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A Century of Human Capital and Hours

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Abstract

An average person born in the United States in the second half of the nineteenth century completed 7 years of schooling and spent 58 hours a week working in the market. By contrast, an average person born at the end of the twentieth century completed 14 years of schooling and spent 40 hours a week working. In the span of 100 years, completed years of schooling doubled and working hours decreased by 30 percent. What explains these trends? We consider a model of human capital and labor supply to quantitatively assess the contribution of exogenous variations in productivity (wage) and life expectancy in accounting for the secular trends in educational attainment and hours of work. We find that the observed increase in wages and life expectancy account for 80 percent of the increase in years of schooling and 88 percent of the reduction in hours of work. Rising wages alone account for 75 percent of the increase in schooling and almost all the decrease in hours in the model, whereas rising life expectancy alone accounts for 25 percent of the increase in schooling and almost none of the decrease in hours of work.

Keywords: schooling, hours of work, productivity, life expectancy, trends, United States.
JEL codes: E1, I25, J11, O4.

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1 Introduction

Over the course of the nineteenth and twentieth century the United States has witnessed a noticeable increase in various measures of educational attainment. For instance, average schooling of generations of the second half of the nineteenth century was about 7 years while is close to 14 years nowadays. Over the same period of time the lifetime labor supply of a typical worker decreased substantially. For instance, the workweek of a typical worker was around 60 hours in the 1870s and is about 40 hours nowadays.[1] What explains these trends? We consider a model of human capital and labor supply that can broadly capture the secular trends in average years of schooling and hours of work. We use the model to quantitatively assess the importance of productivity growth –reflected as an increase in wages– and life expectancy in accounting for the trends in schooling and hours. We find that the observed increase in wages and life expectancy account for 80 percent of the increase in years of schooling and 88 percent of the reduction in hours of work.

The motivation for studying the trends in education and work hours simultaneously is three-fold. First, the trends in education and hours are not specific to the United States but are, instead, common to most developed countries. We argue that over a long period of time the face of western societies have changed quite dramatically because of reduced hours at most margins –the workweek, increased vacations and retirement length– and because of the spread of formal education. Second, other authors such as Heckman (1976) and Blinder and Weiss (1976) have emphasized the importance of jointly modeling labor supply and human capital accumulation. Since some dimensions of human capital investment are not observed, models of human capital accumulation are typically restricted by using data on earnings. Recognizing that the accumulation and utilization of human capital have implications for leisure time, it is an immediate consequence that we can also use observations about leisure

[1] See Figures 1 and 2 for illustrations of the trends in schooling and hours in the United States.

time to bring more discipline to bear on the implications of the model. The study of human capital and labor supply has typically been done in the context of life-cycle frameworks, see for instance the seminal contribution by Blinder and Weiss (1976) and more recent analysis in Guvenen, Kuruscu, and Ozkan (2010). We propose to use the noticeable changes observed over long periods of time as an alternative discipline to these models. Third, our research is connected to the recent literature in macroeconomics on the importance of human capital for understanding inequality across people, time, and countries, e.g. Manuelli and Seshadri (2006), Erosa, Koreshkova, and Restuccia (2010), You (2009), and Guvenen, Kuruscu, and Ozkan (2010). By focusing on a time period with substantial changes in labor supply, we find that abstracting from hours of work critically affects the effective returns to human capital investment.

Our model of human capital accumulation builds on Bils and Klenow (2000) and Restuccia and Vandenbroucke (2011). Individuals live for a finite interval of time and are homogenous within a generation. They choose how long to stay in school as well as spending in educational services and, as a result, accumulate human capital. Individuals also choose to allocate their time between leisure and work. There are two key features of our model. First, preferences feature a taste for schooling. Second, preferences are non-homothetic for consumption goods. In the context of our model, we show that these two features of preferences are critical for schooling to depend on the level of income. We argue these features of preferences are non controversial. Taste for schooling is a common feature in models of schooling such as Heckman, Lochner, and Taber (1998) and Bils and Klenow (2000) and has been found to be empirically relevant in estimated models of human capital accumulation. Non-homothetic preferences are central in theories of structural change.[2] In the model time is continuous and cohorts of constant size (normalized to one) are born at each moment. There are two

[2] See for instance Laitner (2000), Kongsamut, Rebelo, and Xie (2001), and Gollin, Parente, and Rogerson (2002) for models of development; Rogerson (2008) for a model of the allocation of hours across sectors and countries; Greenwood and Vandenbroucke (2008) for a model of the trend in leisure; among others.

exogenous variables: life expectancy and wages per unit of human-capital hour. Each cohort faces a different set of values for these exogenous variables and, therefore, makes different educational and labor supply choices.

We use our model to compute cohort-specific sequences of labor supply and years of schooling. We proceed as follows. First, we restrict the parameters of the model so that it reproduces the years of schooling and hours of work observed in the data for the 1870 generation. We also impose that the model be consistent with the observed 2% rate of growth in income since the nineteenth century. We then conduct a set of counterfactual experiments in which we assess the quantitative importance of the main driving forces in the model. We find that wage growth and life expectancy account for 80 percent of the increase in years of schooling and 88 percent of the decline in hours of work between the 1870 and the 1970 cohorts. Among these forces, the growth in wages explains the bulk of changes in both variables in the model: about 75 percent of the rise in schooling and almost all (97 percent) of the decline in hours. Life expectancy alone accounts for 25 percent of the rise in schooling and 3 percent of the decline in hours. Wage growth also has level effects since it matters for the wealth of any given generation. Thus, absent wage growth, the time path of schooling would not only be flatter than observed, but also it would be substantially lower. Similarly, without wage growth, the time path for hours of work would be almost flat and at a higher level than observed in the data.

We contrast the implications of our model along other dimensions in the data. In particular, we argue that the mechanisms in the model are consistent with the patterns of schooling over time across races and the changes in the distribution of hours over time in the United States.[3] Our model implies that schooling and hours converge faster toward their long-run values at low levels of income than at high levels. We show that this pattern can quantitatively

[3] In Restuccia and Vandenbroucke (2011) we show that differences in productivity and life expectancy can explain most of the schooling patterns across countries and over time.

generate the faster increase in years of schooling observed among blacks relative to whites since the late nineteenth century. We also show that the model can account for the fact that the decline in hours of work in the United States has been more pronounced for individuals at the lower end of the wage distribution than at the upper end as documented by Costa (2000).

The rest of the paper is organized as follows. We next describe the main two facts on years of schooling and hours of work that are the focus of our analysis. Section 3 presents the model in detail. In Section 4 we calibrate the model and state our main results. Section 5 discusses the results. In Section 6 we conclude.

2 Facts

We report the historical trends in average years of schooling and average hours per worker in the United States. Figure 1 reports average years of schooling completed for whites by cohort from Goldin and Katz (2008). The main pattern to take away from Figure 1 is the strong upward trend. Between the 1870 and 1970 generations, average years of schooling increased from 7 to 14.1 years (an increase of a factor of 2). Substantial increases in years of schooling are observed across races and gender. Similarly, Hazan (2009, Figure 1) reports estimates of average years of schooling by birth cohort. He finds a substantial increase in the average years of school completed by cohorts over the course of the last 150 years. There are alternative measures of educational attainment, all pointing to a similar picture of a secular increase in education. For example, school enrollment, defined as enrollment in an institution delivering either an elementary, a high-school, or a college degree, has increased from 47 percent of the 5-19 years old population in 1850 to 92 percent in 1990 and the percentage of persons aged 25 and over with less than 5 years of education decreased from

24 percent in 1910 to less than 2 percent in 2000, while the fraction of people with at least a bachelor's degree increased from 2.7 to 25.6 percent.[4]

Figure 2 shows the trend in the length of the workweek, i.e., the number of hours worked a week per worker. The main pattern from Figure 2 is the downward trend and the slowdown of this trend in the second half of the twentieth century. Besides the reduction in weekly hours of work, individuals exploited other margins in reducing time spent working in the market. Compared with the early 1900s, people work fewer weeks per year and fewer years throughout their life cycle nowadays. Lebergott (1976) reports that 6% of non-farm workers took vacations in 1901 whereas 60% took vacation in 1950 and 80% in 1970. Kopecky (2011) reports that in the 1850s a person could expect to spend about 5% of the adult life in retirement. By 2000 this statistic is close to 30%. Hazan (2009) reports estimates of lifetime hours spent in the labor market per birth cohort. He finds that men born in 1870 spent about 110,000 hours working (total working hours over the lifetime at age 5 by age 79) while men born in 1970 spent less than 74,000 hours: a 33 percent reduction. The pattern of hours displayed in Figure 2 masks some heterogeneity, in particular across gender. It has been well documented that women participation to the labor market increased markedly during the course of the twentieth century. Since the workweek of women tend to be shorter than that of men, the trend in Figure 2 could reflect a change in the composition of the labor force. This compositional effect is small over the 100-year period we consider. Indeed, the trend in the workweek for men displays a pattern similar to the time-series in Figure 2.[5]

Besides the United States, other countries experienced similar changes in the level of educational attainment and labor supply. In the United Kingdom, for example, the number of hours worked per person in 1984 is 51 percent of what it was in 1870. Simultaneously, the

[4]See *Historical Statistics of the United States,* Millennial Edition, series Bc441 and Bc444 and *Digest of Education Statistics*, 2008, Table 8.

