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Applications and Interviews: Firms' Recruiting Decisions in a Frictional Labor Market

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Abstract

I develop a directed search model of the labor market in which firms choose a *recruiting intensity*, determining the number of applicants they will interview. Interviewing applicants is costly but reveals their productivity, allowing the firm to hire better workers. I characterize the equilibrium and find that the uniqueness and cyclicality of recruiting intensity crucially depend on parameter values. Calibration of the model to the US labor market indicates a multiplicity of the equilibrium. An increase in aggregate productivity—given selection of a particular equilibrium—causes recruiting intensity to move counter to unemployment, while a shock to the equilibrium selection rule predicts the opposite pattern.

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1 Introduction

1.1 Motivation and Summary

The process by which workers and firms match in the labor market is a complex one, with either side of the market facing numerous choices and trade-offs. Search models of the labor market based on the Diamond-Mortensen-Pissarides framework have traditionally abstracted from most of these choices. They restrict attention to agents' participation decisions by modeling this process with an aggregate matching function which solely depends on the ratio of the number of vacancies to the number of unemployed. Although this approach has been very fruitful for improving our understanding of modern labor markets, recent quantitative work has emphasized the need for richer models. For example, Davis et al. (2013) argue that firms' *recruiting intensity* varies over the business cycle, counter to unemployment. As they explain, focusing exclusively on the number of vacancies ignores the fact that firms may increase their hiring probability if they "increase advertising or search intensity per vacancy, screen applicants more quickly, relax hiring standards, improve working conditions, and offer more attractive compensation to prospective employees."

The strategic use of wages plays a central role in models of directed search, pioneered by Moen (1997), but screening decisions and hiring standards are still relatively unexplored in the search literature.[1] This is somewhat surprising, because firms spend considerable resources deciding which of their applicants they wish to hire. The Employment Opportunities Pilot Projects (EOPP) data set, which I discuss in detail in section 4, indicates that US firms recruiting for low-skilled jobs in the early eighties on average received 14 applications, conducted 5 interviews and made 1.2 job offers per vacant position, spending a total of 6 hours on the process. Burks et al. (2013) report that a high-tech firm with 1.4 million applicants between 2003 and 2011 subjected 10% to a series of interviews and made a job offer to 0.5%. The absence of a micro-founded equilibrium model in the literature means that various important questions regarding these decisions remain unanswered. For example, what are the trade-offs that firms face in their recruiting decisions? Does firms' recruiting intensity move counter to unemployment in response to changes in aggregate conditions, such as a productivity shock?

This paper aims to shed light on these questions by extending a standard directed search model of the labor market with meaningful recruiting decisions for firms. That is, in addition to workers' compensation, each firm chooses a recruiting intensity, determining how many applicants it will interview.[2] The primary reason for firms to interview applicants is an information friction: applicants differ in their productivity and a firm can learn this productivity

[1] See section 1.2 for a literature review.

[2] Hence, unlike Davis et al. (2013), I reserve the terminology "recruiting intensity" for firms' interviewing decisions and treat workers' compensation as a separate choice. I maintain the standard assumption of one vacancy per firm, because firms' choice of how many vacancies to create has already been analyzed in detail by Kaas and Kircher (2013).

only through a job interview. A larger number of interviews allows a firm to rank more applicants and therefore hire better workers, but comes at a higher cost, capturing the idea that each interview requires time or effort.

In this environment, I characterize equilibrium, derive a rich set of empirical predictions and analyze the above questions. I find that the answers crucially depend on the details of the environment. First, a productivity shock has a direct effect and an indirect effect on recruiting intensity, which work through in increase in the marginal gains of an extra interview for a given number of applicants and a change in the number of applicants itself, respectively. Depending on parameter values, these effects may strengthen or partially offset each other, making the total effect an empirical question. Second, for certain parameter values, the equilibrium may not be uniquely determined, in which case shocks to the equilibrium selection rule can be an additional source of fluctuations in recruiting intensity and unemployment.

The complexity of the equilibrium prevents a precise analytical characterization of these effects. In order to make progress, I therefore demonstrate how the model can be calibrated using data on firms' recruiting process. Although the lack of nationally representative data implies that the results should be interpreted with care, they indicate that the equilibrium is not unique. An increase in aggregate productivity—given selection of a particular equilibrium—causes recruiting intensity to move counter to unemployment, while a shock to the equilibrium selection rule predicts the opposite pattern.

Having provided a rough outline, I now describe the environment and the main results in more detail. As I discuss in section 2, I consider a directed search model of the labor market with a continuum of workers and free entry of firms, each with one vacancy. Workers choose a search intensity, determining the number of applications that they can send, while firms choose a recruiting intensity, determining the number of interviews that they can conduct. Firms have the ability to post information about their terms of trade and their recruiting intensity, helping workers to direct their applications to the firms that offer the highest expected payoff. However, frictions arise because search and recruiting are costly and because agents cannot coordinate their actions. As a result, the number of applications that a firm receives is a random variable with a distribution that depends on the firm's announcement. Subsequently, each firm interviews a number of applicants, consistent with its choice of recruiting intensity. It learns their productivity and ranks them by their profitability in order to make job offers.

In section 3, I first analyze recruiting decisions for the special case in which each worker sends one application per period, as standard in the literature on directed search. I show that a unique equilibrium exists and that this equilibrium is constrained efficient. In equilibrium, all firms choose the same level of recruiting intensity and will generically interview multiple but not all applicants, balancing the cost of an extra interview with the gain of potentially finding a better match. Each firm will make a job offer to its most productive interviewee, as long as his productivity exceeds an endogenous threshold, and—because each worker has only one chance to match—this job offer is always accepted

In this environment, an increase in aggregate productivity has two opposing effects on firms' recruiting intensity. First, there is a direct effect; given a number of applicants, the productivity shock increases firms' payoff of a match, which causes firms to choose a higher recruiting intensity. At the same time however, the productivity shock encourages firm entry, reducing the number of applicants at each firm. This indirect effect causes firms to choose a lower recruiting intensity, as the number of interviews is necessarily bounded by the number of applicants. Hence, the sign of the total impact is ambiguous.

For the magnitude of both the direct effect and the indirect effect, the assumption that workers search for one job at a time clearly matters. If workers apply to multiple jobs simultaneously, they may get multiple job offers and reject all except the best. This potential rejection of job offers may strengthen the direct effect, since a small number of interviews no longer yields a good match with sufficiently high probability, or weaken it, as an extra interview is less likely to result in a better match. Likewise, the way in which workers change their search intensity in response to the productivity shock may strengthen or weaken the indirect effect by increasing or decreasing the expected number of applicants for each firm.

These considerations are important, because the assumption that workers send a single application per period might be pervasive and analytically convenient, it is also inconsistent with various empirical facts. For example, 2012 data from CareerBuilder.com, the largest US employment website, indicates that workers send on average 5.0 applications on days on which they apply at least once.[3] Further, a recent survey by the website among recruiters indicates that although candidates decline job offers relatively infrequently (less than 10%), the main reason for them to do so is because they received another offer (CareerBuilder, 2014).

In the second part of section 3, I therefore relax the assumption that workers apply for one job at a time by embedding the recruiting dimension into a model of multiple applications, in the spirit of Galenianos and Kircher (2009) and Kircher (2009). As in their models, wage dispersion arises. Hence, a worker choosing a higher search intensity incurs a higher cost, but reduces his risk of not getting any job offer and—conditional on getting an offer—increases his chances of finding a job with a high payoff. For firms, the introduction of simultaneous search changes their trade-offs in non-trivial ways. In equilibrium, firms posting different terms of trade choose different recruiting intensities, but the relationship is not necessarily monotonic. Interestingly, equilibrium may not be unique if the cost of recruiting is sufficiently large. This implies that, in addition to productivity shocks, shocks to the equilibrium selection rule may be a second source of fluctuations in recruiting intensity and unemployment. Since these shocks are potentially more persistent than shocks to aggregate productivity, understanding

[3] I thank Ioana Marinescu and Roland Rathelot for this information. A detailed description of the data can be found in Marinescu and Rathelot (2014). Kudlyak et al. (2012) and Faberman and Kudlyak (2014) provide similar evidence of simultaneous search with data from the job search engine SnagAJob, which specializes in hourly jobs.

their respective implications is important.

In section 4, I therefore discuss how the model can be calibrated using aggregate matching and recruiting information from the Current Population Survey (CPS) and the EOPP data set. I show that while the model with one application per worker cannot match the data, the model with multiple applications does a good job and yields reasonable empirical predictions. The calibrated parameter values imply that the equilibrium is not unique. Using these values, I do two counterfactual experiments. First, I analyze the effects of an increase in aggregate productivity and find that—given selection of a particular equilibrium—it leads to a higher recruiting intensity for firms and, not surprisingly, a reduction in unemployment. Hence, recruiting intensity moves counter to unemployment in this case. Second, I study the effect of a shock to the equilibrium selection rule, causing agents to play a different equilibrium. The shock worsens the frictions in the market and although firms increase their recruiting intensity to compensate for this, unemployment increases. Hence, recruiting intensity moves with unemployment in this case. Hence, although better data is required for a definite conclusion, productivity shocks appear to be a better explanation of procyclical movements in recruiting intensity than shocks to the equilibrium selection rule.

Section 5 discusses some of the key assumptions and their impact on the results. In addition, it analyzes constrained efficiency of the equilibrium. Finally, section 6 concludes and the appendix contains all proofs as well as extra details about the calibration.

1.2 Related Literature

This paper contributes to multiple strands of literature. First, this paper adds to the literature on interviewing decisions by firms. Most papers in this literature are primarily empirical and use partial equilibrium models; Barron et al. (1985), Barron et al. (1987) and Burdett and Cunningham (1998) study the determinants of firms' recruiting decisions with the EOPP data, while van Ours and Ridder (1992, 1993) and Abbring and van Ours (1994) analyze Dutch recruiting data and find evidence for simultaneous rather than sequential search by firms. Equilibrium models of job interviews appear to be limited to Villena-Roldan (2012), who studies how interview costs affect inequality. Relative to his work, I emphasize a different question and present a model which features search intensity decisions for workers and active competition among firms.

Also related are papers that abstract from interviewing decisions, but introduce an intensive margin to firms' strategy in a different manner. For example, Pissarides (2000) analyzes recruiting intensity in a reduced-form way in a random search model and finds that the equilibrium intensity is independent of any aggregate variable. Kaas and Kircher (2013) present a model with large firms and interpret recruiting intensity as the decision how many vacancies to create, which they show to be procyclical.

At a technical level, this paper adds to the directed search literature. The papers in this literature have explored various assumptions regarding the process that governs meetings between workers and firms. For example, Moen (1997) and Acemoglu and Shimer (1999) consider bilateral meetings; Shi (2001, 2002, 2006) and Shimer (2005a) use an urn-ball meeting technology in which each worker applies to one firm and each firm faces a Poisson number of applicants; and recent work by Albrecht et al. (2006), Galenianos and Kircher (2009) and Kircher (2009) study extensions of the urn-ball framework by allowing workers to apply to multiple firms. However, in all of these specifications, the number of workers that a firm can contact is exogenously determined and equal to either 1 or the Poisson number of applicants.[4] This paper contributes by making the number of interviews that the firm conducts an endogenous choice.[5]

2 Model

2.1 Setting

Agents. Consider a one-period labor market with a mass s of workers and a positive measure v of firms, determined by free entry. Both types of agents are risk-neutral. All workers supply one indivisible unit of labor, while each firm has a position that can be filled by exactly one worker. As standard in the literature, agents are restricted to symmetric and anonymous strategies.

Timing. The interaction between workers and firms takes place in a number of subsequent phases. First, firms create vacancies and post contracts during the entry phase. After observing all posted contracts, workers choose their search intensity and send job applications during the search phase. Upon receiving these applications, firms interview a number of applicants during the recruitment phase. Matches are formed during the matching phase, after which output is produced in the production phase.

Entry. In the entry phase, firms can enter the market and create a vacancy at cost $k_V > 0$. Each entering firm makes two irreversible decisions. The first decision is a *recruiting intensity* $r \in \mathcal{R} \equiv [0, \infty)$, while the second decision concerns the *terms of trade* with a match, summarized by $w \in \mathcal{W} \equiv [0, 1]$. To simplify exposition, I will often refer to a combination of r and w as—for lack of a better term—a *contract*, denoted by $c = (r, w) \in \mathcal{C} \equiv \mathcal{R} \times \mathcal{W}$. All

[4] Note that this distinction does not coincide with the distinction between bilateral and urn-ball, since some urn-ball models require firms to randomly select one applicant (e.g. Shi, 2001; Albrecht et al., 2006; Galenianos and Kircher, 2009; Gautier and Wolthoff, 2009).

[5] The fact that meeting 1 or all applicants are feasible choices for the firms in my model does not necessarily imply that the cited papers are nested. They generally differ along other dimensions and generate many insights that cannot directly be obtained from my model.

identical contracts will be treated symmetrically by workers and are therefore said to form a *submarket*.

Recruiting Intensity. A firm's recruiting intensity r can be interpreted as a measure for the amount of time or resources that the firm will spend on interviewing workers. A higher value of r increases the probability that the firm will hire a very productive worker, as I will explain below, but is also more costly. I assume that the cost of a recruiting intensity r equals $k_R r \geq 0$. The assumption that a firm chooses r before learning its number of applicants or their types captures the idea that the firm needs to hire a number of recruiters (or sign a contract with a recruiting agency) before knowing how successful its search for a worker is going to be.

Terms of Trade. The terms of trade specify how output will be divided between the firm and the worker that it hires. I assume that the firm compensates its hire for the value of unemployment and additionally pays him a fraction w of the match surplus.[6] This guarantees that a firm and a worker will match only if the match surplus is non-negative and that firms prefer more productive over less productive workers. As I discuss in section 5, efficiency at the aggregate level may require transfers between agents that do not match (such as application fees) for some parameter values. In line with most of the literature, I do not consider such transfers as we rarely observe them in real life, presumably because they could create incentives for 'fake' vacancies to arise.[7]

Search. At the beginning of the search phase, unemployed workers observe the contract of each firm, allowing them to direct their search.[8] Workers initiate contact with the firms by sending applications. I assume the following structure. Each unemployed worker chooses a search intensity $\alpha \in \mathcal{A} \equiv [0, \infty)$ at cost $k_A \alpha$. This search intensity can be interpreted as a measure for the time the worker allocates to job search and it determines the number of applications that he can send. However, the relation between search intensity α and the number of applications a can be stochastic, e.g. because of randomness in the time it takes to

[6] Alternatively, one could allow the firm to post a menu of wages, one for each productivity type, but risk-neutrality implies that workers only care about their expected payoff and the assumption of a surplus-sharing rule is notationally more convenient. Note that this structure resembles the outcome of Nash bargaining, common in random search models (see e.g. Pissarides, 2000), except that w is now an endogenous variable instead of an exogenous parameter representing the worker's bargaining power.

[7] Albrecht et al. (2006) and Galenianos and Kircher (2009) are examples of models in which application fees could improve welfare, but are ruled out by assumption. See Lester et al. (2014) for a detailed discussion of when application fees are useful.

