# University of Toronto Department of Economics



Working Paper 600

# Structural Change and Aggregate Employment Fluctuations in China and the US

By Wen Yao and Xiaodong Zhu

April 14, 2018

# Structural Change and Aggregate Employment Fluctuations in China and the US

Wen Yao Tsinghua University Xiaodong Zhu University of Toronto

First Complete Draft: December 2017. This Draft: April 2018.

#### Abstract

The correlation between the cyclical components of aggregate employment and GDP is highly positive in the US, but close to zero in China. We argue that the difference in the size of the agricultural sector is the reason for the difference in employment-output correlation. We construct a simple two-sector growth model with productivity shocks and non-homothetic preferences and show that the model can simultaneously account for the long-run structural change and short-run employment fluctuations at sector level and in the aggregate for both economies.

JEL Classification: E24, E32, O41

Keywords: Structural Change, Non-homothetic Preferences, Labor Reallocation, Aggregate Fluctuations

Yao, School of Economics and Management, Tsinghua University, email: yaow@sem.tsinghua.edu.cn. Zhu, Department of Economics, University of Toronto, email: xzhu@chass.utoronto.ca. We would like to thank participants at the 2016 PBC-SAIF Conference on Monetary Policy, 2016 Midwest Macro Meetings, 2017 Growth and Institution Program Meeting at Tsinghua University, 2017 China Meeting of the Econometric Society at Wuhan University, and seminars at several universities for their useful comments. Part of the research was done when Xiaodong Zhu was visiting the Shanghai Advanced Institute of Finance (SAIF), and he thanks SAIF for its hospitality and financial support.

## **1** Introduction

One salient feature of business cycles in developed countries is that the aggregate employment is strongly pro-cyclical. This is not the case in China. The correlation of the cyclical components of aggregate employment and output is close to zero. Relative to output, the volatility of aggregate employment is also very low. These puzzling facts about aggregate employment fluctuations in China are present even after we carefully correct for some well-known measurement problems in the official employment series. Some take this as a sign that there are unique institutional constraints that limit the employment variability in China. While it is true that there could be strong employment rigidity in the state-owned enterprises, the labour market for the non-state sector in China is quite flexible – maybe even more flexible than many developed economies due to minimum regulations on hiring and firing workers by non-state firms. Since the non-state sector employment is usually the margin at which the aggregate employment adjusts over the business cycles, the institutional constraints on state-sector employment cannot explain the puzzle

In this paper, we argue that the key to understanding aggregate employment fluctuations in China is its economic structure. We document three new stylized facts for the period from 1978 to 2010. First, the cyclical properties of employment at sector level (agriculture and non-agriculture) in China are very similar to those in the US. In particular, the correlation of the cyclical components of nonagricultural employment and non-agricultural GDP are close to 90 percent in both countries. Second, employments in the agricultural and non-agricultural sectors are negatively correlated in both China and the US. And third, for both economies, the agriculture's share of employment is negatively correlated with the real GDP per worker in both sectors. These similarities between China and the US at the sector level suggest that the key difference between the two economies is the size of the agricultural sector. Between 1978 and 2010, the agriculture's share of total employment averaged around 50% in China, but was less than 3% in the US. Therefore, the labour reallocation between the two sectors could have an important dampening effect on aggregate employment fluctuations in China, but negligible effect in the US. To investigate this possibility, we construct a two-sector growth

model with productivity shocks and non-homothetic preferences, and calibrate it so that the model can account for the secular trend in labour reallocation from agriculture to non-agriculture in both China and the US. We then examine the model's implications for the labour market dynamics at the business cycle frequency. We find that our calibrated model can indeed account for the employment fluctuations at the sector level and in the aggregate for both China and the US. In particular, our model implies a low employment-output correlation for China and, at the same time, a high employment-output correlation for the US.

The model we use in the paper is a standard two-sector growth model, but with a non-homothetic CES utility function proposed by Comin, Lashkari and Mestieri (2015). This utility function allows for income effect at any income level. Our choice of this utility function is motivated by the third stylized fact we discussed above: For both China and the US, the agriculture's share of employment is negatively correlated with real income per worker in both agricultural and non-agricultural sectors. This is a new fact that has not been discussed in the literature, and it shows the importance of income effect in determining labour reallocation between sectors even at the business cycle frequency and for a high income country like the US.

Our paper is related to two strands of literature. There is a rapidly growing literature on structural change. See e.g., Caselli and Coleman (2001), Kongsamult, Rebelo and Xie (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Herrendorf, Rogerson and Valentinyi (2014) for an excellent survey. Most of the studies in this literature focus on understanding the sources of structural change in the long-run; our paper builds on this literature and studies the business cycle implications of structural change. In particular, both Boppart (2014) and Comin, Lashkari and Mestieri (2015) emphasize the importance of income effect in understanding the secular trend of labour reallocation from agriculture to manufacturing and services. We show in this paper that income effect is also important for understanding aggregate employment fluctuations at the business cycle frequency. Our paper is also related to the literature on business cycles in China. Brandt and Zhu (2000) is one of the first papers studying business cycles in China during the reform period. Their focus, however, is on understanding the relationship between GDP growth and inflation over the business cycles in the 1980s and early 1990s. Chang et al. (2016) is a more recent study of business cycles in China, and their focus is on understanding the weak correlation between investment and consumption in China since the late 1990s. Neither of these studies examine the relationship between aggregate employment and output. He, Chong and Shi (2009) carry out an exercise of business cycle accounting for China in the spirit of Chari, Kehoe and McGrattan (2007). They find that most of the fluctuations in aggregate employment can only be accounted for by variations in an unobserved labour wedge, highlighting the inability of the standard one-sector business cycle models in accounting for the employment fluctuations in China. Our paper shows that a standard two-sector model with non-homothetic preferences can account for the aggregate employment fluctuations without introducing a time-varying labour wedge.

There are two studies that are closely related to our paper. Da-Rocha and Restuccia (2006) is the first paper that documents the low correlation between aggregate employment and output in countries with a large agricultural sector. They use a two-sector real business cycle model to examine the role of labour reallocation in accounting for the cyclical behaviour of aggregate employment. To focus on the cyclical fluctuations, they assume that each country is fluctuating around a steady state with a constant employment share of agriculture.<sup>1</sup> Since structural change the secular decline of the agriculture's share of employment - is a very prominent phenomenon in China during the period we study, we think it is important to have an unified model that can account for both the secular trend of structural change and the aggregate employment fluctuations around the trend. In an independent study, Storeslettern, Zhao and Zilibotti (2017) also use a two-sector model to account for both the structural change and aggregate employment fluctuations in China. Their model, however, is very different from ours. They emphasize capital deepening in the agricultural sector rather than income effect as a driving force for the labour reallocation between the agricultural and non-agricultural sectors. We think their study and ours are complementary.

<sup>&</sup>lt;sup>1</sup>Moro (2012) uses a similar method to examine the impact of reallocation from manufacturing to services on the GDP volatility in US.

### 2 Data and Facts

Before presenting our model, we first discuss in detail the data and facts about the employment fluctuations in China and the US. For the US, we directly use the annual sector-level data on real GDP and employment from the Groningen Growth and Development Centre's 10-Sector Database (Timmer, de Vries and de Vries (2015)), and aggregate the nine sectors outside agriculture into one non-agricultural sector. For China, the 10-Sector Database uses the official employment series from China's National Bureau of Statistics (NBS) that are published in the annual China Statistical Yearbook. However, as pointed out by Brandt and Zhu (2010), there are two serious problems with the NBS' employment series that need to be dealt with. We discuss next how we deal with these problems and construct revised annual employment series for China.