[5]See Vandenbroucke (2009, Figure 1).

number of high school graduates went from 2 percent of the 17-year-olds population in 1870 to 69.5 percent in 1960. Similarly, in France, the hours worked per person in 1984 were 53 percent of their 1870 level. On the education front, a man between 15 and 65 years of age in 1895 had completed 6.4 years of schooling while in 1994 this statistic is 12.1 years.[6]

3 The Model

The model follows closely that of Restuccia and Vandenbroucke (2011). Time is continuous. The economy is populated by overlapping generations of individuals living for an interval of time of finite length T_τ, where τ indexes generations. We assume that T_τ is known at the beginning of life. An individual is endowed with one unit of productive time per period, facing a time allocation problem along two margins. First, the individual chooses the time spent in school s which augments human capital and the return to market work. Second, the individual chooses from age s until T_τ how to allocate the time between work and leisure. The wage rate per unit of human-capital-hour is denoted by w_τ and we assume it grows at the constant rate g.[7] We assume that individuals are born with no assets and that there are perfect credit markets on which they can borrow and save at the rate r.

3.1 Preferences

Preferences are defined over sequences of consumption of goods and leisure time, as well as over the time spent in school. The preferences of an individual of generation τ are represented

[6] See Maddison (1987, Table A-9) for the figures on hours per person in France and the U.K. See Goldin and Katz (2008, Table 1.1) for the high school graduation rate for the U.K. See CEPII (Centre d'Etudes Prospectives et d'Informations Internationales); available online at `http://www.cepii.fr`.

[7] In what follows, we refer to w_τ as wages and productivity interchangeably.

by

$$\int_0^{T_\tau} e^{-ru}[U(c) + \alpha V(\ell)]du + \beta W(s), \tag{1}$$

where c is consumption, ℓ is leisure time, and s is time in school. The parameter α is a positive constant while β may be positive or negative. The functions U, V and W are concave and twice continuously differentiable. We assume that the subjective rate of discount equals the rate of interest. This implies that it is optimal for consumption to be constant throughout an individuals' life. We abstract from other life-cycle considerations by restricting leisure time to be constant over an individual's life. Note that s is chosen once and for all and that $\beta W(s)$ is, therefore, the age-0 value of the lifetime utility derived from schooling.

We choose the following functional forms for U, V and W:

$$U(c) = \ln(c - \bar{c}),\ V(\ell) = \ln(\ell),\ W(s) = \ln(s),$$

where $\bar{c} > 0$ is a subsistence level of consumption. We note the following property of U, which will be useful later:

$$U'(c)c \geq 1 \ \text{ and } \ U'(c)c \to 1 \ \text{ as } \ c \to \infty. \tag{2}$$

3.2 Technology

The technology for human capital accumulation is described by

$$H(s, x) = x^\gamma h(s), \tag{3}$$

where s is time devoted to school and x represents the input of educational services in units of goods. We assume the following form for h :

$$h(s) = \exp\left(\frac{\theta}{1-\psi} s^{1-\psi}\right).$$

We choose this functional form following Bils and Klenow (2000). However, we emphasize that it is not critical for our results. As the analysis of the next section will demonstrate, an important restriction on $h(s)$ for the optimal level of s to increase with the level of income is that $h'(s)/h(s)$ is a decreasing function of s. This restriction is satisfied by a large class of functions.

3.3 Optimization

An individual's optimization problem is to choose how much schooling to do, s, as well as consumption and leisure time, c and ℓ, in order to maximize the value of the objective function (1) subject to the lifetime budget constraint given by

$$c \int_0^{T_\tau} e^{-ru} du + x = w_\tau H(s, x)(1-\ell) \int_s^{T_\tau} e^{(g-r)u} du.$$

Note that the purchase of educational services, x, is measured in present value at age 0 and is a once-and-for-all expenditure. Note also that w_τ and T_τ uniquely identify a generation. Thus, within a generation, individuals are identical and make the same choices. Individuals from different generations, however, make different choices because they face different levels of productivity and life expectancy when they are born. In our quantitative exercise we use data on life expectancy and per-capita income to discipline the time series of both w_τ and T_τ.

It is convenient to define

$$a_{c,\tau} = \int_0^{T_\tau} e^{-ru} du,$$

and

$$d_\tau(s) = \int_s^{T_\tau} e^{(g-r)u} du.$$

The first oder condition for optimization can then be written as

$$\begin{aligned}
c &: \quad 0 = U'(c) - \lambda, \\
x &: \quad 0 = 1 - \gamma w_\tau x^{\gamma-1}(1-\ell)h(s)d_\tau(s), \\
\ell &: \quad 0 = \alpha a_{c,\tau} V'(\ell) - \lambda w_\tau x^\gamma h(s) d_\tau(s), \\
s &: \quad 0 = \beta W'(s) + \lambda w_\tau x^\gamma (1-\ell)[h'(s)d_\tau(s) + h(s)d'_\tau(s)],
\end{aligned}$$

where λ is the Lagrange multiplier associated with the lifetime budget constraint. Using the first order conditions with respect to c and x, we can rewrite the first order conditions with respect to ℓ and s as

$$\alpha(1-\gamma)V'(\ell)(1-\ell) = U'(c)c, \tag{4}$$

and

$$\beta(1-\gamma)W'(s) = -a_{c,\tau}U'(c)c\left[\frac{h'(s)}{h(s)} + \frac{d'_\tau(s)}{d_\tau(s)}\right]. \tag{5}$$

3.4 Discussion

Equation (5) describes the costs and benefits associated with the optimal choice of schooling time. Unlike Equation (4) which is fairly common in models with a consumption-leisure tradeoff, Equation (5) deserves some discussion. The marginal benefit of time spent in school has potentially two sources: (1) there is a pecuniary benefit resulting from the increase in income due to the extra units of human capital obtained through schooling and (2) if $\beta > 0$, there is a direct utility benefit from schooling. Similarly, the marginal cost of schooling has potentially two sources: (1) there is a pecuniary cost stemming from the foregone earnings incurred while in school and (2) if $\beta < 0$, there is a direct utility loss from attending school. In what follows we concentrate our discussion on the case where $\beta > 0$ because it is the case prevailing in our quantitative analysis where the value of β is disciplined by data. The pecuniary cost and benefit of schooling are subsumed in the bracketed term in Equation (5). More precisely, $h'(s)/h(s)$ measures the pecuniary part of the marginal benefit of schooling because it directly measures the effect of schooling on earnings per hour.[8] The term $d'_\tau(s)/d_\tau(s)$ measures the marginal cost, that is the effect of s on the time remaining after school to receive the benefit from longer schooling.

We make four remarks. First, if there was no taste for schooling ($\beta = 0$) the optimal schooling choice would be such that it maximizes lifetime income. This would require that the marginal (pecuniary) benefit and cost of schooling time are equal. Thus, setting the term in brackets in Equation (5) to zero would determine the optimal schooling choice. The individual would then use the optimality condition (4) to allocate income between the purchase of leisure time and consumption. It transpires from Equation (5) that in such a case schooling would differ from one generation to the next only to the extent that life expectancy, T_τ, differs across generations. This is an important characterization because it is a well known fact that in the early stages of the nineteenth century (and before) changes in life expectancy occurred mostly because of reductions in early child mortality.[9] Thus, conditional on surviving early childhood, individuals of different generations did not experienced much of an increase in the time they had to benefit from schooling. Yet, there has been a well-documented increase in educational attainment during the early part of the nineteenth century. We consider

[8]The term $h'(s)/h(s)$ is the Mincer return: it is the semi-elasticity of earnings per hour with respect to schooling. To see this, note that earnings per hour are $e = wx^\gamma h(s)$ so that $d\ln(e)/ds = h'(s)/h(s)$.