[8] The assumption that workers also observe recruiting intensity is convenient but not crucial. One can alternatively assume that workers observe the expected productivity of the worker that the firm will hire (in addition to the terms of trade). A formal discussion of this issue requires more notation and is therefore postponed to section 5.1.

apply to a particular position.[9]

Specifically, given a search effort α, a worker can send $a \in \{1, \ldots, A\}$ applications with probability $p_a(\alpha)$, where A is a potentially large but finite integer. I explore two variations of the search process. First, I restrict workers to send one application, i.e. $A = 1$ and $p_1(\alpha) = 1$ for all α. Subsequently, I relax this assumption by allowing workers to send multiple applications, $A > 1$. To make α interpretable as a search intensity in that case, I assume that the cumulative distribution $P_a(\alpha) = \sum_{i=1}^{a} p_i(\alpha)$ satisfies first-order stochastic dominance in α. Further, I assume $p_1(0) = 1$, $p_A(\infty) = 1$ and $p_a(\alpha) \in (0,1)$ for all $a \in \{1, \ldots, A\}$ and $0 < \alpha < \infty$.[10]

Productivity. After the applications have been sent, the productivity x of each potential match is realized, independently across all firm-applicant pairs. An applicant is *unqualified* for the job with probability $1 - q \in [0, 1)$, in which case his productivity x equals zero. With probability q, the applicant is *qualified* and his productivity is a draw from a distribution $Q(x)$ with support $\mathcal{X} \equiv (0, \overline{x}) \subset (0, \infty)$.[11]

Recruitment. The recruitment phase consists of two parts. First, firms costlessly observe whether applicants are qualified or not, allowing them to eliminate all applicants with $x = 0$. Second, the firm interviews a number of qualified applicants. An interview is a prerequisite for hiring and allows firms to learn the productivity x of a match with the applicant. Applicants with productivity below an endogenous threshold $\underline{x}$ generate a negative payoff for the firm and will therefore not be offered the job, even if no other applicants are present at the firm.

The number of interviews n_R that a firm can conduct (its 'interview capacity') depends on its choice of recruiting intensity r. To be precise, I assume that n_R follows a geometric distribution with parameter $\frac{r}{r+1} \in [0, 1]$, such that the probability that a firm can interview $n_R \in \mathbb{N}_1 \equiv \{1, 2, \ldots\}$ applicants is given by $\left(\frac{r}{r+1}\right)^{n_R - 1} \frac{1}{r+1}$ and the expected capacity is exactly one more than the firm's recruiting intensity, $\mathbb{E}[n_R] = r + 1$.[12] The actual number of interviews taking place then equals $\min\{n_Q, n_R\}$, where $n_Q \in \mathbb{N}_0 \equiv \{0, 1, \ldots\}$ is the firm's number of qualified applicants. Since a worker's productivity is unknown ex ante, the firm randomly selects n_R qualified applicants for an interview if $n_Q > n_R$. The firm rejects all applicants which it does not interview or which are too unproductive.

[9] A key advantage of this approach is that it avoids optimization over a discrete variable. See Kaas (2010) for a related setup in a random search model.

[10] Allowing one application at zero cost (by choosing $\alpha = 0$) is not crucial, but avoids that a high k_A dissuades workers from applying and the market collapses.

[11] Theoretically, not much insight is lost by setting $q = 1$. Allowing for $q < 1$ is important when confronting the model with the data in section 4.

[12] Allowing firms to interview one candidate at zero cost (by choosing $r = 0$) eliminates cases without trade, but is otherwise not crucial for the results.

The randomness in the interview capacity is useful because it results in relatively simple expressions for matching probabilities, as I will show in section 2.2. It can be motivated as follows. Firms can generally roughly determine the number of applicants that they will be able to interview by hiring more or fewer recruiters. However, the exact number of interviews may also depend on factors that could not be anticipated. For example, some recruiters may unexpectedly not be available, or the screening of a candidate takes longer or shorter than foreseen.

Matching. In order to understand the mechanism that governs matching, consider workers and firms as nodes in a bipartite network, connected by links representing interviews. Firms' preferences over workers are determined by their productivities x. I assume that workers do not learn their own productivity before matches have been formed. This guarantees that workers' preferences over firms only depend on the terms of trade w, which considerably simplifies the equilibrium analysis.[13] The matching is then assumed to be stable on this network in the sense that no firm matches with a worker with productivity $x \geq \underline{x}$ while one of its preferred interviewed applicants (i.e. with higher productivity) is hired by another firm at worse terms of trade w or remains unemployed. Otherwise, both the firm and the worker could do better by forming a match together. Stability can be motivated by a process in which firms offer their job sequentially to the candidates, and workers are free to reconsider their options.[14] Ties are broken randomly.

Production. In the last phase, production takes place and payoffs are realized. Unemployed workers receive a payoff U_0 from unemployment benefits and/or household production, while firms with vacancies obtain a zero payoff. A match with productivity x produces an output $U_0 + S(x)$, where $S(x)$ denotes the surplus created by the match. Surplus is continuous and strictly increasing in x.[15] To rule out trivial cases, I assume throughout that matches with sufficiently low x generate a negative surplus while matches with sufficiently high x generate a positive surplus, i.e. $S(0) < 0$ and $S(\overline{x}) > 0$. Given terms of trade w, the payoff of a matched worker equals $U_0 + wS(x)$, while the firm for which he works obtains $(1-w)S(x)$.

[13] As I discuss in more detail below, this assumption implies that the equilibrium has a convenient recursive structure, which is lost if workers know their x and may accept a job with a low w but a high x over a job with a high w but a low x. One can interpret the assumption as a convenient way to model the idea that workers starting a new job may know the expected value of the match, but generally face some uncertainty regarding its exact realization (in reality, for example because of uncertainty regarding match duration or wage growth).

[14] This *deferred acceptance* process, first described by Gale and Shapley (1962), converges in finite time for finite economies. Since the labor market described here contains a continuum of agents, I impose stability by assumption, following Kircher (2009).

[15] When analyzing the cyclicality of recruiting intensity in section 3 and 4, I will assume that a change in aggregate productivity causes a vertical shift in $S(x)$.

2.2 Strategies, Queue Lengths, and Payoffs

Firms' Strategies. Each firm faces a number of decisions: whether to enter the market or not, which contract to post, and to which applicant to make a job offer. The first and last decision are straightforward: entry will take place as long as the expected payoff of entry is positive and the firm will always try to hire the best applicant possible as long as his productivity x is high enough to create a positive surplus, i.e. $x \geq \underline{x}$, where $\underline{x}$ solves $S(\underline{x}) = 0$. The optimal contract is less trivial and will be derived below. Denote the equilibrium distribution of contracts by F and its support by $\mathcal{C}^F \subseteq \mathcal{C}$.

Workers' Strategies. Workers make three decisions: which search intensity to choose, to which contracts to apply, and which job offer to accept. Denote the equilibrium distribution of search intensity by $H(\alpha)$ and its support by $\mathcal{A}^H$. Each search intensity translates into a number of applications a. Conditional on this number, a worker solves a portfolio problem to determine to which firms he wishes to apply. By the anonymity assumption, his solution to this problem can be summarized by the a contracts to which he applies, i.e.,

$$\mathbf{c}_a = (c_{1:a}, \ldots, c_{a:a}) = ((r_{1:a}, w_{1:a}), \ldots, (r_{a:a}, w_{a:a})),$$

where, without loss of generality, $w_{1:a} \leq \ldots \leq w_{a:a}$. Although workers send their applications simultaneously, it will occasionally be convenient to refer to the application to $c_{i:a}$ as a worker's i-th application.[16] Conditional on a, let the distribution of workers' application strategies be denoted by G_a with support $\mathcal{C}_a^G$, such that $G_a(\widetilde{\mathbf{c}}_a)$ is the joint probability $\mathbb{P}[r_{1:a} \leq \widetilde{r}_{1:a}, w_{1:a} \leq \widetilde{w}_{1:a}, \ldots, r_{a:a} \leq \widetilde{r}_{a:a}, w_{a:a} \leq \widetilde{w}_{a:a}]$. Likewise, let $G_{i:a}$ represent the marginal distribution of a worker's i-th application with support $\mathcal{C}_{i:a}^G$.

Acceptance Probability. After receiving job offers, workers take the position that provides them with the best terms of trade; assume—again without loss of generality—that they accept the job offer from the application with the higher index in case of a tie. Let $\xi_{i:a}(\mathbf{c}_a)$ denote the probability that a worker with application portfolio $\mathbf{c}_a$ accepts a job offer from a firm posting $c_{i:a}$, conditional on receiving such a job offer. From the point of view of q firm posting c, the probability that its job offer gets accepted depends on the distribution of i and a among the applicants it attracts. Let $\overline{p}_{i:a}(c)$ be the conditional probability that an arbitrary applicant to the firm applied with his i-th out of a applications and let $\overline{G}_{-i:a}(\mathbf{c}_{-i:a}; c)$ the conditional distribution of his remaining applications $\mathbf{c}_{-i:a} = (c_{1:a}, \ldots, c_{i-1:a}, c_{i+1:a}, \ldots, c_{a:a})$. The firm's

[16] Numbering applications from the lowest wage to the highest wage is convenient, as it will turn out that workers who only send a few applications will send them to low wages.

offer will then be accepted with probability

$$\Xi(c) = \sum_{a=1}^{A} \sum_{i=1}^{a} \overline{p}_{i:a}(c) \int \xi_{i:a}(\mathbf{c}_a)\ d\overline{G}_{-i:a}(\mathbf{c}_{-i:a}; c). \tag{1}$$

Gross Queue Length. As well-known in the literature on *urn-ball* models, the number of applications that a firm receives follows a Poisson distribution. The endogenously determined mean of this distribution, which I call the *gross queue length* and denote by $\lambda(c)$, equals the ratio of the total number of applications sent to c to the total number of firms offering c. Formally, for $c = (r, w)$, $\lambda(c)$ is defined by

$$\int_{[1,r]\times[0,w]} \lambda(\widetilde{c})\ d[vF(\widetilde{c})] = s \int_{\mathcal{A}^H} \sum_{a=1}^{A} p_a(\alpha) \sum_{i=1}^{a} G_{i:a}(c)\ dH(\alpha)\ \forall c \in \mathcal{C}. \tag{2}$$

The right-hand side denotes the total mass of applications sent to contracts with terms of trade no higher than w and a recruiting intensity no higher than r. The left-hand side integrates queue lengths across all $vF(r, w)$ firms posting such contracts, yielding the total mass of applications received by these firms. Both masses need to be the same for each possible r and w.[17]

Net Queue Length. A firm does not only care about the number of applicants, but also about their productivity and whether they will accept a potential job offer. Given a gross queue λ, the number of applicants with productivity greater than $x > 0$ and without better offers follows a Poisson distribution as well.[18] Its mean $\mu(x; \lambda, c)$, the *net queue length,* satisfies

$$\mu(x; \lambda, c) = \lambda \Xi(c)\, q\,(1 - Q(x)) \le \lambda. \tag{3}$$

Effective Queue Length. A firm will hire a worker with productivity greater than $x \in [\underline{x}, \overline{x}]$ if and only if it is able to identify a worker from the corresponding net queue among its total number of qualified applicants. To derive an expression for the probability of this event, it is helpful to define a third queue length, the *effective queue length* $\kappa(x; \lambda, c)$, as the weighted average of $q\lambda$ and $\mu(x; \lambda, c)$, with $\frac{1}{r+1}$ and $\frac{r}{r+1}$ being the respective weights. That is,

$$\kappa(x; \lambda, c) = \frac{1}{r+1} q\lambda + \frac{r}{r+1} \mu(x; \lambda, c). \tag{4}$$

Job Offer and Hiring Probability. Despite the complexity of the model, the following proposition then establishes that simple expressions can be derived for two key endogenous variables, which greatly improves tractability. These variables are i) a firm's probability

[17] See Kircher (2009) for a detailed discussion of a similar expression.

[18] Lester et al. (2014) discuss this property of the Poisson distribution (which they call *invariance*) in detail.

$\eta(x;\lambda,c)$ of hiring a worker with productivity exceeding $x>0$; and ii) a worker's probability $\psi(x;\lambda,c)$ of receiving a job offer after drawing a match-specific productivity $x>0$. To be precise, $\eta(x;\lambda,c)$ depends on the firm's contract only through the net queue length and the effective queue length, while $\psi(x;\lambda,c)$ depends on the effective queue length alone, which greatly improves tractability.[19]

Proposition 1. *A firm with net queue length $\mu(x;\lambda,c)$ and effective queue length $\kappa(x;\lambda,c)$ hires a worker with productivity exceeding $x>0$ with probability*

$$\eta(x;\lambda,c)=\begin{cases}\eta(\underline{x};\lambda,c) & \text{for all } x<\underline{x}\\ \frac{\mu(x;\lambda,c)}{\kappa(x;\lambda,c)}\left(1-e^{-\kappa(x;\lambda,c)}\right) & \text{for all } x\geq\underline{x}.\end{cases}\tag{5}$$

The job offer probability $\psi(x;\lambda,c)$ of an applicant with productivity $x>0$ is determined by

$$\psi(x;\lambda,c)=-\frac{\partial}{\partial Q(x)}\frac{1-Q(x)}{\kappa(x;\lambda,c)}\left(1-e^{-\kappa(x;\lambda,c)}\right).\tag{6}$$

If $\kappa(x;\lambda,c)=0$, then, by convention, $\eta(x;\lambda,c)=0$ and $\psi(x;\lambda,c)=\begin{cases}0 & \text{for } x<\underline{x}\\ 1 & \text{for } x\geq\underline{x}.\end{cases}$

Limiting Cases. To build intuition, consider two limiting cases. First, if $r\to 0$, a firm can only interview one random qualified applicant and $\eta(x;\lambda,c)=\Xi(c)(1-Q(x))\left(1-e^{-q\lambda}\right)$. This is the product of (in reverse order) the probability that the firm has at least one qualified applicant, the probability that this applicant draws a productivity larger than x, and the probability that this applicant would accept a job offer. All workers with $x\geq\underline{x}$ are equally likely to get a job offer, hence $\psi(x;\lambda,c)=\frac{1}{q\lambda}\left(1-e^{-q\lambda}\right)$, resembling the simplest urn-ball models.

If $r\to\infty$, a firm can perfectly rank all its applicants. It forms a match with productivity exceeding $x>0$ as long as it attracts at least one applicant who has such productivity and who is willing to accept a job offer, i.e. $\eta(x;\lambda,c)=1-e^{-\mu(x;\lambda,c)}$. An applicant with productivity equal to $x\geq\underline{x}$ will get a job offer as long as no better applicants who would accept the firm's job offer are present. Hence, $\psi(x;\lambda,c)=e^{-\mu(x;\lambda,c)}$, which is strictly increasing in x and equals 1 for the highest productivity type $\overline{x}$.

Properties. The following lemma establishes some properties of $\eta(x;\lambda,c)$ which will be convenient in the equilibrium analysis. These properties imply that a firm is more likely to hire a good worker if it attracts a longer queue or if it chooses a higher recruiting intensity, that

[19] The proposition describes these probabilities conditional on queue lengths. Its scope therefore extends to models with ex ante heterogeneity in worker productivity (e.g. Shimer, 2005a) or models in which applications are sent randomly (e.g. Gautier and Moraga-Gonzalez, 2005; Gautier et al., 2014).

the marginal effect is diminishing, and that the queue and recruiting intensity are complements in hiring.

