University of Toronto
Department of Economics

Working Paper 523

# Policy Distortions and Aggregate Productivity with Endogenous Establishment-Level Productivity

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November 13, 2014

# Policy Distortions and Aggregate Productivity with Endogenous Establishment-Level Productivity*

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November 2014

Abstract

The large differences in income per capita across countries are mostly accounted for by differences in total factor productivity (TFP). What explains these differences in TFP across countries? Evidence suggests that the (mis)allocation of factors of production across heterogenous production units is an important factor. We study factor misallocation in a model with an endogenously determined distribution of establishment-level productivity. In this framework, policy distortions not only misallocate resources across a given set of productive units, but they also worsen the distribution of establishment-level productivity. We show that in our model, compared to the model with an exogenous distribution, the quantitative effect of policy distortions is substantially amplified. Whereas empirically-plausible policy distortions in our model generate TFP that is 14 percent that of a benchmark economy with no distortions, with an exogenous distribution the same policy distortions generate TFP that is 86 percent of the benchmark, a 6-fold amplification factor.

Keywords: distortions, misallocation, investment, endogenous productivity, establishments.
JEL codes: O1, O4.

*For helpful comments we thank Lukasz Drozd, Hugo Hopenhayn, Alex Monge-Naranjo, B. Ravikumar, Loris Rubini, and several participants at 2013 ITAM/INEGI Workshop on Productivity, ITAM, Federal Reserve Bank of St. Louis, SED Meetings in Toronto, North American Summer Meeting of the Econometric Society in Minneapolis, National Bank of Poland. All remaining errors are our own. Restuccia gratefully acknowledges the financial support from the Social Sciences and Humanities Research Council of Canada. Mendes Tavares gratefully acknowledges the financial support of DIFD.

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# 1 Introduction

A crucial question in economic growth and development is why some countries are rich and others poor. A consensus has emerged in the literature whereby the large differences in income per capita across countries are mostly accounted for by differences in labor productivity and in particular total factor productivity (TFP).[1] Hence, a key question is what explains differences in TFP across countries. A recent literature has emphasized the (mis)allocation of factors across heterogenous production units as an important factor.[2] We study factor misallocation in a model with an endogenously determined distribution of establishment-level productivity. In this framework, policy distortions not only misallocate resources across a given set of productive units, but they also worsen the distribution of productivity. This is relevant as the empirical evidence in poor countries indicates not only factor misallocation, but also substantial differences in the distribution of establishment productivity which account for a large portion of differences in aggregate TFP.[3] The importance of the productivity distribution is recognized in a recent literature that endogenizes investment in productivity by establishments.[4] Building on this literature, the key innovation in our paper is to endogenize the distribution of productivity for entering establishments as a function of the economic environment including policy distortions. In our model, compared to the model with an exogenously specified distribution of productivity, the quantitative effect of empirically-plausible policy distortions is substantially amplified, by a factor of 6-fold.

We develop a framework with heterogeneous production units that builds on Hopenhayn (1992) and Restuccia and Rogerson (2008). The framework is extended to allow for an en-

[1]See for instance Klenow and Rodriguez-Clare (1997), Prescott (1998), and Hall and Jones (1999).

[2]See Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Guner et al. (2008), and Hsieh and Klenow (2009). See also surveys of the literature in Hopenhayn (2011), Restuccia and Rogerson (2013), and Restuccia (2013a).

[3]See the empirical evidence in Hsieh and Klenow (2009) for China and India compared to the United States and the expanding literature that has followed using micro data for other countries.

[4]See for instance Restuccia (2013b), Bello et al. (2011), Ranasinghe (2014), Bhattacharya et al. (2013), Gabler and Poschke (2013), and Hsieh and Klenow (2014).

dogenous determination of the distribution of establishment-level productivity. We use this framework to study the impact of policy distortions on misallocation and aggregate measured productivity. There is a large number of homogeneous households with standard preferences over consumption goods. Households accumulate physical capital and supply inelastically their endowment of one unit of time. The key elements of the model are on the production side. A single good is produced in each period. The production unit is the establishment. An establishment has access to a decreasing returns to scale production function with capital and labor as inputs. Establishments are heterogenous with respect to total factor productivity, but differently from the standard framework, the distribution of establishment-level productivity is not exogenous, rather it is determined by endogenous investments by the establishment and the properties of the economic environment such as policy distortions. In other words, entering establishments draw their initial level of productivity from an invariant distribution which is determined endogenously in the model. Establishments are subject to an exogenous exit rate. Following the literature, the economy faces policy distortions which, for simplicity, take the form of output taxes on individual producers. That is, each producer faces an idiosyncratic tax and it is the properties of policy distortions that generate misallocation in the model. Revenues collected from these taxes are rebated back to the households as a lump-sum transfer.

We characterize the closed-form solution of this model in continuous time. In particular, we solve in closed form for the stationary distribution of establishments which is an endogenous object that varies across economies and we show is a Pareto distribution with tail index $\xi$ that depends on policy distortions and the investment response of incumbent establishments to distortions. This allows us to characterize the behavior of aggregate output and TFP across distortionary policy configurations as well as the size and productivity growth rate of establishments, the size distribution of establishments, among other statistics of interest.

To assess the quantitative properties of the model relative to the existing literature, we calibrate the model and provide a set of relevant quantitative experiments. We consider a benchmark economy that faces no distortions, i.e., an economy whose policy distortion process implies no taxes on individual producers, and calibrate the parameters of this economy to data for the United States. Most parameters share the usual targets in the literature, see for instance Restuccia and Rogerson (2008). The parameters that require some discussion are the investment cost in establishment-level productivity, the variance in the distribution of productivity, and the size growth rate of establishments. These parameters are targeted to data on the aggregate growth rate of TFP, the average employment growth of establishments, and the right tail index of the share of employment distribution in the U.S. data. We then perform quantitative analysis by exploring the implications of our model with policy distortions. Our main result is that the quantitative effect of policy distortions on aggregate TFP is substantially larger than that of the model with an exogenous distribution of productivity. That is, the endogenous distribution of establishment productivity amplifies the quantitative effect of distortions on aggregate TFP. This amplification effect is more than 6-fold. In particular, configurations of policy distortions that are reasonable compared to empirical measures of misallocation such as those estimated for China and India in Hsieh and Klenow (2009) generate an aggregate TFP that is 14 percent of that of a benchmark economy without distortions, whereas the same policy distortions imply an 86 percent aggregate TFP in the model with an exogenous distribution of productivity. With ongoing entry and exit of establishments, making the distribution of productivity for entering establishments consistent with the economic environment that changes across countries (as opposed to a constant distribution across countries) is critical in understanding the amplification effect of policy distortions in our framework. We show both analytically and quantitatively that the model generates implications for the distribution of establishments productivity, the employment growth of establishments over time, and the number and size of establishments in the economy. Some of these implications contrast with the standard framework that produces no effects on these objects.

Our paper is related to a large and growing literature on misallocation and productivity. By studying the aggregate impact of policy distortions across countries our paper is closely linked to that of Restuccia and Rogerson (2008) and the related literature. By considering an endogenous distribution of establishment productivity our paper is closely related to the analysis in Restuccia (2013b), Bello et al. (2011), Ranasinghe (2014), Bhattacharya et al. (2013), Gabler and Poschke (2013), and Hsieh and Klenow (2014).[5] The key difference between all these papers and ours is that the distribution of productivity for entering establishments is exogenous in the previous literature whereas it is endogenous in our paper. To perform our analysis and characterize the model in closed form, we use the tools of continuous time and Brownian motion processes. These tools are increasingly popular in the growth literature, for instance Lucas and Moll (2014), Benhabib et al. (2014), Buera and Oberfield (2014), among many others. More closely linked, these tools were prominently used by the seminal work of Luttmer (2007) to study the size distribution of firms in the United States, by Da-Rocha and Pujolas (2011) and Fattal (2014) to study the effect of policy distortions with stochastic productivity and entry/exit decisions, by Gourio and Roys (2014) to study the productivity effects of size-dependent regulations, among others. A key distinction of our work with this existing literature is the emphasis on the amplification effect of policies. This emphasis is quite relevant as Hopenhayn (2013) has pointed out the limited quantitative role of direct specific policies in the standard model in explaining TFP differences across countries. A version of our framework may be better suited to assess the quantitative impact of specific policies on aggregate productivity.

