University of Toronto
Department of Economics

Working Paper 567

Misallocation, Establishment Size, and Productivity

By Pedro Bento and Diego Restuccia

October 27, 2016

Misallocation, Establishment Size, and Productivity[†]

Pedro Bento
Texas A&M University[*]

Diego Restuccia
University of Toronto
and NBER[**]

October 2016

Abstract

We consider a tractable model of heterogeneous production units that features endogenous entry and productivity investment to assess the quantitative impact of policy distortions on aggregate output and establishment size. Relative to the standard factor misallocation framework, policy distortions featuring a positive productivity elasticity of distortions imply larger reductions in output through smaller investments in establishment productivity. A calibrated version of the model implies that when the productivity elasticity of distortions increases from 0.09 in the U.S. to 0.5 in India, aggregate output and average establishment size fall by 53 and 86 percent, compared to 37 and 0 percent in the standard factor misallocation model. Entry productivity investment and factor misallocation contribute equally to the reduction in output, whereas the effect of lower life-cycle productivity growth is fully offset by increased entry and reduced productivity dispersion. Establishment size differences in the model are consistent with evidence from a comprehensive dataset we construct on average establishment size in manufacturing using census data for 134 countries.

Keywords: misallocation, establishment size, productivity, investment, idiosyncratic distortions, life-cycle growth.
JEL codes: O1, O4.

[†]We thank David Atkin, Nick Bloom, Margarida Duarte, Peter Klenow, Virgiliu Midrigan, Michael Peters, Xiaodong Zhu, two anonymous referees, and several seminar and conference participants for helpful comments. All remaining errors are our own. Restuccia gratefully acknowledges the financial support from the Canadian Research Chairs program and the Social Sciences and Humanities Research Council of Canada.

[*]Department of Economics, Texas A&M University, 3056 Allen Building, 4228 TAMU, College Station, TX 77843, USA. E-mail: `pbento@tamu.edu`.

[**]Department of Economics, University of Toronto, 150 St. George Street, Toronto, ON M5S 3G7, Canada. E-mail: `diego.restuccia@utoronto.ca`.

1 Introduction

A consensus view in the literature has emerged where the large variations in income per capita across countries are mostly accounted for by differences in total factor productivity (TFP).[1] What accounts for these TFP differences across countries? A prominent channel emphasized in the literature generating differences in productivity is the misallocation of resources among heterogeneous production units or establishments that differs across countries, e.g. Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). An important finding in the empirical literature on misallocation is that not only is there evidence of large aggregate effects from misallocation, but also that the implied distortions in developing countries feature stronger implicit or explicit taxes on the more productive establishments, what Restuccia and Rogerson (2008) called correlated idiosyncratic distortions.[2] Whereas the misallocation literature has mostly focused on analyzing the effect of such distortions on aggregate productivity for a fixed distribution of establishment-level productivity, in this paper, we broaden this scope by emphasizing the effect that correlated distortions have on productivity investment by establishments and hence on the implied distribution of productivity in the economy. Our emphasis is motivated by the empirical literature that finds substantial cross-country differences in establishment-level productivity and investment in intangible capital.[3] In our framework, policy distortions not only misallocate resources across heterogeneous establishments but also affect the productivity distribution, generating larger effects on aggregate productivity. We show through a calibrated version of the model that correlated distortions generate substantial effects on the productivity distribution such that they roughly double the impact on aggregate productivity of factor misallocation alone. We also show that, unlike in environments with fixed productivity distributions, the effect of correlated distortions on establishment productivity works to reduce

[1]See, for instance, Klenow and Rodriguez-Clare (1997), Prescott (1998), and Jones (2015).

[2]Larger productivity elasticities of distortions in poor countries is also highlighted in Hsieh and Klenow (2014).

[3]For instance, Hsieh and Klenow (2009), Bloom and Van Reenen (2010), Bloom et al. (2010), Pagés (2010), Gal (2013), and Bloom et al. (2013) present evidence of establishment-level productivity differences across countries and Corrado et al. (2012) document cross-country differences in intangible capital.

establishment size, an implication that is consistent with the evidence of smaller establishment sizes in developing countries.

We consider a standard model of heterogeneous production units that builds from Hopenhayn (1992). For comparability, the setup follows closely the monopolistic competition framework used in the empirical analysis of Hsieh and Klenow (2009). The basic framework is extended along three important dimensions in order to address differences in entry and establishment-level productivity. We incorporate an endogenous entry decision of establishments, an initial investment decision determining establishment-level productivity upon entry, and investment over time determining the growth of establishment productivity over the life cycle. There is a large number of potential entrants that draw their idiosyncratic productivity from a known distribution at a cost. Establishments can improve their initial productivity through investment, but only before the realization of their idiosyncratic productivity. In the theory, ex-ante identical entering establishments make the same productivity investment decision but are ex-post heterogeneous in their idiosyncratic productivity. The theory connects policy distortions, institutions, and frictions that discourage establishment-level investment. The key emphasis in the model is the extent to which distortions that effectively penalize more productive relative to less productive establishments—correlated idiosyncratic distortions—discourage productivity investment by all establishments.[4] In the model, we show there is a strong connection between the extent of correlated distortions, entrant productivity, establishment-level productivity growth, and the mass of entrants in the economy. These effects work to lower establishment size, entrant productivity, establishment productivity growth over the life cycle, and aggregate productivity. We emphasize that with no entrant investment and constant establishment productivity, the model would imply no differences in establishment productivity and establishment size from

[4]The set of policies and institutions that effectively create correlated idiosyncratic distortions is very large and has been extensively discussed in the literature, see for instance Restuccia and Rogerson (2013), Restuccia (2013a), and Hopenhayn (2014) for discussions of these policies and institutions. Some examples are small business subsidies, financial constraints, trade restrictions, and the ability of establishments to remain informal to avoid taxes. Hsieh and Klenow (2009, 2014) discuss a number of possible extensions to a standard model through which correlated distortions can emerge *without* adversely affecting productivity investment, such as markups that increase in the size of an establishment as in Melitz and Ottaviano (2008) or Peters (2013), and argue that these alternative mechanisms lead to counterfactual predictions.

factor misallocation. As a result, allowing for entry and investment not only amplifies the losses in output and productivity from misallocation, but also rationalizes the impact of distortions on establishment size as observed in the cross-country data. To the extent that misallocation is reduced within a country over time, the model also contributes to understanding trends in establishment size. In the United States, for example, Poschke (2014) reports a doubling of average firm size since the early twentieth century, while the results in Ziebarth (2013) and Hsieh and Klenow (2009) suggest a significant reduction in misallocation in the U.S. over the same time period.

We calibrate a benchmark economy to U.S. data and show that reasonable variations in the extent of correlated distortions across countries have substantial negative effects on establishment size, entrant productivity, establishment growth over the life cycle, and aggregate output per capita. In particular, compared to the calibrated U.S. benchmark economy, increasing correlated distortions to 0.5—the elasticity between wedges and establishment productivity in India—generates a reduction in establishment size from 22 workers in the U.S. benchmark to 3 workers, which represents an 86 percent reduction in average establishment size and a factor difference in average establishment size between the U.S. and India of 7. The increase in correlated distortions generates a reduction in entrant productivity of 58 percent and in establishment-level productivity growth from 5 to 2 percent, which together with the effect of factor misallocation implies a drop in aggregate TFP of 53 percent. To put it differently, in this experiment, a 1.6-fold difference in aggregate productivity between the U.S. and India generated by static factor misallocation is amplified by 34 percent due to the added effects on productivity investment.[5]

Our model is very tractable and nests the standard model of misallocation without life-cycle growth. As a result, we are able to explicitly decompose the effects of correlated distortions into

[5]Interestingly, our parsimonious measure of correlated distortions generates a reduction in aggregate TFP from factor misallocation that compares in magnitude to the estimates in Hsieh and Klenow (2009) for India using detailed establishment-by-establishment wedges, suggesting that our summary measure of distortions captures the bulk of their effects on factor misallocation.

those working through the entry-investment channel emphasized in our paper, through the life-cycle growth channel analyzed in Hsieh and Klenow (2014), and through the factor misallocation channel analyzed in Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). We show that accounting for the life-cycle investments of establishments does not by itself amplify the impact of correlated distortions on aggregate TFP, relative to the effects of static factor misallocation in Restuccia and Rogerson (2008).[6] Accounting for exogenous life-cycle growth reduces the impact of correlated distortions because of their offsetting effect on entry.[7] Correlated distortions reduce the productivity growth of establishments which lowers aggregate TFP, but the net impact on TFP is negligible (if not positive) because the lower productivity growth encourages entry and compresses the productivity distribution reducing the impact of factor misallocation. Correlated distortions also reduce investment in establishment-level productivity upon entry but the effect on aggregate TFP is not mitigated by reduced misallocation. We show that the reduction in entrant productivity brought about by correlated distortions substantially reduces aggregate TFP, approximately doubling the impact of misallocation relative to the measured impact in an economy without entrant investment, such as that in Hsieh and Klenow (2014). Moreover, relative to the existing literature, our framework generates large establishment size and productivity effects that are more in line with the cross-country data.

To assess the ability of correlated distortions to quantitatively explain productivity differences across a large set of countries, we document evidence from cross-country micro data for the elasticity between distortions (wedges) and establishment productivity, using establishment-level data from the World Bank's Enterprise Surveys. We show that the elasticity of distortions with respect to productivity in the micro data is strongly negatively related to both average establishment size and GDP per capita across 63 countries. We also provide evidence that the establishment size effects in the model are consistent with data on establishment sizes. Evi-

[6] Restuccia and Rogerson (2008) emphasize the impact of misallocation on aggregate TFP in an environment where establishment-level productivity is exogenous and constant and entry is not affected by distortions. In Hsieh and Klenow (2009) entry is constant and establishment-level productivity is also exogenous but given by the micro data in each country.

[7] A similar result in the context of a model with exogenous establishment growth is emphasized in Fattal-Jaef (2015).

dence of the relationship between development and establishment size has been both sparse and inconclusive due to the lack of standardized size data for a large group of countries.[8] We address this by constructing a standardized database on establishment size based on individual-country data from manufacturing censuses and representative surveys and registries. Using hundreds of separate sources, we have assembled data for 134 countries with comparable employment-size data. In contrast to Alfaro et al. (2009) and Bollard et al. (2014), who use international data plagued by cross-country differences in the size of sampled firms, we show that average establishment size is strongly positively correlated with GDP per capita. For instance, whereas average establishment size in U.S. manufacturing is 22 workers, in Benin and Sierra Leone it is about 2 workers, an 11-fold difference. As a summary measure of the relationship between development and size, we compute the income elasticity of establishment size to be 0.29. Similarly, Poschke (2014) reports a positive income elasticity of firm size for the entire economy across a smaller set of countries of 0.45.[9] By constructing a harmonized dataset with comparable numbers for the average establishment size in the manufacturing sector across a large set of countries, we also hope to contribute to the literature by providing data that can be used for calibration exercises and serve as an additional source of discipline to quantitative models.

Our paper is closely related to the broad literature on misallocation and productivity discussed earlier. Specifically, our paper relates to the recent literature emphasizing the impact of misallocation on establishment-level productivity.[10] We highlight two contributions of our work to this literature. First, we emphasize the effect of misallocation on entrant investment, which we find is quantitatively substantial, roughly doubling the impact of factor misallocation. Second,

[8]For instance, in recent studies, Poschke (2014) finds a strong positive relationship between average size and development using two separate economy-wide samples of small/medium firms and large firms, while Alfaro et al. (2009) and Bollard et al. (2014) find a negative relationship between size and development. We discuss further the literature on establishment size in Section 2.

[9]We note that large differences in operational scales are also found in other sectors such as agriculture from Census data where the operational scale of farms in rich countries is 34 times that of poor countries (see for instance, Adamopoulos and Restuccia, 2014).

[10]Early examples of this literature include Restuccia (2013b) and Bello et al. (2011) with more elaborate analysis in Ranasinghe (2014), Bhattacharya et al. (2013), Gabler and Poschke (2013), Hsieh and Klenow (2014), and Da-Rocha et al. (2015). Closely related to our work is Hsieh and Klenow (2014) who consider the impact of correlated distortions on life-cycle productivity growth of establishments and aggregate TFP.

we develop a parsimonious model that allows us to explicitly and analytically disentangle the effects of correlated distortions working through the entry-investment channel, the life-cycle growth channel, and factor misallocation. Investment by entrants has been an under-explored mechanism through which policies and institutions can affect aggregate productivity, but a recent paper by Moreira (2015) suggests it is important. Analyzing the size and investment decisions of entrants over the business cycle in the U.S. data, Moreira finds that the average size of establishments entering during expansions is larger (both at entry and over their life cycle) than those entering during recessions. She concludes that firm investment decisions upon entry depend on the state of the economy and that the productivity effects that result are very persistent over time. In generating differences in establishment size, our paper is related to the seminal work of Lucas Jr. (1978) who showed that an elasticity of substitution less than one may be needed between capital and labor in the production function in order to rationalize the larger operational scales in rich countries. In our framework, even with Cobb-Douglas technology, establishment size can vary with correlated distortions. The view that differences in size across countries can arise from distortions shares with the work of Guner et al. (2008) who emphasize size-dependent distortions, i.e., distortions such as taxes and regulations that apply to establishments above a threshold size in terms of the number of workers. We differ from Guner et al. (2008) in that in our framework any correlated distortion causes productivity at the establishment level to drop for all establishments, adding to the potential factor misallocation effects typically emphasized in the literature. For this reason, the size and productivity impact of correlated distortions in our framework are orders of magnitude larger than those emphasized in Guner et al. (2008). More generally, a number of papers have developed quantitative models that generate differences in establishment size across countries.[11] A common finding in these papers, however, is a relatively small impact of distortions on size. Our model with entrant investment generates large quantitative differences in average establishment across countries, closer to the differences we document in the data. The literature has also explored many specific policies thought to explain income differences across countries such as firing costs, entry costs,

[11]For example, Bhattacharya et al. (2013), Poschke (2014), and Hsieh and Klenow (2014).

or average tax rates. But in a standard framework all of these policies lead to *larger* establishment sizes in poor countries.[12] To the extent that poor countries have both harmful policies *and* correlated distortions, our paper helps to rationalize why establishments are smaller in countries even when facing higher average costs of doing business.

The paper is organized as follows. In the next section, we present the facts from our constructed dataset of 134 countries to establish that establishment size increases substantially with the level of development across countries. Section 3 presents the model and characterizes the qualitative implications. In Section 4, we calibrate the model to data for the United States and show the quantitative implications of the model for hypothetical variations in the extent of correlated distortions. We then construct and document measures of correlated distortions across countries and assess their potential to generate differences in size and productivity. We also discuss our results for reasonable extensions in the model and reasonable variations in key parameter values. We conclude in Section 5.

2 Average Establishment Size across Countries

We describe the construction of a newly-assembled dataset for the average employment size of manufacturing establishments across a large sample of countries using census or representative survey data to show that average establishment size is strongly positively related to the level of development. While this finding is not entirely new or surprising, we discuss how our dataset is able to circumvent some of the limitations from earlier studies on establishment size and development.

[12]See for instance Hopenhayn and Rogerson (1993), Barseghyan and DiCecio (2011), and Moscoso Boedo and Mukoyama (2012), among others.

2.1 Data

We construct a dataset for the average employment size of manufacturing establishments across countries using hundreds of reports from economic censuses and nationally-representative surveys.[13] Our goal in the construction of this dataset is to obtain an internationally-comparable measure of average establishment size for a large sample of countries that is representative of the world income distribution. The challenges of course are data availability—which typically biases country samples towards rich countries— and international comparability—which often involves having data reported using different definitions of employment and production units or having data that disproportionally include larger firms.

Of crucial importance for the assessment of the relationship between establishment size and development is the inclusion in the data of all establishments regardless of whether the establishments are registered or not, and whether the establishments have paid employees or not, as a substantial portion of establishments in poor countries are unregistered and own account businesses and may include unpaid family workers. In Sierra Leone, for example, 83 percent of establishments have no paid employees, and in Ghana, unpaid workers account for almost half of the manufacturing workforce. As a result, excluding non-employer establishments would generate a highly distorted picture of establishment size differences across countries. Throughout our data collection process, we have made an effort to search for evidence from methodology documents and other published reports that small establishments are not included. Any country for which such evidence exists is not included in our sample. We include all countries with publicly-available data representative of all manufacturing establishments or firms.[14] Establishments in the manufacturing sector include businesses with a fixed location. It also includes businesses operating out of households when a sign is posted on the premises.[15]

[13]In Appendix A we provide greater detail about how we construct the dataset. We also provide a list of countries included and a list of the sources used for each country.

[14]We also include in the dataset all territories such as French Guiana, Hong Kong, and Puerto Rico. We use the word "country" solely for ease of exposition.

