The Evolution of Education: A Macroeconomic Analysis

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Abstract 
Between 1940 and 2000 there has been a substantial increase of educational attainment in the United States. What caused this trend? We develop a model of human capital accumulation that features a non-degenerate distribution of educational attainment in the population. We use this framework to assess the quantitative contribution of technological progress and changes in life expectancy in explaining the evolution of educational attainment. The model implies an increase in average years of schooling of 24 percent which is the increase observed in the data. We find that technological variables and in particular skill-biased technical change represent the most important factors in accounting for the increase in educational attainment. The strong response of schooling to changes in income is informative about the potential role of educational policy and the impact of other trends affecting lifetime income.

Keywords: educational attainment, schooling, skill-biased technical progress, human capital. JEL codes: E1, O3, O4.

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1 Introduction

One remarkable feature of the twentieth century in the United States is the substantial increase in educational attainment of the population. Figure 1 illustrates this point. In 1940, 61 percent of white males aged 25 to 29 had not completed high-school education, 31 percent had a high-school degree but did not enroll in or finish college, and 8 percent completed a college degree.\textsuperscript{1} The picture is remarkably different in 2000 when only 8 percent of the same population did not complete high-school, 62 percent completed high-school, and 30 percent completed college. While our focus is on white males, Figure 1 shows that these trends are broadly shared across genders and races. The question we address in this paper is: What caused this substantial and systematic rise of educational attainment in the United States? Understanding the evolution of educational attainment is relevant given the importance of human capital on the growth experience of the United States as well as nearly all other developed and developing countries.

There are several potential explanations for the trends in educational attainment. Our objective is to assess the quantitative relevance of a subset of these, namely the changes in the returns to schooling induced by technical change and life expectancy.\textsuperscript{2} This focus is motivated by empirical evidence that has identified systematic changes in the returns to education between 1940 and 2000 in the United States as well as by quantitative research showing a substantial response of educational attainment to changes in educational policy.\textsuperscript{3} To illustrate the changes in the returns to schooling we use the IPUMS samples for the 1940 to 2000 U.S. Census to compute labor earnings of employed white males of a given cohort.

\textsuperscript{1}In what follows we refer to the detailed educational categories simply as less than high-school, high-school, and college.

\textsuperscript{2}We acknowledge that other potential explanations such as changes in the direct or indirect costs of schooling, changes in credit constraints, and changes in social norms can be important and deserve a quantitative assessment. However, we abstract from these potential alternative explanations in this paper.

\textsuperscript{3}See Heckman, Lochner and Todd (2003) for empirical evidence and Keane and Wolpin (1997), Restuccia and Urrutia (2004), and the references therein for quantitative analysis.
across three educational groups: less than high-school, high-school, and college. Figure 2 shows the corresponding relative labor earnings between college and high-school and between high-school and less than high-school.

Figure 2 deserves a few comments. First, there is a noticeable decrease in both measures of relative earnings during the 1940s, followed by an upward trend until 2000. The decline during the 1940s has been documented elsewhere. Acemoglu (2002, Figure 1) shows a similar pattern for the college premium and Goldin and Katz (2008, Figure 8.1) show a decline in the college graduate and high-school graduate premia that started as early as 1915 and lasted until 1950. Kaboski (2009, Figure 1) reports that the Mincerian returns to schooling exhibit the same pattern as our measure of relative earnings. In addition, other measures of inequality show a \( \cup \)-shape over the course of the twentieth century. For instance, the share of income received by the top 1 (5 and 10) percent of the income distribution was 16 (30 and 40) percent in 1919. It declined to 11 (23 and 33) percent in 1949 and increased to 15 (30 and 41) in 1998.\(^4\) Finally, Kopczuk, Saez and Song (2010, Figure 1) show that the Gini coefficient for earnings of workers, gathered from Social Security data since 1937, exhibits a \( \cup \)-shape too. Second, Figure 2 shows relative *contemporaneous* earnings which provide relevant information, but are only indirect measures of the returns to schooling. Economic theory dictates that rational, forward-looking individuals base their decisions on measures of lifetime rather than contemporaneous income.\(^5\) In particular, in the model we consider in this paper, the decisions of individuals depend on ratios of lifetime earnings measured in present value and compared across possible schooling choices. For such forward-looking individuals year-to-year or decade-to-decade fluctuations in relative contemporaneous earnings are not necessarily indicative of fluctuations in the returns to schooling —at least not of the same magnitude. We use this feature to justify the absence of short-term fluctuations in the


\(^5\)This is the case when individuals have access to perfect credit markets, an assumption we make in our model.
exogenous technological variables driving the returns to schooling in the baseline model. Third, as emphasized in the literature most notably by Heckman, Lochner, and Taber (1998), the earnings information contained in Figure 2 refers to both quantities and prices of skills. As will be emphasized in our quantitative analysis, small variations in skill prices produce large variations in earnings across schooling groups with the amplification being done by human capital accumulation both in the form of quantity of schooling as well as quality.

We develop a model of human capital accumulation that builds upon Becker (1964) and Ben-Porath (1967). The model features overlapping generations of individuals living for a finite number of years. Individuals are heterogeneous with respect to their life expectancy, which is specific to the generation in which they are born and with respect to the (dis)utility from schooling. This last assumption is common in both the macro literature, e.g. Bils and Klenow (2000), as well as the empirical labor literature, e.g. Heckman, Lochner, and Taber (1998).

There are three levels of schooling that individuals can choose from and there is a standard human capital production function that requires the inputs of time (schooling) and goods. The discrete schooling choice is motivated by the fact that the distribution of people across years of schooling is strongly concentrated around years of degree completion. This feature allows the model with three levels of schooling to capture distribution statistics such as those presented in Figure 1. At the aggregate level, the consumption good is produced with a constant-returns-to-scale technology requiring the input of human capital from workers of all generations and all levels of schooling. The human capital of workers with different levels of schooling are imperfect substitutes in production, therefore there are three different wage rates, one for each school level, clearing the labor markets. The exogenous variables are life expectancy, which increases from one generation to the next, total factor productivity (TFP), and skill-biased productivity parameters in the production function.

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6An additional source of heterogeneity may be through “learning ability.” Navarro (2007) finds, however, that individual heterogeneity affects college attendance mostly through the preference channel.
We implement a quantitative experiment to assess the importance of changes in TFP and skill-biased productivity, as well as life expectancy, on the rise of educational attainment. We use data on life expectancy to discipline its model counterpart. TFP and skill-biased technical change are not directly observable. To discipline their paths we observe that earnings data are the empirical counterparts of the product of human capital and wages per unit of human capital, both being endogenous objects determined in an equilibrium of our model. Thus, earnings data can be used to discipline the paths of the neutral and skill-biased productivity variables in the model. More generally, we choose the paths of these variables and other parameters so that the model’s equilibrium wages and schooling choices match a set of key statistics: educational attainment in 1940, the trend in relative earnings across schooling levels from 1940 to 2000, and the average growth rate of gross domestic product (GDP) per worker between 1940 and 2000. This quantitative strategy follows the approach advocated by Kydland and Prescott (1996). In particular, we emphasize that the parameter values are not chosen to fit the data on educational attainment from 1940 to 2000, instead they are chosen to mimic the trends in relative earnings which, as mentioned earlier, provide some relevant information about the returns to schooling.

