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Multi-Dimensional Wellbeing Assessment: A General Method for Index Construction with an Application to Multivariate Deprivation

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Abstract

This paper explores some difficulties encountered in multi-dimensional measures of wellbeing. In particular, it highlights the need for such indices to (i) incorporate information on individual preferences, (ii) be robust with respect to estimation assumptions given the 'curse of dimensionality' problem, and (iii) reflect complementarities and substitutabilities between dimensions. The paper proposes a procedure that enhances the Alkire and Foster (2011) Multi-dimensional Deprivation Index (AFMDI), drawing on recent work in Anderson et al. (2011), which addresses these difficulties. The approach provides the requisite flexibility in the representation of wellbeing component deprivations, whilst admitting the possibility of sub-component substitutability/complementarity in the index, and retains the ability to measure the impact of improvements/worsenings of sub-components within each category. It then provides an application to the measurement and valuation of opportunity in different domains using a unique dataset for working age adults in the U.S.. Empirical findings suggest that freedoms are substitutable, that their values depend on an individual's needs, and that complementarities if they exist are weak. The paper then concludes that such indices are feasible to implement, and holds promise in economic applications ranging from measurement of progress in wellbeing, to the multi-dimensional assessment of poverty.

JEL Code: I31, I32

Key Words: Wellbeing Index

1 Introduction

There is growing interest, particularly in economics literatures on life quality, poverty, development and social policy (Sen 1995; Anand and Sen 1997; Atkinson 2003; Grusky and Kanbur 2006; Stiglitz et al. 2011), in the construction of multi-dimensional indices. Typically such measures are used to assess national progress or identify levels of material deprivation. There have been many approaches to multidimensional wellbeing measurement, three important examples of which are the Human Development Index (United Nations Development Programme 2008), Latent Variable Structural Models of Wellbeing (for example Anand et al. (2011)), and the very popular Alkire-Foster Deprivation Index (Alkire and Foster 2011).

The HDI index, which has a country as its basic agent or unit of analysis, in its original form, is an equally weighted aggregation for each country, of three dimensions; education, life expectancy and GDP per capita. In structural modelling, the approach models happiness as a function of the levels of various functionings an agent possesses, which are limited by the capability set, that is the set of functions the agent could have chosen. By contrast, the AFMDI as an advancement over the former, has an individual as its basic agent, providing a better reflection of the true level of wellbeing within a society. It is a "counting" based measure, incorporating elements of the Foster-Greer-Thorbecke (FGT) index (Foster et al. 1984) that allows it to reflect the complexity of wellbeing, through the number of dimensions that can be considered. These dimensions of individual wellbeing are themselves weighted and they need not be the same, while each individual recieves the same weight towards the calculation of the index.[1]

However, there are several concerns with these measures. Firstly, insofar as the various dimensions of wellbeing are weighted sums, there is an implicit assumption of separability (see for example de la Vega and Urrutia (2011)) which has implications for the structure of the underlying preferences, and may be too restrictive to accurately reflect the true level of wellbeing. Secondly, insofar as many of the parameters within the index are subject to the investigator's choice, these choices may impinge on the index's ability to correctly identify the set of individuals that constitute the poor (see for example Ravallion (2010)). Finally, the breadth and depth granted by its ability to incorporate the multi-dimensional

[1]Some methods used in determining the weights for each dimension of wellbeing include statistical, survey-based, normative-participatory, frequency-based, or a combination of these (See inter alia Atkinson et al. (2002), Brandolini and D'Alessio (1998), Decancq and Lugo (2008) and Sen (1996, 1997)).

nature of individual wellbeing suffers from the "curse of dimensionality" as the number of dimensions considered rises, so that the results may not be robust (see for example Yalonetzky (2012)).

This paper presents a constructive method to address the issues of choice of weights and robustness, which may broaden the applicability of the index. Firstly, we propose that the coefficients used to weight the various dimensions be estimated using non-parametric bounds derived in Anderson et al. (2011) (ACL), and applied to a small number of high-level dimensions. This avoids the need to estimate many more lower-level dimensional weights, and thereby minimises the impact of the curse of dimensionality. Secondly, we propose the use of the overlap measure (see Anderson et al. 2010; Anderson et al. 2012) in the examination of the original AFMDI's, vis-a-vis one with estimated weights, ability to identify the underlying poverty population, thereby assess the robustness of the index. To demonstrate the force of the issues and the potential value of the approach taken here, we use unique data from a recent economic survey concerning life quality in the U.S. (Anand, Roope and Gray (2014 in preparation)).

In the following section, the relationship between our generalisation and the original AFMDI is discussed. Section 3 then goes on to consider robustness issues pertaining to the choice of weight. The informational burden imposed on the index by increasing dimensions is considered, a viable enhancement to the AFMDI is proposed, and a robustness test offered. Essentially, the approach developed provides a way of estimating individual preferences, and incorporating these into the overall index. Section 4 illustrates the approach with an application to data for the U.S., and the results obtained are used as a benchmark for evaluating the AFMDI. Section 5 reports our results and comparisions, and some conclusions are drawn in section 6.

2 Consumer Theory Based Weighting of Alkire-Foster Deprivation Index

The AFMDI has the individual as its basic agent. To see the relationship between the index and Consumer Theory, let there be a sample of N agents with D dimensions that constitute the potential deprivation (poverty/wellbeing) measure's database. In addition, let $K \in \{0, 1, 2, \ldots, D-1\}$ denote the number of dimensions in which the agent fails to meet the threshold for minimum wellbeing. Then the generalized version[2] of the index for the K^{th} level of deprivation may be written as:

$$M_K^\alpha = \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(c_n > K)\sum_{d=1}^{D}\frac{w_d}{D}\mathbb{I}\left(\frac{x_{nd}}{\underline{x}_d} < 1\right)\left(\frac{\underline{x}_d - x_{nd}}{\underline{x}_d}\right)^\alpha \tag{1}$$

where $\mathbb{I}(.)$ is an indicator function (which is 1 when $c_n > K$, and 0 when $c_n \leq K$), c_n is the count of the number of dimensions in which agent $n \in \{1, \ldots, N\}$ is deprived, w_d is a weight attached to the d^{th}, $d \in \{1, \ldots, D\}$, deprivation dimension, $x_{n,d}$ is the deprivation level experienced by the n^{th} agent in the d^{th} dimension, $\underline{x}_d$ is the deprivation threshold on the d^{th} dimension, and $\alpha = \{0, 1, \ldots\}$ is the parameter for the degree of aversion to poverty (deprivation/wellbeing) in the generalized Foster et al. (1984) (FGT) index. The parameter α is most frequently set to zero in practice (and for the empirical analysis that follows), which implies that the index involves an average across agents of a weighted count of the number of dimensions in which an individual has not met the deprivation metric particular to each dimension. Some of the variety of approaches to setting dimension weights include statistical, survey-based, normative-participatory, frequency-based, or a combination of these (See inter alia Atkinson et al. (2002), Brandolini and D'Alessio (1998), Decancq and Lugo (2008) and Sen (1996, 1997)).

On closer examination of the Alkire-Foster index of equation (1), one begins to observe the measure as similar to the aggregation of additively separable utility functions (see for example de la Vega and Urrutia (2011)), where the weights could be construed as the marginal effect of each respective dimension. Consider the familiar *complete union* (failure to meet at least one metric, $c_n = K = 0$) $-$ *complete intersection* (failure to meet all metrics $c_n > K = D - 1$) approaches are extreme versions of the Alkire-Foster index. *Complete union* can be interpreted as the case where all goods are perfect complements in deprivation, so that deprivation in one good implies deprivation in all, whilst *complete intersection* can be interpreted as the case where all goods are perfect substitutes for each other, so that deprivation will only arise when there is deprivation in all.

This suggests that the choice of weights for equation (1) is much like approximating the deprivation/wellbeing boundary, which we denote as $\mathcal{U}^*$, of an individual's preference function $\mathcal{U}(\mathbf{x})$ over $\mathbf{x}$ (in the present context the vector $\mathbf{x}$ represents the various dimensions of functionings, and capabilities of agents). As will be seen in the following section, equation (1) implies a very restrictive form of $\mathcal{U}(.)$, which may not represent or be close

[2]This incorporates a notion of the Foster et al. (1984) class of impoverishment indices.

to the true preferences or needs of agents due to the separability assumptions. Consider an individual's work life wellbeing versus that of his living environment concerns, as manifested in choosing between a good school district for their children that is significantly farther away from the individual's workplace, which consequently affects the individuals work-life balance due to longer commuting times. Or a fresh college graduate having to sacrifice the comforts of suburbia to move to a city center location with higher rents in order to be close to her workplace. Should such incidences be dominant within a population, an index that does not account for these inter-dimensional relationships will not accurately reflect individual wellbeing.