[15]One exception to this rule is the United States. Although U.S. employer data uses a standard definition of "establishment", the data for non-employers (i.e., self-employed) includes businesses with no fixed location like

We collected data for as many years as possible for each country from 2000 to 2012 to construct our dataset. Our standardized definition of size is the average number of persons engaged per establishment. For the vast majority of countries in our sample, the data are reported as total number of persons engaged and total number of establishments. But for some countries, about a quarter of our sample, the data are reported differently as the total number of employees, the total number of full-time equivalents, the total number of firms, or a combination of these instead of persons engaged and establishments. In these cases, we impute persons engaged per establishment using the reported data as follows.[16] To impute the number of persons engaged in countries that only report paid employees, we regress persons engaged on employees using a large set of country-years for which both measures are reported. We then use the resulting coefficient to calculate the number of employees for each year in countries that report only employees. We do a similar imputation of persons engaged for countries that report full-time equivalents or both employees and full-time equivalents. Using our measures of persons engaged (both reported and imputed), the number of establishments, and the number of firms, we then calculate the number of persons engaged per establishment and per firm for each country-year. To impute the number of persons engaged per establishment for countries that only report the number of firms, we first regress persons engaged per establishment on persons engaged per firm using all country-years that report both firm and establishment counts, and then use the resulting coefficient to impute the number of persons engaged per establishment for each year in countries that report only firm counts. We emphasize that not only do these imputations involve a small subset of our sample, but also that they are robust to whether the imputations are based on cross-country relationships for poor vs. rich countries.

In our final dataset we report the average of persons engaged per establishment across all available years for each country, resulting in final sample of 134 countries.

food trucks or sub-contractors in construction. Our focus on manufacturing should prevent this from being an issue, but our reported employment size for the U.S. may as a result be slightly biased downwards.

[16] See Appendix A for greater detail of the imputation procedure.

2.2 Findings

Table 1 reports some descriptive statistics concerning average establishment size from our dataset, GDP per capita, and population for all the countries in our sample.[17]

Table 1: Descriptive Statistics

	Mean	Median	Poorest Decile	Richest Decile
Establishment Size	12	9	6	19
GDP per capita (thousands)	18	13	1.2	55
Population (millions)	32	6	28	25

Notes: "Poorest" and "Richest" deciles refer to the ten percent of countries with the lowest and highest GDP per capita. Data from multiple sources, see text for details.

Figure 1 shows average establishment size for 134 countries in relation to GDP per capita. The data clearly show a positive correlation between average establishment size and GDP per capita. In particular, the elasticity of establishment size with respect to GDP per capita is 0.29. Figure 2 shows that the correlation between size and income is even stronger if we omit small countries with populations less than half of one million. In this case, the elasticity rises to 0.35. Each of these elasticities is remarkably robust to controlling for openness to trade and quality of institutions.[18] Recent models linking market size and markups predict that both GDP per capita and establishment size should increase with population, suggesting that the relationship illustrated in Figures 1 and 2 could be explained by differences in population size across countries.[19] But Figure 3 shows that establishment size is not systematically related to

[17]GDP per capita (adjusted for purchasing power parity, PPP) is from Penn World Table v. 8.0 for 105 countries, the IMF's World Economic Outlook 2013 for 7 countries, and the CIA World Factbook for 17 countries. For four countries (actually overseas departments of France), GDP per capita is from France's National Institute of Statistics and Economic Studies and is made relative to the U.S. GDP per capita using market exchange rates. GDP per capita for Åland Islands is from Statistics and Research Åland, and adjusted for purchasing power parity using Finland's PPP exchange rate from Penn World Table v. 8.0. Population data is from Penn World Table v. 8.0 (105 countries), the World Bank's World Development Indicators (21 countries), the CIA World Factbook (7 countries), and Statistics and Research Åland (for Åland Islands).

[18]Our measure of openness to trade is from Penn World Table v. 8.0. Our measure of institutional quality is the Heritage Foundation's Index of Economic Freedom (2014).

[19]Melitz and Ottaviano (2008) and Desmet and Parente (2012), for example, each develop models in which

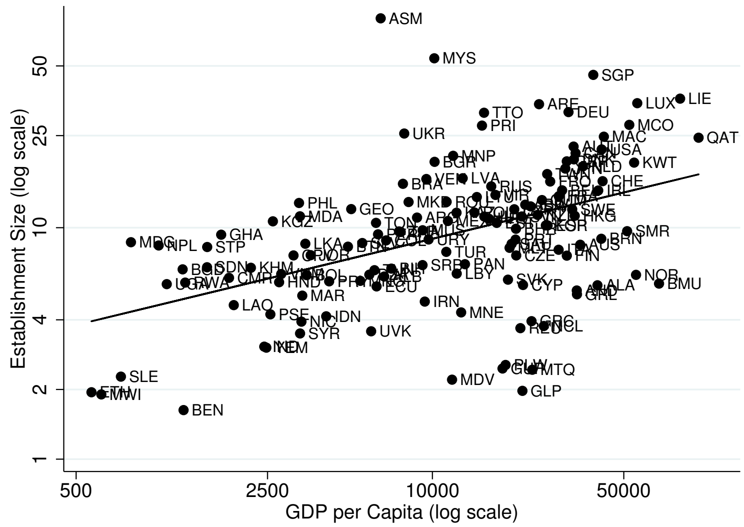

Figure 1: Establishment Size and GDP per Capita

population.[20]

To confirm that the observed relationship between establishment size and GDP per capita (elasticity of 0.29) is not being driven by our construction of establishment size data using imputations, we separately test the relationship between size and GDP per capita for persons engaged per establishment, persons engaged per firm, employees per establishment, and employees per firm, using only the raw source data for each country. The corresponding elasticities are all positive and of comparable magnitude: 0.38 for persons engaged per establishment (data for 64 countries), 0.34 for persons engaged per firm (data for 48 countries), 0.32 for employees per establishment (data for 45 countries), and 0.28 for employees per firm (data for 52 countries).

2.3 Related Literature

We now compare the implications of our data relative to the existing work in the literature with emphasis on the connection between average establishment size and development. There are

larger populations can lead to both higher output per capita and larger establishments.

[20]The regression slope coefficient (standard error) in Figure 1 is 0.29 (0.04) and in Figure 2 is 0.35 (0.04). In Figure 3 the slope coefficient is an insignificant -0.003 (0.03).

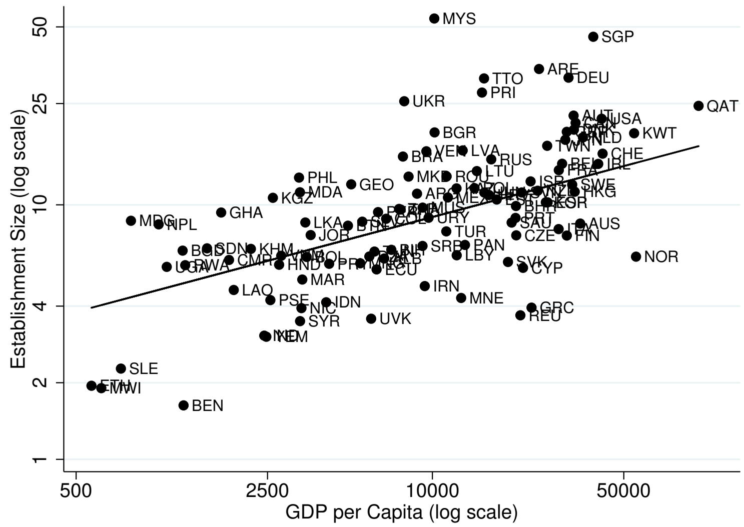

Figure 2: Establishment Size and GDP per Capita (small countries removed)

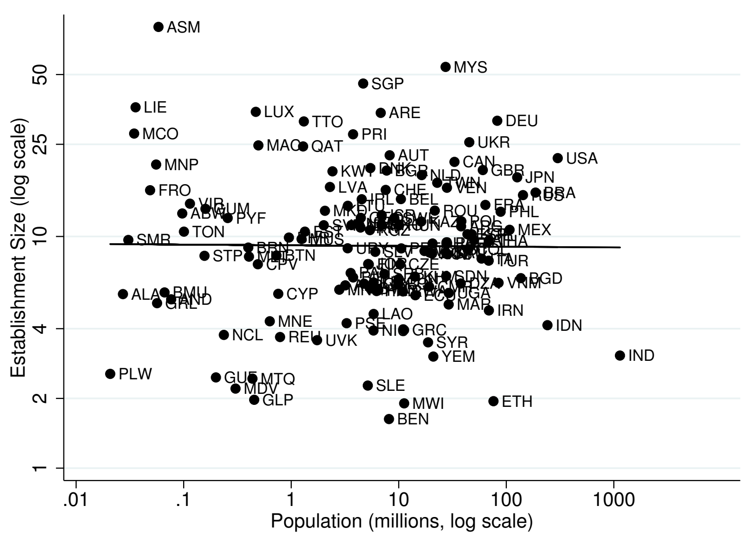

Figure 3: Establishment Size and Population

numerous studies emphasizing firm size across countries, for instance Tybout (2000) surveys a broad literature addressing the size of manufacturing firms in developing countries, but as recognized in this literature, for the vast majority of work the evidence on firm size comes from a relatively small sample of countries and from data sources that may not deal systematically with comparability issues across countries.[21] Similarly, the large empirical literature addressing the misallocation of resources across establishments in developing countries such as that in Hsieh and Klenow (2009, 2014) and Pagés (2010) focus on a handful of countries with varying degrees of data comparability across the countries.[22]

A widely cited reference for the relationship between firm size and income is Alfaro et al. (2009). They use Dun & Bradstreet's WorldBase data (DB) to document a negative relationship between firm size and income per capita across 79 countries. More recently, Bollard et al. (2014) report the same negative relationship using data from the United Nations Industrial Development Organization's (UNIDO) Industrial Statistics Database for 72 countries. These observations are in direct contrast to those just documented from our data.[23] To understand Alfaro et al. (2009), it is useful to first emphasize that DB is comprised of business data aggregated from multiple sources that is typically used to provide credit and market-assessment services. A key issue is that DB has sparse coverage of small firms in poor countries relative to rich countries, with no attempt to make the data representative of all establishments. As a result, when calculating average firm size in poor countries, the under-representation of small firms biases the average upwards. In a sense, Alfaro et al. are comparing average size across most firms in rich countries with the average size of only large firms in poor countries. The UNIDO data used by Bollard et al. (2014) similarly include countries with unbalanced populations of firms, with some countries reporting data for all firms and other countries reporting data

[21] For example, Hsieh and Olken (2014) study comprehensive firm-level data to study firm size distributions across countries but the focus involves three countries: India, Indonesia, and Mexico.

[22] In addition, in the studies of misallocation the data required to measure TFP at the establishment level implies that many small establishments with missing or unreliable observations are excluded.

[23] We note that the negative relationship between average firm size and development found using DB and UNIDO data is also at odds with the relationship found for specific sectors such as agriculture where census data indicates much smaller farm size operations in poor countries relative to rich, see for instance Adamopoulos and Restuccia (2014).

only for larger firms.[24] More importantly, our data contains information for 59 countries from Alfaro et al.'s sample and 59 countries from Bollard et al.'s sample, and the result of a positive relationship between establishment size and development is even stronger in these subsamples than for all 134 countries.

The closest empirical application to our paper is Poschke (2014) who uses two separate datasets, one from the Global Entrepreneurship Monitor (GEM) for small and medium firms in 47 countries, and one from Amadeus for large firms in 34 countries. It is the closest empirical application to us because in choosing these two datasets, Poschke (2014) attempts to harmonize the coverage of firms across the entire cross-country income distribution. Not surprisingly then, Poschke (2014) finds that firm size is strongly positively related with development, consistent with our evidence. Unlike Alfaro et al. (2009) and Bollard et al. (2014), the survey data used in Poschke is constructed to be representative of firms within each size class for each country. Although with a smaller sample of countries, Poschke shows that average firm size in each of these datasets is increasing in development across multiple sectors of the economy. For instance, the implied income elasticities of firm size are 0.45 in both datasets in Poschke (2014), higher than the 0.29 elasticity in our dataset.[25] While the cross-country patterns that arise between size and income are similar, there are important differences between our dataset and the two datasets used in Poschke (2014). First, our dataset provides a number on the average establishment size for each country in a large sample of 134 countries, whereas the data in Poschke (2014) involves two separate numbers for the average establishment size of small and medium firms from GEM and of large firms from Amadeus for a relatively small sample of countries (47 and 34 countries in each case). This distinction is important, because characterizing the relationship between size and development is far from the only use for our data. In many applications a specific measure of establishment size is relevant for calibration and quantitative assessment, as is the case in our paper for evaluating our model's quantitative predictions for establishment size across countries. Second, our dataset provides average establishment size in the manufacturing sector

[24]For this reason, some of the countries used in Bollard et al. (2014) have been excluded from our dataset.

[25]We thank Markus Poschke for generously providing the implied size-income elasticities in his two samples.

across countries, therefore implicitly controlling for changes in the structure of the economy which vary systematically across countries, whereas the average size in Poschke (2014) includes firms in all sectors of the economy.[26]

Comparing the results of our analysis with those of the previous literature makes it clear that analyzing standardized, representative size data for a specific sector, especially with respect to the smallest establishments in poor countries, is crucial to obtaining an accurate measure of the average employment size of establishments across countries and how it varies with the level of development.

3 The Model

We consider an economy where time is discrete and indexed by t. A representative final-good firm uses a variety of imperfectly substitutable inputs from intermediate-good firms to produce the final consumption good, which also serves as the numéraire.[27] There is a stand-in household endowed with a continuum of members (of measure one), each supplying one unit of labor each period. There are a large number of potential intermediate firms who are free to enter, but must pay a fixed entry cost and make a costly productivity-investment decision before producing. Each period after entry firms invest to further increase their productivity. Firms face output distortions which may be correlated with firm-level productivity, and take these distortions into account when investing in productivity. We assume an exogenous probability of exit and, as a result, there is ongoing entry and exit in steady state. We study the steady state of the economy in which firms take the wage, the interest rate, and the size of the economy as given, and free entry ensures the value of entry is driven to zero. We then consider how the extent of correlated distortions affects the number of firms, investment, firm growth, and aggregate

[26] See for instance Duarte and Restuccia (2010) and Herrendorf et al. (2014) for a characterization of differences and changes in the sectoral structure of the economy across countries and over time.

[27] Throughout we use “firm” and “establishment” interchangeably in reference to intermediate-good producers.

output.[28] We begin by describing the environment in more detail.

3.1 Environment

The representative final-good firm produces output using a variety of inputs from intermediate-good firms according to the following production function;

$$Y = \left(\int_0^N y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

where N is the number of intermediate-good firms, y_i the demand for input i, and $\sigma > 1$ the constant elasticity of substitution between varieties.

Each intermediate-good firm has access to the following production function;

$$y = sz\ell,$$

where sz is productivity and ℓ is labor. An entrant's realized z is drawn from a known exogenous distribution, while s is determined in the following way. After paying an entry cost $c_e Y$, but before realizing z, an entrant chooses its initial s_0 by incurring a cost equal to $c_S Y s_0^{\theta}$, where the subscript on s refers to the age of the firm, and both $c_S > 0$ and $\theta > 1$ are exogenous and common to all firms.[29] At the beginning of each period after entry, firms increase their productivity by a factor of $1 + g$ by incurring a cost equal to $c_g(1+g)^{\phi}\Omega(s_{-1}, z)$, where $s_{-1}z$ is a firm's productivity from the previous period, $c_g > 0$, $\phi > 1$, and;[30]

$$\Omega(s_{-1}, z) = \left(\frac{Y_{-1}}{N_{-1}} \right) \cdot \frac{(s_{-1}z)^{\sigma(1-\gamma)-1}}{\frac{1}{N_{-1}} \int_0^{N_{-1}} (s_{-1,i}z_i)^{\sigma(1-\gamma)-1} di}.$$

[28] We refer to the mass of firms as the number of firms for ease of exposition.

[29] Our specification of the entry cost as a multiple of aggregate output is consistent with Bollard et al. (2014) who argue, using time-series data, that entry costs should scale up with secular development. Note that if population were not normalized to one, we would need to make the entry and investment costs scale up with output per capita.

[30] The productivity of an n-year-old firm is therefore equal to $z \cdot s_0 \cdot (1 + g_1) \cdot (1 + g_2) \cdots (1 + g_n)$.

That the cost of per-period productivity investment is increasing in the relative profitability of a firm ensures that Gibrat's Law is satisfied, that is that firm growth is independent of the initial size of a firm.[31] At the end of each period, each intermediate producer faces an exogenous probability of exit equal to λ.