The paper proceeds as follows. In the next section and Section 3, we describe the model and characterize the equilibrium solution. Section 4 calibrates a benchmark economy with no distortions to data for the United States. In section 5 we perform a series of quantitative experiments to assess the impact of policy distortions on aggregate TFP and other relevant

[5] See Rubini (2014) for an analysis of changes in tariffs in the context of a model with endogenous establishment-level productivity. See also Atkeson and Burstein (2010).

statistics. We conclude in section 6.

## 2 Economic Environment

We consider a standard version of the neoclassical growth model as in Restuccia and Rogerson (2008), extended to allow establishments to invest in their own productivity as in Restuccia (2013b).[6] Time is continuous and the horizon is infinite. Establishments have access to a decreasing return to scale technology, pay a one-time fixed cost of entry, and die at an exogenous rate. Establishments hire labor and rent capital services in competitive markets. New entrants draw their productivity from an endogenous distribution. We study a stationary equilibrium in which the economy grows at an exogenous rate. We then analyze policy distortions that affect the allocation of factors across establishments, the investment on productivity of establishments, and therefore, aggregate measured TFP and output. We also compare the effects of the same policy distortions in the environment where productivity investment and the initial distribution of productivity is exogenous. In what follows we describe the environment in more detail.

### 2.1 Baseline Model

There is an infinity-lived representative household with preferences over consumption goods described by the utility function,

$$\max \int_0^\infty e^{-\rho t} u(c) dt,$$

where $c$ is consumption and $\rho$ is the discount rate. The household is endowed with one unit of productive time at each instant and $k_0 > 0$ units of the capital stock at date 0.

[6]See also Bello et al. (2011), Ranasinghe (2014), Bhattacharya et al. (2013), among others.

The unit of production in the economy is the establishment. Each establishment is described by a production function $f(z, k, n)$ that combines capital services $k$ and labor services $n$ to produce output. The function $f$ is assumed to exhibit decreasing returns to scale in capital and labor jointly and to satisfy the usual Inada conditions. The production function is given by:

$$y = z^{2(1-\alpha-\gamma)} k^{\alpha} n^{\gamma}, \quad \alpha, \gamma \in (0, 1), \quad 0 < \gamma + \alpha < 1. \tag{1}$$

Establishment's productivity $z$ follows a Brownian motion but establishments can invest in upgrading their productivity by choosing the drift of the Brownian motion $x_z$. Hence, the establishment productivity $z$ evolves according to:

$$dz = x_z dt + \sigma_z z dw_z,$$

where $x_z$ is the endogenous drift, $\sigma_z$ is the standard deviation, and $dz = \sqrt{dt}$ is the standard Wiener process of the Brownian motion.[7] We assume that investing in upgrading productivity is costly to establishments and that the cost is in units of output, described by a cost function $q(\cdot)$ that is increasing and convex in productivity upgrading $x_z$. Establishments also face an exogenous probability of death $\lambda$.

New establishments can also be created. Entrants must pay an entry cost $c_e$ measured in units of labor. After paying this cost a realization of the initial establishment productivity $z$ is drawn from a distribution, where the pdf is represented by $g(\cdot)$. We assume that there exist an unlimited mass of potential entrants. Feasibility in the model requires:

$$C + I + Q = Y,$$

[7] The drift of the Brownian motion $x_z$ is the policy function and we show later that establishment productivity $z$ follows a Geometric Brownian Motion.

where $C$ is aggregate consumption, $I$ is aggregate investment in physical capital, $Q$ is aggregate cost of investing in establishment productivity, and $Y$ is aggregate output.

## 2.2 Policy Distortions

We next introduce policies that create idiosyncratic distortions to establishment-level decisions as in Restuccia and Rogerson (2008). We focus on output taxes. While the policies we consider are hypothetical, there is a large empirical literature documenting the potential sources of idiosyncratic distortions.[8] In our framework, these policies distort not only the allocation of resources across existing productive units, but also the investment decision in productivity thereby affecting the distribution of productive units in the economy. Specifically, we assume that each establishment faces its own policy distortion. In what follows, we simply refer to this distortion as output tax and use $\tau$ to refer to the establishment-level output tax rate.

We assume that output taxes $\tau$ follow a Geometric Brownian motion with no drift,

$$d\tau = \sigma_\tau \tau dw_\tau,$$

where $\sigma_\tau$ is the standard deviation and $d\tau = \sqrt{dt}$ is the standard Wiener process of the Brownian motion. For tractability, we assume that the output tax and productivity processes are uncorrelated, that is $E(dw_\tau, dw_z) = 0$. We note however that much of the quantitative literature has focused on what Restuccia and Rogerson (2008) call correlated distortions, distortions that apply more heavily on more productive establishments, that have the potential to generate much larger negative aggregate productivity effects. As a result, in our quantitative assessment in Section 5, we also relate our results with other results in the literature arising from uncorrelated distortions.

[8] See for instance Hsieh and Klenow (2009), Bartelsman et al. (2013) and the survey in Restuccia and Rogerson (2013).

The output tax affects the establishment decision of investing in productivity by changing the drift of the Brownian motion that governs the productivity process. In the presence of output taxes, establishment productivity follows the resulting Brownian motion:

$$dz = \frac{x_z}{\tau} z dt + \sigma_z z dw_z.$$

At the time of entry, the establishment-level output tax $\tau$ is not known, its value is revealed after the establishment draws its productivity $z$. From the entering establishment perspective, the relevant information is the joint distribution over these pairs, that is, over the output-tax and productivity. We denote this joint distribution by $g(z, \tau)$ .

A given distribution of establishment-level output tax and productivity may not lead to a balanced budget for the government. As a result, we assume that budget balance is achieved by either lump-sum taxation or redistribution to the representative household. We denote the lump-sum tax by $T$.

# 3 Equilibrium

We focus on a stationary equilibrium where the number of establishments grows at an exogenous rate $\eta$. The stationary equilibrium is characterized by an invariant distribution of establishments $g(z, \tau)$ over productivity $z$ and output tax $\tau$ and a mass of establishments $M$. In equilibrium, the mass of establishments grows over time, however, the rank of the establishments' size distribution is constant. In the stationary equilibrium, the rental prices for labor and capital services are constant and we denote them by $w$ and $r$. Before defining the stationary equilibrium formally, it is useful to consider the decision problems faced by incumbents, entrants, and consumers. We describe these problems in turn.

## 3.1 Incumbent establishments

Incumbent establishments maximize the present value of profits by making static and dynamic decisions. The static problem is to choose the amount of capital and labor services, whereas the dynamic problem involves solving for the amount of investment in establishment productivity. In what follows next, we describe these problems in detail.

**Static problem** At any instant of time an establishment chooses how much capital to rent $k$ and how much labor to hire $n$. These decisions are static and depend on the establishment's productivity $z$, the establishment's output tax rate $\tau_y$, the rental rate of capital $r$, and the wage rate $w$. Formally, the instant profit function $\pi(z, \tau_y)$ is defined by:

$$\pi(z, \tau_y) = \max_{k,n}(1 - \tau_y)y - wn - rk,$$

from which we obtain the optimal demands for capital and labor:

$$n(z, \tau) = \left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha-\gamma}} \tau^2 z^2, \tag{2}$$

$$k(z, \tau) = \left[\left(\frac{\alpha}{r}\right)^{1-\gamma}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}} \tau^2 z^2, \tag{3}$$

where the output tax $\tau$ is a function of the output tax rate $\tau_y$ defined as $\tau = (1 - \tau_y)^{\frac{1}{2(1-\alpha-\gamma)}}$. For future reference, we redefine instant profits as a function of the optimal demand for factors:

$$\pi(z, \tau) = m(w, r)\tau^2 z^2, \tag{4}$$

where $m(w, r) = (1 - \alpha - \gamma)\left[\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right]^{\frac{1}{1-\alpha-\gamma}}$ is a constant across establishments that depends on equilibrium prices.

