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Identification of Biased Beliefs in Games of Incomplete Information Using Experimental Data

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Identification of Biased Beliefs in Games of Incomplete Information Using Experimental Data

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Abstract

This paper studies the identification of players' preferences and beliefs in empirical applications of discrete choice games using experimental data. The experiment comprises a set of games with similar features (e.g., two-player coordination games) where each game has different values for the players' monetary payoffs. Each game can be interpreted as an experimental treatment group. The researcher assigns randomly subjects to play these games and observes the outcome of the game as described by the vector of players' actions. Data from this experiment can be described in terms of the empirical distribution of players' actions conditional on the treatment group. The researcher is interested in the nonparametric identification of players' preferences (utility function of money) and players' beliefs about the expected behavior of other players, without imposing restrictions such as unbiased or rational beliefs or a particular functional form for the utility of money. We show that the hypothesis of unbiased/rational beliefs is testable and propose a test of this null hypothesis. We apply our method to two sets of experiments conducted by Goeree and Holt (2001) and Heinemann, Nagel and Ockenfels (2009). Our empirical results suggest that in the matching pennies game, a player is able to correctly predict other player's behavior. In the public good coordination game, our test can reject the null hypothesis of unbiased beliefs when the payoff of the non-cooperative action is relatively low.

Keywords: Testing biased beliefs; Multiple equilibria; Strategic uncertainty; Coordination game.

JEL classifications: C57, C72.

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1 Introduction

In games of incomplete information, players' behavior depends on their preferences and on their beliefs about the uncertain actions of other players. The empirical researcher is interested in the identification of preferences and beliefs using data from the outcome of multiple realizations of the game. In empirical applications of games using field data, researchers commonly assume that players' actions come from an equilibrium outcome. Under this assumption, a player's beliefs about other players' behavior correspond to the true probability distribution of other players' actions given the information available. Therefore, a player's belief is directly identified by the other player's behavior, and the utility function is identified thereafter.[1] In the experimental economics literature, researchers design laboratory experiments and generate experimental data to study behavior in games. From the point of view of identification, there are clear advantages of having data from a controlled experiment. In particular, the design of the experiment determines players' monetary payoffs such that these payoffs are perfectly known to the researcher. Most of the experimental games literature has exploited this advantage using two alternative approaches. A first approach imposes the restriction that the utility function is equal to the monetary payment (plus a mean-zero private information variable, henceforth, linear utility assumption) and then identifies beliefs using choice data. Examples of this approach include Cheung and Friedman (1997) who estimate a belief-learning process in a repeated game and Nyarko and Schotter (2002) who compare beliefs estimated in this way with elicited beliefs. A second approach assumes that players form equilibrium (unbiased) beliefs and identifies the utility function of money using choice data. An example of this approach is Goeree, Holt and Palfrey (2003) who estimate each player's risk preference under the Quantal Response Equilibrium framework (Mckelvey and Palfrey, 1995 and 1998).

This paper proposes an alternative approach to identify preferences and beliefs in discrete games of incomplete information using data from a controlled experiment. Our approach relaxes the assumption of unbiased or equilibrium beliefs, which is commonly imposed in applications using field data, and it does not impose any parametric restriction on the functional form of the utility function nor needs information of elicited beliefs. Relaxing these restrictions is important in different empirical applications. First, there are multiple reasons why players may have biased beliefs. For instance, playing a Bayesian Nash Equilibrium strategy requires player to determine other players' equilibrium strategy and to be able to integrate it over the other player's private information. Such

[1] Two exceptions are Aradillas-Lopez and Tamer (2008) and Aguirregabiria and Magesan (2015) who relax the assumption of equilibrium beliefs.

calculation is burdensome, and human cognition limits may preclude the equilibrium behavior, particularly in one-shot experimental games. Even in the absence of cognition limits, in games with multiple equilibria, players may have uncertainty about which equilibrium strategy will be chosen by other players. A player may believe that the selected equilibrium is A, while other player may think that it is B. This type of strategic uncertainty has been studied by Van Huyk, Battalio, and Beil (1990), Crawford and Haller (1990), Morris and Shin (2002, 2004), and Heinemann, Nagel, and Ockenfels (2009), among others. Second, assuming that a player's utility is equal to the monetary payoff places strong restrictions on subjects' preferences that are at odds with important empirical findings in the experimental literature. Kahneman and Tversky (1979) note that individuals may respond to loss more sensitively than to gains. Such loss aversion is also ruled out by linear utility assumption. Harrison and Rutström (2008) show that risk aversion is prevalent even for the payoff scale typically found in experimental data.[2] Other relevant features of preferences ruled out by the linear utility assumption include social preferences and heterogeneity across players in their marginal utility of money.[3] Our framework treats a player's utility as an unknown unrestricted function of her monetary payoff and is able to capture both risk preference and loss aversion. Third, there is mixed evidence about the ability of some elicitation processes to reveal players' true beliefs. Recent experimental studies have found a significant discrepancy between elicited beliefs and the beliefs inferred from players' actions (see Costa-Gomes and Weizsäcker, 2008, and Rutström and Wilcox, 2009).

To avoid the estimation biases and the misleading results associated with the failure of these assumptions, we treat both utilities and beliefs as unrestricted (nonparametric) functions to be estimated. Our identification results and tests are based on an exclusion restriction that can be easily generated by the researcher in the design of the experiment. Suppose that the same group of individuals must play K different two-player games such that the (monetary) payoff matrices in these K games are the same for the row player, but they vary across games for the column player. This variation across games in the payoff matrix is what we describe as our exclusion restriction in the sense that it does not affect the payoff function of the row player although it can affect the beliefs of this player about the behavior of the column player. Under this exclusion restriction, the variation across the K games in the empirical distribution of the actions of the row player provides information about this player's beliefs in these games. Without further assumptions and following

[2] Harrison and Rutström (2008) also review different methods to elicit each subject's risk preference.

[3] For examples of social preferences such as fairness, see Güth, Schmittberger and Schwartz (1982), Kahneman, Knetsch and Thaler (1986) and Fehr and Schmidt (1999), among others.

an argument and proof similar to Aguirregabiria and Magesan (2015), we show that this exclusion restriction identifies a function of beliefs. This identification result can be used to test different assumptions on beliefs such as (a) unbiased (equilibrium) beliefs, (b) the validity of elicited beliefs, and (c) monotonicity of the beliefs function with respect to monetary payoff of the other player(s).

The complete identification of utility and beliefs functions requires some additional restrictions. These restrictions are weaker than the ones that have been considered in the literature. In a two-player binary choice game, the researcher needs to impose two restrictions on the beliefs or payoffs. We discuss the different form that these restrictions can take and how the choice of these restrictions can be informed by our tests on beliefs. For instance, the researcher may assume that elicited beliefs are valid or that beliefs are unbiased at two of the K games. How to choose these two games is also an important decision for the researcher, and in this paper, we discuss different criteria that can guide the researcher's decision.

There are several ways to address risk preference in the experimental literature. An influential approach, proposed by Roth and Malouf (1979), involves linearizing the utility function by assigning the payoff as the probability of winning a fixed reward. This mechanism has been applied by Ochs (1995) and Feltovich (2000), among others. In contrast, using the experimental data in the paper by Ochs (1995), Goeree, Holt and Palfrey (2003) shows that this mechanism fails to linearize the utility function. The general validity of Roth and Malouf's approach seems unknown in the literature. Another common method consists in eliciting players' risk preference using a lottery choice with a known objective probability distribution. Such a method is used in Heinemann, Nagel, and Ockenfels (2009), among others. Elicitation introduces an additional cost in the implementation of the experiment and, as mentioned above, there may be different sources of bias in the elicitation of preferences and beliefs. The third approach involves estimating a common parametric function for the utility of money, e.g., a CRRA utility function. This is the approach used by Goeree, Holt and Palfrey (2003). As usual with parametric specification, the misspecification of utility function can generate bias in estimates of beliefs such that, for instance, the researcher may spuriously conclude that players' beliefs are biased (not in equilibrium). For these reasons, we consider a nonparametric specification of both preferences and beliefs functions in this paper.

An alternative way to address biased beliefs is to elicit each player's subjective beliefs. This approach was proposed by Nayarko and Schotter (2002) and applied by Costa-Gomes and Weizsäcker (2008) and Palfrey and Wang (2009), among others. Recently, Karni (2009) and Hossain and Okui (2013) have proposed two scoring rules that can correctly elicit players' beliefs regardless of their risk preference. Nevertheless, Rutström and Wilcox (2009) show that the beliefs elicitation before playing the game can seriously affect players' behavior during the game. In contrast, our approach does not require an additional eliciting process, which reduces the cost of the experiment and avoids this endogeneity problem.

We apply our approach to estimate two types of games that have received substantial attention in the experimental economics literature: the matching pennies game in Goeree and Holt (2001) and the coordination game in Heinemann, Nagel, and Ockenfels (2009). In the matching pennies game, our estimation results suggest that a player can correctly predict other players' behavior when the monetary payoff for the other player varies. In coordination games, we find that the Bayesian Nash Equilibrium cannot explain subjects' behaviors for every treatment of the experiment. Specifically, subjects tend to over-predict the coordination probability when coordination difficulty is high and under-predict it when coordination difficulty is low. In addition, our estimated payoff function is convex when the monetary payoff is low and becomes concave as the monetary payoff increases. This finding suggests that the commonly imposed globally concave utility functions, such as CRRA or logarithmic functions, are not able to capture subject's preference, and a non-parametric specification of the payoff function is more appropriate in this application.

The remainder of this paper is organized as follows. Section 2 describes the model and the experimental design that generates the exclusion restriction. Section 3 presents our identification results. Section 4 describes the two experimental data sets that we use in our empirical analysis and presents the estimation procedure and our empirical results. We summarize and conclude in section 5.

2 Model

2.1 Basic model

For the sake of exposition, we present here a model with two players and binary choice. Our identification result can be generalized to games with more than two players and two actions.[4] There are two roles for players in the game: the "row" player (R) and the "column" player (C). We index player roles by $i, j \in \{R, C\}$. Let $a_R \in \{0, 1\}$ and $a_C \in \{0, 1\}$ be the actions and choice sets for the "row" player and for the "column" player, respectively. Players take their actions

[4] In the empirical applications that we present in section 4, the games can be represented as two-player binary choice games. The matching pennies game is clearly a two-player game. The coordination game in Heinemann, Nagel and Ockenfels (2009) is a game with more than two players. We show inb section 3.4 how our identification approach applies also to this class of games.

simultaneously to maximize their respective expected payoffs. The payoff function of player i is:

$$\Pi_i(a_i, a_j) = \pi\left(m_i(a_i, a_j)\right) + \varepsilon_i(a_i) \tag{1}$$

$m_i(a_i, a_j)$ is the monetary payoff of player i when players take actions (a_i, a_j). $\pi(\cdot)$ is a real-valued function that represents the utility of money. The matrix of monetary payoffs and the utility function $\pi(\cdot)$ are common knowledge to all the players. $\varepsilon_i(a_i)$ represents player i's deviation from the average utility, and it is idiosyncratic for each individual player and is private information of the individual; furthermore, it is independently distributed across subjects with a probability distribution that is public information for all the players.[5]

The asymmetric or incomplete information introduced by variables $\varepsilon_i(a_i)$ implies that players have uncertainty about the behavior of the other player. Each player has beliefs about the action that the other player will take. Let B_i represent the subjective belief of player i about the probability that the other player chooses action $a_j = 1$. Given her payoff function and beliefs, each player chooses the action that maximizes her expected payoff. The expected payoff of player i for action a_i is:

$$\Pi_i^e(a_i, B_i) = [1 - B_i]\ \pi\left(m_i(a_i, 0)\right) + B_i\ \pi\left(m_i(a_i, 1)\right) + \varepsilon_i(a_i) \tag{2}$$

Players maximize their expected payoffs. The best response of player i is alternative $a_i = 1$ if

$$\begin{aligned} &[1 - B_i]\,\pi\left(m_i(1, 0)\right) + B_i\ \pi\left(m_i(1, 1)\right) + \varepsilon_i(1) \\ &\geq [1 - B_i]\,\pi\left(m_i(0, 0)\right) + B_i\ \pi\left(m_i(0, 1)\right) + \varepsilon_i(0) \end{aligned} \tag{3}$$

Integrating this best response function over the private information variables, we obtain player i's best response probability function:

$$Q_i\left(m_i, B_i\right) = F_{\widetilde{\varepsilon}}\left(\alpha_\pi(m_i) + \beta_\pi(m_i)\ B_i\right) \tag{4}$$

where $F_{\widetilde{\varepsilon}}$ is the CDF of $\widetilde{\varepsilon}_i \equiv \varepsilon_i(0) - \varepsilon_i(1)$, $\alpha_\pi\left(m_i\right) \equiv \pi\left(m_i(1, 0)\right) - \pi\left(m_i(0, 0)\right)$, and $\beta_\pi(m_i) \equiv [\pi\big(m_i(1, 1)\big) - \pi\big(m_i(0, 1)\big)] - [\pi\big(m_i(1, 0)\big) - \pi\big(m_i(0, 0)\big)]$. The payoff matrix and the utility function are such that $\beta_\pi(m_i) \neq 0$, i.e., the model is a game and not a single-agent decision problem.