Output distortions are such that each firm retains a fraction $(1 - \tau)$ of its output, and we assume τ depends on firm-level productivity as follows;

$$(1 - \tau) = (sz)^{-\gamma},$$

where the parameter γ is the elasticity of a firm's distortion with respect to its productivity.

Given our assumptions, producers engage in monopolistic competition, each entrant chooses the same s_0, each incumbent chooses the same g each period, all entrants choose to continue operating, and the cross-sectional distribution of firm productivities remain invariant. We abstract from the household's inter-temporal consumption decision and simply assume an exogenous interest rate R.

3.2 Equilibrium

We focus on the steady-state decentralized equilibrium of the economy in which the distributions of prices and allocations are invariant.[32] A *steady-state decentralized equilibrium* is defined as a wage rate w, distributions of firm-level productivities sz, intermediate-good prices P, output y, labor demand ℓ, operating profits π, productivity growth g, number of firms N, and aggregate output Y, such that;

[31] We follow Atkeson and Burstein (2010) in ensuring Gibrat's Law is satisfied.

[32] The equilibrium of this economy differs from its optimal allocation. As noted in Atkeson and Burstein (2010), specifying entry and investment costs in terms of goods rather than labor results in a wedge between the equilibrium allocation and the allocation chosen by a social planner. We solve for the social planner's allocation in Appendix B.

(i) given each P, the final-good firm demands intermediate-good inputs to maximize profits in each period,

(ii) given w, R, and Y, intermediate-good producers choose labor to maximize per-period profits,

(iii) given w, R, and Y, incumbents choose a factor increase in productivity $(1+g)$ to maximize the expected present value of lifetime profits,

(iv) given w, R, and Y, entrants choose initial productivity s_0 to maximize the expected present value of lifetime profits,

(v) free entry ensures the expected present value of lifetime profits for an entrant is equal to the expected present value of all productivity investments plus the entry cost,

(vi) markets clear, i.e., the supply of labor (equal to one) is equal to the quantity of labor demanded by firms.

The final-good firm takes input prices as given and maximizes profits in each period, generating the following inverted demand function for each input i;

$$P_i = Y^{\frac{1}{\sigma}} y_i^{\frac{-1}{\sigma}}.$$

Operating profits in each period for an incumbent firm-i are therefore;

$$\pi_i = (1-\tau_i) Y^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}} - w\ell_i, \quad \text{where} \quad y_i = s_i z_i \ell_i.$$

Firms choose labor to maximize operating profits each period, generating the following demand for labor and optimal output;

$$\ell_i = \frac{(1-\tau_i)^\sigma Y (s_i z_i)^{\sigma-1}}{w^\sigma} \left(\frac{\sigma-1}{\sigma}\right)^\sigma,$$

$$y_i = \frac{(1-\tau_i)^\sigma Y (s_i z_i)^\sigma}{w^\sigma} \left(\frac{\sigma-1}{\sigma}\right)^\sigma.$$

Per-period operating profits for firm-i, given $s_i z_i$, are therefore;

$$\pi_i = \frac{(1-\tau_i)^\sigma Y (s_i z_i)^{\sigma-1} (\sigma-1)^{\sigma-1}}{w^{\sigma-1}\sigma^\sigma}. \tag{1}$$

Combining y_i above with the final-good production function results in the following expression;

$$w = \left(\frac{\sigma-1}{\sigma}\right)\left[\int_0^N (s_i z_i)^{\sigma-1}(1-\tau_i)^{\sigma-1} di\right]^{\frac{1}{\sigma-1}}. \tag{2}$$

Labor-market clearing results in;

$$w^\sigma = Y \left(\frac{\sigma-1}{\sigma}\right)^\sigma \left[\int_0^N (s_i z_i)^{\sigma-1}(1-\tau_i)^\sigma di\right]. \tag{3}$$

Combining equations (2) and (3) and rearranging results in expressions for aggregate output and the wage rate;

$$Y = \left[\int_0^N (s_i z_i)^{\sigma-1}\left(\frac{1-\tau_i}{1-\overline{\tau}}\right)^{\sigma-1} di\right]^{\frac{1}{\sigma-1}}, \tag{4}$$

$$w = (1-\overline{\tau})\left(\frac{\sigma-1}{\sigma}\right) Y, \tag{5}$$

where $(1-\overline{\tau})$ is the weighted average of $(1-\tau_i)$ across all firms, weighted by each firm's share of aggregate output;

$$(1-\overline{\tau}) = \int_0^N \frac{P_i y_i}{Y}(1-\tau_i) di.$$

We digress to more precisely explain the counterfactual experiment we are interested in. In Figure 4 the curve labeled 'low γ' represents the relationship between a firm's distortion τ_i and productivity $s_i z_i$ when γ is relatively low. In log scale, the slope of this curve is equal to γ, the elasticity of distortions with respect to productivity. To investigate the impact of more correlated distortions (a higher γ), we increase γ and then compare the two steady-state equilibriums. While an increase in γ implies a pivoting of the curve in Figure 4, we must choose a point to pivot around. Following Hsieh and Klenow (2009), we choose $(1-\overline{\tau})^{-1}$ as the pivot. This means that as γ is increased in our counterfactual, $\overline{\tau}$ is kept constant. This allows us to focus on the effects of an increase in γ while abstracting from the already well-studied effects of a change in the average distortion.[33]

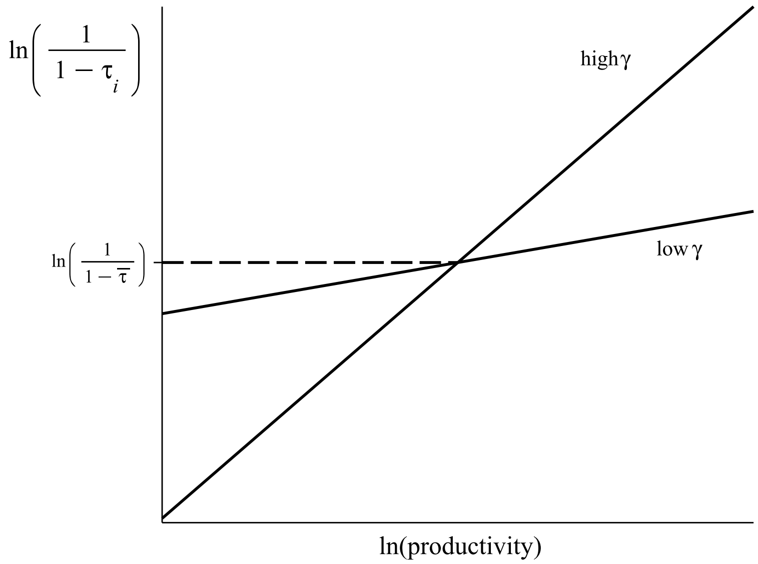

Figure 4: Firm-Level Distortions and Productivity

Notice that aggregate output in equation (4) can be rewritten as;

$$Y = \left[\int_0^N \left(s_i z_i \frac{\overline{MRPL}}{MRPL_i}\right)^{\sigma-1} di\right]^{\frac{1}{\sigma-1}}, \tag{6}$$

where a firm's revenue marginal product of labor and the average revenue marginal product of labor are defined as in Hsieh and Klenow (2009);

$$MRPL_i = \frac{P_i y_i}{\ell_i} \propto \frac{1}{(1-\tau_i)},$$

[33]There are additional reasons to keep $\overline{\tau}$ constant in our counterfactual experiment. First, several of the situations discussed in the literature that contribute to the existence of correlated distortions (for example, subsidies to small or unproductive firms) need not raise the average distortion faced by firms. Second, to the extent that correlated distortions are the result of explicit tax schedules, there does not appear to be a systematic relationship between development and tax revenue as a fraction of GDP (Easterly and Rebelo, 1993). Third, the method developed by Hsieh and Klenow (2009) to infer the distribution of distortions across establishments from micro data (which we use in Section 4.2) can not identify the average distortion faced by establishments.

$$\overline{MRPL} = \left[\frac{1}{N}\int_0^N MRPL_i^{-1} \cdot \frac{P_i y_i}{Y} di\right]^{-1} \propto \frac{1}{(1-\overline{\tau})}.$$

Equation (6) makes clear that if firm-level productivity were exogenous and constant over the life cycle of the firm, removing misallocation by setting each firm's $MRPL_i$ to $\overline{MRPL}$ (bringing γ to zero while maintaining $\overline{\tau}$) would have the same effect on aggregate output as in Hsieh and Klenow (2009), as long as the number of firms N is not affected. To see that N is indeed unaffected if productivity is fixed, we use equations (1) and (2) to derive the expected per-period operating profits of an entrant;[34]

$$\mathbb{E}[\hat{\pi}] = \frac{Y(\sigma-1)^{\sigma-1}}{w^{\sigma-1}\sigma^{\sigma}}\left(\frac{1}{N}\int_0^N (s_i z_i)^{\sigma-1}(1-\tau_i)^{\sigma} di\right) = \frac{Y(1-\overline{\tau})}{\sigma N}. \tag{7}$$

As long as the cost of entry scales up with aggregate output as is the case in our framework and $(1-\overline{\tau})$ is not affected by the removal of misallocation as we assume in our counterfactual experiments, then equation (7) shows that the number of firms is independent of the extent of misallocation when productivity is exogenous and constant.

To determine an incumbent's optimal increase in productivity in any given period, we note that an increase in productivity in the current period affects an incumbent's operating profits and cost of investment in all future periods. Denoting all future increases in productivity by $(1+g')$ and taking into account that $(1-\tau)$ is equal to $(sz)^{-\gamma}$, the value of an incumbent firm is;

$$\begin{aligned} V(s_{-1}, z) = \pi_{-1}(1+g)^{\sigma(1-\gamma)-1} \cdot \Psi - c_g(1+g)^{\phi}\Omega(s_{-1}, z) \\ -c_g(1+g')^{\phi}\Omega(s_{-1}, z) \cdot \frac{(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1+R} \cdot \Psi, \end{aligned} \tag{8}$$

where

$$\Psi \equiv \sum_{t=0}^{\infty}\left[\frac{(1-\lambda)(1+g')^{\sigma(1-\gamma)-1}}{1+R}\right]^t = \frac{1+R}{[1+R-(1-\lambda)(1+g')^{\sigma(1-\gamma)-1}]}.$$

The first term in equation (8) represents the expected present value of current and future oper-

[34] If productivity was assumed to grow at an exogenous rate, then equation (15) shows the number of firms would increase with γ.

ating profits, the second term represents the current cost of investment in productivity growth g, and the third term represents the expected present value of all future costs of investment in productivity growth g'. The subscripts on π and s refer to values from the previous period.

Maximizing equation (8) with respect to $(1+g)$ and taking into account that an incumbent makes the same choice each period (so $g = g'$) results in the following condition after some rearranging;

$$c_g(1+g)^{\phi} = \frac{[\sigma(1-\gamma)-1](1+g)^{\sigma(1-\gamma)-1}(1-\overline{\tau})}{\sigma} \cdot \Theta, \tag{9}$$

where

$$\Theta \equiv \frac{1+R}{\phi(1+R) - [\phi + 1 - \sigma(1-\gamma)](1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}.$$

The value of entry for a potential firm comprises the cost of entry, expected profits upon entry given the choice of entry productivity s_0 and net of the cost of entry investment, and the present discounted value of all future net profits (operational profits net of productivity investment costs). The value of entry can be written as:

$$V_e = -c_e Y + \mathbb{E}[\pi_0|s_0] - c_S Y s_0^{\theta} \tag{10}$$

$$+ \left(\mathbb{E}[\pi_0|s_0](1+g)^{\sigma(1-\gamma)-1} - c_g(1+g)^{\phi}\mathbb{E}[\pi_0|s_0]\frac{\sigma}{1-\overline{\tau}} \right) \left(\frac{1-\lambda}{1+R} \right) \Psi$$

or

$$V_e = -c_e Y - c_S Y s_0^{\theta} + \phi \mathbb{E}[\pi_0|s_0] \cdot \Theta. \tag{11}$$

An entrant chooses its initial productivity s_0 to maximize V_e, resulting in the following condition;

$$c_S Y s_0^{\theta} = \frac{[\sigma(1-\gamma)-1]\phi\mathbb{E}[\pi_0]}{\theta} \cdot \Theta. \tag{12}$$

Free entry guarantees that the value of entry V_e is zero in equilibrium, resulting in the following free-entry condition;

$$c_e Y = \frac{[\theta + 1 - \sigma(1-\gamma)]\phi\mathbb{E}[\pi_0]}{\theta} \cdot \Theta. \tag{13}$$

From equation (1), the expected operating profits of an entrant are equal to;

$$\mathbb{E}[\pi_0] = \frac{Y(\sigma-1)^{\sigma-1} s_0^{\sigma(1-\gamma)-1} \mathbb{E}[z^{\sigma(1-\gamma)-1}]}{w^{\sigma-1}\sigma^{\sigma}}. \tag{14}$$

Using equation (2) for w, this becomes;

$$\mathbb{E}[\pi_0] = \frac{Y(1-\bar{\tau})\left[1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}\right]}{\sigma\lambda N}. \tag{15}$$

Using equations (9), (12), (13), and (15), we can now characterize entrant productivity, per-period firm growth, the number of firms, and aggregate investment in productivity (as a share of output) in a stationary equilibrium;

$$s_0 = \left(\frac{c_e[\sigma(1-\gamma)-1]}{c_S[\theta+1-\sigma(1-\gamma)]}\right)^{\frac{1}{\theta}}, \tag{16}$$

$$(1+g)^{\phi+1-\sigma(1-\gamma)} = \frac{[\sigma(1-\gamma)-1](1-\bar{\tau})}{c_g\sigma} \cdot \Theta, \tag{17}$$

$$N = \frac{[\theta+1-\sigma(1-\gamma)](1-\bar{\tau})\phi[1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}]}{\lambda c_e \sigma\theta} \cdot \Theta, \tag{18}$$

$$\begin{aligned} &\lambda N c_S s_0^{\theta} + (1-\lambda)c_g(1+g)^{\phi} \\ &= \frac{[\sigma(1-\gamma)-1](1-\bar{\tau})}{\sigma} \cdot \Theta \cdot \left(\frac{\phi\xi}{\theta} + (1-\lambda)(1+g)^{\sigma(1-\gamma)-1}\right), \end{aligned} \tag{19}$$

where $\xi = [1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}]$.[35]

[35] We note that since population is constant and normalized to one, the average employment size of firms is simply the inverse of the number of firms.

3.3 Comparative Statics

We are interested in the equilibrium response to changes in the productivity elasticity of distortions γ. Equations (16) through (18) imply that entrant productivity s_0, life-cycle growth g, and average firm size $1/N$ are all decreasing in the productivity elasticity of distortions (γ).[36] To understand the impact on productivity, note that if the productivity elasticity of distortions γ increases, the return to investing in productivity decreases as a given investment results in a larger distortion. As a result, firms enter with lower productivity s_0 and their productivity grows less over their lifecycle g.

To understand the impact of γ on the number of firms N and hence on firm size $1/N$, we note that operating profits of incumbents in equation (7) indicate that, for a given number of firms, lower average productivity induced by lower s_0 and g is exactly offset by a lower wage, that is average operating profits across incumbents does not depend on γ holding N constant. However, lower investment in productivity (both entry and life-cycle investment) induced by a higher γ increases the value of entry which encourages more entry. In addition to the effect on entry through lower investment, a higher productivity elasticity of distortions also has a direct effect on entry. Under the counterfactual we consider, in which we increase γ while keeping the average distortion constant, an increase in γ does not change the average distortion in the economy (by construction) but it reduces the distortions faced by low productivity firms relative to those faced by high productivity firms. In this context, and since entrants are less productive than incumbents in expectation, a higher γ increases the relative operating profits of entrants and thus encourages entry.[37] Both the direct and indirect effects of an increased γ on entry imply a lower average firm size $1/N$.

[36]Appendix C provides proofs of these and other comparative statics.

[37]The positive impact of higher operating profits for entrants on the value of entry generated by increasing γ is somewhat offset by the higher tax that an entrant can expect later in its life cycle. But with positive interest rates, the positive effect of increased operating profits early in the life cycle dominate and the value of entry increases. This higher value of entry encourages entry in equilibrium. Fattal-Jaef (2015) makes this point in the context of a model with exogenous life-cycle growth.

The share of aggregate investment on output in equation (19) may exhibit a non-monotonic relationship with respect to γ because of the differential impact of γ on investment and output. We note however that even if the investment share increases with γ, productivity s_0 and g always fall with increases in γ.

Equations (18) and (19) also indicate that both entry and investment are decreasing in $\overline{\tau}$, and that the number of firms is decreasing in the cost of entry c_e. These are common features of models with free entry, and reinforce the point that many of the policies often emphasized to rationalize low productivity in poor countries would tend to *increase* the average size of firms not reduce it as documented in our newly constructed data set in Section 2.