First, there is a discrete upward jump in total employment in 1990. This jump is due to a change in the official definition of employment after 1990 census which broadened the coverage of the series. The NBS publishes the employment data using the new definition for the years since 1990, but still reports the employment data using the old definition for the years prior to 1990. Brandt and Zhu (2010) use the 1982 census data to adjust the employment data for the years before 1990 so that the entire employment series has a consistent coverage. The official and the revised employment series are plotted in the upper-left panel of Figure 1. The second problem of the NBS employment series is an overestimation of agricultural employment. Brandt and Zhu (2010) find that the official agricultural employment series can be closely approximated by the *Total Rural Employment* minus the *Em*ployment of the Township and Village Enterprises (TVEs). This series clearly overestimates agricultural employment because non-agricultural workers in rural private enterprises and rural individual enterprises (those that employ less than eight employees) are counted as agricultural workers. To better account for employment in agriculture, we follow Brandt and Zhu (2010) and construct the agricultural employment series as the total rural employment minus rural employments in TVEs, private enterprises and individual enterprises. The official and the revised agricultural employment series are plotted in the upper-right panel of Figure 1. Note that



this revised agricultural employment series still has the same problem as the official total employment series for the years prior to 1990. To generate a consistent agricultural employment series for the entire period, for each year we first use the revised agricultural employment and the official total employment to calculate the share of employment in agriculture; we then calculate the final revised agricultural employment as the product of the share and the revised total employment; and finally we calculate the revised non-agricultural employment as the difference between the revised total employment and the revised agricultural employment. The lower panels of Figure 1 plots the revised agricultural and non-agricultural employments and the agricultural employment using the revised data series.

Table 1: Aggregate Moments

	China	US
$\sigma(L)/\sigma(Y)$	0.11	0.70
$\rho\left(L,Y ight)$	0.09	0.87

### 2.1 Employment Fluctuations at the Aggregate Level

Given the revised employment data for China, we now examine the cyclical properties of aggregate employment in China and compare them to those in the US. Our sample runs from 1978 to 2010 for both economies. We present the cyclical properties by calculating several statistics from the hp-filtered time series. Table 1 reports the relative standard deviation of aggregate employment to aggregate output and the correlation of aggregate employment with aggregate output in both China and the US. All variables are normalized by the size of population.

From Table 1, we observe two interesting stylized facts of aggregate employment fluctuations in China:

- 1. The magnitude of fluctuations in the aggregate employment is much lower than that of the aggregate output in China. This is in stark contrast with the well known fact for the US economy, see Cooley and Prescott (1995), where the aggregate employment fluctuates almost as much as the aggregate output.
- 2. Aggregate employment is acyclical in China. The correlation of the aggregate employment and output is close to zero. This is also very different from the established business cycle fact in the US that employment is strongly procyclical.

Figure 2 plots the cyclical movements of the aggregate employment and output for the two economies, which confirm our observations.



### 2.2 Sector-level Employment Fluctuations

The stark differences between the two economies in the aggregate employment fluctuations conceal similarities at the sector level. Panel (A) and (B) in Table 2 present the cyclical properties of the employments in the non-agricultural (*na*) and agricultural (*a*) sectors, respectively. Panel (C) shows the correlation of the cyclical employments in the two sectors and the correlations between the cyclical components of the agriculture's share of employment and the real GDP per worker in the two sectors. Three new stylized facts emerge from the moments presented in Table 2:

- Employment fluctuations at the sector level are very similar between China and the US. For example, in the non-agricultural sector, the relative volatilities of employments in the two economies are of comparable magnitudes. Moreover, the non-agriculture employment in China is as pro-cyclical as that in the US. In the agricultural sector, the employment in China also has nontrivial volatility while the correlations between employment and output are low in both China and the US.
- 2. The correlation between the employments in the two sectors is negative in both China and the US, as shown in the second column of Table 2. This negative correlation suggests a potential important role of labour reallocation

between the two sectors in dampening aggregate employment fluctuations. Of course, the degree to which the fluctuations of the aggregate employment are dampened depends on the relative size of the agricultural sector. Between 1978 and 2010, the agriculture's share of total employment averaged around 50% in China, but was less than 3% in the US. So the labour reallocation between the two sectors could have an important dampening effect on the aggregate employment fluctuations in China, but negligible effect in the US.

3. For both China and the US, the agriculture's share of total employment is negatively correlated with labour productivities (measured as GDP per worker) in both the agricultural and non-agricultural sectors. As far as we know, we are the first in the literature to document this fact. This new fact suggests that income effect, that is, the agricultural good has lower income elasticity than the non-agricultural good, is an important factor for labour reallocation between sectors. Comin, Lashkari and Mestieri (2015) emphasize the importance of income effect in understanding the secular trend of labour reallocation from agriculture to manufacturing and services. Our fact suggests that income effect is also important for labour reallocation at the business cycle frequency.

Motivated by these new stylized facts, we now present our two-sector model with non-homothetic preferences that we will use to quantitatively account for labour market dynamics in both the long-run and short-run.

### **3** The Model

There are two sectors indexed by i = a and na, representing agriculture and nonagriculture, respectively. Each sector produces a consumption good with a linear technology using labour as the only input:

$$Y_{it} = A_{it}N_{it}, \ i = a, na,$$

where  $Y_{it}$ ,  $A_{it}$  and  $N_{it}$  are the output, labour productivity and employment in sector *i*, respectively. There is a stand-in representative household whose preferences

 Table 2: Sector Moments

	China	US
(A) Non-Agriculture Sector		
$\sigma(L_{na})/\sigma(Y_{na})$	0.75	0.71
$\rho\left(L_{na},Y_{na}\right)$	0.88	0.87
(B) Agriculture Sector		
$\sigma(L_a) / \sigma(Y_a)$	0.70	0.33
$\rho\left(L_a,Y_a\right)$	0.24	-0.05
(C) Cross Sector		
$\rho(L_a, L_{na})$	-0.83	-0.23
$ ho(rac{L_a}{L},rac{Y_a}{L_a})$	-0.44	-0.33
$ ho(rac{L_a}{L},rac{Y_{na}}{L_{na}})$	-0.35	-0.42

over a composite consumption good  $C_t$  and working time  $L_t$  are represented by the following utility function:

$$U_t = \frac{1}{1-\lambda}C_t^{1-\lambda} - \frac{B_t}{1+\sigma}L_t^{1+\sigma}$$

Here,  $\lambda$  and  $\sigma$  are both non-negative numbers representing the inverses of the elasticity of intertemporal substitution and the Frisch labour supply elasticity, respectively, and  $B_t > 0$  is a time-varying labour supply parameter that is used to capture the demographic factors (e.g., age structure and gender composition of the labour force) that affect average household's labour supply decisions.<sup>2</sup> Following Comin, Lashkari and Mestieri (2015), the composite consumption  $C_t$  is defined implicitly by the following equation:

$$\left(\varphi_{a}\right)^{\frac{1}{\varepsilon}}C_{t}^{\frac{(1-\varepsilon)\mu_{a}}{\varepsilon}}c_{at}^{\frac{\varepsilon-1}{\varepsilon}}+\left(\varphi_{na}\right)^{\frac{1}{\varepsilon}}C_{t}^{\frac{(1-\varepsilon)\mu_{na}}{\varepsilon}}c_{nat}^{\frac{\varepsilon-1}{\varepsilon}}=1,$$
(1)

<sup>&</sup>lt;sup>2</sup>Note that when  $B_t$  is a constant our utility function is the same as the one proposed by MaCurdy (1981), and it includes the GHH utility function proposed by Greenwood, Hercowitz and Huffman (1988) as a special case ( $\lambda = 0$ ).

where  $\varphi_a$ ,  $\varphi_{na}$ ,  $\mu_a$ ,  $\mu_{na}$  and  $\varepsilon$  are all positive constants. The parameter  $\varphi_i$  represents the household's preference weight on consumption good in sector i ( $\varphi_a + \varphi_{na} = 1$ ),  $\mu_i$  is a parameter that determines the income elasticity of consumption good i and  $\varepsilon$  is the elasticity of substitution between the two consumption goods. The implicit utility function is a generalization of the standard CES utility function by allowing for potentially different income elasticities for the two consumption goods. If  $\mu_a =$  $\mu_{na} = 1$ , then the utility function is reduced to the standard CES utility function. If  $\mu_a < \mu_{na}$ , the income elasticity is smaller for the agricultural good than for the nonagricultural good, and therefore relative demand for the agricultural good declines with income.

