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Inclusive versus Exclusive Markets: Search Frictions and Competing Mechanisms

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Abstract

In a market in which sellers compete for heterogeneous buyers by posting mechanisms, we analyze how the properties of the meeting technology affect the allocation of buyers to sellers. We show that *exclusive* markets (i.e. a separate submarket for each type of buyer) are the efficient outcome if and only if meetings are bilateral. In contrast, an *inclusive* market (i.e. a single market in which all buyer types pool) is optimal if and only if the meeting technology satisfies a novel condition, which we call "love for variety." Both outcomes can be decentralized by sellers posting auctions combined with a fee that is paid by (or to) all buyers with whom the seller meets. Finally, we compare love for variety to two other properties of meeting technologies, invariance and non-rivalry, and explain the differences.

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1 Introduction

One of the fundamental questions in the economic literature concerns the choice of a trading mechanism by a seller who wishes to sell a good. A large literature in mechanism design analyzes this question in the context of a monopolistic seller and often finds that auctions dominate prices. Recent work by Eeckhout and Kircher (2010) however points out that in a market setting in which sellers compete for buyers with private valuations by posting mechanisms, the process that governs meetings between buyers and sellers—i.e., the *meeting technology*—is crucially important for the choice of mechanism. If buyers randomly select a seller among the ones that maximize their expected payoff and sellers are unconstrained in the number of buyers that they can meet (urn-ball meetings), then auctions are indeed useful instruments to identify the buyer with the highest valuation. The efficient equilibrium in this case consists of a single market in which all sellers post auctions and all buyer types pool, as this maximally spreads high-type buyers across sellers. However, if sellers can only meet one buyer at a time (bilateral meetings), low-type buyers may crowd out high-type buyers. In that case, sellers prefer to post prices, which induces perfect separation of buyers into homogeneous submarkets.

Although the assumption of either bilateral or urn-ball meetings is nearly universal in the search literature[1], neither technology is necessarily an adequate description of real-life markets. In many cases, it might be more realistic to consider a technology which allows a seller to meet and learn the type of multiple but not all buyers who are interested in matching with him.[2] This raises the question how robust the above outcomes are. In this paper, we answer this question by deriving necessary and sufficient conditions on the meeting technology under which it is optimal to have (i) a separate market for each type of buyer (*exclusive* markets), or (ii) a single market in which all buyer types pool (*inclusive* market), for all type distributions.[3]

To do so, we analyze an environment based on Eeckhout and Kircher (2010), in which a continuum of buyers and sellers try to trade subject to the frictions generated by an arbitrary meeting technology. Unlike Eeckhout and Kircher (2010), who restrict attention to two types of buyers, we allow for arbitrary distributions of valuations. After describing this environment in detail in section 2, we start our analysis in section 3 by considering the trade-off of a social planner between the desire to spread high-type buyers as much as possible and the risk of them

[1]Bilateral meetings can be found in e.g. Moen (1997), Guerrieri et al. (2010), and Menzio and Shi (2011). Urn-ball is used in e.g. Peters (1997), Burdett et al. (2001), Shimer (2005), and Albrecht et al. (2014).

[2]See Fraja and Sákovics (2001), Lester and Wolthoff (2014), and Wolthoff (2015) for models of the labor market in which screening costs prevent firms from learning the type of all their applicants.

[3]Eeckhout and Kircher (2010) move beyond bilateral and urn-ball by considering general meeting technologies, but stop short of characterizing necessary and sufficient conditions for pooling or separating equilibria. We discuss the connection with their work in detail in section 4. Lester et al. (2015a) and Cai (2015) also consider general meeting technologies, but assume homogeneous buyers and random search, respectively, and are therefore silent on the optimality of inclusive or exclusive markets. Finally, in a companion paper (Cai et al., 2015), we analyze equilibrium outcomes for meeting technologies that yield neither perfect separation nor perfect pooling.

being crowded out by low-type buyers.

Our first result concerns the optimality of exclusive markets. We find that this outcome is not very robust: bilateral meetings are not only sufficient, but also necessary. That is, if one moves away from bilateral meetings by allowing a seller to meet multiple buyers (potentially with arbitrary small probability), then there exist distributions of buyer valuations for which perfect separation is no longer efficient. Intuitively, full separation does not exploit the efficiency gains that arise from sellers ranking multiple buyers: with homogenous submarkets, any meetings beyond the first are irrelevant since they always present the seller with a clone of the buyer that he has already met.

Although the necessity of bilateral meetings for exclusive markets is a new result in the literature, it is perhaps not very surprising. Most of our attention therefore goes out to the optimality of an inclusive market. We show that this outcome is more robust. To be precise, an inclusive market is the efficient outcome if and only if the meeting technology satisfies a novel condition which we call "love for variety." Loosely speaking, this condition guarantees that social surplus can be increased by merging any two submarkets, irrespective of their composition. Love for variety is satisfied by the urn-ball meeting technology, which explains why pooling is the efficient outcome in e.g. Peters and Severinov (1997), but we also describe a number of other meeting technologies that exhibit this property.

In the second half of section 3, we discuss how both the inclusive and the exclusive outcome can be decentralized by each seller posting a second-price auction, combined with a meeting fee to be paid by (or to) each buyer meeting him.[4] Intuitively, in a large market, sellers take buyers' equilibrium payoffs as given, making sellers the residual claimant on any extra surplus that they create and providing them with an incentive to post efficient mechanisms. Auctions guarantee that the good is allocated efficiently, while the meeting fees price any positive or negative externalities in the meeting process, providing all agents with a payoff equal to their social contribution.

We conclude our analysis by comparing our findings to existing results in section 4. In particular, we discuss how love for variety relates to two other properties of meeting technologies described in the literature: (i) invariance as introduced by Lester et al. (2015a), and (ii) non-rivalry as introduced by Eeckhout and Kircher (2010). We show that invariance is a sufficient (but not a necessary) condition for love for variety, while non-rivalry is a necessary (but not a sufficient) condition, and we explain why this is the case. Finally, the appendix contains all proofs and a number of additional results.

[4]As we explain in more detail below, this mechanism reduces to posted prices when meetings are bilateral.

2 Environment

Agents and Preferences. A static economy is populated by a measure 1 of sellers, indexed by $j \in [0,1]$, and a measure $\Lambda > 0$ of buyers. Both types of agents are risk-neutral. Each seller possesses a single unit of an indivisible good, for which each buyer has unit demand.[5] All sellers have the same valuation for their good, which we normalize to zero. A buyer's valuation is an independent draw from a distribution $F(x)$ with $0 \leq x \leq 1$.[6] We impose no additional structure on $F(x)$, although we will sometimes pretend that buyers have either a low valuation x_1 or a high valuation x_2 when describing the intuition behind our results.[7] Buyers observe their valuation before making any decisions. An agent's payoff is the sum of (i) his monetary transfers and (ii) his valuation if he possesses the good at the end of the period (and zero otherwise).

Search. In order to attract buyers, each seller posts and commits to a direct mechanism. A direct mechanism specifies an extensive form game that determines for each buyer i a probability of trade and an expected payment as a function of: (i) the total number n of buyers that meet with the seller; (ii) the valuation x_i that buyer i reports; and (iii) the valuations x_{-i} reported by the $n-1$ other buyers.[8]

All identical mechanisms are treated symmetrically by buyers and are therefore said to form a *submarket*. After observing all submarkets, each buyer chooses the one in which he wishes to attempt to match.[9] As standard in the literature (see e.g. Shimer, 2005), we capture the anonymity of the large market by assuming that: i) sellers can condition their strategies on the actions of buyers but not on their identities, and ii) identical buyers must use identical mixed strategies in equilibrium.

