μ_s) compared to the benchmark economy. Note that while the implementation of policy distortions in poor economies delivers the fact that productivity and employment grow less than in the benchmark economy, we emphasize that this may not be the case in all policy-distortion configurations.

We now comment on the empirical plausibility of these results. The negative effect of distortions on the life cycle growth of firms is consistent with the evidence in Hsieh and Klenow (2014) where the employment and productivity growth of firms is found to be lower in more distorted economies such as India and Mexico than in the United States. We note however that the quantitative magnitude of the life-cycle effect is not as large as in the data. The low growth in productivity and size imply that entrants are more similar to incumbents in distorted economies and, as a result, there is a shift in the distribution of establishment productivity to lower levels. This implication is consistent with the differences in the productivity distributions of plants in the manufacturing sector reported by Hsieh and Klenow (2009) for China and India relative to the United States and with empirical studies of specific policy reforms that find substantial shifts on the productivity distribution, e.g. Pavcnik (2002) on trade reform in Chile, Kirwan et al. (2012) on dismantling of the Tobacco quota in the United States, Bustos (2011) on trade reform and technology upgrading, among others.

6.2 Amplification

To illustrate the quantitative importance of the dynamic effects in amplifying the negative impact of policy distortions on aggregate output and to relate our results with the gains from reallocation in Hsieh and Klenow (2009), in Table 3 we report the gains in aggregate output that arise in each economy when eliminating the dispersion in TFPR in the distorted economy relative to the gains of eliminating distortions in the benchmark economy. We decompose the total effect in aggregate output between the static gains from factor misallocation and the dynamic gains from changes in the productivity distribution. This decomposition follows our characterization of aggregate output between the static effect of factor misallocation and the dynamic effects in equation (20). Hence, the total output gain in Table 3 is the product of the static gains from factor misallocation and the dynamic gains from changes in the productivity distribution. To facilitate comparison with the literature, Table 3 reports the results as the ratio of aggregate output without distortions to output with distortions relative to the benchmark economy in each case. Hence, the inverse of the total effect on output coincides with the relative output reported in Table 2 for each economy.

Table 3: Changes in Policy Distortions

	SD(logTFPR)				
	0.49	0.67	0.74	0.85	1.00
Relative output gains from reduced distortions:					
Static effect	1.00	1.38	1.54	1.79	2.13
Dynamic effect	1.00	1.35	1.45	1.61	2.18
Total effect	1.00	1.87	2.23	2.88	4.65

Notes: Hsieh and Klenow (2009) report that the standard deviation of log TFPR is 0.49 in 1997 United States, 0.67 in 1991 India, and 0.74 in 1998 China. We report the results for two other economies that are more distorted than China and India. Static gains refer to the output gains from reducing log TFPR dispersion (i.e., factor misallocation) to the level in the benchmark economy, holding aggregate factors and the productivity distribution of establishments constant. Endogenous Distribution refers to the change in the endogenous productivity distribution and Total refers to the overall impact on aggregate output. These terms follow the decomposition of aggregate output in equation (20). The inverse of the "Total effect" line corresponds to the "Relative output Y" line in Table 2.

In the economy with dispersion of log TFPR of 0.67, the output gains from eliminating distortions relative to the gains from eliminating distortions in the benchmark economy is 1.87-fold, that is, aggregate output in this economy would increase by 87% when eliminating distortions relative to the corresponding increase in the benchmark economy. Alternatively, this is the increase in aggregate output that results from reducing the dispersion in distortions in this economy to the level of the benchmark economy. Notice that the total increase in output is much larger than that reported in Hsieh and Klenow (2009) for the impact of factor misallocation in India. According to Hsieh and Klenow (2009, Table VI), the output gains in 1991 India from equalizing TFPR relative to 1997 U.S. gains is 41.4%. In our model, the corresponding increase in aggregate output of reducing dispersion in India to the level of the U.S. is 38% (Static gains 1.38 in Table 3). Hence, the increase in output arising from the reduction in factor misallocation is very close to that estimated empirically. But in our model the distribution of productivity changes along with changes in general equilibrium prices, generating a larger increase in output. As a result, the dynamic effect generates substantial amplification over and above the gains from eliminating static misallocation. This amplification effect on output is substantial. For the log TFPR 0.67 economy, whereas the static gain from reducing factor misallocation is 38%, the dynamic effect increases aggregate output by 35%. To put it differently, the dynamic effect accounts for 48% (log(1.35)/log(1.87)) of the gain in aggregate output from reducing misallocation to the levels in the benchmark economy.