### 3.1 Social Planner's Problem

Since we assume that there is no friction nor externality in the economy, the competitive allocation is the same as the social optimal allocation, which is the solution to the following social planner's problem:

$$\max_{C_{at}, C_{nat}, L_{at}, L_{nat}, C_t} \left\{ N_t \left[ \frac{1}{1 - \lambda} C_t^{1 - \lambda} - \frac{B_t}{1 + \sigma} L_t^{1 + \sigma} \right] \right\}$$

subject to (1) and the following constraints:

(

$$c_{at} = A_{at} L_{at}, \tag{2}$$

$$c_{nat} = A_{nat} L_{nat}, \tag{3}$$

$$L_{at} + L_{nat} = L_t. (4)$$

Here,  $N_t$  is the population size and  $L_{it} = N_{it}/N_t$  is the ratio of employment in sector *i* to total population ( $i \in \{a, na\}$ ). In the Appendix A, we show that the optimal consumption of the two goods,  $c_{at}$  and  $c_{nat}$ , and the aggregate employment rate  $L_t$  satisfy the following equations:

$$c_{at} = \frac{\varphi_a A_{at}^{\varepsilon} C_t^{(1-\varepsilon)\mu_a}}{\left(\varphi_a A_{at}^{\varepsilon-1} C_t^{(1-\varepsilon)\mu_a} + \varphi_{na} A_{nat}^{\varepsilon-1} C_t^{(1-\varepsilon)\mu_{na}}\right)^{\frac{\varepsilon}{\varepsilon-1}}},$$
(5)

$$c_{nat} = \frac{\varphi_{na} A_{nat}^{\varepsilon} C_t^{(1-\varepsilon)\mu_{na}}}{\left(\varphi_a A_{at}^{\varepsilon-1} C_t^{(1-\varepsilon)\mu_a} + \varphi_{na} A_{nat}^{\varepsilon-1} C_t^{(1-\varepsilon)\mu_{na}}\right)^{\frac{\varepsilon}{\varepsilon-1}}},\tag{6}$$

$$L_{t} = \left[\frac{\left(\varphi_{a}A_{at}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{a}} + \varphi_{na}A_{nat}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{na}}\right)^{\frac{\varepsilon}{\varepsilon-1}}}{B_{t}\left(\mu_{a}\varphi_{a}A_{at}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{a}+\lambda-1} + \mu_{na}\varphi_{na}A_{nat}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{na}+\lambda-1}\right)}\right]^{\frac{1}{\sigma}}.$$
 (7)

### 3.2 Equilibrium Employment, Consumption and Output

From the goods market clearing conditions, (2), (3), (5), and (6), we have,

$$L_{at} = \frac{\varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a}}{\left(\varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a} + \varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na}}\right)^{\frac{\varepsilon}{\varepsilon - 1}}},$$
(8)

$$L_{nat} = \frac{\varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na}}}{\left(\varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a} + \varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na}}\right)^{\frac{\varepsilon}{\varepsilon - 1}}}.$$
(9)

Hence the aggregate employment to population ratio is

$$L_t = L_{at} + L_{nat} = \left(\varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a} + \varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na}}\right)^{\frac{1}{1 - \varepsilon}}, \qquad (10)$$

and the sector employment shares are

$$l_{at} \equiv \frac{L_{at}}{L_t} = \frac{\varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a}}{\varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a} + \varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na}}},$$
(11)

$$l_{nat} \equiv \frac{L_{nat}}{L_t} = \frac{\varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na}}}{\varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a} + \varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na}}}.$$
 (12)

Equation (11) can also be written as

$$l_{at} \equiv \frac{L_{at}}{L_t} = \frac{\frac{\varphi_a}{1-\varphi_a} \left(\frac{A_{at}}{A_{nat}}\right)^{\varepsilon-1} C_t^{(1-\varepsilon)(\mu_a-\mu_{na})}}{1+\frac{\varphi_a}{1-\varphi_a} \left(\frac{A_{at}}{A_{nat}}\right)^{\varepsilon-1} C_t^{(1-\varepsilon)(\mu_a-\mu_{na})}},$$
(13)

which shows that the agriculture's share of employment is affected by two factors: the relative productivity of agriculture  $A_{at}/A_{nat}$  and the aggregate consumption per capita  $C_t$ . The first factor represents the substitution effect and the second factor the income effect. The substitution effect depends on whether  $\varepsilon$  is smaller or larger than one. If  $\varepsilon$  is less than one, the agriculture's share of employment is a decreasing function of the agricultural sector's productivity and an increasing function of the non-agricultural sector's productivity. The opposite is true if  $\varepsilon$  is greater than one. Therefore the substitution effects of the two sector's labour productivity on the agricultural sector's employment are in the opposite directions as long as the value of  $\varepsilon$  is not equal to one, in which case there is no substitution effect. However, in Table 2 we have documented that the cyclical component of the agriculture's share of employment is negatively correlated with the cyclical components of real labour productivities in both sectors, suggesting that the second factor, income effect, is also important for labour reallocation at the business cycle frequency. If  $\mu_a < \mu_{na}$ , then the agriculture's share of employment is a decreasing function of the aggregate consumption. In this case, since labour productivities in both sectors have positive impact on the aggregate consumption, they both have a negative effect on the agriculture's share of employment.

Equation (10) and (7) can be combined to yield the following equation for the equilibrium value of the aggregate consumption  $C_t$ :

$$\begin{bmatrix} \left(\varphi_{a}A_{at}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{a}}+\varphi_{na}A_{nat}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{na}}\right)^{\frac{\varepsilon}{\varepsilon-1}}\\ B_{t}\left(\mu_{a}\varphi_{a}A_{at}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{a}+\lambda-1}+\mu_{na}\varphi_{na}A_{nat}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{na}+\lambda-1}\right)\end{bmatrix}^{\frac{1}{\sigma}} \qquad (14)$$
$$=\left(\varphi_{a}A_{at}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{a}}+\varphi_{na}A_{nat}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{na}}\right)^{\frac{1}{1-\varepsilon}}$$

Equations (8), (9) and (14) can be used to solve for the equilibrium employment and output in the two sectors as follows. Given the preference parameters and the real labour productivities in the two sectors,  $A_{at}$  and  $A_{nat}$ , equation (14) can be used to solve for  $C_t$ . Given  $C_t$ , equations (8) and (9) can be used to solve for  $L_{at}$  and  $L_{nat}$ . GDP per capita in the two sectors can be calculated as  $Y_{at} = A_{at}L_{at}$  and  $Y_{nat} =$  $A_{nat}L_{nat}$ , respectively. Finally, when the labour productivity levels are normalized so that the relative price of agriculture in some base year is 1, the aggregate real GDP per capita valued with base year prices is simply  $Y_t = Y_{at} + Y_{nat}$ .

### 4 Quantitative Analysis

We now examine quantitatively our model's implication for structural change and aggregate employment fluctuations. We first assume that there is no productivity shocks so that the labour productivities in both sectors are at the respective trend values, and show that our calibrated model can quantitatively account for the secular decline of the agriculture's share of employment in both China and the US. We then introduce productivity shocks into the model, and show that our calibrated model can also quantitatively account for the labour reallocation between the two sectors and the aggregate employment fluctuations around the trend at the business cycle frequency in both economies.