Meeting Technology. Conditional upon the choice of a submarket, meetings between buyers and sellers are governed by a frictional process, the *meeting technology*. To introduce this process, suppose a submarket is visited by a measure b of buyers and a measure s of sellers. Defining $\lambda = b/s$ as the *queue length* (or the inverse of the market tightness) in this submarket,

[5] Although we analyze a goods market, it is straightforward to cast our model in a labor market setting in which homogeneous firms post menus of wages to attract workers who differ in their productivity, as in Shi (2006). All our results carry over to such an environment.

[6] The assumption that all buyers have a (weakly) higher valuation than the seller is standard as well as innocuous. Buyers with lower valuations would simply never trade.

[7] It is worth highlighting that none of our results are driven by the requirement that they should hold for all $F(x)$. That is, our results remain the same if we consider the *weaker* requirement that they should hold for all $F(x)$ with only two points of support.

[8] In line with most of the literature (e.g. Peters, 1997; Eeckhout and Kircher, 2010; Lester et al., 2015a,b), we abstract from mechanisms that condition on other mechanisms present in the market. See Epstein and Peters (1999) for a detailed discussion of such mechanisms.

[9] The assumption that a buyer can meet only one seller per period is standard in the directed search literature. See Albrecht et al. (2006), Galenianos and Kircher (2009), Kircher (2009), Gautier and Holzner (2014) and Wolthoff (2015) for papers that relax this assumption.

we then follow Eeckhout and Kircher (2010) and Lester et al. (2015a) by specifying that a seller meets $n \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$ buyers with probability $P_n(\lambda)$.

Assumptions. We impose a number of restrictions on the meeting technology. First, it must satisfy $\sum_{n=0}^{\infty} nP_n(\lambda) \leq \lambda$, because the number of meetings cannot exceed the number of buyers in the submarket. Second, we assume that $P_n(\lambda)$ is twice-continuously differentiable. Finally, we maintain the assumption of Eeckhout and Kircher (2010) that, within each submarket, the meeting technology allocates buyers to sellers in a way that is independent of types.[10] In other words, if a measure $\mu \in [0, \lambda]$ of the buyers in the submarket are labeled "blue," then the probability for a seller to meet i blue buyers and $n - i$ other buyers equals

$$P_n(\lambda) \binom{n}{i} \left(\frac{\mu}{\lambda}\right)^i \left(1 - \frac{\mu}{\lambda}\right)^{n-i}.$$

Alternative Representation. Cai et al. (2015) show that the analysis of arbitrary meeting technologies is often greatly simplified by using an alternative representation of $P_n(\lambda)$. This alternative representation is the probability $\phi(\mu, \lambda)$ that a seller with a queue μ of blue buyers and a queue $\lambda - \mu$ of other buyers meets at least one blue buyer. We follow this approach here. Given the assumption regarding type-independent allocation of buyers, $\phi(\mu, \lambda)$ equals

$$\phi(\mu, \lambda) = 1 - \sum_{n=0}^{\infty} P_n(\lambda) \left(1 - \frac{\mu}{\lambda}\right)^n. \tag{1}$$

To simplify notation, we will often omit the arguments of ϕ and use subscripts to indicate its partial derivatives.[11]

Examples of Meeting Technologies. For future reference, it will be useful to formally define a few examples of meeting technologies that satisfy all our assumptions.

1. *Urn-Ball.* First explored by Butters (1977) and Hall (1977), this technology specifies that—within a submarket—each buyer is randomly allocated to one of the sellers. As a result, the number of buyers that meet a particular seller follows a Poisson distribution with mean equal to the queue length λ. That is, $P_n(\lambda) = e^{-\lambda} \frac{\lambda^n}{n!}$, which yields $\phi(\mu, \lambda) = 1 - e^{-\mu}$.[12]

[10] Of course, the equilibrium (or planner's) allocation of buyers to *submarkets* can depend on types.

[11] Cai et al. (2015) establish that the relation between $\phi(\mu, \lambda)$ and $P_n(\lambda)$ is one-to-one using the probability-generating function $m(z, \lambda) \equiv \sum_{n=0}^{\infty} P_n(\lambda) z^n = 1 - \phi(\lambda(1 - z), \lambda)$. In particular, it follows from the properties of probability-generating functions that $P_n(\lambda) = \frac{1}{n!} \frac{\partial^n}{\partial z^n} m(z, \lambda)\big|_{z=0} = \frac{(-\lambda)^n}{n!} \frac{\partial^n}{\partial \mu^n} (1 - \phi(\mu, \lambda))\Big|_{\mu=\lambda}$.

[12] To keep the exposition concise, we omit the (straightforward) derivation of $\phi(\mu, \lambda)$ for each example.

2. *Bilateral.* With this technology, each seller meets at most one buyer, i.e. $P_0(\lambda)+P_1(\lambda)=1$ or $\phi(\mu,\lambda)=P_1(\lambda)\frac{\mu}{\lambda}$, with $P_0(\lambda)$ strictly convex. A potential micro-foundation consists of randomly pairing agents and keeping only pairs that consist of one buyer and one seller, yielding $P_1(\lambda)=\frac{\lambda}{1+\lambda}$.[13]

3. *Pairwise Urn-Ball.* This technology, described by Lester et al. (2015a), is a variation on the urn-ball technology. Buyers first form pairs, after which each pair is randomly assigned to a seller in the submarket. That is, $P_n(\lambda)=0$ for $n\in\{1,3,5,\ldots\}$ and $P_n(\lambda)=e^{-\lambda/2}\frac{(\lambda/2)^{n/2}}{(n/2)!}$ for $n\in\{0,2,4,\ldots\}$, which implies $\phi(\mu,\lambda)=1-e^{-\mu\left(1-\frac{1}{2}\frac{\mu}{\lambda}\right)}$.

4. *Multi-Platform.* This technology consists of two platforms or rounds. In the first round, all b buyers and a fraction $0<\alpha<1$ of the s sellers in a submarket attempt to meet according to the random-pairing bilateral technology described above. The $\frac{b}{b+\alpha s}b=\frac{\lambda}{\lambda+\alpha}b$ buyers who fail to meet a seller then participate in the second round, in which they meet the remaining $(1-\alpha)s$ sellers according to an urn-ball process. That is,

$$P_n(\lambda)=\begin{cases}\alpha\frac{\alpha}{\lambda+\alpha}+(1-\alpha)e^{-\xi} & \text{if } n=0\\ \alpha\frac{\lambda}{\lambda+\alpha}+(1-\alpha)\xi e^{-\xi} & \text{if } n=1\\ (1-\alpha)\frac{\xi^n e^{-\xi}}{n!} & \text{if } n\in\{2,3,\ldots\},\end{cases}$$

where $\xi=\frac{\lambda^2}{(1-\alpha)(\lambda+\alpha)}$ is the queue length in the second round. This yields $\phi(\mu,\lambda)=\alpha\frac{\mu}{\lambda+\alpha}+(1-\alpha)\left(1-e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}}\right)$.[14]

3 Planner's Problem and Market Equilibrium

We start by analyzing the problem of a social planner whose objective is to maximize social surplus, while being subject to the frictions generated by the meeting technology. To keep the exposition as simple as possible, we initially assume that the planner knows buyers' valuations, allowing him to provide different types of buyers with different instructions. After deriving the necessary and sufficient conditions for perfect separation and perfect pooling, we then establish

[13] This micro-foundation can be found in the money search literature (see e.g. Kiyotaki and Wright, 1993). Some papers in the labor search literature provide an alternative, consisting of an urn-ball process augmented with the constraint that each seller can only contact one random buyer among the ones that wish to meet him, such that $P_1(\lambda)=1-e^{-\lambda}$ (see e.g. Albrecht et al., 2006; Galenianos and Kircher, 2009; Gautier and Wolthoff, 2009; Gautier et al., 2015).

