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Gains from Trade under Quality Uncertainty

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Abstract

We add quality uncertainty to a two-country trade model with CES preference and monopolistic competition. There are two kinds of firms - low quality and high quality. Quality is perfectly observable in the domestic market but not in the foreign market. Exporters use price to signal their quality. It is now well-established that in such a model with full information, the welfare gains from trade (GFT) can be captured by a sufficient statistic that depends on domestic trade share and the elasticity of substitution. In contrast, in a model with asymmetric information, we show that within the class of separating equilibria, the sufficient statistic always under-estimates GFT, while within the class of pooling equilibria, the sufficient statistic could over-estimate GFT. Nevertheless, GFT are always positive. For an equilibrium refinement, we analyze the determinants of GFT. We show that the actual GFT under asymmetric information could be almost 2.5 times higher than that measured using the sufficient statistic approach.

KEYWORDS : Quality uncertainty, asymmetric information, signalling, gains from trade.

JEL CLASSIFICATION : D83, F10, F19, L15.

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1 Introduction

Uncertainty about product quality is an endemic problem in trade. It is often not possible to assess the true quality of a product before one has consumed it. This problem of incomplete information becomes more severe when products are traded across international boundaries. In such situations, the problem becomes one of asymmetric information, whereby consumers have more information about products sold by domestic firms relative to foreign firms.

In a highly influential paper, Rauch (1999) showed that proximity, common language and colonial ties are more important for differentiated products than for products traded on organized exchanges.[1] Under the hypothesis that differentiated products are those for which quality varies a lot, a possible interpretation of this result is that information facilitates trade. In other words, the absence of complete information may create impediments to trade, especially in goods with varying quality, over and above the standard ones such as distance and policy barriers.

In this paper, we examine how welfare gains from trade (GFT) are affected in the presence of uncertainty about quality of foreign products.[2] We do so by adding quality uncertainty to a canonical model of trade with CES preference and monopolistic competition. In Section 2, we develop the frictionless benchmark. There are two identical countries. In each country, there are two types of firms - firms selling low quality products (L firms) and those selling high quality products (H firms). Quality of a firm is unknown *ex-ante*; it is revealed once a firm has paid a sunk cost and entered the industry. Quality is drawn from an exogenous distribution. Firms can produce by hiring labour, with the marginal cost being an increasing function of the quality of the product. There are no other costs. It is well known by now that in the absence of fixed costs of exporting, GFT in this model are captured by a sufficient statistic that depends on the domestic trade share and the elasticity of substitution. A key requirement is that the mass of active firms in the industry remains unchanged after opening up to trade, a result that follows from free entry and CES preferences.

In Section 3, we analyze the model with asymmetric information. We consider the simplest form of asymmetry - consumers in a country can observe the quality of the products sold by

[1] Other papers to provide evidence of informational asymmetry in international trade, although not necessarily about product quality, include Gould (1994), Head and Ries (1998), Rauch and Trindade (2002), Portes and Rey (2005), Allen (2012) and Steinwender (2014).

[2] By now, it is well established that quality plays an important role in international trade. Research has shown that not only do rich countries export higher quality goods on average (Schott, 2004; Hummels and Klenow, 2005; Khandelwal, 2010), but even within narrowly defined sectors, firms produce and export goods of different quality (Verhoogen, 2008; Kugler and Verhoogen, 2012; Hallak and Sivadasan, 2013).

domestic firms but not foreign firms. Given the static nature of the model, firms try to signal their quality through price. In order to solve the model, we use the notion of perfect Bayesian equilibrium (PBE). We study both separating and pooling equilibrium. In a separating equilibrium, the L firms always charge their first-best (frictionless) price, but the H firms do not. In particular, the H firms charge a price that is strictly higher than their first-best price. The inefficiency arises due to incentive compatibility - the H firms have to charge an "excessively" high price to discourage L firms from mimicking them. A deviation from constant mark-up pricing by the H firms causes the mass of firms in both countries to change after opening up to trade. In particular, the number of firms goes up in the open economy. As a consequence, GFT is no longer captured by the sufficient statistic as in the frictionless benchmark. Rather, the sufficient statistic always under-estimates the GFT. In contrast, in a pooling equilibrium, the sufficient statistic might over-estimate the GFT. Nevertheless, the GFT are always positive.

A standard drawback of PBE is the multiplicity of equilibria owing to the flexibility in choosing off-the-equilibrium beliefs. We apply the intuitive criterion of Cho and Kreps (Cho and Kreps, 1987) to narrow down the set of reasonable equilibria. The intuitive criterion eliminates all pooling and all but one separating equilibria. The resulting separating equilibrium is therefore unique. We go on to examine how the GFT under this equilibrium varies with the parameters of the models such as the probability of being a H firm and the quality gap between the two types of firms. Changing either one parameter changes GFT but at the same time, changes the domestic trade share too. Hence, we also perform a comparative static exercise where we hold the domestic trade share constant. This requires changing multiple parameters simultaneously.[3] We show that the GFT predicted by our model could be as much as 2.5 times the GFT predicted using the sufficient statistic approach.

In an influential paper, Arkolakis et al. (2012), henceforth ACR, show that in an important class of models, the GFT can be captured by a sufficient statistic that depends on only two endogenous objects - (i) the domestic trade share and (ii) the trade elasticity.[4] Their paper has triggered a spate of research in this topic, with researchers trying to understand how widely applicable their result is.[5] One of the conditions required for this result to hold is that aggregate profit equals aggregate expenditure, which in turn guarantees that the measure of firms does not respond to trade shocks. The canonical monopolistic competition model of trade with

[3]A similar point was made recently by Melitz and Redding (2014).

[4]The formula for GFT is slightly more complicated for models with multiple sectors, multiple factors of production or intermediate goods. Nevertheless, GFT can be computed using relatively little information (Costinot and Rodríguez-Clare, 2013).

[5]Some of the papers in this topic include De Blas and Russ (2010); Edmond et al. (2012); Arkolakis et al. (2012b); Redding (2012); Ossa (2012); Holmes et al. (2014).

CES preference and free-entry satisfies this condition. We show that a slight modification to the standard model, arising from some firms not charging the constant mark-up price because of asymmetric information, can modify the formula for GFT. Specifically, the domestic trade share is no longer sufficient to compute GFT - one needs information about all the structural parameters of the model.