### 4.1 Structural Change: Labour Reallocation in the Long-run

We use the hp-filter to filter out the trends of the employment to population ratios in the two sectors and in the aggregate, and the labour productivities in the two sectors. Given the trend aggregate employment rate and trend labour productivities in the two sectors, from equation (10) and (13) we can see that both the trend aggregate consumption and the trend of the agriculture's share of employment are determined by the four implicit utility function parameters,  $\varphi_a$ ,  $\varepsilon$ ,  $\mu_a$  and  $\mu_{na}$ . Therefore we can use the trend data in China to calibrate these parameters. Since the agriculture's share of employment is invariant with respect to the scale of the two income elasticity parameters<sup>3</sup>,  $\mu_a$  and  $\mu_{na}$ , we normalize the scale of the two parameters by setting  $\mu_{na}$  to 1. We discuss next our procedure of calibrating the remaining three parameters of the implicit utility function,  $\varphi_a$ ,  $\varepsilon$  and  $\mu_a$ .

Let  $\overline{x}_t$  denote the hp-filtered trend component of any variable  $x_t$ , and T = 33 the number of years of our sample. First, for any t = 1, ..., T, and given the trend aggregate employment rate  $\overline{L}_t$  and trend labour productivities  $\overline{A}_{at}$  and  $\overline{A}_{nat}$  in the data, from equation (10), we can write the trend aggregate consumption  $\overline{C}_t(\varphi_a, \varepsilon, \mu_a)$  as an implicit function of the three parameters  $(\varphi_a, \varepsilon, \mu_a)$ :

$$\overline{L}_{t} = \left(\varphi_{a}\left(\overline{A}_{at}\right)^{\varepsilon-1}\left(\overline{C}_{t}\right)^{(1-\varepsilon)\mu_{a}} + (1-\varphi_{a})\left(\overline{A}_{nat}\right)^{\varepsilon-1}\left(\overline{C}_{t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}.$$
 (15)

Then, from (11), we can write the trend of the agriculture's share of employment also as a function of  $(\varphi_a, \varepsilon, \mu_a)$ ,

$$\bar{l}_{at}(\varphi_a, \varepsilon, \mu_a) = \frac{\frac{\varphi_a}{1-\varphi_a} \left(\frac{\bar{A}_{at}}{\bar{A}_{nat}}\right)^{\varepsilon-1} (\bar{C}_t)^{(1-\varepsilon)(\mu_a-1)}}{1+\frac{\varphi_a}{1-\varphi_a} \left(\frac{\bar{A}_{at}}{\bar{A}_{nat}}\right)^{\varepsilon-1} (\bar{C}_t)^{(1-\varepsilon)(\mu_a-1)}}.$$
(16)

Finally, we choose the values of  $(\varphi_a, \varepsilon, \mu_a)$  to minimize the following loss function (i.e., non-linear least squares):

$$\sum_{t=0}^{T} \left\{ \left[ \overline{l}_{at}(\varphi_a, \varepsilon, \mu_a) - \overline{L}_{at}/\overline{L}_t \right]^2 \right\}$$
(17)

where  $\overline{L}_{at}$  and  $\overline{L}_t$  are the employment trends from the data. This calibration yields the following results for China:  $\varphi_a = 0.3605$ ,  $\varepsilon = 0.4754$ , and  $\mu_a = 0.1970$ . The calibrated value of the elasticity of substitution ( $\varepsilon$ ) is less than one, implying that the substitution effect is such that the agriculture's share of employment is negatively related to the agriculture's relative productivity. This is consistent with the theoretical assumption of Ngai and Pissarides (2007) and the finding of Herrendorf, Rogerson and Valentinyi (2013). The calibrated value of  $\mu_a$  is significantly less than one, implying that the income effect plays an important role for the decline of the agriculture's share of employment. Figure 3 displays the trend of the agri-

<sup>&</sup>lt;sup>3</sup>See a proof in Appendix B.

culture's share of employment from both the model and the data. The left panel shows that our calibrated model matches well the trend of the agriculture's share of employment in China.

For the US, we keep the values of the two elasticity parameters,  $\varepsilon$  and  $\mu_a$ , the same as the ones for China, but allow the value of  $\varphi_a$  to be different so that the average of the model-implied agriculture's share of employment matches that in the US data. This yields a value of 0.0772 for  $\varphi_a$  in the US.<sup>4</sup> The right panel of Figure 3 displays the trend of the agriculture's share of employment from both the model and the data for the US. Similar to the case of China, our calibrated model also matches well the trend of the agriculture's share of employment in the US. In other words, using the same income and substitution elasticities for both countries and country-specific preference weight  $\varphi_a$ , our simple two-sector model with the non-homothetic CES utility function can quantitatively account for the structural changes in both China and the US. This result is consistent with the finding of Comin, Lashkari and Mestieri (2015) for a panel of countries which does not include China.



The income effect is crucial for our model's ability in matching the speed of

<sup>&</sup>lt;sup>4</sup>The difference in the values of  $\varphi_a$  does not necessarily mean that households in the two countries have different preferences. Rather, it may capture the potential differences in labour intensity of agricultural production, barriers to labour reallocation, and other factors that may influence the average share of employment in agriculture, but are abstracted from our model.

structural change in both economies. To illustrate this, we set  $\mu_a = 1$ , and recalibrate the values of  $\varphi_a$  and  $\varepsilon$  to minimize the same loss function in (17). The resulting value of  $\varepsilon$  is 0, and the values of  $\varphi_a$  are 0.1663 for China and 0.0138 for the US. We plot the model implied trends of the agriculture's share of employment for both China and the US in Figure 3, labelled as homothetic CES. The model with no income effect cannot match the speed of structural change in China nor in the US. This is consistent with the findings of Boppart (2014) for the US and Comin, Lashkari and Mestieri (2015) for other economies.

# 4.2 Labour Reallocation in the Short-run and Aggregate Employment Fluctuations

We now turn to the cyclical properties of our model when there are shocks to productivities in the two sectors. Before presenting the quantitative results, we first discuss our strategies of dealing with the trend in the aggregate employment rate and the calibration of the parameters  $\lambda$  and  $\sigma$ , both of which have direct impact on the cyclical properties of aggregate employment.

### 4.2.1 Detrending the Aggregate Employment Rate

In examining the structural change in the long-run, we have taken the trend of the aggregate employment rate  $\overline{L}_t$  as exogenous. Since our objective here is to investigate our model's implication for aggregate employment fluctuations, we can no longer assume that the aggregate employment rate is exogenously given. Instead, we have to solve  $L_t$  endogenously from the model. This implies that we need to solve the aggregate consumption  $C_t$  from equation (14), which requires the values of the parameters  $\lambda$  and  $\sigma$  as well as the previously calibrated values of ( $\varphi_a, \varepsilon, \mu_a$ ). However, we still calibrate our model so that the model implied trend of the aggregate employment rate matches the trend in the data. Specifically, for any given values of  $\lambda$  and  $\sigma$  and the previously calibrated values of ( $\varphi_a, \varepsilon, \mu_a$ ), we choose the

labour supply parameter  $B_t$  to solve the following equation:

$$\overline{L}_{t} = \left[ \frac{\left(\varphi_{a}\left(\overline{A}_{at}\right)^{\varepsilon-1}\left(\overline{C}_{t}\right)^{(1-\varepsilon)\mu_{a}} + (1-\varphi_{a})\left(\overline{A}_{nat}\right)^{\varepsilon-1}\left(\overline{C}_{t}\right)^{1-\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}}}{B_{t}\left(\mu_{a}\varphi_{a}\left(\overline{A}_{at}\right)^{\varepsilon-1}\left(\overline{C}_{t}\right)^{(1-\varepsilon)\mu_{a}+\lambda-1} + \mu_{na}(1-\varphi_{a})\left(\overline{A}_{nat}\right)^{\varepsilon-1}\left(\overline{C}_{t}\right)^{\lambda-\varepsilon}\right)} \right]^{\frac{1}{\sigma}},$$

where  $\overline{L}_t$ ,  $\overline{A}_{at}$  and  $\overline{A}_{nat}$  are the trends of the aggregate employment rate, the labour productivity in the agricultural and non-agricultural sectors, respectively, and  $\overline{C}_t$  is the trend aggregate consumption solved from equation (15).