[14] While this technology may seem more involved than the other examples, the two-round structure actually resembles the meeting process in various real-life markets: (i) buyers who cannot find a product at the local bazaar may subsequently submit a bid at an online auction site; (ii) workers who have trouble finding a job are often put in touch with firms by a public employment agency; and (iii) singles who fail to meet someone in a bar may subscribe to a dating website. Admittedly, the analogy is not perfect because terms of trade may differ across platforms in reality, which is ruled out here by the definition of a submarket.

that a planner who does not know buyers' valuations can implement the same solution. We do so by showing that this solution can be decentralized as a directed search equilibrium.

3.1 Planner's Problem

The planner's problem consists of two parts. First, the planner has to allocate buyers and sellers to submarkets, creating a queue length and a distribution of buyer types at each seller. Second, the planner has to specify how trade will take place after buyers arrive at sellers. We solve these stages in reverse order.

Trading Rule. Once a number of buyers $n \in \mathbb{N}_1 \equiv \{1, 2, 3, \ldots\}$ has arrived at a seller, surplus is clearly maximized by allocating the good to the buyer with the highest valuation. The following lemma establishes the expected surplus generated by this trading rule.

Lemma 1. *The surplus created by a seller with a queue λ of buyers whose types are distributed according to the distribution $G(x)$ equals*

$$S(\lambda, G) = \int_0^1 \phi(\lambda(1 - G(x)), \lambda)\, dx.$$

Allocation of Buyers. Now consider the allocation of buyers to sellers. For each seller $j \in [0, 1]$, the planner chooses—with a slight abuse of notation—a queue length $\lambda(j)$ and a distribution of buyer types $G(j, x)$ to maximize total surplus $\int_0^1 S(\lambda(j), G(j, x))\, dj$. Of course, the planner cannot allocate more buyers of a certain type than are available. Formally, $\int_0^1 \lambda(j)\nu(j, B)\, dj \leq \Lambda\nu_F(B)$ for any Borel-measurable set B, where ν_F is the measure associated with F and $\nu(j, \cdot)$ is the measure associated with $G(j, \cdot)$.

Exclusive Markets. We first establish that bilateral meetings are a necessary and sufficient condition for the optimality of exclusive markets.

Proposition 1. *Bilateral meetings are a necessary and sufficient condition for the planner to create a separate submarket for each type of buyer under any type distribution $F(x)$.*

Sufficiency of bilateral meetings for full separation is a well-known result in the literature: a separate submarket for each active buyer type avoids the high degree of crowding-out that arises if high-type and low-type buyers visit the same submarket and sellers meet one of both at random.[15] Necessity is however—to the best of our knowledge—a new result.[16] To understand

[15] A planner may of course keep the lowest types out of the market altogether if a marginal seller can generate more surplus in a different submarket (see e.g. Eeckhout and Kircher, 2010).

[16] Note that this result does not contradict Eeckhout and Kircher (2010) who discuss how full separation

the intuition, suppose that a seller can meet two or more buyers with positive probability. With full separation, any meetings beyond the first are irrelevant—as a seller will always meet a clone of the first buyer—and the gain in surplus relative to a bilateral technology is zero. Letting one high-type and one low-type buyer swap submarket, however, provides a way to increase surplus. After all, there is a positive probability that both these buyers meet sellers who meet other buyers as well. In that case, the buyers' joint contribution to surplus was 0 before the swap, but $x_2 - x_1$ after the swap (generated by the high-type buyer; the low-type buyer still contributes 0).[17]

Inclusive Market. To state our main result regarding the optimality of an inclusive market, we define a novel property of meeting technologies, which we call "love for variety."

Definition 1. *A meeting technology exhibits* love for variety *if and only if $\phi(\mu,\lambda)$ is concave in (μ,λ), i.e.*

$$\phi_{\mu\mu}\phi_{\lambda\lambda} \geq \phi_{\mu\lambda}^2, \tag{2}$$

for all $0 \leq \mu \leq \lambda < \infty$.[18]

The following proposition then establishes that love for variety is closely related to the optimality of an inclusive market.

Proposition 2. *Love for variety is a necessary and sufficient condition for the planner to send all agents to the same market under any type distribution $F(x)$.*

The intuition for this result is straightforward.[19] Since the surplus created by a submarket is linear in ϕ, love for variety (i.e. concavity of ϕ) implies that merging any two submarkets always leads to a higher surplus. The condition is not only sufficient but also necessary, because the result should hold for arbitrary type distributions. We provide a detailed discussion of love for variety in section 4, after considering decentralization first.

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dominates full pooling even when meetings are not strictly bilateral. The difference arises because they compare full separation against full pooling for a given (two-type) distribution, while we compare it against arbitrary other allocations for arbitrary distributions, making the set of technologies for which full separation is optimal necessarily smaller.

[17]Of course, this argument is complete only if a buyer's probability to meet a seller is the same in both submarkets, or otherwise the change in surplus associated with changing the buyers' meeting probabilities has to be taken into account. The proof of the proposition therefore focuses on the case in which both types of buyers have valuations that are arbitrarily close.

[18]Condition (2) is necessary and sufficient for concavity, since $\phi_{\mu\mu} < 0$ for all non-bilateral technologies.

[19]Here the advantage of using ϕ becomes apparent; the equivalent condition in terms of P_n, which we derive in appendix B, is far less simple and intuitive.

3.2 Market Equilibrium

The previous subsection analyzed the problem of a planner who knows buyers' valuations. This raises the question whether his solution would be different without that knowledge. In this section, we establish that this is not case by demonstrating that the solution can be decentralized as a directed search equilibrium in which each seller posts a second-price auction combined with a meeting fee τ to be paid by each buyer meeting him.[20] As the argument resembles Cai et al. (2015), we keep the exposition brief.

Equilibrium Definition. To define equilibrium, let $R(\tau, \lambda, G)$ denote the expected payoff of a seller who posts a second-price auction with a meeting fee τ and attracts a queue of buyers (λ, G). Further, let $U(x, \tau, \lambda, G)$ denote the expected payoff of a buyer with valuation x who visits this seller. Each seller aims to maximize his revenue R, but must take into account that his queue (λ, G) is endogenously determined and depends on the fee τ that he posts. To formalize this, let $\overline{U}(x)$ denote the *market utility* of a buyer with valuation x, i.e. the highest expected payoff that this buyer can obtain in equilibrium. In line with the literature, we then impose that a seller expects to be visited by buyers of type x with positive probability only if he offers them their market utility. For many meeting technologies, the market utility assumption uniquely determines the queue. In case of multiplicity, we follow Eeckhout and Kircher (2010) and assume that sellers believe that they will attract the queue that is most favorable to them.[21] We can now define an equilibrium as follows.

Definition 2. *A directed search equilibrium is a second-price auction, a meeting fee* $\tau(j)$ *and a queue* $(\lambda(j), G(j,x))$ *for each seller* $j \in [0,1]$*, combined with a market utility* $\overline{U}(x)$ *for each type of buyer* x*, such that ...*

1. *each tuple* $(\tau(j), \lambda(j), G(j,x))$ *maximizes* $R(\tau, \lambda, G)$ *subject to* $U(x, \tau, \lambda, G) \leq \overline{U}(x)$*, with equality for each* x *in the support of* G*;*

2. *aggregating queues across sellers does not exceed the total measure of buyers of each type.*

Decentralization. The following proposition establishes that the solution to the planner's problem characterized in the previous subsection can always be decentralized as a directed search equilibrium.