It is possible to avoid the issues generated by the separability restrictions, and generate parameter free wellbeing indices. Anderson et al. (2011) showed how two-sided bounds can be placed on a welfare index for each agent using only the assumptions that wellbeing (as measured by the welfare index) be non-decreasing, and weakly quasi-concave with respect to its various dimensions. Unfortunately, from a policy maker's perspective, these ACL bounds are frequently considerably far apart. Furthermore, it is not always clear from such an index what the role or importance of a particular dimension is, so that some sort of parameterization will inevitably be necessary. As will be explored in section 4 in relation to the application, the strategy proposed here is to reduce the dimensional burden and address the functional flexibility requirement in a simple three stage procedure.

1. Firstly, welfare bounds are calculated using the ACL method for higher dimensional (narrowly defined) wellbeing indicators.

2. In the second stage, relationships between the lower dimensional (broadly defined) welfare indicators calculated from the bounds generated by the ACL procedure are estimated to establish relative weights.

3. Calculate the AFMDI using the weights obtained in the second stage.

It is the weights determined in the second stage that facilitate measurement of the relative importance of the various lower dimensional wellbeing concerns, thereby advising public policy.

3 Robustness and the Dimensionality Problem

The issue of robustness of the orderings in international comparisons using aggregating measures such as the AFMDI and HDI arises principally because of the subjective parametric choices of the weights, the wellbeing/poverty cut-offs, and the critical number of dimensions in which an agent is below the cutoff before it is deemed to constitute impoverishment. Within and between society comparisons are commonly performed using stochastic dominance techniques[3] which can be interpreted as providing conditions (the order and direction of dominance) under which certain classes of wellbeing/poverty measures, such as the union and intersection restricted cases of index (1), will be robust in the sense that, if the conditions prevail, all measures in the class will unambiguously reveal a consistent ordering of the distributions. In all cases, they end up comparing some notion of distance between two surfaces. However, when dimensionality increases substantially, these techniques run into practical difficulties not unlike the "curse of dimensionality" in nonparametric economics.

The problem is the rapidly increasing demands placed upon sample size for effective estimation and testing of multi-dimensional functions as dimensions increase, because probability distributions underlying the surfaces being compared lose mass at the center of their distribution as dimensionality increases[4]. This results in the flattening of the distributions over their support, and the surfaces of the distributions inevitably begin to resemble each other, thereby reducing the distance between them, and making it more difficult to discriminate between them. Another way to consider the effect is to note that notionally similar points in D dimensional space grow further apart as D increases. As an example, consider $\phi(\mathbf{0})$ the peak at the center of the joint distribution of D i.i.d. standard normal variables (here $\mathbf{0}$ is the D dimensional null vector), $\phi(\mathbf{0}) = 1/(2\pi)^{D/2}$ which goes to 0 as D increases, and the Euclidean distance between this null vector and the unit vector in D dimensioned space is $\sqrt{D}$, which obviously increases with D. This issue thus fundamentally impinges on the primary ability of the AFMDI to identify poverty, diluting its insights as the dimensions increase.

[3]The classic multi-dimensional stochastic dominance comparisons (See Atkinson and Bourguignon (1982), Duclos et al. (2006), and Anderson (2008) in a continuous framework, and Anderson and Hachem (2012) in a continuous/discrete framework)

[4]This problem is familiar to neural networks researchers, and is referred to as the empty space phenomenon (Verleysen 2003)

Yalonetzky (2012) examines this by developing multivariate stochastic dominance techniques to provide full robustness conditions on $f_A(x)$ and $f_B(x)$ for such "counting" type measures in multi-dimensional settings. He demonstrates that when the number of dimensions considered are greater than 2, generalizations of the Atkinson and Bourguignon (1982) continuous distribution stochastic dominance framework are usually not applicable (an early sign that increasing dimensionality creates complications). Nonetheless, he does obtain some results by restricting the family of measures to particular subclasses (typically the aforementioned union and intersection type measures). His empirical application (based upon the SILC dataset) illustrating the use of these conditions for ordinal variables, found robustness in only 45 of the 325 pairwise comparisons in the two variable case (13.85%), and 38 of the 325 pairwise comparisons (10.8%) in the three variable case (Note the poor and deteriorating robustness when moving from two to three dimensions). All things considered, there is a need to develop alternative methods of discerning robustness of various extensions of the AFMDI.

3.1 "Curse of Dimensionality" & the Functional Form of Preferences

Yatchew (1998, 2008) nicely describes the curse of dimensionality problem, and points to a potential solution. He considers the general equation $y = f(x_1, x_2) + e$, where it is supposed that $f(.,.)$ is a two dimensioned function on the unit square. In order to approximate the function, it is necessary to sample throughout its domain. If T points are distributed uniformly on the unit square, each will "occupy" an area $1/T$, and the typical distance between points will be $1/\sqrt{T}$, so that the approximation error is now $O(1/\sqrt{T})$. Repeating this argument for functions of k variables, the typical distance between points becomes $1/(T^{1/k})$, and the approximation error is $O(1/(T^{1/k}))$. In general, this method of approximation yields error proportional to the distance to the nearest observation. For $T = 100$, the approximation error is 10 times larger in two dimensions than in one, and 40 times larger in five dimensions.

However, if $f(.,.)$ can be assumed to be *additively separable* on the unit square so that $f(x_1, x_2) = f_1(x_1) + f_2(x_2)$, and $2T$ observations are taken (T along each axis), then $f_1(.)$ and $f_2(.)$ can each be approximated with error $O(1/T)$ so that the approximation error for $f(.,.)$ is also $O(1/T)$. In other words, additive separability maintains the approximation error to a multi-dimensional function within a single-dimensional framework, so that the assumption of *additive* (sometimes referred to as *Strong*) *separability* buys much in terms of information requirements. However it is a very strong assumption which needs significant theoretical and empirical justification.

Strongly or additively separable structures for utility functions have played an important role in the empirical development of the theory of consumer behavior, largely as simplifying assumptions for the purposes of facilitating estimable demand equations (for an extensive discussion see Deaton and Muellbauer (1980)) in the context of very limited data sets. A widely used strongly separable utility function is the Stone-Geary utility function underlying the Linear Expenditure System (Stone 1954) which, for goods q_d, $d = \{1, \ldots, D\}$, may be written as:

$$\mathcal{U}(q_1, q_2, \ldots, q_d) = \sum_{d=1}^{D} \beta_d \ln(q_d - \gamma_d) \tag{2}$$

Here, for $\mathcal{U}(.)$ to be appropriately concave, $\beta_d \geq 0$, $q_d > \gamma_d$ for all $d = \{1, \ldots, D\}$, and $\sum_{d=1}^{D} \beta_d = 1$. Usually the γ_d's are interpreted as minimum subsistence requirements defining the lower boundary of the feasible consumption set. Note should be taken here of the mathematical similarity of this structure to (1); AFMDI, is in essence, a strongly separable representation of preferences for (or aversion to) deprivations.

Stone-Geary is frequently referred to as a "want independent" model, since $\partial^2\mathcal{U}/\partial q_1 \partial q_2 = 0$ for $i \neq j$ (note the same is true for (1)). In a similar fashion, the subsistence levels can be considered independent in this system of preferences since the cross partials with respect to the gamma's are also zero. Its attraction is that it requires estimation of only $2n - 1$ parameters[5], much like the deprivation part of $M_K^{\alpha=1}$ in (1) (an unrestricted demand system would require $n^2 - 1$ parameters to be estimated). The problem with (2) in the present context is that it does not admit inferior goods ($\beta_d < 0$ violates concavity), and unfortunately sustaining the concavity assumption also precludes the presence of complementary goods. This is instructive when thinking about the AFMDI, especially when considering the complementarity arguments for using the set intersection ($K = 0$) version of (1). Should an investigator consider estimating a utility function in order to derive the requisite weights for use in the AFMDI, a Stone-Geary utility function would not suffice. What is required is a more general description of the relationship between various sources of wellbeing. Furthermore, if the AFMDI is seen as representing deprivations across groups of goods, with each dimension representing a subgroup, then

[5] Stone (1954) only had 19 observations on the prices of and expenditures on 6 commodities!

additively separable preferences over such subgroups impose extremely strong restrictions over the substitutability between the components across subgroups, especially at the level of subsistence.