3.4 Decomposition of Aggregate Output and Firm Size

The impact of correlated distortions γ on aggregate output can be decomposed in our tractable framework into effects working through the entry-investment channel, which is the focus of our paper; the firm life-cycle investment channel, analyzed in Hsieh and Klenow (2014); and the factor misallocation channel, which is the focus of much of the earlier literature on misallocation such as Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).[38] To disentangle the impact of correlated distortions γ on output and firm size working through the various channels, we characterize aggregate output and average firm size separately for economies with fixed productivity and no investment so as to isolate the impact of factor misallocation; exogenous life-cycle growth and no entry investment to assess the additional impact of exogenous firm growth; endogenous life-cycle productivity growth and no entry investment to assess the additional impact of endogenous firm growth; and both endogenous life-cycle productivity growth and entry productivity investment (our baseline model). We then compare aggregate output and firm size in each case. Below we use g_{US} to denote the growth rate of firm productivity in the benchmark economy of our baseline model, while g depends endogenously on γ.

[38] We continue to consider the counterfactual exercise wherein γ is increased while $\overline{\tau}$ is held constant.

We start with the model with just the factor misallocation channel. In this case, firm productivities are fixed and all investment is shut down. As discussed earlier for this case, the number of firms is unaffected by changes in γ (see equation 7). Let $\hat{z}_i$ denote the productivity of a firm i in the stationary distribution of the benchmark economy in the baseline model. Then aggregate output in this case is;

$$Y_{FM} = \Delta \cdot \frac{\left(\frac{1}{N}\int_0^N \hat{z}_i^{(\sigma-1)(1-\gamma)} di\right)^{\frac{\sigma}{\sigma-1}}}{\frac{1}{N}\int_0^N \hat{z}_i^{\sigma(1-\gamma)-1} di}, \tag{20}$$

where FM refers to 'factor misallocation', and Δ is a constant independent of γ. When firm productivities are fixed and exogenous, then equation (20) represents the impact on aggregate output through the factor misallocation channel analyzed by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).[39] To make a more straightforward mapping with the following three versions of the model that feature firm growth, we can write firm productivity as the product of a fixed component and a life-cycle component, that is if the productivity of each firm is growing at an exogenous rate g_{US}, then the productivity of an a-year old firm is $\hat{z}_i = z_i \cdot (1 + g_{US})^a$. Then we can re-write aggregate output in equation (20) as follows;[40]

$$Y_{FM} = \Delta \cdot \left(\frac{\frac{1}{N}\int_0^N z_i^{(\sigma-1)(1-\gamma)} di}{1-(1-\lambda)(1+g_{US})^{(\sigma-1)(1-\gamma)}}\right)^{\frac{\sigma}{\sigma-1}} \frac{1-(1-\lambda)(1+g_{US})^{\sigma(1-\gamma)-1}}{\frac{1}{N}\int_0^N z_i^{\sigma(1-\gamma)-1} di}. \tag{21}$$

We now extend the the above 'factor misallocation' economy to allow for exogenous productivity growth over the life cycle of firms and assess how firm growth affects aggregate output and firm

[39]Note however that whereas Restuccia and Rogerson (2008) assume the same productivity distribution across economies, the gains from reallocation in Hsieh and Klenow (2009) are computed for a given productivity distribution in each country.

[40]We note that because the exit rate is independent of age, the expression for output does not depend on the age distribution of firms. Note also that Δ is not generally constant with firm growth but in this expression for output we keep it constant to the number of firms in the benchmark economy for comparability to equation (20).

size in conjunction with factor misallocation. In this case, aggregate output is given by;[41]

$$Y_{XG} = \frac{Y_{FM}}{\Delta} \cdot N_{XG}^{\frac{1}{\sigma-1}}, \tag{22}$$

and average firm size is equal to;

$$N_{XG}^{-1} \propto (\xi_{US} \cdot \Psi_{US})^{-1} \propto \frac{1 + R - (1-\lambda)(1+g_{US})^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g_{US})^{\sigma(1-\gamma)-1}}. \tag{23}$$

When life-cycle growth is exogenous and firms cannot invest in productivity, then the factor misallocation caused by correlated distortions has the same impact on aggregate output as in equation (21). But this impact is at least somewhat offset by an increase in the number of firms N_{XG} (as discussed in Section 3.3 above).

We now extend the above 'exogenous firm growth' economy to allow for endogenous investment in productivity over a firm's life cycle, as in Hsieh and Klenow (2014). We refer to this economy as 'endogenous firm growth' and in this case aggregate output is;

$$Y_{NG} = \cdot N_{NG}^{\frac{1}{\sigma-1}} \cdot \left(\frac{\frac{1}{N}\int_0^N z_i^{(\sigma-1)(1-\gamma)} di}{1-(1-\lambda)(1+g)^{(\sigma-1)(1-\gamma)}} \right)^{\frac{\sigma}{\sigma-1}} \frac{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{\frac{1}{N}\int_0^N z_i^{\sigma(1-\gamma)-1} di}, \tag{24}$$

and average firm size is;

$$N_{NG}^{-1} \propto (\xi \cdot \Theta)^{-1} \propto \frac{\phi(1+R) - [\phi + 1 - \sigma(1-\gamma)](1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}. \tag{25}$$

Note that the expression for output is identical to equation (22), except that life-cycle productivity g is now endogenous and decreasing in γ. The numerator in the expression for firm size also differs from that in equation (23). Relative to an economy with exogenous life-cycle growth, the lower average productivity induced by higher γ magnifies the impact of γ on aggregate output. But at the same time, the impact of γ on aggregate output is also dampened

[41] We solve explicitly for the expressions of output and firm size in Appendix D.

in three ways. First, by compressing the productivity distribution, a lower g reduces the impact of factor misallocation.[42] Second, lower growth implies that the average productivity of entrants is closer to the average productivity of incumbents. This matters because firm profits (which depend positively on productivity) increase over the life of a firm. For a given level of average profits over the life cycle, a positive discount rate implies that the discounted value of expected lifetime profits is decreasing in g. By lowering g, a higher γ therefore induces more entry and increases the number of firms N_{NG}, which increases aggregate output. Third, the numerator in equation (25) accounts for the fact that a higher γ reduces firm-level investment in productivity without increasing the average tax burden of firms. For a given number of firms this lower investment *increases* the value of entry, thus encouraging entry further increasing the number of firms.[43]

Our 'baseline model' allows for productivity investment upon entry in addition to life cycle productivity investment. With entry productivity investment, aggregate output can now be expressed as;

$$Y_{BM} = Y_{NG} \cdot \left(\frac{N_{BM}}{N_{NG}} \right)^{\frac{1}{\sigma-1}} \cdot s_0, \tag{26}$$

and average firm size is;

$$N_{BM}^{-1} = N_{NG}^{-1} \cdot [\theta + 1 - \sigma(1-\gamma)]^{-1}, \tag{27}$$

where s_0 is decreasing in γ, as discussed in Section 3.3. The drop in entrant productivity from a higher γ reduces aggregate output, magnifying the impact of correlated distortions. As with life-cycle investment, this magnification in aggregate output is somewhat mitigated by an increase in the number of firms implied by equation (27). But in this case, the additional increase in the number of firms is driven solely by lower investment in initial productivity. Lower entrant productivity does not compress the productivity distribution or change the distortion

[42] See Appendix D for a proof of this proposition.

[43] A more standard tax on investment would be an increase in c_S in the cost function for entrant productivity. Equations (16) and (18) show that such a tax would lower average productivity while leaving the number of firms unchanged. This is because the lower investment by firms would be exactly offset by the higher tax burden they face, leaving the value of entry unchanged.

faced by entrants relative to incumbents which are additional mitigating forces in the life-cycle investment channel.[44] This suggests that the entry-investment channel has the potential to account for a more substantial portion of the impact of correlated distortions than the life-cycle investment channel.

In summary, the decomposition above offers three main points. First, accounting for exogenous productivity growth over the life cycle of firms reduces the impact of correlated distortions relative to the static impact of factor misallocation emphasized in the early misallocation literature. Second, allowing for endogenous firm-level productivity growth can increase the impact of correlated distortions, but this increase is offset through various mechanisms involving a compression of the productivity distribution and increases in the number of firms. Third, extending the model to allow for endogenous entrant productivity has the potential to greatly amplify the impact of correlated distortions. In Section 4 we use the above expressions to decompose our quantitative results and quantify the relative importance of each of these channels.

Even though we did not include a separate term for aggregate productivity in our model, it is worth discussing how variation in aggregate productivity affects outcomes as poor countries may be characterized by having policies and institutions other than correlated distortions that may impact aggregate productivity. Imagine that an entrant-i's productivity is equal to $s_0 \hat{z}_i A$, where $z_i = \hat{z}_i A$ and A is common to all firms. The absence of z in equations (16) through (19) implies that cross-country differences in A do not generate differences in investment, firm productivity growth, or the number of firms in our framework. The only variables of interest affected by A are aggregate output and the real wage rate. This is the result of our assumptions that the costs of entry and investment scale up with development. An increase in A increases these costs and the operating profits of firms proportionately, leaving all firm decisions unchanged. If entry and investment costs were constant and independent of development, as in for example Bhattacharya et al. (2013), then differences in A would affect operating profits but not entry

[44] Lower entrant productivity simply shifts the entire productivity distribution, leaving the relative productivities and relative distortions of entrants to incumbents unchanged.

or investment costs, leading to differences in life-cycle growth and entry. In particular, higher aggregate productivity would lead to the counterfactual prediction of more entry and smaller firms, which is inconsistent with our findings in Section 2. Hence, we conclude that the effects of correlated distortions on entrant productivity, life-cycle growth, and factor misallocation emphasized in our model are separate from other policies and institutions that may contribute to lower capital accumulation and aggregate TFP in poor countries.

Note that if greater misallocation was generated simply by more dispersion in random idiosyncratic distortions (uncorrelated with productivity), then equation (17) implies that life-cycle investment and productivity growth would remain unaffected. Similarly, equations (16) and (18) imply that entrant investment and the number of firms would not be affected. This is the case because random distortions affect neither the average distortion $\overline{\tau}$ nor the marginal return to investment. This echoes the finding of Restuccia and Rogerson (2008) that simple random dispersion in idiosyncratic distortions cannot explain much variation in aggregate TFP, and that the strength of correlated distortions (γ in this paper) is what generates the large potential impact from factor misallocation. While the subsequent literature has shown that Restuccia and Rogerson's finding may not hold for all distributions of productivity and distortions, the importance of correlated distortions for the investment channels is unambiguous, in the context of our model, only correlated distortions (not random distortions) reduce the marginal benefit of investing in productivity, thereby reducing productivity and decreasing average employment across all firms.

4 Quantitative Analysis

In this section, we calibrate the model to U.S. data and show the quantitative implications for establishment size, productivity, and aggregate output of hypothetical variations in the degree of correlated distortions across countries. We decompose the total effect on aggregate output that arises through the different channels in the model: entry investment, life-cycle growth, and factor misallocation. We then use establishment-level data to estimate empirically the extent of correlated distortions across countries and their implications for cross-country variations in establishment size, initial productivity, life-cycle growth, and output. We end the section with a discussion of these results for variations in the model setup as well as some robustness checks on parameter values.

4.1 Calibration

We calibrate the model to manufacturing data for the United States in order to quantify the cross-country effects of correlated distortions on average establishment size, productivity, and aggregate output. The effects of distortions working through the investment channel depend on five key parameters in our model:

- θ: the elasticity of entrant investment with respect to initial productivity,
- ϕ: the elasticity of life-cycle investment with respect to life-cycle productivity growth,
- c_g: a level parameter in the cost of life-cycle investment,
- γ: the elasticity of distortions with respect to productivity,
- σ: the elasticity of substitution between varieties.

In order to keep a close tie with the literature for comparison, we follow Hsieh and Klenow (2009, 2014) in setting $\sigma = 3$. For U.S. manufacturing, Hsieh and Klenow (2014) report $\gamma_{US} = 0.09$. To obtain values for ϕ and c_g, the convexity and level parameters from the life-cycle growth investment function, we target an average rate of productivity growth for U.S. firms of 5 percent from Hsieh and Klenow (2014) and an elasticity of output with respect to R&D equal to 0.028

from Hall et al. (2010).[45] In our model, the elasticity of productivity with respect to life-cycle investment is $[\sigma(1-\gamma)-1]/\phi$. Using the values for σ and γ_{US} discussed above, we obtain a value for ϕ equal to 61.8. We then obtain a value for c_g equal to 0.005 from equation (9).

The productivity elasticity of initial investment, θ, plays a prominent role in determining the aggregate share of output invested in productivity, along with ϕ, c_g, the exit rate λ, the real interest rate R, and the average level of distortions $\overline{\tau}$.[46] Given values for λ, R, and $\overline{\tau}$, we choose a value for θ to match the share of value added invested in intangible capital estimated by McGrattan and Prescott (2010), equal to 0.135. We set λ and R equal to 0.1 and 0.05, standard values in the literature. Our value for $\overline{\tau}$ is taken from the World Bank's Doing Business Surveys, which reports an average tax rate of 9 percent.[47] Given each of these values, we use equation (17) to back out a value for θ of 2.53. We note that this value for θ is relatively close to the value of 2.01 estimated using trade data in Rubini (2014).

The effect of distortions working through the factor misallocation channel depends on the parameters above, as well as on the cross-sectional distribution of productivity in our benchmark economy. In the model, given our parsimonious representation of correlated distortions, there is a simple mapping between productivity sz and employment, such that the demand for labor of establishment i relative to that of establishment j is;

$$\frac{\ell_i}{\ell_j} = \left(\frac{s_i z_i}{s_j z_j}\right)^{\sigma(1-\gamma)-1}.$$

Given values for σ and γ_{US}, we therefore use the above mapping to back out a distribution for sz

[45] Hall et al. (2010, Table 2b) survey several studies estimating within-firm R&D elasticities, and report a median elasticity of 0.028 across studies using recent data (post-1990). We do not include estimates from cross-sectional studies as they generally neglect to control for industry.

[46] Note that while entrant productivity s_0 depends on c_S (the scale parameter in the initial-investment cost function), equation (19) shows that the aggregate share of output invested in productivity is independent of c_S. Since all entry and investment costs scale up with aggregate output, any change in s_0 driven by a change in c_S affects these costs and the operating profits of establishments proportionately. As a result, the entire adjustment to a change in c_S will be through s_0.

[47] The World Bank calculates an average tax rate on revenue, makes an assumption about markups, then reports the implied tax rate on profits. We back out our average tax on revenue by reversing these steps.

using data from the U.S. Census Bureau for the employment distribution of U.S. manufacturing establishments. Once we adjust the data to be representative of both paid and unpaid workers, we obtain a distribution of persons engaged per establishment ranging from 1 to 3,000 persons, and back out a distribution for productivity using $\sigma = 3$ and $\gamma_{US} = 0.09$.[48]

Using the above distribution and our calibrated parameter values, we quantify how average establishment size, productivity, investment, and aggregate output per worker (TFP) change with the extent of correlated distortions, i.e., when γ is increased above the U.S. level (keeping other parameters constant). We follow Hsieh and Klenow (2009) in maintaining a constant average distortion $\overline{\tau}$ as γ is increased. We report the results of this exercise in Table 2. The main finding is that the model implies large variations in average establishment size, productivity, and output per worker across economies with different correlated distortions.[49]

Table 2: Model Results across Correlated Distortions γ

γ	Establishment Size	Entrant Productivity	Life-Cycle Growth (%)	Investment Share (%)	Relative Output
0.09 (γ_{US})	22	1	5.0	13.5	1
0.15	12	0.88	4.6	13.3	0.97
0.2	8.4	0.80	4.3	12.7	0.93
0.3	5.3	0.66	3.7	10.8	0.83
0.4	3.8	0.54	3.0	8.3	0.66
0.5 (γ_{India})	3.0	0.42	2.1	5.4	0.47
0.6	2.4	0.28	0.5	2.3	0.26

Notes: Columns report equilibrium values of average establishment size ($1/N$), entrant productivity (s_0), life-cycle productivity growth (g), share of aggregate output invested in productivity, and aggregate output (Y). Results in columns 2 and 5 are reported relative to the benchmark U.S. economy.

For instance, an economy with $\gamma = 0.4$ features an average establishment size that is 17 percent

[48] We transform the employment distribution into a distribution of persons engaged by assuming non-employer establishments employ between 1 and 3 persons, while employer establishments with 1 to 4 employees are assumed to employ 3 to 5 persons. The employment data contain 10 employment ranges, and we assume establishment-level productivity is uniformly distributed within each range. The last range is open-ended (at least 1,000 employees), so we choose an upper bound of 3,000 to match an average employment size of 2,000 employees.