Proposition 3. *For any meeting technology, the planner's solution* $\{\lambda(j), G(j,x)\}$ *can be decentralized as a directed search equilibrium in which seller* $j \in [0,1]$ *posts a second-price*

[20] The meeting fee can be negative in equilibrium, turning it into a meeting subsidy paid *to* each buyer.

[21] We discuss relaxation of this assumption in detail in Cai et al. (2015).

auction and a meeting fee equal to

$$\tau(j) = -\frac{\int_0^1 \phi_\lambda(\lambda(j)(1-G(j,x)), \lambda(j))\, dx}{\phi_\mu(0, \lambda(j))}. \tag{3}$$

The intuition for this result is similar to the intuition for efficiency in many other directed search models. Since sellers take buyers' equilibrium payoffs as given, they are the residual claimant on any surplus that they create. This provides them with an incentive to post mechanisms that decentralize the planner's solution, which requires efficiency along two margins: (i) the allocation of buyers to sellers, and (ii) the allocation of the good given a queue of buyers.

The second-price auction fulfills the second requirement and provides each buyer with a payoff equal to the extra surplus that he creates when he has the highest valuation. To satisfy the first requirement however, each buyer must receive an expected payoff exactly equal to his marginal contribution to social surplus, which includes the externality that he may impose during the meeting process (e.g. by preventing a buyer with a higher valuation from meeting the seller). Because this externality is type-independent, it can be priced by the meeting fee (3), which equals the (negative of) the spillovers that a buyer imposes on other buyers (the numerator) conditional on the event that he meets a seller (the probability of which is given by the denominator).[22]

Posted Prices versus Auctions. It is worth highlighting that the equilibrium mechanism nests two popular trading mechanisms as special cases. As we discuss in more detail in the next section, technologies that exhibit love for variety give rise to meeting fees that are non-positive. For a subset of those technologies, the equilibrium meeting fee is exactly zero, reducing the equilibrium mechanism to a *standard auction*. In contrast, when meetings are bilateral, the second-price auction does not generate any revenue and the meeting fees, which are then strictly positive, act as *posted prices*.

4 Discussion

Bilateral meetings are well understood, but love for variety is a novel condition and warrants discussion. To better understand this condition, we compare it in this section to two other properties of meeting technologies described in the literature, *invariance* and *non-rivalry*. We show

[22] The equilibrium is of course not unique. Because of risk neutrality and revenue equivalence, a seller could replace the second-price by a first-price auction, in which buyers may either know or not know the number of competing bidders (see e.g. McAfee and McMillan, 1987). Likewise, the seller could condition the meeting fee on the number of buyers that shows up. However, all equilibrium mechanisms give rise to the same *expected* payoffs. See e.g. Peters and Severinov (1997), Albrecht et al. (2014), Lester et al. (2015a) for detailed discussions of efficiency in related models.

that invariance is a sufficient (but not a necessary) condition, while non-rivalry is a necessary (but not a sufficient) condition. Figure 1 summarizes this discussion.

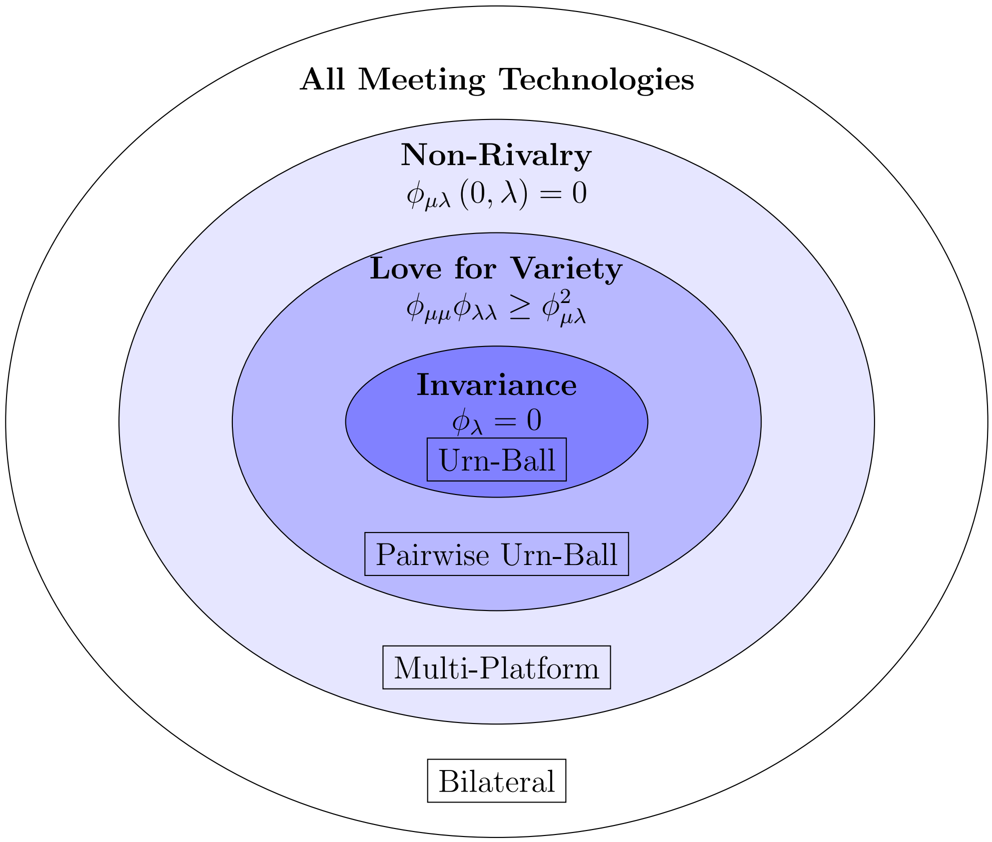

Figure 1: Classification of Meeting Technologies

Invariance. Introduced by Lester et al. (2015a), an invariant technology is one in which the queue of blue buyers μ at a seller is a sufficient statistic for the distribution of the number of meetings between blue buyers and that seller.[23] Formally,

$$\sum_{N=n}^{\infty} P_N(\lambda) \binom{N}{n} \left(\frac{\mu}{\lambda}\right)^n \left(1 - \frac{\mu}{\lambda}\right)^{N-n} = P_n(\mu), \tag{4}$$

for all $0 \leq \mu \leq \lambda < \infty$ and $n \in \mathbb{N}_0$. In the proof of Lemma 2, we establish that if (4) holds for $n = 0$, then it holds for all n. That is, invariance can alternatively be defined as the condition that the probability that a seller meets *at least one* of the μ blue buyers is independent of the number of other buyers visiting the same submarket.

Lemma 2. *A meeting technology is invariant if and only if* $\phi_{\lambda}(\mu, \lambda) = 0$ *for all* $0 \leq \mu \leq \lambda < \infty$.

[23] Cai (2015) shows that this property drives the efficiency of vacancy creation with wage posting if unemployed and employed workers search equally efficient in the model of Gautier et al. (2010).

Perhaps the best-known example of an invariant technology is the urn-ball technology.[24]. As shown by e.g. McAfee (1993), Peters and Severinov (1997) and Albrecht et al. (2014), this technology leads to an inclusive market in equilibrium.[25] The following lemma extends this result to all invariant technologies and establishes that while invariance is a sufficient condition for love for variety, it is not a necessary condition.