This model includes the Bayesian Nash Equilibrium as a particular case.

Definition. *The model is consistent with Bayesian Nash Equilibrium if players' beliefs about other players' actions are equal to these players' best response probabilities: $B_i = Q_j\left(m_j, B_j\right)$ and $B_j = Q_i\left(m_i, B_i\right)$.* ■

[5] In field data, researchers typically observe a vector of state variables that remains fixed for all action profiles such that utility for action profile (a_i, a_j) can be defined as $\Pi_i(x_i, a_i, a_j)$ where x_i is the vector of state variables affecting subject i's payoff. The identification results in this paper can be extended to a utility function $\pi\left(m_i(a_i, a_j), x_i\right)$ as long as the experimental design provides randomized variation in the vector of state variables (x_R, x_C).

This framework is related to the *Quantal Response Equilibrium (QRE)* proposed by Mckelvey and Palfrey (1995, 1998). In particular, the Bayesian Nash equilibrium can be interpreted as a QRE.

QRE interprets $\varepsilon_i(a_i)$ as player i's decision error. If the player's behavior departs from the Nash equilibrium, the QRE interprets it as the player making a mistake when calculating her expected payoff, although the player can still form the correct belief about the opponent's action. Our framework relaxes two assumptions with respect the existing empirical applications of QRE models. First, subjects are not restricted to having the correct belief when they play the game. They may have biased beliefs, and the bias can be heterogeneous across subjects. Second, instead of assuming that the researcher knows the payoff function π and that it is equal to the monetary payoff, we treat it as an unknown to be estimated from the data.[6]

2.2 Experimental design and subject heterogeneity

Given the game described above, the experimental researcher chooses T different matrices of monetary payoffs that are indexed by $t \in \{1, 2, ..., T\}$. Let $\mathbf{m}_t = (\mathbf{m}_{Rt}, \mathbf{m}_{Ct})$ be the $t - th$ matrix of monetary payoffs, where $\mathbf{m}_{Rt}$ and $\mathbf{m}_{Ct}$ represent the matrices of payoffs for the row and the column player, respectively. There is a sample of N subjects indexed by $n \in \{1, 2, ..., N\}$. Subjects are randomly assigned to $2T$ possible treatments. A treatment in this experiment is defined as a pair (i, t), where t is the index of the payoff matrix in the game the subject has to play, and $i \in \{R, C\}$ represents the player role of the subject in that game (i.e., either row or column player). The random allocation of players to treatments is anonymous such that each subject does not have any information about who is the other subject he is playing against. Once subjects have been allocated to treatments, they play their respective games. We use the categorical variable $d_n \in \{R, C\} \times \{1, 2, ..., T\}$ to represent the treatment (i, t) received by subject n, and the binary variable $a_n \in \{0, 1\}$ is used to represent the subject's actual choice in the game. Therefore, the data from this randomized experiment can be described in terms of the observations $\{d_n, a_n : n = 1, 2, ..., N\}$.

Subjects can be heterogeneous in preferences and beliefs. Variables $\varepsilon_{nit}(0)$ and $\varepsilon_{nit}(1)$ represent the idiosyncratic components of the payoff function for subject n if he is assigned to treatment (i, t). Similarly, the probability B_{nit} represents the subjective belief of subject n when assigned to treatment (i, t). We make the following assumption of additive separability and independence on

[6] See Mckelvey and Palfrey (1995), Mckelvey, Palfrey and Weber (2000) and Goeree, Holt and Palfrey (2003) among others. Goeree, Holt and Palfrey (2003) relax the assumption that the utility function π is equal to the monetary payoff and estimate a parametric model for this function. In this paper, we do not impose any functional form for the utility, other than being an increasing function.

subjects' heterogeneity in preferences and beliefs.

Assumption 1. *(A) All the heterogeneity in preferences across subjects is captured by the private information variables $\varepsilon_{nit}(0)$ and $\varepsilon_{nit}(1)$ that have zero mean, are independently distributed across (n,i,t), and independent of $\mathbf{m}_t$. (B) Subject n's beliefs in treatment (i,t) are $B_{nit} = \overline{B}_{it} + \xi_{nit}$, where $\overline{B}_{it}$ represents the mean beliefs across all subjects conditional on treatment (i,t), and ξ_{nit} is subject n idiosyncratic component in beliefs that is private information of this subject, has zero mean, it is independently distributed across (n,i,t), and independent of $\mathbf{m}_t$, $\overline{B}_{it}$, and $\overline{B}_{jt}$.* ■

Assumption 2. *Define the random variable $\omega_{nit} \equiv \varepsilon_{nit}(0) - \varepsilon_{nit}(1) - \xi_{nit}\ \beta_\pi(m_{it})$, where $\beta_\pi(m_{it})$ has been defined above. Conditional on m_{it}, the random variable ω_{nit} has a probability distribution $F_{\omega|m_{it}}$ that is known to the researcher and it is strictly monotonic in $\mathbb{R}$.* ■

Consider subject n that has been assigned to treatment t as a player of type i. The best response probability of this subject is $Q_{nit} \equiv Q_i(m_{it}, B_{nit})$, and using the definition in equation (4) this best response probability is equal to $F_{\widetilde{\varepsilon}}(\alpha_\pi(m_{it}) + \beta_\pi(m_{it})\ B_{nit})$. This best response probability depends on the idiosyncratic beliefs of subject n, B_{nit}. Under Assumptions 1-2, we can integrate this best response probability function over the idiosyncratic component of beliefs, ξ_{nit}. We obtain the *Conditional Choice Probability (CCP)* function:

$$P_{it} \equiv P_i\left(m_{it}, \overline{B}_{it}\right) = F_{\omega|m_{it}}\left(\alpha_\pi(m_{it}) + \beta_\pi(m_{it})\ \overline{B}_{it}\right) \tag{5}$$

There is a substantial empirical literature in behavioral and experimental economics that studies players' non-equilibrium behavior and heterogeneous beliefs. Level-k models by Nagel (1995) and Stahl and Wilson (1994, 1995) and the cognitive hierarchy model by Camerer, Ho, and Chong (2004) are some important contributions in this literature.[7] Our model relaxes some restrictions in these previous studies. We do not impose BNE, QRE, or level-k rationalizability, and ξ_{nit} captures heterogeneity in beliefs across player roles and across subjects in the same role. We also consider a nonparametric specification of the utility of money. In addition, our framework allows that the standard deviation of error term ω_{nit} depends on the monetary payoff matrix m_{it}.

Assumption 2 imposes the restriction that the researcher knows the distribution of the unobservable variable ω_{nit}. In section 3, we relax Assumption 2 and show that this distribution can be nonparametrically identified if the experimental design includes a *special regressor*.[8] Suppose that

[7] For a survey of papers in this field, see Crawford, Costa-Gomes and Iriberri (2013).

[8] See Matzkin (1992) and Lewbel (2000) for the nonparametric identification of the distribution of the unobservable variables in single-agent discrete decision models using special regressors. Lewbel and Tang (2015) have used this idea for the estimation of games of incomplete information under equilibrium restrictions.

the payoff function has the following structure:

$$\Pi_{nit}(a_{nit}, a_j) = a_{nit}\ z_{nit} + \pi\left(m_{it}(a_{nit}, a_j)\right) + \varepsilon_{nit}(a_{nit}) \tag{6}$$

where z_{nit} is a non-monetary payoff (e.g., extra points in the grade of student n) and it is private information of subject n. That is, the experimental design includes a payment z_{nit} that subject n obtains if he chooses alternative 1 regardless the action of the other player. The payment z_{nit} is not subject to any strategic interaction with the other player.[9] Furthermore, the experimental design is such that z_{nit} is independent of the unobservable ω_{nit} and it is independently and identically distributed over subjects and treatments with a distribution F_z that has continuous support over a compact set $\mathcal{Z} \in \mathbb{R}$. Payments z_{Rnt} and z_{Cnt} are random draws from the distribution F_z.[10] In this model, the Conditional Choice Probability for player-treatment (i, t) is:

$$P_i\left(z_{it}, m_{it}, \overline{B}_{it}\right) = F_{\omega|m_{it}}\left(z_{it} + \alpha_\pi\left(m_{it}\right) + \beta_\pi\left(m_{it}\right)\ \overline{B}_{it}\ \right) \tag{7}$$

In section 3, we also present identification results for this model.

3 Identification

The dataset consists of N observations $\{d_n, a_n\}$, one for each subject, where d_n represents the treatment received by subject n, and a_n is his action in the game. Each subject n is randomly assigned to one of the $2T$ treatments such that d_n is independent of the unobservables in ω_n. Let π_i be the vector of payoff parameters for player i in the experiment, $\pi_i \equiv \{\pi(m_{it}(a_R, a_C)) : (a_R, a_C) \in \{0,1\}^2$ and $t = 1, 2, ..., T\}$. Similarly, let $\overline{\mathbf{B}}_i \equiv \{\overline{B}_{it} : t = 1, 2, ..., T\}$ be the vector of average belief parameters for player i in the experiment. The researcher is interested in using this experimental data to estimate preferences and beliefs parameters π_R, π_C, $\overline{\mathbf{B}}_R$, and $\overline{\mathbf{B}}_C$.

Let $\mathcal{M}_T \equiv \{\mathbf{m}_t = (\mathbf{m}_{Rt}, \mathbf{m}_{Ct}) : t = 1, 2, ..., T\}$ be the set of payoff matrices in the T treatments of the randomized experiment. Assumption 3 establishes a condition on the set $\mathcal{M}_T$ that plays a fundamental role in our identification results.

Assumption 3. *The set $\mathcal{M}_T$ of payoff matrices in the randomized experiment is such that there are at least two treatments, say t_1 and t_2, such that: (A) player i has the same payoffs in the two*

[9] A key difference between the two payments is that m_{it} is subject to strategic uncertainty (i.e., it depends on the unknown action of the other player) while z_{it} is not. Based on this difference, the specification of the payoff function with the special regressor assumes that payment z_{it} is valued under risk neutrality (linear and additive) while payment m_{it} may be subject to risk aversion/risk loving considerations under the utility function π.