Our paper is admittedly stylized. A more realistic model should have one or more of the following features: (1) more than two types of firms, (2) consumers receiving noisy signals about the quality of foreign products, (3) a dynamic model where product quality is revealed gradually, (4) endogenous quality choice, to name just a few. The purpose of this paper, however, is not to study a general model of international trade under asymmetric information, but rather, to show how the insights about GFT obtained from one of the simplest trade models is modified under quality uncertainty. We believe that at least qualitatively, the insights from our model should carry over to more general settings.

Besides the literature on GFT, our paper also contributes to the theoretical literature on quality in international trade. Many of the papers in this literature have been concerned with explaining the pattern of trade in vertically differentiated goods,[6] or the systematic difference in unit value of exports across firms.[7] Our work is closely related to those papers that have explored the role of asymmetric information about product quality in an international trade context.[8] Finally, our paper contributes to the literature on uncertainty in international trade. Most of the papers in this literature focus on how countries formulate trade policy in the presence of uncertainty (See Ruffin, 1974, for example). Our paper is closer in spirit to those papers that also study uncertainty in trade policy, but focuses on firms' response to such uncertainty.[9]

2 The Model

We introduce quality heterogeneity in the simplest possible way to the canonical monopolistic competition model of Krugman (1980). There are two symmetric countries populated by L consumers. Trade costs take the form of iceberg costs, with one unit of a good in one country requiring $\tau > 1$ units to be shipped from the other country. We begin by laying down the preference of consumers and technology faced by firms.

[6] See Flam and Helpman (1987); Stokey (1991); Murphy and Shleifer (1997); Fajgelbaum et al. (2011)

[7] See Schott (2004); Khandelwal (2010); Baldwin and Harrigan (2011); Kugler and Verhoogen (2012); Johnson (2012); Hallak and Sivadasan (2013)

[8] See Grossman and Horn (1988); Bagwell and Staiger (1989); Raff and Kim (1999); Chisik (2003)

[9] See Chisik (2012); Handley and Limao (2012)

Preference : Consumers have CES preference over varities:

$$U = \Big[\int_{\Omega} q(i)c(i)^{1-\frac{1}{\sigma}} di \Big]^{\frac{\sigma}{\sigma-1}},$$

where $i \in \Omega$ indexes a variety that is available to consumers in a country, while $c(i)$ and $q(i)$ are consumption and quality of variety i. The above preference implies the following demand for variety i:

$$c(i) = q(i)^{\sigma} p(i)^{-\sigma} Y / P^{1-\sigma},$$

where Y is aggregate income and P, the ideal price index, is given by

$$P = \Big[\int_{\Omega} q(i)^{\sigma} p(i)^{1-\sigma} di \Big]^{\frac{1}{1-\sigma}}.$$

Technology : A potential entrant needs to hire S workers to enter the industry, and draws a quality after entry. Without incurring any additional cost, a firm can also produce a new variety. We assume that quality can be of two types - low (q_L) and high (q_H), and define $\xi = q_L/q_H$ as the quality gap. Henceforth we use i to index quality. To produce a unit of a variety with quality $q_i (i = L, H)$, a firm requires q_i^{γ} workers. Therefore, the marginal cost is wq_i^{γ}. Finally, the exogenous probability of drawing quality q_H is η.

Before analyzing the equilibrium under asymmetric information about product quality, we first solve for the full information benchmark. The equilibrium price of a q_i firm is

$$p_i^f = \frac{\sigma}{\sigma - 1} w q_i^{\gamma}, \tag{1}$$

where w denotes wage. Setting the wage as the numeraire, firms' profits are

$$\pi_i^f = \frac{\alpha Y}{\sigma (P_A^f)^{1-\sigma}} q_i^{\gamma + \sigma(1-\gamma)}, \tag{2}$$

where $\alpha = [\sigma/(\sigma - 1)]^{1-\sigma}$ is a constant and the super-script f denotes full information variables. Total income in a country, Y, is the sum of total variable labor costs and total profits. The latter, in turn, must be equal to total sunk costs because of free entry. As a result, in equilibrium, $Y = L$. The full information price index under autarky, P_A^f is given by

$$P_A^f = \alpha^{\frac{1}{1-\sigma}} (M_A^f)^{\frac{1}{1-\sigma}} \big[\eta q_H^{\gamma + \sigma(1-\gamma)} + (1 - \eta) q_L^{\gamma + \sigma(1-\gamma)} \big]^{\frac{1}{1-\sigma}}, \tag{3}$$

where M_A^f is the equilibrium measure of firms under autarky. Firms enter the industry until their expected profit, given by

$$E[\pi^f] = \frac{\alpha Y}{\sigma(P_A^f)^{1-\sigma}}\left[\eta q_H^{\gamma+\sigma(1-\gamma)} + (1-\eta)q_L^{\gamma+\sigma(1-\gamma)}\right], \tag{4}$$

equals the sunk entry cost S. Replacing P_A^f from (3) in (4), we can solve for M_A^f:

$$M_A^f = \frac{L}{\sigma S}. \tag{5}$$

As in the homogenous firm model, the measure of firms is completely pinned down by the size of the market, with a fraction η being H firms. In an open economy, $p_i^f = \sigma/(\sigma-1)\tau q_i^\gamma$ and the aggregate price index, P_T^f is given by

$$P_T^f = (1+\tau^{1-\sigma})^{\frac{1}{1-\sigma}}\left(\frac{M_T^f}{M_A^f}\right)^{\frac{1}{\sigma-1}} P_A^f,$$

where M_T^f is the equilibrium measure of firms under trade. Then the gains from trade (GFT) can readily be computed as the ratio of the real wages after and before trade:

$$\frac{\mathcal{W}_T^f}{\mathcal{W}_A^f} = (1+\tau^{1-\sigma})^{\frac{1}{\sigma-1}}\left(\frac{M_T^f}{M_A^f}\right)^{\frac{1}{\sigma-1}}.$$

Now, conditional on the measure of firms, the expected profit of a potential entrant under trade is the same as under autarky.[10] As the sunk entry costs do not change due of trade, the free entry condition yields $M_T^f = M_A^f$. Consequently, the second term on the right-hand side of the above equation equals one. The gains from trade are always positive (because τ is less than infinity). Following the recent literature on gains from trade (Arkolakis et al. (2012)), let us define λ_s ($s = A, T$) as the share of a country's expenditure that goes towards its own goods, i.e., its domestic trade share. Noting that $\lambda_A = 1$, the gains from trade under this alternative formulation is given by

$$\frac{\mathcal{W}_T^f}{\mathcal{W}_A^f} = \left(\frac{1}{\lambda_T}\right)^{\frac{1}{\sigma-1}}.$$

Therefore, even with heterogenous quality, the domestic trade share is a sufficient statistic for computing the gains from trade when there is full information about product quality.[11] This

[10] As we discuss later, CES preferences are crucial for this result.