**Dynamic problem** Incumbent establishments choose how much to invest in upgrading their productivity $x_z$. The cost of investing in upgrading productivity is in units of output, described by a cost function $q(\cdot)$ that is increasing and convex in the productivity parameter $x_z$, specifically we assume $q(x_z) = c_\mu \frac{x_z^2}{2}$. The optimal decision of upgrading productivity is characterized by maximizing the present value of profits subject to the Brownian motion governing the evolution of productivity and the Brownian motion governing the evolution of output taxes. Formally, incumbent establishments solve the following dynamic problem:

$$W(z,\tau) = \max_{x_z} \left\{ m(w,r)z^2\tau^2 - q(x_z) + \frac{1}{1+\lambda+R} E_{z,\tau} W(z+dz, \tau+d\tau) \right\},$$

$$\begin{aligned} s.t. \quad dz &= \frac{x_z}{\tau}dt + \sigma_z z dw_z, \\ d\tau &= \sigma_\tau \tau dw_{\tau,} \end{aligned}$$

where $\lambda$ is the exogenous probability of death and $R$ is the stationary equilibrium interest rate which is equal to the rental rate of capital $r$ minus the depreciation rate $\delta_k$. Next, we define the Hamilton-Jacobi-Bellman of the stationary solution,

$$(\lambda + R)W(z,\tau) = \max_{x_z} \left\{ m(w,r)\tau^2 z^2 - \frac{c_\mu}{2}x_z^2 + \frac{x_z}{\tau}W_z' + \frac{\sigma_z^2}{2}z^2 W_{zz}'' + \frac{\sigma_\tau^2}{2}\tau^2 W_{\tau\tau}'' \right\}.$$

From the first order conditions, we find that the optimal investment rate $x_z$ is a function of the output tax $\tau$, the investment cost $c_\mu$, and the marginal present value profits $W_z'$,

$$x_z = \frac{W_z'}{c_\mu \tau}.$$

By guessing and verifying, we find that the optimal Hamilton-Jacobi-Bellman equation is given by $W(z,\tau) = A(w,r)z^2\tau^2$, where the constant $A(w,r)$ is the solution of the polynomial:

$$\frac{A(w,r)^2}{c_\mu} - \left[ \frac{\lambda+R}{2} - \frac{\sigma_z^2}{2} - \frac{\sigma_\tau^2}{2} \right] A(w,r) + \frac{m(w,r)}{2} = 0. \tag{5}$$

There are two possible solutions of this polynomial and below we restrict the solution to the negative root. This root has the desired property that an increase in profits $m(w, r)$ increases the present value of an operating establishment. In the following Lemma 1 we characterize formally the value function of operating incumbents.

**Lemma 1.** *Given an output tax $\tau$, a productivity level $z$, and operating profits $m(w, r)$, the value function that solves the establishment dynamic problem is given by $W(z, \tau) = A(w, r)\tau^2 z^2$, where the constant $A(w, r)$ is characterized by the following expression:*

$$A(w, r) = \left(\Theta - \sqrt{\Theta^2 - \frac{2m(w, r)}{c_\mu}}\right) \frac{c_\mu}{2},$$

*and*

$$\Theta = \frac{\lambda + R}{2} - \frac{\sigma_z^2}{2} - \frac{\sigma_\tau^2}{2}.$$

*Then expected growth rate of establishment's productivity $z$ follows Gibrath's law:*

$$\frac{dz}{z} = \frac{2A(w, r)}{c_\mu} dt + \sigma_z dw.$$

The proof of Lemma 1 is straightforward from equation (5). From Lemma 1, we find that the growth rate of productivity does not depend on the intrinsic characteristics of individual establishments, that is, it does not depend on the establishment productivity $z$ or the output tax $\tau$. As a consequence Gilbrath's law holds and productivity growth does not depend on the establishment size, which is supported by empirical evidence.[9] Now that we have fully characterized the incumbent establishment's problem, we can discuss the entrant's establishment problem.

[9] For more discussion, see Luttmer (2010).

### 3.2 Entering establishments

Potential entering establishments face an entry cost $c_e$ in units of labor and make their entry decision knowing that they face a distribution over potential draws for the pairs $(z, \tau)$ of productivity and taxes. Therefore, the expected value of an entering establishment is equal to

$$W_e = \int_{z \times \tau} W(z,\tau) g(z,\tau) d(z,\tau) - wc_e.$$

In an equilibrium with entry, $W_e$ must be equal to zero (free-entry condition), otherwise additional establishments would enter.

### 3.3 Stationary distribution of establishments

Given the optimal decisions of incumbents and entering establishments, we are now ready to characterize the stationary distribution $g(z, \tau)$ over productivity $z$ and output tax $\tau$. The first step to characterize this distribution is to rewrite the Brownian motions of productivity $z$ and output tax $\tau$ as a function of $s$, where $s = \tau z$. The resulting $s$ Brownian motion is given by:

$$\frac{ds}{s} = \mu dt + \sigma dw_s, \tag{6}$$

where the drift $\mu$ is equal to the drift of the productivity Brownian motion $\mu_z$. It is important to remember that the drift of the productivity Brownian motion $\mu_z$ is an endogenous object and is given by the solution of the incumbent establishment's dynamic problem, $\mu_z = \frac{2A(w,r)}{c_\mu}$. The variance $\sigma^2$ of the $s$ Brownian motion is the sum of the variance of the output tax Brownian motion $\sigma_\tau^2$ and the variance of the productivity Brownian motion $\sigma_z^2$.

In order to characterize the stationary distribution over $s$, it is useful to rewrite the model in logarithms. Let $x$ denote the logarithm of $s$, that is $x = \log(s)$. Now we can rewrite the

Geometric Brownian motion in equation (6) as a Brownian motion in the logarithm of $s$,

$$dx = \hat{\mu} dt + \hat{\sigma} dw_x,$$

where $\hat{\mu} = \mu_z - \frac{1}{2}(\sigma_z^2 + \sigma_\tau^2)$, and $\hat{\sigma}^2 = \sigma_z^2 + \sigma_\tau^2$. Let $F(x, t)$ be the measure of producing establishments at time $t$ with characteristics $x$. $F(x, t)$ satisfies the Kolmogorov forward equation (also known as the Fokker-Planck equation), associated with the Brownian process given by:

$$\frac{\partial F(x,t)}{\partial t} = -\hat{\mu}\frac{\partial F(x,t)}{\partial x} + \frac{\hat{\sigma}^2}{2}\frac{\partial^2 F(x,t)}{\partial x^2} - \lambda F(x,t). \tag{7}$$

The Kolmogorov forward equation describes the evolution over time of the distribution of operating establishments, taking into account the exogenous probability of death $\lambda$. Let $M(t)$ denote the mass of establishments operating at time $t$, that is, $M(t)$ is the integral of all establishment operating at time $t$,

$$M(t) = \int_0^\infty F(x,t)dx.$$

$M(t)$ describes the evolution of the mass of operating establishments over time. Next, we rewrite the measure of operating establishment with characteristic $x$ at time $t$, as the product of the mass of establishment operating at time $t$, $M(t)$, and the probability density of establishments with characteristic $x$ at time time $t$, $f(x, t)$, that is

$$F(x,t) = M(t)f(x,t), \tag{8}$$

where the probability density satisfies the usual property that $\int_0^\infty f(x,t)dx = 1$.