[10] For notational simplicity, here we assume that the distribution of z_{nit} is the same for any treatment (i, t). However, all the results trivially extend to an experimental design where the distribution of z_{nit} varies across treatments (i, t), as long as the random draws of z_{nit} are independent across (n, i, t).

treatments but the payoffs of player $j \neq i$ are different, i.e., $\mathbf{m}_{it_1} = \mathbf{m}_{it_2}$ and $\mathbf{m}_{jt_1} \neq \mathbf{m}_{jt_2}$; (B) player i's conditional choice probabilities vary across the two treatments. ■

Assumption 3(A) establishes that the experimental design generates a particular variation in monetary payoffs across treatments: the payoff matrix of player j varies while the payoff matrix of player i remains constant. We show below that this condition provides an *exclusion restriction* that can be used to identify player i's beliefs from this player's observed behavior. Assumption 3(B) is a "Relevance condition" that is necessary for identification. Since conditional choice probabilities are nonparametrically identified, Assumption 3(B) is testable from the data. Assumption 3(B) can be also interpreted as a *Rationalizability assumption*, i.e., player i's knows that player j maximizes expected payoff given beliefs. Since player j's payoff matrix varies across treatments t_1 and t_2, player i's beliefs about player j's behavior also varies between the two treatments, and given that his own payoff matrix did not change, his actual behavior should be different as long as his behavior depends on beliefs.

We show below that under Assumptions 1 to 3 we can test for the null hypothesis of unbiased (equilibrium) beliefs without parametric assumption on the utility function and average beliefs. Then, we present additional conditions for the full nonparametric identification of the model. Finally, we relax Assumption 2 and present identification results for the model where the distribution function $F_{\omega|m_{it}}$ is unknown to the researcher and nonparametrically specified and the experiment includes a randomized special regressor z.

3.1 Tests of unbiased beliefs

Under the conditions in Assumption 1 the choice probabilities P_{it} are identified for every player-role and treatment (i, t). In particular, the probability P_{it} is equal to $\mathbb{E}[a_n \mid d_n = (i, t)]$ and we can estimate P_{it} consistently using the frequency estimator:

$$\widehat{P}_{it} = \frac{\sum_{n=1}^{N} a_n \ 1\{d_n = (i,t)\}}{\sum_{n=1}^{N} 1\{d_n = (i,t)\}} \tag{8}$$

where $1\{.\}$ is the indicator function. For the identification results in this section, we treat the choice probabilities P_{it} as known. Let $F_{\omega|m_{it}}^{-1}(.)$ be the inverse function of the CDF of $F_{\omega|m_{it}}$. This inverse function exits because the strict monotonicity of the CDF. Under Assumption 2, the inverse function $F_{\omega|m_{it}}^{-1}(P_{it})$ is identified for every treatment (i, t). For notational simplicity, we use the variable S_{it} to represent $F_{\omega|m_{it}}^{-1}(P_{it})$. The model implies that:

$$S_{it} = \alpha_{\pi}\left(m_{it}\right) + \beta_{\pi}\left(m_{it}\right) \ \overline{B}_{it} \tag{9}$$

Let t_1 and t_2 be the two treatments in Assumption 3. Let $\mathcal{T}_{i,t_1}$ be the subset of treatments in the experiment where player i has the same monetary payoffs as in treatment t_1 i.e., $\mathcal{T}_{i,t_1} \equiv \{t : \mathbf{m}_{it} = \mathbf{m}_{it_1}\}$. For any treatment $t \in \mathcal{T}_{i,t_1}$, we have that

$$S_{it} - S_{it_1} \quad = \quad \beta_\pi \left(m_{it_1} \right) \ \left[\overline{B}_{it} - \overline{B}_{it_1} \right] \tag{10}$$

Assumption 3(B) and the strict monotonicity of the CDF $F_{\omega|m_{it}}$ imply that $S_{it_2} - S_{it_1} \neq 0$. Therefore, given that $\beta_\pi(m_{it}) \neq 0$, equation (10) implies that $\overline{B}_{it_2} - \overline{B}_{it_1} \neq 0$. Taking this into account, we have that for any treatment $t \in \mathcal{T}_{i,t_1}$,

$$\frac{S_{it} - S_{it_1}}{S_{it_2} - S_{it_1}} \quad = \quad \frac{\overline{B}_{it} - \overline{B}_{it_1}}{\overline{B}_{it_2} - \overline{B}_{it_1}} \tag{11}$$

This expression shows that, under assumptions 1-3, the observed behavior of subjects playing type-i identifies the beliefs ratio $(\overline{B}_{it} - \overline{B}_{it_1})/(\overline{B}_{it_2} - \overline{B}_{it_1})$ for any treatment $t \in \mathcal{T}_{i,t_1}$. That is, observed behavior can identify an object that depends only on beliefs and not on preferences. This result implies that the assumption of unbiased or equilibrium (average) beliefs is testable.

Under the restriction of equilibrium beliefs, the ratio $(\overline{B}_{it} - \overline{B}_{it_1})/(\overline{B}_{it_2} - \overline{B}_{it_1})$ should be equal to the ratio of the choice probabilities of the other player (subject playing type-j), i.e., $(P_{jt} - P_{jt_1})/(P_{jt_2} - P_{jt_1})$. This provides a testable restriction.

Proposition 1. *Under Assumptions 1 to 3, for any treatment $t \in \mathcal{T}_{i,t_1}$, the hypothesis of equilibrium (unbiased) beliefs implies the restriction:*

$$\frac{S_{it} - S_{it_1}}{S_{it_2} - S_{it_1}} \quad = \quad \frac{P_{jt} - P_{jt_1}}{P_{jt_2} - P_{jt_1}} \tag{12}$$

with $S_{it} \equiv F^{-1}_{\omega|m_{it}}(P_{it})$. Given that the choice probabilities P_{it} and P_{jt} are identified, this restriction is testable when the number of treatments in the set $\mathcal{T}_{i,t_1}$ is at least three. ■

For experiments where the payoff matrix of player i has a particular structure, it is possible to construct a test of unbiased beliefs that requires only two treatments in the set $\mathcal{T}_{i,t_1}$. Suppose that the matrix of monetary payoffs of player i is symmetric and diagonal-constant (Toeplitz matrix) such that $m_i(0,0) = m_i(1,1)$ and $m_i(0,1) = m_i(1,0)$. For instance, this is form of the payoff matrix in a matching pennies game. Under this condition, we have that $\beta_\pi \left(m_i \right) = -2 \ \alpha_\pi \left(m_i \right)$ and equation (9) becomes $S_{it} = \alpha_\pi \left(m_{it} \right) \left[1 - 2 \ \overline{B}_{it} \right]$. Therefore, under Assumption 3, for treatments t_1 and t_2 we have that

$$\frac{S_{it_2}}{S_{it_1}} \quad = \quad \frac{1 - 2 \ \overline{B}_{it_2}}{1 - 2 \ \overline{B}_{it_1}} \tag{13}$$

This condition provides a different test for the null hypothesis of unbiased beliefs.

Proposition 2. *Under Assumptions 1 to 3 and the condition that the matrix of monetary payoffs of player i is symmetric and diagonal-constant, the hypothesis of equilibrium (unbiased) beliefs implies the testable restriction:*

$$\frac{S_{it_2}}{S_{it_1}} = \frac{1 - 2\ P_{jt_2}}{1 - 2\ P_{jt_1}} . \qquad \blacksquare \tag{14}$$

3.2 Identification of utility and beliefs

We now consider the identification of utility parameters $\alpha_\pi(m_{it})$ and $\beta_\pi(m_{it})$ and belief parameters $\overline{B}_{it}$ for any treatment t in the set of treatments $\mathcal{T}_{i,t_1}$. Later we discuss the identification of the utility function from the functions $\alpha_\pi(m_{it})$ and $\beta_\pi(m_{it})$.

Equations (9) and (11) imply that, for any treatment $t \in \mathcal{T}_{i,t_1}$, preferences and beliefs of player i are identified up to two constants. To see this, define the constant parameters μ and λ as $\mu \equiv \overline{B}_{it_1}$ and $\lambda \equiv \overline{B}_{it_2} - \overline{B}_{it_1}$. And for any treatment $t \in \mathcal{T}_{i,t_1}$, define the ratio $R_{it} \equiv (S_{it} - S_{it_1})/(S_{it_2} - S_{it_1})$ that is identified from the data. Note that by definition $R_{it_1} = 0$ and $R_{it_2} = 1$. Then, we can write equation (11), that describes the model restrictions on beliefs, as:

$$\overline{B}_{it} = \mu + \lambda\ R_{it} \tag{15}$$

Similarly, for any treatment $t \in \mathcal{T}_{i,t_1}$ we can write equation (9) as:

$$S_{it} = \alpha_\pi(m_{it_1}) + \beta_\pi(m_{it_1})\ \ [\mu + \lambda\ R_{it}] \tag{16}$$

Operating in this equation we can obtain the following expressions for the preference parameters in terms of identified objects and the unknown constants μ and λ. For any $t \in \mathcal{T}_{i,t_1}$,

$$\beta_\pi(m_{it}) = \beta_\pi(m_{it_1}) = \frac{1}{\lambda}\ (S_{it_2} - S_{it_1}) \tag{17}$$

$$\alpha_\pi(m_{it}) = \alpha_\pi(m_{it_1}) = S_{it_1} - \frac{\mu}{\lambda}\ (S_{it_2} - S_{it_1}) \tag{18}$$

Equations (15), (17) and (18) show that the vector of parameters $\theta_{t_1} \equiv \{\overline{B}_{it}, \alpha_\pi(m_{it}), \beta_\pi(m_{it}) : t \in \mathcal{T}_{i,t_1}\}$ is identified up to the two constants μ and λ.

The model implies an additional restriction on the sign of $\alpha_\pi(m_{it_1})$. Remember that $\alpha_\pi(m_i) \equiv \pi(m_i(1,0)) - \pi(m_i(0,0))$. Since the utility of money is an increasing function, we have that the sign of $\alpha_\pi(m_{it_1})$ is equal to the sign of the money difference $m_{it_1}(1,0) - m_{it_1}(0,0)$, such that:

$$sign\left\{S_{it_1} - \frac{\mu}{\lambda}\ (S_{it_2} - S_{it_1})\right\} = sign\{\ m_{it_1}(1,0) - m_{it_1}(0,0)\ \} \tag{19}$$

Suppose that $m_{it_1}(1,0) - m_{it_1}(0,0) \geq 0$ and that $S_{it_2} - S_{it_1} > 0$. This is without loss of generality because we can always label the two choice alternatives such that $m_{it_1}(1,0) - m_{it_1}(0,0) \geq 0$, and we can label treatments t_1 and t_2 such that $S_{it_2} - S_{it_1} > 0$. The sign restriction in (19) implies the following inequality constraint for μ/λ:

$$\frac{\overline{B}_{it_1}}{\overline{B}_{it_2} - \overline{B}_{it_1}} \equiv \frac{\mu}{\lambda} \leq \frac{S_{it_1}}{S_{it_2} - S_{it_1}} \tag{20}$$

Note that this inequality also provides a testable restriction for the null hypothesis of unbiased (equilibrium) beliefs: under this null hypothesis, we should have that $P_{jt_1}/(P_{jt_2} - P_{jt_1}) \leq S_{it_1}/(S_{it_2} - S_{it_1})$.