Lemma 1. *The hiring probability satisfies the following properties:*

1. $\eta(x;\lambda,c)$ *is strictly increasing in* λ *and* r*;*
2. $\eta(x;\lambda,c)$ *is strictly concave in* λ *and* r*, with* $\frac{\partial^2}{\partial\lambda\partial r}\eta(x;\lambda,c) > 0$*.*

Notation. The definitions (3) to (6) take the gross queue λ as exogenously given. While this will prove convenient for much of the analysis, the gross queue length is of course an equilibrium object which depends on a firm's contract c, as made explicit in (2). Hence, a firm's hiring probability equals $\eta(x;\lambda(c),c)$ in equilibrium. To keep the exposition as simple as possible, I will often slightly abuse notation and refer to this probability as $\eta(x;c)$. Likewise, I will write $\mu(x;c) \equiv \mu(x;\lambda(c),c)$, $\kappa(x;c) \equiv \kappa(x;\lambda(c),c)$ and $\psi(x;c) \equiv \psi(x;\lambda(c),c)$.

Firms' Payoffs. Given the hiring probability (5), the maximization problem of a firm can be written as $\max_c V(c)$, where $V(c)$ represents the value of creating a vacancy and posting a contract c. That is,[20]

$$V(c) = -(1-w)\int_{\underline{x}}^{\overline{x}} S(x)\; d\eta(x;c) - k_V - k_R r. \tag{7}$$

Free entry implies that firms will continue to enter as long as the expected payoff is strictly positive. Hence, in equilibrium, $\max_{c\in\mathcal{C}} V(c) \le 0$ and $v \ge 0$, with complementary slackness.

Workers' Payoffs. Workers' payoffs can divided into two parts. First, given a realization of a number of applications a, an application portfolio $\mathbf{c}_a$, and a productivity draw $x > 0$, a worker will end up in a position offering terms of trade $w_{i:a}$ with probability $\xi_{i:a}(\mathbf{c}_a)\,\psi(x;c_{i:a})$. He chooses $\mathbf{c}_a$ to maximize his expected payoff $U_a^* = \max_{\mathbf{c}_a} U_a(\mathbf{c}_a)$, where[21]

$$U_a(\mathbf{c}_a) = U_0 + \sum_{i=1}^{a} \xi_{i:a}(\mathbf{c}_a)\, w_{i:a}\mathbb{E}\left[\psi(x;c_{i:a})\, S(x)\right]. \tag{8}$$

Anticipating this, the worker chooses his search intensity α to maximize his expected payoff net of search costs, i.e. his value of search, $U^* = \max_\alpha U(\alpha)$, where

$$U(\alpha) = p_1(\alpha)\, U_1^* + p_2(\alpha)\, U_2^* - k_A\alpha. \tag{9}$$

[20]The minus sign in front of the integral appears because $\eta(x;c)$ represents the probability mass *above* x.
[21]Recall that $\psi(x;c) = 0$ for all $x < \underline{x}$.

Recursive Formulation. A worker's acceptance probability must satisfy

$$\xi_{i:a}\left(\mathbf{c}_a\right)=\prod_{j=i+1}^{a}\left(1-\mathbb{E}\left[\psi\left(x;c_{j:a}\right)\right]\right), \tag{10}$$

where $\mathbb{E}\left[\cdot\right]$ denotes the expectation over all possible values of x.[22] Substitution of this expression in (8) reveals that—as in Galenianos and Kircher (2009) and Kircher (2009)—we can write workers' payoff in a recursive way. That is,

$$U_i^*=\max_c \mathbb{E}\left[\psi\left(x;c\right)\left(U_0+wS\left(x\right)\right)+\left(1-\psi\left(x;c\right)\right)U_{i-1}^*\right], \tag{11}$$

where $U_0^*=U_0$. This formulation implies that a worker's payoff from his i-th application does not depend on the total number of applications that he sends, which considerably simplifies the equilibrium analysis. Going forward, it will often be convenient to interpret U_{i-1}^* as the outside option corresponding to a worker's i-th application.

3 Equilibrium

3.1 Equilibrium Definition

Out-of-Equilibrium Beliefs. Before defining the equilibrium, note that firms must form beliefs regarding the acceptance probability $\Xi\left(c\right)$ and the gross queue length $\lambda\left(c\right)$ for every $c\in\mathcal{C}$, as they must be able to determine their expected payoff for any feasible contract. While equation (1) and (2) can be used to determine $\Xi\left(c\right)$ and $\lambda\left(c\right)$ for all contracts posted in equilibrium, firms' beliefs off the equilibrium path require extra care. I follow the solution of Galenianos and Kircher (2009)—who discuss this issue in detail—by imposing that $\Xi\left(c\right)=\lim_{\varepsilon\to 0}\Xi_\varepsilon\left(c\right)$ and $\lambda\left(c\right)=\lim_{\varepsilon\to 0}\lambda_\varepsilon\left(c\right)$, where $\Xi_\varepsilon\left(c\right)$ and $\lambda_\varepsilon\left(c\right)$ are the acceptance probability and gross queue in a perturbed economy in which a fraction ε of firms posts a random contract $c\in\mathcal{C}$, according to some distribution $\widetilde{F}\left(c\right)$ with full support. In this economy, workers face a distribution of contracts $F_\varepsilon\left(c\right)=\left(1-\varepsilon\right)F\left(c\right)+\varepsilon\widetilde{F}\left(c\right)$ and will choose application portfolios and search intensities according to $\{G_{1,\varepsilon}\ldots,G_{A,\varepsilon}\}$ and H_ε, respectively. Given these distributions, $\Xi_\varepsilon\left(c\right)$ and $\lambda_\varepsilon\left(c\right)$ can be calculated from (the updated versions of) (1) and (2) in a straightforward manner. It follows from the results in Galenianos and Kircher (2009) that the exact form of $\widetilde{F}\left(c\right)$ does not matter as long as it has full support.

Equilibrium Definition. I now define an equilibrium as follows.

Definition 1. An equilibrium is a measure of vacancies v, a distribution of contracts F, application distributions $\{G_1,\ldots G_A\}$, a search intensity distribution H, a gross queue lengths

[22] Hence, $\mathbb{E}\left[\psi\left(x;c_{j:a}\right)\right]=\left(1-q\right)\psi\left(0;c_{j:a}\right)+q\int_0^{\overline{x}}\psi\left(x;c_{j:a}\right)\,dQ\left(x\right)=q\int_0^{\overline{x}}\psi\left(x;c_{j:a}\right)\,dQ\left(x\right)$.

$\lambda(c)$, an acceptance probability $\Xi(c)$, and market utilities $\{U_1^*, \ldots, U_A^*\}$, such that:

1. Optimal Contracts: $V(c) = V^* \equiv \max_{\widetilde{c} \in \mathcal{C}} V(\widetilde{c})$, $\forall c \in \mathcal{C}^F$;
2. Optimal Entry: $V^* = 0$ if $v > 0$ and $V^* \leq 0$ if $v = 0$;
3. Optimal Applications: $U_a(\mathbf{c}_a) = U_a^* \equiv \max_{\widetilde{\mathbf{c}}_a} U_a(\widetilde{\mathbf{c}}_a)$, $\forall \mathbf{c}_a \in \mathcal{C}_a^G$ $\forall a \in \{1, \ldots, A\}$;
4. Optimal Search Intensity: $U(\alpha) = U^* = \max_{\widetilde{\alpha}} U(\widetilde{\alpha})$, $\forall \alpha \in \mathcal{C}^H$;
5. Off-the-Equilibrium Beliefs: $\Xi(c) = \lim_{\varepsilon \to 0} \Xi_\varepsilon(c)$ and $\lambda(c) = \lim_{\varepsilon \to 0} \lambda_\varepsilon(c)$;
6. Consistency: equations (1) to (11) hold.

3.2 One Application

I first analyze the model under the assumption that all workers send one application, i.e. $A = 1$ and $p_1(\alpha) = 1$ for all α. This assumption, which is pervasive in the literature on directed search, simplifies the equilibrium derivation considerably, since it implies that all job offers will be accepted with probability $\Xi(c) = \xi_{i:a}(\mathbf{c}_a) = 1$ and that workers will trivially choose a search intensity $\alpha = 0$.

Existence and Uniqueness. Proposition 2 establishes existence and uniqueness of the equilibrium. The derivation of these results is relatively standard and is therefore relegated to the proof in the appendix. In short, one can rewrite the firm's optimization problem in terms of the queue length λ and recruiting intensity r. By virtue of lemma 1, the resulting objective function is strictly concave in these two variables, which, in combination with the free entry condition, implies a unique solution $(\lambda_1^*, r_1^*, w_1^*)$. The measure of firms v^* subsequently follows from equation (2).

Proposition 2. *For $A = 1$, a unique equilibrium exists. In this equilibrium, all $v^* > 0$ firms that enter the market post the same equilibrium contract (r_1^*, w_1^*), each attracting a gross queue $\lambda_1^* = \frac{s}{v^*}$.*

As common in directed search models with one application and ex ante homogeneous agents, all firms and workers participate in the same submarket in equilibrium and they share the surplus, i.e. $0 < w_1^* < 1$. Unless the recruiting cost is very large, the heterogeneity in productivity provides an important incentive for firms to choose $r_1^* > 0$ and interview multiple workers (in expectation). As a result, the distribution of productivity in realized matches first-order stochastically dominates $Q(x)$.

Productivity Shock. Next, consider the effect of an increase in aggregate productivity, which is modeled as a parallel upward shift of the surplus function $S(x)$.[23] Proposition 3 establishes that this productivity shock has two opposing effects on recruiting intensity r_1^*, a positive direct effect for a given λ_1^* and a negative indirect effect through a change in λ_1^*. After all, for a given λ_1^*, the upward shift of the surplus function increases firms' payoff from a match and lowers the match-specific productivity cutoff $\underline{x}$ below which a firm does not want to hire. This increases the potential gains from interviewing, causing firms to choose a higher recruiting intensity. At the same time, the decrease in $\underline{x}$ makes entry more attractive, which lowers the equilibrium queue length λ_1^* for a given r_1^*. As established in lemma 1, λ and r are complements in hiring, so the shorter queues provide firms with an incentive to lower their recruiting intensity, making the total effect ambiguous.

Proposition 3. *For $A = 1$, the direct effect of an increase in aggregate productivity on r_1^* is positive and the indirect effect is negative.*

3.3 Multiple Applications

While the assumption of a single application per worker was convenient for deriving the results in the previous subsection, it simultaneously limits their scope. Allowing for multiple applications per worker is not only empirically relevant, as argued in the introduction, but also important from a theoretical point of view. For example, when workers can send multiple applications, their choice of search intensity will be an important determinant of how queue lengths respond to an increase in aggregate productivity. Likewise, the potential rejection of job offers determines how attractive it is for firms to increase their recruiting intensity in response to the productivity shock. In this subsection, I therefore re-introduce the possibility for workers to send more than one application, assuming throughout that the search cost k_A is sufficiently small such that they indeed choose $\alpha > 0$.[24]

Contract Dispersion. Simultaneous search complicates the equilibrium analysis considerably, as it makes the acceptance probability $\Xi(c)$ an endogenous variable. Fortunately, one can build on the insights provided by Galenianos and Kircher (2009) and Kircher (2009). They prove for related environments—but without choices of recruiting intensity or productivity differences among workers—that equilibrium must have a very specific structure. In particular, if workers send at most A applications, A different wages emerge in equilibrium, $w_1^* < w_2^* < \ldots < w_A^*$, and workers apply at most once to each wage. The following lemma extends these results to the environment of this paper.

[23] An increase in aggregate productivity may also change the relative gains from hiring a good versus a bad worker. I leave this extension for future research, as little evidence is available on the degree of complementarity / substitutability between match productivity and aggregate productivity.

[24] Alternatively, one could assume that $p_1'(\alpha) \to -\infty$.

Lemma 2. *For $A > 1$, any equilibrium consists of A different contracts, $(c_1^*, \ldots, c_A^*)$ satisfying $w_1^* < \ldots < w_A^*$. Workers apply at most once to each contract.*

To understand lemma 2, suppose all workers send exactly A applications. These applications differ along two dimensions. First, the applications differ in their outside options, $U_0^* < \ldots < U_{A-1}^*$. Second, the probability with which a worker accepts a job offer differs across the applications, $\xi_{1:A}^* < \ldots < \xi_{A:A}^*$. Which application a worker uses to apply to a particular firm is private information. However, if recruiting is costly, firms have incentive to try to induce revelation of this information, as the likelihood of rejection of job offers affects their entry decisions. The difference in the applications' outside option makes this feasible, as it implies that the worker is willing to accept a larger decline in matching probability in exchange for a higher wage while sending his j-th application than while sending his i-th application, for all $j > i$.[25] Hence, firms design the wages such that perfect separation of a worker's applications arises in equilibrium.

Existence and Uniqueness. The results of Galenianos and Kircher (2009) and Kircher (2009) regarding existence and uniqueness of the equilibrium do not carry over equally easily. The reason is somewhat technical. The requirement of perfect separation gives rise to a large number of incentive compatibility (IC) constraints; a worker must not strictly prefer to send application i to contract c_j^* for $j \neq i$. Of these constraints, the most restrictive one is for $j = i + 1$. That is, if a worker weakly prefers to send his i-th application to c_i^* rather than c_{i+1}^*, for all $i \in \{1, \ldots, A-1\}$, then incentive compatibility holds for other j as well.

In equilibrium, the IC constraint between c_i^* and c_{i+1}^* will hold. However, it depends on the fine details of the environment whether this IC constraint binds or not. Galenianos and Kircher (2009) show that it binds for all $i \in \{1, \ldots, A-1\}$ in their model with $r_i^* = r_{i+1}^* = 0$. In contrast, Kircher (2009) demonstrates that it is not binding for $r_i^*, r_{i+1}^* \to \infty$. For $0 < r_i^*, r_{i+1}^* < \infty$, as is generically the case in my model, one cannot establish analytically which of the two cases applies, which impedes equilibrium characterization.[26]

Proposition 4 therefore focuses on the case $A = 2$, for which these complications are manageable as there is effectively only one IC constraint,

$$U_1^* \geq U_1^{dev} \equiv U_0 + w_2^* \mathbb{E}\left[\psi\left(x; c_2^*\right) S\left(x\right)\right], \tag{12}$$

where U_1^{dev} denotes the payoff from sending the first application to c_2^*. The proposition establishes that while existence is always guaranteed, uniqueness—in this environment in which some workers apply once while others apply twice—crucially depends on whether (12) binds

[25] That is, the outside option affects the worker's marginal rate of substitution between wage and matching probability.