Our findings can be summarized as follows. First, our baseline experiment shows that changes in life expectancy, TFP, and skill-biased technical variables, generate a substantial increase in educational attainment. Specifically, the model implies a substantial increase in college attainment while under-predicting the decrease in the less-than-high school group and under-predicting the increase in the high-school group. In terms of the average years of schooling attained by generations reaching 25-29 between 1940 and 2000, the model generates an increase of 24.3 percent, from 10.8 in 1940 to 13.4 in 2000, which is the same as the corresponding statistics computed using U.S. data. Second, we conduct a set of counterfactual experiments revealing that the most relevant driving variables generating the rise in educational attainment are, in order of importance, the “college-biased” technical variable,
the “high-school-biased” technical variable, and life expectancy. If the “college-biased” technology did not rise as in our baseline experiment, average years of schooling would increase by 14.2 percent instead of 24.3 percent in the baseline. Interestingly, if the “high-school-biased” technology did not rise as in the baseline, average years of schooling would increase faster than in the baseline (26.8 percent). The reason for this result is that the lack of productivity growth at the high-school level implies large returns to college versus high-school, driving a strong increase in college attainment. When life expectancy remains constant educational attainment still increases by 22.8 percent between 1940 and 2000 versus 24.3 percent in the baseline. Hence, changes in life expectancy only explain about 6 percent of the increase in average years of schooling in the model. Overall, we find that schooling responds strongly to skill-biased technical change, a result driven by a large change in relative earnings and not from an implausibly large elasticity of educational attainment. This response of schooling is informative for educational policy and for the evaluation of related trends. For instance, the effect on educational attainment of the gaps in lifetime earnings associated with racial or gender differences or of rising costs of education.

Our paper contributes to a literature on human capital, schooling and income inequality. It complements this literature by analyzing the trends in schooling in a model that remains consistent with the general trends in wage inequality, while the existing literature’s main focus is on explaining rising wage inequality. As discussed earlier, certain patterns of wage inequality are not as relevant for schooling decisions. To the best of our knowledge there has been no systematic attempt to quantify and decompose the forces that lead to changes in educational attainment since 1940. Prominent papers such as Heckman, Lochner, and Taber (1998) and Guvenen and Kuruscu (2009) focus on the determinants of rising wage inequality since the late 1960s or early 1970s rather than on educational attainment itself. More importantly, these models are not suited for studying the longer upward trend in the

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7For a review of the evidence on inequality, see Topel (1997) and Acemoglu (2002).
evolution of educational attainment which can be traced back to at least 1940. In addition, these papers do not explicitly decompose the forces explaining the trends in schooling. Lee and Wolpin (2006) and Lee and Wolpin (2010) propose and estimate dynamic equilibrium models of the labor market with schooling and occupational choices. The main focus in these papers is on accounting for intersectoral mobility and the evolution of wages and employment structure in the U.S. between 1968 and 2000. Although the models have also implications for the evolution of schooling, there is no explicit decomposition of the causes of the changes in schooling over time.

Our work also contributes to a literature in macroeconomics assessing the role of technical progress on a variety of trends. It also relates to the labor literature emphasizing the connection between technology and education such as Goldin and Katz (2008) and the literature on wage inequality emphasizing skill-biased technical change. We recognize that changes in the returns to education may not be the only explanation for rising education during this period. For instance Glomm and Ravikumar (2001) emphasize the rise in public-sector provision of education. We also recognize that educational attainment was rising well before 1940 and that changes in the returns to education may not be a contributing factor during the earlier period. Our focus on the period between 1940 and 2000 follows from data restrictions and the emphasis in the labor literature on rising returns to education as the likely cause of rising wage inequality. In the broader historical context, other factors may be

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8Heckman, Lochner, and Taber (1998, page 33) report that their model produces large jumps in college enrollment, inconsistent with the path observed in Figure 1. They discuss how their model could be modified to alleviate this concern. Guvenen and Kuruscu (2009, Section 4.4) report that their model does not capture the behavior of college enrollment in the short run, and that the level of hours supplied by college-educated individuals is significantly below the counterpart in the U.S. data.

9He and Liu (2008) propose a model of skill accumulation to study rising wage inequality but do not explicitly model schooling, rather the accumulation of the stock of skills (human capital). He (2011) considers a model of discrete schooling choices with a focus on separating the contribution of demographic and technological factors in understanding the patterns of wage inequality. Moreover, the model in He (2011), while consistent with the evolution of wage inequality, produces only a small increase in educational attainment compared to the data.

10See Greenwood and Seshadri (2005) and the references therein.

11See for instance Juhn, Murphy, and Pierce (1993) and Katz and Autor (1999).
more important such as the development of educational institutions and declines in schooling costs.\textsuperscript{12}

The rest of the paper proceeds as follows. In the next section we describe the model. In Section 3 we calibrate the model and conduct the main quantitative experiments. Section 4 discusses our main results in the context of various alternative specifications and experiments. We conclude in Section 5.

2 Model

2.1 Environment

Time is discrete. We use $t$ to refer to calendar time. The economy is inhabited by overlapping generations of individuals living for a finite number of periods. The size of each generation is normalized to one. We use $\tau$ to refer to a generation, i.e. generation $\tau$ is composed of individuals of age 1 at date $t = \tau$. Individuals of the same generation are heterogeneous with respect to the intensity of their (dis)taste for schooling which is represented by $a \in \mathbb{R}$, and is distributed according to the time-invariant cumulative distribution function $A$.\textsuperscript{13} Individuals of different generations differ by their life expectancy, $T(\tau)$. Define $\mathcal{A} = \mathbb{R} \times \{0, 1, \ldots, \infty\}$ as


\textsuperscript{13}We abstract from ability to learn and earn across individuals as another potential source of differences in educational attainment. We recognize however that ability may be important for schooling choices. There are three main motivations for our abstraction. First, there is no complete agreement in the literature as to how ability is actually changing over time for the different schooling groups. Depending upon assumptions, the mapping of ability and schooling may imply that, over time, average ability decreases, stays constant, or even increases. Second, empirical estimates from structural models that have included both preference variation and ability variation seem to find that ability selection is less relevant in determining schooling choices than the preference variation (see for instance Navarro (2007)). Third, for the type of experiment we perform in the paper, namely that we infer growth in technology to reproduce paths of earnings, changes in average ability would require different growth in technology variables, mitigating the potential different impact on schooling over time.
the set of individual types, i.e., pairs composed of a schooling taste parameter, $a \in \mathbb{R}$, and a
generation index $\tau \in \{0, 1, \ldots, \infty\}$. We assume that $a$ and $T(\tau)$ are observed by individuals
before any decisions are made. We also assume that there is no uncertainty and that credit
markets are perfect. We use $r$ to denote the gross rate of interest which is exogenously given
and equals the inverse of the subjective discount factor.

An individual can accumulate human capital through schooling before entering the labor
market. Let $h(s, e)$ denote his human capital once he starts working. It is a function of $s$, the number of years he spent in school, and $e$ which represents services affecting the quality
of education purchased by the individual while in school. Both $s$ and $e$ are choice variables.
There are three levels of schooling labeled 1, 2 and 3. To complete level $i$ an individual has
to spend $s_i \in \{s_1, s_2, s_3\}$ periods in school and, therefore, cannot work before reaching age
$s_i + 1$. We assume $s_1 < s_2 < s_3$. Thus, level 1 is the model’s counterpart to the less than
high-school category discussed in Section 1. Similarly, level 2 corresponds to the high-school
category and level 3 to college.