Proposition 4. *Invariance implies love for variety, but love for variety does not imply invariance.*

It is straightforward to see why invariance is sufficient. Invariance implies that the presence of low-type buyers in the submarket has no effect on the meetings between high-type buyers and sellers. Surplus is therefore maximized by spreading high-type buyers evenly across all sellers, as opposed to concentrating them at a subset, in order to maximize the number of high-type buyers that will trade. A single market results.

To understand why invariance is not necessary, consider the pairwise urn-ball technology. As explained by Lester et al. (2015a), this technology is not invariant. Intuitively, when there are very few low-type buyers in the submarket, most buyer pairs consist of two high types, making it likely that a seller will meet an even number of buyers with high valuations. Adding additional low-type buyers to this submarket increases the probability that a buyer pair will consist of one low and one high type, and that a seller will meet an odd number of high-type buyers. This makes it more likely that a seller will meet at least one high-type buyer, i.e. $\phi_\lambda > 0$. This feature violates invariance, but not love for variety: the fact that the addition of low-type buyers to the submarket helps to spread the high-type buyers better across sellers strengthens the incentive to send all buyers to the same market.[26]

Non-Rivalry. Eeckhout and Kircher (2010) define a (purely) non-rival technology as one in which a buyer's probability to meet one of the sellers is not affected by the presence of other buyers in the market. We first establish that their definition is equivalent to $\phi_{\mu\lambda}(0, \lambda) = 0$ for all $0 \leq \lambda < \infty$.[27]

Lemma 3. *A meeting technology is non-rival if and only if* $\phi_{\mu\lambda}(0, \lambda) = 0$ *for all* $0 \leq \lambda < \infty$.

To understand this expression, recall that $\phi(\mu, \lambda)$ represents the probability that a seller meets at least one blue buyer, which is clearly zero if $\mu = 0$. The partial derivative $\phi_\mu(0, \lambda)$

[24]Recall that urn-ball implies $\phi(\mu, \lambda) = 1 - e^{-\mu}$. As discussed in Lester et al. (2015a), a second example of an invariant technology is the geometric distribution $P_n(\lambda) = \frac{\lambda^n}{(1+\lambda)^{n+1}}$, which yields $\phi(\mu, \lambda) = \frac{\mu}{1+\mu}$.

[25]Shi (2006) derives a similar result in a labor-market setting.

[26]This may raise the question how $\phi_\lambda \geq 0$ relates to love for variety. We prove in appendix B that it is a necessary but not a sufficient condition.

[27]Note that non-rivalry uses the point of view of a buyer. It is therefore *not* equivalent to a seller's probability to meet at least one buyer, which is $1 - P_0(\lambda)$, being independent of λ.

captures how this changes if a single buyer (or more precisely, an arbitrarily small measure of buyers) in the queue becomes blue and must therefore equal the probability that this blue buyer succeeds in meeting the seller. Since meetings are type-independent, the same expression applies to all λ buyers in the queue, irrespective of how many of them are blue. Non-rivalry then says that this meeting probability should be independent of λ.

It is easy to verify that the above examples of technologies that exhibit love for variety, i.e. urn-ball and pairwise urn-ball, both satisfy non-rivalry. This is not a coincidence. As the following proposition establishes, all technologies that exhibit love for variety are non-rival. However, not all non-rival technologies exhibit love for variety.

Proposition 5. *Love for variety implies non-rivalry, but non-rivalry does not imply love for variety.*

To understand why non-rivalry is a necessary condition for love for variety, consider a submarket with a single high-type buyer with valuation $x_2 > 0$ and a number of low-type buyers with valuation $x_1 \to 0$, such that surplus only depends on the trading probability of the high-type buyer. Violation of non-rivalry would imply that this probability could be increased by sending either some low-type buyers (if $\phi_{\mu\lambda}(0, \lambda) < 0$) or some sellers (if $\phi_{\mu\lambda}(0, \lambda) > 0$) to a different submarket, contradicting the optimality of the single market associated with love for variety.

To see why non-rivalry is not sufficient, consider the multi-platform technology. Clearly, every buyer meets a seller with probability 1, which means that this technology is non-rival. However, the presence of low-type buyers in the submarket increases the chances for high-type buyers to be crowded out at one of the αs sellers in the first round, concentrating them at the $(1 - \alpha) s$ second-round sellers in higher numbers than optimal. It is therefore better to send at least some low types to a separate submarket. Hence, non-rivalry does not imply love for variety.[28]

5 Conclusion

We study an environment in which sellers compete for heterogeneous buyers by posting mechanisms. Buyers can direct their search to the mechanism that maximizes their expected payoff, but may experience frictions in meeting a particular seller. We derive necessary and sufficient conditions on the technology that governs these meetings under which either exclusive markets (a separate submarket for each type of buyer) or an inclusive market (a single market in

[28]This contradicts proposition 5 in Eeckhout and Kircher (2010) which states that non-rivalry is a sufficient condition for a single market. The discrepancy originates in the fact that the proof of their proposition implicitly assumes invariance rather than non-rivalry when treating the trading probability for high-type buyers as independent of the queue of low-type buyers.

which all buyers pool) are optimal. We find that exclusive markets are the efficient equilibrium outcome if and only if meetings are bilateral, while an inclusive market arises if the meeting technology satisfies a novel property, which we call "love for variety."

Appendix A Proofs

Proof of Lemma 1. The maximum valuation at a seller who meets $n \in \mathbb{N}_1$ buyers is an order statistic, distributed according to $G^n(x)$. Taking the expectation over x and n, followed by integration by parts and using the Dominated Convergence Theorem to interchange summation and integration, yields

$$S(\lambda, G) = \sum_{n=1}^{\infty} P_n(\lambda) \int_0^1 x\, dG^n(x) = \int_0^1 \left(1 - \sum_{n=0}^{\infty} P_n(\lambda) G^n(x)\right) dx.$$

The result then follows because the rightmost integrand equals $\phi(\lambda(1 - G(x)), \lambda)$. □

Proof of Proposition 1. Part 1 (bilateral meetings imply full separation): Suppose that there exists a submarket with a measure s of sellers and a queue (λ, G). Because of lemma 1 and the fact that meetings are bilateral, the surplus created in this submarket equals

$$sS(\lambda, G) = sP_1(\lambda) \int_0^1 (1 - G(x))\, dx. \tag{5}$$

Now suppose the planner would decompose this submarket into a separate submarket for each type of buyer, allocating sellers in such a way that the queue length in each new submarket remains λ. A seller in the submarket for type x then creates a surplus $P_1(\lambda)x$, such that surplus across all submarkets equals

$$s \int_0^1 P_1(\lambda)\, x\, dG(x). \tag{6}$$

Clearly, (5) and (6) are equal to each other. The result then follows because the allocation of sellers in (6) is suboptimal—the planner can increase surplus by allocating more sellers to the submarkets in which buyers have high valuations than to the submarkets where they have low valuations.

Part 2 (full separation implies bilateral meetings): We prove this result by showing that if meetings are not bilateral for some $\Lambda > 0$, i.e. $P_0(\Lambda) + P_1(\Lambda) < 1$, then there exists a two-type distribution of buyers such that full separation is not optimal.[29] To do so, suppose the market

[29]Of course, if $P_0(\Lambda) + P_1(\Lambda) < 1$, then—by continuity—there exists a small neighborhood of Λ for which $P_0 + P_1 < 1$.

is populated by a measure 1 of sellers, a measure b_1 of buyers with valuation x_1, and a measure b_2 of buyers with valuation x_2, satisfying $b_1 + b_2 = \Lambda$ and $x_2 > x_1$.