[26] In principle, the IC constraint may bind for some but not all $i \in \{1, \ldots, A-1\}$.

or not.[27]

Proposition 4. *For $A = 2$, an equilibrium exists. The equilibrium is unique when $U_1^* > U_1^{dev}$, while a continuum of equilibria arises when $U_1^* = U_1^{dev}$. In an equilibrium, all workers choose the same search intensity α^* and all $v^* > 0$ firms entering the market post one of two contracts c_1^* and c_2^*, satisfying $w_1^* < w_2^*$. Workers who apply twice send one application to each type of contract. Workers applying once send their application to c_1^* with probability 1 if $U_1^* > U_1^{dev}$ and with probability γ if $U_1^* = U_1^{dev}$, with each equilibrium corresponding to a different value of γ.*

Multiplicity. Key for understanding proposition 4 is to realize that heterogeneity in the number of applications breaks the perfect correlation between applications' outside options and acceptance probabilities.[28] There now exist three different types of applications: i) the applications of workers who only apply once; ii) the first application of workers who apply twice; and iii) the second application of workers who apply twice. Denote these types by 1:1, 1:2, and 2:2, respectively. Workers who send only one application have an outside option U_0^* (as is the case for the *first* application of workers who apply twice), but will accept a job offer with probability $\xi_{1:1}^* = 1$ (as is the case for the *second* application of workers who apply twice). The equivalence in outside option implies that the applications of type 1:1 cannot be separated from applications of type 1:2. This means that IC constraint (12), which separates applications of type 1:2 from applications of type 2:2, plays a crucial role. If this constraint is slack, workers who send one application must send it to c_1^*. However, if the constraint binds, a worker applying once is exactly indifferent between c_1^* and c_2^* and therefore happy to randomize between them with probability γ and $1 - \gamma$, respectively.[29]

Both cases are potentially relevant. By continuity, it follows from the results in Kircher (2009) that the IC constraint is slack and the equilibrium is unique for sufficiently small k_R. Similarly, the results in Galenianos and Kircher (2009) imply that the constraint binds and a continuum of equilibria arises when k_R is sufficiently large and the degree of heterogeneity in the number of applications is sufficiently small. Although establishing the relevant case for arbitrary k_R is not feasible analytically, it is straightforward to do so numerically. If a multiplicity occurs, the free entry condition guarantees that firms receive the same expected payoff (i.e., zero) in each equilibrium. All other equilibrium outcomes—including worker's search intensities and payoffs as well as firms' recruiting intensities—will however vary across the equilibria.

[27] Galenianos and Kircher (2005) conjecture on the basis of numerical simulations that "there may be multiplicity of equilibria" if some workers send N applications and other workers send $N + 1$ applications, but do not provide a characterization.

[28] In that sense, the source of the multiplicity resembles Chang (2014) and Guerrieri and Shimer (2014), who study two-dimensional private information in—otherwise very different—models of asset markets.

[29] The assumption of symmetric strategies implies that all workers use the same γ in a particular equilibrium.

This means that, in addition to productivity shocks, shocks to the equilibrium selection rule can be a second source of fluctuations in recruiting intensity and unemployment in this case. For example, a jump from $\gamma = 1$ to $\gamma = 0.5$ will cause fewer workers applying once to send their application to c_1^*. This lowers the acceptance probability ($\xi^*_{1:2} < \xi^*_{1:1} = 1$), reducing firm entry into this submarket, which through the IC constraints has an effect on the outcomes in all other submarkets.

More Applications. Although extending proposition 4 to arbitrary values of A is difficult, the insight that a multiplicity can arise is robust to such an extension. For example, if the IC constraint binds between c_i^* and c_{i+1}^* for all $i \in \{1, \ldots, A-1\}$, then a worker sending $a \in \{1, \ldots, A-1\}$ applications is indifferent between the application portfolios $\{c_1^*, \ldots, c_a^*\}$ and $\left\{c_2^*, \ldots, c_{a+1}^*\right\}$. Numerical calculation of the equilibrium for arbitrary A is straightforward, as I demonstrate below.

Shocks. The complexity of the equilibrium hinders a clear characterization of the implications of shocks to aggregate productivity and—if a multiplicity arises—the equilibrium selection rule. For aggregate productivity, the effects described in section 3.2 are, ceteris paribus, still present in each of the submarkets. However, any statement regarding the response of firms' recruiting intensity should not only take into account aggregation across submarkets, but also the fact that the productivity shock will change workers' search intensities, their acceptance probabilities and the set of incentive compatibility constraints, all of which affect firms' choices. The same holds for a jump to a different equilibrium. For this reason, I will address the effects of these shocks by calibrating the model in the next section.

4 Calibration

In this section, I demonstrate how the model can be calibrated.[30] The calibration takes place in two steps. First, some parameters are set exogenously, after which the remaining parameters are chosen to match aggregate data from the Current Population Survey (CPS) and the Employment Opportunities Pilot Projects (EOPP) 1982 data set, which—to my knowledge—is the only source of detailed recruiting information for a sample of US firms. Subsequently, I describe some of the equilibrium outcomes. After establishing that the model performs well, I analyze the effect of a shock to aggregate productivity as well as a shock to the equilibrium selection rule, which causes agents to play a different equilibrium.

A caveat is in order: since the EOPP data mostly covers firms hiring for low-skilled jobs between January 1980 and September 1981, the results and conclusions of the calibration

[30] A previous draft of this paper estimated a version of the model with binary heterogeneity using Maximum Likelihood. The estimates for key parameters (e.g., search and recruiting costs) were similar to the ones presented in this section. See Wolthoff (2012) for details.

exercise may not automatically apply to other samples. For this reason, I keep the exposition brief and refer to appendix B for additional details regarding the data and the calibration.[31]

4.1 Exogenous Parametrization

Confrontation of the model with the data requires that the model is extended beyond a single period, endogenizing the number of searchers s, the value of unemployment U_0 and the surplus function $S(x)$. A full analytical characterization of equilibrium in such an environment is beyond the scope of this paper[32], but can mostly be circumvented for the purpose of the calibration by targeting s, U_0 (or $\underline{x}$), and $S(x)$ directly, as I describe below.

Period Length, Discounting and Retirement. I focus on the steady state in an infinite-horizon, discrete-time labor market in which agents play the game described in section 2 in every period. In line with most of the literature (e.g. Menzio and Shi, 2011), I exogenously fix the period length to one month.[33] Both firms and workers discount future payoffs at a factor $\beta = 0.96^{1/12}$ per period. Firms are infinitely-lived, but an exogenous fraction τ of the workers retires at the end of each period. I assume $\tau = 0.0021$, which corresponds to an average career of 40 years. Retired workers obtain zero future payoff and are replaced by new unmatched workers in order to keep the size of the labor force equal to 1.[34]

Job Destruction, Matching and Searchers. Besides retirement, job destruction is a second source of separations. At the end of each period, an exogenous fraction $\delta \in (0, 1)$ of the matches gets dissolved. Employed workers hit by the shock become unmatched. In the next period, they can—like the u unemployed, i.e. workers who failed to find a job during the current period—search for a new job, which they find with endogenous probability Ψ. Hence, a worker who is employed in the current production phase may be employed at a different firm in the production phase of the next period after being unmatched for a very short amount of time. Discreteness of the data may cause this to look like a job-to-job transition, even though the model does not allow for on-the-job search in the classic sense.[35] In appendix B,

[31] By performing one aggregate calibration, I implicitly assume that all workers are identical or that labor markets at different skill levels are scale replicas of each other (see Wolthoff, 2012, for such an implementation). The EOPP data set is not rich enough to relax this assumption, but fundamental parameters may of course vary considerably with skill; analyzing this seems a promising avenue for future research.

[32] Galenianos and Kircher (2005) provide a characterization of equilibrium for an infinite-horizon version of their (arguably simpler) model and discuss the analytical complexities that arise.

[33] In theory, the data contains some information on the period length, since one application per week gives rise to a different equilibrium than two applications per two weeks. However, identification of the period length along these dimensions cannot be established easily and would be very indirect at best. See Wolthoff (2014) for a model in which the period length is a parameter.

[34] This feature of the model and the 'job-to-job' transitions below are important as workers' search intensity will be inferred from the total number of workers searching and the total number of applications sent.

[35] See Menzio and Shi (2011) for regular on-the-job search in a directed search model with one application per period. The combination of simultaneous search and on-the-job search is computationally challenging.

I demonstrate how data from the Current Population Survey can be used to calibrate Ψ and δ. This yields $\Psi = 0.413$ and $\delta = 0.063$.[36] These values—combined with the steady state assumption—imply that a mass $s = 0.144$ of workers applies to jobs in each search phase.

Application Distribution. For the number of applications, I assume a (truncated) Poisson distribution with search intensity α as its parameter. To be precise, the fraction of workers $p_i(\alpha)$ that sends i applications satisfies:[37]

$$p_i(\alpha) = \begin{cases} e^{-\alpha} \frac{\alpha^{i-1}}{(i-1)!} & \text{for } i \in \{1, \ldots, A-1\} \\ 1 - \sum_{j=1}^{A-1} p_j(\alpha) & \text{for } i = A. \end{cases}$$

A Poisson distribution is a natural choice, since it is the discrete-time equivalent of the Poisson process typically used in continuous-time models. I set $A = 15$, which will prove not to be restrictive given the equilibrium value of α.

Productivity and Surplus. As I discuss below, the fraction of qualified applicants q is important for the calibration of the recruiting costs k_R. Unfortunately however, no good evidence on the value of q is available. I will therefore present results for two scenarios, one in which all applicants are qualified ($q = 1$) and one in which half of all applicants are qualified ($q = 0.5$).[38]

The distribution $Q(x)$ is assumed to be log-normal with scale parameter normalized to 0 and shape parameter σ, so median productivity is 1. Periodical match output equals the sum of match productivity x and aggregate productivity y. In the calibration, y will be normalized to 0, but when analyzing the effect of an increase in aggregate productivity, y will take the value 0.01. Given this setup, the surplus $S(x)$ of a match satisfies

$$U_0 + S(x) = x + y + \beta(1-\tau)\left[(1-\delta)(U_0 + S(x)) + \delta U^*\right],$$

which, using $S(\underline{x}) = 0$, yields

$$S(x) = \frac{x - \underline{x}}{1 - \beta(1-\tau)(1-\delta)}.$$

[36]Note that δ cannot directly be compared to the estimates for the 'employment-exit probability' by Shimer (2012). First, δ only describes a transition from employment to the pool of workers who will search for a job. Unemployment only follows in case that search fails. Second, Shimer (2012) needs to control for time aggregation because he uses a continuous-time model, whereas here the period length in the model corresponds to the frequency of the data (1 month). Finally, I control for the age structure by calculating δ for seven different age cohorts and averaging over them using the corresponding sample fractions.

[37]Recall that workers can send one application for free.

[38]For the EOPP data, q is bounded below by the fact that firms on average interview 5 out of 14 applicants.

Home Production. Unemployed workers are assumed to receive a periodical payoff h from home production. At the beginning of the next period, they can search for a job, which provides them with a value U^*. Hence, the value of unemployment U_0 equals $h + \beta U^*$.

Equilibrium Selection. Given the above assumptions, the insights from section 3 continue to hold. Hence, $A = 15$ different contracts will be offered in equilibrium and a multiplicity may arise when the incentive compatibility constraint between any of them binds. I assume that if this is the case, the agents select the equilibrium that arises for $\gamma = 1$.[39]

4.2 Matching Moments

Remaining Parameters. After making the above assumptions, five parameters (k_V, k_R, k_A, h, and σ) remain for which no direct evidence regarding their value is available. I calibrate these parameters in a two-step procedure The key to understanding this procedure is to realize that the equilibrium depends on k_A and h only through α and $\underline{x}$ and that the calculation of the former two parameters from the latter two is easier than the other way around.[40] For this reason, I calibrate $(k_V, k_R, \alpha, \underline{x}, \sigma)$ and calculate k_A and h afterwards.

Moments. The parameters are chosen to simultaneously match the following five data moments regarding recruiting and matching: 1) a monthly matching probability of 0.413 for a worker; 2) a monthly hiring probability of 0.936 for a firm; 3) an average of 14.06 applicants per firm, conditional on hiring; 4) an average of 4.97 interviews per firm, conditional on hiring; and 5) an average of 6.16 hours spent on recruiting, conditional on hiring. As explained above, the first moment follows from the CPS. The remaining four moments can be calculated from the EOPP data, as I describe in the appendix.

Calibrated Values. Despite the high degree of non-linearity, the five parameters can match the five moments perfectly. This fact is not obvious and adds credibility to the model with multiple applications. After all, a model with $A = 1$ cannot simultaneously match (a subset of) these moments, since 14 applicants or 5 interviews per firm imply upper bounds on workers' matching probability equal to 0.07 and 0.20, respectively. The solution of the calibration is displayed in table 1, for each of the two values of q.

Discussion. A few values warrant discussion. First, the estimate for the recruiting cost k_R is increasing in q, while the shape parameter σ is decreasing. This is in line with what one would

[39] That is, when workers are indifferent between two application portfolios, they apply to the one with the lower wages.

[40] The parameters k_A and h need to be calculated only once, while the parameters α and $\underline{x}$ need to be calculated in each iteration.

Table 1: Calibrated Parameter Values

	$q = 0.5$	$q = 1$
Entry cost k_V	0.139	0.122
Recruiting cost k_R	0.005	0.009
Search intensity α	5.162	5.150
Cutoff type $\underline{x}$	0.989	0.992
Shape parameter σ	0.010	0.006
Search cost k_A	0.006	0.002
Home production h	0.973	0.987

expect. Firms interview on average only 5 out of 14 applicants, which can be either because many applicants are not qualified for the job and therefore dismissed without an interview, or because the recruiting cost is high relative to the variance in match productivity.

Second, the estimated search intensity α is almost completely insensitive to the value of q. This is because the number of searchers and the number of vacancies are fully determined by the targeted data moments and because the (unconditional) number of applicants per firm must be close to 14.[41] Given a virtually constant search intensity, the implied search cost k_A is naturally increasing in the degree of heterogeneity in match quality.

Finally, the value of home production is relatively high compared to Shimer (2005b) and Hall and Milgrom (2008) but, accounting for the fact that average match productivity in my model is larger than 1, close to the estimate by Hagedorn and Manovskii (2008). This should not be a surprise. After all, Hagedorn and Manovskii (2008) rely on the same information in their calibration: time spent on recruiting as reported in the EOPP data. The fact that this number is small implies that match surplus must be small, or otherwise, in terms of my model, firms would be willing to spend more on recruiting and interview more workers to further increase their matching probability. Assuming that many applicants are unqualified—i.e. the alternative rationale for the relatively small number of interviews—lowers the estimate of h, but only slightly, since that fails to explain why firms do not simply increase their queue lengths by posting more attractive terms of trade.[42]

4.3 Equilibrium Outcomes

Given the above values for the exogenous parameters, the equilibrium can be calculated. This yields a large number of empirical predictions, some of which are displayed in table 2. It turns out that for the calibrated parameter values, the IC constraint between c_i^* and c_{i+1}^* binds for all $i \in \{1, \dots, A-1\}$, such that many equilibria exist. In line with the calibration, the

[41] The calibration forces the average number of applicants per firm conditional on hiring to be 14.06. The relation between the unconditional and the conditional number of applicants is endogenous, but severely constrained by the fact that firms' matching probability is close to 1.

[42] The same logic explains why the dispersion in productivity is not larger.

Table 2: Equilibrium Outcomes

	$q = 0.5$	$q = 1$
Aggregate variables		
Unemployment	0.085	0.085
Number of vacancies	0.064	0.064
Search variables		
Search intensity	5.16	5.15
Avg. number of applications	6.16	6.15
Avg. search time (hours)	5.24	1.59
Recruiting variables		
Avg. number of applicants	13.96	13.94
Avg. recruiting intensity	7.49	4.19
Avg. number of interviews	4.75	4.74
Avg. number of offers	1.18	1.17
Avg. recruiting time (hours)	6.15	6.15

equilibrium outcomes that I present here are for $\gamma = 1$. I analyze the effects of a shock to γ in section 4.4.