Proposition 3. *Under Assumptions 1 to 3 and monotonicity of the payoff function, the hypothesis of equilibrium (unbiased) beliefs implies the inequality restriction:*

$$\frac{P_{jt_1}}{P_{jt_2} - P_{jt_1}} \quad \leq \quad \frac{S_{it_1}}{S_{it_2} - S_{it_1}} \tag{21}$$

Given that the choice probabilities P_{it} and P_{jt} are identified, this restriction is testable as long as the set $\mathcal{T}_{i,t_1}$ contains at least two treatments. ■

Suppose that we have an empirical application where the number of treatments in the set $\mathcal{T}_{i,t_1}$ is greater than two. Suppose that for any treatments t different than t_1 and t_2 we reject the null hypothesis in Proposition 2, but that for treatments t_1 and t_2 we cannot reject the null hypothesis in Proposition 3. Therefore, we cannot reject the null hypothesis that player i has unbiased beliefs at treatments t_1 and t_2 but has biased beliefs at other treatments in the set $\mathcal{T}_{i,t_1}$. Given this condition, the whole vector of structural parameters $\theta_{t_1} \equiv \{\overline{B}_{it},\ \alpha_\pi(m_{it})\ \beta_\pi(m_{it}) : t \in \mathcal{T}_{i,t_1}\}$ is point identified.

Proposition 4. *Under Assumptions 1 to 3 and the condition that player i has unbiased beliefs in treatments t_1 and t_2, the vector of structural parameters $\theta_{t_1} \equiv \{\overline{B}_{it},\ \alpha_\pi(m_{it}),\ \beta_\pi(m_{it}) : t \in \mathcal{T}_{i,t_1}\}$ is point identified.* ■

Proof: If beliefs at treatments t_1 and t_2 are unbiased, we have that $\mu \equiv \overline{B}_{it_1} = P_{jt_1}$ and $\lambda \equiv \overline{B}_{it_2} - \overline{B}_{it_1} = P_{jt_2} - P_{jt_1}$ such that constants μ and λ are identified. Then, equations (15), (17) and (18) imply that the parameters $\overline{B}_{it}$, $\alpha_\pi(m_{it})$, and $\beta_\pi(m_{it})$ are identified for any $t \in \mathcal{T}_{i,t_1}$. ■

Note that the selection of the baseline treatments t_1 and t_2 in the set $\mathcal{T}_{i,t_1}$ should be based on the test in Proposition 3.

When the matrix of monetary payoffs of player i is symmetric and diagonal-constant, we can construct a different version of the inequality test in Proposition 3 and of the identification result of beliefs and payoffs in Proposition 4. Under this structure of the payoff matrix, there is only one unknown constant to determine beliefs and payoff parameters. Taking into account that $S_{it} = \alpha_\pi(m_{it})\,[1-2\,\overline{B}_{it}]$ and $\mu \equiv \overline{B}_{it_1}$, it is straightforward to show that for any treatment $t \in \mathcal{T}_{i,t_1}$,

$$\alpha_\pi(m_i) \quad = \quad \frac{S_{it_1}}{1-2\,\mu} \tag{22}$$

and

$$\overline{B}_{it} = \frac{1}{2}\left[1-(1-2\,\mu)\,\frac{S_{it}}{S_{it_1}}\right] \tag{23}$$

Given these conditions, we have versions of Propositions 3 and 4 for games with a symmetric and diagonal-constant matrix of monetary payoffs.

Proposition 5. *Under Assumptions 1 and 2, monotonicity of the payoff function, and a symmetric and diagonal-constant matrix of monetary payoffs, the hypothesis of equilibrium (unbiased) beliefs implies the testable inequality restrictions:*

$$\frac{S_{it}}{1-2P_{jt}} \quad \geq \quad 0 \tag{24}$$

for any $t \in \mathcal{T}_{i,t_1}$. ■

Proposition 6. *Under Assumptions 1 to 3, a symmetric diagonal-constant matrix of monetary payoffs, and the condition that player i has unbiased beliefs in one of the treatments in set* T_{i,t_1}*, the vector of structural parameters* $\theta_{t_1} \equiv \{\overline{B}_{it},\ \alpha_\pi(m_{it}),\ \beta_\pi(m_{it}) : t \in \mathcal{T}_{i,t_1}\}$ *is point identified.* ■

Proof: Suppose (without loss of generality) that the treatment with unbiased beliefs is t_1. Then, we have that $\mu \equiv \overline{B}_{it_1} = P_{jt_1}$ such that constants μ is identified. Then, equations (22) and (23) imply that the parameters $\overline{B}_{it}$, $\alpha_\pi(m_{it})$, and $\beta_\pi(m_{it})$ are identified for any $t \in \mathcal{T}_{i,t_1}$. ■

3.3 Identification of the distribution of private information

So far, we have assumed that the researcher knows the distribution function $F_{\omega|m_{it}}$ of the unobservable variable in the payoff function. We now relax this assumption and replace Assumption 2 with conditions for the nonparametric identification of this distribution function.

Assumption 4. *(A) The payoff function is* $\Pi_i(a_i, a_j) = a_i\ z_i + \pi(m_i) + \varepsilon_i(a_i)$*. (B) In the experiment, each subject n is randomly assigned to a treatment that consists of: a game t with payoff matrices* $(\mathbf{m}_{Rt}, \mathbf{m}_{Ct})$*; a player role, i.e., row or column player; and a non-monetary payment* z_{nit} *that is a random draw from the probability distribution* F_z *and it is private information of subject n. (C) The probability distribution* F_z *is continuous and strictly increasing over the real line. (D)*

The unobservable variable $\omega_{nit} \equiv \varepsilon_{nit}(0) - \varepsilon_{nit}(1) - \xi_{nit}\ \beta_\pi(m_{it})$ *has median zero and is median independent of the monetary payoff* m_{it}. ■

The dataset consists of N subjects and an observation for subject n consists of $\{d_n, a_n, z_n\}$, where d_n represents the player-role and monetary treatment for subject n, z_n is the non-monetary treatment, and a_n is the player's action in the game.

Proposition 7. *Under Assumptions 1, 3, and 4, the cumulative distribution function* $F_{\omega|m_{it}}(\omega)$ *is nonparametrically identified for any treatment* (i,t) *and any value* $\omega \in \mathbb{R}$. ■

Proof: Given treatment (i,t) and $z \in \mathbb{R}$, define the conditional choice probability $P_{it}(z) \equiv \Pr(a_n = 1 \mid d_n = (i,t), z_n = z)$. For any (i,t,z), this choice probability function is nonparametrically identified. Assumptions 1 and 4 imply that $P_{it}(z) = F_{\omega|m_{it}}(z + \alpha_\pi(m_{it}) + \beta_\pi(m_{it})\ \overline{B}_{it})$. By Assumption 4, there is a value z such that the choice probability function takes a value equal to 1/2. Let z^*_{it} be this value of z for treatment (i,t), i.e., $P_{it}(z^*_{it}) = 1/2$. For any treatment (i,t), the value z^*_{it} is identified from the data. Since the random variable ω_{nit} has median equal to zero, we have that the condition $F_{\omega|m_{it}}(\omega) = 1/2$ implies that $\omega = 0$. Therefore, $P_{it}(z^*_{it}) = 1/2 = F_{\omega|m_{it}}(z^*_{it} + \alpha_\pi(m_{it}) + \beta_\pi(m_{it})\ \overline{B}_{it})$ implies that:

$$z^*_{it} + \alpha_\pi(m_{it}) + \beta_\pi(m_{it})\ \overline{B}_{it} = 0 \tag{25}$$

This expression shows that $\alpha_\pi(m_{it}) + \beta_\pi(m_{it})\ \overline{B}_{it}$ is equal to $-z^*_{it}$, and therefore it is identified. Therefore, for any value $\omega \in \mathbb{R}$ we have that:

$$\begin{aligned} P_{it}(\omega + z^*_{it}) &= F_{\omega|m_{it}}(\omega + z^*_{it} + \alpha_\pi(m_{it}) + \beta_\pi(m_{it})\,\overline{B}_{it}) \\ &= F_{\omega|m_{it}}(\omega) \end{aligned}$$

Such that $F_{\omega|m_{it}}(\omega)$ is identified. ■

The proof of Proposition 7 provides also a straightforward approach to estimate the value $S_{it} = \alpha_\pi(m_{it}) + \beta_\pi(m_{it})\ \overline{B}_{it}$. This value is equal to $-z^*_{it}$ and can be estimated by looking at the value of z that makes the choice probability function $P_{it}(z)$ equal to one-half.

4 Empirical application

In this section, we apply the model and the identification results in section 2 and 3 to datasets from two laboratory experiments that incorporate the exclusion restriction in Assumption 3. We start by describing these experiments.

4.1 Experiment 1: Matching pennies

Table 1 presents the payoff matrices in the experiment by Goeree and Holt (2001, henceforth GH). Each player simultaneously chooses between two possible actions, 0 or 1. The pairs of numbers between brackets, $[m_R, m_C]$, represent the monetary payoffs of row player and the column player, respectively, measured in cents. The experiment contains three games or treatments. The only difference across treatments is in the monetary payoff of the row player under action profile $(a_R, a_C) = (0, 0)$. It is clear that this experimental design satisfies the exclusion restriction in Assumption 3(A). Furthermore, note that the payoff matrix of the column player is symmetric and diagonal-constant. Therefore, for this game we can apply the test and identification result in Propositions 2, 5, and 6.

Table 1: Matching Pennies Experiment (Goeree and Holt, 2001)

Treatment 1

		Player C $a_C = 0$	$a_C = 1$
Player R	$a_R = 0$	[80 , 40]	[40 , 80]
	$a_R = 1$	[40 , 80]	[80 , 40]

Treatment 2

		Player C $a_C = 0$	$a_C = 1$
Player R	$a_R = 0$	[320 , 40]	[40 , 80]
	$a_R = 1$	[40 , 80]	[80 , 40]

Treatment 3

		Player C $a_C = 0$	$a_C = 1$
Player R	$a_R = 0$	[44 , 40]	[40 , 80]
	$a_R = 1$	[40 , 80]	[80 , 40]

The experiment includes 50 subjects ($N = 50$): five cohorts of ten subjects who were undergraduates in an economic class from University of Virginia. They were randomly matched and assigned as row or column player. In addition, the ordering of treatments is alternated for different sessions. Each subject records his/her decision of the game described by table 1 in an instruction sheet. In addition to this matching pennies game, subjects are also asked to play other nine different games which are not the focus of this paper. In this experiment each subject is paid \$6 for showing up. The average earnings for a two-hour session is about \$35 with range from \$15 to \$60 for all 10 games.

Half of the subjects are randomly selected as row players and the remaining subjects are column players. Each subject plays all three treatments once, and his role as either row or column player is fixed across treatments.[11] Table 2 presents the frequencies or players' choice probabilities from this experiment and the corresponding standard errors. The behavior of both players varies across treatments. In particular, though the payoff matrix of the column player is the same in the three treatments the behavior of this player varies considerably. According to the model, the change in the behavior of the column player should be attributed to the change in this player's beliefs on the behavior of the row player. This evidence is consistent with the "relevance" restriction in Assumption 3(B) that establishes that player R's behavior varies across treatments. We will exploit this source of variation in this experiment to test for unbiased beliefs of the column player and to identify beliefs and utilities for this player. Since the experiment does not provide the same source of variation for the row player, we cannot identify beliefs and preferences for this other player.

Table 2: Matching Pennies Game Experiment
Empirical Choice Probabilities: $N = 50$
(Standard errors in parentheses)

	Player R $[\widehat{P}_{R,t}]$		*Player C* $[\widehat{P}_{C,t}]$	
Treatment 1	0.52	(0.100)	0.52	(0.10)
Treatment 2	0.04	(0.039)	0.84	(0.073)
Treatment 3	0.92	(0.054)	0.20	(0.080)

Note: For player-type i, $\widehat{P}_{it} = \left[\sum_{n=1}^{N} a_n \ 1\{d_n = (i,t)\}\right] / \left[\sum_{n=1}^{N} 1\{d_n = (i,t)\}\right]$

Goeree and Holt (2001) treat this matching pennies game as one with complete information. They conclude that while subjects' behaviors are consistent with mixed strategy Nash Equilibrium in treatment 1, their behaviors departures considerably from the theoretical prediction in treatment 2 and 3. In contrast, our framework treats subject's preference as private information and it allows for players' biased beliefs and nonlinear utility of money.