[49] Table 2 reports the impact of increasing γ as high as 0.6. Equation (1) makes it clear that γ must be less than $(\sigma - 1)/\sigma$ to ensure that the after-tax profitability of establishments remains an increasing function of establishment productivity.

of that in the United States. This economy also features an entry-level productivity that is 54 percent of the benchmark due to lower initial investment in productivity. As a result, output per capita is 66 percent of the benchmark economy. These are large differences in size and productivity compared to the findings in the broad literature on misallocation. The γ's in Table 2 are hypothetical, but the range is plausible. As a point of reference, consider that Hsieh and Klenow (2014) report $\gamma = 0.5$ for India using a large micro dataset of manufacturing plants. Given this value for γ, the model predicts an average establishment size of 3 workers, close to the value of 3.1 workers found in the data, and a growth rate of establishment-level productivity of 2.1 percent, also very close to that reported for India by Hsieh and Klenow (2014). The model also predicts India should have an aggregate output of about 47 percent of the U.S. level, generating a factor-difference in output about 40 percent higher than that found from static factor misallocation only between the U.S. and India in Hsieh and Klenow (2009).

It is important to note that the effects of correlated distortions on establishment size and productivity work solely through the entry and life-cycle investment channels, while the impact on aggregate output works also through the factor misallocation channel. In Table 3 we decompose the impact on aggregate output into the effects working through each channel. The numbers in each column are derived from the corresponding expressions for aggregate output in Section 3.4 (equations 21, 22, 24, and 26). The first column reports the impact of correlated distortions on output solely through factor misallocation. The second column shows that the impact of factor misallocation on aggregate output is offset by increased entry with exogenous life-cycle productivity growth. Allowing for endogenous life-cycle productivity growth (column 3) increases the implied impact of distortions (relative to the model with exogenous firm growth), but the net impact on output is still weaker than that calculated for the factor misallocation economy without life-cycle growth in column 1. The impact of γ in the baseline model is substantially larger because of the entry productivity investment channel. In particular, consider the impact on aggregate output of increasing γ from the U.S. level (0.09) to that of India (0.5). Factor misallocation reduces output to 63% of the U.S. economy, while adding exogenous life-cycle growth reduces it to 91% which implies that exogenous firm growth increases output by a factor 1.44 (0.91/0.63), partially compensating the reduction through factor misallocation. Allowing for endogenous growth, whereby firms in India would growth slower than in the United States, reduces output to 70% of the U.S. economy, still implying a net increase in output due to endogenous firm growth by a factor 1.11 (0.70/0.63). Allowing for entry productivity investment (our baseline model) generates a strong reduction in aggregate output to 47% of the U.S. economy. The entry productivity channel reduces output by a factor of 0.67 (0.47/0.70), roughly doubling the contribution of factor misallocation to the reduction in aggregate output in this economy.

Table 3: Decomposition: γ and Aggregate Output

γ	Factor Misallocation	+ Exogenous Life-Cycle Growth	+ Endogenous Life-Cycle Investment	+ Entry Productivity Investment
0.09 (γ_{US})	1	1	1	1
0.15	0.98	1.10	0.99	0.97
0.2	0.94	1.13	0.97	0.93
0.3	0.86	1.13	0.93	0.83
0.4	0.74	1.02	0.82	0.66
0.5 (γ_{India})	0.63	0.91	0.70	0.47
0.6	0.52	0.77	0.54	0.26

Notes: Columns 1 through 4 report the cumulative impact on aggregate output (Y) of distortions (higher γ's) working through the factor misallocation channel, through entry when life-cycle growth is exogenous, through the life-cycle investment channel, and through the entry productivity channel in the baseline model. All results are relative to the benchmark U.S. economy.

Note that the impact of distortions working through the static factor misallocation channel for India is the same as that estimated for India by Hsieh and Klenow (2009), who use comprehensive micro data to back out distributions of distortions and productivity. We interpret this finding as suggestive that our parsimonious representation of idiosyncratic distortions through γ (the elasticity of distortions with respect to productivity) works extremely well as a summary measure of empirical distortions (actual wedges in the data).

Table 4: Decomposition: γ and Establishment Size

γ	Factor Misallocation	+ Exogenous Life-Cycle Growth	+ Endogenous Life-Cycle Investment	+ Entry Productivity Investment
0.09 (γ_{US})	22	22	22	22
0.15	22	17	14	12
0.2	22	15	12	8.4
0.3	22	13	9.4	5.3
0.4	22	11	8.2	3.8
0.5 (γ_{India})	22	11	7.5	3.0
0.6	22	9.9	7.1	2.4

Notes: Columns 1 through 4 report the cumulative impact on average establishment size ($1/N$) of distortions (higher γ's) working through the factor misallocation channel, through entry when life-cycle growth is exogenous, through the life-cycle investment channel, and through the entry productivity channel in the baseline model. All results are relative to the benchmark U.S. economy.

Table 4 reports the contribution of each channel to the total effect of γ on average establishment size. Again taking India as an example, increasing γ from 0.09 to 0.5 results in a substantial drop in average establishment size in a model with exogenous growth in life-cycle productivity but no investment. Allowing for life-cycle investment amplifies this effect moderately, while extending the model to include investment at entry substantially increases the impact of γ on average size. The combined effect of these channels is to decrease establishment size by a factor of 7.

We note that the quantitative impact of correlated distortions on aggregate output via life-cycle growth and factor misallocation is fairly stable across different model configurations. For instance, in our model, Gibrat's law holds in all economies and hence correlated distortions reduce growth for all establishments in the same proportion, whereas in Hsieh and Klenow (2014), Gibrat's law only holds in an economy without distortions. So in their setup, correlated distortions reduce growth for all establishments but more so for high productivity establishments, compressing the productivity distribution. While this compression in the productivity distribution reduces the impact of factor misallocation, it generates less of a positive impact on the number of establishments, leading to similar quantitative effects. To put it differently, our decomposition of the life-cycle growth effect contain two opposing effects on aggregate output (one working through the number of establishments and the other one through reduced factor misallocation) that roughly offset each other in different configurations of life-cycle growth. The implications of these configurations for average establishment size are quite different. Whereas the life-cycle growth in Hsieh and Klenow (2014) implies modest reductions in size via changes in γ (a reduction of about 15 percent in India relative to the United States), in our model establishment size is reduced through the life-cycle channel by a factor of three.

An important takeaway from Tables 3 and 4 is that accounting for the growth of establishments over their life cycle enriches our understanding of establishment dynamics and its interaction with misallocation, but does little to amplify the overall impact of misallocation on aggregate output and TFP. In contrast, accounting for investment decisions at entry approximately doubles the impact of correlated distortions on aggregate output.

4.2 Correlated Distortions

The calibrated model shows how correlated distortions encourage smaller establishments, lower aggregate output, and lower investment in productivity. In this section, we provide systematic evidence that the productivity elasticity of distortions is indeed higher in poor countries. We then provide evidence consistent with the mechanism highlighted in Section 3, using cross-country data to show that both average establishment size and aggregate R&D investment are decreasing in the extent of correlated distortions.

Our measure of correlated distortions is constructed using establishment-level data from the World Bank's Enterprise Surveys. Enterprise Surveys is an ongoing project of the World Bank to collect establishment-level data from mostly low and middle-income countries through face-to-face surveys. The dataset contains standardized information about sales, intermediate purchases, inputs, and a host of other variables for establishments in over 100 countries for at least one year since 2002. In each country, between 150 and more than 1000 establishments have been surveyed, and efforts have been made to make these samples representative of the population of establishments with at least five employees.[50] Importantly for our purposes, manufacturing establishments are classified into fifteen industries. From this dataset, we use observations containing values for industry classification, sales, number of employees, total wage bill, and purchases of materials and intermediate goods, for all countries which are also in our dataset for establishment size described in Section 2.

We back out our measure of establishment-level distortions and productivity for each establishment within a country-industry following Hsieh and Klenow (2009), except that we do not use capital data. Abstracting from capital allows us to increase the number of usable countries substantially, as a large number of establishments in the Surveys do not report capital (more on this below). From Section 3, labor productivity for some establishment i is;

$$\frac{P_i y_i}{\ell_i} = \frac{w}{(1-\tau_i)}\left(\frac{\sigma}{\sigma-1}\right) \propto \frac{1}{(1-\tau_i)},$$

where $P_i y_i$ is an establishment's value added (sales minus intermediate inputs) and ℓ_i is employment.[51] As in Hsieh and Klenow, we remove the constant in the above expression by using labor productivity *relative to* the weighted average of labor productivity across all establishments within the same industry.[52] We infer an establishment's productivity $s_i z_i$ by exploiting

[50] Given the absence of very small establishments in the Enterprise Surveys data, we need to assume (as we do in Section 3) that the elasticity of distortions with respect to productivity is constant.

[51] Following Hsieh and Klenow (2009), we use an establishment's total wage bill (including benefits) in our computations instead of employment in order to control for differences in human capital across establishments.

[52] More precisely, we measure the distortion faced by establsihment i as;

$$\frac{P_i y_i}{\ell_i} \sum_{i'=1}^{N}\left[\left(\frac{P_{i'} y_{i'}}{\ell_{i'}}\right)^{-1} \frac{P_{i'} y_{i'}}{\sum_{i'=1}^{N} P_{i'} y_{i'}}\right].$$

Productivity $s_i z_i$ is similarly measured relative to $\left(\sum_{i'=1}^{N}(s_{i'} z_{i'})^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$.

the following relationship;

$$s_i z_i = \frac{y_i}{\ell_i} \propto \frac{(P_i y_i)^{\frac{\sigma}{\sigma-1}}}{\ell_i}.$$

Following Hsieh and Klenow (2014), we then do a simple OLS regression of logged distortions on logged productivity to obtain each country's productivity elasticity of distortions (γ).[53] Some countries have data for two or even three years, so we average elasticities over all years, weighting by the number of observations in each year. We obtain elasticities for 93 countries, 62 of which are included in the establishment-size data from Section 2.[54] Among these 62 countries, elasticities range from 0.22 to 0.74, averaging 0.52. Among all 93 countries the average elasticity remains a close 0.51. It is reassuring to note that our computed elasticity for India is 0.56, close to the value Hsieh and Klenow (2014) obtain using much more comprehensive micro data. To check the sensitivity of our measures to abstracting from capital, we also calculate elasticities using Hsieh and Klenow's (2009) TFPR and TFPQ as our measures of distortions and productivity. Among the 50 countries which satisfy the criteria above, the average elasticity is 0.56. If we recalculate these elasticities abstracting from capital data (but only using observations that report capital) we find the same average, and the correlation between the two measures is 0.89.[55]

Figures 5 and 6 show how GDP per capita and average establishment size are related to the productivity elasticity of distortions in 63 countries. The elasticity for the U.S. of 0.09 is taken from Hsieh and Klenow (2014).[56] The data show a clear link between the elasticity and both GDP per capita and average size, consistent with the model. Our measure of each country's distortion elasticity is estimated from a regression, so we also have information about the standard error associated with each country's estimate. For robustness we perform a weighted

[53] Hsieh and Klenow (2014) perform this procedure for the U.S., India, and Mexico. Before doing the regressions, we first trim the 1 percent tails of both distortions and productivity for each country to remove outliers. We then recalculate the averages as above.

[54] We do not use countries with fewer than 100 observations. Over the 62 countries with size data, we use a total of 37,410 establishment-level observations in our regressions.

[55] This is consistent with Gal (2013, Table 9), who calculates both labor productivity and TFPR for firms in a handful of OECD countries and reports correlations between the two statistics ranging from 0.8 to 0.9.

[56] The regression coefficients (standard errors) in Figures 5 and 6 are -3.02 (0.80) and -1.95 (0.46).

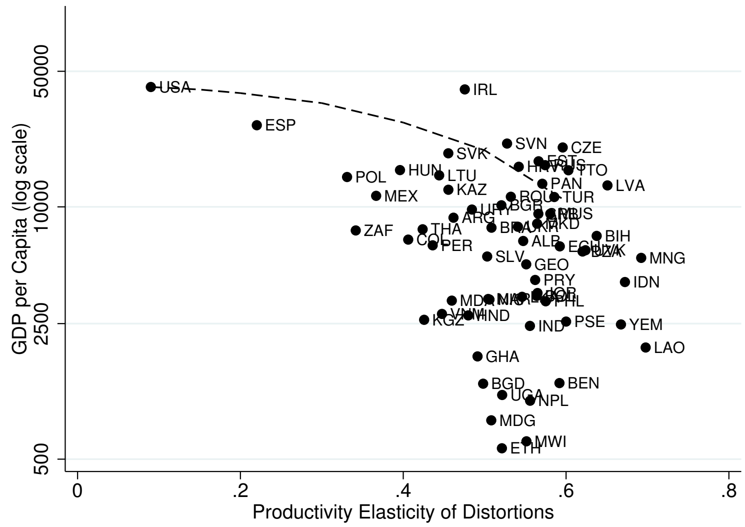

Figure 5: GDP per Capita and Correlated Distortions

least squares regression on the observations in both Figures 5 and 6, weighting each observation by the inverse of its standard error. The resulting coefficients are very similar and remain significant at the one percent level. The dashed lines in Figures 5 and 6 show the analogous relationships predicted by the model. In Figure 5 the model matches the relationship between the elasticity and GDP per capita well, although it does not capture the entire magnitude of the differences in GDP per capita across countries. The dashed line in Figure 6 shows that while the model comes close to predicting India's average establishment size, it predicts average sizes for most countries lower than those reported in the data. As discussed previously, to the extent that output per capita and establishment size are affected by country features other than correlated distortions (such as capital accumulation and entry costs differences) it is reassuring that although correlated distortions are able to account for a large portion of the cross-country patterns, it does not account for all the patterns leaving room for other plausible and relevant explanations.

In the model developed in Section 3, the mechanism through which correlated distortions reduce establishment size is the disincentive to invest in productivity. As a consequence, the model also predicts the share of output invested in productivity should be lower in economies with

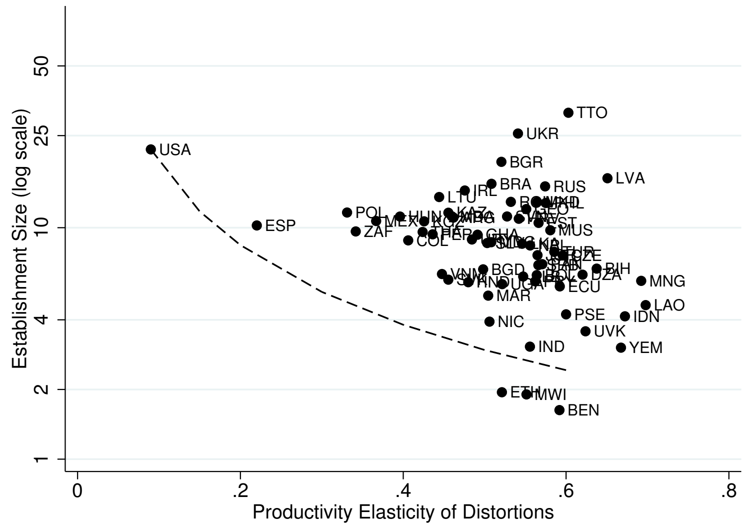

Figure 6: Establishment Size and Correlated Distortions

high γ. Broad measures of investment in intangible capital have not yet been collected for a large number of countries, but R&D intensity should provide a fair proxy. R&D is a significant component of life-cycle investment, and Corrado et al. (2012) show that differences in life-cycle investment (including R&D) across countries are highly correlated with proxies for entrant investment (like early-stage venture capital investment, for example). Figure 7 shows how establishment size across countries varies with R&D intensity, while Figure 8 shows how R&D intensity is related to γ.[57] The relationships illustrated in both figures are clearly consistent with the predictions of the model. Again using India as a point of reference, the model predicts an investment share in India about 40 percent of the U.S. level. In the data, India's R&D intensity is a relatively close 29 percent of the U.S. level. The dashed line in Figure 8 shows that the investment share predicted by the model matches the rest of the data fairly well.

[57]The regression coefficients (standard errors) in Figures 7 and 8 are 0.16 (0.04) and -4.23 (0.97). The coefficient (standard error) from a weighted least squares regression corresponding to Figure 8 (see above) is -3.30 (1.12). R&D data is taken from UNESCO, and is calculated as total investment in R&D as a share of GDP, relative to the U.S.