Aggregate Variables. The unemployment rate implied by the model is $u = 0.085$, which is close to the one observed in the data (0.082), taking into account the age structure of the sample. Hence, the steady assumption is reasonable despite the recessive state of the US economy in 1980 and 1981. A fraction $(\delta + \tau - \delta\tau)\,\Psi = 0.027$ of the employed workers moves from one job to the next without intermittent unemployment spell. This value is consistent with estimates of monthly job-to-job transition rates by Fallick and Fleischman (2004), Nagypal (2008) and Moscarini and Thomsson (2007), which range from 2.2% to 3.2%. The measure of vacancies is 0.064, which is somewhat higher than estimates by Davis et al. (2013) for the relevant time period.[43] This difference seems to merely reflect that my definition of a vacancy is wider than theirs, since any match in the model requires a preceding vacancy.

Search Variables. Workers choose a search intensity α which allows them to send slightly more than 6 applications on average. If we assume that a year consists of 50 weeks of 5 days of 8 productive hours, the time unemployed workers spend on search can be calculated as $\frac{2000}{12}\frac{\alpha k_A}{h}$, which yields 5.24 hours (of foregone home production) for $q = 0.5$ and 1.59 hours for $q = 1$. No estimates of this number are available in the literature for the workers in my sample, but the values I find are broadly in line with the conclusion of Krueger and Mueller

[43] Roughy 3%, based on an approach which combines the Conference Board's Help-Wanted Index with the Job Openings and Labor Turnover Survey (JOLTS).

(2008) that unemployed workers do not dedicate a large of amount of time to search. Dividing the total search time by the number of applications implies that the average time cost of an application equals 0.85 hours for $q = 0.5$ and 0.26 hours for $q = 1$.

Recruiting Variables. Firms receive on average almost 14 applications and choose a recruiting intensity which implies an average of almost 5 interviews. Not surprisingly, both these numbers are (slightly) smaller than the equivalent numbers conditional on hiring, used as targets in the calibration, since firms with more applicants or interviews are (slightly) more likely to hire. Dividing recruiting time by the number of interviews implies that the average time cost of an interview equals 1.30 hours of production. It is worth emphasizing that the average number of job offers a firm makes (almost 1.2) is consistent with the EOPP data (see appendix), even though this moment was not targeted in the calibration.

The first row of figure 1 plots the gross queue λ, the recruiting intensity r and the expected number of interviews as a function of firms' terms of trade. In equilibrium, the terms of trade vary between $w_1^* = 0.29$ and $w_{15}^* = 0.45$ for $q = 0.5$ and between $w_1^* = 0.09$ and $w_{15}^* = 0.24$ for $q = 1$. Firms posting better terms of trade receive more applications, which—ceteris paribus—causes them to choose a higher recruiting intensity.[44] However, firms' gains from recruiting also depend on the acceptance probability, which makes the overall relation between terms of trade and recruiting intensity (or the number of interviews) potentially non-monotonic. Quantitatively, the variation in recruiting intensity or the number of interviews is relatively small across the contracts, which the average firm always interviewing between 3.9 and 5.5 workers.

Matching. The second row of figure 1 displays firms' hiring density $-\frac{\partial}{\partial x}\eta\left(x;c\right)$ for each of the 15 equilibrium contracts. A few things stand out. First, firms that offer better terms of trade hire with larger probability, as indicated by the fact that the mass under the graph of $-\frac{\partial}{\partial x}\eta\left(x;c_i^*\right)$ is increasing in i. Second, these firms also end up hiring better workers in the sense of first-order stochastic dominance. For example, for $q = 0.5$, a firm posting c_1^* matches with probability 0.88, while a firm posting c_{15}^* hires with probability 0.98. Their average hires are respectively 0.76 and 1.10 standard deviations better than a random qualified applicant.[45]

The last row of figure 1 shows workers' job offer probability $\psi\left(x;c\right)$. The graph confirms that applications to better terms of trade are less likely to result in a job offer than applications to worse terms of trade. More importantly, the graph reveals that the difference across terms of trade is relatively small compared to the variation caused by productivity differences. In

[44] Preliminary work by Faberman and Menzio (2010) suggested a negative relationship between wages and applications in the EOPP data, but as discussed in Marinescu and Wolthoff (2012), the absence of detailed occupational information makes omitted variable bias in this result likely.

[45] The corresponding numbers for $q = 1$ are 0.72 and 0.94 standard deviations, respectively.

other words, the main determinant of whether an application to a position is successful is how good of a fit the worker is for the position.

4.4 Counterfactual Experiments

After establishing that the calibrated model yields reasonable outcomes, I now perform two counterfactual experiments. First, I analyze the effects of an increase aggregate productivity. Subsequently, I study the model's response to a shock to the equilibrium selection rule, causing the agents to play a different equilibrium. The results of these exercises are displayed in table 3, along the outcomes of the original equilibrium to facilitate comparison.

Productivity Shock. The first counterfactual exercise is an increase in aggregate productivity y from 0 to 0.01, such that the median applicant becomes 1% more productive. The new steady state outcomes for $q = 0.5$ and $q = 1$ are displayed in column II of 3. Both values of q give qualitatively similar results, although the magnitude of all effects is larger for $q = 1$.

As can be seen from the table, the productivity shock causes unemployment to fall and the number of vacancies to rise. The change in unemployment is large relative to e.g. Shimer (2005b). Two factors contribute to this. First, the relatively high value of h creates amplification, as pointed out by Hagedorn and Manovskii (2008). Second, in line with Davis (2011), workers' search intensity goes up in response to the productivity shock, magnifying the effect on workers' matching probability.

Firms' recruiting intensity goes up as well. Both the direct and the indirect effect described in section 3 contribute to this, since the average firm receives not fewer but more applications after the productivity shock. This increase in the gross queue, despite a decline in unemployment and an increase in entry of vacancies, is again due to the increase in workers' search intensity.

Equilibrium Shock. The second experiment is a drop in the value of γ to 0.5, causing the agents to play a different equilibrium. The lower value of γ implies that workers who are indifferent between two application portfolios $(c_1^*, \ldots, c_a^*)$ and $\left(c_2, \ldots, c_{a+1}^*\right)$ apply more frequently to the latter than before. Ceteris paribus, this lowers Ξ_1, worsening the frictions in the submarket with the lowest wage, which through the binding IC constraints affects all other submarkets in a non-trivial way. Column III of table 3 presents the results.[46] In response to the shock, workers marginally reduce their search intensity, while firms slightly increase their recruiting intensity. On balance, unemployment rises by 0.4 to 1.0 percentage points. This means that if one believes recruiting intensity to be mostly procyclical, as documented by Davis et al. (2013), then the model suggests that aggregate productivity shocks are a more

[46]Other values of γ provide qualitatively similar results. The magnitude of the effects is decreasing in γ.

Figure 1: Equilibrium Outcomes

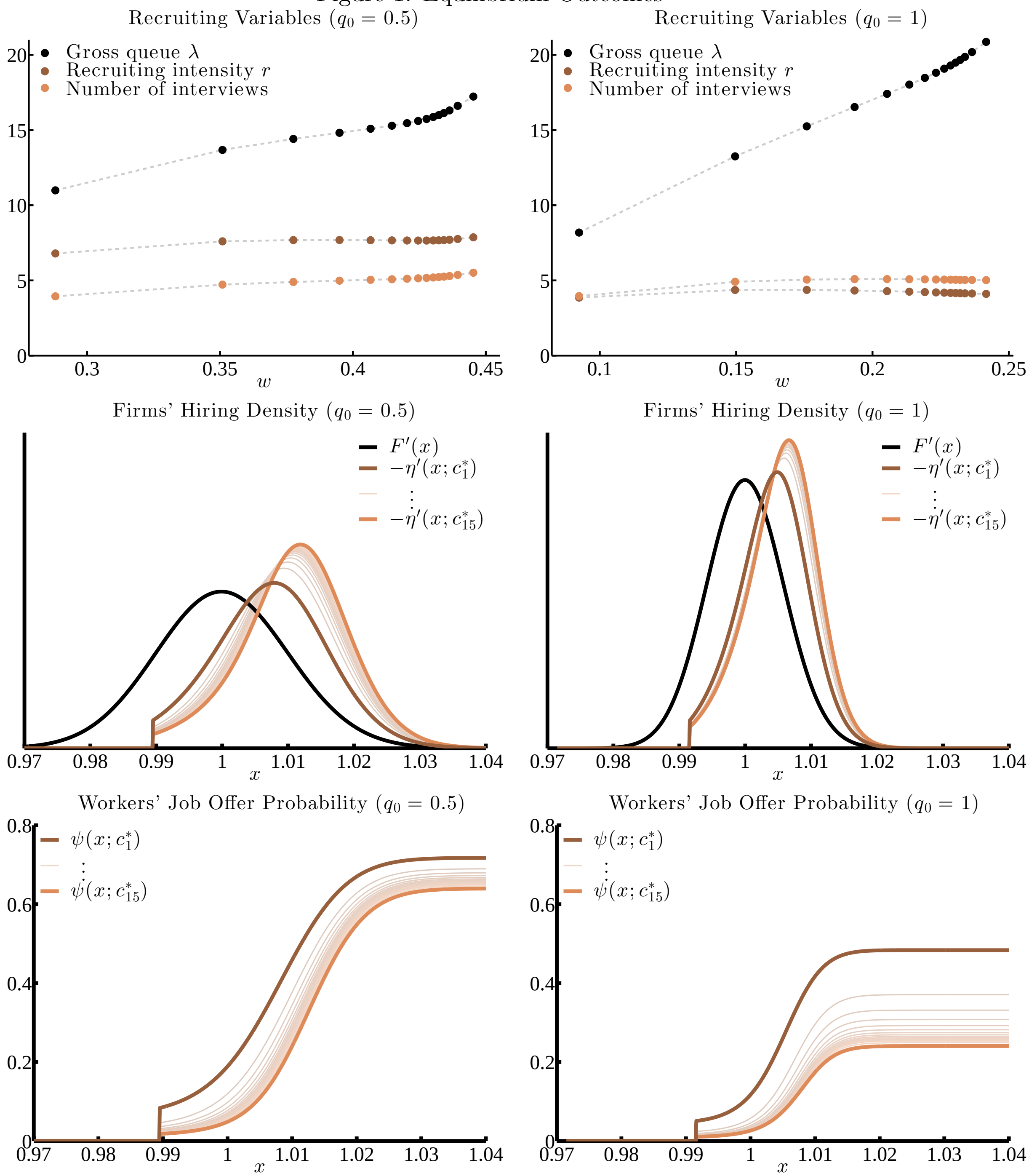

Table 3: Calibration Results—Equilibrium and Comparative Statics
Baseline: $\gamma = 1$ and $y = 0$; Prod. Shock: $\gamma = 1$ and $y = 0.01$; Eqm. Shock: $\gamma = 0.5$ and $y = 0$.

	I (Baseline)		II (Prod. Shock)		III (Eqm. Shock)	
	$q = 0.5$	$q = 1$	$q = 0.5$	$q = 1$	$q = 0.5$	$q = 1$
Aggregate variables						
Unemployment	0.085	0.085	0.061	0.047	0.089	0.095
Number of vacancies	0.064	0.064	0.065	0.067	0.063	0.063
Worker variables						
Search intensity	5.16	5.15	7.06	9.89	5.13	5.07
Avg. number of applications	6.16	6.15	8.05	10.72	6.13	6.07
Avg. search time (hours)	5.24	1.59	7.16	3.05	5.21	1.57
Matching probability	0.413	0.413	0.502	0.567	0.400	0.382
Firm variables						
Avg. number of applicants	13.96	13.94	14.98	17.42	14.45	14.93
Avg. recruiting intensity	7.49	4.19	7.94	4.67	7.61	4.26
Avg. number of interviews	4.75	4.74	5.06	5.28	4.88	4.88
Avg. number of offers	1.18	1.17	1.27	1.31	1.18	1.16
Avg. recruiting time (hours)	6.15	6.15	6.53	6.87	6.26	6.26
Hiring probability	0.936	0.936	0.934	0.922	0.941	0.941

important source of fluctuations in recruiting intensity than shocks to the equilibrium selection rule.

5 Discussion

This section discusses several of the key assumptions, along with a few potentially interesting extensions.

5.1 Observability of Recruiting Intensity

The assumption that workers can observe the recruiting intensity r of each firm (in addition to the terms of trade w) before choosing where to apply played an important role in deriving the results of this paper. Since firms typically do not advertise recruiting intensities in real life, one may wonder whether this assumption is too strong. However, note that workers do not care about a firm's recruiting intensity r per se. Instead, they care about the probability ψ of getting a job offer. It seems reasonable to say that in real life job seekers possess some information about this probability, even before making application decisions. What matters in terms of the model is whether the information that the worker observes is sufficient for him to deduce ψ. Assuming that he observes r (and w) is an analytically convenient way to guarantee that this is the case, but is not crucial. That is, there exists other pieces of information which the worker can observe instead, such that the results remain unchanged. For example, the following lemma establishes that this is the case for the expected productivity $x^{\mathbb{E}}$ of the worker that the firm will hire, i.e.,

$$x^{\mathbb{E}} = \frac{-\int_{\underline{x}}^{\overline{x}} x \, d\eta\left(x;c\right)}{\eta\left(\underline{x};c\right)}.$$

Proposition 5. *The results in this paper remain remain unchanged if workers observe $x^{\mathbb{E}}$ (and w) instead of r (and w) for each firm.*

The idea that firms communicate the type of worker that they expect to hire should not seem unrealistic. Casual observation of job ads on CareerBuilder.com suggests that the vast majority of them contains statements on the characteristics that a successful applicant is expected to have.[47]

5.2 Constrained Efficiency

A traditional concern in much of the search literature is whether the market equilibrium is constrained efficient. To answer this question, consider the problem of a social planner whose objective is to maximize net social surplus, subject to the frictions of the environment. The planner's problem can be broken down into multiple components. First, the planner determines entry of firms and search intensity for each workers. Second, he allocates the applications to the firms and chooses a recruiting intensity for each firm. Finally, he specifies a matching rule for agents to follow after the interviews have taken place. Proposition 6 analyzes these decisions for $A = 1$ and establishes that the equilibrium described in proposition 2 decentralizes the planner's solution.

Proposition 6. *For $A = 1$, the equilibrium is constrained efficient.*

Efficiency in a directed search model with one application per worker is not a novel result in itself. For example, Moen (1997), Shimer (2005a) and Menzio and Shi (2011) show how the planner's solution can be decentralized for a variety of settings. The intuition is the same in each of these models: since workers' equilibrium payoffs must equal the market utility, a firm is the residual claimant on any additional surplus it creates, creating an incentive to post a contract implementing the efficient allocation to maximize this surplus. Hence, the crucial question is often not whether efficient contracts exist, but rather what they look like and whether the contract space is flexible enough to allow them. The existing literature is silent on this topic in the context of choices of recruiting intensity.[48] The efficiency result of

[47] See Marinescu and Wolthoff (2012) for a detailed analysis of job ads on CareerBuilder.com.