### **4.2.2** Calibration of $\lambda$ and $\sigma$

We set  $\sigma = 0.6$  so that the Frisch elasticity of labour supply is 1.7, a value used by Greenwood, Hercowitz, and Huffman (1988) and many others in the business cycle literature. For the parameter  $\lambda$ , we calibrate it to the US data by choosing the value of  $\lambda$  such that when the model is applied to the US, it can generate the correlation between the cyclical components of the aggregate employment rate and output that matches the corresponding correlation in the US data. The exact procedure is the following:

- (1) Given any value of  $\lambda$ , choose  $\{B_t^{US}\}_{t=1,...,T}$  to match the trend aggregate employment rates in the US data as discussed above.
- (2) Take  $\{B_t\}_{t=1,...,T}$  and the actual US labour productivities  $\{A_{at}^{US}\}_{t=1,...,T}$  and  $\{A_{nat}^{US}\}_{t=1,...,T}$ , which include both the trend and the realized productivity shocks, solve the sector-level and aggregate employment rates and GDP in equilibrium using the method described at the end of Section 3
- (3) Detrend all employment and GDP series using hp-filter to retrieve the cyclical components of these series.
- (4) Calculate the correlation between the cyclical components of the equilibrium aggregate employment rate and GDP.
- (5) Continue step (1) through (4) with different values of  $\lambda$  until finding a value at which the correlation in Step (4) matches the data.

Using this calibration procedure, we find a value of 0.8 for the parameter  $\lambda$ . We summarize all the calibration results in Table 3. Given these calibrated values of parameters. We follow step (1) through (4) above to calculate the model-implied moments of the cyclical components for both China and the US. Table 4 presents the results from the calibrated model.

Parameter	Description	Target	China Value	US Value
$\varphi_a$	preference weight	average of agriculture's	0.3605	0.0772
	of agriculture	employment share		
ε	elasticity of substitution between two goods	trend of agriculture's employment share in China	0.4754	0.4754
	U	1 2		
$\mu_a$	income elasticity of agricultural good	trend of agriculture's employment share in China	0.1970	0.1970
$\mu_{na}$	income elasticity of non-agricultural good	normalization	1	1
σ	inverse of Frisch elasticity of labour supply	literature	0.6	0.6
λ	inverse of the intertemporal elasticity of substitution	correlation of US aggregate employment and output	0.8	0.8

Table 3: Benchmark Calibration

### 4.2.3 Benchmark Results

The first and second columns of Table 4 present the business cycle statistics calculated from the Chinese data and the simulated time series from the model, and the third and fourth columns present the corresponding results for the US. Panel A shows the relative standard deviations of the aggregate employment to output and the correlation between the aggregate employment and output, panel B the sector level correlations and relative standard deviations, and panel C the correlation between sector employment and the correlations of the agriculture's share of employment with sector labor productivities.

	China		US	
	Data	Model	Data	Model
(A) Aggregate				
$\sigma(L)/\sigma(Y)$	0.11	0.13	0.70	0.23
$\rho\left(L,Y ight)$	0.09	-0.01	0.87	0.87
(B) Within Sector				
$\sigma(L_a)/\sigma(Y_a)$	0.70	0.81	0.33	1.08
$\sigma(L_{na})/\sigma(Y_{na})$	0.75	0.55	0.71	0.24
$\rho\left(L_a,Y_a\right)$	0.24	-0.92	-0.05	-0.99
$\rho\left(L_{na},Y_{na}\right)$	0.88	0.83	0.87	0.86
(C) Cross Sector				
$\rho(L_a,L_{na})$	-0.83	-0.82	-0.23	-0.28
$\rho(\frac{L_a}{L}, A_a)$	-0.44	-0.99	-0.33	-1.00
$\rho(\frac{L_a}{L}, A_{na})$	-0.35	-0.27	-0.42	-0.13

Table 4: Benchmark Results

**Results for China**. Overall, the model does a good job in matching both the aggregate and sector moments in the Chinese data. From panel A, we see that the model produces a relative aggregate employment volatility of 0.13, which is very close to 0.11 in the data. The model also generates an acyclical employment series, with its correlation with output close to zero. From panel B we see that the model generates relative employment volatilities in the two sectors that are comparable to those in the data. The model-implied non-agriculture employment is strongly pro-cyclical, as in the data. However, for the agricultural sector, employment is negatively correlated with output in the model and slightly positive in the data. The model-implied negative correlation between employment and output in the agricultural employment and agricultural labour productivity is strongly negative. We explain below that this is

due to a strong income effect. Panel C shows the labor reallocation between the two sectors. The correlation of employments in the two sectors is -0.82, which is very close to the data, indicating strong reallocation between the sectors. Moreover, the model implies that the agriculture's share of employment is negatively correlated with labor productivities in both sectors. When the labor productivity in the agricultural sector increases, the relative price of the agricultural good falls. Given that the agricultural and non-agricultural goods are complements ( $\varepsilon < 1$ ), substitution effect leads to a fall of the agriculture's share of employment. In addition, higher agricultural labor productivity also raises aggregate consumption. Because  $\mu_a < 1$ , the income effect is such that the agriculture's share of employment declines. Thus, both the substitution and income effects lead to negative correlation between the agriculture's share of employment and labor productivity. This explains why the model implies a very strong negative correlation of -0.99. When the labor productivity in the non-agricultural sector increases, the relative price of the agricultural good rises and the substitution effect is such that the agriculture's share of employment increases, but the income effect still leads to a fall of the agriculture's share of employment. Overall, the income effect dominates, leading to a correlation of -0.27.

**Results for the US**. Our model also does a good job in replicating the US business cycle facts. In the aggregate, the model can generate highly pro-cyclical aggregate employment by construction. The model produces a relative employment volatility that is lower than that in the data. This problem is common for standard real business cycle model, as pointed out by Cooley and Prescott (1995), that without additional labor market frictions these models have difficulty in generating sizable employment variations. Panel B and C illustrate the sector level correlations and labor reallocations across sectors, which are broadly consistent with the data. It is worth emphasizing that, as shown in panel C, the model is able to produce a negative correlation between the two sectors' employments and negative correlations in both sectors.

In summary, despite being highly stylized, our model economy can match well the employment fluctuations in both China and the US at sector level and in the aggregate. Similar to the case for the long-run structural change, the key to the success of our model is the income effect generated by the non-homothetic preferences. Because the income elasticity of the agricultural good is less than that of the non-agricultural good, the income effect on the employments in the agricultural and non-agricultural sectors are in the opposite directions. When the agricultural sector is large, this income-effect-induced negative correlation between employments in the two sectors dampens the aggregate employment volatility and reduces the correlation between the aggregate employment and output. In the sensitivity analysis below, we will examine the quantitative implications of the two-sector model when the two consumption goods are aggregated by a standard homothetic CES utility function with no income effect, and we will show that the model cannot match the aggregate employment fluctuations in China.

#### 4.2.4 Sensitivity Analysis

We first illustrate the importance of income effect by showing the results for the case of homothetic CES utility function, and then conduct some additional sensitivity analysis to show the robustness of our benchmark model with income effect.

**Homothetic CES utility function**. When  $\mu_a = \mu_{na} = 1$ , our model has the standard homothetic CES utility function, which is also the utility function used by Da-Rocha and Restuccia (2006). We have already shown in Section 4.1 that without income effect the model cannot match the long-run structural change in the data for either economy. We now investigate if the model can account for the aggregate employment fluctuations in China if we follow the common practice in the business cycle literature to detrend the data and focus on the cyclical part. We follow the calibration strategy of Da-Rocha and Restuccia (2006) by choosing a country-specific value of  $\varphi_a$  to match the average of the agriculture's share of employment in the data for each of the two economies, and choosing the value of  $\varepsilon$  to match the ratio of the volatility of agricultural employment to that of non-agricultural employment in the US. Table 5 presents the business cycle statistics of the calibrated model without income effect.