[11] Note, however, that this result requires that there are no additional fixed cost of exporting (Melitz and Redding,

is not surprising given that with CES preference and monopolistic competition, a model with heterogenous quality is isomorphic to one with heterogenous costs, and as already shown by ACR, gains from trade can be fully captured by the domestic trade share in this class of models.

In the next section, we explore how GFT are affected in the presence of asymmetric information about product quality. In particular, we ask if GFT can still be computed using only the domestic trade share and the elasticity of substitution.

3 Quality Uncertainty

In this section, we consider a scenario where consumers in a country perfectly observe the quality of the domestic products but do not observe the quality of the foreign products. In the absence of full information, producers try to signal the quality of their product in the export market through their prices.[12] After observing prices, consumers in the export market form beliefs about the quality of different products. The timing of the game in the export market is as follows:

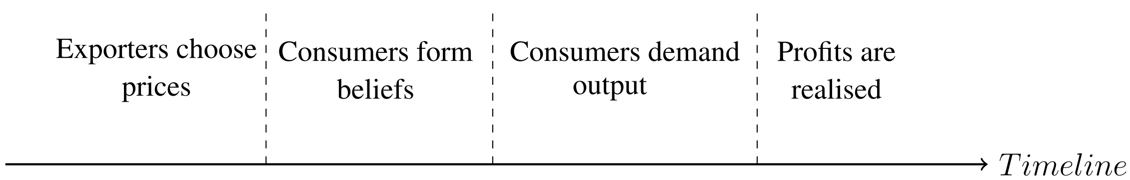


Figure 1: *Timing of the game in the export market*

Given the structure of the problem, we use the concept of a perfect Bayesian equilibrium (PBE). PBE requires a strategy profile for the agents and posterior beliefs about the type of the agents. In this model, the strategy for a consumer (both home and foreign) is to demand a variety, while the strategy for a firm is to choose a price for each market. The posterior belief held by the consumers, $\mu(q|p)$, is about the quality of the variety sold by a firm. Formally,

Definition 1. *A PBE of the model consists of strategies for the consumers and firms, and posterior beliefs such that:*
(a) Consumers maximize utility,
(b) Firms maximize profits,
(c) $\mu(q|p)$ is formed from the prior distribution using Bayes' rule whenever possible.

2014).

[12] We assume that guarantees, certifications, etc. are imperfect tools for revealing quality.

Let the quality expected by consumers on observing a price p be denoted by $\bar{q}$. The profit of a i firm charging p is then given by

$$\pi_i(p;\bar{q}) = (p - q_i^\gamma)\bar{q}^\sigma p^{-\sigma}\frac{Y}{P^{1-\sigma}},$$

where the aggregate price index under asymmetric information, P, is usually different from that under full information. The iso-profit curves for the L and H firms are shown in Figure 2. The important thing to observe is that for a given $(p,\bar{q})$, the slope of the iso-profit of the L firm is greater than that of the H firm. This "single-crossing property" is key for some of the results that we shall derive later. Note that *conditional* on $\bar{q}$, profit is still maximized for $p_i = p_i^f$. A higher $\bar{q}$ simply acts as a demand shifter that increases the profit levels at all prices.

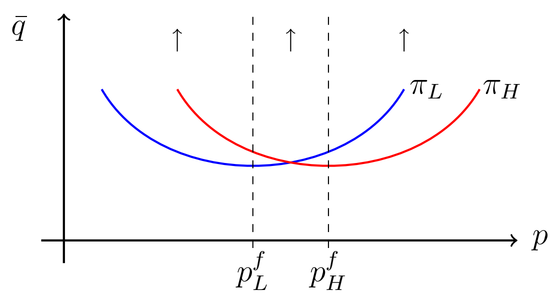


Figure 2: *Iso-profit curves for L and H firms*

We begin by analyzing the class of separating equilibria. Then we consider pooling equilibria. Finally, we apply notions of equilibrium refinement to narrow down the equilibrium set.

3.1 Separating equilibrium

We seek two prices, p_L and p_H, charged by the L and H firms respectively such that foreign consumers, upon observing $q_L(q_H)$ believe that the firm is of quality L(H). *Individual rationality* implies that $p_L \geq \tau q_L^\gamma$ and $p_H \geq \tau q_H^\gamma$. A separating equilibrium also has a unique price for L firms as shown in the following lemma.

Lemma 1. *In a separating equilibrium, $p_L = p_L^f$.*

Thus, asymmetric information about product quality does not prevent the L firms from charging the first-best price. Intuitively, a lack of information cannot hurt low quality firms; it can only make them weakly better-off relative to a full information world.

Incentive compatibility requires that L firms must not want to mimic H firms and similarly for H firms, i.e., $\pi_L(p_L; q_L) > \pi_L(p_H; q_H)$ and $\pi_H(p_H; q_H) > \pi_H(p_L; q_L)$. Observe that if $p_L^f < \tau q_H^\gamma$, H firms will never mimic L firms. But L firms might still want to mimic H firms. Suppose a L firm charges p_H^f. Then it will face a demand of q_H. The corresponding profit is

$$\begin{aligned}\pi_L(p_H^f; q_H) &= \frac{\alpha Y}{P_T^{1-\sigma}} q_H^{\gamma+\sigma(1-\gamma)} \big(\sigma - (\sigma-1)\xi^\gamma\big) \\ &> \frac{\alpha Y}{P_T^{1-\sigma}} q_L^{\gamma+\sigma(1-\gamma)}\end{aligned}$$

where the inequality follows from the fact that $\xi = q_L/q_H < 1$. But the expression on the second line is nothing but $\pi_L(p_L^f; q_L)$. Hence, if consumers assign beliefs $\mu(\bar{q} = q_H | p = p_H^f) = 1$, the L firms will deviate and mimic H firms. Hence, in a separating equilibrium, *H firms will no longer be able to charge their first-best price.*[13]

Next, let p_1 be such that $\pi_L(p_1; q_H) = \pi_L(p_L^f; q_L)$, i.e., p_1 makes a L firm indifferent between signalling that it is indeed a L firm and mimicking a H firm. If beliefs are such that $\mu(\bar{q} = q_H | p < p_1) = 1$, a L firm will always mimic a H firm. Similarly, define p_2 as the price satisfying $\pi_H(p_2; q_H) = \pi_H(p_L^f; q_L)$, i.e., p_2 makes a H firm indifferent between signalling that it is indeed a H firm and mimicking a L firm. If beliefs are such that $\mu(\bar{q} = q_H | p > p_2) = 1$, a H firm will always mimic a L firm. Hence, in a separating equilibrium, we must have $p_1 \le p_H \le p_2$.