To put it more succinctly, the AFMDI does not admit tradeoffs between the domains. In the analysis of wellbeing, this is obviously not the case. For instance, it does not admit the possibility that an individual might be willing to trade some domestic sources of life quality for those produced at work as suggested in the previous discussion. If one wished to accommodate such inter-relationships in the AFMDI, one could contemplate a modification of (1) to the following form:

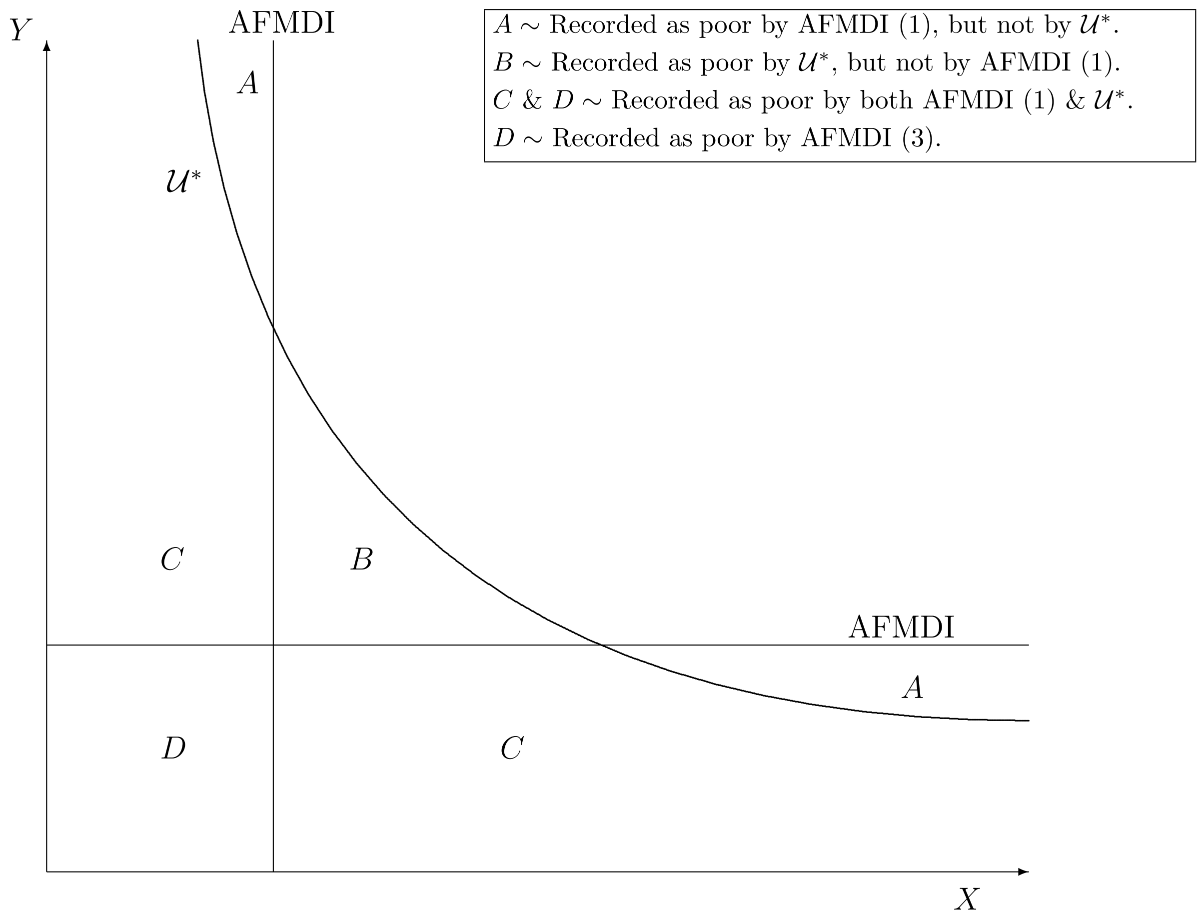
$$M_K^\alpha = \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(c_n > K)\sum_{d=1}^{D}\frac{w_d}{D}\left\{\begin{array}{c}\mathbb{I}\left(\frac{x_{nd}}{\underline{x}_d} < 1\right)\left(\frac{\underline{x}_d - x_{nd}}{\underline{x}_d}\right)^\alpha \\ \times \left[1 + \sum_{h=d+1}^{D}\frac{w_{dh}}{D}\mathbb{I}\left(\frac{x_{nh}}{\underline{x}_h} < 1\right)\left(\frac{\underline{x}_h - x_{nh}}{\underline{x}_h}\right)^\alpha\right]\end{array}\right\} \quad (3)$$

where w_{dh}'s are chosen to reflect the complementary ($w_{dh} > 0$) or substitutability ($w_{dh} < 0$) nature of capabilities x_d and x_h, $d \neq h$, d, $h \in \{1, \dots, D\}$. It remains nonetheless, an empirical question whether the AFMDI of equation (1) or (3) best reflects the aggregate deprivations of a society. As noted above, the question of robustness is complicated by the dilution of such indices as dimensions considered increase. The following section discusses an alternative method for examining robustness of the AFMDI, and measures of welfare that focuses on the identity of the observations the researcher is interested in examining.

3.2 Empirical Examination of the Robustness Issue

Taking a step back from the traditional way of considering robustness through examining ordering across populations, it is proposed here that one should consider instead the "identity" of observations classified as belonging to a group. In other words, the extent to which alternative parameterizations of the AFMDI identify the same underlying groups is considered, since robust parameterizations should largely identify very similar groups. This idea in two dimensions is illustrated in diagram 1 below. $\mathcal{U}^*$ denotes the deprivation frontier estimated under a relevant parameterization, while the lines denoted by AFMDI represents the alternative frontier according with Alkire and Foster (2011) under another relevant set of parameterization (the AFMDI frontiers based on equation (1) are horizontal and vertical lines due to the fact that it is essentially a count measure for a particular level of deprivation). The groups demarcated by these lines, from A to D are differing groups of individuals with differing levels of wellbeing. Robustness in this context is then a question of the extent to which sets $(A \cup C \cup D)$, and $(C \cup B \cup D)$ are equivalent, which thus reduces the question to whether sets A and B are indeed empty sets[6].

Figure 1: AFMDI versus $\mathcal{U}^*$ in Two Dimensions



This may be examined by a combination of overlap measures (using Anderson et al. (2010), and Anderson et al. (2012)), one which reports the proportion of agents *defined* as poor by $\mathcal{U}^*$ that are *reported* poor by Alkire-Foster (a value of 1 indicates Alkire-Foster fully covers $\mathcal{U}^*$), and one which reports the proportion of agents *defined* as poor by Alkire-Foster that are *reported* poor by $\mathcal{U}^*$ (a value of 1 indicates $\mathcal{U}^*$ fully covers Alkire-Foster).

[6]Note that when wellbeing is measured by a single variable, the difference between equation (1) and another alternative is in the weight applied, so that either one distribution will first order stochastically dominate the other depending on the weight, and one of these sets will always be empty. This is another sign of the complications associated with increasing dimensionality, since the likelihood of non-empty sets A and B arising increases as the number of dimensions increases.

In other words, the two respective measures can be written as,

$$\mathbf{OV}_{(\mathcal{U}^* vs.\text{AFMDI})|\text{AFMDI}} = \sum_{\delta \in \Delta_{\text{AFMDI}}} \min \left\{ f_{\text{AFMDI}}(z_\delta), \frac{n_{\mathcal{U}^*}}{n_{\text{AFMDI}}} f_{\mathcal{U}^*}(z_\delta) \right\} \tag{4}$$

$$\mathbf{OV}_{(\mathcal{U}^* vs.\text{AFMDI})|\mathcal{U}^*} = \sum_{\delta \in \Delta_{\mathcal{U}^*}} \min \left\{ \frac{n_{\text{AFMDI}}}{n_{\mathcal{U}^*}} f_{\text{AFMDI}}(z_\delta), f_{\mathcal{U}^*}(z_\delta) \right\} \tag{5}$$

where $n_j \in \{n_{\text{AFMDI}}, n_{\mathcal{U}^*}\}$ is the number of observations classified as poor under the respective method j, z_δ is the K vector of variable realizations, and Δ_j is the set of cells classified as poor. Further, the overlap index of Anderson et al. (2010) is used to examine the overlap or joint coverage of the two poverty samples generated.

$$\mathbf{OV}_{cov} = \sum_{\delta \in \Delta_{\mathcal{U}^* \cup \text{AFMDI}}} \min \{ f_{\text{AFMDI}}(z_\delta), f_{\mathcal{U}^*}(z_\delta) \} \tag{6}$$

A robustness index[7] on the $[0, 1]$ interval is provided by taking an average of the two overlap measures, with 1 implying absolute robustness, and 0 implying no connection between the two poverty groups.