[48] The random search literature has analyzed recruiting intensity more often, but only obtains efficiency under a very specific functional form in which the equilibrium recruiting intensity exclusively depends on the associated cost and is independent of worker behavior or any aggregate variable (see the discussion in Pissarides, 2000).

proposition 6 is new in that regard.[49]

With $1 < A < \infty$, it immediately follows from the results in Kircher (2009) that the equilibrium is efficient if and only if firms' hiring probability depends on the gross queue λ only through the net queue $\mu(\cdot)$. Inspection of proposition 1 reveals that this requirement is equivalent to $r \to \infty$ for all equilibrium contracts, which is the case if and only if $k_R = 0$. The intuition for this result is the following.[50] A firm is only interested in applicants that will end up accepting their job offers, the queue of which is described by $\mu(\cdot)$. Applicants that reject the firm's job offer may crowd out these more desirable candidates, unless the firm interviews *all* qualified applicants ($r \to \infty$). This crowding out is a negative externality which needs to be priced in order for constrained efficiency to prevail. However, since applicants that turn down the job offer never match with the firm, a wage payment is not able to fulfill this role. Instead, as also pointed out Lester et al. (2014) in a different but related context, firms would need to extend their contracts with application fees, to be paid by all applicants.[51]

5.3 Ex Ante Heterogeneity

An important assumption in the model is that workers are ex ante homogeneous in terms of their productivity. This simplifies the analysis by avoiding that workers' productivity affects their search decisions. To see this, note that the literature has analyzed ex ante heterogeneity only for two special cases. These cases are 1) firms randomly select among their applicants, who each applied once (corresponding to $A = 1$ and $k_R \to \infty$) and 2) firms perfectly rank all their applicants, who each applied once ($A = 1$ and $k_R \to 0$). In the first case, a separating equilibrium arises; workers will sort themselves across different submarkets based on their productivity to avoid the crowding out of the most productive workers (see e.g. Shi, 2001). In contrast, crowding out cannot occur in the second case, such that firms have an incentive to attract all types of workers, leading to a pooling equilibrium (see e.g. Shimer, 2005a; Shi, 2006).

Work in progress (Lester and Wolthoff, 2014) suggests that the equilibrium is not quite as tractable for intermediate values of k_R, even when maintaining $A = 1$. When firms can rank some but not all applicants, equilibrium must feature partial pooling and partial separation across a potentially large number of submarkets; types that are sufficiently close will be pooled, but very low types must be separated from very high types in order to prevent too much crowding out. Allowing for simultaneous search adds another layer of complexity, as it is not obvious whether workers who pool with one application will also pool with other applications.

[49] See also Lester et al. (2013, 2014) for a detailed discussion of efficiency in directed search models and for intuition that the contracts that I consider remain optimal if we give firms access to a larger set of mechanisms. In particular, firms cannot improve their payoffs by using applications fees or subsidies when $A = 1$.

[50] See Kircher (2009) for a detailed discussion.

[51] Note that when the firm's hiring probability depends on λ only through $\mu(\cdot)$, the distribution of the number of interviews satisfies the *invariance* property of Lester et al. (2014).

Ex ante heterogeneity is therefore left for future research.

6 Conclusion

I analyze firms' recruiting intensity in a model in which firms compete for workers in a frictional labor market. In particular, I focus on firms' decision how many of their applicants to subject to a job interview. An interview is valuable because it reveals the productivity of the applicant. Hence, interviewing more workers allows the firm to rank more applicants and form a better match, but a higher cost, since each interview takes time or resources away from production. I characterize the equilibrium and show that the model's predictions regarding firms' recruiting intensity crucially depend on parameter values. For example, equilibrium recruiting intensity may increase or decrease in response to a shock to aggregate productivity. Further, it may be subject to non-fundamental shocks because the equilibrium is not uniquely determined for a subset of the parameter space.

To shed some first light on these effects, I demonstrate how the model can be calibrated to aggregate data. The results of this exercise indicate that the equilibrium is not unique. Given selection of a particular equilibrium, recruiting intensity moves counter to unemployment in response to a productivity shock, but with unemployment in response to a shock to the equilibrium selection rule. These results suggest that procyclical movements in recruiting intensity, as documented for the US labor market by Davis et al. (2013), can better be explained with changes in aggregate productivity than with shocks to the equilibrium selection rule. Of course, a more conclusive answer requires both more detailed data regarding firms' recruiting decisions in recent years and a richer model.[52] This seems a promising avenue for future research.

[52] For example, Davis et al. (2013) document large heterogeneity in the number of vacancies and the hiring probabilities across firms. See Kaas and Kircher (2013) for a model of these observations.

A Proofs

A.1 Proof of Proposition 1

Consider a firm with recruiting intensity $r \geq 0$ and a queue $q\lambda > 0$ of qualified applicants, and suppose that a qualified applicant has a certain characteristic with probability ϕ.[53] We will first derive an expression for the probability that the firm interviews at least 1 applicant with the characteristic and then use that to prove the lemma. To simplify notation, define $\ell = q\lambda$ and $\rho = \frac{r}{r+1}$.

Number of Interviews. Given a number of qualified applicants $n_Q \sim \text{Poi}(\ell)$ and an interview capacity $n_R \sim \text{Geo}(\rho)$, the actual number of interviews equals $n_I = \min\{n_Q, n_R\}$. This equals 0 with probability $\mathbb{P}[\min\{n_Q, n_R\} = 0] = e^{-\ell}$ and $n \in \mathbb{N}_1$ with probability

$$\begin{aligned}
\mathbb{P}[\min\{n_Q, n_R\} = n] &= \sum_{i=n+1}^{\infty} e^{-\ell}\frac{\ell^i}{i!}\rho^{n-1}\frac{1}{r+1} + e^{-\ell}\frac{\ell^n}{n!}\sum_{i=n}^{\infty}\rho^{i-1}\frac{1}{r+1} \\
&= \rho^{n-1}\left[e^{-\ell}\frac{\ell^n}{n!} + \frac{1}{r+1}\sum_{i=n+1}^{\infty} e^{-\ell}\frac{\ell^i}{i!}\right].
\end{aligned}$$

Expectation. Conditional on n interviews, the probability to meet at least 1 applicant with the characteristic is $1 - (1-\phi)^n$. Taking the expectation over n yields

$$\begin{aligned}
\sum_{n=0}^{\infty}[1-(1-\phi)^n]\,\mathbb{P}[\min\{n_Q, n_R\} = n] &= \sum_{n=1}^{\infty}\mathbb{P}[\min\{n_Q, n_R\} = n] \\
&\quad - \sum_{n=1}^{\infty}(1-\phi)^n\rho^{n-1}e^{-\ell}\frac{\ell^n}{n!} \\
&\quad - \frac{1}{r+1}\sum_{n=1}^{\infty}(1-\phi)^n\rho^{n-1}\sum_{i=n+1}^{\infty}e^{-\ell}\frac{\ell^i}{i!}.
\end{aligned}$$

Consider each of the three terms on the right-hand side separately. The first term simplifies to $1 - e^{-\ell}$. The second term equals

$$\sum_{n=1}^{\infty}(1-\phi)^n\rho^{n-1}e^{-\ell}\frac{\ell^n}{n!} = \frac{1}{\rho}\left(e^{-\ell(1-\rho+\rho\phi)} - e^{-\ell}\right).$$

[53] In this proof, I omit arguments as much as possible to keep notation simple.

Finally, apply a change in the order of summation in the third term to get

$$\frac{1}{r+1}\sum_{n=1}^{\infty}(1-\phi)^{n}\rho^{n-1}\sum_{i=n+1}^{\infty}e^{-\ell}\frac{\ell^{i}}{i!}=\frac{1}{r}\left[\frac{\rho(1-\phi)-e^{-\ell(1-\rho+\rho\phi)}}{1-\rho+\rho\phi}+e^{-\ell}\right].$$

Combining the three terms gives, after some simplification,

$$\frac{\phi}{1-\rho+\rho\phi}\left(1-e^{-\ell(1-\rho+\rho\phi)}\right). \tag{13}$$

Hiring Probability. Consider now the original problem. To hire a worker with productivity greater than x, satisfying $x \geq \underline{x}$, a firm needs to interview at least one such applicant who, in addition, must accept the job offer. That is, $\phi = \Xi(c)(1-Q(x)) = \frac{\mu(x;\lambda,c)}{q\lambda}$. Substituting this into (13) yields (5). Workers with $x < \underline{x}$ will never be hired, hence $\eta(x;\lambda,c) = \eta(\underline{x};\lambda,c)$ for those productivity draws.

Job Offer Probability. The job offer probability $\psi(x;\lambda,c)$ must satisfy the accounting equation $\psi(x;\lambda,c)\ d\mu(x;\lambda,c) = d\eta(x;\lambda,c)$. Substitution of (3) yields the desired expression. □

A.2 Proof of Lemma 1

Monotonicity of the Hiring Probability. A few straightforward calculations yield

$$\frac{\partial}{\partial\lambda}\eta(x;\lambda,c)=\frac{\mu(x;\lambda,c)\,e^{-\kappa(x;\lambda,c)}}{\lambda}>0$$

and

$$\frac{\partial}{\partial r}\eta(x;\lambda,c)=\frac{(\lambda q-\mu(x;\lambda,c))\,\mu(x;\lambda,c)\left(1-e^{-\kappa(x;\lambda,c)}-\kappa(x;\lambda,c)\,e^{-\kappa(x;\lambda,c)}\right)}{(r+1)^{2}\left[\kappa(x;\lambda,c)\right]^{2}}>0.$$

Hence, $\eta(x;\lambda,c)$ is strictly increasing in λ and r.

Concavity of the Hiring Probability. In a similar fashion, one obtains

$$\frac{\partial^{2}}{\partial\lambda^{2}}\eta(x;\lambda,c)=-\frac{\mu(x;\lambda,c)\,\kappa(x;\lambda,c)}{\lambda^{2}}e^{-\kappa(x;\lambda,c)}<0$$

and

$$\frac{\partial^{2}}{\partial\lambda\partial r}\eta(x;\lambda,c)=\frac{(\lambda q-\mu(x;\lambda,c))\,\mu(x;\lambda,c)\,e^{-\kappa(x;\lambda,c)}}{\lambda(r+1)^{2}}>0,$$

while the determinant of the Hessian of $\eta(x;\lambda,c)$ with respect to λ and r equals

$$2\left(\frac{\mu(x;\lambda,c)}{r+1}\right)^3 \frac{\lambda q-\mu(x;\lambda,c)}{\lambda^2\left[\kappa(x;\lambda,c)\right]^2} e^{-\kappa(x;\lambda,c)}\left(1-e^{-\kappa(x;\lambda,c)}-\kappa(x;\lambda,c)\,e^{-\kappa(x;\lambda,c)}\right).$$

This determinant is strictly positive, so $\eta(x;\lambda,c)$ is strictly concave in λ and r with a positive cross-partial. □

A.3 Proof of Proposition 2

Simplification. As $\Xi(c)=1$ for all c, expression (3) for the net queue length simplifies to $\lambda(c)\,q\,(1-Q(x))$. That is, $\mu(x;\lambda,c)$ depends on the terms of trade w only through the queue length $\lambda(c)$. The same is then true for $\eta(x;\lambda,c)$ and $\psi(x;\lambda,c)$. I will slightly abuse notation to reflect that by writing $\eta(x;\lambda,r)$ and $\psi(x;\lambda,r)$ throughout this proof.

Constraint and Payoffs. Firms posting a contract c expect to attract a queue $\lambda(c)$, implicitly determined by

$$U_1^* = U_0 + w\mathbb{E}\left[\psi(x;\lambda(c),r)\,S(x)\right]. \tag{14}$$

Solving this constraint for w yields

$$w = \frac{U_1^*-U_0}{\mathbb{E}\left[\psi(x;\lambda(c),r)\,S(x)\right]}. \tag{15}$$

Substituting this expression into the firm's optimization problem yields, after some simplification

$$V(c) = -\int_{\underline{x}}^{\overline{x}} S(x)\;d\eta(x;\lambda(c),r) - \lambda(c)\left(U_1^*-U_0\right) - k_V - k_R r. \tag{16}$$

As is common in directed search models, this payoff only depends on w through the queue length $\lambda(c)$. Hence, we can analyze the firm's optimization problem in terms of λ and r, instead of w and r.

First Order Conditions. Integrating (16) by parts yields

$$V(c) = \int_{\underline{x}}^{\overline{x}} \eta(x;\lambda(c),r)\;dS(x) - \lambda(c)\left(U_1^*-U_0\right) - k_V - k_R r.$$

By lemma 1, this is a strictly concave function of λ and r. Combined with the fact that the firm's payoff tends to $-\infty$ for $\lambda\to\infty$ or $r\to\infty$, strict concavity implies that a unique solution to the firm's optimization problem exists for any U_1^*. Using subscripts to indicate partial derivatives, this solution must satisfy the FOC with respect to λ,

$$\int_{\underline{x}}^{\overline{x}} \eta_\lambda (x; \lambda, r) \; dS(x) = U_1^* - U_0, \tag{17}$$

and either the the boundary point $r = 0$ or the FOC with respect to r,

$$\int_{\underline{x}}^{\overline{x}} \eta_r (x; \lambda, r) \; dS(x) - k_R = 0. \tag{18}$$

Optimal Queue and Recruiting Intensity. Equilibrium requires in addition that U_1^* and the chosen solution are consistent with the free entry condition, which after substitution of (17) simplifies to

$$\int_{\underline{x}}^{\overline{x}} \left[\eta (x; \lambda, r) - \lambda \eta_\lambda (x; \lambda, r)\right] \; dS(x) - k_V - k_R r = 0. \tag{19}$$

To see that there exists a unique combination of λ_1^* and r_1^* that solves these conditions, distinguish two cases, $r_1^* = 0$ and $r_1^* > 0$.

First, consider $r_1^* = 0$. In that case, the derivative of (19) with respect to λ equals

$$-\lambda \int_{\underline{x}}^{\overline{x}} \eta_{\lambda\lambda} (x; \lambda, 1) \; dS(x) > 0.$$

Hence, the Intermediate Value Theorem implies that there exists a unique solution λ_1^*. Equations (15) subsequently imply r_1^* and w_1^*, respectively.

Second, consider $r_1^* > 0$. In this case, (18) holds. Its solution $\widehat{r}(\lambda)$ varies with λ. By the Implicit Function Theorem, we get

$$\frac{\partial \widehat{r}(\lambda)}{\partial \lambda} = -\frac{\int_{\underline{x}}^{\overline{x}} \eta_{\lambda r} (x; \lambda, \widehat{r}(\lambda)) \; dS(x)}{\int_{\underline{x}}^{\overline{x}} \eta_{rr} (x; \lambda, \widehat{r}(\lambda)) \; dS(x)} > 0. \tag{20}$$

The derivative of (19) with respect to λ therefore equals

$$\int_{\underline{x}}^{\overline{x}} \left[-\lambda \eta_{\lambda\lambda} (x; \lambda, \widehat{r}(\lambda)) + \eta_r (x; \lambda, \widehat{r}(\lambda)) \frac{\partial \widehat{r}(\lambda)}{\partial \lambda} - \lambda \eta_{\lambda r} (x; \lambda, \widehat{r}(\lambda)) \frac{\partial \widehat{r}(\lambda)}{\partial \lambda}\right] dS(x) - k_R \frac{\partial \widehat{r}(\lambda)}{\partial \lambda},$$

which after substitution of (18) and (20) simplifies to

$$-\lambda \int_{\underline{x}}^{\overline{x}} \eta_{\lambda\lambda} (x; \lambda, \widehat{r}(\lambda)) \; dS(x) + \lambda \frac{\left[\int_{\underline{x}}^{\overline{x}} \eta_{\lambda r} (x; \lambda, \widehat{r}(\lambda)) \; dS(x)\right]^2}{\int_{\underline{x}}^{\overline{x}} \eta_{rr} (x; \lambda, \widehat{r}(\lambda)) \; dS(x)}.$$

This expression is again strictly positive by lemma 1. Hence, the Intermediate Value Theorem implies that there exists a unique solution λ^*. Equations (18) subsequently implies the equilibrium recruiting intensity $r_1^* = \widehat{r}(\lambda_1^*)$.