We are interested in finding the stationary distribution, that is the probability density function that is independent of time, letting $f_\infty(x)$ denote such stationary distribution and noting that on the stationary equilibrium $M(t) = e^{\eta t}M$ we obtain:

$$F(x,t) = e^{\eta t} M f_\infty(x). \tag{9}$$

The measure of establishments with characteristic $x$ at time $t$ is a linear function of the mass of establishments $M$ and the stationary distribution $f_\infty(x)$. As a result, on the stationary equilibrium, the mass of establishments grows, however, the rank is fixed. We can now characterize the stationary distribution $g(s)$.

**Lemma 2.** *Given wages $w$ and the rental rate of capital $r$, the stationary distribution $g(s)$ is a Pareto distribution:*

$$g(s) = (\xi - 1)s^{-\xi},$$

*the tail index of the Pareto distribution is given by*

$$\xi = -\left(\frac{\mu}{\sigma^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{4}\right)^2 + \frac{2(\eta+\lambda)}{\sigma^2}},$$

*and the net entry rate is given by* $\varepsilon = \eta + \lambda - \left(\mu - \frac{1}{2}\sigma^2 + \frac{3\sigma^2}{2}\xi\right)\xi.$

We leave the poof of Lemma 2 to the appendix. However, we emphasize that the characterization of the stationary distribution is one of the main results of paper. This distribution implicitly depends on policy distortions and on the incumbent's response to this policy through the drift $\mu$. In addition, new entrants draw their productivity from this distribution.

Next, we characterize other aggregate variables and their dynamics on the stationary equilibrium. Aggregate output is given by the mass of establishments multiplied by the integral over the stationary distribution of output. In order to find the stationary distribution of output, we apply the same methodology as in Lemma 2.[10] The stationary distribution of

[10]Substituting the demand for labor in equation (2) and the demand for capital in equation (3) onto

output is also a Pareto distribution and the expression is given by:

$$g_Y(s) = (\xi_Y - 1)s^{-\xi_Y},$$

and the tail index $\xi_Y$ is given by,

$$\xi_Y = -\left(\frac{\mu_Y}{2\sigma_Y^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu_Y}{2\sigma_Y^2} - \frac{1}{4}\right)^2 + \frac{(\eta + \lambda)}{2\sigma_Y^2}}.$$

The tail index of the invariant distribution of output has the same functional form of the tail index of the invariant distribution $g(\cdot)$. However, the drift $\mu_Y$ and the standard deviation $\sigma_Y$ are not the same, they also depend on the capital share $\alpha$ and labor share $\gamma$. Solving for aggregate per capita output we find that:

$$Y = M\left(\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\right)\left(\frac{\xi_Y - 1}{\xi_Y - 3}\right).$$

Thus, in the stationary equilibrium per capita output grows at the same rate as the mass of establishments $\eta$.

## 3.4 Household's problem

The stand-in household seeks to maximize lifetime utility subject to the law of motion of wealth given by:

$$(RK + w + T + \Pi - c)\,dt,$$

equation (1) and integrating over all operating establishments, we obtain aggregate per-capita output:

$$Y = M\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}}\int_1^\infty s_Y ds_Y,$$

where $s_Y = z^2\tau^{2(\alpha+\gamma)}$. Then Using Ito lemma's we find that the Brownian motion that governs the process $s_Y$ is given by $\frac{dF}{F} = (\alpha+\gamma)\left[\frac{\mu_z}{\alpha+\gamma} - (1-\alpha-\gamma)\sigma_\tau\right]dt + [(\alpha+\gamma)\sigma_\tau + \sigma_z]\,dw_F$. The drift of this Brownian motion $\mu_Y$ is equal to $\mu_Y = (\alpha+\gamma)\left[\frac{\mu_z}{\alpha+\gamma} - (1-\alpha-\gamma)\sigma_\tau\right]$ and the standard deviation $\sigma_Y$ is equal to $\sigma_Y = (\alpha+\gamma)\sigma_\tau + \sigma_z$.

where $w$ is the wage rate, $R$ is the is the interest rate which in equilibrium is the rental price of capital minus capital depreciation ($R = r - \delta_k$), $T$ is the lump-sum tax levied by the government, $\Pi$ is the total profit from the operations of all establishment, and $c$ is consumption.

We assume that households have log utility, $u(c) = \log(c)$, and we characterize the equilibrium interest rate by solving the household's problem. We define total wealth as:

$$a = K + \frac{w}{R} + \frac{T}{R} + \frac{\Pi}{R},$$

and we rewrite the law of motion of wealth as:

$$da = (Ra - c)dt.$$

The household solves the following Hamilton-Jacobi-Bellman equation:

$$\rho V(a) = \max_c \left\{ \log(c) + [Ra - c] V'(a) \right\}.$$

In Lemma 3 we show that in the stationary equilibrium the interest rate $R$ is equal to the discount rate $\rho$.

**Lemma 3.** *In the stationary equilibrium the interest rate is equal to the discount rate $R = \rho$.*

We assume that households grow at same exogenous rate as establishments $\eta$. This assumption guarantees that wages $w$ are constant in the stationary equilibrium. Since the growth in population is exogenous, we define the stationary equilibrium in per capita terms.

## 3.5 Stationary equilibrium

**Definition** Given incumbents' and households' exogenous growth rate $\eta$ and initial capital stock $k_0$, a stationary equilibrium for this economy is a stationary distribution $g(\cdot)$, a mass of establishments $M$, a net entry rate $\varepsilon$, a value function for incumbents $\{W(\cdot)\}$, a value a function for new entrants $\{W_e(\cdot)\}$, policy functions $\{k(\cdot), n(\cdot), x_z(\cdot), c(\cdot)\}$, prices $\{r, w\}$, and transfer $\{T\}$ such that:

i) Given prices and transfer, the households' policy function $\{c(\cdot)\}$ solves the household dynamic problem.

ii) Given prices, the incumbents' policy functions $\{k(\cdot), n(\cdot)\}$ solve the incumbents' static problem.

iii) The incumbents' policy function $\{x_z(\cdot)\}$ together with the value function $\{W(\cdot)\}$ solve the incumbents' dynamic problem.

iv) The stationary distribution $\{g(\cdot)\}$ and the net entry rate $\varepsilon$ solve the Kolmogorov forward equation.

v) The entering establishments' value function $\{W_e(\cdot)\}$ solves the free-entry condition.

vi) Market Clearing:

   a) Capital: $K = M \int_1^\infty k(s, w, r) g(s) ds$,

   b) Labor: $M \int_1^\infty n(s, w, r) g(s) ds = 1 - M\varepsilon c_e$.

vi) The government budget constraint is satisfied, $T = M \int_1^\infty \tau_y y(s) g(s) ds$.

The stationary equilibrium is a fixed point in measure and it is very simple to compute. From the household's problem, we solve for the stationary interest rate $R$ and hence pin down the rental rate of capital $r$. From the incumbents' static problem, we solve the labor and capital demand as a function of prices $\{r, w\}$ and policies $\{\tau\}$. Given the solution of the static problem, incumbents solve the dynamic problem of investing in productivity. The solution of this problem is a policy function $x_z(\cdot)$ that determines the Geometric Brownian motion for productivity of the entire economy. Given the Geometric Brownian motion for productivity, we solve for the stationary distribution $g(\cdot)$ and the net entry rate $\varepsilon$ that solve the Kolmogorov forward equation. After solving for the stationary distribution $g(\cdot)$, the wages $w$ must solve the free-entry condition, and markets must clear. There are two market clearing conditions: capital and labor. Capital market clearing is straightforward. Labor market clearing guarantees that labor demand is equal to labor supply. Labor demand is the integral over the labor demand of all operating establishments. Labor supply is equal to the measure one of labor supplied inelastically by households minus the aggregate entry cost, which the product of the mass of operating establishment $M$, the net entry rate $\varepsilon$, and the entry cost $c_e$. The mass of operating establishments $M$ guarantees that the labor market clearing condition is satisfied. Last the government budget constraint must be satisfied.