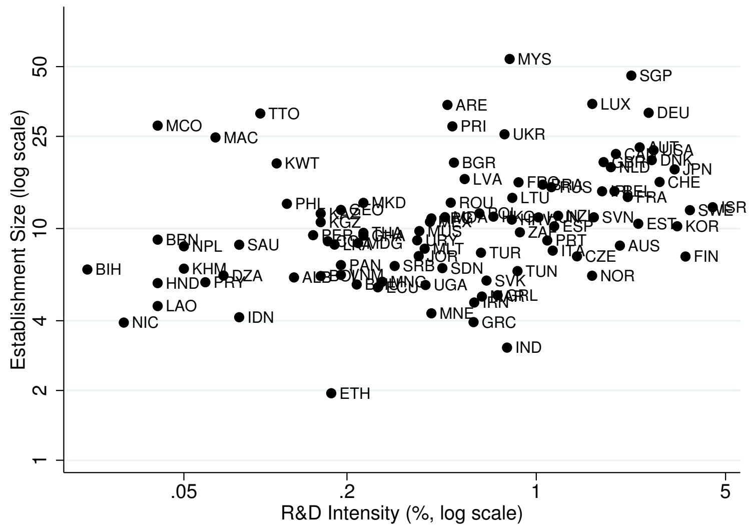

Figure 7: Establishment Size and R&D Intensity

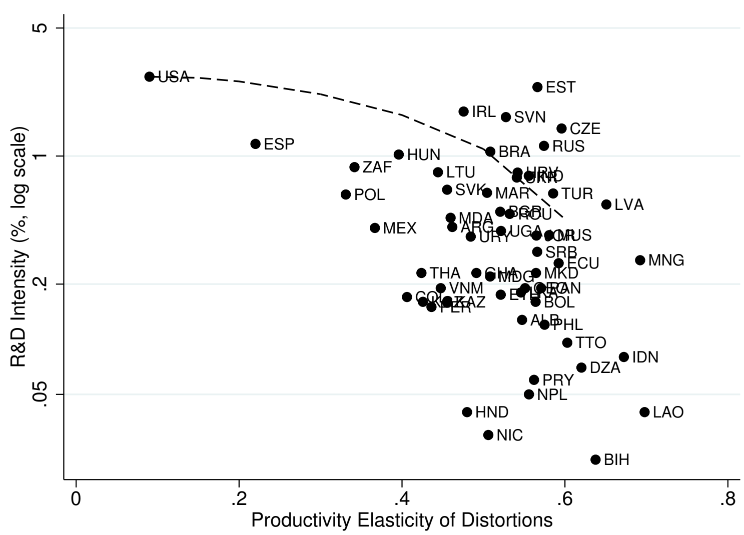

Figure 8: R&D Intensity and Correlated Distortions

4.3 Measurement Error

It is worth considering whether and to what extent measurement error may be driving the relationship between correlated distortions and development reported in Section 4.2. If measurement error causes us to overestimate the elasticity of distortions with respect to productivity, then we might simply be picking up a negative relationship between measurement error and development. There are two reasons why measurement error may be more prevalent in poor countries. First, statistical agencies may be under-funded or otherwise less efficient. This source of error should not be as important in our context, as the Enterprise Surveys data is collected by the World Bank using a common methodology for all included countries. Second, weaker management practices in poor countries may result in less-accurate record keeping by firms, resulting in larger reporting errors (Bloom et al., 2012). We focus on this second source of error by considering how measurement error is related to our measure of γ across the countries in our sample.

Our estimate of a country's elasticity of distortions with respect to productivity (γ) is from a regression of each establishment's (logged) inferred distortion on each establishment's (logged) inferred productivity. We cannot separate measurement error associated with estimated distortions from the true dispersion in idiosyncratic distortions. But our inferred measure of productivity should be a combination of only an establishment's true productivity and measurement error, so we can test whether the variance of (logged) inferred productivity is higher in countries with higher estimated γ's. In our model, the variance of true logged productivity across all establishments is equal to the variance of $\log(z)$, and so is independent of γ. If the variance of inferred productivity is correlated with our estimates of γ, then this can be interpreted as evidence that measurement error may be driving our results.

One complication with this test is that the Enterprise Surveys data we use to estimate γ only includes establishments with at least five workers. This is important because a higher γ in our model decreases the size of establishments. This means that for a given productivity distribution, the threshold productivity above which an establishment will be included in the data should be increasing in γ. For our calibrated distribution of productivity, this leads to a higher variance in logged productivity for included establishments.[58] Looking at the data, we find that the variance of logged productivity does indeed increase with γ. But this increase is roughly in line with what the model predicts when only establishments with at least 5 workers are included.[59] We interpret this result as suggesting that measurement error, which we admit may affect our average measure of γ across all countries, does not seem to be decreasing in development and so does not seem to be driving the relationship between γ and development.

4.4 Discussion

We discuss our main results for reasonable extensions of the model and different values of key parameters.

Model Extensions Extending the model to include capital and capital accumulation does not change our results, as long as we interpret our baseline impact on aggregate output as an impact on TFP. The total impact on aggregate output would be magnified in the usual way through a change in the steady-state capital stock. Extending the model to allow entrants to learn the exogenous portion of their productivity (z) before investing would generate a richer relationship between γ and the productivity distribution across establishments, as the incentive for more productive establishments to invest more than less productive establishments would be dampened. We leave this as an interesting topic for future theoretical and empirical research.

[58] We calculate the implied variance of $\log(z)$ in the model by calculating the productivity at which establishments employ five workers in our benchmark U.S. economy, then calculating how this threshold increases when average establishment size decreases due to γ, and then calculating the variance of $\log(z)$ across establishments above each threshold. Note that these calculations do not take into account how the distribution of productivity changes with life-cycle growth, which may dampen the implied increases in variance.

[59] In fact, the model predicts that the variance in logged productivity increases with γ even faster than what we observe in the data.

Robustness on θ A critical parameter determining the impact of correlated distortions on establishment size and aggregate output is the elasticity of investment in entrant productivity θ. Our baseline calibrated value for θ is 2.53. We explore the cross-country implications of the model for two alternative values for θ: a 20 percent higher value than in the baseline calibration which implies $\theta = 3.03$, and a 20 percent lower value than the baseline which implies $\theta = 2.02$. We report the results of increasing correlated distortions from $\gamma_{US} = 0.09$ to $\gamma_{India} = 0.5$ on establishment size, entrant productivity, life-cycle growth, investment, and aggregate output in Table 5. Note that the calibrated value for ϕ is independent of θ, and so remains the same as in the baseline calibration. As a result, the effect of higher γ's on life-cycle growth does not differ from the baseline case in Table 2. Note also that since we calibrated θ to match an aggregate share of intangible investment to output, the variations in θ amount to changes in the aggregate share of intangible investment in the benchmark economy ranging from 12.7 to 14.7 percent (relative to our calibration target of 13.5 percent). While the quantitative magnitude of changes in γ on the variables of interest depends sensibly on θ, overall the amplification effect of entrant productivity on aggregate output remains quantitatively important.

Table 5: Model Results for Alternative Values of θ

γ	Establishment Size	Entrant Productivity	Life-Cycle Growth (%)	Investment Share	Relative Output
			$\theta = 3.03$		
0.09 (γ_{US})	22	1	5.0	12.7	1
0.5 (γ_{India})	3.9	0.53	2.1	4.8	0.52
			$\theta = 2.02$		
0.09 (γ_{US})	22	1	5.0	14.7	1
0.5 (γ_{India})	1.4	0.24	2.1	6.4	0.38

Notes: Columns report equilibrium values of average establishment size ($1/N$), entrant productivity (s_0), life-cycle productivity growth (g), share of aggregate output invested in productivity, and aggregate output (Y). Results in columns 2 and 5 are reported relative to the benchmark U.S. economy.

Robustness on ϕ The elasticity of life-cycle investment with respect to productivity growth ϕ is an important determinant of both life-cycle growth and establishment size. In Table 6 we show how our variables of interest are affected by correlated distortions when we increase and decrease the calibrated value for ϕ by 20 percent. Note that each alternative value for ϕ implies a different value for θ from our calibration. When $\phi = 74.1$, θ must be lower than our benchmark value in order to generate our target investment share. When $\phi = 49.4$, θ must be higher. As a result, a higher ϕ results in less variation in life-cycle growth, but more variation in entrant productivity. The opposite holds for a lower value of ϕ. The net impact on aggregate output is therefore close to the benchmark case.

Table 6: Model Results for Alternative Values of ϕ

γ	Establishment Size	Entrant Productivity	Life-Cycle Growth (%)	Investment Share	Relative Output
			$\phi = 74.1$		
0.09 (γ_{US})	22	1	5.0	13.5	1
0.5 (γ_{India})	1.9	0.29	2.6	5.9	0.42
			$\phi = 49.4$		
0.09 (γ_{US})	22	1	5.0	13.5	1
0.5 (γ_{India})	4.2	0.60	1.5	4.7	0.53

Notes: Columns report equilibrium values of average establishment size ($1/N$), entrant productivity (s_0), life-cycle productivity growth (g), share of aggregate output invested in productivity, and aggregate output (Y). Results in columns 2 and 5 are reported relative to the benchmark U.S. economy.

5 Conclusion

We have assembled from hundreds of sources a unique dataset of manufacturing establishments to document a strong positive association between average establishment size and GDP per capita. The cross-country income elasticity of establishment size is 0.29 in our sample of 134 countries. We considered an otherwise standard model of heterogenous establishments with endogenous entry and investment in establishment productivity. We showed that, in a reasonably calibrated version of the model, cross country variation in the degree of correlation between establishment distortions and productivity generates substantial differences in establishment size consistent with our reported data and aggregate productivity. We also documented cross-country variation in the degree of correlated distortions from micro data for a large set of countries. Overall, the analysis in this paper puts us closer to understanding the patterns in operational scale and productivity observed across countries.

Our model captures several mechanisms highlighted in the literature through which correlated distortions can affect productivity and establishment size. By keeping the model tractable, we have been able to analytically and quantitatively decompose the effects of correlated distortions into those working through entry, entrant investment, life-cycle investment, and factor misallocation. We found that accounting for life-cycle investment allows us to rationalize the relationship between correlated distortions and lower life-cycle investment in productivity, but does not amplify the effects of misallocation relative to those calculated in a setting without life-cycle growth. In contrast, accounting for entrant investment substantially increases the estimated impact of correlated distortions.

Our analysis has abstracted from many factors which may be worth exploring further. For example, we have abstracted from different forms of entry and operation costs that seem to hinder the operation of establishments in many poor countries. We have also abstracted from policies and institutions that may generate differences in the productivity distribution of entrants across countries. We leave a detailed exploration of these factors for future research.

References

Adamopoulos, Tasso and Diego Restuccia, "The Size Distribution of Farms and International Productivity Differences," *American Economic Review*, 2014, *104* (6), 1667–1697.

Alfaro, Laura, Andrew Charlton, and Fabio Kanczuk, "Plant-Size Distribution and Cross-Country Income Differences," in "NBER International Seminar on Macroeconomics 2008" University of Chicago Press 2009, pp. 243–272.

Atkeson, Andrew and Ariel Tomás Burstein, "Innovation, Firm Dynamics, and International Trade," *Journal of Political Economy*, 2010, *118* (3), 433–484.

Barseghyan, Levon and Riccardo DiCecio, "Entry costs, industry structure, and cross-country income and TFP differences," *Journal of Economic Theory*, 2011, *146* (5), 1828–1851.

Bello, Omar D., Juan S. Blyde, and Diego Restuccia, "Venezuela's growth experience," *Latin American Journal of Economics*, 2011, *48* (2), 199–226.

Bhattacharya, Dhritiman, Nezih Guner, and Gustavo Ventura, "Distortions, Endogenous Managerial Skills and Productivity Differences," *Review of Economic Dynamics*, 2013, *16* (1), 11–25.

Bloom, Nicholas and John Van Reenen, "Why Do Management Practices Differ Across Firms and Countries?," *Journal of Economic Perspectives*, 2010, *24* (1), 203–224.

_ , Aprajit Mahajan, David McKenzie, and John Roberts, "Why Do firms in Developing Countries Have Low Productivity?," *American Economic Review*, 2010, *100* (2), 619–623.

_ , Benn Eifert, Aprajit Mahajan, David McKenzie, and John Roberts, "Does Management Matter? Evidence from India," *Quarterly Journal of Economics*, 2013, *128* (1), 1–51.

_ , Raffaella Sadun, and John Van Reenen, "Management as a Technology?," 2012. Unpublished manuscript, London School of Economics.

Boedo, Hernan J. Moscoso and Toshihiko Mukoyama, "Evaluating the Effects of Entry Regulations and Firing Costs on International Income Differences," *Journal of Economic Growth*, 2012, *17* (2), 143–170.

Bollard, Albert, Peter J Klenow, and Huiyu Li, "Entry Costs Rise with Development," 2014. Unpublished manuscript, Stanford University.

Corrado, Carol, Jonathan Haskel, Cecilia Jona-Lasinio, and Massimiliano Iommi, *Intangible Capital and Growth in Advanced Economies: Measurement Methods and Comparative Results*, IZA, 2012.

Da-Rocha, José-Marıa, Marina Mendes Tavares, and Diego Restuccia, "Inaction, Size Dependent Policies and Aggregate Productivity with Endogenous Establishment-Level Productivity," 2015. Working Paper, University of Toronto.

Desmet, Klaus and Stephen L Parente, "The Evolution of Markets and the Revolution of Industry: A Unified Theory of Growth," *Journal of Economic Growth*, 2012, *17* (3), 205–234.

Duarte, Margarida and Diego Restuccia, "The Role of the Structural Transformation in Aggregate Productivity," *Quarterly Journal of Economics*, 2010, *125* (1), 129–173.

Easterly, William and Sergio Rebelo, "Fiscal Policy and Economic Growth," *Journal of Monetary Economics*, 1993, *32*, 417–458.

Fattal-Jaef, Roberto N., "Entry, Exit and Misallocation Frictions," 2015. Unpublished manuscript, World Bank.

Gabler, A. and M. Poschke, "Experimentation by Firms, Distortions, and Aggregate Productivity," *Review of Economic Dynamics*, 2013, *16* (1), 26–38.

Gal, Peter N, "Measuring Total Factor Productivity at the Firm Level Using OECD-ORBIS," 2013. Working Paper, World Bank.

Guner, N., G. Ventura, and Y. Xu, "Macroeconomic Implications of Size-Dependent Policies," *Review of Economic Dynamics*, 2008, *11* (4), 721–744.

Hall, Bronwyn H., Jacques Mairesse, and Pierre Mohnen, "Measuring the Returns to R&D," *Handbook of the Economics of Innovation*, 2010, *2*, 1033–1082.

Herrendorf, Berthold M., Richard Rogerson, and Ákos Valentinyi, "Growth and Structural Transformation," *Handbook of Economic Growth*, 2014, *2*, 855–941.

Hopenhayn, Hugo A., "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 1992, *60*, 1127–50.

Hopenhayn, Hugo A, "Firms, Misallocation, and Aggregate Productivity: A Review," *Annual Review of Economics*, 2014, *6* (1), 735–770.

Hopenhayn, Hugo A. and Richard Rogerson, "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, 1993, pp. 915–938.

Hsieh, Chang-Tai and Benjamin Olken, "The Missing" Missing Middle"," *Journal of Economic Perspectives*, 2014, *28* (3), 89–108.

_ **and Peter J. Klenow**, "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, November 2009, *124* (4), 1403–1448.

_ **and** _ , "The Life Cycle of Plants in India and Mexico," *Quarterly Journal of Economics*, August 2014, *129* (3), 1035–1084.

Jones, Charles I, "The Facts of Economic Growth," Technical Report, National Bureau of Economic Research 2015.

Jr., Robert E. Lucas, "On the Size Distribution of Business Firms," *Bell Journal of Economics*, 1978, pp. 508–523.

Klenow, Peter J. and Andreas Rodriguez-Clare, "The Neoclassical Revival in Growth Economics: Has it Gone Too Far?," in B. Bernanke and J. Rotemberg, eds., *NBER Macroeconomics Annual*, Cambridge: MIT Press, 1997.

McGrattan, Ellen R. and Edward C. Prescott, "Technology Capital and the US Current Account," *American Economic Review*, 2010, *100*, 1493–1522.

Melitz, Marc J. and Giancarlo I.P. Ottaviano, "Market Size, Trade, and Productivity," *Review of Economic Studies*, 2008, *75* (1), 295–316.

Moreira, Sara, "Firm Dynamics, Persistent Effects of Entry Conditions, and Business Cycles," 2015. Unpublished manuscript, University of Chicago.

Pagés, Carmen, *The Age of Productivity: Transforming Economies from the Bottom Up*, Palgrave Macmillan, 2010.

Peters, Michael, "Heterogeneous mark-ups, growth and endogenous misallocation," Technical Report, London School of Economics and Political Science, LSE Library 2013.

Poschke, Markus, "The Firm Size Distribution Across Countries and Skill-Biased Change in Entrepreneurial Technology," 2014. Discussion Paper, IZA.

Prescott, Edward C., "Needed: A Theory of Total Factor Productivity," *International Economic Review*, 1998, *39*, 525–52.