4.2 Experiment 2: Coordination game

The second experiment deals with a coordination game. Heinemann, Nagel, and Ockenfels (2009, henceforth HNO) study and measure the strategic uncertainty that appears in games with multiple

[11] For detailed instruction of this experiment, visit http://www.people.virginia.edu/~cah2k/trdatatr.pdf.

equilibria when players have non-coordinated beliefs about the selected equilibrium. To study this phenomenon, they design and implement a randomized experiment using a set of coordination games with different group sizes, monetary payoffs and coordination difficulty.

Table 3 presents the payoff matrices in the different treatments of their experiment. There are G players in the game. Players simultaneously choose between action 0 and 1. Action $a = 0$ is a safe action that gives the player m_0 Euros regardless of other players' decisions. Action $a = 1$ is a risky action that yields 15 Euros if at least a fraction λ of other players also choose action $a = 1$, but it yields zero monetary payoff otherwise. Even though this experiment is a game with more than two players, table 3 shows that we can treat it as a two player game in which each player plays with an aggregate player. In this game, m_0 is a measure of the opportunity cost of coordination, and λ measures coordination difficulty. We expect that as m_0 and λ increase, coordination behavior becomes more unlikely to be maintained.

Table 3: Coordination Game Experiment (Heinemann, Nagel and Ockenfels, 2009)

		Other players $q =$ fraction of other players choosing $a = 1$	
		$q < \lambda$	$q \geq \lambda$
Player R	$a_R = 0$	m_0	m_0
	$a_R = 1$	0	15 Euros

Treatments

$T = 81$ treatments. Set of treatment consists of every combination (G, m_0, λ) with:

G (# players) $\in \{4, 7, 10\}$

$m_0 \in \{j * 1.5 : j = 1, 2, ..., 9\}$ Euros

$\lambda \in \{1/3,\ 2/3,\ 1\}$

The experiment was conducted in different locations: Frankfurt, Barcelona, Bonn and Cologne. Heinemann, Nagel and Ockenfels (2009) report that there are substantial differences among subject pools. For instance, subjects in Frankfurt are more risk averse than students from other locations. Therefore, it is reasonable to believe that those subjects from different locations are from different populations. Accordingly, our analysis focuses on Frankfurt as it contains most of the subjects and treatments.

The experiment was run at a computer laboratory in the Economics Department of the University of Frankfurt between May and July 2003. Most of subjects were undergraduates in business and economics. There are 90 treatments or games according to all the possile values of the parameters G, m_0, and λ with $G \in \{4, 7, 10\}$, $\lambda \in \{1/3, 2/3, 1\}$, and any value of m_0 between 1.5 Euros and 13.5 Euros with an incremental unit of 1.5 Euros.[12] Subjects were randomly assigned into a group G, where G is 4, 7 or 10. Then, given the selection of group size G, a subject participates in all the treatments / games for every value of λ and m_0. Therefore, each subject participates in 30 treatments. To prevent learning, Heinemann, Nagel and Ockenfels (2009) do not give feedback between blocks. At the end of a session, only 1 of 40 situations is randomly selected to determine subject's earning. This avoids potential hedging and each decision situation can be treated as independent. The duration of a session is about 40-60 minutes with an average earning of 16.68 Euros per subject.

The experiment is conducted using different populations of students from Frankfurt, Barcelona, Bonn and Cologne. Heinemann, Nagel and Ockenfels (2009) report that there is substantial differences in risk preferences between subject pools. For instance, subjects in Frankfurt are more risk averse than students from other locations. Therefore, it is reasonable to believe that those subjects come from different populations. Accordingly, our analysis focuses on Frankfurt as it contains most of the subjects and treatments.[13]

Unlike the GH experiment, the HNO coordination game does not have a variable that shifts one player's monetary payoff while has no impact on other players' utility. Note that the parameter m_0 shifts all players' monetary payoff and cannot be an exclusion restriction. However, changes in the coordination difficulty parameter λ and group size G play the same role as our exclusion restriction. In particular, a change in λ or G does not shift the payoff matrix of any player but it affects the beliefs that players have about the behavior of other players. With exogenous (randomized) variation in λ and G, all the identification results in previous section hold in HNO coordination game.

The number of subjects N in this experiment is 64, 42, or 40 depending on the treatment. Table 4 presents players' empirical choice probabilities and their corresponding standard errors for each of the 81 treatments. Note that for any value of the parameters G and m_0, the choice probability of the risky action ($a = 1$) always declines when the parameter λ (the *coordination difficulty*) increases. This implies that λ is a *relevant* instrument because it affects players' beliefs without affecting their own payoff matrix, i.e., it satisfies Assumption 3. Note that for each value of the

[12] We have not used treatments with $m_0 = 15$ in our analysis because subjects' choice probabilities are very imprecisely estimated for these treatments.

[13] For details about this experiment, see section 3 in Heinemann, Nagel and Ockenfels (2009). The experimental instructions are available on the supplements page of the *Review of Economic Studies* website at http://www.restud.org.

safe monetary payoff m_0, there are nine treatments (i.e. 9 combinations of (G, λ)). For illustration purpose, we index these nine treatments by t in HNO experiment.

Table 4: Coordination Game Experiment
Empirical Choice Probabilities (Probability of choosing risky action)
(Standard errors in parentheses)

	$G = 4$			$G = 7$			$G = 10$		
	$\lambda = 1$	$\lambda = \frac{2}{3}$	$\lambda = \frac{1}{3}$	$\lambda = 1$	$\lambda = \frac{2}{3}$	$\lambda = \frac{1}{3}$	$\lambda = 1$	$\lambda = \frac{2}{3}$	$\lambda = \frac{1}{3}$
$m_0 = 1.5$	0.7813	0.8750	0.9531	0.7143	0.8333	0.8810	0.6750	0.8500	0.9000
	(0.0517)	(0.0413)	(0.0264)	(0.0697)	(0.0575)	(0.0500)	(0.0741)	(0.0565)	(0.0474)
$m_0 = 3.0$	0.7188	0.8750	0.9688	0.6429	0.7143	0.8333	0.6250	0.8500	0.9250
	(0.0562)	(0.0413)	(0.0217)	(0.0739)	(0.0697)	(0.0575)	(0.0765)	(0.0565)	(0.0416)
$m_0 = 4.5$	0.6094	0.8438	0.9531	0.5000	0.7381	0.8571	0.4000	0.8000	0.9000
	(0.0610)	(0.0454)	(0.0264)	(0.0772)	(0.0678)	(0.0540)	(0.0775)	(0.0632)	(0.0474)
$m_0 = 6.0$	0.4375	0.7031	0.8750	0.3810	0.5714	0.8333	0.3250	0.5250	0.8500
	(0.0620)	(0.0571)	(0.0413)	(0.0749)	(0.0764)	(0.0575)	(0.0741)	(0.0790)	(0.0565)
$m_0 = 7.5$	0.2813	0.4688	0.8125	0.2619	0.4286	0.7143	0.2750	0.3750	0.8250
	(0.0562)	(0.0624)	(0.0488)	(0.0678)	(0.0764)	(0.0697)	(0.0706)	(0.0765)	(0.0601)
$m_0 = 9.0$	0.1719	0.2656	0.6406	0.1667	0.3333	0.6190	0.2250	0.2500	0.6000
	(0.0472)	(0.0552)	(0.0600)	(0.0575)	(0.0727)	(0.0749)	(0.0660)	(0.0685)	(0.0775)
$m_0 = 10.5$	0.1406	0.1250	0.4375	0.0714	0.1667	0.4286	0.1250	0.2250	0.4500
	(0.0435)	(0.0413)	(0.0620)	(0.0397)	(0.0575)	(0.0764)	(0.0523)	(0.0660)	(0.0787)
$m_0 = 12.0$	0.0781	0.1094	0.2656	0.0714	0.0476	0.2619	0.1250	0.1500	0.3500
	(0.0335)	(0.0390)	(0.0552)	(0.0397)	(0.0329)	(0.0678)	(0.0523)	(0.0565)	(0.0754)
$m_0 = 13.5$	0.0781	0.0781	0.1875	0.0476	0.0238	0.1667	0.1000	0.1250	0.2500
	(0.0335)	(0.0335)	(0.0488)	(0.0329)	(0.0235)	(0.0575)	(0.0474)	(0.0523)	(0.0685)
Subjects per treatment	64			42			40		

In this game, $P_{m_0,t}$ represents the choice probability of the risky action when the treatment is (m_0, t) where t denotes a treatment index for (G, λ). Given $G-1$ of the players (all except one), let $g_{m_0,t}$ be the number of these players who choose the risky action. According to the model, $g_{m_0,t}$ is a Binomial random variable with parameters $G-1$ and $P_{m_0,t}$. Therefore, the probability that at least a fraction λ of the other players choose the risky action is:

$$CP_{m_0,t} \equiv \Pr\left(g_{m_0,t} \geq \lambda[G-1]\right) = 1 - BIN\left(\lambda[G-1]\ ; G-1, P_{m_0,t}\right) \tag{26}$$

where $BIN(n; N, P)$ is the CDF of a Binomial with parameters (N, P). In this game, $\overline{B}_{m_0,t}$ represents beliefs about the probability that at least a fraction λ of the other players choose the risky action. Therefore, the condition of unbiased beliefs is not $\overline{B}_{m_0,t} = P_{m_0,t}$ but instead $\overline{B}_{m_0,t} = CP_{m_0,t}$.

4.3 Tests and estimation

4.3.1 Hypothesis testing

In the two applications we present tests and estimation results under three alternative parametric specifications for the distribution of the unobserved variable ω_{nit}: (a) standard normal (Probit); (b) double exponential (Logit); and (c) exponential with zero median. The form of the inverse function $S_{it} \equiv F^{-1}_{\omega|m_{it}}(P_{it})$ for these three distributions is: (a) for the Probit model, $S_{it} = \Phi^{-1}(P_{it})$, where Φ^{-1} is the inverse CDF of the standard normal; (b) for the Logit model, $S_{it} = \ln(P_{it}) - \ln(1 - P_{it})$; and (c) for the exponential model, $S_{it} = -\ln(2[1 - P_{it}])$.