[9]The life expectancy at age 5 in 1850 was 49.5 and 50.8 for men and women, respectively. In 1880 these numbers were 51 and 51.3, respectively. In contrast, in the following 30 years, life expectancy at age five increased to 56.1 and 58.5.

this evidence supportive of an income effect being potentially important in the historical evolution of schooling.

Second, when there is a taste for schooling, that is when $\beta \neq 0$, the level of income matters for the schooling choice. This transpires in Equation (5) through the level of consumption. Similarly, Equation (4) reveals that leisure time depends on the level on income as well. In the appendix we show formally that, when $\beta > 0$, the optimal choices for schooling and leisure time are non-decreasing functions of the level of productivity w_τ. We summarize this in the following proposition.

Proposition 1 *Assume that $\beta > 0$. The optimal length of schooling and leisure time are increasing in w_τ. That is: $ds/dw_\tau > 0$ and $d\ell/dw_\tau > 0$.*

Proof 1 *See appendix.*

To understand this result note that schooling is a time allocation decision. Thus, an increase in productivity raises the opportunity cost of schooling inducing individuals to choose lower levels of schooling whereas an increase in productivity also implies an income effect whereby individuals with a positive taste for schooling ($\beta > 0$) use the extra income to acquire more schooling. The net effect of these opposing forces is ambiguous and generally depends upon preferences, particularly the rate at which the marginal utility of consumption decreases when consumption rises, i.e., the term $U'(c)c$. Our functional form assumptions imply that, when $\beta > 0$, the income effect dominates the substitution effect unambiguously –see appendix. The critical parameter driving the strength of the income effect in our analysis is $\bar{c}$. If $\bar{c} = 0$ then it is immediate, from Equations (4) and (5), that leisure and schooling time are independent of the level of income. This is a well known result: income and substitution effects offsets each other with logarithmic preferences. When $\bar{c} > 0$ an increase in productivity, and the subsequent increase in consumption, imply a "fast" decline of the marginal utility of consumption. This, in turn, implies that the opportunity cost of time does not increases as fast as productivity, hence the income effect dominates. The same logic applies for the optimal choice of leisure time. Interpreting $\bar{c}$ as a minimum consumption requirement, the intuition for this discussion can be summarized as follows. At low levels of productivity individuals have to work long hours in order to finance the consumption of $\bar{c}$. So, leisure and schooling are low. As productivity increases the minimum work required to finance the consumption of $\bar{c}$ is lower. This allows individuals to work less hours and invest more in schooling. So, leisure time and schooling increase.

Third, we note that in the long-run, that is as $c \to \infty$, Equation (4) implies that leisure time and individual hours worked are constant. This stems from a property of U described in Equation (2), namely that $U'(c)c \to 1$ as $c \to \infty$. This asymptotic property of the model is consistent with models displaying balanced growth and has been motivated in the literature by the relative constancy of hours during the second half of the twentieth century – see for instance Prescott (1986) and King, Plosser and Rebelo (1988).[10] Following a standard practice in the literature, we will use this asymptotic property of leisure to calibrate some the model's parameters. Thus, we introduce a notation for the long-run value of leisure time, $\tilde{\ell}$, which must satisfy

$$\alpha(1-\gamma)V'(\tilde{\ell})(1-\tilde{\ell}) = 1. \tag{6}$$

A similar property holds for schooling time. As productivity and life expectancy increase one can verify that schooling converges to a long-run value, $\tilde{s}$, which satisfies

$$\beta(1-\gamma)W'(\tilde{s}) = -\frac{1}{\rho}\left[\theta\tilde{s}^{-\psi} + g - \rho\right], \tag{7}$$

[10]The real business literature often refers to leisure per capita while our motivating data is about hours per worker. McGrattan and Rogerson (2004) show that hours worked by men workers during the 1950-2000 period exhibit little trend. Hours decrease slightly between 1950 and 1970 and increase slightly between 1970 and 2000.

where the facts that $U'(c)c \to 1$ as $c \to \infty$ and $a_{c,\tau} \to 1/\rho$ and $d'_\tau(s)/d_\tau(s) \to g - \rho$ as $T_\tau \to \infty$ have been used.[11] We use this asymptotic property of schooling time to calibrate some of the model's parameters.

Fourth, and as transpires from the previous discussion, the fact that $U'(c)c$ is a decreasing function of c is critical for the time series properties of the model. There are many specifications for U which deliver this property. Our specification for U is guided by the same principle as in modern business cycles and growth theory where long-run increases in wealth have cancelling income and substitution effects on labor supply. Specifically, our utility function allows $U'(c)c$ to decrease, but converging asymptotically to a positive constant, namely 1. The convergence of $U'(c)c$ to 1 implies that s converges to a constant in $(0, T_\tau)$ as opposed to approaching T_τ and leisure converges to a constant in $(0, 1)$ instead of 1.[12]

4 Quantitative Analysis

We construct a quantitative experiment by computing sequences of hours and years of schooling for generations starting in 1870 up to 1970 taking as exogenous sequences of wages w_τ and life expectancy T_τ. The model is calibrated to long-run restrictions and U.S. data for 1870. We assess the quantitative importance of increases in wages and life expectancy in explaining the paths of hours and schooling in the U.S. data in the last century.

[11] See the appendix for a derivation of the properties of $d'_\tau(s)/d_\tau(s)$.

[12] Convergence of $U'(c)c$ to a positive constant is a consequence of the logarithmic utility specification and the non-homothetic term $\bar{c} > 0$.

4.1 Calibration

To perform our quantitative experiment we need to restrict the parameters of the model and the time series for the two exogenous variables. We proceed as follows. For wages, the constant-growth-rate assumption implies that the time series w_τ can be represented as:

$$w_\tau = w_{1870} \times e^{g(\tau-1870)},$$

where w_{1870} is an initial condition for which we adopt the normalization $w_{1870} = 1$. To build a time series for life expectancy we proceed as in Restuccia and Vandenbroucke (2011). That is, we note that T_τ corresponds in the model to the sum of years spent in school and years spent on the labor market. Thus, we add Hazan (2009)'s measure of years spent on the labor market by cohort to Goldin and Katz (2008)'s figures for years of schooling attained by each cohort. We construct the time series for T_τ by estimating a linear time trend on the constructed time series. We obtain,

$$T_\tau = 0.1716 \times \tau - 279.38,$$

which implies the following values of T_τ for cohorts in 1900 and 2000: $T_{1900} = 47$ and $T_{2000} = 64$.

We let the rate of interest (and time discount) be 4 percent: $\rho = 0.04$. We also choose $\gamma = 0.1$, following measures of the share of goods and time in the production of human capital. In Section 5.3 we conduct a sensitivity analysis with respect to this parameter. We now turn to the remaining parameters: ψ and θ, the parameters of the human capital technology, g the growth rate of the wage per unit of human capital, and $\bar{c}$, α and β, the remaining preference parameters. Bils and Klenow (2000) suggest a range of estimates for

ψ between 0 and 0.6. We choose $\psi = 0.3$, the middle of this range, and do sensitivity with respect to this parameter in Section 5.3. We denote by λ the 5×1 vector of parameters left to be determined:

$$\lambda = [\theta, g, \bar{c}, \alpha, \beta]' .$$

We use 5 restrictions to discipline these parameters. The first and second restrictions impose that the model's predictions for years of schooling and hours exactly match the U.S. data for the 1870 generation. That is, we impose that the first generation of the model chooses to stay in school for 7 years and to work 58 hours per week. The number 58 corresponds to the workweek in 1905, which is when the 1870 cohort reaches age 35.[13] We assume that there is a total of $24 - 8$ hours of discretionary time each day, which implies a total of 112 hours per week, so 58 hours translates into $\ell_{1870} = 1 - 58/112$. As productivity and life expectancy increase our model predicts that hours of work converge to $1 - \tilde{\ell}$ and that years of schooling converge to $\tilde{s}$. We use these properties to construct two additional restrictions. We impose that $\tilde{\ell} = 1 - 40/112$, that is we impose that in the long-run hours of work are constant at 40 hours per week. This restriction is consistent with the behavior of the workweek in Figure 2. We also impose that $\tilde{s} = 21$. We choose this number as the total years required to complete the highest degree in the current educational system.[14] The last restriction we impose is that the model reproduces an average increase in income of 2 percent per year.[15]

Formally, our procedure can be described as solving a system of 5 equations in 5 unknowns. For a given λ, we compute years of schooling, labor supply and income for a sequence of 100 generations born between 1870 and 1970. Our targets for γ are summarized below. Thus,

[13] Recal that the data on years of schooling are about schooling completed at age 35. See Figure 1.