For the case of China, the model performs poorly in the aggregate level, with

a model-implied employment-output correlation of 1. We show in Appendix C that this is actually a general property of the model with a homothetic CES utility function. We prove that for any  $\sigma \ge 0$ ,  $\varepsilon > 0$  and  $0 \le \lambda < 1$ , the model-implied aggregate employment and aggregate consumption are perfectly correlated. In this model with no investment, the aggregate GDP and the aggregate consumption are identical if the nominal GDP is deflated using the ideal price index. The real GDP (in the data and in our model) is slightly different because it is measured using the prices in a base year, but it is quantitatively very similar to the real GDP deflated using the ideal price index. So it is not surprising that the correlation of the aggregate employment and the measured real aggregate GDP in the model is also 1.5 Da-Rocha and Restuccia (2006) also uses a CES utility function, but they were able to generate a low correlation between the aggregate employment and output because they introduced independent expost shocks to the agricultural productivity (weather shocks). In the version of the model without ex post shocks, their model's implied employment-output correlation is 0.95.<sup>6</sup> It is slightly smaller than one because in their model there is investment so that output and consumption are not perfectly correlated. In contrast, our benchmark model with income effect can generate low employment-output correlation without introducing any ex post shock.

The homothetic CES model without income effect also performs poorly at sector level. It generates a high correlation (0.98) between the agricultural employment and non-agricultural employment, which contradicts with the negative correlation in the data. Moreover, the model-implied correlation of the agriculture's share of employment and the labour productivity in the non-agricultural sector is positive (0.19) while the correlation in the data is negative (-0.35). Even for the US, the model also performs poorly at the sector level. Again, the correlation of the employments in the two sectors and the correlation of the agriculture's share of employment and the non-agricultural sector's labour productivity are both positive in the model, but negative in the data.

We now conduct some additional sensitivity analysis to show the robustness of

<sup>&</sup>lt;sup>5</sup>If  $\lambda > 1$ , then we show in Appendix C that the aggregate employment and the aggregate consumption has a correlation of -1, which is also inconsistent with the data.

<sup>&</sup>lt;sup>6</sup>See Table 9 on page 477 of Da-Rocha and Restuccia (2006).

	China			US		
		Our	Homothetic		Our	Homothetic
	Data	Model	CES	Data	Model	CES
(A) Aggregate						
$\sigma(L)/\sigma(Y)$	0.11	0.13	0.12	0.70	0.23	0.13
$\rho\left(L,Y ight)$	0.09	-0.01	1.00	0.87	0.87	1.00
(B) Within Sector						
$\sigma(L_a)/\sigma(Y_a)$	0.70	0.81	0.08	0.33	1.08	0.03
$\sigma(L_{na})/\sigma(Y_{na})$	0.75	0.55	0.16	0.71	0.24	0.13
$\rho\left(L_{a},Y_{a} ight)$	0.24	-0.92	0.88	-0.05	-0.99	-0.65
$\rho\left(L_{na},Y_{na}\right)$	0.88	0.83	0.71	0.87	0.86	0.99
(C) Cross Sector						
$\rho(L_a,L_{na})$	-0.83	-0.82	0.98	-0.23	-0.28	0.57
$\rho(\frac{L_a}{L}, A_a)$	-0.44	-0.99	-0.85	-0.33	-1.00	-0.98
$ ho(rac{L_a}{L},A_{na})$	-0.35	-0.27	0.19	-0.42	-0.13	0.05

Table 5: Comparison with Homothetic CES Utility Function

our benchmark model with income effect.

Aggregate labour supply parameter  $B_t$ . Since the long-run change of the aggregate employment rate is determined by demographic factors, which are not the interest of our paper, we used exogenous  $B_t$  to match the trend of aggregate employment rate in our calibration. This is effectively a detrending method for our non-homothetic model. We now test the robustness of our model by imposing a constant  $B_t$ . In particular, we use B to match the long-run average of the aggregate employment rate, which gives B = 4.71 for China and B = 4.86 for the US. Table 6 shows the simulation results, where column (2) and (7) are benchmark results for China and the US and column (3) and (8) are the robustness checks when B is a constant. We see that holding  $B_t$  constant barely affects the business cycle properties of our model. All the conclusions from our benchmark model carry through.

**Elasticity of labour supply**. The parameter  $\sigma$  governs the elasticity of labor

supply, which affects directly the aggregate employment volatility. In line with the literature, we choose this parameter to be 0.6 in our benchmark calibration. We now check the sensitivity of our model to this parameter by changing the value of  $\sigma$ . In column (4), (5), (9), (10) of Table 6, we report the simulation results for different values of  $\sigma$  in China and the US. It can be seen that higher labor elasticity, or lower value of  $\sigma$ , implies higher aggregate employment volatility. Aggregate employment remains acyclical for China and pro-cyclical for the US under different values of  $\sigma$ . While there is some minor differences in the results across different value of  $\sigma$ , the properties of sector-level fluctuations and the labour reallocation between the two sectors of the benchmark model still hold.

## 5 Conclusion

The cyclical behavior of aggregate employment differs significantly between China and the US. This sharp difference at the aggregate level conceals similar behavior of cyclical properties of employments at sector level. We argue that the main difference between China and the US is the size of the agricultural sector, which results in quantitatively different impacts of labour reallocation between sectors on the aggregate employment. We show that a simple two-sector growth model with productivity shocks and non-homothetic preferences can simultaneously account for the structural change in the long-run and the employment fluctuations in the short-run in both China and the US.

and the US	
China	
Analysis -	
Sensitivity	
Table 6:	

	(1) China	(2)	(3)	(4)	(2)	(9) US	(2)	(8)	(6)	(10)
	Data	Benchmark	Constant B	$\sigma = 0.1$	$\sigma = 2$	Data	Benchmark	Constant B	$\sigma = 0.1$	$\sigma = 2$
(A) Aggregate										
$\sigma(L)/\sigma(Y)$	0.11	0.13	0.12	0.18	0.09	0.70	0.23	0.13	0.26	0.21
$ ho\left(L,Y ight)$	0.09	-0.01	-0.02	0.04	-0.04	0.87	0.87	0.97	0.91	0.79
(B) Within Sector										
$\sigma\left(L_{a} ight)/\sigma\left(Y_{a} ight)$	0.70	0.81	0.82	0.88	0.76	0.33	1.08	1.09	1.08	1.07
$\sigma\left(L_{na} ight)/\sigma\left(Y_{na} ight)$	0.75	0.55	0.52	0.51	0.58	0.71	0.24	0.14	0.27	0.23
$ ho\left(L_{a},Y_{a} ight)$	0.24	-0.92	-0.93	-0.89	-0.94	-0.05	-0.99	-0.99	-0.99	-0.99
$ ho\left(L_{na},Y_{na} ight)$	0.88	0.83	0.84	0.85	0.82	0.87	0.86	0.95	0.92	0.77
(C) Cross Sector										
$\rho(L_a, L_{na})$	-0.83	-0.82	-0.87	-0.76	-0.86	-0.23	-0.28	-0.44	-0.20	-0.36
${oldsymbol{ ho}}(rac{L_a}{L},A_a)$	-0.44	-0.99	-0.99	-1.00	-0.99	-0.33	-1.00	-1.00	-1.00	-1.00
${oldsymbol{ ho}}(rac{L_a}{L},A_{na})$	-0.35	-0.27	-0.28	-0.30	-0.25	-0.42	-0.13	-0.12	-0.13	-0.12

# Appendix

# A Derivation of Formulas

The FOCs of the social planner's maximization problem with respect to  $L_{at}$  and  $L_{nat}$  are:

$$\frac{\partial C_t}{\partial c_{at}} C_t^{-\lambda} A_{at} - B_t L_t^{\sigma} = 0$$
(18)

$$\frac{\partial C_t}{\partial c_{nat}} C_t^{-\lambda} A_{nat} - B_t L_t^{\sigma} = 0$$
<sup>(19)</sup>