Terms of Trade and Entry. Given λ_1^* and r_1^*, the equilibrium terms of trade w_1^* follow directly from equation (15). The measure v^* of firms entering the market is determined by $v^* = \frac{s}{\lambda_1^*}$. $\square$

A.4 Proof of Proposition 3

To formalize the idea that the increase in aggregate productivity y causes a parallel upward shift in the surplus function, define—with a slight abuse of notation—$S(x) = \widetilde{S}(x) + y$. For a given λ_1^*, applying the Implicit Function Theorem to equation (18) then yields

$$\frac{\partial r_1^*}{\partial y} = -\frac{\eta_r\left(\underline{x};\lambda_1^*, r_1^*\right)}{\int_{\underline{x}}^{\overline{x}} \eta_{rr}\left(x;\lambda_1^*, r_1^*\right)\, dS(x)} > 0,$$

since $\frac{\partial S(\underline{x})}{\partial x}\frac{\partial \underline{x}}{\partial y} = -1$. Likewise, for a given r_1^*, equation (19) implies

$$\frac{\partial \lambda_1^*}{\partial y} = \frac{\left[\eta\left(\underline{x};\lambda_1^*, r_1^*\right) - \lambda_1^*\eta_\lambda\left(\underline{x};\lambda_1^*, r_1^*\right)\right]}{\lambda_1^*\int_{\underline{x}}^{\overline{x}} \eta_{\lambda\lambda}\left(x;\lambda_1^*, r_1^*\right)\, dS(x)} < 0.$$

A.5 Proof of Lemma 2

Ordering of Queue Lengths. I start by showing that the indifference curves of applicants pin down queue lengths for each contract. Given U_i^* and acceptance probabilities $\Xi(c)$, define $\lambda_i(c) \geq 0$ as the gross queue length that solves

$$U_i^* \geq \mathbb{E}\left[\psi(x;c)\left(U_0 + wS(x)\right) + \left(1-\psi(x;c)\right)U_{i-1}^*\right], \tag{21}$$

with complementary slackness. As workers' job offer probabilities are strictly decreasing in λ, a unique solution exists for each i and c. Denote the corresponding effective queue length by $\kappa_i(x;c)$. As applicants are willing to apply to a firm until (21) binds, a firm posting a contract c expects to attract a gross queue $\lambda(c) = \max_i\{\lambda_i(c)\}$. Likewise, define $\kappa(x;c) = \max_i\{\kappa_i(x;c)\}$.[54]

Single Crossing. Next, consider how the queue lengths vary with w. While $\Xi(c)$ and $\lambda_i(c)$ do not need to be differentiable with respect to w, $\kappa_i(x;c)$ must be, because the right-hand side of (21) depends on endogenous variables only through this variable. Implicit differentiation of

[54]Note that the ordering of $\kappa_i(x;c)$, $i \in \{1,\ldots,A\}$, is independent of x as $\lambda(c)$ does not depend on x.

(11) reveals that $\kappa_i(x;c)$ is increasing in w.[55] Moreover, the slope is increasing in i, because U^*_{i-1} is increasing in i. This implies a single-crossing condition: if $\kappa_i(x;c) = \kappa_{i+1}(x;c)$ for some x and $c = (r, w)$, then $\kappa_{i+1}(x;\widetilde{c}) > \kappa_i(x;\widetilde{c})$ for all $\widetilde{c} = (r, \widetilde{w})$ satisfying $\widetilde{w} > w$.

Application Regions. We can then define subsets $\mathcal{C}_i$ of the contract space, consisting of the contracts for which $\kappa_i(x;c)$ is strictly larger than all other $\kappa_j(x;c)$, i.e.

$$\mathcal{C}_i = \left\{ c \in \mathcal{C} | \kappa_i(x;c) > \max_{j \neq i} \{\kappa_j(x;c)\} \right\}.$$

These subsets have a number of useful properties. First, by construction, if there exist contracts in $\mathcal{C}_i$ which are part of an equilibrium, they can only receive the i-th application of workers. Second, since a worker's payoff only depends on r through κ, each $\mathcal{C}_i$ can be written as $\mathcal{R} \times \mathcal{W}_i$, where $\mathcal{W}_i$ is an appropriately defined subset of $\mathcal{W}$. Third, the single-crossing condition implies that each $\mathcal{W}_i$ is a open interval, which can be denoted by $(\overline{w}_{i-1}, \overline{w}_i)$, where $\overline{w}_0$ is the lowest wage that would receive applications, $\overline{w}_i$ is the highest wage that provides workers applying i times with a payoff U^*_i, and $\overline{w}_A = 1$.[56]

Application Behavior. By construction, a worker must now send his i-th application to contracts in the closure of $\mathcal{C}_i$, i.e. $\mathrm{cl}(\mathcal{C}_i) = \mathcal{R} \times [\overline{w}_{i-1}, \overline{w}_i]$. After all, a contract $\widetilde{c} = (\widetilde{r}, \widetilde{w})$ that offers $\widetilde{w} > \overline{w}_i$ or $\widetilde{w} < \overline{w}_{i-1}$ attracts an effective queue $\kappa(x;\widetilde{c}) > \kappa_i(x;\widetilde{c})$. Such a contract would provide the worker with an expected payoff that is strictly lower than U^*_i, which violates the equilibrium conditions.

Acceptance Probability. Firms and workers need to form beliefs regarding the probability $\Xi(c)$ that a job offer from a firm posting c will get accepted. While it is straightforward to derive this probability for the equilibrium contracts, its value off the equilibrium path requires a more careful analysis. Galenianos and Kircher (2009) analyze this issue in detail and prove formally using the trembles in firms' behavior described in section 3.1 that the acceptance probability is constant within each $\mathcal{W}_i$. Although their model does not include a choice of recruiting intensity, their proof trivially extends to all contracts in $\mathcal{C}_i = \mathcal{R} \times \mathcal{W}_i$, as acceptance decisions are independent of the recruiting intensity.

A more informal—but perhaps more intuitive—argument is the following. Any contract $c \in \mathcal{C}_i$, whether part of an equilibrium or not, can expect to attract a queue $\lambda_i(c)$ of applicants who will all be sending their i-th application. Hence, each of these contracts provides applicants with the same payoff U^*_i. Conditional on U^*_i, a worker's decision where to send application

[55]Contracts for which $\kappa(x;c) = 0$ can never be part of an equilibrium and will therefore be ignored in the remainder of the proof.

[56]For notational convenience, I do not include $w = 1$ in the subsets. This is without loss of generality as $w = 1$ can never be part of an equilibrium.

$j > i$ does not depend on the exact contract $c \in \mathcal{C}_i$ to which he sent his i-th application. Hence, all contracts $c \in \mathcal{C}_i$ face the same acceptance probability $\Xi(c) = \Xi_i$.

Further, if a contract $(r, \overline{w}_i)$ is part of an equilibrium, then $\Xi(r, \overline{w}_i)$ must equal Ξ_{i+1}, or else a deviation to $\widetilde{c} = (r, \overline{w}_i + \varepsilon)$ would be profitable by providing the firm a discretely higher hiring probability. To see this, note that a firm's hiring probability can be written as

$$\eta(x; \lambda, c) = \frac{\Xi(c)(1 - Q(x))}{\frac{1}{r} + \frac{r-1}{r}\Xi(c)(1 - Q(x))}\left(1 - e^{-\kappa(x;\lambda,c)}\right).$$

Since $\kappa(x; c)$ is continuous in $\overline{w}_i$, right-continuity of $\eta(x; \lambda, c)$ in $\overline{w}_i$ requires right-continuity of $\Xi(c)$ in this point.

Optimization Problem. The above results imply that equilibrium will give rise to at least one contract in each $\underline{\mathcal{C}}_i \equiv \mathcal{R} \times [\overline{w}_{i-1}, \overline{w}_i)$. We can write the optimization problem of a firm posting a contract in $\underline{\mathcal{C}}_i$ as

$$\begin{aligned} \max\nolimits_{c \in \underline{\mathcal{C}}_i} \quad & -(1-w)\int_{\underline{x}}^{\overline{x}} S(x)\, d\eta(x; \lambda_i(c), r) - k_V - k_R r \\ \text{s.t.} \quad & U_i^* = \mathbb{E}\left[\psi(x; \lambda_i(c), r)(U_0 + wS(x)) + (1 - \psi(x; \lambda_i(c), r))U_{i-1}^*\right]. \end{aligned} \tag{22}$$

Solving the constraint for w yields

$$w = \frac{\mathbb{E}\left[\Delta_i^* + \psi(x; \lambda_i(c), r)\sum_{j=1}^{i-1}\Delta_j^*\right]}{\mathbb{E}\left[\psi(x; \lambda_i(c), r)S(x)\right]}, \tag{23}$$

where Δ_i^* represents the marginal payoff of the i-th application, i.e. $\Delta_i^* \equiv U_i^* - U_{i-1}^*$. Substituting this into the objective (22) and applying (6) yields

$$\max_{c \in \underline{\mathcal{C}}_i} - \int_{\underline{x}}^{\overline{x}} \left(S(x) - \sum_{j=1}^{i-1}\Delta_j^*\right) d\eta(x; \lambda_i(c), r) - \lambda_i(c)\Xi_i\Delta_i^* - k_V - k_R r. \tag{24}$$

Properties. Expression (24) resembles (16) in two important ways. First, it only depends on w through the queue length $\lambda(c)$, such that we can again analyze the firm's optimization problem in terms of λ and r, instead of w and r. Second, by lemma 1, it is strictly concave in λ and r, implying that exactly one contract will arise in each $\underline{\mathcal{C}}_i$ in equilibrium. □

Proof of Proposition 4

High-Wage Contract. In order to derive the optimal contract in $\underline{\mathcal{C}}_2$, it is useful to distinguish two cases, one in which the IC constraint binds ($w_2^* = \overline{w}_1$) and one in which it does not ($w_2^* > \overline{w}_1$). I will consider these cases in reverse order.

Interior Solution. First, consider the case in which $w_2^* > \overline{w}_1$. The optimal contract $c_2^* \in \underline{\mathcal{C}}_2$ can then be derived in a way analogous to the proof of proposition 2, except for the fact that the solution will depend on the endogenous variable Δ_1. That is, given Δ_1, the optimal λ_2^* and r_2^* are uniquely determined by the first order conditions of (24) with respect to λ and r , subject to the restriction $r_1^* \geq 0$, and the free entry condition that (24) should equal zero. The terms of trade w_2^* follow from (23). A higher value of Δ_1 diminishes entry as well as recruiting intensity in $\underline{\mathcal{C}}_2$, lowering Δ_2^*.

Boundary Solution. Next, consider the case in which $w_2^* = \overline{w}_1$. The optimal contract $c_2^* \in \underline{\mathcal{C}}_2$ is determined—for a given Δ_1—by the boundary constraint $U_1^* = U_1(c_2^*)$, the first order condition of (24) with respect to r, subject to the restriction $r_1^* \geq 0$, and the free entry condition that (24) should equal zero. To see this, note that $U_1^* = U_1(c_2^*)$ implies

$$w_2^* = \frac{\Delta_1}{\mathbb{E}\left[\psi\left(x; \lambda_2^*, c_2^*\right) S\left(x\right)\right]}$$

and $\Delta_2^* = \left(1 - \mathbb{E}\left[\psi_2\left(x; \lambda\left(c\right), r\right)\right]\right) \Delta_1$. Substitution of Δ_2^* into (24) yields

$$-\int_{\underline{x}}^{\overline{x}} S\left(x\right)\, d\eta_2\left(x; \lambda, r\right) - \lambda\Delta_1 - k_V - k_R r = 0.$$

This expression is strictly decreasing in λ, and together with the first order condition of (24) with respect to r, it implies a unique solution (r_2^*, λ_2^*) for a given Δ_1.

Low-Wage Contract. The optimal contract in $\underline{\mathcal{C}}_1$ is always interior and can be derived in a similar fashion as the corresponding solution for c_2^*, except that it depends on the endogenous variable Ξ_1 instead of Δ_1. That is, given Ξ_1, the solution of a firm posting a contract in $\underline{\mathcal{C}}_1$ is uniquely determined by the first order conditions of (24) with respect to λ and r, subject to the restriction $r_1^* \geq 0$, and the free entry condition that (24) should equal zero. A higher value of Ξ_1 stimulates entry and increases Δ_1. Hence, we can write $\Delta_1 = \widehat{\Delta}\left(\Xi_1\right)$ with $\widehat{\Delta}'\left(\Xi_1\right) > 0$. Note that $\widehat{\Delta}\left(0\right) = 0$ and that $\widehat{\Delta}\left(1\right)$ equals some intermediate value.

Acceptance Probability. The acceptance probability for firms posting contracts in $\underline{\mathcal{C}}_1$ crucially depends on whether the interior or the boundary solution for c_2^* emerges. If $w_2^* = \overline{w}_1$, workers who apply once are indifferent between c_1^* and c_2^*, i.e. $U_1^* = U_0 + w_2^* \mathbb{E}\left[\psi\left(x; c_2^*\right) S\left(x\right)\right]$. Let γ denote the fraction of these workers applying to c_1^*. In that case, Ξ_1^* equals

$$\Xi_1^* = \frac{\gamma p_1\left(\alpha^*\right) + p_2\left(\alpha^*\right)\left(1 - \mathbb{E}\left[\psi\left(x; \lambda_2^*, c_2^*\right)\right]\right)}{\gamma p_1\left(\alpha^*\right) + p_2\left(\alpha^*\right)}.$$

However, if $w_2^* > \overline{w}_1$, workers who apply once must send their application to c_1^* and γ must equal 1.

Through λ_2^* and c_2^*, the expression for Ξ_1^* is increasing in Δ_1. Hence, we can write $\Xi_1 = \widehat{\Xi}(\Delta_1; \alpha, \gamma)$, with $\widehat{\Xi}'(\Delta_1; \alpha, \gamma) > 0$. The acceptance probability is strictly positive for Δ_1 equal to zero and reaches 1 when applicants to c_1^* capture the entire surplus and no entry in $\underline{\mathcal{C}}_2$ is feasible.

Solution. The solutions for c_1^* and c_2^* provide two relations between Δ_1 and Ξ_1. Although both relations are positive, $\widehat{\Delta}^{-1}(\Delta_1)$ is steeper than $\widehat{\Xi}(\Delta_1; \alpha, \gamma)$ and a unique intersection (Δ_1^*, Ξ_1^*) exists. This intersection and the above results pin down the equilibrium contracts c_1^* and c_2^* for a given α and γ.

Search Intensity. Equilibrium search intensity must maximize (9). Substituting $p_2(\alpha) = 1 - p_1(\alpha)$ in (9) and rewriting the resulting expression yields

$$U(\alpha) = U_2^* - p_1(\alpha)\Delta_2^* - k_A\alpha.$$

Hence, given Δ_2^*, all workers choose the same search intensity α^*, satisfying the first order condition

$$-p_1'(\alpha^*)\Delta_2^* = k_A. \tag{25}$$

This solution and $\widehat{\Xi}(\Delta_1; \alpha, \gamma)$ provide two relations between Δ_1 and α. A higher value of Δ_1 implies a smaller value of Δ_2^* and therefore a decrease in α^*. Hence a unique intersection exists.