[14] We note that the procedure of restricting preference and technology parameters to generate asymptotic values in the model is reminiscent of the development literature in setting targets for long-run share of food consumption or the long-run share of employment in agriculture.

[15] This increase in income is motivated by data on real Gross Domestic Product per capita from Historical Statistics, see Carter et al (2006, Table Ca-C). However, we note that a similar restriction would follow from using wage data, see for instance Williamson (1995).

we solve for the zero of the function $F(\lambda)$ defined by

$$F(\lambda) = \begin{bmatrix} s_{1870} - 7 \\ \ell_{1870} - 0.48 \\ \tilde{s} - 21 \\ \tilde{\ell} - 0.64 \\ y_{1970}/y_{1870} - e^{0.02\times 100} \end{bmatrix},$$

where y_τ is the period income of a particular generation.[16] Although the system $F(\lambda) = 0$ determines all parameters simultaneously, some parameters are more important for some targets than for others. In particular, the growth rate g has a first order effect on the growth rate of income, and parameters such as $\bar{c}$, θ and β also matter in pinning down the initial level of schooling and hours and the long-run level of schooling. Finally, we emphasize that the restrictions on the long-run level of hours is independent of the other restrictions. This can be seen from Equation (6) which implies that, given γ and $\tilde{\ell}$, we have

$$\alpha = \frac{1}{(1-\gamma)V'(\tilde{\ell})(1-\tilde{\ell})}.$$

It is worth emphasizing this property of the model because it implies that the only preference parameter pertaining to leisure, that is α, is disciplined by a long-run restriction on hours. Therefore, the initial level of hours imposes discipline on other aspects of the model and, in particular, on the human capital technology. This is the sense in which the historical trend in hours of work imposes additional discipline on the human capital accumulation technology. Similarly, the taste parameter for schooling can be derived from Equation (7) given ρ, ψ, θ, γ and $\tilde{s}$:

$$\beta = -\frac{1}{\rho}\frac{\theta\tilde{s}^{-\psi} + g - \rho}{(1-\gamma)W'(\tilde{s})}.$$

[16]For a generation born when the wage per unit of human capital is w_τ, we have $y_\tau = w_\tau e^{gs} H(s,x)(1-\ell)$.

4.2 Baseline Results

Table 1 reports the parameter values resulting from the calibration of the model. Figures 3 and 4 plot the trend predicted by the model against U.S. data for years of schooling and hours of work. The first two columns of Table 2 summarize the changes in years of schooling and hours in the U.S. and in the baseline version of the model. The first lesson from Table 2 (and Figures 3 and 4) is that the model replicates the bulk of the increase in years of schooling and the decrease in hours. The model predicts that years of schooling increase by 81.5 percent (from 7 to 12.7) while in the data the increase is 101.5 percent (from 7 to 14.1). Thus, the model accounts for 80 percent (81.5/101.5) of the rise in schooling between the 1870 and 1970 generations. In terms of hours, the model predicts a 27.5 percent decline (from 58 to 42 hours) while the U.S. data shows a 31.2 percent drop (from 58 to 40 hours). Hence, the model accounts for 88 percent (27.5/31.2) of the decline in hours.

A critical parameter for our results is the non-homotheticity parameter $\bar{c}$. This parameter plays a key role to set the level of hours and years of schooling for the 1870 generation in our calibration procedure. It is also critical, in the time series, for the determination of the increase in schooling and the decrease in hours of work. This can be seen from the term $U'(c)c$ in Equations (4) and (5). Our calibration procedure finds that $\bar{c} = 0.30$. In order to gauge how reasonable this number is, we compute the ratio of $\bar{c}$ to income per capita for the 1870 and the 1970 generations. We find that this ratio decreases from 37 percent to 5 percent. We compare these numbers to data on final expenditures on food relative to GDP. The data is for the 1996 Benchmark study of the International Comparison Program. For the United States, the share of food is 5.2%. For the average of a set of rich countries (i.e. countries with a GDP per capita no less than 90 percent of that of the U.S.) this share is 5.7%, whereas for a set of poor countries that are between 8 and 10 percent of the U.S. GDP per capita this share is 40.2%. We note that the U.S. in 1870 was about 13 percent of the U.S. in 1970 if growth was around 2% per year. We also note that Maddison (2009) reports that GDP per capita, between 1 and 1500 was between 450 and 771 at constant 1990 dollars. This represents a range of 2 to 4.5 percent of the 1970 GDP per capita. If we interpret the period 1-1500 as one when western Europe was close to subsistence, that is GDP per capita was close to $\bar{c}$, then we conclude that our calibrated value for $\bar{c}$ appears to be within a reasonable range.

4.3 Decomposing the Forces

We quantify the importance of the two driving forces in the model explaining the rise in educational attainment and the reduction in hours by running counterfactual experiments and comparing them to the result of the baseline model. We compute the path of years of schooling and hours under the counterfactual that each driving force is "shut down." That is, in the first experiment we keep w_τ constant at its 1870 value while T_τ increases as in the baseline. In the second experiment we keep T_τ constant at its 1870 value while w_τ grows at rate g as in the baseline.

The last two columns of Table 2 summarize our results. When w_τ remains constant and only T_τ increases, years of schooling increase by 13 percent vis à vis 81.5 percent in the baseline. When T_τ remains constant, years of schooling increase by 39.5 percent. To compute the contribution of w_τ alone to the rise in schooling we note that 13 and 39.5 do not add up to 81.5. This means that there is a positive interaction between the rise in w_τ and the rise in T_τ. The reason for this is that an increase in life expectancy when the wage rate remains constant represents a smaller increase in wealth than an increase in life expectancy when the wage rate keeps growing over the additional years. The measure of the contribution of changes in w_τ alone depends on how we impute the interaction term between w_τ and T_τ. But we can say that this contribution is not less than $39.5/81.5 = 49$ percent of the total effect in the model. We choose to impute the interaction term proportionately between productivity and life expectancy. We find that productivity alone accounts for $39.5/(39.5 + 13.0) = 75$ percent of the increase in years of schooling while life expectancy accounts for 25 percent. In terms of the decline in hours, we find that the main driving force is w_τ which accounts for at least 98 percent of the total effect in the model. The interaction between life expectancy and productivity, in terms of hours is small. Nonetheless, imputing it proportionately to each factor, we find that productivity accounts for $98/(98 + 3) = 97$ percent of the decline in hours while life expectancy accounts for the 3 percent.

Figure 5 and Figure 6 display, graphically, the results of the experiments summarized in Table 2. We emphasize from these figures that, in addition to a growth effect, the growth rate of w_τ has a noticeable level effect on both schooling and hours. The reason for this is as follows. Consider the 1870 generation in the baseline model and in the counterfactual where w_τ does not grow. Although the initial value of w_τ is the same for each of them, i.e., $w_{1870} = 1$, the present value of w_τ is lower for the 1870 generation in the counterfactual exercise because it remains constant over its lifetime. Hence all generations, including the initial one, are poorer in the counterfactual than in the baseline, which explain the lower schooling and higher hours. In the exercise where T_τ does not grow, the initial generations are identical in the baseline and the counterfactual, hence they make the same schooling and labor supply decisions.

4.4 Other Implications

We show that the model can be used to shed light on two noticeable changes that occurred during the last century in the U.S. labor market at a more disaggregate level. First, the increase in average years of schooling exhibit remarkable differences across races, with schooling raising much faster for blacks than for whites. Second, there has been a substantial change in the dispersion in hours of work across the population, with the hours of the lowest paid workers decreasing much faster than the hours of the highest paid workers which declined only slightly.