From equation (1), we have

$$\mu_{a}(\varphi_{a})^{\frac{1}{\varepsilon}} c_{at}^{\frac{\varepsilon-1}{\varepsilon}} C_{t}^{\frac{(1-\varepsilon)\mu_{a}-\varepsilon}{\varepsilon}} \frac{\partial C_{t}}{\partial c_{at}} + \mu_{na}(\varphi_{na})^{\frac{1}{\varepsilon}} c_{nat}^{\frac{\varepsilon-1}{\varepsilon}} C_{t}^{\frac{(1-\varepsilon)\mu_{na}-\varepsilon}{\varepsilon}} \frac{\partial C_{t}}{\partial c_{at}} - (\varphi_{a})^{\frac{1}{\varepsilon}} c_{at}^{-\frac{1}{\varepsilon}} C_{t}^{\frac{(1-\varepsilon)\mu_{a}}{\varepsilon}} = 0,$$

$$\mu_{a}(\varphi_{a})^{\frac{1}{\varepsilon}}c_{at}^{\frac{\varepsilon-1}{\varepsilon}}C_{t}^{\frac{(1-\varepsilon)\mu_{a}-\varepsilon}{\varepsilon}}\frac{\partial C_{t}}{\partial c_{nat}}+\mu_{na}(\varphi_{na})^{\frac{1}{\varepsilon}}c_{nat}^{\frac{\varepsilon-1}{\varepsilon}}C_{t}^{\frac{(1-\varepsilon)\mu_{na}-\varepsilon}{\varepsilon}}\frac{\partial C_{t}}{\partial c_{nat}}-(\varphi_{na})^{\frac{1}{\varepsilon}}c_{nat}^{-\frac{1}{\varepsilon}}C_{t}^{\frac{(1-\varepsilon)\mu_{na}}{\varepsilon}}=0.$$

Thus, we have

$$\frac{\partial C_t}{\partial c_{at}} = \frac{(\varphi_a)^{\frac{1}{\varepsilon}} c_{at}^{-\frac{1}{\varepsilon}} C_t^{\frac{(1-\varepsilon)\mu_a}{\varepsilon}}}{D_t},$$
(20)

$$\frac{\partial C_t}{\partial c_{nat}} = \frac{(\varphi_{na})^{\frac{1}{\varepsilon}} c_{nat}^{-\frac{1}{\varepsilon}} C_t^{\frac{(1-\varepsilon)\mu_{na}}{\varepsilon}}}{D_t},$$
(21)

where

$$D_{t} = \mu_{a} \left(\varphi_{a}\right)^{\frac{1}{\varepsilon}} c_{at}^{\frac{\varepsilon-1}{\varepsilon}} C_{t}^{\frac{(1-\varepsilon)\mu_{a}-\varepsilon}{\varepsilon}} + \mu_{na} \left(\varphi_{na}\right)^{\frac{1}{\varepsilon}} c_{nat}^{\frac{\varepsilon-1}{\varepsilon}} C_{t}^{\frac{(1-\varepsilon)\mu_{na}-\varepsilon}{\varepsilon}}, \tag{22}$$

Substituting equations (20) and (21) into (18) and (19), respectively, and solving for  $c_{at}$  and  $c_{nat}$ , we have the following:

$$c_{at} = \varphi_a \left( \frac{A_{at}}{D_t B_t L_t^{\sigma} C_t^{\lambda}} \right)^{\varepsilon} C_t^{(1-\varepsilon)\mu_a}, \tag{23}$$

$$c_{nat} = \varphi_{na} \left( \frac{A_{nat}}{D_t B_t L_t^{\sigma} C_t^{\lambda}} \right)^{\varepsilon} C_t^{(1-\varepsilon)\mu_{na}}.$$
(24)

Substituting these two equations into (1) we have

$$\varphi_a \left(\frac{A_{at}}{D_t B_t L_t^{\sigma} C_t^{\lambda}}\right)^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a} + \varphi_{na} \left(\frac{A_{nat}}{D_t B_t L_t^{\sigma} C_t^{\lambda}}\right)^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na}} = 1,$$

which implies that

$$\left(D_{t}B_{t}L_{t}^{\sigma}C_{t}^{\lambda}\right)^{1-\varepsilon}\left(\varphi_{a}A_{at}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{a}}+\varphi_{na}A_{nat}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{na}}\right)=1,$$

$$D_{t}B_{t}L_{t}^{\sigma}C_{t}^{\lambda}=\left(\varphi_{a}A_{at}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{a}}+\varphi_{na}A_{nat}^{\varepsilon-1}C_{t}^{(1-\varepsilon)\mu_{na}}\right)^{\frac{1}{\varepsilon-1}}.$$
(25)

Substituting (25) into (23) and (24) and solving for  $c_{at}$  and  $c_{nat}$  yield the solution in equations (5) and (6). Substituting (5) and (6) into (22) and simplifying yields the following:

$$D_t = \frac{\mu_a \varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a - 1} + \mu_{na} \varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na} - 1}}{\varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_a} + \varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\mu_{na}}}.$$

From (25), then, we have

$$L_{t} = \left[ \frac{\left( \varphi_{a} A_{at}^{\varepsilon - 1} C_{t}^{(1 - \varepsilon)\mu_{a}} + \varphi_{na} A_{nat}^{\varepsilon - 1} C_{t}^{(1 - \varepsilon)\mu_{na}} \right)^{\frac{\varepsilon}{\varepsilon - 1}}}{B_{t} \left( \mu_{a} \varphi_{a} A_{at}^{\varepsilon - 1} C_{t}^{(1 - \varepsilon)\mu_{a} + \lambda - 1} + \mu_{na} \varphi_{na} A_{nat}^{\varepsilon - 1} C_{t}^{(1 - \varepsilon)\mu_{na} + \lambda - 1} \right)} \right]^{\frac{1}{\sigma}}.$$
(26)

# **B** Invariance of the Agriculture's Share of Employment to the Scale of $\mu_a$ and $\mu_{na}$

We prove here that for any exogenously given  $L_t$ , the solution of the agriculture's share of employment from equation (10) and (13),  $l_{at}(\varphi_a, \varepsilon, \mu_a, \mu_{na})$  is invariant to the common scale of  $(\mu_a, \mu_{na})$ . First, let  $C_t^*(\varphi_a, \varepsilon, \mu_a, \mu_{na})$  be the solution to equation (10) for the given  $L_t$ . It can be shown that the solution is unique and the corresponding agriculture's share of employment is

$$l_{at} = \frac{\frac{\varphi_a}{1-\varphi_a} \left(\frac{A_{at}}{A_{nat}}\right)^{\varepsilon-1} C_t^{*(1-\varepsilon)(\mu_a-\mu_{na})}}{1+\frac{\varphi_a}{1-\varphi_a} \left(\frac{A_{at}}{A_{nat}}\right)^{\varepsilon-1} C_t^{*(1-\varepsilon)(\mu_a-\mu_{na})}}.$$
(27)

Let  $\mu'_a = \eta \mu_a$  and  $\mu'_{na} = \eta \mu_{na}$  for an arbitrary positive constant  $\eta$ . Equation (10) and (13) now become

$$L_t = L_{at} + L_{nat} = \left(\varphi_a A_{at}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\eta\mu_a} + \varphi_{na} A_{nat}^{\varepsilon - 1} C_t^{(1 - \varepsilon)\eta\mu_{na}}\right)^{\frac{1}{1 - \varepsilon}},$$

and

$$l_{at}' = \frac{\frac{\varphi_a}{1-\varphi_a} \left(\frac{A_{at}}{A_{nat}}\right)^{\varepsilon-1} C_t^{(1-\varepsilon)\eta(\mu_a-\mu_{na})}}{1+\frac{\varphi_a}{1-\varphi_a} \left(\frac{A_{at}}{A_{nat}}\right)^{\varepsilon-1} C_t^{(1-\varepsilon)\eta(\mu_a-\mu_{na})}}.$$