Entry. Given λ_i^* and α^*, the measure v_i^* of firms entering submarket i follows from

$$v_i^* = \frac{[\gamma_i p_1(\alpha^*) + p_2(\alpha^*)]\, s}{\lambda_i^*},$$

which concludes the proof.

□

A.6 Proof of Proposition 5

In order to obtain the directed search equilibrium, the information that a firm provides must allow a worker to infer the firm's gross queue λ. When the worker observes both the recruiting intensity r and the terms of trade w, he can immediately infer λ. This is no longer possible if r is unobserved, since various combinations of λ and r imply the same expected payoff for given terms of trade w. It follows from earlier results that these combinations describe a positive relationship between the two variables.

To prove the lemma, it is then sufficient to show that only one of these combinations will imply an expected productivity equal to $x^{\mathbb{E}}$. To show that this is the case, I prove that a constant expected productivity describes a negative relationship between λ and r, such that only one point of intersection between the two curves exists. Again borrowing the notation $\eta(x;\lambda,r) = \eta(x;\lambda,r,w)\ \forall w$ from the proof of proposition 2 and using integration by parts, the expected productivity of a hire can be written as

$$x^{\mathbb{E}} = \underline{x} + \int_{\underline{x}}^{\overline{x}} \frac{\eta(x;\lambda,r)}{\eta(\underline{x};\lambda,r)}\, dx.$$

Omitting arguments to simplify notation, some tedious algebra yields

$$\frac{\partial^2}{\partial\lambda\partial x}\log\eta = e^{-\kappa}\frac{\kappa - 1 + e^{-\kappa}}{(1-e^{-\kappa})^2}\frac{r}{r+1}F' > 0$$

and

$$\frac{\partial^2}{\partial r\partial x}\log\eta = \frac{\left((1-e^{-\kappa})^2 - e^{-\kappa}\kappa^2\right)\kappa_x\kappa_r - (1-e^{-\kappa})\,\kappa\,(1-e^{-\kappa}-\kappa e^{-\kappa})\,\kappa_{rx}}{(1-e^{-\kappa})^2\,\kappa^2} > 0.$$

Consequently, $\frac{\partial x^{\mathbb{E}}}{\partial\lambda} > 0$ and $\frac{\partial x^{\mathbb{E}}}{\partial r} > 0$, after which the Implicit Function Theorem yields the desired result. □

A.7 Proof of Proposition 6

As described in the main text, the planner's problem can be broken down into decisions regarding 1) entry and search intensity; 2) queue lengths and recruiting intensity; and 3) matching. I solve these decisions in reverse order, borrowing the notation $\eta(x;\lambda,r) = \eta(x;\lambda,r,w)\ \forall w$ from the proof of proposition 2.

Matching Rule. After one or more interviews, surplus is clearly maximized when the firm matches with the most productive applicant it has identified, as long as the match surplus is positive. Hence, the threshold productivity below which no matching takes place solves $S(\underline{x}) = 0$, which coincides with the market equilibrium.

Queue Lengths and Recruiting Intensity. Moving one step back, consider the choice of queue lengths and recruiting intensity. Let Λ denote the average number of applicants per firm. Given a number of firms v and a total number of applications $v\Lambda$, the planner chooses λ_j and r_j for each firm $j \in [0,v]$ to maximize total surplus, subject to the constraint that the integral of the queue lengths must equal the number of applications, $\int_0^v \lambda_j dj = v\Lambda$. Hence, ignoring sunk costs, the planner's objective function equals

$$\max_{\{\lambda_j, r_j\}} \int_0^v \left[\int_{\underline{x}}^{\overline{x}} \eta\left(x; \lambda_j, r_j\right) \, dS\left(x\right) - k_R r_j \right] dj. \tag{26}$$

It follows from the proof of proposition 2 that this expression is strictly concave in λ and r. Hence, the planner chooses the same recruiting intensity r^P and the same queue length λ^P at each firm. This queue length equals $\lambda^P = \Lambda$, after which the recruiting intensity r^P equals either the the boundary point, $r^P = 0$, or satisfies the FOC with respect to r, which coincides with (18).

Search Intensity and Entry. Finally, the planner chooses entry v by solving

$$\max_{v, \alpha} v \left[\int_{\underline{x}}^{\overline{x}} \eta\left(x; \lambda^P, r^P\right) \, dS\left(x\right) - k_V - k_R r^P \right],$$

where $\lambda^P = \Lambda = \frac{s}{v}$. Denote the solution by v^P. By the envelope theorem, the FOC with respect to v equals

$$\int_{\underline{x}}^{\overline{x}} \left[\eta\left(x; \lambda^P, r^P\right) - \lambda^P \eta_\lambda\left(x; \lambda^P, r^P\right)\right] \, dS\left(x\right) - k_V - k_R r^P = 0, \tag{27}$$

which is identical to the equilibrium condition (19). Hence, a unique solution exists and this solution corresponds to the market equilibrium, $\lambda^P = \lambda_1^*$, $r^P = r_1^*$, and $v^P = v^*$. That is, the market equilibrium is efficient. □

B Calibration

B.1 EOPP Data

Background. The Employment Opportunities Pilot Project—developed by the Office of the Assistant Secretary for Policy, Evaluation and Research, and funded by the Department of State's Employment and Training Administration—was introduced in the summer of 1979.[57] It consisted of an intensive job search program combined with a work and training program, organized at 10 pilots sites throughout the country. Each pilot site consisted of a small number of neighboring counties. The program was aimed at unemployed workers with a low family income, and tried to place eligible workers in private-market jobs at one of the pilot sites during a job search assistance program. If these attempts failed, the worker was offered a

[57] Several other authors have used the EOPP data set, e.g. Barron et al. (1985), Barron et al. (1987), and Burdett and Cunningham (1998). See these papers and Wolthoff (2012) for additional information about the survey.

federally-assisted work or training position. The program was in full operation by the summer of 1980, but was phased out during 1981 by the new Administration.

In order to evaluate the program, a survey was sent to firms at the ten pilot sites and twenty control sites which where selected on the basis of their similarity to the pilot sites. The first wave of the survey took place between March and June 1980. The second wave was conducted between February and July 1982 and aimed to re-interview all respondents to the first survey. The response rate was about 70%.

Representativeness. The data set is not representative of the entire US labor market. Workers in the sample are relatively young and due to the nature of the labor market program, low incomes are overrepresented. Moreover, the pilot sites are disproportionally concentrated in Gulf Coast cities and underrepresent cities in the Northeast of the US. Further, the data set does not include workers in government and non-profit organizations. The probability for a firm in one of the sites to be included in the survey depended on its size and location and varied between 0.006 for the smallest establishments to close to 1 for establishments with more than 200 employees (see Barron et al., 1985, for more details). The data set contains sample weights to account for the heterogeneity in the sampling probability, which I use throughout.

Data and Sample. The survey sent to the firms included various questions on the recruitment process for the last hired worker who was a participant in the labor market program and for the last hired worker who was not a participant. I only use the latter group in my analysis. The second survey was a lot more comprehensive than the first one. For this reason, I only utilize the 1982 data in this paper, which contains detailed information on recruitment process for the last hire, conditional on the hiring taking place between January 1980 and September 1981. The data set contains information on both the last subsidized and the last non-subsidized hire. I restrict the sample to the latter group. For example, the data reports 1) the number of applications the firm received; 2) the number of interviews the firm conducted; 3) the number of job offers the firm made; 4) the wage the firm paid to the worker it hired; 5) the amount of time the firm spent screening applicants; 6) the vacancy duration; and 7) some characteristics of the hire. I restrict the sample by omitting observations with missing or unreliable values.[58] This results in a sample of 640 observations. Table 4 presents means and standard deviations for some of the key variables.[59]

[58]I omit outliers (top 0.5% of the distribution) in the number of applications, interviews and job offers. This does not affect the results.

[59]A larger set of descriptive statistics is available in Wolthoff (2012).

Variable	Mean	Std.dev.
Number of applicants	14.06	22.66
Number of interviews	4.97	6.80
Number of offers	1.20	0.60
Hours spent on screening	6.16	7.48
Fraction of firms hiring within 30 days	0.936	0.246

Table 4: Descriptive statistics of the EOPP sample.

B.2 Derivations

Job Destruction and Matching. Using the subscript t to indicate time, unemployment evolves according to $u_{t+1} = u_t (1 - \Psi_t) + u^s_{t+1}$, where $u^s_{t+1} \equiv (1 - u_t)(\delta_t + \tau - \delta_t \tau)(1 - \Psi_{t+1})$ denotes the number of short-term unemployed workers at time $t + 1$, i.e. the number of workers who were not unemployed yet at time t. Using data from the Current Population Survey, Shimer (2005b, 2012) constructs time series for both u_t and u^s_t. These can be used to calculate the workers' matching probability Ψ_{t+1} and the job destruction rate δ_t according to

$$\Psi_{t+1} = 1 - \frac{u_{t+1} - u^s_{t+1}}{u_t} \tag{28}$$

and

$$\delta_t = \frac{1}{1-\tau} \left(\frac{u^s_{t+1}}{1 - u_t} \frac{u_t}{u_{t+1} - u^s_{t+1}} - \tau \right),$$

respectively. After averaging over the relevant time interval and taking into account the age structure of the sample, I find $\Psi = 0.413$ and $\delta = 0.063$.

Unemployment and Searchers. Steady state unemployment u follows from equating outflow, $u\Psi$, to inflow, $(1 - u)(\delta + \tau - \delta\tau)(1 - \Psi)$. This implies

$$u = \frac{(\delta + \tau - \delta\tau)(1 - \Psi)}{\Psi + (\delta + \tau - \delta\tau)(1 - \Psi)}.$$

Evaluating this in the chosen values for δ, τ and Ψ gives $u = 0.085$. The number of searchers is then equal to

$$\begin{aligned} s &= u + (1 - u)(\delta + \tau - \delta\tau) \\ &= \frac{\delta + \tau - \delta\tau}{\Psi + (\delta + \tau - \delta\tau)(1 - \Psi)}, \end{aligned}$$

which yields $s = 0.144$.

Number of Interviews, Conditional on Hiring. Let v_j^* denote the mass of firms posting equilibrium contract c_j^* for all $j \in \{1, \ldots, A\}$ and let n_I denote the number of interviews that a firm conducts. Applying Bayes' Rule twice then implies that the expected number of interviews conditional on hiring equals

$$\begin{aligned}\mathbb{E}\left[n_I|\text{hire}\right] &= \frac{\sum_{j=1}^{A}\sum_{n=1}^{\infty} n\mathbb{P}\left[n_I = n|\text{hire}, c_j^*\right]\mathbb{P}\left[\text{hire}|c_j^*\right]v_j^*}{\sum_{j=1}^{A} v_j^*\mathbb{P}\left[\text{hire}|c_j^*\right]} \\ &= \frac{\sum_{j=1}^{A} v_j^*\sum_{n=1}^{\infty} n\mathbb{P}\left[\text{hire}|n_I = n, c_j^*\right]\mathbb{P}\left[n_I = n|c_j^*\right]}{\sum_{j=1}^{A} v_j^*\mathbb{P}\left[\text{hire}|c_j^*\right]}.\end{aligned}$$

Note that $\mathbb{P}\left[\text{hire}|n_I = n, c\right]$ and $\mathbb{P}\left[n_I = n|c\right]$ were provided in the proof of proposition 1 and $\mathbb{P}\left[\text{hire}|c\right]$ equals $\eta\left(\underline{x}; c\right)$.

Number of Interviews. The (unconditional) expected number of interviews per firm equals

$$\mathbb{E}\left[n_I\right] = \frac{\sum_{j=1}^{A}\sum_{n=1}^{\infty} n\mathbb{P}\left[n_I = n|c_j^*\right]v_j^*}{\sum_{j=1}^{A} v_j^*},$$

where $\mathbb{P}\left[n_I = n|c\right]$ is again provided in the proof of proposition 1. Some algebra gives

$$\sum_{n=1}^{\infty} n\mathbb{P}\left[n_I = n|c\right] = (r+1)\left(1 - e^{-\frac{\lambda(c)}{r+1}}\right) \leq r + 1.$$

Number of Applications, Conditional on Hiring. In a similar fashion, the expected number of applicants n_A conditional on hiring equals

$$\begin{aligned}\mathbb{E}\left[n_A|\text{hire}\right] &= \frac{\sum_{j=1}^{A} v_j^*\sum_{n=1}^{\infty} n\mathbb{P}\left[\text{hire}|n_A = n, c_j^*\right]\mathbb{P}\left[n_A = n|c_j^*\right]}{\sum_{i=1}^{A} v_i^*\mathbb{P}\left[\text{hire}|c_j^*\right]} \\ &= \frac{\sum_{j=1}^{A} v_j^*\sum_{n=1}^{\infty} n\sum_{i=1}^{n}\mathbb{P}\left[\text{hire}|n_I = i, c_j^*\right]\mathbb{P}\left[n_I = i, n_A = n|c_j^*\right]}{\sum_{j=1}^{A} v_j^*\mathbb{P}\left[\text{hire}|c_j^*\right]}.\end{aligned}$$

To calculate $\mathbb{P}\left[n_I = i, n_A = n|c\right]$, let n_Q and n_R denote the firm's number of qualified applicants and interview capacity, respectively. Then

$$\mathbb{P}\left[n_I = i, n_A = n|c\right] = \mathbb{P}\left[n_I = i|n_A = n, c\right]\mathbb{P}\left[n_A = n|c\right],$$

where $\mathbb{P}\left[n_A = n|c\right] = e^{-\lambda(c)} \frac{[\lambda(c)]^n}{n!}$ and

$$\begin{aligned}
\mathbb{P}\left[n_I = i|n_A = n, c\right] &= \mathbb{P}\left[n_Q = i, n_R \geq i|n_A = n, c\right] + \mathbb{P}\left[n_Q > i, n_R = i|n_A = n, c\right] \\
&= \left(\frac{r}{r+1}\right)^{i-1} \left[\binom{n}{i} q^i \left(1-q\right)^{n-i} + \sum_{m=i+1}^{n} \binom{n}{m} q^m \left(1-q\right)^{n-m} \frac{1}{r+1}\right].
\end{aligned}$$

Recruiting Cost, Conditional on Hiring. A firm posting a contract c spends $k_R r$ on recruiting. Assume that a year consists of 50 weeks of 5 days of 8 productive hours and let $x_{avg}(c)$ denote the average match quality given c, such that the time cost of recruiting can be calculated as

$$T(c) = \frac{2000}{12} \frac{k_R r}{x_{avg}(c)}.$$

Then, the expected time spent on recruiting conditional on hiring equals

$$\mathbb{E}\left[T|\text{hire}\right] = \frac{\sum_{j=1}^{A} v_j^* \mathbb{P}\left[\text{hire}|c_j^*\right] T\left(c_j^*\right)}{\sum_{j=1}^{A} v_j^* \mathbb{P}\left[\text{hire}|c_j^*\right]}.$$

Number of Job Offers. The expected number of job offers n_O per firm equals

$$\mathbb{E}\left[n_O\right] = \frac{\sum_{j=1}^{A} \sum_{n=1}^{\infty} n \mathbb{P}\left[n_O = n|c_j^*\right] v_j^*}{\sum_{j=1}^{A} v_j^*},$$

where aggregate consistency requires $\sum_{n=1}^{\infty} n \mathbb{P}\left[n_O = n|c\right] = \lambda(c) \mathbb{E}\left[\psi(x; c)\right]$.

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