Consider the GH experiment. Let $\widehat{P}_{Rt}$ and $\widehat{P}_{Ct}$ be the estimated choice probabilities in table 2 for $t = 1, 2, 3$, and let $\widehat{S}_{Ct}$ be $F^{-1}_{\omega}(\widehat{P}_{Ct})$. Based on Proposition 1, we could construct the statistic $\frac{\widehat{S}_{C3}-\widehat{S}_{C1}}{\widehat{S}_{C2}-\widehat{S}_{C1}} - \frac{\widehat{P}_{R3}-\widehat{P}_{R1}}{\widehat{P}_{R2}-\widehat{P}_{R1}}$ and its standard error to test for the null hypothesis of unbiased beliefs using a t-test. This test is asymptotically valid. However, this test does not have good small-sample properties when one, or both, of the values in the denominator, $\widehat{S}_{C2} - \widehat{S}_{C1}$ and $\widehat{P}_{R2} - \widehat{P}_{R1}$, are close to zero. To deal with this issue, we use instead the following statistic:

$$\widehat{\delta} = \left(\widehat{S}_{C3} - \widehat{S}_{C1}\right)\left(\widehat{P}_{R2} - \widehat{P}_{R1}\right) - \left(\widehat{S}_{C2} - \widehat{S}_{C1}\right)\left(\widehat{P}_{R3} - \widehat{P}_{R1}\right) \tag{27}$$

We also use the bootstrap method to calculate the standard error $se(\widehat{\delta})$ (Horowitz, 2001). Since in this experiment the matrix of monetary payoffs of the column player is symmetric and diagonal-constant, we can also apply the test of unbiased beliefs in Proposition 2. We can construct the statistics[14]

$$\begin{aligned} \widehat{\delta}_{12} &= \widehat{S}_{C2}\ (1 - 2\widehat{P}_{R1}) - \widehat{S}_{C1}(1 - 2\widehat{P}_{R2}) \\ \widehat{\delta}_{13} &= \widehat{S}_{C3}\ (1 - 2\widehat{P}_{R1}) - \widehat{S}_{C1}\ (1 - 2\widehat{P}_{R3}) \end{aligned} \tag{28}$$

$\widehat{\delta}_{12}$ is a test statistic for the unbiased belief in treatments 1 and 2, and $\widehat{\delta}_{13}$ is the same type of test statistic but for treatments 1 and 3. Define the vector $\hat{\delta}_1 = (\hat{\delta}_{12}, \hat{\delta}_{13})'$. Under the null hypothesis of unbiased beliefs in treatments 1, 2, and 3, the statistic $\hat{\delta}_1' \cdot \widehat{Var}(\hat{\delta}_1) \cdot \hat{\delta}_1$ has a Chi-square distribution with two degrees of freedom where $\widehat{Var}(\hat{\delta}_1)$ is an estimate of the variance-covariance matrix of $\hat{\delta}_1$.

We apply the same test to the HNO experiment but with the following adjustments. Recall that $P_{m_0,t}$ represents the choice probability of the risky action when the treatment is (m_0, t), where t denotes a treatment index for (G, λ). And $CP_{m_0,t}$ is the probability that at least a fraction λ of the other players choose the risky action. Therefore, for a single m_0, we can always find there

[14] Note that the restrictions $\delta_{12} = 0$ and $\delta_{13} = 0$ imply the restriction $\delta_{23} = 0$, and therefore this third restriction is redundant. Also, note that the restrictions $\delta_{12} = 0$ and $\delta_{13} = 0$ imply δ defined in equation (27) is also zero.

treatments $t_1,\ t_2$ and t and construct the following test-statistic:

$$\begin{aligned}\widehat{\delta}_{m_0,(t_1,t_2,t)} &= \left(\widehat{S}_{m_0,t_2} - \widehat{S}_{m_0,t_1}\right)\left(\widehat{CP}_{m_0,t} - \widehat{CP}_{m_0,t_1}\right) \\ &- \left(\widehat{S}_{m_0,t} - \widehat{S}_{m_0,t_1}\right)\left(\widehat{CP}_{m_0,t_2} - \widehat{CP}_{m_0,t_1}\right)\end{aligned} \tag{29}$$

$\widehat{\delta}_{m_0,(t_1,t_2,t)}$ is the unbiased beliefs test statistic for treatments $t_1,\ t_2$ and t. For given $(m_0, t_1,\ t_2)$, there are seven other combinations of (G, λ) for treatment t, such that we can construct seven different statistics $\widehat{\delta}_{m_0,(t_1,t_2,t)}$. We use these seven $\widehat{\delta}$'s to construct a Chi-Square test for testing the hypothesis that players have unbiased belief in all treatments (G, λ) for a given m_0.

4.3.2 Estimation

For the estimation of payoffs and beliefs in the GH experiment, we can exploit the symmetry of the payoff matrix of the column player to identify payoffs and beliefs parameters with only one restriction of unbiased beliefs (Proposition 6). Suppose that we impose the restriction of unbiased beliefs in treatment $t = 1$. This implies that we can estimate beliefs of the column player at treatments $t = 2, 3$ using the estimator:

$$\widehat{\widehat{B}}_{Ct} = \frac{1}{2}\left[1 - \left(1 - 2\ \widehat{P}_{R1}\right)\frac{\widehat{S}_{Ct}}{\widehat{S}_{C1}}\right] \tag{30}$$

And we can estimate the payoff parameter of the column player using the estimator:

$$\widehat{\alpha}_{\pi}\left(m_C\right) = \pi\left(8\widehat{0) - \pi}\left(40\right) \quad = \quad \frac{\hat{S}_{C1}}{1 - 2\hat{P}_{R1}} \tag{31}$$

In the experiment for the coordination game, we impose the restriction that beliefs are unbiased for treatment t_1 and t_2 for m_0. Under this restriction, we can estimate belief for treatment (m_0, t):

$$\widehat{\widehat{B}}_{m_0,t} = \widehat{CP}_{m_0,t_1} + \left(\widehat{CP}_{m_0,t_2} - \widehat{CP}_{m_0,t_1}\right)\left(\frac{\widehat{S}_{m_0,t} - \widehat{S}_{m_0,t_1}}{\widehat{S}_{m_0,t_2} - \widehat{S}_{m_0,t_1}}\right) \tag{32}$$

Given estimated beliefs $\widehat{\widehat{B}}_{m_0,t}$ for every treatment, we apply OLS to the regression-like equation

$$\hat{S}_{m_0,t} = \frac{\pi(m_0)}{\sigma_{\omega,m_0}} + \frac{\pi(15)}{\sigma_{\omega,m_0}}\widehat{\widehat{B}}_{m_0,t}$$

to estimate the utility parameters $\pi(m_0)/\sigma_{\omega,m_0}$ and $\pi(15)/\sigma_{\omega,m_0}$, where σ^2_{ω,m_0} is the variance of the unobservable ω that we allow to be heteroscedastic with respect to m_0. We use a bootstrap re-sampling method to calculate standard errors that account for the two-step feature of the estimation method. Given estimates $\pi(m_0)/\sigma_{\omega,m_0}$ and $\pi(15)/\sigma_{\omega,m_0}$, we obtain the the normalized payoff $\widetilde{\pi}(m_0) = 15 * [\pi(m_0)/\sigma_{\omega,m_0}]/[\pi(15)/\sigma_{\omega,m_0}] = 15 * \pi(m_0)/\pi(15)$ such that $\widetilde{\pi}(m_0)$ does not depend on the variance of the unobservable and $\widetilde{\pi}(15)$ is normalized to 15.

4.4 Empirical results: GH experiment

The monetary payment for player R in outcome $(0,0)$ is higher in treatment 2 compared to treatment 1. Therefore, alternative $a_R = 0$ becomes more attractive to R in treatment 2. If player C has rational beliefs, she should predict that player R will choose $a_R = 0$ with higher probability in treatment 2 than in treatment 1. The best response to such belief is to choose $a_C = 1$ more frequently. A similar argument applies to the comparison of treatments 1 and 3. The estimated choice probabilities in Table 2 are consistent with this argument: P_{C2} [= 0.84] > P_{C1} [= 0.52] > P_{C3} [= 0.20], and these inequalities are statistically significant. However, this argument is not a formal and rigorous test of unbiased beliefs. Without taking into account players' preferences and their degree of risk aversion/loving, we do not know whether or not these changes in the choice probability are consistent with unbiased beliefs. Here we implement formal tests of unbiased beliefs that takes into account these considerations.

Table 5: Tests of Unbiased Beliefs
Matching Pennies

	Probit	*Logit*	*Exponential*
Treatments 1,2, & 3			
$H_0 : \delta = 0$: $\widehat{\delta}$ [s.e]	0.0503 [0.3753]	0.0726 [0.6479]	-0.1942 [0.3683]
(p-value)	(0.8818)	(0.9032)	(0.5426)
Treatments 1 & 2			
$H_0 : \delta_{12} = 0$: $\widehat{\delta}_{12}$ [s.e]	-0.0859 [0.3242]	-0.1400 [0.5402]	-0.0831 [0.3454]
(p-value)	(0.7932)	(0.7926)	(0.7926)
Treatments 1 & 3			
$H_0 : \delta_{13} = 0$: $\widehat{\delta}_{13}$ [s.e]	0.0758 [0.2886]	0.1277 [0.4760]	0.0531 [0.2102]
(p-value)	(0.7876)	(0.7868)	(0.7820)
Treatments 1, 2, & 3			
$H_0 : \delta_{12} = \delta_{13} = 0$: Chi-square	0.1392	0.1336	0.1217
(p-value)	(0.9328)	(0.9354)	(0.9410)

Note: Standard error is calculated using bootstrap with 5,000 replications.

Table 5 presents results for our tests of unbiased beliefs in the GH experiment. We report results from four different tests: the test from Proposition 1 for $\delta = 0$, and three different tests from Proposition 2 for $\delta_{12} = 0$ and $\delta_{13} = 0$ separately and for the joint restriction. Standard errors are calculated by bootstrap with 5,000 bootstrap samples. All the tests are consistent with the hypothesis that the column player has unbiased beliefs in the three treatments. All the p-values are greater than 0.5 and highly insignificant. Goeree and Holt (2001) conclude that column player tends to predict row players' behaviors correctly based on an observation of choice probability. In this paper, we verify their qualitative observation in a framework with incomplete information. They also conclude that row player seems to simply responds to monetary payoff without considering column player's response. However, we cannot verify this point as there is a lack of exclusion restriction for the row player. Take treatment 2 as an example, row player's high choice probability of action 0 can be explained by either row player predicts column player would choose action 0 with a sufficiently high probability or row player values 320 cents far more than 80 cents and 40 cents or both. Without an exclusion restriction, we cannot distinguish these two effects.

Table 6 presents estimates of the preference parameter $\alpha_\pi = \pi(80) - \pi(40)$. We report estimates under the three models for the unobservables (Probit / Logit / Exponential), and under the condition of unbiased beliefs at each of the treatments.[15] All the estimates are significantly greater than zero which implies the strict monotonicity of the payoff function. Furthermore, the estimates under the three different distributional assumptions on the unobservable are very close after adjusting their scales.[16] In this experiment, player C receives only two possible monetary payoffs, and therefore, we cannot study possible departures from the restriction of linear utility function.

Table 6: Estimation of Payoff Parameters
Matching Pennies

Parameter	*Probit*	*Logit*	*Exponential*
Unbiased beliefs at $t = 2$			
$\pi(80) - \pi(40)$	1.0809***	1.8024**	1.2385*
(s.e.)	(0.3899)	(0.7464)	(0.6478)
Unbiased beliefs at $t = 3$			
$\pi(80) - \pi(40)$	1.0019**	1.6504*	0.5595*
(s.e.)	(0.4321)	(0.7849)	(0.1554)

***, **, * indicate significance at 1%, 5%, and 10% levels.

[15] In Table 6, we do not include the estimate of $\pi(80) - \pi(40)$ under the restriction of unbiased beliefs in treatment 1. This is because, in treatment 1, both players' choice probabilities are close to 50%, the exclusion restriction has little power, and as a result the estimated preference is very imprecise.

[16] Recall that the standard deviation of the error is 1 for Probit model, $\sqrt{\frac{2}{3}}\pi$ for Logit model, and $\sqrt{2}$ for exponential model.

4.5 Empirical results: HNO experiment

The HNO experiment includes many treatments with rich randomized variation players' monetary payoffs. Such design enables us to address not only the test of unbiased beliefs but also possible non-linearity of the utility function of money.