### 3.6 Exogenous TFP distribution

The main difference between our model and previous models in the literature is that in our model new entrants draw their productivity from the stationary distribution, which is endogenous, while in the previous literature new entrants draw their productivity from an exogenous distribution. In order to quantify how much the endogenous distribution contributes to explain differences in TFP, we need to develop a version of the model where the distribution of TFP is exogenous. We follow Restuccia and Rogerson (2008) and Bhattacharya et al. (2013), and set the exogenous distribution as the distribution generated by the model in a benchmark economy without policy distortions, that is when $\sigma_\tau = 0$. Applying Lemma 2 to the case where there is no policy distortions, we obtain the endogenous distribution of the undistorted economy $g(\cdot)$, which is a Pareto distribution with tail index $\xi_z$ given by:

$$\xi_z = -\left(\frac{\mu_z}{\sigma_z^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu_z}{\sigma_z^2} - \frac{1}{4}\right)^2 + \frac{2(\eta+\lambda)}{\sigma_z^2}}. \tag{10}$$

When the distribution of TFP is exogenous and the productivity process is independent of the output tax process, the joint distribution of productivity and output tax is the product of these two distributions. From Lemma 2 we have the stationary distribution of productivity. The stationary distribution of output tax $h(\cdot)$ is characterized by Lemma 4 below.

**Lemma 4.** *Given $\tau \in (0,1]$, the stationary distribution of output tax $h(\tau)$ is a Pareto distribution:*

$$h(\tau) = (\xi_\tau + 1)\tau^{\xi_\tau},$$

*and the tail index $\xi_\tau$ is given by:*

$$\xi_\tau = -\frac{1}{4} + \sqrt{\left(-\frac{1}{4}\right)^2 + \frac{(\eta+\lambda)}{2\sigma_\tau^2}}.$$

After we characterize both stationary distributions, we can characterize the joint distribution of productivity $z$ and output tax $\tau$, $v(\cdot)$.

**Lemma 5.** *The joint stationary distribution of output tax and productivity $v(s)$ is a Pareto distribution:*

$$v(s) = \frac{s^{-\xi_z}}{(\xi_z + \xi_\tau) + 1}.$$

*and the tail index $\xi_z$ is given by equation (10).*

**Lemma 6.** *If the distribution of TFP is exogenous and the productivity process is independent of the output tax process, the rank of firm size distribution is independent of output taxes.*

The proof of Lemma 6 is straightforward from Lemma 5. An important result from Lemma 6 is that the Gini coefficient of the stationary distribution is independent of the output tax. As a result, in this environment changes in output tax $\tau$ do not affect the rank of firm size distribution.

The definition of equilibrium for this economy is almost the same as the previous one, however, the main difference is that the distribution of productivity is exogenous.

# 4 Calibration

We calibrate a benchmark economy with no distortions $\sigma_\tau = 0$ to data for the United States. Our main objective is to study the quantitative impact of policy distortions on aggregate TFP and GDP per worker in an economy that is relatively more distorted than the United States in the same spirit of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).

We start by selecting a set of parameters that are standard in the literature. These parameters have either well-known targets which we match or the values have been well discussed in the literature. Following the literature, we assume decreasing returns in the establishment-level production function and set $\alpha + \gamma = 0.85$, e.g., Restuccia and Rogerson (2008). Then we split it between $\alpha$ and $\gamma$ by assigning 1/3 to capital and 2/3 to labor, implying $\alpha = 0.283$ and $\gamma = 0.567$. We select the annual exit rate $\lambda$ to 10 percent from the estimates of exit rates in the literature, e.g., Davis et al. (1998).

We calibrate all other parameters by solving the equilibrium of the model and making sure the equilibrium statistics match some targets. All these parameters are selected simultaneously but some parameters have first-order impact on some targets so we discuss them in turn. To calibrate the exogenous growth rate of the mass of establishments $\eta$, we use the property of the model that the aggregate growth rate of TFP over time is proportional to the growth rate of the mass of establishments.[11] Since the growth rate of TFP in the United States in the last 100 years is roughly equal to 2 percent, $\eta$ is given by:

$$\eta = \frac{dM/dt}{M} = \frac{d\text{TFP}/dt}{\text{TFP}} \frac{1}{(1-\alpha-\gamma)} = \frac{0.02}{0.15} = 0.0946.$$

We calibrate the investment cost $c_\mu$ to match the employment growth rate that we calculate from the model. Employment in the model is given by the solution of the establishment static problem, more precisely equation (2). As a result, in the model the growth rate of employment also follows a Geomtric Brownian motion:

$$\frac{dn}{n} = (2\mu_z + \sigma^2)dt + 2\sigma dw,$$

where in the case with no distortions $\sigma = \sigma_z$.[12] The drift $\mu_z$ is a function of the cost of investing in productivity $c_\mu$. We calibrate $c_\mu$ to match the establishment's employment growth rate, which is approximately equal to two percent, according to Hsieh and Klenow (2014).

The volatility of the productivity process $\sigma_z$ is calibrated to match the Gini coefficient of the establishment size distribution in the United States. We calculate the Gini coefficient from the employment invariant distribution $g_N(\cdot)$. Applying the same methodology as in Lemma 2, we calculate the invariant distribution of employment:

$$g_N(n) = \frac{(\xi+1)}{2} n^{-(\xi+1)/2}.$$

[11]We calculate aggregate TFP in the model as $\frac{Y}{K^\alpha N^\gamma}$, it is straightforward to show that $TFP = M^{1-\alpha-\gamma}\left(\frac{\xi_Y-1}{\xi_Y-3}\right)\left(\frac{\xi-1}{\xi-3}\right)^{-(\alpha+\gamma)}$. The two terms between parenthesis are constant and do not change over time. The only term that changes over time is M. So, from this expression we get $(1-\alpha-\gamma)\frac{dM/dt}{M} = \frac{d\text{TFP}/dt}{\text{TFP}}$.

[12]From the static problem of establishments, we find the labor demand from equation (2), which is given by $n(z,\tau) = \left((\alpha/r)^\alpha (\gamma/w)^{1-\alpha}\right)^{1/(1-\alpha-\gamma)} \tau^2 z^2$. Labor demand follows a Geometric Brownian motion: $\frac{dn}{n} = (2\mu+\sigma^2)dt + 2\sigma dw$. It is straightforward that the distribution $g_N(\cdot)$ is a monodic transformation of the distribution $g(\cdot)$. Thus, the employment stationary distribution is given by: $g_N(n) = \frac{(\xi+1)}{2} n^{-(\xi+1)/2}$. Therefore, the Gini index is equal to $\text{Gini}(n) = 1/(\xi-2)$.

Therefore, we can calculate the Gini index, which is equal to Gini$(n) = \frac{1}{(\xi-2)}$ and Luttmer (2010) finds that is equal to 1.06 in the United States.

We calibrate the discount rate $\rho$ to match a long term interest rate of 4 percent. The depreciation of capital $\delta_k$ is calibrated to match the capital output ratio to 2.3, and we calibrate the entry cost $c_e$ to normalize the mass of establishments to one. The parameters values are summarized in Table 1, and in the next section we perform a comparative static exercise with respect to policy distortions.

Table 1: Benchmark Calibration to U.S. Data

| Parameter | Value | Target |
|---|---|---|
| $\sigma_z$ | 0.03 | Gini coefficient |
| $c_\mu$ | 21,280 | Employment growth rate |
| $c_e$ | 21.548 | Normalization of M |
| $\eta$ | 0.09 | TFP growth rate |
| $\rho$ | 0.04 | Interest rate |
| $\delta_k$ | 0.0069 | Capital output ratio |

# 5 Quantitative Experiments

We quantify the impact of policy distortions on aggregate TFP, aggregate output, productivity investment, and other relevant aggregates by comparing these statistics in each distorted economy relative to the benchmark economy which is assumed to have no distortions. We highlight the quantitative impact of policy distortions in our model with investment in establishment-level productivity and an endogenous distribution of productivity relative to versions of the model where these features are absent. We show that empirically-plausible policy distortions generate substantial negative effects on aggregate output and TFP and that endogenizing the productivity distribution is the feature of the model that provides a substantial quantitative amplification effect of policy distortions on aggregate TFP. Hence, in our framework, empirically-plausible policy distortions can generate differences in output per capita across countries that are closer in line with evidence relative to the existing literature with exogenous distributions of productivity.