4 Incorporating a Flexible Representation of Agent Preferences into the Calculus

4.1 Data

To explore empirically how our procedure works and the insights it can generate, we draw on a data set specifically designed to measure different aspects and dimensions of human wellbeing (Anand, Roope and Gray (2013 in preparation)). The 2011 data set contains national samples of working age adults in the U.S., U.K. and Italy, though for illustrative purposes we focus on the U.S. sample. This sample comprises data drawn equally from four regional areas of the U.S. yielding 1061 observations, and is designed to be roughly representative with respect to age, gender, and social class.

The overall index of wellbeing (Y) is based on three indicators obtained from the responses to question 1, parts (a), (b), and (d). These are the dimensions that the utility function are monotonically increasing in. Aside from the overall index of wellbeing, satisfaction with respect to six categories of wellbeing were surveyed (respondents were asked

[7]Generally these Overlap measures are approximately $N(p, (p(1-p)/n))$, which facilitates inference.

to report satisfaction on a $0-10$ scale, with 0 denoting "strongly disagree", and 1 denoting "strongly agree".), the details of specific questions are in appendix A. The variables are interpreted as domain assessments, based on self-assessed capability scores. In other words, the notion of wellbeing corresponds to the freedom based notion of advantage.

The dimensions are as follows:

1. $X_1 \equiv X_{\text{exfam}} \sim$ Social relationships *external to the family* (3 dimensions)
2. $X_2 \equiv X_{\text{infam}} \sim$ Social relationships *internal to the family* (7 dimensions)
3. $X_3 \equiv X_{\text{work}} \sim$ *Work opportunities* and constraints (6 dimensions)
4. $X_4 \equiv X_{\text{comm}} \sim$ *Local social opportunities* and constraints (4 dimensions)
5. $X_5 \equiv X_{\text{env}} \sim$ *Environmental* concerns (5 dimensions)
6. $X_6 \equiv X_{\text{svc}} \sim$ Access to *services* (7 dimensions)

Due to non-response to question 4 regarding local social opportunities and constraints, the final data used in the analysis was reduced to 725. Although unreported here, results from the full data set with the exclusion of question 4, as well as a smaller random set were investigated, and the results found here are robust.

4.2 Structuring the Estimating Equations

In general, when applying multi-dimensional indices to wellbeing, it will often be desirable to incorportate information about the relative importance individuals assign to different dimensions. Since the data set employed here records responses on satisfaction with respect to overall wellbeing, as well as responses to the various dimensions of wellbeing, it presents an opportunity to examine the importance of the various dimensions of wellbeing that constitute overall happiness, $\mathcal{U}(\mathbf{X})$. In general, overall happiness $\mathcal{U}(\mathbf{X})$ has I dimensions, in other words $\mathcal{U}(\mathbf{X}) \equiv \mathcal{U}(U_1(X_1), U_2(X_2), \ldots, U_I(X_I))$, where with a slight abuse in notation, X_i, $i \in \{1, \ldots, I\}$ are each J_i vectors of responses to questions on wellbeing on the i^{th} dimension, with typical elements x_{ij}, $j \in \{1, \ldots, J_i\}$, so that $\mathbf{X} = [X_1, \ldots, X_I]'$ is a $\sum_{i=1}^{I} J_i$ dimensional vector.[8] Thus $\mathcal{U}$ is assumed to be weakly

[8]For discussions of the satisfaction, happiness and utility nexus see Clark et al. (2008), Clark et al. (2009), Kahneman and Deaton (2010), Kimball and Willis (2006), Layard (2005).

separable in the sub-components of each dimension of wellbeing[9].

Unlike conventional empirical approaches to consumer theory, where the nature of preferences (the structure of $\mathcal{U}(.)$) has to be inferred from agents' observed incomes, expenditures on goods, and the prices they face, here direct observations on $\mathcal{U}(\mathbf{X})$, and estimates of its various sub-components $U_i(X_i)$, $i = \{1, \ldots, I\}$, are available. Thus the structure of $\mathcal{U}(.)$ can be estimated directly, which will in turn provide information on the structure of indices such as (1), or provide alternative indices. Estimating an unrestricted wellbeing function over all of the sub-components within each of the $(I = 6)$ dimensions of wellbeing in this data would require 496 parameters, which given a sample size of 725 leaves very few degrees of freedom. Therefore, in lieu of the previously discussed concerns regarding increasing dimensionality, we propose here that for the sub-component functions, $U_i(.)$, ACL be employed to provide an aggregating wellbeing index of the sub-components in each dimension for each observation, as a first stage procedure. The wellbeing indices derived for each dimension are essentially defined in relation to a (arbitrary) social planner's view of an observation's welfare, W_i on the i^{th} dimension(Anderson et al. 2011), and the level sets associated with W_i therefore represents the manner in which the social planner would trade between each dimension, or sub-component in the current application, on behalf of an observation.

For each of the $(I = 6)$ dimensions of wellbeing of an observation, the ACL technique generates an upper and lower bound, $\overline{D}_i$ and $\underline{D}_i$ respectively, $i \in \{1, \ldots, I = 6\}$, for the distance function which measures the amount by which the Euclidean norm of the component vector of the observation, $||X_i||$, has to be scaled to reach a reference welfare level, W_i. This idea is illustrated for two dimensions below in figure 2 for an agent a, on the i^{th} wellbeing dimension with 2 sub-components. X_i^b and X_i^c are adjacent observations of agent b and c, whose information are used to derive the bounds. Anderson et al. (2011) showed that these two-sided bounds can be placed on each dimension of wellbeing for each observation, using only the assumptions that each wellbeing component be non-decreasing and weakly quasi-concave with respect to the sub-components. The bounds encompass the entire set of wellbeing sub-components consistent with monotonicity and quasi-concavity, and as such they provide a region within which any parametric index

[9] This is tantamount to assuming a weakly separable wellbeing function with respect to the sub-components, whilst admitting more standard substitutability and complementarity properties between their aggregates (Gorman 1968; Kannai 1980).

with these properties can be expected to lie[10].

Here, the wellbeing index of the individual from the perspective of the social planner, $U_i(X_i)$, is defined as,

$$\begin{aligned} U_i(X_i) &= \left(1 - \frac{\overline{D}_i + \underline{D}_i}{2}\right) ||X_i|| \\ &= \left(1 - \frac{\overline{D}_i + \underline{D}_i}{2}\right) \sqrt{\sum_{j=1}^{J_i} x_{ij}^2} \end{aligned} \tag{7}$$

In other words, the wellbeing index can be thought of as measuring the "size" of X_i relative to the reference welfare level W_i, which without loss of generality, is here defined as midway between the two bounds. To be precise,

$$W_i = \left(\frac{\overline{D}_i + \underline{D}_i}{2}\right) ||X_i|| \tag{8}$$

The approach is applied directly to our data, and is fully nonparametric in the sense that it does not require any further assumptions on the functional form of the welfare function within each dimension, nor does it require the estimation of any functions of the data. Indeed the method can be applied to very small datasets (as well as to large ones) where statistical techniques − and especially nonparametric statistical techniques − could not be relied upon. A useful feature is that the methodology is easily replicable requiring nothing more complex than standard linear programming techniques.