There is a single consumption good produced with a constant-returns-to-scale technology
using three inputs: the human capital of workers who completed $s_1$ years of schooling, the
human capital of workers who completed $s_2$ years of schooling and, finally, the human capital
workers who completed $s_3$ years. The three types of human capital are not perfect substitutes
in production, thus the wage rate per unit of human capital is specific to each schooling level:
$w_t(s)$ at date $t$. 
2.2 Individuals

Preferences are defined over consumption sequences and time spent in school. They are represented by the following utility function for an individual of type \((a, \tau)\):

\[
\sum_{t=\tau}^{\tau+T(\tau)-1} \beta^{t-\tau} \ln(c_t) - as,
\]

where \(\beta \in (0, 1)\) is the subjective discount factor, \(c_t\) is consumption during period \(t\) and \(s\) represents years of schooling. Note that \(a\) can be positive or negative so that schooling provides either a utility benefit or a cost. The distribution of \(a\) is normal with mean \(\mu\) and standard deviation \(\sigma\):

\[
A(a) = \Phi \left( \frac{a - \mu}{\sigma} \right),
\]

where \(\Phi\) is the cumulative distribution function of the standard normal distribution.

The optimization problem of an individual of type \((a, \tau)\) is as follows:

\[
\max_{s \in \{s_1, s_2, s_3\}} \{V_\tau(s) - as\},
\]

where \(V_\tau(s)\), the lifetime utility from consumption conditional on a schooling choice, is defined by

\[
V_\tau(s) = \max_{e, \{c_t\}_{t=\tau}^{\tau+T(\tau)-1}} \left[ \sum_{t=\tau}^{\tau+T(\tau)-1} \beta^{t-\tau} \ln(c_t) \right],
\]

subject to

\[
\sum_{t=\tau}^{\tau+T(\tau)-1} \left( \frac{1}{r} \right)^{t-\tau} c_t + e = h(s, e)W_\tau(s),
\]

where

\[
W_\tau(s) = \sum_{t=\tau+s}^{\tau+T(\tau)-1} \left( \frac{1}{r} \right)^{t-\tau} w_t(s),
\]
and

\[ h(s, e) = s^\eta e^{1-\eta}, \]  

(5)

where \( \eta \in (0, 1) \). Note that lifetime income, in the right-hand side of the intertemporal budget constraint (3), stipulates that labor income is earned from age \( s + 1 \) onward. Thus, this optimization program summarizes the various costs associated with acquiring education: the utility cost, the time cost, and the resource cost. We assume that \( s \) and \( e \) are chosen once and for all at the beginning of an individual’s life. Therefore \( e \) is the present value of educational expenditures.

Define \( I_\tau(s) \), the lifetime income net of the cost of educational expenditures for an individual of generation \( \tau \) conditional on schooling level \( s \):

\[ I_\tau(s) = \max_e \{ h(s, e)W_\tau(s) - e \}. \]

(6)

Given our assumption that the rate of interest and the subjective rate of discount are the same, i.e., \( \beta r = 1 \), the optimal consumption sequence is constant throughout an individual’s life and equal to \( I_\tau(s)(1 - \beta)/(1 - \beta^{T_\tau}) \). Hence,

\[ V_\tau(s) = \ln \left( \frac{I_\tau(s)}{1 - \beta^{T_\tau}} \right) \frac{1 - \beta^{T_\tau}}{1 - \beta}. \]

We can now turn to the characterization of an individual’s optimal choice of schooling. Note that the value functions \( V_\tau(s) - a \) are decreasing in \( a \) with slopes that are increasing (in absolute value) in \( s \). Figure 3 displays these functions and describes the schooling choice of individuals from generation \( \tau \). Specifically, an individual of type \( (a, \tau) \) chooses schooling level \( i \) over \( j \) \((i > j)\) whenever his utility cost of schooling is low enough, that is whenever \( a < a_{ij,\tau} \); where \( a_{ij,\tau} \) is the threshold value of \( a \) for which the individual is indifferent between
Given the form of $V_\tau(s)$, the threshold values $a_{ij,\tau}$ are
\[
a_{ij,\tau} = \frac{1 - \beta T(\tau)}{1 - \beta} \times \frac{1}{s_i - s_j} \times \ln \left( \frac{I_\tau(s_i)}{I_\tau(s_j)} \right).
\] (7)

The infinite support for $a$ ensures that each pair of value functions has a single intersection and that there are always non-empty sets of individuals choosing the first and third level of schooling. It is possible, however, that no individual finds it optimal to choose the second level. This situation is illustrated in panel b of Figure 3. The schooling decision of an individual of type $(a, \tau)$ is therefore described as
\[
s(a, \tau) = \begin{cases} 
  s_3 & \text{if } a \leq \min\{a_{32,\tau}, a_{31,\tau}\} \\
  s_1 & \text{if } a \geq \max\{a_{21,\tau}, a_{31,\tau}\} \\
  s_2 & \text{otherwise.}
\end{cases}
\] (8)

In summary, the optimal choices of an individual of type $(a, \tau)$ are a schooling choice $s(a, \tau)$, an educational expenditure $e(a, \tau)$ and a consumption sequence $\{c_t(a, \tau)\}_{t=\tau}^{\tau+T(\tau)-1}$ solving the optimization problem (1)–(5). The specific form of the schooling choice is described in (8), the educational expenditure is the maximizer of (6), i.e.,
\[
e(a, \tau) = s(a, \tau) \left( (1 - \eta) W_\tau(s(a, \tau)) \right)^{1/\eta},
\] (9)
implying that the human capital of the individual is
\[
h(s(a, \tau), e(a, \tau)) = s(a, \tau) \left( (1 - \eta) W_\tau(s(a, \tau)) \right)^{(1-\eta)/\eta},
\] (10)
finally, the consumption sequence satisfies

\[ c_t(a, \tau) = I_\tau(s(a, \tau)) \frac{1 - \beta}{1 - \beta^I(\tau)}. \]

### 2.3 Technology

At each date, there is a single good produced with a constant returns to scale technology. We specify the production technology along lines suggested by Goldin and Katz (2008, page 294). That is we assume a CES production function between college human capital and a CES aggregate of high-school and less than high-school human capital:

\[ F(H_{1,t}, H_{2,t}, H_{3,t}) = z_t \left( \left( z_1 t H_{1,t}^\rho + z_2 t H_{2,t}^\theta \right)^{\rho/\theta} + z_3 t H_{3,t}^\rho \right)^{1/\rho}, \]

where \( H_{i,t} \) is the demand, at date \( t \), for human capital supplied by individuals who completed the \( i \)th level of schooling. The parameters \( \rho \) and \( \theta \) govern the elasticity of substitution between inputs. Specifically, the elasticity of substitution between \( H_{1,t} \) and \( H_{2,t} \) is \( 1/(1 - \theta) \) while the elasticity of substitution between \( H_{3,t} \) and the aggregate \( (z_1 t H_{1,t}^\rho + z_2 t H_{2,t}^\theta)^{1/\theta} \) is \( 1/(1 - \rho) \). The parameters \( z_{i,t} \) \((i = 1, 2, 3)\) are school-specific, exogenous productivity terms. We label the term \( z_t \) as TFP, but we note that if \( \rho \) is different than \( \theta \) changes in \( z_t \) are not neutral because they affect the marginal products of the \( H_{i,t} \)’s differently. Since our focus is on long-run trends we assume constant rates of growth. Thus,

\[ z_{i,t} = z_{i,0} \times g_i^t \quad \text{for } i = 1, 2, 3 \quad \text{and } z_t = z_0 \times g^t, \]

where the \( g_i \)’s are the (gross) rates of growth of the \( z_{i,t} \)s, and the \( z_{i,0} \)s are initial conditions.
The firm’s objective is to maximize its profit:

$$\max_{H_{1t}, H_{2t}, H_{3t}} F(H_{1t}, H_{2t}, H_{3t}) - \sum_{i=1,2,3} w_t(s_i)H_{it},$$

implying that the following three first order conditions must hold at each date $t$:

$$0 = \frac{\partial}{\partial H_{it}} F(H_{1t}, H_{2t}, H_{3t}) - w_t(s_i) \quad \text{for } i = 1, 2, 3. \quad (11)$$