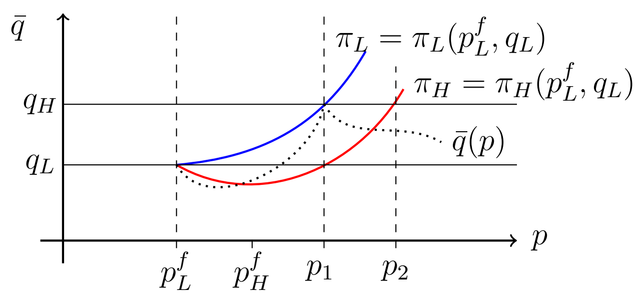


Figure 3: *A separating equilibrium*

The prices p_1 and p_2 are shown in Figure 3. In drawing the figure, we have assumed that the parameter values are such that $\pi_H(p_L^f, q_L) > 0$.[14] Otherwise, p_2 does not exist. What value of p_H is chosen in equilibrium? The flexibility in choosing off-equilibrium beliefs implies that any price between p_1 and p_2 can be sustained in equilibrium as the price charged by H firms.

[13]One can draw parallels between this model and the job market signalling model of Spence (1973). As in his model, we have two types of agents with unobserved attributes and the "high" type choosing a costly action to try to distinguish itself from the "low" type.

[14]As discussed above, this requires that $p_L^f > \tau q_H^\gamma$.

A possible equilibrium belief is shown by the dotted, black line in Figure 3. Of course, as we discuss in Section 3.3, not all of these beliefs are *intuitive*. This allows us to narrow down the set of equilibria considerably.

The absence of information friction in the domestic market means that L firms charge the first-best price in both markets while H firms charge the first-best price only in the domestic market. The aggregate price index in an open economy is given by

$$P_T = (\alpha\sigma)^{\frac{1}{1-\sigma}} M_T^{\frac{1}{1-\sigma}} \left[\eta q_H^{\gamma+\sigma(1-\gamma)}\left(1 + (\frac{p_H}{q_H^\gamma}\frac{\sigma-1}{\sigma})^{1-\sigma}\right) + (1-\eta) q_L^{\gamma+\sigma(1-\gamma)}(1+\tau^{1-\sigma})\right]^{\frac{1}{1-\sigma}}, \quad (6)$$

where M_T is the measure of firms in either country under a separating equilibrium. Comparing (3) with (6), one can see that the difference between the two price indices arises because $p_H \neq \frac{\sigma}{\sigma-1}\tau q_H^\gamma$. Using the value for P_T, we obtain an expression for expected profits:

$$E[\pi] = \frac{L}{\sigma M_T} \times \frac{\Phi + (p_H - \tau q_H^\gamma) p_H^{-\sigma}\Psi}{\Phi + \frac{1}{\sigma} p_H^{1-\sigma}\Psi},$$

where

$$\Phi = q_H^{\gamma+\sigma(1-\gamma)}[\eta + (1-\eta)\xi^{\gamma+\sigma(1-\gamma)}](1+\tau^{1-\sigma}),$$
$$\Psi = \sigma\eta q_H^\sigma/\alpha.$$

The free-entry condition, $E[\pi] = S$, allows us to solve for the equilibrium measure of firms under trade:

$$M_T = \frac{L}{\sigma S}\frac{\Phi + (p_H - \tau q_H^\gamma) p_H^{-\sigma}\Psi}{\Phi + \frac{1}{\sigma} p_H^{1-\sigma}\Psi}.$$

Now, the measure of firms under autarky, M_A, is the same as in Section 2, because a closed economy is not plagued by information asymmetry. The next lemma derives a relation between the measure of firms under autarky and trade.

Lemma 2. $M_T > M_A$.

Unlike in the full information scenario, the measure of firms under asymmetric information is actually *higher* under trade. When a country opens up to trade, there are two forces that act on firms' profits - a positive market size effect and a negative competition effect. Because firms now sell to a bigger market, firm's profits go up conditional on the aggregate price index. But under trade, firms in each country face more competition. This tends to reduce the aggregate price index thereby reducing profits.

In a full information world, both L and H firms charge a constant mark-up over marginal cost. Under this pricing rule, the market size and competition effects exactly cancel each other out, whereby, conditional on the measure of firms, there is no change in profits of individual firms, and accordingly, in expected profits. But when the H firms diverge from constant mark-up pricing, the two effects no longer cancel each other out. Rather the positive market size effect dominates, which tends to increase expected profits conditional on the measure of firms. This triggers entry. Hence, the standard result of a constant measure of firms is no longer valid under quality uncertainty.

As before, the gains from trade can be computed as

$$\frac{\mathcal{W}_T}{\mathcal{W}_A} = \left(\frac{1}{\lambda_T}\right)^{\frac{1}{\sigma-1}} \left(\frac{M_T}{M_A}\right)^{\frac{1}{\sigma-1}}. \tag{7}$$

Lemma 2 then implies that the own trade share, λ_T, is *no longer a sufficient statistic for computing welfare gains from trade.* Rather, simply looking at λ_T leads to an under-estimation of the GFT. In a separating equilibrium, measurement of welfare gains requires the knowledge of the measure of firms, which in turn depends on the price being charged by the H firms in the foreign market, p_H, among other things. We summarize in the following proposition:

Proposition 1. *Under a separating equilibrium, the domestic trade share under-estimates the true GFT.*