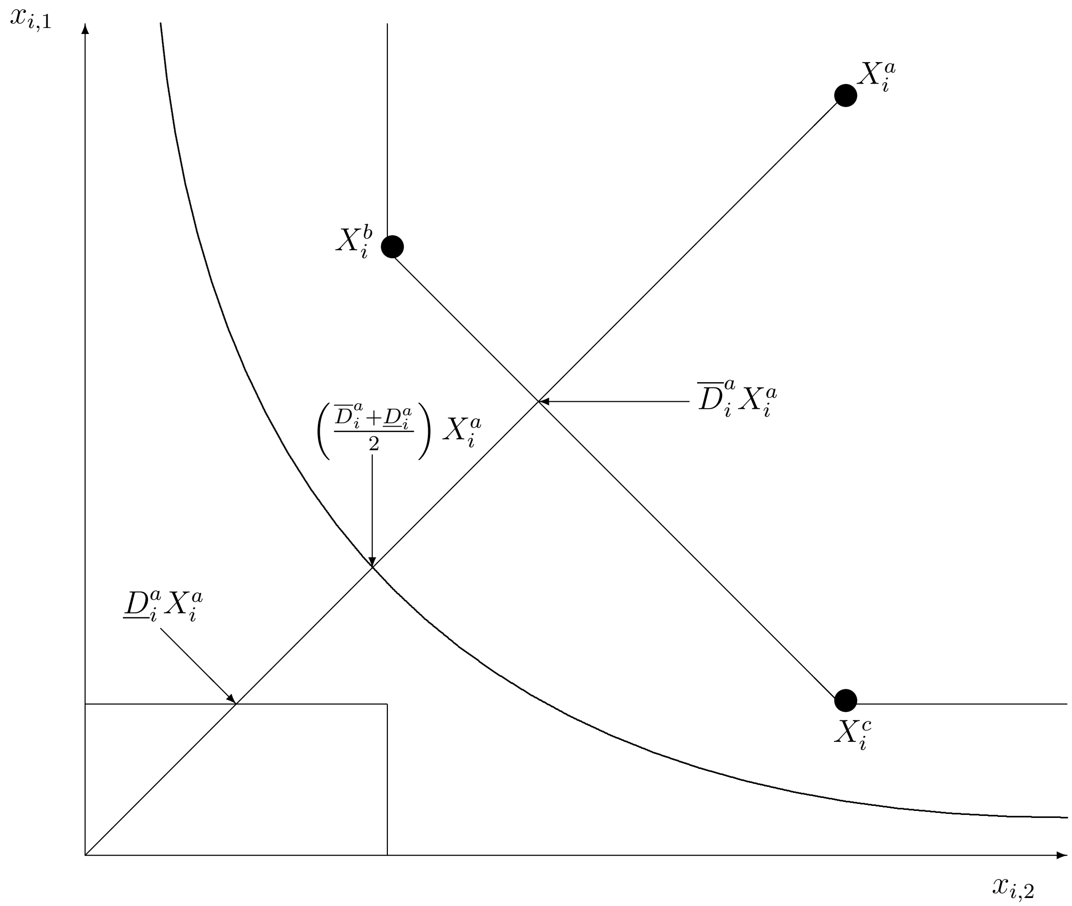
To address the issue of flexibility in modelling the inter-relationships between the various dimensions of overall wellbeing, ($\mathcal{U}$) is assumed to be quadratic in the components of wellbeing (X_i's) in the second stage of our procedure. Thus for each agent, her vector of component wellbeing outcomes is $\mathbf{X} = [X_1, X_2, \ldots, X_6]'$, and her overall satisfaction is:

$$\mathcal{U}(\mathbf{X}) = \mathbf{XAX} \tag{9}$$

In addition, let $|\mathbf{A}| < 0$, and $\mathcal{U}$ be monotonically non-decreasing, and concave in $\mathbf{X}$ (essentially the standard assumptions in consumer theory). Note the caveat that concave preferences are usually only required in consumer theory because budget sets are weakly convex, and concavity facilitates constrained optimization solutions. Technically all that is required is for preferences to be less convex than the budget sets (or more concave

[10] The bounds' span can be shown to diminish with the wellbeing index (bounds are tighter on high wellbeing, and loose on low wellbeing), and generally increase with dimensionality.

Figure 2: Two-Sided Bounds on Distance Function



than the budget sets if they are concave). Further, adopting Young's theorem, **A** will be symmetric, (indeed all that can be identified in a regression version of (9) is a symmetric version of **A**), and a version of **A** consistent with an AFMDI structure would have **A** be a diagonal matrix. Note that the model adopted at this stage is dependent on the research, thus affording the prescribed technique significant flexibility to adapting to the variables on hand.

The assumption of quadratic structure of overall wellbeing is a simple way of examining the various dimensions' and sub-components' effect on overall wellbeing. To understand the impacts of the various components, first note that:

$$\begin{aligned}\frac{\partial U_i(X_i)}{\partial x_{ik}} &= \left(1-\frac{\overline{D}_i+\underline{D}_i}{2}\right)\frac{x_{ik}}{\sqrt{\sum_{j=1}^{J_i} x_{ij}^2}} \\ &= \frac{x_{ik}W_i}{\frac{\left(\overline{D}_i+\underline{D}_i\right)}{2}\left(\sum_{j=1}^{J_i} x_{ij}^2\right)^{3/2}} > 0\end{aligned}$$

where J_i denotes the total number of sub-components within component i. Then the measure of marginal wellbeing with respect to the k^{th} sub-component of the i^{th} wellbeing dimension, and its degree of complementarity/substitutability with the h^{th} sub-component on the g^{th} wellbeing dimension are respectively:

$$\frac{\partial \mathcal{U}}{\partial x_{ik}} = 2\sum_{m=1}^{6} a_{im}X_m \frac{x_{ik}W_i}{\left[\frac{\left(\overline{D}_i+\underline{D}_i\right)}{2}\left(\sum_{j=1}^{J_i} x_{ij}^2\right)\right]^{3/2}} > 0 \quad \text{if} \quad \sum_{m=1}^{6} a_{im}X_m > 0 \tag{10}$$

and

$$\frac{\partial^2 \mathcal{U}}{\partial x_{ik}\partial x_{gh}} = 2a_{ig}\frac{x_{ik}W_i}{\left[\frac{\left(\overline{D}_i+\underline{D}_i\right)}{2}\left(\sum_{j=1}^{J_i} x_{ij}^2\right)\right]^{3/2}}\frac{x_{gh}W_g}{\left[\frac{\left(\overline{D}_g+\underline{D}_g\right)}{2}\left(\sum_{j=1}^{J_g} x_{gj}^2\right)\right]^{3/2}} \tag{11}$$

where the sign of the latter is governed by the sign of a_{ig}, the $\{i,g\}$ element of the matrix $\mathbf{A}$. The crucial point here is the central role that the coefficient matrix $\mathbf{A}$ plays in determining not just the sign of the marginal benefit derived from each dimension of wellbeing, but the joint effect that the differing dimensions have on overall wellbeing.

It should be emphasized that the above combination of the ACL technique in reducing the burden of increasing dimensionality, and the use of a simple representation of preference structure, required the estimation of only 21 parameters, in contrast with the 496 parameters without the use of ACL. Yet the technique still admits a measure of substitutability between the various dimensions of overall wellbeing which can be transported to the AFMDI.

This is achieved in the final step of the procedure by substituting the empirical counterpart of the marginal benefit and cross partials, $\frac{\partial \mathcal{U}}{\partial X_i}$ and $\frac{\partial^2 \mathcal{U}}{\partial X_i \partial X_j}$, $i, j \in \{1, \ldots, 6\}$, for w_d and w_{dh} in equation (3) respectively, in the calculation of the generalized version of the AFMDI. The empirical counterpart can be calculated at the means (used in the application below), or medians, or be evaluated at relevant values of X_i, for the various dimensions of overall wellbeing. In consequence, deprivation in functionings and capabilities may now be considered in terms of a threshold, such as some percentile of the population who have satisfaction wellbeings below some particular metric (for example those below a wellbeing measure reflecting some percentage of the median level or average level in each dimension). Since $\mathcal{U}(.)$ can be computed, standard univariate deprivation, inequality and polarization measures can all be employed using this calculation. In summary, the attraction of the approach is that it provides maximal flexibility in the representation of overall wellbeing, whilst admitting the possibility of component (and sub-component) substitutability/complementarity in the overall index, and retaining the ability to measure the impact of improvements/worsenings of each dimension (and sub-component), and yet reduce the increasing dimensional burden on the AFMDI when the researcher seeks to model the myriad concerns that impinge on individual wellbeing.

5 Empirical Analysis

5.1 Summary Statistics

Drawing on the preceding discussion, an analysis of multi-dimensional wellbeing in the U.S. follows. At the heart of the analysis are summary indices for overall wellbeing, and indices for each of the components of wellbeing for each agent, based upon Anderson et al. (2011) (ACL). Table 1 reports the statistical properties of the ACL indices, while Table 2 reports the properties of the ACL bounds. From Table 1, the distribution of each component across agents is left skewed (that is their means are less than medians, implying the distribution is dense in the upper range of the index, but sparse in the lower range), and are similar across the domains.

As noted above, the ACL bounds encompass the entire set of possible welfare functions (W_i's) consistent with monotonicity and quasi-concavity, and can be used as a computationally convenient robustness check on parametric methods. Thus the difference between the high and low bounds provides an idea of the range within which the true W_i's lie. As can be seen from Table 2, the distribution of the differences between the bounds are right skewed, and negatively correlated with the ACL indices themselves, implying that the bounds on high levels of wellbeing are tight (i.e. narrow), whereas bounds on low levels of wellbeing are loose.