Ranasinghe, Ashantha, "Impact of Policy Distortions on Firm-Level Innovation, Productivity Dynamics and TFP," *Journal of Economic Dynamics and Control*, 2014, *46*, 114–129.

Restuccia, Diego, "Factor Misallocation and Development," in Online Edition, ed., *The New Palgrave Dictionary of Economics*, Palgrave Macmillan: Steven N. Durlauf and Lawrence E. Blume, 2013.

_ , "The Latin American Development Problem: An Interpretation," *Economía*, 2013, *13* (2), 69–100.

_ **and Richard Rogerson**, "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*, 2008, *11* (4), 707–720.

_ **and** _ , "Misallocation and Productivity," *Review of Economic Dynamics*, 2013, *16* (1), 1–10.

Rubini, Loris, "Innovation and the Trade Elasticity," *Journal of Monetary Economics*, 2014, *66*, 32–46.

Tybout, James R, "Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?," *Journal of Economic Literature*, 2000, *38* (1), 11–44.

Ziebarth, Nicolas L., "Are China and India Backward? Evidence from the 19th Century US Census of Manufactures," *Review of Economic Dynamics*, 2013, *16* (1), 86–99.

On-line Appendix (Not for Publication)

A Establishment Size Data

We describe in more detail how we construct the establishment size data for the manufacturing sector. Our standardized definition of establishment size is the number of persons engaged per establishment. Persons engaged is defined as the average number of persons working for an establishment, both paid and unpaid. A manufacturing establishment is defined as a physical location where the primary activity is manufacturing. Establishments include households who have signs posted on the property indicating commercial activity. Not all countries report persons engaged or the number of establishments, so we also use data on the number of paid employees, the number of full-time equivalent employees, and the number of firms (collections of one or more establishments under common ownership) to impute persons engaged and establishments for these countries. We explain in detail the exact procedure for these imputations but we note that imputations are only involved in about one quarter of our sample of countries.

The source data for each country is from economic censuses, as well as surveys which use comprehensive business registries to create sampling frames and as a result are representative of the population of establishments.[60] We use all publicly available data for the years 2000 through 2012.[61] In an effort to maintain consistency across countries, we do not use data unless efforts were made by a statistical agency to make the data representative of an economy's entire population of manufacturing establishments. We exclude any data collected without accounting for small establishments, except in cases where only establishments without paid employees are excluded. In the later case, we use U.S. data to adjust measured establishment size (this is the case for eight countries). Further, we include data for any country that excludes

[60] For some countries data is from EUROSTAT or OECD's Structural Business Statistics, but we check each country's methodology to confirm the consistency of definitions.

[61] In some cases countries have published only press releases or bulletins describing the census data. We include these countries when the data meets our criteria.

establishments with low revenue, as long as the revenue threshold is lower than the country's GDP per capita (this is the case for four countries). Two countries (Algeria and Honduras) do not report employment, but do report the distribution of establishments across multiple employment tranches. In these two cases we estimate total employment by using an average employment within each tranche consistent with data in comparable countries.[62] We are left with 134 countries with useable data for at least one year, with an average of six years per country.[63] Table 7 reports the total number of countries reporting each variable for at least one year, as well as the total number of poor countries and the total number of rich countries (defined as having GDP per capita below and above the median) doing the same.

Table 7: Sample of Countries

Variable	Total Number of Countries	Number of Poor Countries	Number of Rich Countries
persons engaged	101	54	47
employees	86	34	52
engaged and employees	53	21	32
full-time equivalents	25	2	23
establishments	83	45	38
firms	67	26	41
establishments and firms	16	4	12

Note: 'Poor' and 'Rich' refer to countries with GDP per capita below and above the median. Data from multiple sources, see text for details.

We construct our standardized measure of persons engaged per establishment as follows. First, the total number of persons engaged is reported for 101 countries. For the remaining 33 countries, we impute persons engaged based on each country's reported data for the number of employees and/or the number of full-time equivalent employment. We estimate the relationship between persons engaged and employment from a regression of persons engaged on employees and/or full-time equivalent employment using country-year data for the more than 50 countries

[62] We assume average employment within a tranche to be one third of the distance from the lower to the upper threshold. For the last open-ended tranche (for example, 200 or more employees) we assume an average employment equal to twice the lower threshold.

[63] Although size data is also available for Norfolk Island, it has been dropped for lack of any reliable measure of GDP per capita.

that report all these variables. We then multiply the estimated coefficients by the reported country-year data to obtain persons engaged for those countries. Hence, this first step produces a number for the total persons engaged for each country-year in our sample. Second, we compute persons engaged per establishment (83 countries) and persons engaged per firm (67 countries). This allowed us to estimate the coefficient from a regression of persons engaged per establishment on persons engaged per firm for country-years that report both and then use the estimated coefficient to impute persons engaged per establishment using the data of countries that only report persons engaged per firm.

Each of the above regressions use all country-years which report the relevant variables. The results of the four regressions described above are;

- persons engaged $= 1.44 \cdot$ employees $- 0.40 \cdot$ full-time equivalents
- persons engaged $= 1.07 \cdot$ employees
- persons engaged $= 1.12 \cdot$ full-time equivalents
- persons engaged per establishment $= 0.89 \cdot$ persons engaged per firm

There is a small number of countries for which the data exclude non-employer establishments or that report a combination of manufacturing, extraction, and energy instead of just manufacturing. For these countries we do the following. To adjust persons engaged per establishment in countries which exclude non-employer establishments (this is the case for eight countries), we multiply these values by a factor equal to the average ratio of persons engaged per establishment to persons engaged per establishment with paid employees across all years in the U.S. data (this ratio is 0.51 in U.S. data). We similarly standardize persons engaged per establishment for manufacturing for five countries which report statistics for a combination of manufacturing, extraction, and energy using U.S. data for the ratio of persons engaged per establishment in manufacturing relative to manufacturing, extraction and energy (this ratio is 1.14 in U.S. data).

In our final dataset, the resulting measures of persons engaged per establishment are averaged over all years for each of the 134 countries.

Table 8 lists each country in the final dataset, the number of years for which data is available, and the sources from which data has been collected.

Table 8: List of Countries and Sources

Country	Code	Years	Sources
Âland Islands	ALA	9	Statistics and Research Âland: Statistical Yearbooks of Âland 2006-2010 and 2013, and www.asub.ax
Albania	ALB	8	Instituti i Statistikave: www.instat.gov.al/en/figures/statistical-databases.aspx
Algeria	DZA	1	Office National des Statistiques, Alger: Premier recensement économique -2011- Résultats définitifs
American Samoa	ASM	2	U.S. Census Bureau: 2002, 2007 County Business Patterns, and 2002, 2007 Nonemployer Statistics
Andorra	AND	12	Departament d'Estadística: 2010 Statistical Yearbook, and www.estadistica.ad
Argentina	ARG	1	Instituto Nacional de Estadística y Censos: 2005 Economic Census
Aruba	ABW	1	Central Bureau of Statistics: Business Count 2003
Australia	AUS	5	Australian Bureau of Statistics: Counts of Australian Businesses 2003-2007, Labour Force Surveys (Quarterly)
Austria	AUT	12	Statistik Austria: statcube.at, and OECD's SDBS Structural Business Statistics
Bahrain	BHR	2	Kingdom of Bahrain Central Informatics Organization: Population, Housing, Buildings, Establishments and Agriculture Census
Bangladesh	BGD	1	Bangladesh Bureau of Statistics: Economic Census 2001 & 2003
Belgium	BEL	11	Eurostat, and OECD's SDBS Structural Business Statistics
Benin	BEN	1	Institut National de la Statistique et de l'Analyse Economique: General Census of Companies, and Les Entreprises Artisanales au Benin
Bermuda	BMU	11	Department of Statistics: www.govsubportal.com
Bhutan	BTN	4	National Statistics Bureau: Statistical Yearbooks 2010-2013
Bolivia	BOL	1	Instituto Nacional de Estadística: Structural Statistics of the Manufacturing Industry, Trade and Services - 2010, and Results of the Quarterly Survey of Micro and Small Business 2010
Bosnia and Herzegovina	BIH	8	Institute for Statistics of FB&H: Statistical Yearbooks 2008-2013
Brazil	BRA	13	Brazilian Institute of Geography and Statistics: Cadastro Central de Empresas
Brunei	BRN	1	Department of Economic Planning and Development: Brunei Darussalam Statistical Yearbook 2010
Bulgaria	BGR	12	Eurostat
Cambodia	KHM	2	National Institute of Statistics: Economic Census 2011, and Establishment Listing 2009
Cameroon	CMR	1	Institut National de la Statistique du Cameroun: Recensement Général des Entreprises 2009
Canada	CAN	7	Statistics Canada: CANSIM
Cape Verde	CPV	4	Instituto Nacional de Estatística: Business Census 2007, and Annual Business Surveys 2008-2009
Columbia	COL	1	Departamento Administrativo Nacional de Estadística: Encuesta Annual Manufacturera, and www.dane.gov.co
Croatia	CRV	4	Eurostat

Table 8: List of Countries and Sources

Country	Code	Years	Sources
Cyprus	CYP	12	Eurostat
Czech Republic	CZE	10	Eurostat, and OECD's SDBS Structural Business Statistics
Denmark	DNK	12	Eurostat, and OECD's SDBS Structural Business Statistics
Ecuador	ECU	1	Instituto Nacional Estadística y Censos: National Economic Census 2010
El Salvador	SLV	1	Ministerio de Economica: Tomo I de los VII Censos Económicos Nacionales 2005
Estonia	EST	1	Statistics Estonia: Statistical Yearbooks 2011-2013, and pub.stat.ee
Ethiopia	ETH	1	Central Statistical Agency: Report on Small Scale Manufacturing Industries Survey 2005/6, Report on Large and Medium Scale Manufacturing and Electricity Industries Survey 2005/6, and Labour Force Survey 2005
Faroe Islands	FRO	12	Statistics Faroe Islands: www.hagstova.fo
Finland	FIN	1	Statistics Finland: Labour Force Survey 2013, and www.stat.fi
France	FRA	9	Institut National de la Statistique et des Études Économiques: Tableaux de l'Économie Française - Édition 2005-6, 2010-2014, L'industrie en France - édition 2007, 2008, and www.insee.fr
French Guiana	GUF	1	Institut national de la statistique et des études économiques: Caractéristiques des entreprises et établissements
French Polynesia	PYF	13	Institut de la Statistique de la Polynésie Française: www.ispf.pf
FYR Macedonia	FYR	5	State Statistical Office: www.stat.gov.mk
Georgia	GEO	11	National Statistics Office of Georgia: Statistical Yearbooks 2009-2013, and www.geostat.ge
Germany	DEU	12	Eurostat, and OECD's SDBS Structural Business Statistics
Ghana	GHA	1	Ghana Statistical Service: National Industrial Census 2003
Greece	GRC	6	Eurostat, and OECD's SDBS Structural Business Statistics
Greenland	GRL	5	Statistics Greenland: bank.stat.gl
Guadeloupe	GLP	1	Institut national de la statistique et des études économiques: Caractéristiques des entreprises et établissements
Guam	GUM	7	U.S. Census Bureau: 2008-2011 County Business Patterns, and 2002, 2007, 2012 Economic Census of Island Areas
Honduras	HND	1	Instituto Nacional de Estadística y Censos: Directorio de Establecimientos Económicos
Hong Kong	HKG	13	Census and Statistics Department: Annual Survey of Industrial Production, and www.statistics.gov.hk
Hungary	HUN	11	Eurostat, and OECD's SDBS Structural Business Statistics
India	IND	1	Central Statistics Office: 2005 Economic Census
Indonesia	IDN	3	Statistics Indonesia: Statistical Yearbook 2013
Iran	IRN	1	Statistical Centre of Iran: Statistical Yearbook 1382
Israel	ISR	9	Central Bureau of Statistics: www1.cbs.gov.il, Eurostat, and OECD's SDBS Structural Business Statistics
Italy	ITA	12	Eurostat, and OECD's SDBS Structural Business Statistics
Japan	JPN	3	Statistics Japan: Establishment and Enterprise Censuses 2001, 2004, 2006

Table 8: List of Countries and Sources

Country	Code	Years	Sources
Jordan	JOR	8	Department of Statistics: www.dos.gov.jo
Kazakhstan	KAZ	1	Committee on Statistics: www.stat.gov.kz
Korea	KOR	9	Statistics Korea: Censuses on Establishments 2007, 2009, 2011, 2012
Kosovo	UVK	6	Statistical Agency of Kosovo: Statistical Register of Business
Kuwait	KWT	10	Central Statistical Bureau: Annual Surveys of Establishments 2002-2011
Kyrgyzstan	KGZ	1	National Statistical Committee of Kyrgyz Republic: stat.kg
Laos	LAO	1	Lao Statistics Bureau: Economic Census 2006
Latvia	LVA	10	Central Statistical Bureau of Latvia: www.csb.gov.lv, and Eurostat
Libya	LBY	2	Bureau of Statistics and Census Libya: bsc.ly
Liechtenstein	LIE	6	Statistical Office: Statistical Yearbooks 2007/2008, 2009-2012
Lithuania	LTU	7	Eurostat
Luxembourg	LUX	12	Eurostat
Macau	MAC	13	Statistics and Census Service: www.dsec.gov.mo
Madagascar	MDG	1	Institut National de la Statistique: Rapport de l'enquete sur les Entreprises a Madagascar
Malawi	MWI	6	National Statistical Office: www.nsomalawi.mw
Malaysia	MYS	6	Department of Statistics Malaysia: Statistics Yearbooks 2007-2012
Maldives	MDV	1	Department of National Planning: Economic Survey 2007/2008
Malta	MLT	7	Eurostat
Martinique	MTQ	1	Institut national de la statistique et des études économiques: Caractéristiques des entreprises et établissements
Mauritius	MUS	2	Statistics Mauritius: Censuses of Economic Activity 2002, 2007, Phases I and II
Mexico	MEX	2	Instituto Nacional de Estadstica y Geografía: Censos Economicos 2004, 2009
Moldova	MDA	8	Statistica Moldovei: www.statistica.md
Monaco	MCO	13	Monaco Statistics: Observatoire de l'Economie 2012, 2013
Mongolia	MNG	2	National Statistical Office of Mongolia: Monthly Bulletins of Statistics 2011, 2012
Montenegro	MNE	3	Statistical Office of Montenegro: www.monstat.org
Morocco	MAR	1	Haut-Commissariat au Plan du Maroc: 2001-2 Economic Census
Nepal	NPL	1	Central Bureau of Statistics: Census of Manufacturing Establishments 2006/7, Survey of Small Manufacturing 2008/9
Netherlands	NLD	11	Eurostat, Statistics Netherlands: Statistical Yearbooks 2004-2013
New Caledonia	NCL	13	Institut de la Statistique et des Etudes Economique: www.isee.nc
New Zealand	NZL	13	Statistics New Zealand: www.stats.govt.nz
Nicaragua	NIC	1	Instituto Nacional de Información de Desarrollo: Urban Economic Census
Norfolk Island	NFK	1	Australian Business Statistics: www.ausstats.abs.gov.au
Northern Mariana Islands	MNP	6	U.S. Census Bureau: 2008-2011 County Business Patterns, and 2007, 2012 Economic Census of Island Areas
Norway	NOR	8	Eurostat, and OECD's SDBS Structural Business Statistics

Table 8: List of Countries and Sources

Country	Code	Years	Sources
Palau	PLW	1	Office of Planning and Statistics: 2012 - 2nd, 3rd Quarters Economic Indicators
Palestinian Territories	PSE	7	Palestinian Central Bureau of Statistics: Establishment Censuses 2004, 2007, 2012, and Comparison Study on Industrial Activities 1999-2004
Panama	PAN	1	Instituto Nacional de Estadística y Censo: Preliminary Results of Economic Census 2012
Paraguay	PRY	1	Direccin General de Estadística, Encuestas y Censos: National Economic Census 2011
Peru	PER	1	Instituto Nacional de Estadística e Informática: IV Censo National Economico 2008
Philippines	PHL	2	National Statistics Office: NSO's 2012 List of Establishments, and 2003 Annual Survey of Philippine Business and Industry (ASPBI)
Poland	POL	12	Central Statistical Office of Poland: Statistical Yearbook 2011, 2012, Eurostat, and OECD's SDBS Structural Business Statistics
Portugal	PRT	11	Eurostat, and OECD's SDBS Structural Business Statistics
Puerto Rico	PRI	7	U.S. Census Bureau: 2006-2011 County Business Patterns, and 2002 Economic Census of Island Areas
Qatar	QAT	3	Ministry of Development Planning and Statistics: Establishment Censuses 2004, 2008, 2010
Réunion	REU	3	Institut national de la statistique et des études économiques: Caractéristiques des entreprises et établissements
Romania	ROU	6	National Institute of Statistics: Statistical Yearbooks 2006-2012
Russia	RUS	3	Federal State Statistics Service: Industry of Russia 2008, 2009, 2011, and Small and Medium Businesses in Russia 2008, 2009, 2011
Rwanda	RWA	1	National Institute of Statistics of Rwanda: Establishment Census - 2011
San Marino	SMR	8	Ufficio Informatica, Tecnologia, Dati e Statistica: www.statistica.sm
São Tomé and Príncipe	STP	2	Instituto Nacional de Estatísticas de São Tomé e Príncipe: Business Statistics 2006, 2007
Saudi Arabia	SAU	1	Central Department of Statistics and Information: 2010 Economic Census
Serbia	SRB	3	Republika Srpska Institute of Statistics: Statistical Yearbook of Republika Srpska 2011, 2012, 2013
Sierra Leone	SLE	1	Statistics Sierra Leone: Report of the Census of Business Establishments 2005
Singapore	SGP	10	Department of Statistics Singapore: Census of Manufacturing Activities 2012
Slovak Republic	SVK	2	Eurostat
Slovenia	SVN	12	Eurostat, and OECD's SDBS Structural Business Statistics
South Africa	ZAF	12	Statistics South Africa: Annual Financial Statistics 2010, 2012, and Survey of Employers and the Self-Employed 2013
Spain	ESP	12	Eurostat, and OECD's SDBS Structural Business Statistics