For each m_0, there exist nine combinations of (G, λ) which provide exogenous variation for unbiased belief test. However, we cannot use all nine combinations as some of them do not satisfy the "relevance" condition of Assumption 3(B). To see this, suppose that the opportunity cost, as measured by m_0, is large enough such that the choice probability of the risky action becomes close to zero regardless $\lambda = 1$ or $\lambda = 2/3$. This implies that, for high values of m_0, subjects' beliefs are not affected by changes in λ and those treatments do not provide enough exogenous variation to identify biased beliefs. We address this issue by choosing the largest subset of nine treatments for each m_0 such that any two treatments do not share same choice probabilities (i.e. reject an equal choice probability at 10 percent significance level). We then conduct the unbiased belief test using such subset based on equation (29).[17]

Table 7 presents the results of all these tests. The fifth column shows which subset of treatments is used to conduct the test of unbiased beliefs for the corresponding m_0 and the degrees of freedom for the test. Table 7 shows that there are several subsets of treatments where the test rejects the hypothesis of unbiased beliefs, implying that Bayesian Nash equilibrium is inconsistent with observed subjects' behavior. Specifically, when m_0 is very low (i.e. $m_0 = 1.5$, or 3, or 4.5) subjects' beliefs appear as biased. In contrast, for the largest values of (i.e. $m_0 = 12$, or 13.5) beliefs are close to the actual coordination probability and the test statistic cannot reject the hypothesis of unbiased beliefs. For middle range values of m_0 the results are mixed. values is insignificant.[18] Note that this conclusion should not be interpreted as a comparative statistics exercise on m_0 because the subsets of treatments to conduct unbiased belief test for each m_0 are different. We postpone the discussion about comparative statistics on beliefs after the estimation of players' payoff function and beliefs below.

[17] Admitedly, this procedure might be affected by pre-testing bias.

[18] Heinemann, Nagel and Ockenfels (2009) also consider a Bayesian game which is different than the one in this paper. The Bayesian Nash Equilibrium in their framework predicts that the equilibrium probability of the risky action increases monotonically with m_0 and λ, and decreases with G. This prediction is clearly rejected by the empirical choice probabilities, and therefore, they conclude that BNE is not appropriate in this experiment. In contrast, the BNE in our framework does not predict their comparative statistics and it requires a formal test of unbiased beliefs.

Table 7: Tests of Unbiased Beliefs
Coordination Game (Chi-Square Test)
(p-value in parentheses)

	Probit Model	*Logit Model*	*Exponential Model*	*Treatment Index*
$m_0 = 1.5$	3.1898**	2.6966	2.5239	1,2,3
	(0.0741)	(0.1006)	(0.1121)	(d.f. = 1)
$m_0 = 3.0$	4.5180**	4.0588**	3.9355**	1,2,3
	(0.0335)	(0.0439)	(0.0473)	(d.f. = 1)
$m_0 = 4.5$	5.6385**	4.7488*	3.4600	1,2,3,7
	(0.0597)	(0.0931)	(0.1773)	(d.f. = 2)
$m_0 = 6.0$	3.3787	3.0128	3.4123	2,3,7,8
	(0.1846)	(0.2217)	(0.1816)	(d.f. = 2)
$m_0 = 7.5$	1.5463	1.3931	4.4916**	1,2,3
	(0.2137)	(0.2379)	(0.0341)	(d.f. = 1)
$m_0 = 9.0$	3.7684*	3.5429*	3.6324*	1,3,5
	(0.0522)	(0.0598)	(0.0567)	(d.f. = 1)
$m_0 = 10.5$	3.7707*	2.9759*	4.6426**	3,4,5
	(0.0.0522)	(0.0845)	(0.0312)	(d.f. = 1)
$m_0 = 12.0$	1.7664	1.5882	1.6675	5,8,9
	(0.1838)	(0.2076)	(0.1966)	(d.f. = 1)
$m_0 = 13.5$	1.6626	1.6349	1.1114	5,8,9
	(0.1973)	(0.2010)	(0.2918)	(d.f. = 1)
Number of subjects	64	42	40	

***, **, * indicate significance at 1%, 5%, and 10% levels.

We estimate payoff parameters using the two-step lest squares estimator described above in section 4.3. To select the unbiased beliefs treatments, we apply the following procedure. First (step 1), for each m_0, we construct all the possible subsets of three treatments satisfying the condition that any pair of treatments within the subset has significantly different choice probabilities (using $p = 0.1$ as the cutoff for the p-value of the test of equal choice probabilities). This step tries to account for the potential problem of "weak instruments" in the test of unbiased beliefs. Second (step 2), using equation (29), we conduct the unbiased beliefs test for every subset of three treatments that passes the selection criterion in step 1. We select the subset with the highest p-value and impose the restriction that beliefs are unbiased in these three treatments. Third (step 3), under the restriction in step 2, we estimate the rest of the beliefs parameters and utility parameters. For values of m_0 such that the unbiased belief test rejects the null hypothesis for every possible subset of three treatments, we choose the two treatments by applying Proposition 3 that exploits the restriction of strict monotonicity of the payoff function.

Table 8: Estimation of Payoff Parameters
Coordination Game
(standard error in parentheses)

	Probit Model		*Logit Model*		*Exponential Model*	
	Payoffs	Unbiased Belief Treatments	Payoffs	Unbiased Belief Treatments	Payoffs	Unbiased Belief Treatments
$\pi(1.5)$	0.3803	$t = 1, 3$	1.4103	$t = 1, 3$	2.9421	$t = 1, 3$
	(4.7402)	p=n.a.	(4.2894)	p=n.a.	(3.4681)	p= n.a.
$\pi(3)$	1.3150	$t = 1, 3$	2.0244*	$t = 1, 3$	3.1003***	$t = 1, 3$
	(1.0919)	p=n.a.	(1.1733)	p=n.a.	(1.0925)	p= n.a.
$\pi(4.5)$	1.6543*	$t = 1, 3, 7$	1.6358**	$t = 1, 3, 7$	1.4417**	$t = 1, 7, 9$
	(0.9241)	p=0.5830	(0.8837)	p=0.7892	(0.7237)	p=0.8798
$\pi(6)$	4.1466***	$t = 3, 7, 8$	4.0637***	$t = 3, 7, 8$	4.1527***	$t = 2, 7, 8$
	(1.0131)	p=0.8904	(1.0035)	p=0.9692	(1.0412)	p=0.6866
$\pi(7.5)$	7.8417***	$t = 2, 4, 6$	7.7179***	$t = 1, 2, 6$	4.4342***	$t = 3, 4, 5$
	(1.3819)	p=0.9654	(1.2272)	p=0.9514	(0.7336)	p=0.6264
$\pi(9)$	10.1793***	$t = 1, 2, 3$	10.2377***	$t = 1, 2, 3$	8.7978***	$t = 1, 2, 3$
	(1.1710)	p=0.5070	(1.1775)	p=0.4692	(1.5230)	p=0.5548
$\pi(10.5)$	13.8134***	$t = 3, 4$	13.6697***	$t = 3, 4$	15.2276***	$t = 3, 4$
	(0.9597)	p=n.a.	(0.8517)	p=n.a.	(2.4847)	p= n.a.
$\pi(12)$	13.8952***	$t = 5, 8, 9$	13.4586***	$t = 5, 8, 9$	17.9552***	$t = 5, 8, 9$
	(1.6660)	p=0.1642	(1.7893)	p=0.1764	(1.9996)	p=0.1588
$\pi(13.5)$	10.5192***	$t = 5, 8, 9$	9.7867***	$t = 5, 8, 9$	17.6108	$t = 5, 8, 9$
	(2.8859)	p=0.1402	(2.8718)	p=0.1538	(11.8893)	p=0.1714
$\pi(15)$	15	n.a.	15	n.a.	15	n.a.
	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

***, **, * indicate significance at 1%, 5%, and 10% levels.

Table 8 presents the estimation results. The column titled "Unbiased Beliefs Treatments" reports the subset of treatments where we impose the restriction of unbiased beliefs and its corresponding p-value. Note that, as explained at the end of section 4.3.2, we account for heterocedasticity in the unobservable and then we need to incorporate a scale normalization in the utility function: we normalize $\pi(15) = 15$. Before we comment our main estimation results, we want to draw attention on the estimate of the utility parameter $\pi(13.5)$. The monotonicity of the payoff function is violated at $m_0 = 13.5$. Note that the p-value of the unbiased belief test at this value of m_0 is no more than 0.18 for the distributional assumptions on the unobservable. Therefore, we interpret this non-monotonicity result as evidence that the selected combination of treatments $t = 5, 8, 9$ does not satisfy the unbiased belief assumption. For the Probit model, the estimated utility function is strictly increasing at all the other values of money.[19]

[19] The monotonicity condition also fails for $\pi(3)$ for Logit model and $\pi(3)$, $\pi(10.5)$, $\pi(12)$ for exponential model. We interpret it as exponential error term is inadequate to explain subjects' behaviors especially when m_0 is relatively large.

Figure 1: Estimated Payoff (Probit Model)

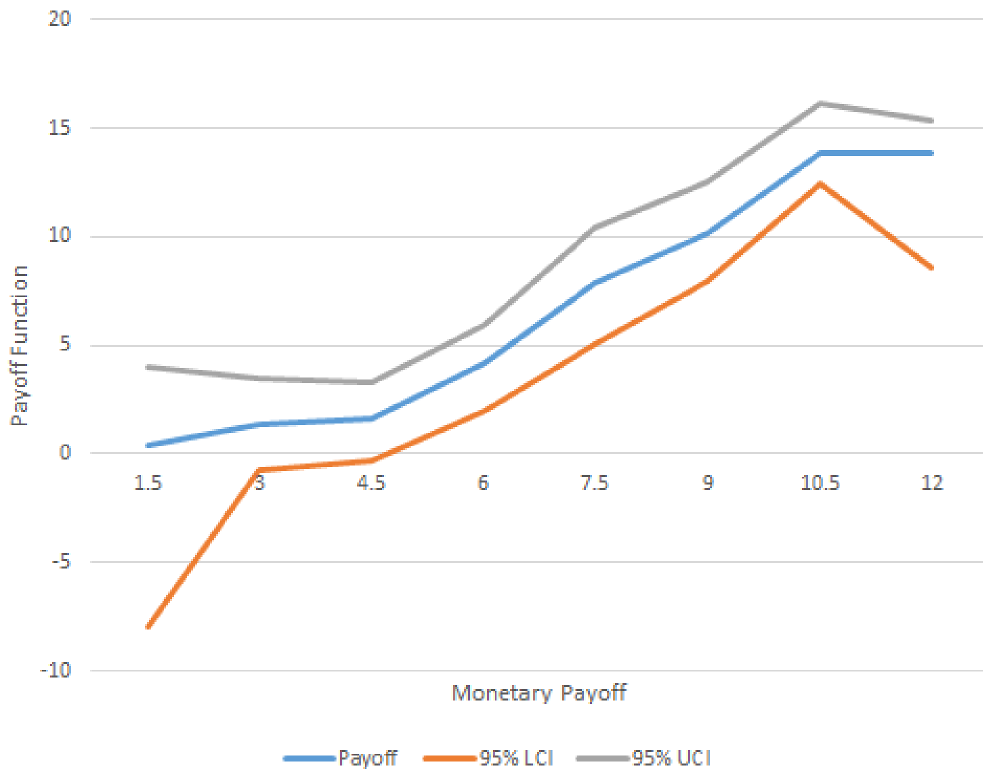

Figure 1 presents the estimated utility function for the Probit model, and 95% confidence bands. This estimated utility function has an inverted S-shape: it is convex for relatively low values of money and concave for large monetary payoffs. It indicates that subjects are risk loving when receiving small monetary payoffs but they turn to be risk averse when the payoff increases. This function is significantly different to the standard specification that restricts utility to be equal to the monetary payoff (i.e., $\pi(m_0) = m_0$). The linear specification over-estimates (under-estimates) utility for values of m_0 smaller (larger) than 7.5 Euros. Imposing the restriction that the utility is equal to the monetary payoff can generate incorrect conclusions on beliefs. Furthermore, our estimates suggest that the specification of a globally concave utility function may generate also important biases because this concavity does not hold at small monetary payoff. This indicates that the conventional functional forms adopted for the utility function in many applications (i.e. linear, logarithmic, CRRA, or CARA functions) can be mis-specified. In contrast, this paper provides a method that estimate payoff functions without imposing any functional form assumption. Of course, this flexibility has the price of less precise estimates of the parameters. However, the nonparametric specification can be considered an exploratory approach to search for the correct parametric specification.