Table 1: ACL Indices (Number of Factors in each Domain in Parenthesis)

Summary Stat.	$Y(3)$	$X_{\text{exfam}}(3)$	$X_{\text{infam}}(7)$	$X_{\text{work}}(6)$	$X_{\text{comm}}(4)$	$X_{\text{env}}(5)$	$X_{\text{svc}}(7)$
Means	8.3142	8.1796	7.8711	7.7131	8.2496	8.0729	8.1737
Medians	8.6250	8.5334	8.4722	8.4722	8.6032	8.6643	8.6250
Minimums	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Maximums	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
Std. Dev.	1.0933	1.1949	1.5053	1.7341	1.2395	1.5192	1.3774

Table 2: Bounds for ACL Indices $\left(\overline{D}_i - \underline{D}_i\right)$

Index (Dimensions)	Mean Difference	Median Difference	Standard Deviation	Index/Bounds Correlation
$Y(3)$	0.5385	0.2222	1.2209	-0.6594
$X_{\text{exfam}}(3)$	1.0077	0.3493	1.9795	-0.8858
$X_{\text{infam}}(7)$	1.8707	0.7334	2.7586	-0.9415
$X_{\text{work}}(6)$	2.1042	0.6112	3.1338	-0.9258
$X_{\text{comm}}(4)$	1.1063	0.375	2.0585	-0.8224
$X_{\text{env}}(5)$	1.5594	0.375	2.8352	-0.9284
$X_{\text{svc}}(7)$	1.395	0.4584	2.6192	-0.9533

The domain indices were then employed to estimate a symmetric version of **A** of equation (9) using the regression equation:

$$Y_n^2 = \sum_{i=1}^{6}\sum_{j=1}^{6} a_{ij} X_{in} X_{jn} + e_n \qquad \text{for} \quad n = 1, \ldots, N$$

Here $\mathcal{U}(.)$ is set to Y^2 for scaling reasons, the standard regression assumptions on e and the X's are adopted, and n is the number of agents in the sample (725 in this case). Based on these results[11], Table 3 reports the "marginal wellbeing" at various values of the component indices (means, medians and minimums). These evaluated derivatives (where $\widehat{\ }$ indicates estimated values, for a_{ij} at the mean, median and minimum of the

[11] The complete results are not reported here but are available from the authors on request.

X_i's respectively) may be written as:

$$\frac{\partial Y^2}{\partial \widehat{X}_j} = 2\sum_{i=1}^{6} \widehat{a}_{ij}\widehat{X}_i \qquad \text{for} \quad \forall j$$

They may be construed as reflecting respectively the "average", the "median", and the poorest person's marginal benefit for each dimension of wellbeing, and provides the basis for the weights in the AFMDI. Notice that these are not based on "needs independent" equations (i.e. the incremental benefit accruing to an incremental change in the i^{th} dimension depends upon the status of the j^{th} dimension).

Table 3: Marginal Benefits from the Six Domains

Derivative	Sample Means	Sample Medians	Sample Minimums
$\partial\mathcal{U}/\partial X_{\text{exfam}}$	3.0859	3.0771	0.3253
$\partial\mathcal{U}/\partial X_{\text{infam}}$	2.5161	2.6878	0.2945
$\partial\mathcal{U}/\partial X_{\text{work}}$	1.0160	1.2512	0.1306
$\partial\mathcal{U}/\partial X_{\text{comm}}$	1.6600	1.6744	0.1680
$\partial\mathcal{U}/\partial X_{\text{env}}$	-0.1846	-0.3052	-0.0338
$\partial\mathcal{U}/\partial X_{\text{svc}}$	0.9950	1.1880	0.1205

$\det(A) = -0.0971$

From the results of Table 3, note first the substantial differences in the weights across the X_i's, arguing for something other than equal weighting (Atkinson et al. (2002)) across the domains. Since the determinant of $\mathbf{A}$ is negative, the wellbeing function is concave (albeit slightly). It is interesting to note that both external relationships (first factor), and household relationships and concerns (second factor) dominate overall wellbeing. However note the problem with the fifth factor (Local Social Opportunities and Constraints), with its negative values across all average, median and poorest agents, indicating that $\mathcal{U}$ is not monotonically non-decreasing in this particular domain. This is likely due to multicollinearity between this dimension and the others, which is unsurprising given the proximity of concerns pertaining to living environment reflected in the fifth factor versus community concerns reflected in fourth. Thus, based upon the elimination of the fifth factor, a 5 factor model is considered in the following.

5.2 5 Factor Model & Optimal Weights

Table 4 presents the full set of estimated coefficients for the 5 factor model. First note that overall wellbeing is convex in all its components, with it being strongest in terms of social opportunities external and internal to the family (X_{exfam} and X_{infam}). A particularly significant finding is the existence of statistically significant substitution trade-offs for both concerns external and internal to the family vis-à-vis all the other dimensions. Interestingly there are even hints of some complementarities, though only the relationship between work concerns and local social opportunities is statistically significant. It is important to note that the AFMDI of equation (1) would not have been able to accommodate these relationships, since the AFMDI essentially reflects "needs independent" preferences on the part of agents. As far as we know, this is the first instance of empirical evidence about the extent of substitutability and complementarity between aspects of freedom.

Table 5 reports the marginal benefits in this model. As may be observed, the preference structure is convex ($|\mathbf{A}| > 0$), though only marginally so, but now overall satisfaction is monotonically non-decreasing in all domains. Further, note the very different marginal effects relative to those of Table 3, which again provides a counter argument against equal weighting in an AFMDI type index. Nonetheless, marginal benefit remains strongest along both the dimensions of social concerns external and internal to the family.

To examine the effects of the various specifications of deprivation, an arbitrary cutoff of $0.7\times$ median is considered. Poverty counts for each domain would be for $X_{\text{exfam}} \sim 0.1055$, $X_{\text{infam}} \sim 0.1545$, $X_{\text{work}} \sim 0.0927$, $X_{\text{comm}} \sim 0.1127$ and $X_{\text{svc}} \sim 0.1164$. Table 6 reports an *Intersection Rule* ($c_n = K + 1$) poverty measure, and three variations of the AFMDI measures; the first of which is with equal weighting, the second with needs independent utility weighting based upon weights from a separate regression with the off diagonal elements of $\mathbf{A}$ set to zero, which are essentially *Union Rule* measures (since $c_n > K$), and the final one based on the full utility weighting from table 4 using equation (3). Note the substantial change in the measures when the weighting scheme changes from equal to "utility" weightings, which to a considerable degree supports the concerns expressed in Ravallion (2010). It is also important to point out the robustness of the fully weighted AFMDI measure (AFMDI_{WF}) as the number of impoverished domains ($K + 1$) that constitute poverty increases, relative to all other versions of the index. This feature combined with its robustness in identifying poverty, evident in the following discussion,

Table 4: The Five Variable Model Coefficients

Variable	Coefficient	Std. Error	$\lvert Z\rvert$	$1-F(\lvert Z\rvert)$
X^2_{exfam}	1.5013 ***	0.1495	10.0428	0.0000
$X_{\text{exfam}} \times X_{\text{infam}}$	-0.5085 ***	0.2079	2.4455	0.0072
$X_{\text{exfam}} \times X_{\text{work}}$	-0.5307 ***	0.2020	2.6270	0.0043
$X_{\text{exfam}} \times X_{\text{comm}}$	-0.0617	0.2199	0.2805	0.3896
$X_{\text{exfam}} \times X_{\text{svc}}$	-1.2253 ***	0.2476	4.9477	0.0000
X^2_{infam}	1.3284 ***	0.1552	8.5573	0.0000
$X_{\text{infam}} \times X_{\text{work}}$	-0.4169 ***	0.1763	2.3655	0.0090
$X_{\text{infam}} \times X_{\text{comm}}$	-0.7205 ***	0.2016	3.5734	0.0002
$X_{\text{infam}} \times X_{\text{svc}}$	-0.3155 *	0.2155	1.4639	0.0716
X^2_{work}	0.3811 ***	0.1475	2.5835	0.0049
$X_{\text{work}} \times X_{\text{comm}}$	0.2396 *	0.1536	1.5598	0.0594
$X_{\text{work}} \times X_{\text{svc}}$	0.1760	0.1984	0.8871	0.1875
X^2_{comm}	0.2535 **	0.1260	2.0129	0.0221
$X_{\text{comm}} \times X_{\text{svc}}$	0.1692	0.1736	0.9745	0.1649
X^2_{svc}	0.7359 ***	0.1320	5.5730	0.0000

$R^2 = 0.4651$, and $\sigma^2 = 117.8034$.

Note: ***, **, and * denote statistical significance at 1%, 5% and 10%.