### 2.4 Equilibrium

An equilibrium of the labor market involves equating the supply and demand for human capital of individuals with schooling level $i$. To specify these market-clearing conditions note that the youngest individual employed at $t$ with $s$ years of schooling is of age $1$ at date $t - s$, hence, this individual is from generation $t - s$. Thus, with a utility cost of schooling of $a$, human capital of this individual at date $t$ is $h(s, e(a, t - s))$. The oldest worker at date $t$ is of age $1$ at a date $\tau^*(t)$ such that $\tau^* + T(\tau^*) - 1 = t$. Hence, the total human capital supplied at date $t$ by individuals having completed schooling level $i$ is

$$\sum_{\tau = \tau^*(t)}^{t - s_i} \int h(s_i, e(a, \tau))I\{s(a, \tau) = s_i\}A(da),$$

where $I$ is the indicator function. In this expression the discrete summation aggregates over generations working at date $t$ while the integral aggregates over the taste for schooling within each generation. The labor market clearing conditions are then

$$H_{it} = \sum_{\tau = \tau^*(t)}^{t - s_i} \int h(s_i, e(a, \tau))I\{s(a, \tau) = s_i\}A(da) \quad \text{for } i = 1, 2, 3. \quad (12)$$

**Definition 1** Given $r$, a competitive equilibrium of the labor market is composed of (i) allo-
cations for individuals of each type: \( \{ \{ c_t(a, \tau) \}_t=\tau^{T(\tau)-1}, e(a, \tau), s(a, \tau) \}_{(a, \tau) \in A} \), and allocations
for firms: \( \{ H_{1t}, H_{2t}, H_{3t} \}_{t=0}^\infty \); (ii) prices: \( \{ w_t(s_1), w_t(s_2), w_t(s_3) \}_{t=0}^\infty \); such that

1. Optimization:
   - The allocations \( \{ \{ c_t(a, \tau) \}_t=\tau^{T(\tau)-1}, e(a, \tau), s(a, \tau) \}_{(a, \tau) \in A} \) solve the optimization problem of each individual \((a, \tau) \in A\) given prices;
   - The allocation \( \{ H_{1t}, H_{2t}, H_{3t} \}_{t=0}^\infty \) solves the firm’s optimization problem given prices;

2. Market clearing:
   - Equations (12) hold at each \( t \).

It is useful to define a notation for the educational attainment of generation \( \tau \). Let then \( p_{i, \tau} \) be the proportion of members of generation \( \tau \) having completed schooling level \( i \):

\[
p_{i, \tau} = \int \mathbb{I}\{s(a, \tau) = s_i\} A(da).
\]

(13)

\( p_{i, \tau} \) is the model counterpart of the educational attainment statistics of Figure 1.\(^{14}\)

2.5 Discussion

Equations (7) and (8) establish that the schooling decision of an individual depends upon the semi-elasticity of net lifetime income across schooling levels. For instance, an increase in the

\(^{14}\)Note that for a given level of schooling the education expenditures are constant across members of the same generation regardless of their taste parameter \( a \) – see Equation (9). This implies that the human capital of individuals of the same generation with the same level of schooling is also constant across individuals regardless of their taste for schooling – see Equation (10). Thus the labor market clearing condition can also be written as

\[
H_{it} = \sum_{\tau=\tau^*(i)}^{t-s_i} p_{i, \tau} h \left( s_i, s_i \left( 1 - \eta \right) \frac{W_{\tau}(s_i)}{q_\tau} \right)^{1/\eta} \quad \text{for } i = 1, 2, 3.
\]
lifetime income from college relative to high-school, $I_\tau(s_3)/I_\tau(s_2)$, raises the threshold utility cost of entering college, $a_{32,\tau}$ and, therefore, increases college attainment. The magnitude of the increase depends upon the magnitude of the change in the threshold as well as the shape of the distribution of utility-schooling costs, $A$, as can be seen from Figure 3. More specifically, the proportion of individuals from generation $\tau$ completing college can be written, using Equation (13), as $p_{3,\tau} = A(a_{32,\tau})$ which implies that the evolution of college attainment through time is given by,

$$\frac{dp_{3,\tau}}{d\tau} = A'(a_{32,\tau}) \frac{da_{32,\tau}}{d\tau}, \quad (14)$$

and similarly, $p_{1,\tau} = 1 - A(a_{21,\tau})$ so that

$$\frac{dp_{1,\tau}}{d\tau} = -A'(a_{21,\tau}) \frac{da_{21,\tau}}{d\tau}. \quad (15)$$

Equations (14) and (15) deserve a few comments. First, the distribution of schooling-utility cost, $A$, is relevant for the quantitative impact of changes in income on educational attainment. Given this, one concern may be our assumption that $A$ is a Normal distribution. We have experimented with more flexible distributions and found that our quantitative results are essentially unchanged when the parameters of the distribution are calibrated as we do in Section 3.15 Second, changes in the critical values $a_{32,\tau}$ and $a_{21,\tau}$ are driven by changes in life expectancy and relative lifetime incomes. The relative lifetime income $I_\tau(s_i)/I_\tau(s_j)$ for an individual of generation $\tau$ contemplating school levels $i$ and $j$ writes, after some algebra, as

$$\frac{I_\tau(s_i)}{I_\tau(s_j)} = \frac{s_i}{s_j} \left( \frac{\sum_{t=\tau+s_i}^{\tau+T(\tau)-1} \left( \frac{1}{T} \right)^{t-\tau} w_t(s_i)}{\sum_{t=\tau+s_j}^{\tau+T(\tau)-1} \left( \frac{1}{T} \right)^{t-\tau} w_t(s_j)} \right)^{1/\eta}. \quad (16)$$

15We performed these experiments using the Beta distribution for two reasons. First, because the Beta has two parameters, allowing us to maintain our calibration strategy. Second, because the density of the Beta distribution can accommodate many different shapes such as uniform, bell-shaped or U-shaped and not necessarily symmetric. Our main finding is that when we let $A$ be a Beta distribution our calibration procedure implies parameters such that its density is numerically very close to the normal density we calibrate in Section 3, see Restuccia and Vandenbroucke (2010).
This expression differs from one generation to the next, implying different schooling choices, because life expectancy and the future path of wages per unit of human capital differ across generations. Skill-biased technical change has a first order effect on educational attainment because it can change disproportionately the wages $w_t(s_i)$ and $w_t(s_j)$. However, our model rules out the possibility that skill-neutral technical change matters. To see this, imagine that $w_t(s_i)$ and $w_t(s_j)$ are both multiplied by the same number for each $t$, then the ratio $I_{\tau}(s_i)/I_{\tau}(s_j)$ remains constant and educational attainment is unchanged. Whether the $w_t(s)$’s, which are endogenous, actually behave in such a way as to generate changes in educational attainment in line with the data is a quantitative question that we address in the next section.

3 Quantitative Analysis

3.1 Calibration

The first stage of our calibration strategy is to assign values to some parameters using a-priori information. We let a period represent one year. We set the gross interest rate to $r = 1.04$ and the subjective discount factor to $\beta = 1/r$. For the number of years required by each level of education we use $s_1 = 9$, $s_2 = 13$ and $s_3 = 16$. Following Bils and Klenow (2000), we choose $\eta = 0.9$. For $T(\tau)$, the life expectancy of generation $\tau$, we add Hazan (2009) measure of years spent on the market by cohort to Goldin and Katz (2008)’s figures for years of schooling achieved

\footnote{Using the IPUMS samples for the 1940 and 2000 U.S. Census, we find that the average number of years spent in school was 8.9 and 12.9 for individuals with less than high school and with high school completion. Since to solve our model we need to compute discrete sums of the form given in Equation (4), we rounded these figures to $s_1 = 9$ and $s_2 = 13$. We assume $s_3 = 16$ for all years.}
by cohort. We find $T(1900) = 47$ and $T(2000) = 64$ and assume a linear time trend to determine the life expectancy of other generations.