We divide the quantitative experiments in three parts. (1) We study the impact of policy distortions in the economy by comparing statistics of this economy relative to the benchmark economy. More precisely, we quantify the impact of changes in policy distortions via changes in $\sigma_\tau$ that create misallocation. (2) We quantify the contribution to these results of the endogenous component of the distribution of establishment-level TFP by comparing the predictions of the model to two versions of the model where the distribution of TFP is exogenous. The first version of the model has an exogenous distribution of TFP for entering establishments but establishments still invest in the evolution of their productivity over time. The second version of the model has exogenous distribution of entering firm's TFP in addition to exogenous investment in productivity by establishments which is kept constant to the level of that of the undistorted economy. This version of the model is the closest comparable to the work of Restuccia and Rogerson (2008) where the distribution of firm-level productivities are exogenous and the same across all economies. (3) We quantify the contribution in our results of general equilibrium effects on wages relative to the benchmark economy.

## 5.1 Policy Distortions

We study the quantitative impact of changes in policy distortions via changes in $\sigma_\tau$ on the economy and report statistics relative to the undistorted benchmark economy. We note that the closest counterpart of this configuration of distortions is the setting of uncorrelated idiosyncratic distortions in Restuccia and Rogerson (2008) whereby policy distortions are uncorrelated to establishment-level productivity. We report the results in Table 2 for a number of statistics such as aggregate output, aggregate TFP, productivity investment, growth rate of establishment size, wages, a measure of the dispersion of real establishment-level productivity (ofter referred to as quantity TFP or TFPQ), and a measure of the extent of distortions (revenue productivity or TFPR).[13] All statistics reported are relative to the benchmark economy in percent except for the standard deviation of log TFPR which is 0 in the economy with no distortions. For this statistic we compare the difference in the standard deviation of log TFPR across economies as a gauge of the extent of distortions relative to the available evidence for some countries.

Table 2: Effects of Changes in Policy Distortions $\sigma_\tau$

| | $\sigma_\tau$ | | |
|---|---|---|---|
| Variable (%) | 0.10 | 0.50 | 1.00 |
| Relative Y | 69.0 | 30.1 | 14.0 |
| Relative TFP | 69.0 | 30.3 | 14.5 |
| Relative prod. investment $\mu_z$ | 99.7 | 91.5 | 65.8 |
| Relative establishment size $n$ | 100.0 | 100.1 | 100.5 |
| Relative size growth rate $\Delta(n)/n$ | 99.8 | 96.1 | 84.5 |
| Relative wages $w$ | 100.1 | 102.2 | 110.6 |
| Relative SD(logTFPQ) | 100.0 | 99.4 | 97.7 |
| SD(logTFPR) | 0.3 | 1.4 | 2.8 |

As is apparent from Table 2, policy distortions that generate misallocation through variations in $\sigma_\tau$ have a substantial impact on aggregate output and TFP. For instance, the economy with $\sigma_\tau = 1$ has an aggregate output and TFP that is around 14 percent of that of the benchmark economy (a 7-fold difference). Differences in output and TFP are increasing in the amount of distortions. More distorted economies feature lower firm-size growth relative to the undistorted economy as distortions discourage investment in productivity. This implication goes in the same direction as the findings in Hsieh and Klenow (2014) of lower firm size growth in India and Mexico relative to establishments in the United States. We note that the substantial effect of policy distortions on TFP does not arise from unreasonable amounts

[13]We present in the Appendix analytical expressions in the model for TFPQ and TFPR following Hsieh and Klenow (2009).

of misallocation. To gauge that, following the literature, we have computed the dispersion in log TFPR as a summary measure of distortions in the economy. This measure of distortions is increasing in $\sigma_\tau$ but the most distorted economy has distortions that are 2.8 percentage points higher than the benchmark economy. The difference in the standard deviation of log TFPR between China or India and the United States is between 14 and 29 percentage points depending on the year analyzed as reported by Hsieh and Klenow (2009).

## 5.2 Endogenous Productivity Distribution

Next, we quantify the importance of the endogenous distribution of establishment-level productivity in our framework. We consider two variations of our model in order to better place the results relative to the existing literature.

First, we consider the model where the distribution of entering-establishment's productivity is exogenous and set to the distribution of establishment-level TFP generated by the model for the benchmark economy (i.e., the economy without policy distortions). In this economy, entering establishments draw productivity from the same distribution in every economy regardless of distortions but incumbent establishments invest in their productivity and this investment can differ across economies with different policy distortions. This version of the model broadly captures the recent literature that has endogenized productivity investment across establishments, for example, Restuccia (2013b), Bello et al. (2011), Ranasinghe (2014), Bhattacharya et al. (2013), and Gabler and Poschke (2013). We refer to this version of the model as "Endogenous Investment". We note, however, that whereas this literature emphasizes correlated distortions in their quantitative applications, this version of our model features the same uncorrelated distortions as in our baseline model. This distinction is critical in understanding the output and productivity effects in this version of our model with exogenous distribution.

Second, we consider a version of the model where, in addition to fixing the productivity distribution of entering establishments, we also fix the amount of productivity investment by establishments $\mu_z$, which we set exogenously to the level of that in the benchmark economy. This implies that the distribution of establishment-level TFP is the same in all economies regardless of the policy distortions. This case is closest to the framework in Restuccia and Rogerson (2008) where the distribution of productive units is kept constant in all economies and uncorrelated distortions are evaluated. We refer to this version of the model as "Exogenous".

Table 3 reports the results of these versions of the model for $\sigma_\tau = 1$ along with the statistics for our baseline distorted economy where both productivity investment and the productivity distribution of entrants are endogenous. Table 3 allows us to decompose the impact of policy distortions in our framework into three components. (1) Misallocation of inputs across producers, which is the standard static-misallocation effects emphasized in the literature, e.g. Restuccia and Rogerson (2008), described by the last column in Table 3. (2) Inefficient productivity investment, e.g., Restuccia (2013b) and Bhattacharya et al. (2013), described in the third column in Table 3. (3) Changes in the distribution of establishment-level TFP which is the contribution of the two features together in our framework ("Endogenous Baseline").

Table 3: Effects of Endogenous Distribution ($\sigma_\tau = 1$)

| Variable | Endogenous Baseline | Endogenous Investment | Exogenous |
|---|---|---|---|
| Relative Y | 14.0 | 86.5 | 86.3 |
| Relative TFP | 14.5 | 86.9 | 86.8 |
| Relative prod. investment $\mu_z$ | 65.8 | 100.2 | 100.0 |
| Relative establishment size $n$ | 100.5 | 99.0 | 100.0 |
| Relative size growth rate $\Delta(n)/n$ | 84.5 | 100.0 | 100.0 |
| Relative wages $w$ | 110.6 | 100.2 | 100.0 |
| Relative SD(logTFPQ) | 97.7 | 100.0 | 100.0 |
| SD(logTFPR) | 2.8 | 2.8 | 2.8 |

The main result from Table 3 is that economies with exogenous productivity distributions are not able to generate substantial changes in output per capita and in TFP. Whereas the baseline model with an endogenous productivity distribution generates a distorted economy whose output per worker and TFP is 14 percent of that of the benchmark economy, the same policy configuration in the model with an exogenous distribution generates economies with output per worker and TFP that are 86 percent of that of the benchmark economy. This is the case even for the economy with endogenous productivity investment. As a result, the endogenous productivity distribution is a fundamental mechanism that amplifies the impact of policy distortions, and this amplification effect is substantial, the drop in output and TFP is a factor of 6-fold larger than in the economies with an exogenous productivity distribution. We emphasize that this large amplification effect arises in this framework for the same policy distortions. Moreover, the large output and TFP effects in the baseline model arise with relatively small policy distortions since the standard deviation of log revenue productivity (TFPR) is only a fraction of those found empirically in China and India in Hsieh and Klenow (2009).