In the economy with dispersion in log TFPR of 0.74, the total increase in aggregate output from the reduction in the dispersion of distortions relative to the benchmark economy is 2.2-fold with the dynamic effect accounting again for 47% of this increase. Economies with more distortions feature larger increases in relative output and a similar contribution of the dynamic effect. In these numerical experiments, the dynamic effect of misallocation is as important as the static loss from factor misallocation, doubling the quantitative impact of distortions on aggregate output and TFP. Although the purpose of these quantitative experiments is not to account for any particular country experience, to gauge the relative importance of the amplification mechanism derived from the dynamic effect in our model, it is useful to recall that Hsieh and Klenow (2009) crudely estimate the gap in manufacturing productivity between the United States and China and India to be a factor between 2.3 and 2.6-fold to show that static misallocation is an important component of the factor difference. Our results indicate that the dynamic effect can potentially account for the bulk of the remaining difference.

7 Conclusions

We develop a tractable dynamic model of heterogeneous producers to study the effect of distortions on the distribution of establishment-level productivity across economies. The model tractability allows us to obtain closed-form solutions that are useful in identifying the response of distortions on aggregate output via static factor misallocation and dynamic effects. In this framework, policy distortions not only generate differences in factor misallocation (static effect) as emphasized in a large literature, but also on the distribution of establishment-level productivity and general equilibrium prices (dynamic effect). We show that empirically-reasonable policy distortions have substantial negative effects on aggregate output and TFP in this economy, roughly doubling the effect of distortions in models with constant exogenous distributions.

It would be interesting to explore specific policies and institutions—such as size-dependent policies, firing taxes, financial frictions—in the context of our framework with dynamic effects of distortions. These explorations of specific policies in our framework may help reconcile the empirically large effects found in the literature relative to models with a constant exogenous distribution. Broadly speaking, we argue that our dynamic framework is better suited to explore the data. As a result, further progress aimed at broadening the empirical mapping of the model to the data may provide useful insights. Our analytical solution of the productivity distribution as a function of distortions is a critical first step in this mapping. But the mapping requires reliable and comparable panel data of producers across countries. While these data are increasingly available for some countries, comparability across countries remains an important limitation. We leave these interesting and important explorations for future work.

References

Acemoglu, D., Akcigit, U., Alp, H., Bloom, N., and Kerr, W. (2018). Innovation, reallocation, and growth. *American Economic Review*, 108(11):3450–91.

Atkeson, A. and Burstein, A. (2010). Innovation, firm dynamics, and international trade. *Journal of Political Economy*, 118(3):433–484.

Axtell, R. L. (2001). Zipf distribution of u.s. firm sizes. *Science*, 293(5536):1818–1820.

Banerjee, A. and Duflo, E. (2005). Growth theory through the lens of development economics. *Handbook of Economic Growth*.

Bartelsman, E., Haltiwanger, J., and Scarpetta, S. (2013). Cross-country differences in productivity: The role of allocation and selection. *The American Economic Review*, 103(1):305–334.

Bello, O., Blyde, J., and Restuccia, D. (2011). Venezuela's growth experience. *Latin american journal of economics*, 48(2):199–226.

Benhabib, J., Perla, J., and Tonetti, C. (2014). The growth dynamics of innovation, diffusion, and the technology frontier. manuscript,New York University.

Bento, P. and Restuccia, D. (2017). Misallocation, establishment size, and productivity. *American Economic Journal: Macroeconomics*, 9(3):267–303.

Bhattacharya, D., Guner, N., and Ventura, G. (2013). Distortions, endogenous managerial skills and productivity differences. *Review of Economic Dynamics*, 16(1):11–25.