Our specification of the production function followed Goldin and Katz (2008). These authors provide estimates to the required elasticity parameters. They consider three values for the elasticity of substitution between the first and second level of schooling: 2, 3 and 5. We use the intermediate value 3 so that $\theta = 1 - 1/3$. They also consider 3 values for the elasticity of substitution between the third level of education and the aggregate of the first two levels: 1.4, 1.64 and 1.84. We use the intermediate value 1.64 so that $\rho = 1 - 1/1.64$.

The list of remaining parameters is

$$\theta = (\mu, \sigma, z_{1,0}, z_{2,0}, z_{3,0}, g_1, g_2, g_3),$$

which consists of the distribution parameters for the utility cost of schooling, and initial levels and growth rates for the productivity variables. We normalized the initial level of $z$ to 1:

$$z_0 = 1.$$

To assign a value to $\theta$ we build a measure of the distance between statistics generated by the model and their empirical counterparts in the U.S. data. We target the following statistics:

1. The educational attainment of the 25-29 years old in the 1940 Census, that is 61 percent of individuals attaining less than high-school and 31.2 percent of individuals attaining high-school education. The model counterpart to these statistics is educational attainment of the 1921 generation which corresponds to individuals reaching age 25 in 1940 in the U.S. data.

2. The time path of relative earnings from 1940 to 2000, displayed in Figure 2.
3. The growth rate of GDP per worker from 1940 to 2000 which was on average 2 percent. This target ensures that the growth rate of productivity does not imply unreasonable levels of economic growth.

Even though the parameter values are chosen simultaneously to match the data targets, each parameter has a first-order effect on some target. The levels of the technology parameters and their growth rates are important in matching the time path of relative earnings across schooling groups. The growth rate of TFP is important in matching growth in average labor productivity, but because TFP is not neutral it also matters for matching relative earnings. The distribution parameters, \( \mu \) and \( \sigma \), are critical in matching the distribution of educational attainment in 1940.

Formally, the calibration strategy can be described as follows. Given a value for \( \theta \) we compute an equilibrium trajectory and define the following objects. First,

\[
\hat{E}_{ij,t}(\theta) = \frac{w_i(s_i)h(s_i, t - 25)}{w_j(s_j)h(s_j, t - 25)}
\]

is the ratio of earnings between members of group \( i \) and \( j \) at the age of 25 at date \( t \) in the model. The empirical counterpart of \( \hat{E}_{32,t}(\theta) \) is the relative earnings between college and high-school, for white males between the age of 25 and 29, denoted by \( E_{32,t} \). Similarly, \( \hat{E}_{21,t}(\theta) \) is the model counterpart of \( E_{21,t} \), the relative earnings between high-school and less than high-school. Second, we define

\[
M(\theta) = \begin{bmatrix}
    p_{1,21} - 0.61 \\
    p_{2,21} - 0.31 \\
    Y_{00}/Y_{40} - 1.02^{60}
\end{bmatrix}.
\]
Then, to assign a value to $\theta$ we solve the following minimization problem

$$
\min_{\theta} \sum_{t \in T} \left( \frac{\hat{E}_{32,t}(\theta)}{E_{32,t}} - 1 \right)^2 + \left( \frac{\hat{E}_{21,t}(\theta)}{E_{21,t}} - 1 \right)^2 + M(\theta)^	op M(\theta),
$$

where $T \equiv \{1940, 1950, \ldots, 2000\}$. The first part of the objective function is the distance between the relative contemporaneous earnings implied by the model and their empirical counterpart at Census dates between 1940 and 2000. The second part of the function is the distance between the model and the rest of the targets in our procedure: educational attainment in 1940 and the growth rate of the economy between 1940 and 2000.

At this stage it is important to describe the computational experiment leading to the determination of a particular equilibrium trajectory of our model. First, observe that the decisions of generation $\tau$ depend on the path of wages from date $\tau$ onward, as summarized in $W_\tau(s)$ in Equation (4). These wages depend on the stocks of aggregate human capital at each date which, in turn, depend on the decisions of generations prior to $\tau$—see Equations (11) and (12). To circumvent this we make two assumptions. First, we assume that the economy is in a steady state until date 0 (corresponding to 1860 in the data). That is, the first step in the determination of an equilibrium trajectory is to determine educational attainment and human capital when the exogenous parameters (productivity and life expectancy) are constant at their initial values. Under this assumption we can solve for wages at any date $t$ by assuming that the human capital, at $t$, of individuals born prior to date 1 is the steady state human capital. Second, we assume that exogenous variables start growing from date 50 (corresponding to 1910 in the data). The motivation for this second assumption is to avoid a discontinuity in the schooling and human capital choices between the generations up to to date 0 and the date 1 and subsequent generations. Thus exogenous variables are constant from $t = -\infty$ to $t = 50$ and then start growing at the rate prescribed by the $g_i$'s for productivity and as prescribed by the linear trend of $T$ for life expectancy.
The calibrated parameters are presented in Table 1. The model is able to match well its calibration targets. The growth rate of output per worker is 2 percent per year. Also, the model implies a smooth path of relative earnings that captures the trend observed in the data as illustrated in Figure 4. In the initial steady state, 2 percent of a generation completed college education, 11 percent completed high school, and 87 percent attained less than high school.

### 3.2 Baseline Experiment

The main quantitative implications of the model are the time paths for the distribution of educational attainment for the three categories considered: less than high-school, high-school, and college. Figure 5 reports these implications of the model. The model implies a substantial increase in college attainment, but it under-predicts the decrease in the less than high-school group and the increase in the high-school group. The fraction of 25 to 29 year-olds with college education increases in the model by 33 percentage points from 1940 to 2000, while in the data the increase is 20 percentage points. For high-school, the model implies an increase from 31 to 39 percent between 1940 and 2000 whereas, in the data, the increase is from 31 to 61.5 percent.

To summarize our quantitative findings, we calculate average years of schooling of a given generation in the model and the U.S. data as

\[
\sum_{i=1,2,3} s_i p_{i,\tau},
\]

where \( s_i \) are the three constant schooling categories and \( p_i \) is the fraction of the relevant population that attains each schooling level. In the U.S. data average years of schooling increased by 24.3 percent, from 10.8 in 1940 to 13.4 in 2000. The model reproduces the
average years of schooling in 1940 as a calibration target and implies an average years of schooling in 2000 of 13.4. Thus, by this measure, the model accounts for 100 percent of the increase in average years of schooling between 1940 and 2000.

The path of relative earnings in the data is fairly irregular compared to the calibrated path (see again Figure 4). This may raise concerns as to the sensitivity of our results to the choice of constant growth rates for the productivity variables. We note, however, that education decisions are forward looking and in particular, in our model, individuals consider the lifetime earnings associated with each schooling choice. That relative earnings fall or increase sharply in a particular decade in the data is only relevant to the extent that it affects relative lifetime earnings calculations for some cohorts. The constant-growth-rate assumption is a useful and parsimonious way to capture the relevant patterns for relative earnings in the data. In fact, we show in section 4.2 that alternative assumptions about the variability of skill prices leaves our results almost unaltered. The most relevant deviation from our quantitative results is on the extent of foresight of individuals for the future path of skill prices. However, we show in section 4.1 that even the most conservative form of expectations—where wages are assumed to be constant throughout the lifetime—technical progress still accounts for almost 40 percent of the observed increase in educational attainment.