Tables 9 to 11 present our estimates of beliefs on the probability that at least a proportion λ of other players choose the risky action, and compare these beliefs with the actual probability. Each table corresponds to a particular value for the number of players G. Table 9 shows some interesting features of subject's beliefs. In general, subject's belief about successful coordination decreases as either m_0, G, or λ increase. For $G = 7$, subjects' beliefs underestimate the actual coordination probability when m_0 is small (i.e., smaller than 6 Euros) and λ is also relatively low (i.e. $\lambda = \frac{2}{3}, \frac{1}{3}$). However, subjects tend to over-predict the probability of successful coordination when the coordination difficulty is high (i.e. $\lambda = 1$). A similar pattern is also found for experiments with $G = 4$ and $G = 10$, though the experiment with four players presents stronger evidence in favor of the hypothesis equilibrium beliefs.

Table 9: Comparison Between Belief and Actual Coordination Probability
Probit Model, $G = 7$
(standard error in parentheses)

	$\lambda = 1$		$\lambda = \frac{2}{3}$		$\lambda = \frac{1}{3}$	
	Beliefs	True Choice Probability	Beliefs	True Choice Probability	Beliefs	True Choice Probability
m_0= 1.5	0.3544**	0.1328	0.5879***	0.9377***	0.7114***	0.9999***
s.e.	(0.1428)	(0.0863)	(0.1573)	(0.0593)	(0.1570)	(0.0011)
Equality Test	p-value=0.0428		p-value=0.0352		p-value=0.0960	
m_0= 3	0.2670**	0.0706	0.3649***	0.7703***	0.5615***	0.9993***
s.e.	(0.1068)	(0.0571)	(0.1096)	(0.1189)	(0.1285)	(0.0027)
Equality Test	p-value=0.0128		p-value=0.0004		p-value=0.0082	
m_0= 4.5	0.1103	0.0156	0.4461***	0.8113***	0.6726***	0.9997***
s.e.	(0.1007)	(0.0201)	(0.1315)	(0.1089)	(0.1536)	(0.0017)
Equality Test	p-value=0.3586		p-value=0.0010		p-value=0.0508	
m_0= 6	0.0872	0.0031	0.3889***	0.4852	0.8807***	0.9993***
s.e.	(0.1083)	(0.0063)	(0.1406)	(0.1455)	(0.1270)	(0.0028)
Equality Test	p-value=0.5518		p-value=0.1586		p-value=0.5166	
m_0= 7.5	0.0000	0.0003	0.3745***	0.2210*	0.9890***	0.9913***
s.e.	(0.0334)	(0.0014)	(0.1227)	(0.1164)	(0.0404)	(0.0142)
Equality Test	p-value=0.5124		p-value=0.0620		p-value=0.9586	
m_0= 9	0.0000	0.0000	0.3579**	0.1001	0.9042***	0.9671***
s.e.	(0.0901)	(0.0002)	(0.1651)	(0.0770)	(0.1254)	(0.0360)
Equality Test	p-value=0.4884		p-value=0.0322		p-value=0.6640	
m_0= 10.5	0.0000	0.0000	0.3129***	0.0087	0.8078 ***	0.8085***
s.e.	(6.89×10^{-6})	(0.0000)	(0.1194)	(0.0170)	(0.1248)	(0.1065)
Equality Test	p-value=1		p-value=0.0154		p-value=0.9840	
m_0= 12	0.0000	0.0000	0.0000	0.0001	0.4899***	0.4941***
s.e.	(0.0559)	(0.0000)	(0.0155)	(0.0010)	(0.1473)	(0.1498)
Equality Test	p-value=0.4490		p-value=0.3248		p-value=0.9432	
m_0= 13.5	0.0000	0.0000	0.0000	0.0000	0.2680**	0.2632*
s.e.	(0.0418)	(0.0000)	(0.0067)	(0.0002)	(0.1173)	(0.1308)
Equality Test	p-value=0.4212		p-value=0.2638		p-value=0.9456	

***, **, * indicate significance at 1%, 5%, and 10% levels.

Table 10: Comparison Between Belief and Actual Coordination Probability

Probit Model, $G = 4$

(standard error in parentheses)

	$\lambda = 1$		$\lambda = \frac{2}{3}$		$\lambda = \frac{1}{3}$	
	Beliefs	True Choice Probability	Beliefs	True Choice Probability	Beliefs	True Choice Probability
m_0= 1.5	0.4768**	0.4768***	0.6943***	0.9570***	0.9999***	0.9999***
s.e.	(0.0953)	(0.0963)	(0.1008)	(0.0285)	(0.0003)	(0.0003)
Equality Test	p-value=1		p-value=0.0090		p-value=1	
m_0= 3	0.3713***	0.3713***	0.6511***	0.9570***	1.0000***	1.0000***
s.e.	(0.0880)	(0.0883)	(0.1131)	(0.0281)	(0.0001)	(0.0002)
Equality Test	p-value=1		p-value=0.0166		p-value=1	
m_0= 4.5	0.2566***	0.2263	0.6423***	0.9344***	0.9931***	0.9999***
s.e.	(0.0922)	(0.0680)	(0.1255)	(0.0369)	(0.0103)	(0.0003)
Equality Test	p-value=0.5664		p-value=0.0162		p-value=0.5734	
m_0= 6	0.1782	0.0837**	0.6096***	0.7879***	1.0000***	0.9934***
s.e.	(0.1099)	(0.0371)	(0.1092)	(0.0714)	(0.0336)	(0.0061)
Equality Test	p-value=0.3000		p-value=0.0172		p-value=0.8374	
m_0= 7.5	0.0457	0.2220	0.4582***	0.4532***	1.0000***	0.9934***
s.e.	(0.1121)	(0.0145)	(0.1454)	(0.0927)	(0.0336)	(0.0061)
Equality Test	p-value=0.8926		p-value=0.9620		p-value=0.1734	
m_0= 9	0.0000	0.0051	0.2124**	0.1742***	0.9468***	0.9536***
s.e.	(0.0236)	(0.0052)	(0.0873)	(0.0645)	(0.0282)	(0.0243)
Equality Test	p-value=0.2902		p-value=0.4926		p-value=0.4944	
m_0= 10.5	0.2437	0.0028	0.1979	0.0430	0.8220***	0.8220***
s.e.	(0.1536)	(0.0034)	(0.1493)	(0.0283)	(0.0602)	(0.0599)
Equality Test	p-value=0.1714		p-value=0.3456		p-value=1	
m_0= 12	0.0000	0.0005	0.0843	0.0333	0.4977***	0.6039***
s.e.	(0.0819)	(0.0010)	(0.1132)	(0.0238)	(0.1275)	(0.0900)
Equality Test	p-value=0.4740		p-value=0.7522		p-value=0.1060	
m_0= 13.5	0.0663	0.0005	0.0663	0.0174	0.3040***	0.4636***
s.e.	(0.0787)	(0.0010)	(0.0789)	(0.0159)	(0.1109)	(0.0963)
Equality Test	p-value=0.5514		p-value=0.5942		p-value=0.0470	

***, **, * indicate significance at 1%, 5%, and 10% levels.

Table 11: Comparison Between Belief and Actual Coordination Probability
Probit Model, $G = 10$
(standard error in parentheses)

	$\lambda = 1$		$\lambda = \frac{2}{3}$		$\lambda = \frac{1}{3}$	
	Beliefs	True Choice Probability	Beliefs	True Choice Probability	Beliefs	True Choice Probability
m_0= 1.5	0.2892**	0.0291	0.6280***	0.9661***	0.7706***	1.0000***
s.e.	(0.1453)	(0.0445)	(0.1611)	(0.0555)	(0.1565)	(0.0001)
Equality Test	p-value=0.0640		p-value=0.0454		p-value=0.2086	
m_0= 3	0.2437**	0.0146	0.5953***	0.9661***	0.7927***	1.0000***
s.e.	(0.1071)	(0.0267)	(0.1357)	(0.0569)	(0.1411)	(0.0001)
Equality Test	p-value=0.0196		p-value=0.0170		p-value=0.2234	
m_0= 4.5	0.0000	0.0003	0.5536***	0.9144***	0.7853***	1.0000***
s.e.	(0.0204)	(0.0016)	(0.1416)	(0.0928)	(0.1517)	(0.0001)
Equality Test	p-value=0.3812		p-value=0.0102		p-value=0.2388	
m_0= 6	0.0000	0.0000	0.3156**	0.3055*	0.9238***	1.0000***
s.e.	(0.0235)	(0.0004)	(0.1284)	(0.1583)	(0.1205)	(0.0010)
Equality Test	p-value=0.4954		p-value=0.0888		p-value=0.7010	
m_0= 7.5	0.0303	0.0000	0.2603	0.0740	1.0000***	0.9999***
s.e.	(0.1234)	(0.0002)	(0.1765)	(0.0835)	(0.0384)	(0.0016)
Equality Test	p-value=0.8776		p-value=0.1772		p-value=0.4126	
m_0= 9	0.1161	0.0000	0.1764	0.0100	0.8673***	0.9750***
s.e.	(0.1375)	(0.0000)	(0.1492)	(0.0256)	(0.1395)	(0.0406)
Equality Test	p-value=0.5530		p-value=0.3092		p-value=0.4606	
m_0= 10.5	0.1979	0.0000	0.4461***	0.0058	0.8419 ***	0.8505***
s.e.	(0.1638)	(0.0000)	(0.1493)	(0.0180)	(0.1226)	(0.1172)
Equality Test	p-value=0.3196		p-value=0.0196		p-value=0.8380	
m_0= 12	0.1388	0.0000	0.2168*	0.0006	0.6625***	0.6627***
s.e.	(0.1298)	(0.000)	(0.1271)	(0.0051)	(0.1703)	(0.1667)
Equality Test	p-value=0.4020		p-value=0.0802		p-value=0.5810	
m_0= 13.5	0.1274	0.0000	0.1861*	0.0002	0.3992***	0.3993***
s.e.	(0.1043)	(0.0000)	(0.1125)	(0.0026)	(0.1754)	(0.1772)
Equality Test	p-value=0.2762		p-value=0.0786		p-value=0.5324	

***, **, * indicate significance at 1%, 5%, and 10% levels.

5 Conclusion

A common approach to study risk aversion and biased beliefs in experimental games is to directly eliciting preferences and beliefs. Recent papers have shown that the elicitation process may affect players' behavior in games. This paper complements the existing literature by treating utility and beliefs as unknowns and estimating them directly from choice data. Our approach requires an experimental design with multiple treatments where payoff matrices vary across treatments for some players but not others. This revealed preference/beliefs approach avoids endogeneity issues and the elicitation process, which can reduce the experimental burden. We propose different tests for the null hypothesis of unbiased (equilibrium) beliefs and present identification results on beliefs and payoff function.

We apply our test and identification results to experimental data from a matching pennies game conducted by Goeree and Holt (2001) and a coordination game studied by Heinemann, Nagel, and Ockenfels (2009). Our empirical results show that in the matching pennies game, subjects tend to correctly predict other players' behavior when other players' monetary payoffs change. In the coordination game, the estimated utility function of money has an inverted S-shape, indicating that subjects are risk loving when receiving small monetary payoffs but they become risk averse when the payoff increases. Estimated beliefs exhibit a U-shape with respect to the opportunity cost of coordination. Players' beliefs are unbiased when the opportunity cost is in the middle range while the belief becomes biased when the cost is either low or high.

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