Let  $C'_t = C^{\eta}_t$ . Then, we can rewrite the two equation as

$$L_{t} = L_{at} + L_{nat} = \left(\varphi_{a}A_{at}^{\varepsilon-1}C_{t}^{\prime(1-\varepsilon)\mu_{a}} + \varphi_{na}A_{nat}^{\varepsilon-1}C_{t}^{\prime(1-\varepsilon)\mu_{na}}\right)^{\frac{1}{1-\varepsilon}},$$
(28)

1

and

$$l'_{at} = \frac{\frac{\varphi_a}{1-\varphi_a} \left(\frac{A_{at}}{A_{nat}}\right)^{\varepsilon-1} C_t^{\prime(1-\varepsilon)(\mu_a-\mu_{na})}}{1+\frac{\varphi_a}{1-\varphi_a} \left(\frac{A_{at}}{A_{nat}}\right)^{\varepsilon-1} C_t^{\prime(1-\varepsilon)(\mu_a-\mu_{na})}}.$$
(29)

Since equation (28) has a unique solution, we have  $C'_t = C^*_t$ . From (27) and 29, then, we know that  $l'_{at} = l_{at}$ .

# C Solution of the Model with Homothetic CES Utility Function

Since we are focusing on the cyclical property of the model with no trend, there is no need for a time-varying  $B_t$ . We set it as a constant and for simplicity normalize it to one. With  $\mu_a = \mu_{na} = 1$  and  $B_t = 1$ , from (8), (9) and (10), we have

$$L_{at} = \frac{\varphi_a A_{at}^{\varepsilon - 1} C_t}{\left(\varphi_a A_{at}^{\varepsilon - 1} + \varphi_{na} A_{nat}^{\varepsilon - 1}\right)^{\frac{\varepsilon}{\varepsilon - 1}}},\tag{30}$$

$$L_{nat} = \frac{\varphi_{na} A_{nat}^{\varepsilon - 1} C_t}{\left(\varphi_a A_{at}^{\varepsilon - 1} + \varphi_{na} A_{nat}^{\varepsilon - 1}\right)^{\frac{\varepsilon}{\varepsilon - 1}}},\tag{31}$$

and

$$L_t = \frac{C_t}{A_t}.$$
(32)

Here,

$$A_t = \left( \varphi_a A_{at}^{\varepsilon - 1} + \varphi_{na} A_{nat}^{\varepsilon - 1} \right)^{\frac{1}{\varepsilon - 1}}.$$

Therefore,

$$c_{at} = Y_{at} = A_{at}L_{at} = \varphi_a \left(\frac{A_{at}}{A_t}\right)^{\varepsilon} C_t$$
$$c_{nat} = Y_{nat} = A_{nat}L_{nat} = \varphi_{na} \left(\frac{A_{nat}}{A_t}\right)^{\varepsilon} C_t$$

From (7), we have

$$L_t = A_t^{\frac{1}{\sigma}} C_t^{-\frac{\lambda}{\sigma}}.$$
(33)

Comparing (32) and (33) yields the following solutions for  $C_t$  and  $L_t$ :

$$C_{t} = A_{t}^{\frac{1+\sigma}{\lambda+\sigma}},$$

$$L_{t} = A_{t}^{\frac{1-\lambda}{\lambda+\sigma}}$$

$$Y_{t} = \frac{\varphi_{a}A_{at}^{\varepsilon} + \varphi_{na}A_{nat}^{\varepsilon}}{A_{t}^{\varepsilon}}C_{t} = (\varphi_{a}A_{at}^{\varepsilon} + \varphi_{na}A_{nat}^{\varepsilon})A_{t}^{\frac{1+\sigma}{\lambda+\sigma}-\varepsilon}$$

We can see that as long as  $\lambda < 1$ ,  $\ln(L_t)$  and  $\ln(C_t)$  are perfectly correlated. In this model with no investment, the real aggregate GDP, when measured using the ideal price index, is identical to the aggregate consumption  $C_t$ . Since the measured aggregate GDP  $Y_t$  using a fixed base year is very similar to the aggregate GDP measured using the ideal price index, it is not surprising that the correlation of the aggregate employment and the measured aggregate GDP is also close to 1. If  $\lambda > 1$ , then  $\ln(L_t)$  and  $\ln(C_t)$  has a correlation of -1.

## References

- [1] Acemoglu, Daron, and Veronica Guerrieri. 2008. "Capital Deepening and Non-Balanced Economic Growth." *Journal of Political Economy*, *116*, 467-498.
- [2] Boppart, Timo. 2014. "Structural Change and The Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences." *Econometrica* 82 (6), 2167-2196.
- [3] Brandt, Loren, and Xiaodong Zhu. 2000. "Redistribution in a Decentralized Docomy: Growth and Inflation in China under Reform." *Journal of Political Economy* 108 (2), 422-439.
- [4] Brandt, Loren, and Xiaodong Zhu. 2010. "Accounting for China's Growth." University of Toronto Working Paper No. 394.
- [5] Caselli, Francesco and Coleman, Wilbur John, II. 2001. "The US Structural Transformation and Regional Convergence: A Reinterpretation." *Journal of Political Economy 109*, 584-616.
- [6] Chang, Chun, Kaiji Chen, Daniel F. Waggoner, and Tao Zha. 2016. "Trends and Cycles in China's Macroeconomy." *NBER Macroeconomics Annual 30*, 1-84.
- [7] Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2007. "Business Cycle Accounting." *Econometrica* 75, 781-837.
- [8] Cooley, Thomas F., and Prescott, Edward C. 1995. "Economic Growth and Business Cycles." in the *Frontiers of Business Cycle Research*, edited by Thomas F. Cooley, Princeton University Press.
- [9] Comin, Diego, Danial Lashkari, and Marti Mestieri. 2015. "Structural Change with Long-run Income and Price Effects." NBER Working Paper No 21595.
- [10] Da-Rocha, José M., and Diego Restuccia. 2006. "The Role of Agriculture in Aggregate Business Cycles." *Review of Economic Dynamics* 9, 455–482.
- [11] Greenwood, Jeremy, Zvi Hercowitz, and Gregory Huffman. 1988. "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review* 78 (3).
- [12] He, Qing, Terence Tai-Leung Chang, and Kang Shi. 2009. "What accounts for Chinese Business Cycle?" *China Economic Review* 20, 650–661.
- [13] Herrendorf, Berthold, Richard Rogerson, and Akos Valentinyi. 2014. "Growth and Structural Transformation." in the *Handbook of Economic Growth, Volume 2*. Elsevier.
- [14] Kongsamut, Piyabha, Sergio Rebelo and Danyang Xie. 2001. "Beyond Balanced Growth." *Review of Economic Studies*, 68(4), pp. 869-82.
- [15] MaCurdy, T. E. 1981. "An Empirical Model of Labor Supply in a Life-Cycle Setting." *Journal of Political Economy*, 89(6), pp.1059–1085.

- [16] Moro, Alessio. 2012. "The Structural Transformation between Manufacturing and Services and the Decline in the US GDP Volatility." *Review of Economic Dynamics*, Vol 12 (3), pp.402-415.
- [17] Ngai, L. Rachel and Christopher A. Pissarides. 2007. "Structural Change in a Multisector Model of Growth." *American Economic Review*, 97, 429-443.
- [18] Storesletten, Kjetil, Bo Zhao, and Fabrizio Zilibotti. 2017. "Fluctuating Like China: Business Cycles during Structural Change." Unpublished.
- [19] Timmer, Marcel P., Gaaitzen de Vries, and Klaas de Vries. 2015. "Patterns of Structural Change in Developing Countries." in the *Routledge Handbook of Industry and Development*. pp. 65-83. Routledge.