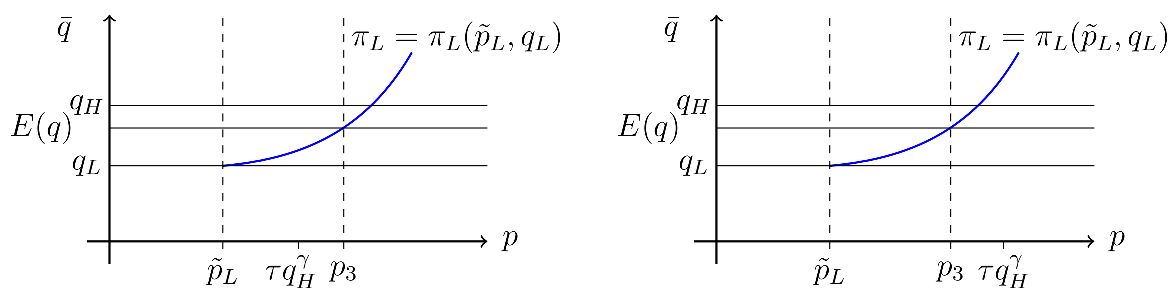
Why does the ACR result not apply in our setup? One of the key macro-level restrictions in ACR is that aggregate profit must be proportional to aggregate revenue. This restriction results in the measure of firms being pinned down by fundamentals of the model such as endowments, costs and preference parameters. The proof proceeds in three steps. First, with labour being the only factor of production, aggregate revenue is equal to aggregate income. In a one sector model, the latter is simply the labour endowment (assuming the nominal wage to be the numeraire). Second, under free entry, aggregate profit is proportional to the measure of firms. Third, aggregate profit is assumed to be proportional to aggregate revenue. Combining, we have the measure of firms to be proportional to the labour endowment, which is exogenously given. This third assumption is true when preferences are CES and all firms engage in constant mark-up pricing, as in our model under full information. But this assumption is violated for a subset of firms under asymmetric information. As a result, the measure of firms is no longer constant as an economy moves from autarky to trade.[15] This reasoning applies in the case of a pooling equilibrium too, as we explore next.

[15] To see this formally, note that free entry implies that aggregate profit, $\bar{\Pi} = MS$. The macro-level assumption

3.2 Pooling equilibrium

Under a pooling equilibrium, we seek a unique price $\bar{p}$ that is charged by both L and H firms in the foreign market. Upon observing this price, consumers must also believe that the expected quality $\bar{q}$ is equal to $E(q) = \eta q_H + (1-\eta) q_L$.

Individual rationality now requires that $\bar{p} \geq \tau q_H^{\gamma}$. Observe that L firms can always charge a price of p_L^f and earn their first-best profits. Let us define p_3 such that $\pi_L(p_L^f; q_L) = \pi_L(p_3; E(q))$, i.e., p_3 makes a L firm indifferent between choosing the pooling equilibrium price and the first-best price. Any pooling equilibrium must therefore have a price that is bounded above by p_3. Combined with the individual rationality condition, this constraint presents us with two scenarios as shown in Figure 4. When $\tau q_H^{\gamma} > p_3$, a pooling equilibrium does not exist. Otherwise, a pooling equilibrium exists. As before, there are a large number of beliefs that can sustain the price $\bar{p}$ in equilibrium. The reasonableness of such beliefs is discussed in Section 3.3.



(a) *Pooling equilibrium exists* **(b)** *Pooling equilibrium does not exist*

Figure 4: *Pooling equilibrium*

The aggregate price index in an open economy is given by

$$P_T = (\alpha\sigma)^{\frac{1}{1-\sigma}} M_T^{\frac{1}{1-\sigma}} \left[\eta q_H^{\gamma+\sigma(1-\gamma)} + (1-\eta) q_L^{\gamma+\sigma(1-\gamma)} + \frac{[E(q)]^{\sigma} \bar{p}^{1-\sigma}}{\alpha\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{8}$$

where M_T is now the measure of firms in either country under a pooling equilibrium. Using the value for P_T, we can obtain an expression for expected profits as before. Combining with the free-entry condition, this allows us to solve for M_T:

$$M_T = \frac{L}{\sigma S} \times \frac{\Phi + \big(\bar{p} - E(q^{\gamma})\big)\bar{p}^{-\sigma}\Psi}{\Phi + \frac{1}{\sigma}\bar{p}^{1-\sigma}\Psi},$$

implies that $\bar{\Pi} \approx L$.Therefore, $M \approx L/S$. Under CES preference and constant mark-up pricing, $\bar{\Pi} = L/\sigma$, whereby $M = L/(\sigma S)$.

where

$$\Phi = q_H^{\gamma+\sigma(1-\gamma)}[\eta + (1-\eta)\xi^{\gamma+\sigma(1-\gamma)}],$$
$$\Psi = \sigma[E(q)]^\sigma/\alpha.$$

The following lemma provides a comparison between M_T and M_A.

Lemma 3. *M_T is higher (lower) than M_A accordingly as $\bar{p}$ is higher (lower) than $\frac{\sigma}{\sigma-1}E(q^\gamma)$.*

Under what condition does the measure of firms always go up due to trade? Because $\bar{p}$ is bounded below by τq_H^γ, Lemma 3 implies that M_T must be greater than M_A if

$$\tau q_H^\gamma > \frac{\sigma}{\sigma-1}E(q^\gamma).$$

The above condition is more likely to be satisfied when ξ is small (i.e., q_H is much larger than q_L), γ is large (i.e., marginal cost of production rises fast with quality) or η is small (i.e., the measure of H firms is small). When η is small, for example, most of the entrants into the industry are L firms. For these firms, the possibility of being treated as a H firm is very attractive, leading to a large inflow following the opening up of the economy.

The GFT continue to be given by (7). But now, the possibility of the measure of firms going down after trade implies that focussing only on the domestic trade share, λ_T, could over-estimate the GFT. Can the gains from trade be overturned? From (8) we have (ignoring constants) $P_T^{1-\sigma} = M_T(\Phi + \frac{1}{\sigma}\bar{p}^{1-\sigma}\Psi)$. Replacing the value of M_T from above, we have

$$P_T^{1-\sigma} = M_A[\Phi + \big(\bar{p} - E(q^\gamma)\big)\bar{p}^{-\sigma}\Psi],$$
$$> M_A\Phi,$$

where the last line follows from the fact that $\bar{p} > E(q^\gamma)$ (because $\bar{p} > \tau q_H^\gamma$). But $M_A\Phi$ is equal to $P_A^{1-\sigma}$. Hence, even if there is a decline in the measure of firms in each country, the total number of varieties available to consumers in both countries under trade is higher than that under autarky, generating GFT. We summarize the above finding in the following proposition:

Proposition 2. *Under a pooling equilibrium, the domestic trade share might under-estimate or over-estimate the true GFT.*

3.3 Equilibrium refinement

As discussed above, there is a multiplicity of separating and pooling equilibria owing to the flexibility allowed by PBE in choosing off-the-equilibrium beliefs (Fudenberg and Tirole, 1991). But not all such beliefs are reasonable. To see this, let us consider a separating equilibrium. Assume that foreign consumers have the following beliefs: $\mu(q = q_H|p = p_e) = 1$ and $\mu(q < q_H|p \neq p_e) = 1$ where the price p_e is shown in Figure 5. Suppose foreign consumers observe a price $p_e - \epsilon$. How would they react?