Buera, F., Moll, B., and Shin, Y. (2013). Well-intended policies. *Review of Economic Dynamics*, 16(1):216–230.

Buera, F. and Oberfield, E. (2014). The global diffusion of ideas. manuscript, Princeton University.

Buera, F. J. and Fattal-Jaef, R. N. (2018). The dynamics of development: Innovation and reallocation.

Bustos, P. (2011). Trade liberalization, exports, and technology upgrading: Evidence on the impact of mercosur on argentinian firms. *The American economic review*, 101(1):304–340.

Chen, C., Restuccia, D., and Santaeulàlia-Llopis, R. (2017). The effects of land markets on resource allocation and agricultural productivity.

Cirera, X., Fattal Jaef, R. N., and Maemir, H. B. (2017). Taxing the good? distortions, misallocation, and productivity in sub-saharan africa. Technical report, The World Bank.

Da-Rocha, J. M. and Pujolas, P. (2011). Policy distortions and aggregate productivity: The role of idiosyncratic shocks. *The BE Journal of Macroeconomics*, 11(1).

Da-Rocha, J.-M., Restuccia, D., and Tavares, M. M. (2019). Firing costs, misallocation, and aggregate productivity. *Journal of Economic Dynamics and Control*, 98:60–81.

Davis, S., Haltiwanger, J., and Schuh, S. (1998). *Job Creation and Destruction*, volume 1 of *MIT Press Books*. The MIT Press.

Fattal, R. N. (2014). *Entry, Exit and Misallocation Frictions*. Manuscript, IMF.

Gabaix, X. (2009). Power laws in economics and finance. *Annual Review of Economics*, 1(1):255–294.

Gabler, A. and Poschke, M. (2013). Experimentation by firms, distortions, and aggregate productivity. *Review of Economic Dynamics*, 16(1):26–38.

Gourio, F. and Roys, N. (2014). Size-dependent regulations, firm size distribution, and reallocation. *Quantitative Economics*, 5:377–416.

Guner, N., Parkhomenko, A., and Ventura, G. (2018). Managers and productivity differences. *Review of Economic Dynamics*, 29:256–282.

Guner, N., Ventura, G., and Xu, Y. (2008). Macroeconomic implications of size-dependent policies. *Review of Economic Dynamics*, 11(4):721–744.

Hall, R. and Jones, C. (1999). Why do some countries produce so much more output per worker than others? *Quarterly Journal of Economics*, pages 83–116.

Hopenhayn, H. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica*, 60:1127–50.

Hopenhayn, H. (2014). Firms, misallocation, and aggregate productivity: A review. *Annu. Rev. Econ.*, 6(1):735–770.

Hsieh, C. and Klenow, P. (2009). Misallocation and manufacturing tfp in china and india. *Quarterly Journal of Economics*, 124(4):1403–1448.

Hsieh, C.-T. and Klenow, P. J. (2014). The life cycle of plants in india and mexico. *The Quarterly Journal of Economics*, 129(3):1035–1084.

Kirwan, B. E., Uchida, S., and White, T. K. (2012). Aggregate and farm-level productivity growth in tobacco: before and after the quota buyout. *American journal of agricultural economics*, 94(4):838–853.

Klenow, P. and Rodriguez-Clare, A. (1997). The neoclassical revival in growth economics: Has it gone too far? In Bernanke, B. and Rotemberg, J., editors, *NBER Macroeconomics Annual*. MIT Press, Cambridge.

Lucas, R. and Moll, B. (2014). Knowledge growth and the allocation of time. *Journal of Political Economy*, 122(1):1–51.

Luttmer, E. (2007). Selection, growth, and the size distribution of firms. *Quarterly Journal of Economics*, 122(3):1103–1144.

Lyuu, Y.-D. (2002). *Financial Engineering and Computation: Principles, Mathematics, Algorithms*. Cambridge University Press.

Pavcnik, N. (2002). Trade liberalization, exit, and productivity improvements: Evidence from chilean plants. *The Review of Economic Studies*, 69:245–276.