4.4.1 Races

There are substantial differences in educational attainment across races. Even more striking is the pace at which these differences changed over time. Goldin and Katz (2008, Figure 1.6) show that, for a generation born in the 1870s, the schooling gap between blacks and whites was about a factor of 1.84, that is a white individual of this generation at age 35 had completed 84% more years of schooling than a black individual from the same generation. This gap in schooling has closed substantially through time. For the 1915 generation, the racial gap in schooling is 43% and for the 1970 generation only 7%. (See Figure 7.) This closing of the schooling gap across races results from the much faster increase in years of schooling for blacks.

Our model suggests two potential explanations for the racial gap in schooling and its dramatic reduction over time. First, the potential for a racial gap in life expectancy and its reduction over time. Second, the potential for a racial gap in wages and its reduction over time. Differences in life expectancy across races have limited quantitative importance. The data show that in 1900 the life expectancy at age 5 of black men was 4.5 years below that of white men. In 1980 the difference in life expectancy increased slightly to 6.3 years. Thus, the differences in life expectancy between whites and blacks increased implying that the model cannot ascribe the closing of the racial schooling gap to the change in life expectancy.[17] More-

[17]For data on life expectancy, see Carter et al (2006), series Ab670-Ab695. Unfortunately, there are no

over, the actual racial gap in life expectancy does not quantitatively generate a substantial racial gap in schooling in the model.

To assess the importance of racial differences in income we note that racial differences in earnings are not measured in the data much earlier than the 1940s. As a result, we proceed as follows. We construct time series of schooling and hours for whites and blacks based on two exogenous differences. For whites we use the baseline model. For blacks we assume a lower life expectancy T_τ of 5 years for each generation (e.g. we assume a constant difference in life expectancy across races). We also assume that blacks face a lower wage per human capital-hour than whites. We choose the racial gap in w_τ in order to match the years of schooling for the 1870 generation of blacks as in the data (3.8 years). Hence, the gap in wages is used to match the 1870 gap in schooling across races. We assume that w_τ grows as in the baseline model for both whites and blacks.

Table 3 shows the results of the experiment. We first note that years of schooling increase faster for blacks than for whites (1.11 v. 0.61% per year). This implies that the gap in years of schooling by age 35 shrinks over time. In 2000, this gap is 15% in the model (v. 7% in the data). Since the initial schooling gap is 84% in both the model and the data, the model accounts for $(15-84)/(7-84) = 89\%$ of the closing of the schooling gap between whites and blacks. We emphasize that the only factor accounting for the closing of the gap is the initial difference in wages since both wage growth and the change in life expectancy are assumed to be constant across races in the experiment. We think it is reasonable to attribute the difference in initial conditions to factors that are outside the model e.g., slavery and the fact that the civil war ended just a few years before the period of analysis. The model implies a stronger decline in the hours of work of blacks than whites. As a consequence, the earnings of blacks do not increase as fast as for whites (1.80% v. 2.01% per year). The ratio of earnings

figures for the 1860s and 1870s. In addition, the same data on total years of schooling and labor hours we use in the baseline model from Goldin and Katz (2008) and Hazan (2009) is not available across races.

per labor hour between whites and blacks shrinks from 2.72 in 1905 to 2.56 in 2000. In the data for the United States, the ratio of wages between whites and blacks in 1940 and 1950 for men with 1 to 5 years of experience was 2.14 and 1.62.[18] We conclude that differences in income across races may be at the core of understanding the large racial gap in schooling at the beginning of the twentieth century and its subsequent reduction over time and provides additional evidence of the importance of income in explaining the rise in schooling and the fall in hours of work in the U.S. economy in the last century.

4.4.2 The Distribution of Hours

Costa (2000) documents substantial differences in hours of work across individuals in the wage distribution in the 1890s, with workers at the top of the wage distribution working less hours than those at the bottom of the wage distribution. She also shows that the differences in hours of work have essentially disappeared by the 1970s and 1990s. For instance, among men aged 25-64 in 1890, individuals at the bottom decile of the wage distribution worked 10.99 hours per day versus 8.95 hours for individuals in the top decile of the wage distribution. This amounts to a ratio of hours worked between the top and bottom deciles of 0.81. For 1890, comparing instead the middle and bottom deciles yields an hours ratio of 0.96, the top and middle decile yields a ratio of 0.90 and the top two deciles a ratio of 0.99. Hence, the fact in 1890 is that better paid individuals worked less hours and the hours differences were smaller between individuals with high and similar income. In 1973 the distribution of hours is quite different. Individuals in the top of the wage distribution worked 8.22 hours per day versus 8.83 hours for those in the bottom decile. The ratio of hours is only 0.93 (0.99 when comparing the top and middle decile, 0.94 between the middle and bottom decile, and 1.0 between the top two deciles). The data from Costa (2000) also reveals that the striking

[18] See Carter et al (2006), series Ba4432.

reduction in the dispersion of hours over time is, for the most part, accounted for by the decrease in the hours of work for low-paid individuals (a 20% decline) and only marginally by the decrease in the hours of the highly-paid workers (a 8% decline).[19]

In the context of our model, an interpretation of these findings is that individuals with higher income allocate fewer hours to work. To assess the potential quantitative role of this channel we make the following modification of the model. We assume that each generation is made of two types of individuals: high and low "ability" to earn. Lifetime income for an individual would be $aw(1-\ell)H(s,x)d(s)$ where $a \in \{a^h, a^l\}$ stands for ability to earn.[20] It transpires from Proposition 1 that the high-ability individuals would work less and be more educated than the low-ability individuals. Furthermore, when faced with the same growth in income w_τ, low-ability individuals would reduce hours more than high-ability individulals. Thus, qualitatively, our model has the potential for being consistent with the features of the distribution of hours described above. We construct the following quantitative exercise. We choose a^h and a^l to reproduce the hours of worked in 1890 by the individuals in the top and bottom deciles of the wage distribution as reported by Costa (2000).[21] All other variables and parameters are as in the baseline model including the growth in wages which we assume constant across high and low ability individuals. The results of this experiment are reported in Table 4. The first part of the table reports the results of the calibration where, in 1895, the hours of work of the two types are matched to the hours of the bottom and top deciles of the wage distribution. Hence, a ratio of hours of 0.81. The high ability obtain more schooling and more earnings than the low ability. In particular, the ratio of wages (earnings per hour) between high and low ability is 2.05 in the model. The ratio of earnings between the 90 to 10 percentile of the earnings distribution is 2.81 in Goldin and Katz (2001, Table 2.1). The

[19] Costa (2000) reports similar findings across industries and occupations.

[20] Similar results would arise if we assume ability to learn where the ability parameter appears in the human capital production function.

[21] We compute decision for the 1860 generation which is of age 35 in 1895.

second part of the table shows the implications for hours, schooling and wages in 1975, that is for individuals born in 1940 which are 35 year old in 1975. The difference with the 1895 result is that individuals face a longer life expectancy and a higher level of productivity. Both are chosen in line with the growth rate of w_τ and T_τ implied by the results of Section 4.1. The ratio of hours between the two groups declines to 0.95 (vs. 0.93 in Costa (2000)). Both types have reduced their hours and increased their years of schooling. As can be seen from the last column, however, the increase in schooling and the decline in hours is more pronounced for the low type than for the high type. The differences in schooling, which was a factor of 1.29 in 1895 has decreased to a factor 1.05. The gap in hours is also reduced since the high ability types, which used to work 0.81 of the hours of the low-ability types work, in 1975, 0.95 of the hours of low-ability types. The high-ability types have reduced their hours by $7.09/8.95 - 1 = 21\%$ while the low-ability types reduced theirs by 32%. Thus, the narrowing of the distribution of hours is mostly accounted for by the reduction of the hours of the low-paid workers, as in the U.S. data.

5 Discussion

5.1 The Cost of Education

In the model, schooling has two costs: a time cost since individuals do not work while in school and a goods cost since individuals purchase educational services x. We assumed in our baseline model that the relative price of goods services is constant over time and equal to one. Recalling that years of schooling implied by the model depart from the data around 1920, we note that this is around the time when the high-school movement started. Indeed, Goldin and Katz (2008, chapter 6) place the high school movement between 1910 and 1940.

Goldin and Katz emphasize that a significant aspect of the movement was the increased number of educational institutions, both private and public, during this period. We thus interpret this movement as a reduction in the cost of schooling.