According to the refinement proposed by Cho and Kreps (1987), the consumers should be able to reason as follows: If a H firm deviates from p_e and charges a price $p_e - \epsilon$, then there is a *possibility* that his profit might go up in the event that consumers still believe that he is a H firm. But if a L firm deviates from p_L^f and charges $p_e - \epsilon$, then his profits will always go down, no matter what off-the-equilibrium beliefs are. This is because at any price above p_1, the profit of the L firm is strictly less than his equilibrium profit even if consumers believe that he has quality q_H. This suggests that the belief $\mu(q < q_H|p \neq p_e) = 1$ is *not intuitive*. When observing a price like $p_e - \epsilon$, consumers should still believe that the firm is H type. But then, a H firm should deviate from p_e. In fact, one can see that any price for H firm that is greater than p_1 cannot be part of an equilibrium strategy if one uses the intuitive criterion of Cho and Kreps (1987). The only price that survives the equilibrium refinement is $p_H = p_1$.

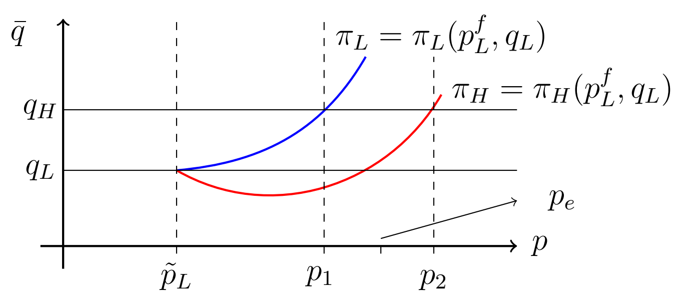


Figure 5: *A separating equilibrium*

Next consider any pooling equilibrium price, $\bar{p}$, as shown in Figure 6. The single-crossing property implies that we can always find a price $\bar{p}+\epsilon$ such that $\pi_L(\bar{p}+\epsilon, q_H) < \pi_L(\bar{p}, E(q))$ but $\pi_H(\bar{p} + \epsilon, q_H) > \pi_H(\bar{p}, E(q))$. This suggests that a L firm will never deviate from $\bar{p}$, no matter what consumers' off-the-equilibrium beliefs are, while a H firm could deviate. Accordingly, if consumers see a firm deviating to $\bar{p} + \epsilon$, they should believe that this is a H firm. Because this is true for any $\bar{p}$, the intuitive criterion rules out all pooling equilibria. This allows us to state the following result.

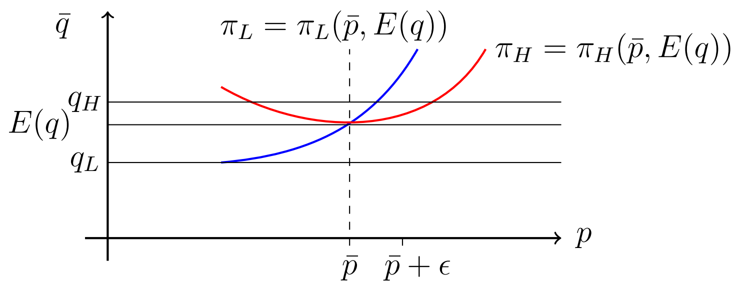


Figure 6: *A pooling equilibrium*

Proposition 3. *Using the Cho and Kreps' intuitive criterion, the unique equilibrium is one where the L firms charge p_L^f while the H firms charge p_1.*