Table 5: Marginal Benefits from the Five domains

Derivative	Sample Means	Sample Medians	Sample Minimums
$\partial\mathcal{U}/\partial X_{\text{exfam}}$	3.3140	3.1962	0.3382
$\partial\mathcal{U}/\partial X_{\text{infam}}$	2.9188	3.1849	0.3477
$\partial\mathcal{U}/\partial X_{\text{work}}$	0.9298	1.1106	0.1151
$\partial\mathcal{U}/\partial X_{\text{comm}}$	0.6415	0.6932	0.0669
$\partial\mathcal{U}/\partial X_{\text{svc}}$	1.2617	1.3991	0.1381

$\det(A) = 0.0324$

makes it the preferred measure.

Table 6: Overlap Comparison Between Intersection Rule vs. Variations of AFMDI Poverty Measures

	Minimum Number of Domains ($K+1$) in the AFMDI				
	1	2	3	4	5
Intersection Rule – Equal Weight	0.1917	0.0455	0.0262	0.0166	0.0069
AFMDI – Equal Weight (AFMDI)	0.2869	0.0952	0.0497	0.0234	0.0069
AFMDI – Utility Weight, Needs Independent (AFMDI$_W$)	0.7251	0.5266	0.4376	0.4297	0.2000
AFMDI – Full Utility Weights (AFMDI$_{WF}$)	0.4566	0.3347	0.3698	0.3539	0.2000

5.3 Comparison of the Alkire-Foster and Utility Based Index

The enhancements to the AFMDI manifested in the utility based index is best understood in the following comparison of the extent to which the utility based index ($\mathcal{U}$ derived from the regressions), and the various versions of AFMDI cover each other. For this exercise the "utility based" ($\mathcal{U}$) measure considers the wellbeing levels enjoyed by individuals at 0.7, 0.8, 0.9, and 1.0 of the domain medians as metrics for poorness. The deprivation levels of wellbeing (associated with the $\mathcal{U}$ measure), and deprivation rate (associated with the AFMDI) pairings at 70%, 80%, 90%, and 100% of domain medians are respectively (44.2568, 0.0400), (57.8048, 0.0618), (73.1591, 0.1382) and (90.3199, 0.4309).

Tables 7, 8 and 9 report the various degrees to which the AFMDI, and the $\mathcal{U}$ deprivation index proposed here cover the same group of agents using equations (4), (5), and (6). Each column reports the comparison of the two types of measures under the assumption that impoverishment in $K+1$ domains constitute poverty. The 2 dimensional figure 1 in section 3.2 illustrates the issue, there agents in areas C and D are recorded as poor by both indices, agents in areas B are recorded as poor by $\mathcal{U}^*$ but not by AFMDI, and agents in areas A are recorded as poor by AFMDI but not by $\mathcal{U}^*$. These results highlight the fact that there are profound differences in who are identified as impoverished by the $\mathcal{U}$ approach, and the AFMDI approach. Similarly the weighting system in the AFMDI has a profound effect on who is deemed poor. In other words, in the context of figure 1, sets A and B are non-trivial in terms of the sample considered. Ultimately, the choice of

Table 7: Coverage of $\mathcal{U}$ & an Equally Weighted AFMDI

	Minimum Number of Domains ($K+1$) in the AFMDI				
	1	2	3	4	5
Coverage ($\mathcal{U}$ vs. AFMDI) | AFMDI					
$\mathcal{U}$(0.7× Domain Medians)	0.0817	0.1739	0.3055	0.4706	0.6000
$\mathcal{U}$(0.8× Domain Medians)	0.1010	0.2029	0.3333	0.5294	0.6000
$\mathcal{U}$(0.9× Domain Medians)	0.2404	0.4348	0.5833	0.6471	0.6000
$\mathcal{U}$(1.0× Domain Medians)	0.5529	0.7101	0.8056	0.8824	0.8000
Coverage ($\mathcal{U}$ vs. AFMDI) | $\mathcal{U}$					
$\mathcal{U}$(0.7× Domain Medians)	1.0000	0.7059	0.6471	0.4706	0.1765
$\mathcal{U}$(0.8× Domain Medians)	0.7778	0.5185	0.4444	0.3333	0.1111
$\mathcal{U}$(0.9× Domain Medians)	0.7042	0.4225	0.2958	0.1549	0.0423
$\mathcal{U}$(1.0× Domain Medians)	0.3734	0.1591	0.0942	0.0487	0.0130
Joint Coverage: Utility Based vs. Equal Weighting †					
$\mathcal{U}$(0.7× Domain Medians)	0.5409	0.4399	0.4763	0.4706	0.3882
$\mathcal{U}$(0.8× Domain Medians)	0.4394	0.3607	0.3889	0.4314	0.3556
$\mathcal{U}$(0.9× Domain Medians)	0.4723	0.4287	0.4396	0.4010	0.3211
$\mathcal{U}$(1.0× Domain Medians)	0.4631	0.4346	0.4499	0.4655	0.4065

†An index of the commonality of the groups being covered, where proximity to 1 implies identical groups are covered, while proximity to 0 implies groups are completely segmented.

measure between the AFMDI or the utility based $\mathcal{U}$ is dependent on the research question in hand, and we make no prescription for one method over another. Tables 7, 8 and 9 do highlight that implications from the AFMDI do not necessarily carry to a more general approach.

The main contribution here is that the utility based weighting of the AFMDI that incorporate notions of complementarity and substitutability of wellbeing dimensions is significantly more robust and consistent in identifying poverty groups, and generally the full utility weighted Alkire Foster index (AFMDI$_{WF}$) and the Utility model ($\mathcal{U}$) have the greatest compatibility. Indeed if the Utility model were the true model, the AFMDI$_{WF}$

Table 8: Coverage of $\mathcal{U}$ & a Needs Independent Weighted Alkire-Foster Index (AFMDI_W)

	Minimum Number of Domains ($K+1$) in the AFMDI_W				
	1	2	3	4	5
Coverage ($\mathcal{U}$ vs. AFMDI_W) | AFMDI_W					
$\mathcal{U}$(0.7× Domain Medians)	0.1354	0.2075	0.2903	0.6364	0.6000
$\mathcal{U}$(0.8× Domain Medians)	0.1667	0.2453	0.3226	0.6364	0.6000
$\mathcal{U}$(0.9× Domain Medians)	0.3750	0.4906	0.5484	0.8182	0.6000
$\mathcal{U}$(1.0× Domain Medians)	0.7083	0.7358	0.8065	0.9091	0.8000
Coverage ($\mathcal{U}$ vs. AFMDI_W) | $\mathcal{U}$					
$\mathcal{U}$(0.7× Domain Medians)	0.7647	0.6471	0.5294	0.4118	0.1765
$\mathcal{U}$(0.8× Domain Medians)	0.5926	0.4815	0.3704	0.2593	0.1111
$\mathcal{U}$(0.9× Domain Medians)	0.5070	0.3662	0.2394	0.1268	0.0423
$\mathcal{U}$(1.0× Domain Medians)	0.2208	0.1266	0.0812	0.0325	0.0130
Joint Coverage: Utility Based vs. Needs Independent Weighting †					
$\mathcal{U}$(0.7× Domain Medians)	0.4501	0.4273	0.4099	0.5241	0.3882
$\mathcal{U}$(0.8× Domain Medians)	0.3796	0.3634	0.3465	0.4478	0.3556
$\mathcal{U}$(0.9× Domain Medians)	0.4410	0.4284	0.3939	0.4725	0.3211
$\mathcal{U}$(1.0× Domain Medians)	0.4646	0.4312	0.4438	0.4708	0.4065

†An index of the commonality of the groups being covered, where proximity to 1 implies identical groups are covered, while proximity to 0 implies groups are completely segmented.

provides the best approximation to it. Robustness as far as AFMDI indices are concerned is best examined by comparing how well different versions of the indices cover each other. Table 10 reports the results of these comparisons. The Full Utility Weights index (which includes the substitutability components) fully covers both the more restrictive equally weighted, and *needs independent* utility weighted versions of the index, whereas the reverse is not generally true ("needs independent" and equally weighted versions do a reasonable job of covering each other). To put it another way, with the full utility weighting augmented AFMDI, regardless of the number of impoverishment dimensions that constitute poverty are considered, the utility weighted AFMDI is very robust in its ability to cover

Table 9: Coverage of $\mathcal{U}$ & a Full Utility Weighted Alkire-Foster Index (AFMDI_{WF})