Suppose the planner fully separates the two types of buyers and optimally allocates s_i sellers to the submarket for valuation x_i, where $s_1 + s_2 = 1$. Define queue lengths $\lambda_i = \frac{b_i}{s_i}$. Clearly, if $x_1 \to x_2$, then $s_1 \to \frac{b_1}{b_1+b_2}$ and $s_2 \to \frac{b_2}{b_1+b_2}$, which implies $\lambda_1 \to \Lambda$ and $\lambda_2 \to \Lambda$.

Let now a measure ε of buyers with valuation x_1 and an equally large measure of buyers with valuation x_2 swap submarket, such that—in both submarkets—the queue lengths stay the same, but the composition of types becomes marginally more diverse. Again by lemma 1, social surplus of this new allocation equals

$$\begin{aligned}\mathcal{S}(\varepsilon) &= s_2\left[(x_2 - x_1)\,\phi\left(\frac{b_2-\varepsilon}{s_2}, \lambda_2\right) + x_1\phi(\lambda_2, \lambda_2)\right] \\ &\quad + s_1\left[(x_2 - x_1)\,\phi\left(\frac{\varepsilon}{s_1}, \lambda_1\right) + x_1\phi(\lambda_1, \lambda_1)\right].\end{aligned}$$

Clearly, $\varepsilon = 0$ corresponds to full separation. To analyze whether this is the optimal outcome, consider the change in surplus associated with a marginal increase in ε, i.e.

$$\mathcal{S}'(0) = (x_2 - x_1)\left(\phi_\mu(0, \lambda_1) - \phi_\mu(\lambda_2, \lambda_2)\right) \tag{7}$$

Note that

$$\phi_\mu(\mu, \lambda) = \frac{P_1(\lambda)}{\lambda} + \frac{1}{\lambda}\sum_{n=2}^{\infty} nP_n(\lambda)\left(1 - \frac{\mu}{\lambda}\right)^{n-1},$$

which implies that $\phi_\mu(0, \lambda) \geq \frac{P_1(\lambda)}{\lambda} + \frac{2(1-P_0(\lambda)-P_1(\lambda))}{\lambda}$ and $\phi_\mu(\lambda, \lambda) = \frac{P_1(\lambda)}{\lambda}$. Substitution into (7) yields

$$\frac{1}{x_2 - x_1}\mathcal{S}'(0) \geq \frac{P_1(\lambda_1)}{\lambda_1} + \frac{2(1 - P_0(\lambda_1) - P_1(\lambda_1))}{\lambda_1} - \frac{P_1(\lambda_2)}{\lambda_2}.$$

Let $x_1 \to x_2$, such that $\lambda_1 \to \Lambda$ and $\lambda_2 \to \Lambda$. Then

$$\frac{1}{x_2 - x_1}\mathcal{S}'(0) \to \frac{2(1 - P_0(\Lambda) - P_1(\Lambda))}{\Lambda} > 0.$$

Hence, full separation is not optimal and social surplus can be increased by slightly mixing the submarkets. □

Proof of Proposition 2. Part 1 (love for variety implies perfect pooling): To prove this result, suppose that there are two submarkets, indexed by $i \in \{1, 2\}$, consisting of $s_i > 0$ sellers who each have a queue (λ_i, G_i). By lemma 1, total surplus across the two submarkets is equal to

$$s_1 \int_0^1 \phi\left(\lambda_1\left(1 - G_1(x)\right), \lambda_1\right)\, dx + s_2 \int_0^1 \phi\left(\lambda_2\left(1 - G_2(x)\right), \lambda_2\right)\, dx. \tag{8}$$

We show a higher surplus can be generated by merging the two submarkets, creating one market with $s_0 = s_1 + s_2$ sellers, each with a queue $\lambda_0 = \frac{s_1\lambda_1 + s_2\lambda_2}{s_1+s_2}$ of buyers whose valuations are distributed according to

$$G_0(x) = \frac{s_1\lambda_1 G_1(x) + s_2\lambda_2 G_2(x)}{s_1\lambda_1 + s_2\lambda_2}.$$

Again by lemma 1, this combined market will create a surplus $s_0 \int_0^1 \phi\left(\lambda_0\left(1 - G_0(x)\right), \lambda_0\right)\, dx$, which is larger than (8) because concavity of $\phi(\mu, \lambda)$ implies that

$$s_1\phi(\mu_1, \lambda_1) + s_2\phi(\mu_2, \lambda_2) \le s_0\phi\left(\frac{s_1\mu_1 + s_2\mu_2}{s_1+s_2}, \frac{s_1\lambda_1 + s_2\lambda_2}{s_1+s_2}\right).$$

Hence, a single market is optimal for technologies that exhibit love for variety.

Part 2 (perfect pooling implies love for variety): We prove this result by showing that if ϕ is not concave, there exists a two-type distribution of buyers such that one market is not optimal. Note that if ϕ is not concave, then—by the definition of concavity—there exist values α, μ_1, μ_2, λ_1 and λ_2, such that

$$\alpha\phi(\mu_1, \lambda_1) + (1-\alpha)\phi(\mu_2, \lambda_2) > \phi(\mu_0, \lambda_0), \tag{9}$$

where $\mu_0 = \alpha\mu_1 + (1-\alpha)\mu_2$ and $\lambda_0 = \alpha\lambda_1 + (1-\alpha)\lambda_2$.

Consider now a market in which buyers' valuations are either x_1 or x_2, with $0 < x_1 < x_2$. Set the measure of high-type buyers equal to μ_0 and the measure of low-type buyers equal to $\lambda_0 - \mu_0$, while maintaining the assumption that the measure of sellers equals 1. Then by Lemma 1, the social surplus of creating a single market is $\mathcal{S}_1 = (x_2 - x_1)\phi(\mu_0, \lambda_0) + x_1\phi(\lambda_0, \lambda_0)$.

Now, decompose the single market into two submarkets A and B, with seller measures α and $1-\alpha$, total queue lengths λ_1 and λ_2, and high-type queue lengths μ_1 and μ_2, respectively.[30] The social surplus per seller for the two submarkets is

$$S_2^A = (x_2 - x_1)\phi(\mu_1, \lambda_1) + x_1\phi(\lambda_1, \lambda_1)$$
$$S_2^B = (x_2 - x_1)\phi(\mu_2, \lambda_2) + x_1\phi(\lambda_2, \lambda_2)$$

and total surplus across the two submarkets equals $\mathcal{S}_2 = \alpha S_2^A + (1-\alpha)S_2^B$.

In the limit $x_1 \to 0$, the two submarkets create more surplus than the single market, i.e.

[30] This is possible because $\mu_0 = \alpha\mu_1 + (1-\alpha)\mu_2$ and $\lambda_0 = \alpha\lambda_1 + (1-\alpha)\lambda_2$.

$\mathcal{S}_2 > \mathcal{S}_1$, if and only if

$$x_2\left(\alpha\phi(\mu_1,\lambda_1)+(1-\alpha)\phi(\mu_2,\lambda_2)\right) > x_2\phi(\mu_0,\lambda_0),$$

which holds because it is exactly equation (9). Hence, love for variety is a necessary condition for a single market. □

Proof of Proposition 3. This result follows directly from Cai et al. (2015). For completeness, we also give a short proof here. This proof consists of two parts. First, we consider a seller who can choose the length and composition of his queue directly in a competitive market ("relaxed maximization problem"). By the first welfare theorem, the equilibrium in this market is Pareto optimal, which necessarily implies that it maximizes social net output as there is only one consumption good. Subsequently, we establish that a seller who posts the proposed equilibrium mechanism to attract an endogenously determined queue of buyers ("constrained maximization problem") implements the same solution.