Peters, M. (2015). Heterogeneous mark-ups, growth and endogenous misallocation. Unpublished manuscript, Yale University.

Prescott, E. C. (1998). Needed: A theory of total factor productivity. *International Economic Review*, 39:525–52.

Ranasinghe, A. (2014). Impact of policy distortions on firm-level innovation, productivity dynamics and tfp. *Journal of Economic Dynamics and Control*.

Restuccia, D. (2013a). Factor misallocation and development. In Edition, O., editor, *The New Palgrave Dictionary of Economics*. Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan.

Restuccia, D. (2013b). The latin american development problem: An interpretation. *Economía*, 13(2):69–100.

Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics*, 11(4):707–720.

Restuccia, D. and Rogerson, R. (2013). Misallocation and productivity. *Review of Economic Dynamics*, 16(1):1–10.

Restuccia, D. and Rogerson, R. (2017). The causes and costs of misallocation. *Journal of Economic Perspectives*, 31(3). forthcoming, summer.

Restuccia, D. and Santaeulalia-Llopis, R. (2017). Land misallocation and productivity. NBER Working Paper, w23128.

Rubini, L. (2014). Innovation and the trade elasticity. *Journal of Monetary Economics*, 66:32–46.

A Appendix

This appendix presents the proofs of Lemma 1 and Lemma 2, the detailed steps to calculate aggregate output, and the characterization of the distribution of TFPR.

A.1 Proof Lemma 1

From the first order condition for the productivity drift in equation (5), we can solve for the productivity drift μ_z as a function of the determinants of costs and benefits such as distortions τ, cost scale c_μ, and the marginal present value profits W_z'. In particular, equating the marginal cost and benefit from productivity growth implies,

$$c_\mu (z\tau)^\theta \mu_z^{\theta-1} = zW_z'.$$

By guessing and verifying, we find that the optimal Hamilton-Jacobi-Bellman equation is given by $W(z,\tau) = A(w,r)z^\theta \tau^\theta$, where the constant $A(w,r)$ is the solution of the polynomial:

$$\begin{aligned} &\left[\frac{(\lambda+R)}{(\theta-1)} - \frac{\theta\mu_\tau}{(\theta-1)} - \frac{\theta(\sigma_z^2+\sigma_\tau^2)}{2} - \frac{\theta^2\sigma_z\sigma_\tau\rho_{z,\tau}}{(\theta-1)}\right] A(w,r) \\ &\qquad - \left[\frac{\theta}{c_\mu}\right]^{\dfrac{1}{\theta-1}} A(w,r)^{\dfrac{\theta}{\theta-1}} = \frac{m(w,r)}{(\theta-1)}. \end{aligned} \tag{A.1}$$

Given the solution to this polynomial, the optimal productivity drift μ_z is independent of establishment characteristics τ and z:

$$\mu_z = \left[\frac{\theta A(w,r)}{c_\mu}\right]^{\dfrac{1}{\theta-1}}.$$

A.2 Proof Lemma 2

The stationary pdf is the solution of the boundary-value problem that consists of solving

$$\begin{aligned} f''(x) - \gamma_1 f'(x) - \gamma_2 f(x) &= 0 && \text{if} \quad x \neq 0, \\ f''(x) - \gamma_1 f'(x) - \gamma_2 f(x) &= -\gamma_3 \delta(x-0) && \text{if} \quad x = 0, \end{aligned}$$

where the constants γ_1, γ_2, and γ_3 are given by

$$\gamma_1 = \frac{2\mu_x}{\sigma^2} < 0, \quad \gamma_2 = \frac{2(\lambda+\eta)}{\sigma_x^2} > 0, \quad \gamma_3 = \frac{2\hat{b}}{\sigma_x^2} > 0.$$

We solve the boundary-value problem using Laplace transforms. Laplace transforms are given by

$$\begin{aligned} \mathscr{L}[f'(x)] &= s\mathscr{L}[f(x)] - f(0), \\ \mathscr{L}[f''(x)] &= s^2\mathscr{L}[f(x)] - sf(0) - f'(0). \end{aligned}$$