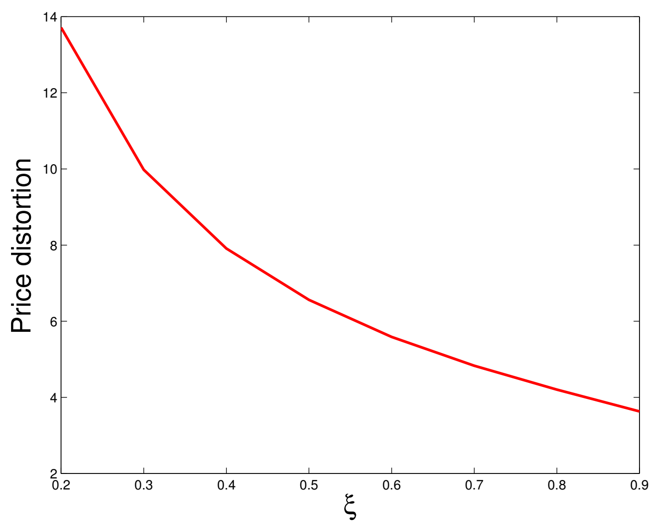


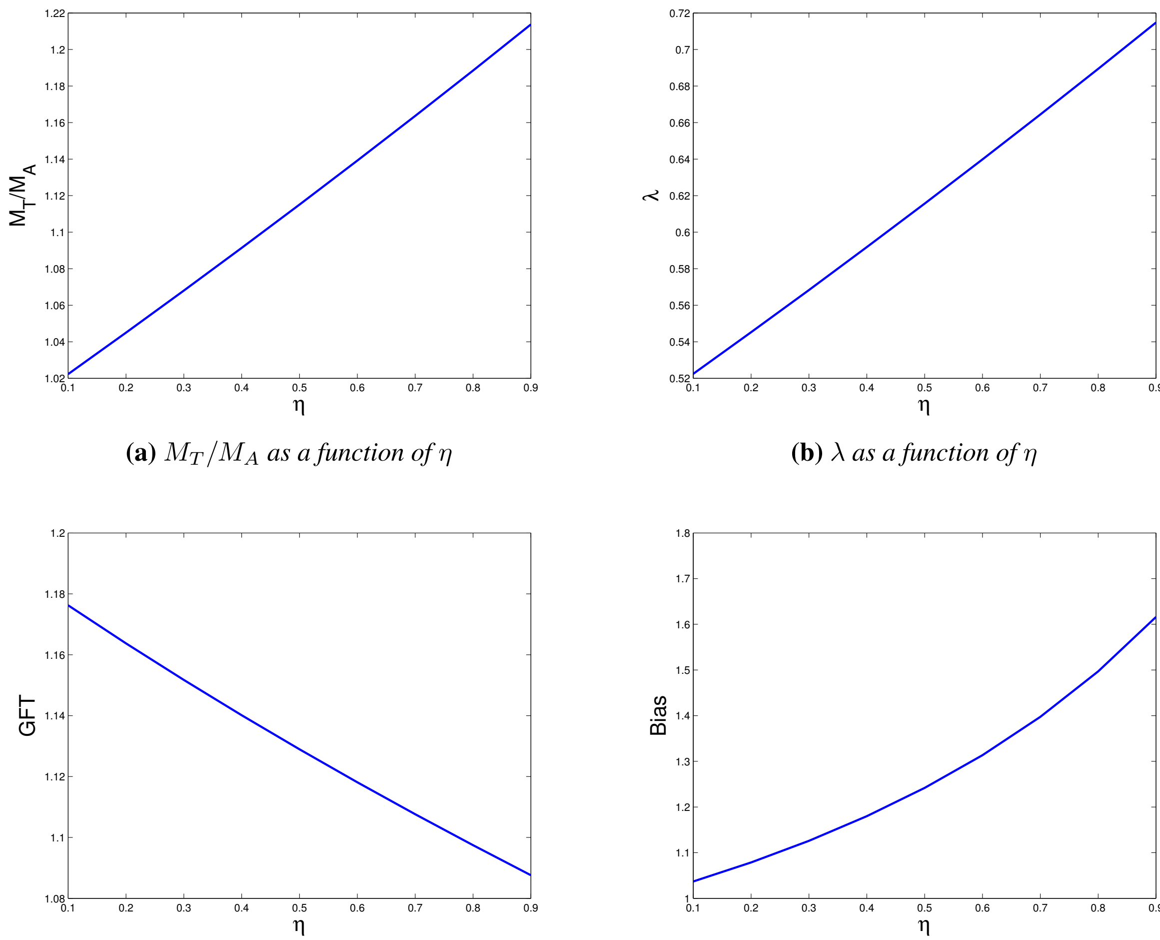
Figure 7: *Price distortion of H firms*

Recall that p_1 solves $\pi_L(p_1; q_H) = \pi_L(p_L^f; q_L)$. Re-arranging, the solution to p_1 is given by the following implicit equation:

$$\chi(p_1)^\sigma - p_1 + \tau\xi^\gamma q_H^\gamma = 0, \tag{9}$$

where $\chi = \alpha\tau^{1-\sigma}\xi^{\gamma+\sigma(1-\gamma)}q_H^{\gamma(1-\sigma)}$. Because in equilibrium, p_1 must always be greater than p_H^f, the first-best price of the H firms, the ratio p_1/p_H^f could be thought of as a measure of inefficiency created by asymmetric information. Under perfect information, the price charged by the H firms is independent of q_L; it depends only on q_H. But in this model, even conditional on q_H, the price distortion varies with ξ due to the incentive compatibility constraint. Figure 7 shows the relation between p_1/p_H^f and ξ.

Lemma 4. *M_T is increasing in η.*



(a) *M_T/M_A as a function of η* **(b)** *λ as a function of η*

(c) *GFT as a function of η* **(d)** *Bias in GFT as a function of η*

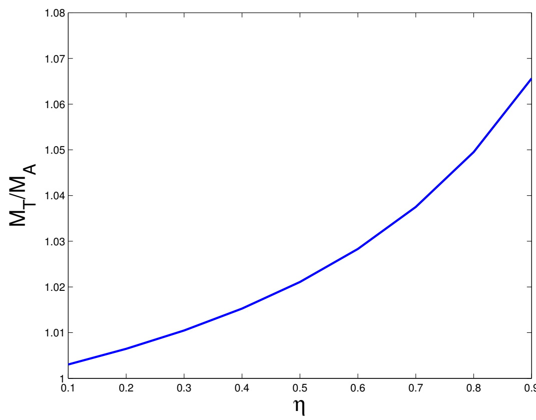
Figure 8: *Comparative statics with respect to η ($\xi = 0.9$)*

Recall that the measure of firms under a separating equilibrium is given by

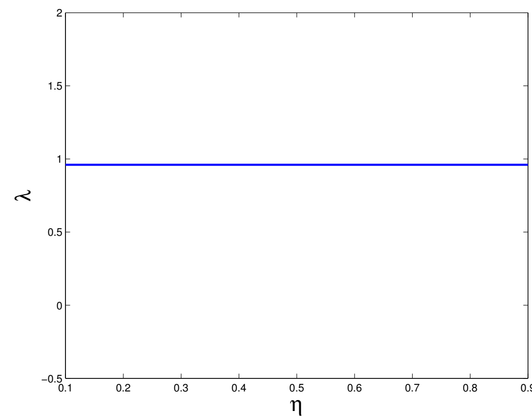
$$M_T = \frac{L}{\sigma S} \frac{\Phi + (p_1 - \tau q_H^{\gamma})(p_1)^{-\sigma}\Psi}{\Phi + \frac{1}{\sigma}(p_1)^{1-\sigma}\Psi},$$

where Φ and Ψ have been defined before. We now explore how M_T changes with η. From (9), it is clear that p_1 is independent of η. Accordingly, η affects M_T directly. The following lemma characterizes this relationship.

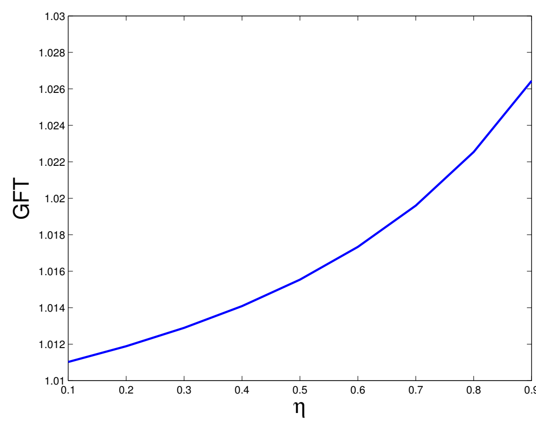
As η increases, the expected profit of entering the industry, conditional on the measure of firms, rises. Restoration of equilibrium then requires more entry. As discussed earlier, when H firms charge p_1 instead of the first-best price p_H^f, the convenient property of a constant mark-up pricing is lost. The implication is the following: *even controlling for the domestic trade share*, λ, the gains from trade, GFT, are increasing in η.