3.3 Decomposing the Forces

We quantitatively decompose the contributions of exogenous variables to the changes in educational attainment. We proceed by running a set of counterfactual experiments. The first three experiments are designed to assess the effect of productivity biased toward the each education group, $z_{it}$. We compute an equilibrium trajectory of the model under the calibration derived in Section 3.1 with one difference: we impose that $z_{it}$ remains constant
at its initial level while all other exogenous variables grow as in the baseline exercise. That is we assume \( g_i = 1 \) for each \( i \) in these experiments. In the fourth experiment, \( z_t \) is fixed at the initial level, i.e., \( g = 1 \). In the fifth experiment we fix life expectancy at its initial level while all productivity terms grow as in the baseline. In the last experiment, \( z_{1t}, z_{2t} \) and \( z_{3t} \) are held constant at their initial level and only life expectancy and neutral productivity \( z_t \) increase.

**Effect of** \( z_1 \) This experiment assumes \( g_1 = 1 \). We decompose the predictions of the model into two steps. First, the level of educational attainment measured by average years of schooling is lower in 1940 in this experiment than in the baseline: 10.5 (v. 10.8 in the baseline). This lower attainment results from a larger fraction of individuals having completed less than high school education in 1940: 68 percent (v. 61 in the baseline), and a lower fraction of people having completed high school: 24 percent (v. 31 in the baseline). Second, the evolution of educational attainment (from a lower initial value) is quite similar to the baseline. Average years of schooling increase by 23.8 percent between 1940 and 2000 (v. 24.3 in the baseline). The small role of growth in \( z_1 \) on educational attainment is due to the fact that \( g_1 = 0.996 \) in the baseline calibration is fairly close to 1 as in this counterfactual experiment. The growth rate of the economy is 2.05 percent per year.

**Effect of** \( z_2 \) This experiment assumes \( g_2 = 1 \). The level of educational attainment in 1940 in this experiment is lower than in the baseline and in the previous experiment: 10.2 (v. 10.8 in the baseline). This is due to a large number of individuals having completed less than high school: 76 percent (v. 61 in the baseline), and a low number of individuals having completed high school: 16 percent (v. 31 in the baseline). Through time, however, educational attainment increases faster in this experiment than in the baseline: average years of schooling increase by 26.8 percent between 1940 and 2000 (v. 24.3 in the baseline). This
increase is driven mostly by the fact that, holding $z_{2,t}$ constant produces an increase in the college to high-school earnings ratio while reducing the change in the high-school to less than high-school earnings ratio. Specifically, in the baseline experiment the college to high-school earnings ratio increased by 4.46 percent between 1940 and 2000 while in this experiment it increases by 5.45 percent. The high-school to less than high-school earnings ratio, which increased by 5.35 percent in the baseline increases by only 2.40 percent in this experiment. As a result college attainment increases from 8 to 50 percent while the fraction of individuals with less than high school falls from 76 to 38 percent (v. 61 to 20 percent in the baseline), driving the strong increase in average years of schooling. The growth rate of the economy is 1.63 percent per year (v. 2 in the baseline). This lower growth rate relative to the baseline despite a stronger increase in years of schooling is the result of lower productivity growth.

**Effect of $z_3$** This experiment assumes $g_3 = 1$. The level of educational attainment measured by average years of schooling in 1940 in this experiment is similar to that of the first experiment: 10.5 (v. 10.8 in the baseline). The fraction of individuals having completed college education is lower than in the baseline: 2 percent (v. 8 in the baseline), while the proportions of those with less than high school and high school are slightly higher than in the baseline: 65 and 33 percent (v. 61 and 31 in the baseline). Through time, the college to high-school earnings ratio decreases by 2.84 percent between 1940 and 2000, implying a stagnation of college attainment: 2 percent in 2000 as in 1940. The increase in the high-school to less than high-school earnings ratio is 7.05 percent and, as a consequence, average years of schooling increases thanks to high-school enrollment. But the increase in average years of schooling between 1940 and 2000 is noticeably smaller than in the baseline: 14.2 percent (v. 24.3 percent in the baseline).
Effect of $z$  This experiment assumes $g = 1$. There is little effect on educational attainment. In 1940 average years of schooling is 10.7 (v. 10.8 in the baseline). The difference is due to the smaller high-school and college attainment: 30 and 7 percent (v. 31 and 8 in the baseline). The evolution of educational attainment is quite similar to the baseline with average years of schooling increasing by 23.8 percent between 1940 and 2000 (v. 24.3 percent in the baseline).

Effect of $T$  When $T$ remains constant at its initial level, educational attainment in 1940 is higher than in the baseline: 11.9 years (v. 10.8). This difference with the baseline is the result of stronger growth in relative earnings through equilibrium effects. Unlike in the baseline model where increases in productivity are mitigated in equilibrium by increases in population size due to increasing life expectancy, in this experiment there is no offset of the productivity effects. Through time, however, the effect of $T$ is moderate. Between 1940 and 2000 average years of schooling increases by 22.8 percent (v. 24.3 in the baseline). The constant population implies that the economy grows faster in this experiment: 2.52 percent (v. 2 in the baseline).

Effect of $z_1$, $z_2$ and $z_3$  The previous experiments suggest that $T$ and $z$ have a small contribution to the change in educational attainment over time. To assess the joint contribution of the productivity parameters biased toward various educational groups, that is $z_1$, $z_2$ and $z_3$, we conduct an experiment where these three are set constant to their initial levels while $T$ and $z$ behave as in the baseline model. First, we observe that the level of educational attainment in 1940 is quite lower than in the baseline: 9.6 years instead of 10.8. Second, the increase in educational attainment through time is almost null since average years of schooling increase by 0.2 percent over the 1940–2000 period (v. 24.3 in the baseline). This is the result of the decrease in relative earnings implied by the absence of productivity growth combined with the increase in human capital due to growing life expectancy.
4 Discussion

4.1 Expectations

In our model, lifetime income is a critical determinant of educational attainment and human capital accumulation as illustrated in equations (7)-(8). A key underlining assumption in our quantitative assessment of skill-biased technical change in the baseline model is that individuals have perfect foresight about the evolution of wages and hence the lifetime income implied in each schooling level. To the extent that part of the observed changes in income are difficult to forecast, a reasonable question to ask is whether our quantitative assessment hinges critically on the perfect-foresight assumption. While we view the baseline model with perfect foresight as an appropriate benchmark to quantitatively assess the implications of changes in technology and life expectancy on schooling, our aim in this section is to provide reasonable alternatives that illustrate the quantitative importance of the perfect-foresight assumption. There are in principle many alternatives to perfect foresight but some of these have limited applicability in the particular context of our model. In our model, individuals make once-and-for-all educational decisions at the beginning of their life taking into account the future evolution of wages and their impact in lifetime income in each of the educational choices. Hence, in this context, we investigate alternative assumptions about the perceived path of wages that individuals use in comparing educational choices.