By applying Laplace transforms in equation (8), we obtain:

$$(s^2 - \gamma_1 s - \gamma_2)\mathscr{L}[f(x)] - (s-\gamma_1)f(0) - f'(0) = -\gamma_3\mathscr{L}[\delta(x-0)].$$

Using the boundary condition $f(0) \geq 0$ and $\mathscr{L}[\delta(x-0)] = 1$ we find:

$$(s^2 - \gamma_1 s - \gamma_2)Y(s) = f'(0) + (s-\gamma_1)f(0) - \gamma_3,$$

where

$$Y(s) = \frac{f'(0) - \gamma_3 + (s-\gamma_1)f(0)}{(s^2 - \gamma_1 s - \gamma_2)}.$$

We obtain the solution by solving the Laplace inverses when $x \neq 0$ given by:

$$\begin{aligned} \mathscr{L}^{-1}\left[\frac{1}{(s-r_1)(s-r_2)}\right] &= \frac{1}{(r_1-r_2)}\left(e^{r_1 x} - e^{r_2 x}\right), \\ \mathscr{L}^{-1}\left[\frac{(s-\gamma_1)}{(s-r_1)(s-r_2)}\right] &= \frac{1}{(r_1-r_2)}\left[(r_1-\gamma_1)e^{r_1 x} - (r_2-\gamma_1)e^{r_2 x}\right], \end{aligned}$$

where the two roots (one positive and one negative) are given by $r = \dfrac{\gamma_1 \pm \sqrt{\gamma_1^2 + 4\gamma_2}}{2}$. We can rewrite the final solution for this case as:

$$\begin{array}{lll} f(x) = \frac{f'(0)}{(r_1 - r_2)} \left(e^{r_1 x} - e^{r_2 x}\right) + \frac{f(0)}{(r_1 - r_2)} \left[(r_1 - \gamma_1)e^{r_1 x} - (r_2 - \gamma_1)e^{r_2 x}\right] & \text{if} & x \neq 0, \\ f(x) = \frac{f'(0) - \gamma_3}{(r_1 - r_2)} \left(e^{r_1 x} - e^{r_2 x}\right) + \frac{f(0)}{(r_1 - r_2)} \left[(r_1 - \gamma_1)e^{r_1 x} - (r_2 - \gamma_1)e^{r_2 x}\right] & \text{if} & x = 0. \end{array}$$

When $x \neq 0$ (that is $\forall x \in (-\infty, 0) \cup (0, \infty)$), we have

$$f(x) = \begin{cases} C_1 e^{r_1 x} + C_2 e^{r_2 x}, & \text{if} \quad x < 0, \\ \\ C_1 e^{r_1 x} + C_2 e^{r_2 x}, & \text{if} \quad x > 0, \end{cases}$$

where

$$\begin{aligned} C_1 &= \frac{1}{(r_1 - r_2)} \left[f'(0) + f(0)(r_1 - \gamma_1)\right], \\ C_2 &= \frac{-1}{(r_1 - r_2)} \left[f'(0) + f(0)(r_2 - \gamma_1)\right], \end{aligned}$$

and $r_1 > 0$ and $r_2 < 0$. When $x > 0$ in order to $f(\cdot)$ be a pdf, it is necessary that $C_1 = 0$ and

$$f'(0) = -f(0)(r_1 - \gamma_1) \Rightarrow C_2 = \frac{-1}{(r_1 - r_2)} \left[f(0)(\gamma_1 - r_1) + f(0)(r_2 - \gamma_1)\right] = f(0).$$