Part 1 (relaxed maximization problem): For a given market utility function $\overline{U}(x)$, a seller chooses the queue (λ, G) that maximizes his expected payoff, which equals the difference between surplus $S(\lambda, G)$ and the expected payoff that the seller has to offer to each of the buyers. That is,

$$\int_0^1 \phi\left(\lambda\left(1-G\left(z\right)\right),\lambda\right)\,dz - \int_0^1 \overline{U}\left(z\right)\,d\lambda G(z).$$

Because the seller takes the market utility function as given, he is a residual claimant on any extra surplus that he creates. Hence, the seller will compare the marginal cost $\overline{U}(x)$ of attracting a buyer with valuation x to this buyer's marginal contribution to surplus $T(x)$. To calculate $T(x)$, increase the measure of buyers with values around x, formally $[x, x+\Delta x]$, by ε and denote the new market tightness and buyer value distribution as λ' and G' respectively. That is, $\lambda' = \lambda+\varepsilon$, while $\lambda'(1-G'(z)) = \lambda(1-G(z))$ for $z > x$ and $\lambda'(1-G'(z)) = \lambda(1-G(z))+\varepsilon$ for $z < x$. By Lemma 1, the average contribution to surplus by buyers with values around x is

$$\begin{aligned}\frac{S(\lambda',G')-S(\lambda,G)}{\varepsilon} &= \frac{1}{\varepsilon}\left(\int_0^x \phi(\lambda(1-G(x))+\varepsilon,\lambda+\varepsilon)-\phi(\lambda(1-G(x)),\lambda)\right)\\ &+\frac{1}{\varepsilon}\left(\int_x^1 \phi(\lambda(1-G(x)),\lambda+\varepsilon)-\phi(\lambda(1-G(x)),\lambda)\right)\end{aligned}$$

Let $\varepsilon \to 0$, then the above equation converges to

$$T(x) = \int_0^1 \phi_\lambda\left(\lambda\left(1-G\left(z\right)\right),\lambda\right)\,dz + \int_0^x \phi_\mu\left(\lambda\left(1-G\left(z\right)\right),\lambda\right)\,dz. \tag{10}$$

The solution to the relaxed maximization problem must therefore satisfy

$$\overline{U}(x) \geq T(x) \text{ for all } x, \text{ with equality for all } x \in \operatorname{supp} G \tag{11}$$

Part 2 (constrained maximization problem): Consider now a seller who posts a second-price auction and a meeting fee τ, attracting a queue (λ, G). A buyer with valuation x in the support of G meets the seller together with $n-1$ other buyers with probability $\frac{nP_n(\lambda)}{\lambda}$.[31] Hence, he pays the meeting fee τ with probability $\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda) = \phi_\mu(0,\lambda)$ and trades with probability $\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda)\, G(x)^{n-1} = \phi_\mu(\lambda(1-G(x)),\lambda)$. As a result, his expected payoff is

$$U(x,\tau,\lambda,G) = -\phi_\mu(0,\lambda)\,\tau + \int_0^x \phi_\mu(\lambda(1-G(y)),\lambda)\, dy, \tag{12}$$

where the second term is the payoff from the auction, which—by standard results in auction theory—equals the integral over the trading probabilities (see e.g. Peters, 2013). A queue (λ, G) is therefore compatible with an auction with fee τ if and only if

$$\overline{U}(x) \geq U(x,\tau,\lambda,G) \text{ for all } x, \text{ with equality for all } x \in \operatorname{supp} G \tag{13}$$

Clearly, if a queue (λ, G) satisfies (11), then by setting the entry fee τ in equation (12) equal to

$$\tau = -\frac{\int_0^1 \phi_\lambda(\lambda(1-G(x)),\lambda)\, dx}{\phi_\mu(0,\lambda)},$$

it also satisfies (13). Therefore, any queue chosen by an unconstrained seller who can buy queues directly at a price $\overline{U}(x)$ is also compatible with an auction with an entry fee. □

Proof of Lemma 2. **Part 1** (invariance implies $\phi_\lambda = 0$): Evaluating the definition of invariance (4) in $n = 0$ yields

$$\sum_{N=0}^{\infty} P_N(\lambda)\left(1-\frac{\mu}{\lambda}\right)^N = P_0(\mu). \tag{14}$$

The left-hand side of this equation is $1-\phi(\mu,\lambda)$ and the right-hand side is independent of λ. Hence, $\phi_\lambda(\mu,\lambda) = 0$ for all $0 \leq \mu \leq \lambda < \infty$.

Part 2 ($\phi_\lambda = 0$ implies invariance): Note that $\phi_\lambda(\mu,\lambda) = 0$ for all $0 \leq \mu \leq \lambda < \infty$ implies that $\phi(\mu,\lambda) = \phi(\mu,\mu)$. By equation (1), $\phi(\mu,\mu) = 1 - P_0(\mu)$. Consequently, equation (14) must hold for all $0 \leq \mu \leq \lambda < \infty$. By standard results from analytic function theory (see e.g. Ahlfors, 1979, p.32), we can differentiate both sides of this equation n times with respect to μ,

[31] See Eeckhout and Kircher (2010) or Lester et al. (2015a).

which yields

$$\sum_{N=n}^{\infty} \frac{N!}{(N-n)!} P_N(\lambda) \left(-\frac{1}{\lambda}\right)^n \left(1-\frac{\mu}{\lambda}\right)^{N-n} = P_0^{(n)}(\mu), \tag{15}$$

for all $0 \leq \mu \leq \lambda < \infty$. For $\mu = \lambda$, this gives $P_0^{(n)}(\mu) = \frac{n!}{(-\mu)^n} P_n(\mu)$. Substitute this into the right hand side of equation (15) and rearrange the term $\frac{n!}{(-\mu)^n}$ to the left hand side gives (4). Hence, $\phi_\lambda = 0$ implies invariance. □

Proof of Proposition 4. Part 1 (invariance implies love for variety): This result follows immediately from lemma 2: $\phi_\lambda(\mu, \lambda) = 0$ for all $0 \leq \mu \leq \lambda < \infty$ implies that $\phi_{\lambda\lambda}(\mu, \lambda) = \phi_{\mu\lambda}(\mu, \lambda) = 0$ for all $0 \leq \mu \leq \lambda < \infty$, which in turn implies that equation (2) is satisfied.

Part 2 (love for variety does not imply invariance): Consider the pairwise urn-ball technology, which satisfies $\phi(\mu, \lambda) = 1 - e^{-\mu\left(1-\frac{1}{2}\frac{\mu}{\lambda}\right)}$. Since $1 - e^{-y}$ is an increasing, concave function, a sufficient condition for $\phi(\mu, \lambda)$ to be concave is that the map $(\mu, \lambda) \to \mu(1 - \frac{1}{2}\frac{\mu}{\lambda})$ is concave.[32] The Hessian of this map is indeed negative semi-definite. However, the technology is not invariant, as

$$\phi_\lambda(\mu, \lambda) = \frac{1}{2}\frac{\mu^2}{\lambda^2} e^{-\mu\left(1-\frac{1}{2}\frac{\mu}{\lambda}\right)} > 0.$$

Hence, love for variety does not imply invariance. □

Proof of Lemma 3. As shown by Lester et al. (2015a), non-rivalry is satisfied if and only if $\frac{\partial}{\partial\lambda}\frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda) = 0$, for all $0 \leq \lambda < \infty$. The desired result then follows directly from observing that $\phi_\mu(0, \lambda) = \frac{1}{\lambda}\sum_{n=1}^{\infty} nP_n(\lambda)$. □

Proof of Proposition 5. Part 1 (love for variety implies non-rivalry): Suppose a technology does not satisfy non-rivalry, i.e. $\phi_{\mu\lambda}(0, \lambda) \neq 0$. Then

$$\phi_{\mu\mu}(0, \lambda)\,\phi_{\lambda\lambda}(0, \lambda) - \phi_{\mu\lambda}^2(0, \lambda) < \phi_{\mu\mu}(0, \lambda)\,\phi_{\lambda\lambda}(0, \lambda) = 0,$$

since $\phi(0, \lambda) = 0$ for all $0 \leq \lambda < \infty$. In words, ϕ is not concave. Hence, love for variety implies non-rivalry.