Before we describe what we do, it is important to note that individuals in our model need information on wages while in the data we have data on earnings which combines wages and human capital. The solution to our baseline model has predictions for earnings and human capital from which we extract wage information. We use these paths for wages to construct alternative assumptions about lifetime income for individuals. More concretely, we assume
that individuals make educational decisions from an estimate of lifetime income that is based on past wages, we denote this estimate of lifetime income \( \hat{I}_\tau(s) \) given by:

\[
\hat{I}_\tau(s) = \max_e \left\{ h(s,e)\hat{W}_\tau(s) - q_\tau e : \hat{W}_\tau(s) = w_\tau(s) \sum_{t=\tau+s}^{\tau+T(\tau)-1} \left( \frac{g_{s,\tau,n}}{r} \right)^{t-\tau} \right\},
\]

where \( g_{s,\tau,n} \) is a constant annual gross growth rate of wages in education level \( s \) for an individual of generation \( \tau \) that is based on wages in the last \( n \) periods. Current wages across educational categories are observed before educational decisions are made for each generation \( \tau \). Note that this estimate of lifetime income differs from the lifetime income assumed in the baseline model with perfect foresight \( I_\tau(s) \) implied by equations (4)-(6). The estimated growth rates of wages is a historical average over the \( n \) previous periods:

\[
g_{s,\tau,n} = \begin{cases} 
\left( \frac{w_\tau(s)}{w_{\tau-n}(s)} \right)^{1/n} & \text{for } n > 0 \\
1 & \text{for } n = 0 
\end{cases}
\]

We refer to these expectations as “history-based.” Individuals make educational decisions as before, dictated by equations (7)-(8) where the ratio of lifetime income \( I_\tau(s_i)/I_\tau(s_j) \) in those equations is replaced now by the estimate \( \hat{I}_\tau(s_i)/\hat{I}_\tau(s_j) \) in equation (16). In what follows, we generically refer to these ratios of lifetime income across educational categories as returns to education.

We compute educational attainment for each generation under different specifications of \( n \) for the history-based expectations but otherwise assuming the same parameter values as in the baseline model. We compare the implications on educational attainment against the baseline model. We start by computing the case where individuals use a long historical time series of wages to predict future income. We assume \( n = 100 \). Table 3 reports the results for average years of schooling in 1940 and 2000 as well as the rate of change during the
period. We also report the implied returns to education in each case. We emphasize that while average years of schooling increase at a slower pace in this experiment relative to the baseline model, there is still a substantial increase in schooling, an increase of 20.6 percent between 1940 and 2000 versus the 24.3 percent increase in the baseline model. We note that in this experiment average years of schooling in 1940 is 9.6 years compared to 10.8 years in the baseline model and data. The reason for this discrepancy is the fact that the model is not re-calibrated to the targets in the baseline model and, as a result, the expected increase in lifetime income is systematically lower than the baseline increase in lifetime income (on average expected increases in lifetime income in this experiment are 9 percent lower than in the baseline).

For completeness, we also conduct an experiment where individuals assume that wages remain constant throughout their lifetime, that is we compute the case for $n = 0$. Table 3 reports the results for this case. Not surprisingly, individuals underestimate the returns to schooling both for college and high-school relative to the baseline and this has implications for the levels of educational attainment. In this experiment, average years of schooling increase by 9.3 percent between 1940 and 2000 (versus 24.3 percent in the baseline). Even though individuals expect wages to remain constant throughout their lifetime, each new cohort observes current relative wages when making educational decisions which affect their choices. This effect alone explains 38 percent ($9.3/24.3$) of the increase in average years of schooling between 1940 and 2000.$^{18}$

$^{17}$To the extent that the level of educational attainment is useful in pinning down the elasticity of schooling on income changes, as discussed previously, we find it of interest to also evaluate the implications of the same experiment but with a recalibration of the distribution of schooling costs ($\mu$ and $\sigma$) to match the average years of schooling in 1940 in the data. We note that the ratio of lifetime income in the model is independent of $\mu$ and $\sigma$ and as a result, the implications on lifetime income in this experiment are the same as in the model without the recalibration. We find in this case that the experiment generates an increase in average years of schooling of 23.3 percent which is larger than the one without recalibration and closer to the baseline experiment.

$^{18}$When adjusting the parameters of the cost-of-schooling distribution to match the average years of schooling in 1940, this experiment produces an increase in average years of schooling of 12.6 percent compared to the 9.3 percent without recalibrating and 24.3 percent in the baseline.
We conclude that while the extent of foresight of future wages is important for the contribution of skill-biased technical progress to the increase in educational attainment, in the most conservative scenario where individuals assume constant wages throughout, the impact of skill-biased technology still implies a substantial increase in educational attainment, almost 40 percent of the observed increase in average years of schooling.

4.2 Time-Series Variability in Relative Earnings

Another potentially relevant feature in the time series of relative earnings across schooling groups is the decade-to-decade variation (see again Figure 4). While in the baseline model we captured the long-run path for these series, a relevant question is whether fluctuations in relative earnings around these trends are important for the overall contribution of skill-biased technological progress to the increase in schooling. To the extent that educational decisions by individuals are made based on measures of lifetime income, we expect that variability around the trend doesn’t have a first-order impact on the overall contribution of skill biased technical progress to the increase in schooling. Nevertheless, the objective in this section is to assess the quantitative importance of the fluctuations in relative earnings for the results.

As discussed earlier, there is not a simple decomposition of relative earnings in the data between wages and human capital. Broadly speaking, to circumvent this issue, we feed in a series of wages in the model such that the implied optimal human capital decisions of individuals generate the relative earnings we observe in the data. We emphasize two main findings. First, we find, perhaps no so surprisingly, that we need small deviations in trend growth for wages to capture the fluctuations in relative earnings seen in the data. Second, educational attainment implied by the model is fairly close to the baseline. For instance, in 1940, 63 percent of individuals attained less than high-school (v. 61 percent in the baseline)
and 28 percent attained high-school (v. 31 percent in the baseline), implying 10.75 years of schooling in 1940 (v. 10.8 in the baseline). The model also implies an increase in average years of schooling between 1940 and 2000 of 25.5 percent (v. 24.3 percent in the baseline).

We conduct the same experiments related to the perfect-foresight assumption as in the previous section to assess the impact of fluctuations in relative earnings. In the long history case where \( n = 100 \), that is when individuals use a long time series of data to evaluate the growth rate of wages, educational attainment measured by average years of schooling increases by 20.5 percent (v. 20.6 percent with trend wages and 24.3 percent in the baseline). When individuals expect no changes in relative earnings, the case where \( n = 0 \), average years of schooling increases by 9 percent (v. 9.3 percent with trend wages).

We conclude that while decade-by-decade fluctuations in relative earnings may be potentially important for educational decisions, we find that these fluctuations are not quantitatively important in reducing the overall contribution of skill-biased technical change on schooling, in the perfect foresight case as well as the more conservative type of expected income.

### 4.3 Other Items

The model delivers an elasticity of educational attainment to changes in lifetime income which is driven mostly by skill biased technical change. As argued previously, a critical aspect of the discipline of this elasticity in our calibration is from the utility cost of schooling to match the distribution of educational attainment. We discuss the reasonableness of the implied elasticity by looking at closely related empirical and model-based evidence. We also connect the implications of the implied elasticity for educational policy and other trends that may affect relative incomes. Next, we discuss the relevance for our results of considering human capital accumulation on the job.
The Elasticity of Educational Attainment  There is a large empirical literature assessing the impact of educational policy on schooling. The typical focus is on assessing the response of college attainment to changes in subsidies or other factors that alter lifetime incomes such as Dynarski (2002, 2003), van der Klaauw (2002), and Keane and Wolpin (1997). While there is no complete agreement on the exact magnitude of these elasticities, the evidence suggests that they are large and we use this evidence to provide a benchmark against which to assess the magnitude implied by our quantitative results. For instance, Keane and Wolpin (1997) estimate a life-cycle model of schooling and career choices. Their structural estimates imply that subsidizing college costs by about 50 percent increases college completion from 28.3 to 36.7 percent. To construct an elasticity, we calculate that the subsidy represents about 1 percent of lifetime income.\(^\text{19}\) This implies an elasticity of college completion of \(\ln(36.7/28.3)/\ln(1.01) = 26\). We calculate that in our model a subsidy of the same size, that is 1 percent of the lifetime income from college for the 1981 generation (the generation whose educational attainment is measured in 2000) yields an increase in college attainment from 40.8 to 45.5, that is an elasticity of \(\ln(45.5/40.8)/\log(1.01) = 11\). Dynarski (2003) studies an exogenous change in education policy –namely the elimination of the Social Security Student Benefit program in the United States in 1981– that affected some students but not others. Dynarski found that $1,000 (dollars of year 2000) in college subsidy generates an increase in college enrollment of 3.6 percentage points. We can also relate to this finding by performing the same policy experiment. To obtain an increase in college enrollment of 3.6 percentage points in 2000 in the model, a subsidy to college equivalent to 0.8 percent of lifetime income is needed which, we argue, is a larger number than $1,000. We conclude from these experiments that the strong effect of skill-biased technical change on educational attainment in the baseline model comes from strong changes in relative earnings and not from an implausibly large elasticity of educational attainment. The quantitative magnitude