Symmetrically when $x < 0$ we need $C_2 = 0$. Therefore

$$f'(0) = -f(0)(r_2 - \gamma_1) \Rightarrow C_1 = \frac{1}{(r_1 - r_2)} \left[f(0)(\gamma_1 - r_2) + f(0)(r_1 - \gamma_1)\right] = f(0),$$

and

$$f(x) = \begin{cases} f(0)e^{r_1 x} & \text{if} \quad x < 0, \\ \\ f(0)e^{r_2 x} & \text{if} \quad x \geq 0, \end{cases}$$

where $f(0) = \left(\frac{r_1 r_2}{r_2 - r_1}\right)$. Finally we need to prove that: 1) for $x > 0$, $f'(0) = -f(0)(r_1 - \gamma_1)$ (i.e. $C_1 = 0$), and 2) for $x < 0$, $f'(0) = -f(0)(r_2 - \gamma_1)$ (i.e. $C_2 = 0$); Given that when $x > 0$ $f'(0) = r_2 f(0)$ (and when $x < 0$ $f'(0) = r_1 f(0)$) this is equivalent to show that

$$(r_2 + r_1) f(0) = \left(\frac{\gamma_1 - \sqrt{\gamma_1^2 + 4\gamma_2}}{2} + \frac{\gamma_1 + \sqrt{\gamma_1^2 + 4\gamma_2}}{2} \right) f(0) = f(0)\gamma_1.$$

When $x = 0$ we have

$$f(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x},$$

where

$$\begin{aligned} C_1 &= \frac{1}{(r_1 - r_2)} \left[f'(0) - \gamma_3 + f(0)(r_1 - \gamma_1) \right], \\ C_2 &= \frac{-1}{(r_1 - r_2)} \left[f'(0) - \gamma_3 + f(0)(r_2 - \gamma_1) \right]. \end{aligned}$$

Therefore

$$f(0) = C_1 + C_2 = \frac{1}{(r_1 - r_2)} \left[r_1 f(0) - r_2 f(0) \right] = f(0).$$

Using $s = s_e e^x$, we can recover the size distribution $g(s)$. That is

$$g(s) = \frac{1}{s} f(\ln(s/s_e)) = \begin{cases} f(0) \dfrac{s^{r_1 - 1}}{s_e^{r_1}} & \text{if} \quad s < s_e, \\ \\ f(0) \dfrac{s^{r_2 - 1}}{s_e^{r_2}} & \text{if} \quad s \geq s_e. \end{cases}$$

Note that this solution is equivalent to the guess and verify solution obtained by solving the characteristic equation $\frac{\sigma_x^2}{2}\xi^2 + \left(\mu_s - \frac{\sigma_x^2}{2}\right)\xi - (\lambda + \eta) = 0$ with $r_1 = -\xi_-$ and $r_2 = -\xi_+$.

Finally, average establishment size $\bar{s}$ is given by

$$\bar{s} = s_e \frac{-\xi_- \xi_+}{(\xi_+ - 1)(1 - \xi_-)} = s_e \frac{\eta + \lambda}{\eta + \lambda - \mu_s}. \blacksquare$$

A.3 Derivation of Aggregate Output

Recall that establishment level output is given by the production function,

$$y = z^{\theta(1-\alpha-\gamma)} k^{\alpha} n^{\gamma}.$$

From the static profit maximization problem, we derive the demand of capital and labor for each establishment, $k(z,\tau)$ and $n(z,\tau)$. After substituting these equations into the production function we obtain:

$$y(z,\tau) = \left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\gamma}{w}\right)^{\gamma} \right]^{\frac{1}{1-\alpha-\gamma}} \tau^{\theta(\alpha+\gamma)} z^{\theta}. \tag{A.2}$$

Note from this expression that output at the establishment level is not just a function of productivity but also depends on distortions. Because the demand of inputs depends linearly on $(z\tau)^{\theta}$ we define size $s \equiv (z\tau)^{\theta}$ and we can therefore express output as a function of size and distortions. Note also that the term in square brackets is a constant across establishments that depends on prices and production parameters. Labor market clearing (aggregating the demand for labor across establishments and equating to 1) implies,

$$\left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\gamma}{w}\right)^{(1-\alpha)} \right]^{\frac{1}{1-\alpha-\gamma}} \bar{s} = 1.$$

Then, after a few algebra manipulations, we can express the constant in equation (A.2) as proportional to average size as follows:

$$\left[\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\gamma}{w}\right)^{\gamma} \right]^{\frac{1}{1-\alpha-\gamma}} \propto \frac{1}{\bar{s}^{\gamma/(1-\alpha)}}.$$