	Minimum Number of Domains ($K+1$) in the AFMDI_{WF}				
	1	2	3	4	5
Coverage ($\mathcal{U}$ vs. AFMDI_{WF}) | AFMDI_{WF}					
$\mathcal{U}$(0.7× Domain Medians)	0.0817	0.0817	0.0817	0.0914	0.0914
$\mathcal{U}$(0.8× Domain Medians)	0.1010	0.1010	0.1010	0.1143	0.1143
$\mathcal{U}$(0.9× Domain Medians)	0.2404	0.2404	0.2404	0.2800	0.2800
$\mathcal{U}$(1.0× Domain Medians)	0.5529	0.5529	0.5529	0.5886	0.5886
Coverage ($\mathcal{U}$ vs. AFMDI_{WF}) | $\mathcal{U}$					
$\mathcal{U}$(0.7× Domain Medians)	1.0000	1.0000	1.0000	0.9412	0.9412
$\mathcal{U}$(0.8× Domain Medians)	0.7778	0.7778	0.7778	0.7407	0.7407
$\mathcal{U}$(0.9× Domain Medians)	0.7042	0.7042	0.7042	0.6901	0.6901
$\mathcal{U}$(1.0× Domain Medians)	0.3734	0.3734	0.3734	0.3344	0.3344
Joint Coverage: Utility Based vs. Full Utility Weighting †					
$\mathcal{U}$(0.7× Domain Medians)	0.5409	0.5409	0.5409	0.5163	0.5163
$\mathcal{U}$(0.8× Domain Medians)	0.4394	0.4394	0.4394	0.4275	0.4275
$\mathcal{U}$(0.9× Domain Medians)	0.4723	0.4723	0.4723	0.4851	0.4851
$\mathcal{U}$(1.0× Domain Medians)	0.4631	0.4631	0.4631	0.4615	0.4615

†An index of the commonality of the groups being covered, where proximity to 1 implies identical groups are covered, while proximity to 0 implies groups are completely segmented.

the other versions of AFMDI, or other definitions of poverty, which combined with the fact that the utility based weights include notions of substitutability and complementarity between the components of wellbeing, makes the augmented AFMDI very versatile, and therefore a desirable generalisation.

Table 10: Pairwise Coverage Between Different Versions of Alkire-Foster

Pairwise Comparisons	Minimum Number of Domains $(K+1)$ in AFMDI				
	1	2	3	4	5
Coverage (AFMDI vs. AFMDI$_W$) | AFMDI$_W$	1.0000	1.0000	0.8065	0.9091	1.0000
Coverage (AFMDI vs. AFMDI$_W$) | AFMDI	0.4615	0.7681	0.6944	0.5882	1.0000
Joint Coverage: Equal vs. Needs Independent Weighting	0.7308	0.8841	0.7505	0.7487	1.0000
Coverage (AFMDI$_W$ vs. AFMDI$_{WF}$) | AFMDI$_{WF}$	0.4615	0.2548	0.1490	0.0629	0.0286
Coverage (AFMDI$_W$ vs. AFMDI$_{WF}$) | AFMDI$_W$	1.0000	1.0000	1.0000	1.0000	1.0000
Joint Coverage: Needs Independent vs. Full Weighting	0.7308	0.6274	0.5745	0.5314	0.5143
Coverage (AFMDI vs. AFMDI$_{WF}$) | AFMDI$_{WF}$	1.0000	0.3317	0.1731	0.0971	0.0286
Coverage (AFMDI vs. AFMDI$_{WF}$) | AFMDI	1.0000	1.0000	1.0000	1.0000	1.0000
Joint Coverage: Equal vs. Full Weighting	1.0000	0.6659	0.5865	0.5486	0.5143

Note: AFMDI $\sim$ equally weighted AFMDI of equation (1), AFMDI$_W$ $\sim$ needs independent weighted AFMDI of equation (1), and AFMDI$_{WF}$ $\sim$ Full utility weighted AFMDI from equation (3)

6 Conclusions

The move toward understanding wellbeing in terms of functionings, capabilities, and happiness has fostered considerable interest in multi-dimensional wellbeing measurement, with increasing dimensionality being high on the agenda. This, in turn, has led to proposals for measures of deprivation in the context of the functionings and capabilities approach which "count" the numbers of dimensions or domains in which agents are deprived, and considering only those agents who reach at least a prespecified number of domain deprivations as being poor. These socalled "Mashup" indices have not been without their critics whose concern has largely been with the robustness of these indices to vagaries in the choice of parameters, which are largely at the behest of investigators. There are also technical concerns with increasing dimensionality which have hitherto been overlooked both in terms of the informational requirements placed upon data, and the over-simplistic view of the inter-relationships between the domains of satisfaction inherent in "Mashup" type indices.

Here a new approach is proposed which addresses some of these issues. The attraction of the approach is that it affords a significant level of flexibility and generality in the representation of sub-component deprivations, whilst admitting the possibility of reflecting domain substitutability/complementarity in the overall index, and retaining the ability to measure the impact of improvements/worsenings of domain sub-components on measures of overall deprivation. The demonstration of the approach is facilitated by a unique data set which records overall satisfaction on the part of agents, as well as their satisfaction with sub-domains of functioning and capability, which permits estimation of the inter-relationship between the sub-domains and overall satisfaction. In this work, the U.S. sample of respondents, drawn equally from four regions of the U.S., and are representative of working age adults in terms of age, gender and social class in 2011, were used. Exploration of this dataset revealed significant inter-relationships between domain satisfactions, and considerable differences between the groups of individuals captured by the various measures as deprived or impoverished. The proposed generalized version of the Alkire-Foster Multi-Dimensional Deprivation Index, which admits notions of substitutability and complementarity between the various aspects of functioning and capability impoverishment in the index, was found to have superior robustness properties to other more restrictive versions of the index.

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A Appendix

A.1 Wellbeing Variables Used in this Study

1. Please could you say how you would rate each of the following aspects of your life? Please rate on a scale of 0 to 10, where 0 indicates the lowest rating you can give and 10 the highest.

   (a) Overall, how satisfied are you with your life nowadays?

   (b) Overall, how happy did you feel yesterday?

   (c) Overall, how anxious did you feel yesterday?

   (d) Overall, to what extent do you feel that the things you do in your life are worthwhile?

2. Please indicate how satisfied, or dissatisfied you are with the following on a scale of 0 to 10, where 0 indicates you are very dissatisfied and 10 that you are very satisfied.

   (a) Friendships

   (b) Relationships with colleagues at work

   (c) The neighborhood in which you live

3. Here are some questions about the opportunities and constraints that you face. For each of the following statements, please indicate how much you agree, or disagree on a scale of 0 to 10, where 0 indicates you strongly disagree and 10 that you strongly agree.

   (a) I am able to share domestic tasks within the household fairly

   (b) I am able to socialise with others in the family as I would wish

   (c) I am able to make ends meet

   (d) I am able to achieve a good work-life balance

   (e) I am able to find a home suitable for my needs

   (f) I am able to enjoy the kinds of personal relationships that I want

   (g) I have good opportunities to feel valued and loved

4. Here are some questions about the opportunities and constraints that you face. For each of the following statements, please indicate how much you agree, or disagree on a scale of 0 to 10, where 0 indicates you strongly disagree and 10 that you strongly agree.

   (a) I am able to find work when I need to

(b) I am able to use my talents and skills at work

(c) I am able to work under a good manager at the moment

(d) I am always treated as an equal (and not discriminated against) by people at work

(e) I have good opportunities for promotion or recognition at work

(f) I have good opportunities to socialize at work

5. For each of the following statements, please indicate how much you agree, or disagree on a scale of 0 to 10, where 0 indicates you strongly disagree and 10 that you strongly agree.

   (a) I have good opportunities to take part in local social events

   (b) I am treated by people where I live as an equal (and not discriminated against)

   (c) I am able to practice my religious beliefs (including atheism/agnosticism)

   (d) I am able to express my political views when I wish

6. For each of the following statements, please indicate how much you agree, or disagree on a scale of 0 to 10, where 0 indicates you strongly disagree and 10 that you strongly agree.

   (a) I am able to walk in my local neighborhood safely at night

   (b) I am able visit parks or countryside whenever I want

   (c) I am able to work in an environment that has little pollution from cars or other sources

   (d) I am able to keep a pet or animals at home with ease if I so wish

   (e) I am able to get to places I need to without difficulty

7. Moving on to think about access to services. Again please indicate how much you agree, or disagree on a scale of 0 to 10, where 0 indicates you strongly disagree and 10 that you strongly agree.

   (a) Make use of banking and personal finance services

   (b) Get my garbage cleared away

   (c) Get trades people or the landlord to help fix problems in the house

   (d) Be treated by a doctor or nurse

   (e) Get help from the police

   (f) Get help from a solicitor

   (g) Get to a range of shops