In order to capture this phenomena and gauge its quantitative importance in the context of the model we proceed as follows. We label the relative price of educational services by q, i.e., in order to purchase x units of educational services an individual must give up qx units of consumption. Thus, the individual's inter-temporal budget constraint is of the form

$$c \int_0^{T_\tau} e^{-ru} du + q_\tau x = w_\tau H(s,x)(1-\ell) \int_s^{T_\tau} e^{(g-r)u} du.$$

We then assume that q_τ is constant and equal to 1 until 1920 and that it declines at the rate g_q thereafter. We contemplate different values for g_q. Figure 8 reports the results for years of schooling and hours of work for $g_q = 20$ percent. A substantial reduction in the relative cost of education can bring the implications of the model for years of schooling and hours of work much closer to data. In this case the model accounts for 86 percent of the increase in years of schooling (v. 80 in the baseline). This result illustrates the potential importance of the relative price of educational services in a model of the trend in schooling for the period starting around 1920. We note that our model, augmented with a relative price for educational services, implies that the ratio w_τ / q_τ^γ is critical for the individual's decision on years of schooling and hours of work.[22] Hence, one interpretation of the results in this experiment is that both a decline in the relative cost of education (represented by a decline in q_τ) starting around 1920 as well as a faster increase in w_τ resulting from skill-biased technical change starting around 1940 could be explaining the faster increase in years of schooling and

[22]The intertemporal budget constraint of an individual, after solving out for educational expenditures, is

$$c = \frac{\kappa}{a_{c,\tau}} \left[\frac{w_\tau}{q_\tau^\gamma} (1-\ell) h(s) d_\tau(s) \right]^{\frac{1}{1-\gamma}}.$$

decline in hours observed in the data since 1920 relative to the baseline model. Explicitly modeling and measuring these sources of variation over time are important elements that we leave for future research.[23]

5.2 Importance of Labor Supply

An implication of substantial changes in labor supply is a relatively low income elasticity of schooling. This elasticity is of interest in a variety of contexts (e.g., the development and labor literatures). A critical aspect of the low elasticity arises from the substantial decline in hours of work during the period of analysis. For a given increase in wages, a reduction in hours of work amounts to, other things equal, a reduction in the return to human capital accumulation.

To illustrate the importance of changes in labor supply during the period of analysis, we calibrate a version of the model where $\alpha = 0$ (individuals do not value leisure) and we set a constant labor supply to 49 hours per week, which corresponds to the average observed in the U.S. data between 1870 and 1970 (from 58 to 40). We choose $\bar{c}$, g and θ in order to match the following three targets: (i) 7 years of schooling for the 1870 generation; (ii) 2 percent growth in income per capita; and (iii) a ratio of subsistence consumption to income per capita of 41 percent for the 1870 generation. The first two targets were part of the baseline calibration. The last target deserves an explanation since it was not part of the baseline calibration strategy. By keeping labor supply constant, we are effectively loosing an observation on labor supply that can be used to restrict a parameter in the human capital technology. In the spirit of Blinder and Weiss (1976), we view this property as a virtue of our baseline calibration. The additional target we consider is the ratio of subsistence consumption to

[23] See Restuccia and Vandenbroucke (2012) for a detailed analysis of the importance of skill-biased technical change in the U.S. economy since 1940.

income which corresponds to the statistic implied by the baseline calibration. Thus, the spirit of this exercise is to be as close as possible to our baseline calibration.

We find that the model without a consumption-leisure tradeoff generates a faster increase in years of schooling: a 94 percent change between instead of a 81.5 percent change in the baseline. As a consequence, the model accounts for 92 percent of the actual increase in years of schooling versus 80 percent in the baseline. However, in order to generate the 2 percent annual increase in income, we only need the wage per unit of human capital to increase at a rate of 1.5 percent per year, as opposed to 1.9 percent in the baseline calibration. Hence, abstracting from the substantial decline in labor supply yields an income elasticity of schooling that is 37 percent larger than in the baseline.

5.3 Sensitivity

Our baseline calibration uses $\psi = 0.3$. To assess the sensitivity of our results to this choice we consider two alternative values, $\psi = 0.35$ and $\psi = 0.25$. For each value of ψ, we recalibrate the model using the same method as described in Section 4.1. The results are displayed in Table 5. The main conclusions from our baseline calibration remain unaltered. First, for schooling, the baseline simulation (when both w_τ and T_τ increase) accounts from 68 to 86 percent of the increase in schooling versus 80 percent in our baseline calibration. For hours the results are remarkably close to those in the baseline calibration. Decomposing the forces, we find that the contribution of life expectancy to the rise in schooling is somewhat sensitive to ψ: from 11 to 29 percent of the increase in schooling versus 16 percent in our baseline calibration. The contribution of w_τ to the rise in schooling is from 32 to 55 percent versus 49 in the baseline. In terms of hours there are no noticeable differences in the contribution of life expectancy as ψ changes. Since in this exercise we recalibrate the model it is important to identify which parameters are adjusted for the different values of ψ. The main difference with the baseline calibration is in the value of β, the weight of schooling time in the utility function, while $\bar{c}$ the subsistence level of consumption and g the growth rate of w_τ remain almost the same.

Our second sensitivity exercise is with respect to the choice of γ, the share of goods in the human capital technology. We used $\gamma = 0.1$ in our baseline calibration. We recalibrate the model under two alternative values: $\gamma = 0.0$ and $\gamma = 0.2$. Table 6 shows the results. We find that, jointly life expectancy and productivity account from between 75 to 85 percent of the observed trend, versus 80 in the baseline calibration. For hours the results are the same as in the baseline. The respective contribution of w_τ and T_τ differ from our baseline but the main message remains that T_τ matters little for the change in hours and that its contribution to the change in schooling is noticeable, but lower than that of w_τ.

6 Conclusions

We considered a model of human capital accumulation and labor supply to quantitatively assess the contribution of exogenous variations in productivity and life expectancy in accounting for the secular increase in educational attainment and the decrease in hours of work observed between 1870 and 1970. We find that the increase in wages and life expectancy account for 80 percent of the increase in years of schooling and 88 percent of the reduction in hours of work. Wages alone account for the bulk of the increase in schooling (75 percent) and the decline in hours (97 percent). Life expectancy plays a significant role in the increase in schooling, accounting by itself for 25 percent of the increase, but its contribution to the decline in hours is small. The income effect embedded in our model also generates predictions that are in line with more disaggregate observations. We show that our model can shed light on the faster increase in schooling experienced by blacks relative to whites since the late nineteenth century in the United States. We also show that the model is consistent with the more pronounced decline in hours of work for individuals at the lower end of the income distribution relative to individuals at the higher end of the income distribution. Finally, we argued that abstracting from the substantial decline in hours of work during this time period critically affects the connection between labor income and schooling.

Admittedly, the model does not account for all the increase in years of schooling. Other forces may be at work. A reduction in the cost of acquiring education in the form of higher-education institutions as emphasized by Goldin and Katz (2008) may be important, specially since the schooling implications of the model start to depart from the data around 1920. We also abstracted from skill-biased technical change which may be important after 1940. We plan to incorporate some of these important features in our future research.

A Proof of Proposition 1

Define $G(\ell) \equiv V'(\ell)(1-\ell)$, $Z(c) \equiv U'(c)c$ and note that $G, Z > 0$ and that $G', Z' < 0$. After substituting the optimal value of x into the inter-temporal budget constraint, define consumption as a function of s and ℓ along the inter-temporal budget constraint:

$$C(s,\ell) = \frac{\kappa}{a_{c,\tau}} \left[w(1-\ell) h(s) d(s) \right]^{\frac{1}{1-\gamma}}$$

where $\kappa = \gamma^{\gamma/(1-\gamma)} - \gamma^{1/(1-\gamma)}$. Finally, define $A_\tau(s) = h'(s)/h(s) + d'_\tau(s)/d_\tau(s)$, and note that $C_s = C(s,\ell)A(s)/(1-\gamma)$ and $C_\ell = -C(s,\ell)(1-\ell)^{-1}/(1-\gamma) < 0$.