We note that in our framework with exogenous productivity distributions, as in Restuccia and Rogerson (2008), uncorrelated policy distortions do not generate substantial negative effects on output and TFP. Moreover, policy distortions generate similar output and productivity effects in the exogenous and in the endogenous investment versions of the model. This is a consequence of uncorrelated distortions. For instance, Bhattacharya et al. (2013) obtain a small amplification effect by focusing on correlated distortions that discourage the investment on managerial ability by establishments during their life cycle. Correlated distortions can generate larger productivity effects in the endogenous investment version relative to the exogenous case as emphasized in the literature with this feature, but correlated distortions will also generate larger effects when both productivity investment and the distribution of entering productivity are endogenous in our baseline.

## 5.3 General Equilibrium Effects

We quantify the importance of general equilibrium changes in wages in our framework both for the magnitude of the result as well as for the decomposition of the effects by comparing the results with the exogenous distribution versions of the model (endogenous investment and exogenous). In order to make this comparison, we fix wages in all versions of the model to the wages of the benchmark economy. In Table 4 we report the results for a policy distortion of $\sigma_\tau = 1.0$, which is the same value as in Table 3.

Table 4: General Equilibrium Effects of Policy Distortions ($\sigma_\tau = 1$)

| | Fixed Wages | | | General Eqm Wages | | |
|---|---|---|---|---|---|---|
| Variable | End. Bas. | End. Inv. | Exo. | End. Bas. | End. Inv. | Exo. |
| Relative Y | 0.2 | 86.3 | 86.3 | 14.0 | 86.5 | 86.3 |
| Relative TFP | 0.2 | 86.4 | 86.4 | 14.5 | 86.9 | 86.8 |
| Relative wages $w$ | 100.0 | 100.0 | 100.0 | 110.6 | 100.2 | 100.0 |
| Relative prod. investment $\mu_z$ | 101.0 | 101.0 | 100.0 | 65.8 | 100.2 | 100.0 |
| Relative establishment size $n$ | 226.7 | 100.0 | 100.0 | 100.5 | 99.0 | 100.0 |
| Relative mass $M$ | 1.0 | 100.0 | 100.0 | 100.2 | 101.5 | 100.0 |
| Relative SD(logTFPQ) | 100.0 | 100.0 | 100.0 | 97.7 | 100.0 | 100.0 |
| SD(logTFPR) | 2.8 | 2.8 | 2.8 | 2.8 | 2.8 | 2.8 |

We find that the general equilibrium effect on wages attenuates the impact of policy distortions. We can observe this by comparing output and TFP for the same set of policy distortions in Table 4. Although in the three models "Endogenous Baseline", "Endogenous Investment", and "Exogenous", wages attenuate the effect of policy distortions, the magnitude of the effect of wages is different. In the model with an endogenous distribution this effect is significant, while in the models with exogenous distributions the effect is negligible. The different impact of wages between the endogenous and exogenous distributions is that in the models with exogenous distributions establishments have a constant size. As a result, even when wages are flexible, policy distortions have a small impact on labor demand

and consequently on wages. However, in the model with an endogenous distribution, policy distortions have a significant impact on establishment size, and consequently on labor demand and wages. When wages are flexible, an increase in the average size of establishments increases labor demand, and hence wages. Since wages are higher, establishments invest less in productivity. When wages are fixed, establishments do not adjust their investment in productivity, as a result, establishments are on average larger than in the model with flexible wages, and because of decreasing to returns to scale technology output and TFP are even lower than in the model with flexible wages.

This result demonstrates the importance of general equilibrium effects, specially in models with endogenous investment in productivity. But the result also shows that if the model was extended to allow for complementary features to explain low establishment size in poor countries and hence lower wages, the amplification effects would be larger than emphasized in our baseline experiments. This is relevant as the evidence is that poor countries are characterized by low average establishment sizes relative to rich countries.[14] Small establishment sizes in poor countries may be the result of high entry costs as emphasized in the literature. It may also be due to correlated idiosyncratic distortions as found in Hsieh and Klenow (2009) and Bento and Restuccia (2014).

# 6 Conclusions

We developed a tractable dynamic model that endogenizes the distribution of establishment-level TFP across economies, what Hsieh and Klenow (2009) and the related literature call TFPQ. The model tractability allows us to find closed-form solutions that can be used to identify distortions from establishment-level data. In this framework, policy distortions not only generate differences in productivity investment across establishments as in the recent

[14] See for instance the evidence on average establishment size in the manufacturing sector across countries in Bento and Restuccia (2014).

literature endogenizing life-cycle investments, but also the distribution of establishment-level productivity where new establishments draw their productivity from. We showed that empirically-reasonable policy distortions have substantial negative effects on aggregate TFP in this economy, an effect that is orders of magnitude larger than in the same model with exogenous distributions of establishment-level TFP.

We have considered policy distortions that are uncorrelated to establishment-level productivity and nevertheless have found that these distortions have substantial negative effects on aggregate TFP. Since the literature has emphasized the substantially larger productivity impact of idiosyncratic distortions that affect more productive larger producers, it would be interesting to explore the implications of correlated distortions in our framework. This requires a non-trivial extension of the theory and for this reason we leave this important exploration for future work. We also think that it would be interesting to explore specific policies or institutions such as size-dependent policies, firing taxes, financial frictions, among many others in the literature in the context of this model with endogenous establishment-level TFP. These explorations of specific policies in our framework may reconcile the puzzle in the literature of the apparent disparity in findings between indirect measurement and direct quantitative assessments of misallocation and productivity discussed in Hopenhayn (2013). Broadly speaking, we argue that our framework with an endogenous productivity distribution may be better suited to explain the data. As a result, further progress aimed to broaden the empirical mapping of the model to the data may yield high returns. We leave these interesting and important extensions for future work.

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# A Proofs

This appendix presents the proofs of Lemma 2, Lemma 4 and Lemma 5.

## Proof of Lemma 2

From the Kolmogorov forward equation (7), from our guess of the probability density function (8), and after some algebraic manipulation, we find that the dynamics of the probability density of operating establishments, $f(\cdot,\cdot)$ satisfies the following Kolmogorov forward equation:

$$\frac{\partial f(x,t)}{\partial t} = -\hat{\mu}\frac{\partial f(x,t)}{\partial x} + \frac{\sigma^2}{2}\frac{\partial^2 f(x,t)}{\partial x^2} + \left[\frac{(\Delta J(t) - dM(t)/dt)}{M(t)} - \lambda\right] f(x,t)$$

where

$$\Delta J(t) = J(x,t)|_0^\infty = \left(-\hat{\mu}F(x,t) + \frac{\sigma^2}{2}\frac{\partial F(x,t)}{\partial x} + \sigma^2\frac{\partial f(x,t)}{\partial x}\frac{F(x,t)}{f(x,t)}\right)\Bigg|_0^\infty .$$

From the expression above, we observe that the dynamics of the probability density of operating establishments is a function of the rate of departure of establishments (first two terms in the right-hand side), and net rate of new entrants (last right-hand-side term), where both are a function of the expected profitability and the aleatory shocks captured in the Kolmogorov forward process.

In the stationary equilibrium, the density of operating establishments, $f_\infty(x)$, does not vary over time, i.e. $\frac{\partial f(x,t)}{\partial t} = 0$ . As a result we can rewrite the Kolmogorov forward equation in equilibrium as follows:

$$\left[\eta + \lambda - \frac{\Delta J}{M}\right] f_\infty(x) = -\hat{\mu}\frac{\partial f_\infty(x)}{\partial x} + \frac{\hat{\sigma}^2}{2}\frac{\partial^2 f_\infty(x)}{\partial x^2}$$

where

$$\frac{\Delta J}{M} = -\hat{\mu} f_\infty(x) + \frac{3\hat{\sigma}^2}{2}\frac{\partial f_\infty(x)}{\partial x}\Big|_0^\infty .$$

The Kolmogorov forward equation in the stationary equilibrium implies that the rate of departure of establishments is equal to the rate of new entrants at each profitability level $x$.