Part 2 (non-rivalry does not imply love for variety): Consider the multi-platform technology. Starting from the expression for $\phi(\mu, \lambda)$ for this technology, one can derive

$$\phi_{\mu\lambda} = -\left(1 - e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}} - \frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)} e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}}\right) \frac{\alpha}{(\lambda+\alpha)^2} \leq 0,$$

which equals 0 (only) for $\mu = 0$. Hence, the multi-platform technology is non-rival.

[32]See, for example, Theorem 5.1 in Rockafellar (1970, p.32).

Further, we get

$$\phi_\lambda = -\left(1 - e^{-\frac{\lambda\mu}{(1-\alpha)(\lambda+\alpha)}}\right) \frac{\mu\alpha}{(\lambda+\alpha)^2},$$

which is strictly negative for all $0 < \mu \leq \lambda < \infty$ and $\alpha > 0$. As we show in the online appendix, $\phi_\lambda \geq 0$ is a necessary condition for love for variety. Hence, the multi-platform technology does not exhibit this property. □

Appendix B Additional Results

Love for Variety Using P_n

In the main text, we define love for variety in terms of ϕ, but an equivalent condition in terms of P_n, the actual primitive of the model, can be derived.[33] Starting from the definition (1), taking partial derivatives of ϕ yields

$$\phi_{\mu\mu} = -\sum_{n=0}^{\infty}(n+2)(n+1)\frac{P_{n+2}}{\lambda^2}\left(1-\frac{\mu}{\lambda}\right)^n,$$

$$\phi_{\mu\lambda} = \sum_{n=0}^{\infty}\left[(n+1)\frac{\lambda P'_{n+1} - P_{n+1}}{\lambda^2} + (n+2)(n+1)P_{n+2}\frac{\mu}{\lambda^3}\right]\left(1-\frac{\mu}{\lambda}\right)^n,$$

$$\phi_{\lambda\lambda} = -\sum_{n=0}^{\infty}\left[P''_n + 2\mu(n+1)\frac{\lambda P'_{n+1} - P_{n+1}}{\lambda^3} + \frac{(n+2)(n+1)P_{n+2}\mu^2}{\lambda^4}\right]\left(1-\frac{\mu}{\lambda}\right)^n.$$

Using the fact that $\sum_{n=0}^{\infty} a_n y^n \sum_{n=0}^{\infty} b_n y^n - \left(\sum_{n=0}^{\infty} c_n y^n\right)^2 = \sum_{n=0}^{\infty}\sum_{i=0}^{n}\left(a_i b_{n-i} - c_i c_{n-i}\right) y^n$, condition (2) can then be written as

$$\phi_{11}\phi_{22} - \phi_{12}^2 = \sum_{n=0}^{\infty} Z_n \left(1-\frac{\mu}{\lambda}\right)^n \geq 0,$$

where, after some simplification, Z_n equals

$$Z_n = \sum_{i=0}^{n}\left[\frac{(i+2)(i+1)P_{i+2}P''_{n-i}}{\lambda^2} - (i+1)(n-i+1)\frac{\lambda P'_{i+1} - P_{i+1}}{\lambda^2}\frac{\lambda P'_{n-i+1} - P_{n-i+1}}{\lambda^2}\right].$$

Necessity and Insufficiency of $\phi_\lambda \geq 0$

The fact that the pairwise urn-ball technology satisfies $\phi_\lambda \geq 0$ as well as love for variety may raise the question how these two properties are related. The following proposition establishes that that $\phi_\lambda(\mu,\lambda) \geq 0$ for all $0 \leq \mu \leq \lambda < \infty$ is a necessary but not a sufficient condition for

[33] To save on notation, we suppress the argument of P_n throughout this derivation.

love for variety.

Proposition 6. *Love for variety implies $\phi_\lambda \geq 0$, but $\phi_\lambda \geq 0$ does not imply love for variety.*

Proof. **Part 1** (love for variety implies $\phi_\lambda \geq 0$): We prove this result by contradiction. Suppose that there exists a meeting technology for which $\phi(\mu,\lambda)$ is concave in (μ,λ), but $\phi_\lambda(\mu_0,\lambda_0) < 0$ in some point (μ_0,λ_0). Note that $\phi_{\mu\mu} < 0$ for all technologies that exhibit love for variety, hence $\phi(\mu,\lambda)$ must also be concave in λ alone, i.e. $\phi_{\lambda\lambda} \leq 0$. In other words, $\phi_\lambda(\mu,\lambda)$ is a non-increasing function of λ, such that $\phi_\lambda(\mu_0,\lambda) \leq \phi_\lambda(\mu_0,\lambda_0) < 0$ for all $\lambda > \lambda_0$. This implies that $\phi(\mu_0,\lambda) \leq \phi(\mu_0,\lambda_0) + \phi_\lambda(\mu_0,\lambda_0)(\lambda-\lambda_0)$ for all $\lambda > \lambda_0$. Let $\lambda \to \infty$ and thus $\phi_\lambda(\mu_0,\lambda_0)(\lambda-\lambda_0) \to -\infty$, such that $\phi(\mu_0,\lambda) \to -\infty$. Since ϕ is a probability, this leads to the required contradiction. Hence, concavity of $\phi(\mu,\lambda)$, i.e. love for variety, implies $\phi_\lambda \geq 0$.

Part 2 ($\phi_\lambda \geq 0$ does not imply love for variety): Consider the following technology.

> *Minimum Demand.* This technology consists of two rounds. In the first round, the b buyers in the submarket are allocated to the s sellers according to the urn-ball technology. In the second round, each seller draws a minimum demand requirement and operates only if the number of buyers that came to him weakly exceeds this minimum.[34] We assume that the minimum demand requirements follow a geometric distribution, such that the minimum is weakly less than $n \in \mathbb{N}_1$ with probability $1-(1-\psi)^n$ for $0<\psi<1$. Hence, $P_n(\lambda) = e^{-\lambda}\frac{\lambda^n}{n!}\left(1-(1-\psi)^n\right)$ for $n \in \mathbb{N}_1$ and $P_0(\lambda) = 1 - \sum_{n=1}^{\infty} P_n(\lambda) = e^{-\psi\lambda}$.

This technology gives $\phi(\mu,\lambda) = 1 - e^{-\mu} - e^{-\psi\lambda} + e^{-\lambda\psi-\mu(1-\psi)}$. Hence, $\phi_\lambda = \psi e^{-\psi\lambda}\left(1-e^{-\mu(1-\psi)}\right)$, which is strictly positive. However, the determinant of the Hessian of ϕ, evaluated in $\mu = 0$, equals $-\psi^2(1-\psi)^2 e^{-2\lambda\psi} < 0$, which means that ϕ is not concave. Hence, $\phi_\lambda \geq 0$ does not imply love for variety. □

[34] Geromichalos (2012) analyzes minimum demand requirements in a different context. Minimum class size requirements are also common in the matching between students and schools.

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