The first order conditions (4) and (5) can now be expressed as

$$\alpha(1-\gamma)G(\ell) - Z(C(s,\ell)) = 0,$$
$$\beta(1-\gamma)W'(s) + a_{c,\tau}Z(C(s,\ell))A_\tau(s) = 0$$

Note that $A < 0$ and $A' < 0$. The first inequality is derived from the first order condition with respect to s when $\beta > 0$. The second inequality derives from the functional form of A. Namely, $h'(s)/h(s) = \theta s^{-\psi}$ which is decresing in s as long as θ and ψ are positive (as is the case in our calibration). Also, we have $d_\tau(s) = \int_s^{T_\tau} e^{(g-\rho)u} du = \left[e^{(g-\rho)T_\tau} - e^{(g-\rho)s} \right] / (g-\rho)$ so that $d'_\tau(s) = -e^{(g-\rho)s}$ and

$$\frac{d'_\tau(s)}{d(s)} = \frac{g-\rho}{1-e^{(g-\rho)(T_\tau - s)}}.$$

which is negative and decreasing in s. Note that $d'_\tau(s)/d_\tau(s) \to g-\rho$ as $T_\tau \to \infty$.

Implicitly differentiating the first order conditions with respect to s, ℓ and w_τ yields

$$\begin{aligned} 0 &= \alpha(1-\gamma)G'd\ell - Z'\left(C_s ds + C_\ell d\ell + \frac{\partial C(s,\ell)}{\partial w} \right) \\ 0 &= \beta(1-\gamma)W''ds + a_{c,\tau}AZ'\left(C_s ds + C_\ell d\ell + \frac{\partial C(s,\ell)}{\partial w} \right) + a_{c,\tau}ZA'ds. \end{aligned}$$

After rearranging one obtains

$$\frac{ds}{dw} = -\frac{\partial C(s,\ell)/\partial w}{\Delta} \text{ and } \frac{d\ell}{dw} = -\frac{\partial C(s,\ell)/\partial w}{\Phi},$$

where

$$\Delta = \overbrace{C_s}^{-} - \overbrace{\underbrace{\left(\overbrace{C_\ell}^{-} - \alpha\overbrace{(1-\gamma)\frac{G'}{Z'}}^{+}\right)}_{-}\frac{\overbrace{\frac{a_{c,\tau}}{1-\gamma}ZA' + \beta W''}^{-}}{\underbrace{\alpha a_{c,\tau}AG'}_{+}}}^{+} < 0$$

and

$$\Phi = \underbrace{C_\ell - \alpha(1-\gamma)\frac{G'}{Z'}}_{-} - \underbrace{\frac{\alpha a_{c,\tau}AG'}{\frac{a_{c,\tau}}{1-\gamma}ZA' + \beta W''}}_{+}C_s < 0$$

and where the signs of the elements of Δ and Φ are are derived from the properties of the functions A, C, G, Z and W. Since $\partial C(s,\ell)/\partial w > 0$, it follow that $ds/dw > 0$ and $d\ell/dw > 0$.

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Table 1: Calibration

Preferences	$\rho = 0.04$, $\alpha = 2$, $\beta = 4.95$
	$\bar{c} = 0.30$
Technology	$\gamma = 0.1$
	$\theta = 0.05$, $\psi = 0.3$
Productivity	$g = 0.019$, $w(1870) = 1$
Demography	$T(\tau) = 0.1716 \times \tau - 279.38$

Table 2: Percentage change in schooling, hours, and income

	U.S.	Model		
	Data (1)	Baseline (2)	only T_τ grows	only w_τ grows
Schooling	+101.5	+81.5	+13.0	+39.5
relative to (1)	1.00	0.80		
relative to (2)		1.00	0.16	0.49
Hours	−31.2	−27.5	−0.7	−27.0
relative to (1)	1.00	0.88		
relative to (2)		1.00	0.03	0.98
Income	2.00	2.00	0.02	1.86

Note: The numbers for income are annualized rates of growth.

Table 3: Racial Differences in Schooling and Hours

	1905	1950	2000	Annualized change (%)
Schooling (years):				
White	7.0	10.0	12.5	0.61
Black	3.8	7.6	10.9	1.11
Ratio	1.84	1.33	1.15	
Hours (per week):				
White	58.2	47.2	42.5	-0.33
Black	81.8	56.9	46.0	-0.60
Ratio	0.71	0.83	0.92	
Wages:				
White	1.5	4.5	14.1	2.35
Black	0.6	1.7	5.5	2.42
Ratio	2.72	2.63	2.56	

Table 4: The Distribution of Hours

	Ability		Ratio
	High	Low	(High/Low)
1895:			
Hours (per day)	8.95	10.99	0.81
Schooling (years)	7.4	5.8	1.29
Wages	3.61	1.76	2.05
1975:			
Hours (per day)	7.09	7.49	0.95
Schooling (years)	11.8	11.2	1.05
Wages	21.25	10.52	2.02

Table 5: Percentage change in schooling, hours, and income under alternative ψ

		U.S.	Model		
		Data (1)	Baseline (2)	only T_τ grows	only w_τ grows
$\psi = 0.35$	Years of school	+101.5	+69.1	+20.1	+22.1
	relative to (1)	1.00	0.68		
	relative to (2)		1.00	0.29	0.32
	Hours	−31.2	−27.7	−0.9	−27.2
	relative to (1)	1.00	0.89		
	relative to (2)		1.00	0.03	0.98
	Income	2.00	2.00	0.04	1.81
$\psi = 0.25$	Years of school	+101.5	+87.3	+9.8	+48.0
	relative to (1)	1.00	0.86		
	relative to (2)		1.00	0.11	0.55
	Hours	−31.2	−27.4	−0.7	−26.9
	relative to (1)	1.00	0.88		
	relative to (2)		1.00	0.02	0.98
	Income	2.00	2.00	0.01	1.88

Note: The numbers for income are annualized rates of growth.

Table 6: Percentage change in schooling, hours, and income under alternative γ

		U.S.	Model		
		Data (1)	Baseline (2)	only T_τ grows	only w_τ grows
$\gamma = 0.2$	Years of school	+101.5	+75.7	+19.2	+29.3
	relative to (1)	1.00	0.75		
	relative to (2)		1.00	0.25	0.39
	Hours	−31.2	−27.6	−1.2	−27.0
	relative to (1)	1.00	0.88		
	relative to (2)		1.00	0.04	0.98
	Income	2.00	2.00	0.04	1.81
$\gamma = 0.0$	Years of school	+101.5	+86.0	+8.4	+47.8
	relative to (1)	1.00	0.85		
	relative to (2)		1.00	0.10	0.56
	Hours	−31.2	−27.4	−0.4	−27.0
	relative to (1)	1.00	0.88		
	relative to (2)		1.00	0.01	0.99
	Income	2.00	2.00	0.00	1.90

Note: The numbers for income are annualized rates of growth.

2.1).

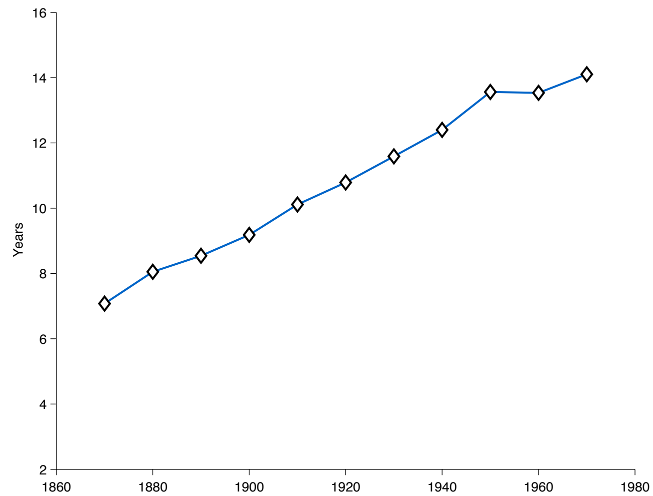

Figure 1: Years of school completed by whites at age 35 by birth cohort

Source: Goldin and Katz (2008, Figure 1.5).