Now we guess that the invariant probability distribution has the following functional form $f_\infty(x) = Ke^{-\xi x}$, consequently the first derivative is equal to $f'_\infty(x) = -\xi f_\infty(x)$ and the second is equal to $f''_\infty(x) = \xi^2 f_\infty(x)$. Since $f$ is a probability distribution, it must satisfies that its measure is equal to one, i.e. $\int_0^\infty f_\infty(x)dx = 1$, which implies that $K = \xi$. Therefore, $\frac{\Delta J}{M} = -\hat{\mu} f_\infty(x) + \frac{3\hat{\sigma}^2}{2}\frac{\partial f_\infty(x)}{\partial x}\Big|_0^\infty = (\hat{\mu} + \frac{3\hat{\sigma}^2}{2}\xi)\xi$ and the tail index $\xi$ is found by solving the following polynomial,

$$\xi^2 + \frac{\hat{\mu}}{\hat{\sigma}^2}\xi - \frac{(\eta+\lambda)}{2\hat{\sigma}^2} = 0.$$

Substituting $\hat{\mu}$, from its definition, $\hat{\mu} = \mu - \frac{1}{2}\sigma^2$, and $\hat{\sigma}$ from its definition, $\hat{\sigma} = \sigma$, we find the following polynomial,

$$\xi^2 + \left(\frac{\mu}{2\sigma^2} - \frac{1}{4}\right)\xi - \frac{(\eta+\lambda)}{2\sigma^2} = 0. \tag{11}$$

We restrict $\xi$, which is the tail index of the invariant distribution, to the positive root that solves equation (11), which is $\xi = -\left(\frac{\mu}{2\sigma^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu}{2\sigma^2} - \frac{1}{4}\right)^2 + \frac{2(\eta+\lambda)}{\sigma^2}}$.■

## Proof of Lemma 4

We first guess the steady state distribution, $f_\infty(y) = Ke^{\xi_\tau y}$, $y = e^\tau$, as a result $f'_\infty(y) = \xi_\tau f_\infty(y)$ and $f''_\infty(y) = \xi_\tau^2 f_\infty(y)$. Moreover, $\int_{-\infty}^0 f_\infty(y)dy = 1$ implies that $K = \xi_\tau$.

The invariant distribution $f_\infty(\cdot)$ must satisfy the Kolmogorov forward equation:

$$\left[\eta+\lambda-\frac{\Delta J}{M}\right] f_\infty(x) = -\hat{\mu}_\tau \frac{\partial f_\infty(x)}{\partial x} + \frac{\hat{\sigma}_\tau^2}{2}\frac{\partial^2 f_\infty(x)}{\partial x^2}.$$

By substituting our guess of the invariant distribution into the Kolmogorov equation, we find a polynomial in $\xi_\tau$ given by:

$$\xi_\tau^2 - \frac{\hat{\mu}_\tau}{\hat{\sigma}_\tau}^2 \xi - \frac{(\eta+\lambda)}{2\hat{\sigma_\tau}^2} = 0.$$

Substituting $\hat{\mu} = \mu - \frac{1}{2}\sigma^2$ and $\hat{\sigma} = \sigma$, and selecting the positive root of the polynomial, we find $\xi_\tau = \left(\frac{\mu_\tau}{2\sigma_\tau^2} - \frac{1}{4}\right) + \sqrt{\left(\frac{\mu_\tau}{2\sigma_\tau^2} - \frac{1}{4}\right)^2 + \frac{(\eta+\lambda)}{2\sigma_\tau^2}}$ and this concludes the proof. ∎

## Proof of Lemma 5

Given that the productivity $z$ and output tax $\tau$ are independent stochastic variables, the join distribution of productivity and output tax $v(\cdot)$ is the product of the stationary distribution of productivity $g(\cdot)$ and the stationary distribution of output tax $h(\cdot)$. The joint distribution $v(\cdot)$ is given by:

$$v(z,\tau) = (\xi_z - 1) z^{-\xi_z} (\xi_\tau + 1)\tau^{\xi_\tau}.$$

From the point of view of establishments the important statistic is $s = \tau z$. By applying a simple change of variables, we can rewrite $v(\cdot)$ and a function of $s$. The final expression for $v(\cdot)$ is given by:

$$v(s) = \int_0^1 \left(\frac{s}{\tau}\right)^{-\xi_z} \tau^{\xi_\tau} d\tau = \frac{s^{-\xi_z}}{(\xi_z+\xi_\tau)+1}.$$

The joint invariant distribution is also a Pareto distribution with tail index $\xi_z$. ∎

# B Definitions of TFPQ and TFPR

This appendix presents the definition of TFPQ and TFPR in the context of our model.

**Definition of TFPQ** We calculate TFPQ at the establishment level as in Hsieh and Klenow (2009),

$$\text{TFPQ}(z) = \frac{y}{k^{\alpha} n^{\gamma}} \propto \frac{z^2 \tau^{2(\alpha+\gamma)}}{z^{2(\alpha+\gamma)} \tau^{2(\alpha+\gamma)}} = z^{2(1-\alpha-\gamma)}.$$

Therefore, TFPQ at establishment level depends on distortions given and on $\mu_z$, which is an endogenous object. The log of TFPQ follows an exponential distribution $f(x) = \xi_z e^{-\xi_z x}$, where $\xi_z$ is defined as:

$$\xi_z = -\left(\frac{1}{8} + \frac{\mu_z}{2\sigma_z^2}\right) + \sqrt{\left(\frac{1}{8} + \frac{\mu_z}{2\sigma_z^2}\right)^2 + \frac{(\eta+\lambda)}{2\sigma_z^2} + \frac{\mu_z}{2\sigma_z^2} - \frac{1}{4}}.$$

Therefore, the standard deviation of the log TFPQ is equal to the variance of an exponential distribution, which is given by:

$$\text{SD}[\log \text{TFPQ}] = \frac{1}{\xi_z}.$$

**Definition of TFPR** We calculate TFPR in the model as in Hsieh and Klenow (2009):

$$\text{TFPR}(z, \tau) = \frac{1}{(1-\tau_y)} = \tau^{-2(1-\alpha-\gamma)}.$$

Therefore log TFPR$(z, \tau)$ follows an exponential distribution $f(x) = \xi_{\text{TFPR}} e^{-\xi_{\text{TFPR}} x}$ with

$$\xi_{\text{TFPR}} = -\left(\frac{1}{8} + \frac{\mu_{\text{TFPR}}}{2\sigma_{\text{TFPR}}^2}\right) + \sqrt{\left(\frac{1}{8} + \frac{\mu_{\text{TFPR}}}{2\sigma_{\text{TFPR}}^2}\right)^2 + \frac{(\eta+\lambda)}{2\sigma_{\text{TFPR}}^2} + \frac{\mu_{\text{TFPR}}}{2\sigma_{\text{TFPR}}^2} - \frac{1}{4}},$$

where (applying Ito's calculus)

$$\begin{aligned}
\mu_{\mathrm{TFPR}} &= -2(1-\alpha-\gamma)\left[\mu_\tau - \frac{1}{2}(3-2\alpha-2\gamma)\sigma_\tau^2\right], \\
\sigma_{\mathrm{TFPR}}^2 &= 4(1-\alpha-\gamma)^2\sigma_\tau^2.
\end{aligned}$$

Therefore, SD of the log TFPR is equal to

$$\mathrm{SD}[\log \mathrm{TFPR}] = \frac{1}{\xi_{\mathrm{TFPR}}},$$