The proportionality is because there is a constant term that depends on the real interest rate which is constant in all our economies. Using this expression into (A.2), output is given by

$$y(z,\tau;\bar{s}) \propto \frac{1}{\bar{s}^{\gamma/(1-\alpha)}} \tau^{\theta(\alpha+\gamma)} z^{\theta}.$$

Then aggregate output Y is obtained by integrating over all establishments. Following Lyuu (2002), aggregate output is given by

$$Y \propto \frac{1}{\bar{s}^{\gamma/(1-\alpha)}} \int_0^{+\infty} z^\theta g_z(z) dz \int_0^{+\infty} \tau^{\theta(\alpha+\gamma)} g_\tau(\tau) d\tau.$$

Because z and τ are Brownian motions with distributions that are double Pareto with drifts $\mu_{z^\theta} = \theta\mu_z + \theta(\theta-1)\frac{\sigma_z^2}{2}$ and $\mu_{\tau^{\theta(\alpha+\gamma)}} = \theta(\alpha+\gamma)\mu_\tau + \theta(\alpha+\gamma)(\theta(\alpha+\gamma)-1)\frac{\sigma_\tau^2}{2}$; and standard deviations $\sigma_{z^\theta} = \theta\sigma_z$ and $\sigma_{\tau^{\theta(\alpha+\gamma)}} = \theta(\alpha+\gamma)\sigma_\tau$; we use the same methodology as in Lemma 2 to write aggregate output as proportional to:

$$Y \propto \frac{1}{\bar{s}^{\gamma/(1-\alpha)}} \left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{z^\theta}} \right) z_e^\theta \right] \left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{\tau^{\theta(\alpha+\gamma)}}} \right) \tau_e^{\theta(\alpha+\gamma)} \right].$$

Using the definition of s, we substitute $z_e^\theta = s_e/\tau_e^\theta$ to obtain,

$$Y \propto \frac{1}{\bar{s}^{\gamma/(1-\alpha)}} \left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{z^\theta}} \right) \frac{s_e}{\tau_e^\theta} \right] \left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{\tau^{\theta(\alpha+\gamma)}}} \right) \tau_e^{\theta(\alpha+\gamma)} \right].$$

Rearranging the τ_e terms, and using the definition of τ, we substitute $\tau_e^{\theta(1-\alpha-\gamma)} = (1-\tau_{y,e})$ and obtain,

$$Y \propto \frac{1}{\bar{s}^{\gamma/(1-\alpha)}} \left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{z^\theta}} \right) s_e \right] \left[\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{\tau^{\theta(\alpha+\gamma)}}} \right) \frac{1}{(1-\tau_{y,e})} \right]. \tag{A.3}$$

From equation (A.3) we emphasize that aggregate output Y depends on two key terms. The first term in square brackets represents the static output gain from equalizing TFPR across establishments,

$$\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{\tau^{\theta(\alpha+\gamma)}}} \right) \frac{1}{(1-\tau_{y,e})}.$$

The second term represents the output gain from the change in the (endogenous) productivity distribution,

$$\left(\frac{\eta+\lambda}{\eta+\lambda-\mu_{z^\theta}} \right) \frac{s_e}{\bar{s}^{\gamma/(1-\alpha)}}.$$

A.4 Stationary TFPR Equilibrium

The distribution of distortions $g_\tau(\tau)$ is a Double Pareto. Therefore, log TFPR follows a Double Exponential with roots $\xi_{TFPR,-}$ and $\xi_{TFPR,+}$ that solve the characteristic equation:

$$\frac{\sigma_{TFPR}^2}{2}\xi^2 + \left(\mu_{TFPR} - \frac{\sigma_{TFPR}^2}{2}\right)\xi - (\lambda + \eta) = 0,$$

where $\mu_{TFPR} = -\theta(1-\alpha-\gamma)\mu_\tau - [\theta(1-\alpha-\gamma)+1]\frac{\sigma_\tau}{2}$ and $\sigma_{TFPR}^2 = \theta^2(1-\alpha-\gamma)^2\sigma_\tau^2$.