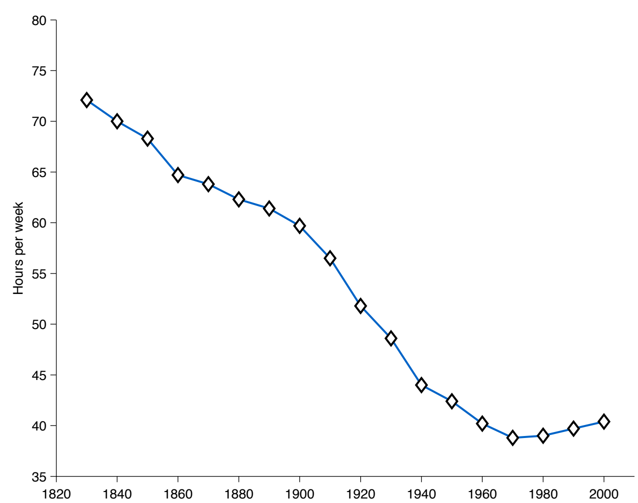

Figure 2: Weekly hours worked per worker

Source: Kendrick (1961, Table A-IX), McGrattan and Rogerson (2004, Table 1) and Whaples (1990, Table 2.1).

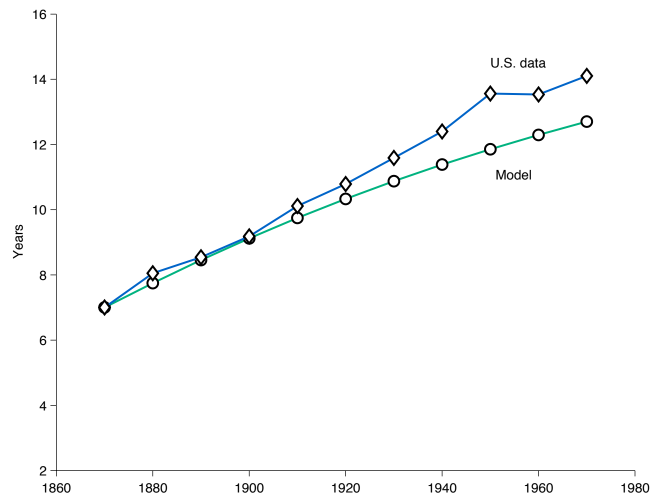

Figure 3: Years of schooling by birth cohort – model and U.S. data

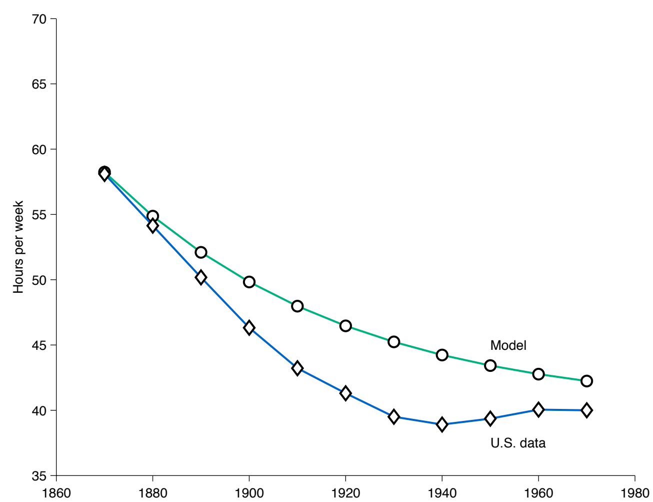

Figure 4: Hours by birth cohort – model and U.S. data

Note: We associate the hours worked of an individual of cohort τ with the workweek observed in $\tau + 35$. For example, the 1900 cohort reaches age 35 in 1935, so we compare the model's prediction for hours of this cohort with the workweek in 1935, which we derive from Figure 2.

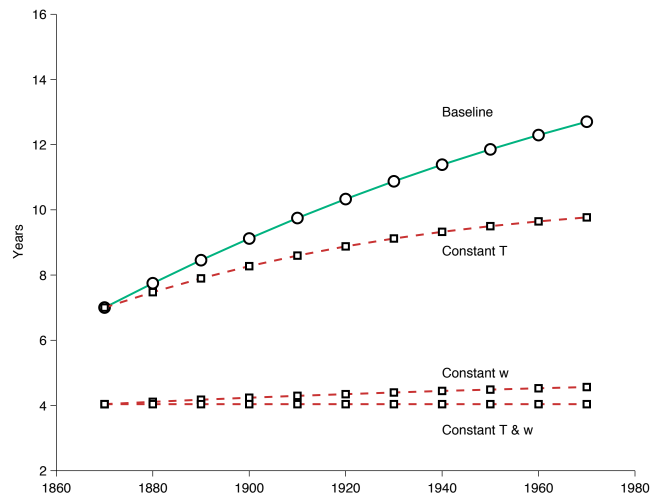

Figure 5: Years of schooling by birth cohort – baseline and counterfactual experiments

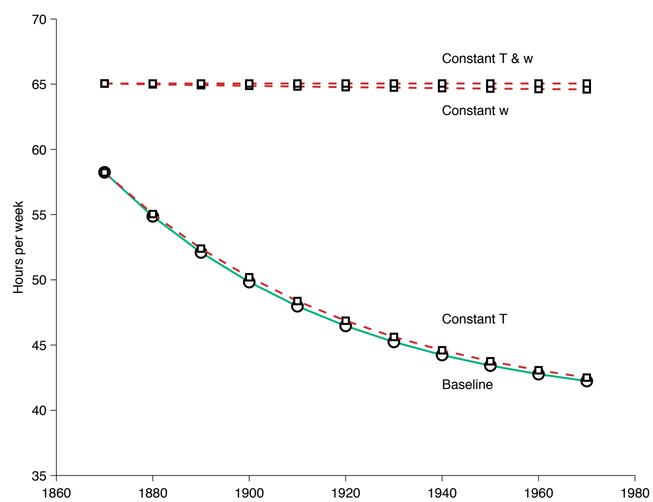

Figure 6: Hours by birth cohort – baseline and counterfactual experiments

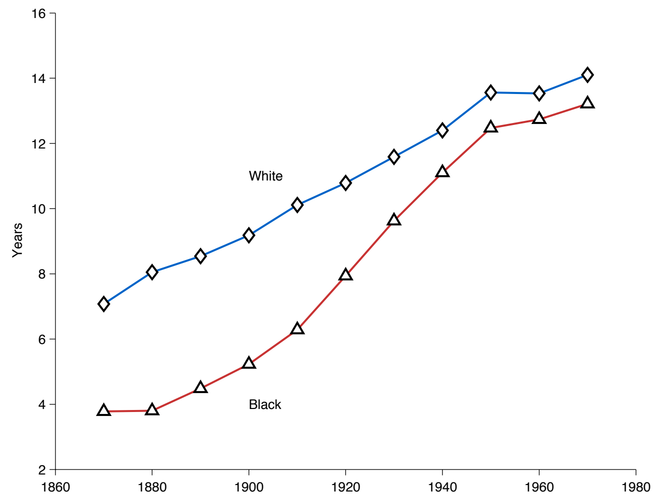

Figure 7: Years of schooling completed by birth cohort by race

Source: Goldin and Katz (2008, Figure 1.6).

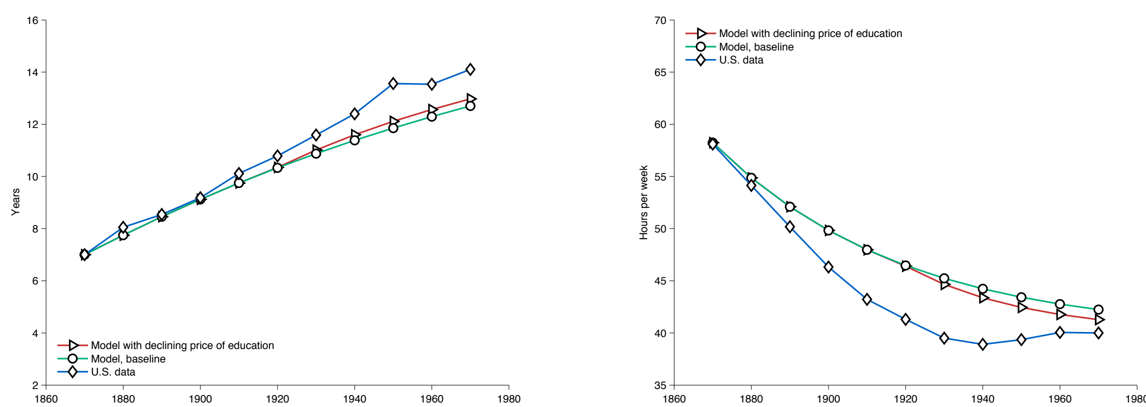

Figure 8: Years of schooling and hours by birth cohort – U.S. data, baseline model and experiment with declining relative price of educational services