Table 8: List of Countries and Sources

Country	Code	Years	Sources
Sri Lanka	LKA	1	Department of Census and Statistics - Sri Lanka: Census of Industry 2003/4
Sudan	SDN	1	Central Bureau of Statistics: Statistical Year Book for the Year 2009
Sweden	SWE	12	Eurostat, and OECD's SDBS Structural Business Statistics
Switzerland	CHE	3	Swiss Statistics: www.bfs.admin.ch/bfs/portal/en/index.html
Syria	SYR	4	Central Bureau of Statistics: www.cbssyr.sy
Taiwan	TWN	3	National Statistics: Industry, Commerce and Service Censuses 2001, 2006, 2011
Thailand	THA	2	National Statistical Office: Industrial Censuses 2007, 2012
Tonga	TON	7	Tonga Department of Statistics: Manufacturing Output, Employment and Wages/Salaries 2000-2003, 2001-2005, 2002-2006
Trinidad and Tobago	TTO	7	Central Statistical Office: Business Establishments in T & T by Industry Economic Activity 2005-2007
Tunisia	TUN	12	Institut National de la Statistique: Statistiques Issues du Répertoire des Entreprises
Turkey	TUR	8	Eurostat, and OECD's SDBS Structural Business Statistics
Uganda	UGA	2	Uganda Bureau of Statistics: Report on the Census of Business Establishments 2010/2011, and Business Register 2001/02
Ukraine	UKR	3	State Statistics Service of Ukraine: www.ukrstat.gov.ua
United Arab Emirates	ARE	1	National Bureau of Statistics: www.uaestatistics.gov.ae
United Kingdom	GBR	12	Eurostat, and OECD's SDBS Structural Business Statistics
United States	USA	11	U.S. Census Bureau: 2002-2011 County Business Patterns, and 2002-2011 Nonemployer Statistics
Uruguay	URY	9	Instituto Nacional de Estadística: Anuario Estadístico 2000-2012
U.S. Virgin Islands	VIR	2	U.S. Census Bureau: County Business Patterns, and 2002, 2012 Economic Census of Island Areas
Venezuela	VEN	1	Instituto Nacional de Estadística: IV Censo Económico
Vietnam	VNM	3	General Statistics Office: Establishment Censuses 2002, 2007, and 2012
Yemen	YEM	2	Central Statistical Organization: Results of Economic Surveys 2005-2006

B Social Planner

We solve the social planner's problem for our model economy. In each period, the social planner chooses the number of entrants (which we denote by E), entrant productivity s_0, the growth rate of productivity for incumbents g, and labor for each producer (ℓ) to maximize the discounted present value of an infinite stream of consumption (C). Given s_0, g, and the number of firms N, the planner chooses ℓ_i for each producer i in each period to maximize;

$$C = Y \cdot (1 - I) = \left(\int_0^N y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \cdot (1 - I),$$

$$\text{subject to} \quad y_i = s_i z_i \ell_i \text{ and } 1 = \int_0^N \ell_i d_i,$$

where I is the investment rate, or the fraction of aggregate output spent to finance entry, initial investment, and life-cycle investment each period. The optimal quantity of labor for each firm i is;

$$\ell_i = \frac{(s_i z_i)^{\sigma-1}}{\int_0^N (s_j z_j)^{\sigma-1} d_j}.$$

Let hatted variables denote variables chosen in previous or future periods. The planner chooses E, s_0, and g to maximize;

$$Y_0 \cdot \left(1 - E(c_e + c_S s_0^{\theta})\right) - Y_{-1}(1-\lambda)c_g(1+g)^{\phi}$$
$$+ \sum_{t=1}^{\infty} \left[\frac{Y_t}{(1+R)^t} \left(1 - \hat{E}(c_e + c_S \hat{s_0}^{\theta})\right) - \frac{Y_{t-1}}{(1+R)^t}(1-\lambda)c_g(1+\hat{g})^{\phi} \right].$$

A bit of rearranging results in;

$$\begin{aligned} & - Y_{-1}(1-\lambda)c_g(1+g)^{\phi} \\ & + Y_0 \left(1 - E(c_e + c_S s_0^{\theta}) - \frac{(1-\lambda)c_g(1+\hat{g})^{\phi}}{(1+R)}\right) \\ & + \sum_{t=1}^{\infty} \frac{Y_t}{(1+R)^t} \left(1 - \hat{E}(c_e + c_S \hat{s_0}^{\theta}) - \frac{(1-\lambda)c_g(1+\hat{g})^{\phi}}{(1+R)}\right), \end{aligned} \tag{28}$$

$$\text{where} \quad Y_{-1} = \left(\frac{\lambda N_{-1} \overline{z^{\sigma-1}} \hat{s_0}^{\sigma-1}}{1 - (1-\lambda)(1+\hat{g})^{\sigma-1}} \right)^{\frac{1}{\sigma-1}},$$

$$Y_{t\geq 0}^{\sigma-1} = \left(\frac{\lambda N_{-1}\overline{z^{\sigma-1}}\hat{s_0}^{\sigma-1}}{1-(1-\lambda)(1+\hat{g})^{\sigma-1}}\right)(1+g)^{\sigma-1}(1-\lambda)^{t+1}(1+\hat{g})^{t(\sigma-1)}$$

$$+ E\overline{z^{\sigma-1}}s_0^{\sigma-1}(1-\lambda)^t(1+\hat{g})^{t(\sigma-1)}$$

$$+\sum_{T=1}^{t}\hat{E}\overline{z^{\sigma-1}}\hat{s_0}^{\sigma-1}(1-\lambda)^T(1+\hat{g})^{T(\sigma-1)},$$

$$\text{and} \quad \overline{z^{\sigma-1}} \equiv \frac{1}{N}\int_0^N z_i^{\sigma-1}di.$$

Considering the fact that the planner's choices of E, s_0, and g are identical for each period, the first order conditions for this problem are;

$$(E): \quad Y\left(c_e + c_S s_0^\theta\right) = \sum_{t=0}^{\infty}\frac{\partial Y_t/\partial E}{(1+R)^t}\left(1 - c_e E - c_S s_0^\theta E - \frac{(1-\lambda)c_g(1+g)^\phi}{(1+R)}\right) \tag{29}$$

$$(s_0): \quad Y\theta c_S s_0^{\theta-1} = \sum_{t=0}^{\infty}\frac{\partial Y_t/\partial s_0}{(1+R)^t}\left(1 - c_e E - c_S s_0^\theta E - \frac{(1-\lambda)c_g(1+g)^\phi}{(1+R)}\right) \tag{30}$$

$$(g): \quad Y\frac{\phi(1-\lambda)c_g(1+g)^{\phi-1}}{\sigma} = \sum_{t=0}^{\infty}\frac{\partial Y_t/\partial g}{(1+R)^t}\left(1 - c_e E - c_S s_0^\theta E - \frac{(1-\lambda)c_g(1+g)^\phi}{(1+R)}\right) \tag{31}$$

Combined with the condition that $E = \lambda N$ in steady state, the following conditions characterize the planner's optimal allocation;

$$(E): \quad \lambda N\left(c_e + c_S s_0^\theta\right) = \frac{\left[1-(1-\lambda)(1+g)^{\sigma-1}\right]}{\sigma-1}\cdot\Psi\cdot(1-I) \tag{32}$$

$$(s_0): \quad \lambda N c_S s_0^\theta = \frac{\left[1-(1-\lambda)(1+g)^{\sigma-1}\right]}{\theta}\cdot\Psi\cdot(1-I) \tag{33}$$

$$(g): \quad \frac{(1-\lambda)c_g(1+g)^\phi}{\sigma} = \frac{(1-\lambda)(1+g)^{\sigma-1}}{\phi}\cdot\Psi\cdot(1-I) \tag{34}$$

$$\text{where} \quad (1-I) \equiv \left(1 - c_e E - c_S s_0^\theta E - \frac{(1-\lambda)c_g(1+g)^\phi}{(1+R)}\right)$$

$$\text{and} \quad \Psi \equiv \frac{1+R}{1+R-(1-\lambda)(1+g)^{\sigma-1}}.$$

The investment rate I is;

$$I = \frac{(1+R)[\phi - (\phi+1-\sigma)(1-\lambda)(1+g)^{\sigma-1}]}{(1+R)[\sigma\phi - (\phi+1-\sigma)(1-\lambda)(1+g)^{\sigma-1}] - \phi(\sigma-1)(1-\lambda)(1+g)^{\sigma-1}}, \tag{35}$$

which is higher than the equilibrium investment rate in an undistorted economy. The social planner chooses the same entrant productivity s_0 but allocates more resources to establishment entry and life-cycle productivity growth relative to the equilibrium allocation. This wedge between the optimal and equilibrium allocations is common in models with endogenous life-cycle growth when costs are specified in terms of goods rather than labor (e.g., Atkeson and Burstein, 2010).

C Comparative Statics

We show that entrant productivity s_0, life-cycle growth g, and average firm size $1/N$ are all decreasing in γ.

From equation (16), initial productivity is clearly decreasing in γ;

$$\frac{\partial(s_0^\theta)}{\partial\gamma} = -\Delta \cdot \frac{\sigma\theta}{[\theta+1-\sigma(1-\gamma)]^2} < 0,$$

where Δ contains all terms that are independent of γ.

The effect of γ on firm productivity growth g can be analyzed from equation (17). We fully differentiate equation (17) and rearrange to obtain the following expression;

$$\frac{\partial(1+g)}{\partial\gamma} = \frac{-(1+g)\sigma}{\phi+1-\sigma(1-\gamma)} \cdot \left(\phi \ln(1+g)\Psi + \frac{1}{\sigma(1-\gamma)-1}\right).$$

Given $\sigma(1-\gamma) > 1$ and $\phi > \sigma(1-\gamma) - 1$, productivity growth is unambiguously decreasing in γ.

Average firm size from equation (18) is equal to;

$$1/N = \Delta \cdot [\theta + 1 - \sigma(1-\gamma)]^{-1} \cdot \left(\frac{\phi(1+R) - [\phi + 1 - \sigma(1-\gamma)](1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}} \right),$$

or

$$1/N = \Delta \cdot [\theta + 1 - \sigma(1-\gamma)]^{-1}\phi \cdot \left(\frac{1+R-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}} \right)$$

$$+\Delta \cdot \left(\frac{\sigma(1-\gamma)-1}{\theta+1-\sigma(1-\gamma)} \right) \cdot \left(\frac{(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}} \right).$$

Given g is decreasing in γ, average firm size is also decreasing in γ.

D Decomposition

We describe and solve two simplified variants of our model. The first is a model with exogenous productivity growth over a firm's life cycle, as in Fattal-Jaef (2015), with no firm investments in productivity. The second is a model with endogenous productivity growth over a firm's life cycle but without a productivity investment at entry, as in Hsieh and Klenow (2014).

D.1 Exogenous Life-Cycle Growth

From equation (15), the expected operating profits of an entrant are equal to;

$$\mathbb{E}[\pi_0] = \frac{Y(1-\overline{\tau})}{\sigma\lambda N} \cdot \xi_{US},$$

$$\xi_{US} \equiv 1 - (1-\lambda)(1+g_{US})^{\sigma(1-\gamma)-1},$$

where g_{US} is the exogenous growth rate of firm productivity. With no investments in productivity, free entry requires that the present value of expected life-time profits be equal to the cost of entry;

$$c_e = \frac{1-\overline{\tau}}{\sigma\lambda N} \cdot \xi_{US} \cdot \Psi_{US},$$

$$\Psi_{US} \equiv \sum_{t=0}^{\infty} \left(\frac{(1-\lambda)(1+g_{US})^{\sigma(1-\gamma)-1}}{1+R} \right)^t = \frac{1+R}{1+R-(1-\lambda)(1+g_{US})^{\sigma(1-\gamma)-1}}.$$

The above free entry condition can be rearranged to express average firm size $1/N$ as;

$$N^{-1} = \frac{\sigma\lambda c_e}{1-\overline{\tau}} \cdot (\xi_{US} \cdot \Psi_{US})^{-1}.$$

As in Fattal-Jaef (2015) an increase in γ when life-cycle growth is exogenous leads to an increase in N, and therefore a decrease in firm size. Given that aggregate output is increasing in N, this partially offsets the effect of misallocation on output through factor misallocation.

D.2 Endogenous Life-Cycle Growth

We extend the model of exogenous life-cycle productivity growth to allow for investment in productivity in each period after a firm enters (but not at entry). From equation (11), the expected discounted value of life-time operating profits for a firm net of investments in life-cycle productivity is;

$$\mathbb{E}[\pi_0] \cdot \phi \cdot \Theta,$$

$$\Theta \equiv \frac{1+R}{\phi(1+R) - [\phi + 1 - \sigma(1-\gamma)](1-\lambda)(1+g)^{\sigma(1-\gamma)-1}},$$

where $\mathbb{E}[\pi_0]$ is defined as above in Section D.1. With no initial investment in entrant productivity, free entry requires that the above net profits be equal to the cost of entry;

$$c_e = \frac{\phi(1-\overline{\tau})}{\sigma\lambda N} \cdot \xi \cdot \Theta.$$

Average firm size can now be expressed as;

$$N^{-1} = \frac{\sigma\lambda c_e}{\phi(1-\overline{\tau})} \cdot (\xi \cdot \Theta)^{-1}.$$

Equation (9) shows that g is decreasing in γ. To prove that average size $1/N$ is decreasing in γ, we therefore show that average size is decreasing in γ given g, and increasing in g given γ.

$$\frac{\partial(N^{-1})}{\partial\gamma} = \frac{\partial}{\partial\gamma}\left(\Delta\frac{\phi(1+R) - [\phi + 1 - \sigma(1-\gamma) - 1](1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}\right)$$

$$= -\Delta\frac{\sigma(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}}{\left[1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}\right]^2}\cdot\left[\xi + \ln(1+g)\left(\phi(1+R) - [\phi+1-\sigma(1-\gamma)]\right)\right].$$

Given $\xi > 0$, $\ln(1+g) > 0$, and $\gamma < (\sigma-1)/\sigma$, the above derivative is indeed negative. And the following expression shows that average firm size is indeed increasing in g, given γ;

$$\frac{\partial(N^{-1})}{\partial g} = \Delta\frac{[\sigma(1-\gamma)-1](1-\lambda)(1+g)^{\sigma(1-\gamma)-2}}{\xi^2}\cdot\left(\phi(1+R) - [\phi+1-\sigma(1-\gamma)]\right) > 0.$$

We now prove that (as we discuss in Section 3.4) a decrease in life-cycle growth g from an increase in γ dampens the effect of factor misallocation on aggregate output by compressing the productivity distribution. Using equation (21), the percentage decrease in Y through factor misallocation from a small increase in γ is;

$$\frac{\partial Y_{FM}}{\partial\gamma}Y_{FM}^{-1} = \Delta\cdot\frac{-\left[(1+g)^{(\sigma-1)(1-\gamma)} - (1+g)^{\sigma(1-\gamma)-1}\right]}{\left(1-(1-\lambda)(1+g)^{\sigma(1-\gamma)-1}\right)\left(1-(1-\lambda)(1+g)^{(\sigma-1)(1-\gamma)}\right)}\cdot(1-\lambda)\sigma\ln(1+g)$$

where Δ is independent of g. The magnitude of the decrease in Y through factor misallocation from an increase in γ is clearly higher when g is higher. It follows that the lower g induced by γ dampens the impact of factor misallocation.