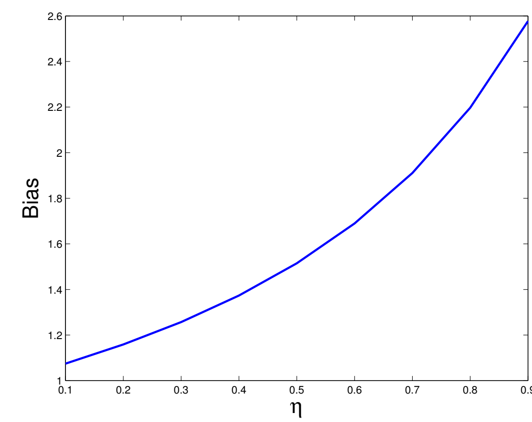
(a) *M_T/M_A as a function of η*



(b) *λ as a function of η*



(c) *GFT as a function of η*



(d) *Bias in GFT as a function of η*

Figure 9: *Comparative statics with respect to η (variable ξ)*

The effect of η on GFT is displayed in Figure 8. Observe that η not only changes M_T, it also changes the domestic trade share, λ, as shown in Figure 8b. This result is different from that under full information, where λ is independent of η. A higher λ translates into lower GFT, despite an increase in M_T (Figure 8c). But the bias in the GFT due to ignoring the endogenous measure of firms is increasing in η.[16] Even for small differences in quality (in this example, $q_L = 0.9q_H$), whereby the distortion from asymmetric information is small, the GFT predicted by our model could be 1.5 times higher than that predicted using the sufficient statistic approach, as shown in Figure 8d.[17]

Next, we perform a different counterfactual. As we increase η, we also adjust ξ so as to keep λ constant at a level of 0.93.[18] This is approximately the share of domestic trade for the U.S. (see ACR). Now the GFT are increasing in η (Figure 9c). This follows from the result that M_T is increasing in η while λ is unchanged. Under these conditions, the GFT predicted by our model could be almost 2.5 times higher than that predicted using the sufficient statistic approach, as shown in Figure 9d. This has the following implication: even controlling for λ, the actual GFT could vary significantly depending on the combination of the underlying structural parameters η and ξ.

4 Conclusion

We add quality uncertainty to a two-country trade model with CES preference and monopolistic competition. Exporters use price to signal their quality. In such a model with full information, the welfare gains from trade (GFT) can be captured by a sufficient statistic that depends on domestic trade share and the elasticity of substitution. In contrast, we show that within the class of separating equilibria, the sufficient statistic always under-estimates GFT, while within the class of pooling equilibria, the sufficient statistic could over-estimate GFT. For an equilibrium refinement, we analyze the determinants of GFT. We show that the actual GFT under asymmetric information could be almost 2.5 times higher than that measured using the sufficient statistic approach.

[16] We measure bias as $(GFT - 1)/(GFT_{suf} - 1)$, where GFT_{suf} is the gains computed using the sufficient statistic approach.

[17] For this and the following comparative static exercise, we chose $\sigma = 5$, $\gamma = 0.5$ and $q_H = 5$. The value of σ is standard in the literature while the results are insensitive to γ and q_H.

[18] As pointed out by Melitz and Redding (2014), for two models with different productivity distributions to generate the same domestic trade share, the structural parameters should be different across the models.

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Appendix

Proof of Lemma 1. We prove by contradiction. Let there be a price $p' \neq p_L^f$ that L firms charge in equilibrium. When $p = p'$, $\bar{q} = q_L$, i.e., on observing a price of p', consumers must believe that the firm is of type L. If a L firm deviates and charges $p = p_L^f$, its profit goes up. This is because $\bar{q}$ is bounded below by q_L, and p_L^f maximizes the profit of a L firm irrespective of what $\bar{q}$ is. □

Proof of Lemma 2. Consider the expression for M_T:

$$M_T = \frac{L}{\sigma S} \frac{\Phi + (p_H - \tau q_H^\gamma) p_H^{-\sigma} \Psi}{\Phi + (\frac{1}{\sigma} p_H) p_H^{-\sigma} \Psi}.$$

In equilibrium, $p_H > p_H^f$. Because $p_H^f = \frac{\sigma}{\sigma-1} \tau q_H^\gamma$, it follows that $p_H - \tau q_H^\gamma > \frac{1}{\sigma} p_H$. Therefore, $M_T > \frac{L}{\sigma S} = M_A$. □

Proof of Lemma 3. Consider the expression for M_T:

$$M_T = \frac{L}{\sigma S} \frac{\Phi + (\bar{p} - E(q^\gamma)) \bar{p}^{-\sigma} \Psi}{\Phi + (\frac{1}{\sigma} \bar{p}) \bar{p}^{-\sigma} \Psi}.$$

The numerator of the second term on the right-hand side of the above expression is greater than one if $\bar{p} - E(q^\gamma) > \frac{1}{\sigma} \bar{p}$. Re-arranging, the condition becomes $\bar{p} > \frac{\sigma}{\sigma-1} E(q^\gamma)$. □

Proof of Lemma 4. Consider the part of the expression for M_T that depends on η:

$$M_T \approx \frac{\Phi + (p_H - \tau q_H^\gamma) p_H^{-\sigma} \Psi}{\Phi + (\frac{1}{\sigma} p_H) p_H^{-\sigma} \Psi}.$$

Observe that $\Phi = q_H^{\gamma+\sigma(1-\gamma)}[\eta + (1-\eta)\xi^{\gamma+\sigma(1-\gamma)}](1+\tau^{1-\sigma})$ depends on η while Ψ does not. Now, $\frac{dM_T}{d\eta} = \frac{dM_T}{d\Phi} \cdot \frac{d\Phi}{d\eta}$. It is easy to check that $\frac{d\Phi}{d\eta} > 0$. Furthermore, the sign of $\frac{dM_T}{d\Phi}$ depends of the sign of $1 - \frac{\sigma}{\sigma-1} \frac{\tau q_H^\gamma}{p_H}$. Because the latter is always positive, we conclude that $\frac{dM_T}{d\eta} > 0$. □