\(^{19}\text{Keane and Wolpin (1997)'s subsidy is }$2,000\text{ per year for four years. The present value of a }$35,000\text{ annual income for 40 years, discounted at }4\%\text{ is about }$730,000. \text{So, we found }2000 \times 4/730,000 = 0.01.\)
of the response of schooling to changes in relative lifetime income is relevant for educational policy and for the impact evaluation of other related trends. For example, it is often viewed that the racial gap in schooling is related primarily to under investments in education in the early part of life. Our results suggest that another important factor is the gap in lifetime earnings associated with racial differences. Similarly, there have been recent discussions on the rising costs of college in explaining a recent slowdown in college enrolment. In the context of our model, rising costs of education can have implications for schooling to the extent that they affect the relative lifetime earnings (net of educational costs).

**Experience**  Our model abstracts from human capital accumulated on the job. The data suggest that there are considerable returns to experience. The age profile of earnings, for example, are increasing in the data while our baseline model implies that they are decreasing. Returns to experience may affect educational decisions. First, if they increase with education—as has been documented is the case in the data—then this provides an additional return to schooling, reinforcing the effects of skill-biased technical change. Second, substantial returns to experience implies that, other things equal, individuals would have an incentive to enter the labor market sooner. Because of these opposing effects, it is a quantitative question whether on-the-job human capital accumulation affects the evolution of educational attainment over time. We have experimented adding on-the-job human capital accumulation to the model and found that under a reasonable calibration of the returns to experience, our baseline results remain after adding experience.\(^{20}\)

\(^{20}\)For more details on these experiments see Restuccia and Vandenbroucke (2010).
5 Conclusion

We developed a model of human capital accumulation to address the role of changes in the returns to education on the rise of educational attainment in the United States between 1940 and 2000. The model features discrete schooling choices and individual heterogeneity so that people sort themselves into the different schooling groups. In the model, skill-biased technical change increases the returns to schooling thereby creating an incentive for more people to attain higher levels of schooling. We find that changes in the returns to education generate a substantial increase in educational attainment and that this quantitative importance is robust to relevant variations in the model. We also found that the substantial changes in life expectancy in the data turn out to explain a small portion of the change in educational attainment in the model.

There are several issues that would be worth exploring further. First, we have not addressed the factors that may contribute to the slowdown in college attainment since the late 70s. Assessing the contribution of rising college costs together with tighter borrowing constraints may be important. Second, it would be interesting to assess the role of changes in the returns to education on educational attainment in other contexts such as across genders, races, and countries. For instance, it would be relevant to investigate changes in the returns to schooling in countries with different labor-market institutions. Institutions that compress wages may reduce the incentives for schooling investment and perhaps, holding other institutional aspects constant, this wage compression may explain the lower educational attainment in some European countries compared to the United States. Similarly, it is relevant to explore the changes in the returns to education for women in conjunction with the observed increase in labor market participation and the reduction in the gender wage gap. A glance at the data suggests that changes in the returns to schooling for women have been similar to that of men. Hence, together with faster overall wage growth and an increase in labor market hours
for women may explain the larger increase in educational attainment observed for women between 1940 and 2000. Third, our analysis has taken the direction of technical change as given. It would be interesting to study quantitatively the process of human capital accumulation allowing for endogenous technical change in the spirit of Galor and Moav (2000). We leave all these relevant explorations for future research.
References


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Table 1: Calibration

| Preferences | $\beta = 1/1.04$, $\mu = 0.72$, $\sigma = 0.66$ |
| Life Expectancy | Linear trend with $T(1900) = 47$ and $T(2000) = 64$ |
| Technology | $\eta = 0.9$ |
| Productivity | $z_0 = 1$ (normalization), $g = 1.012$ |
| | $z_{1,0} = 0.41$, $g_1 = 0.996$ |
| | $z_{2,0} = 0.20$, $g_2 = 1.001$ |
| | $z_{3,0} = 0.05$, $g_3 = 1.027$ |

Table 2: Decomposition

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Exp. 1</th>
<th>Exp. 2</th>
<th>Exp. 3</th>
<th>Exp. 4</th>
<th>Exp. 5</th>
<th>Exp. 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years of schooling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1940</td>
<td>10.8</td>
<td>10.5</td>
<td>10.2</td>
<td>10.4</td>
<td>10.7</td>
<td>11.9</td>
<td>9.6</td>
</tr>
<tr>
<td>2000</td>
<td>13.4</td>
<td>13.0</td>
<td>13.0</td>
<td>11.9</td>
<td>13.2</td>
<td>14.7</td>
<td>9.7</td>
</tr>
<tr>
<td>% change</td>
<td>24.3</td>
<td>23.8</td>
<td>26.8</td>
<td>14.2</td>
<td>23.8</td>
<td>22.8</td>
<td>0.2</td>
</tr>
<tr>
<td>% Change in relative earnings 2000–1940</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College/HS</td>
<td>4.46</td>
<td>5.09</td>
<td>5.45</td>
<td>-2.84</td>
<td>3.79</td>
<td>9.21</td>
<td>-0.74</td>
</tr>
<tr>
<td>HS/Less HS</td>
<td>5.35</td>
<td>4.96</td>
<td>2.40</td>
<td>7.05</td>
<td>4.03</td>
<td>15.65</td>
<td>-1.26</td>
</tr>
<tr>
<td>Annual Growth of GDP per Worker (%)</td>
<td>2.00</td>
<td>2.05</td>
<td>1.63</td>
<td>1.29</td>
<td>1.06</td>
<td>2.52</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Note – In the first experiment $z_{1,t}$ is kept constant at its initial level while all over exogenous variables are growing as in the baseline experiment. In the second experiment $z_{2,t}$ is constant while other exogenous are growing. In the third experiment $z_{3,t}$ is constant. In the fourth $z_t$ is constant. In the fifth experiment $T(\tau)$ is constant. In the sixth experiment $z_{1,t}$, $z_{2,t}$ and $z_{3,t}$ are kept constant.
Table 3: Educational Attainment and Lifetime Income in the Baseline Model with Perfect Foresight and in the Model with “History-Based” Expectations

<table>
<thead>
<tr>
<th>Years of schooling</th>
<th>Baseline</th>
<th>n = 100</th>
<th>n = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1940</td>
<td>10.8</td>
<td>9.6</td>
<td>9.6</td>
</tr>
<tr>
<td>2000</td>
<td>13.4</td>
<td>11.5</td>
<td>10.5</td>
</tr>
<tr>
<td>% change</td>
<td>24.3</td>
<td>20.6</td>
<td>9.3</td>
</tr>
</tbody>
</table>

% change in relative lifetime income, 1940 to 2000

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>n = 100</th>
<th>n = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>College/HS (actual)</td>
<td>10.6</td>
<td>10.6</td>
<td>10.6</td>
</tr>
<tr>
<td>College/HS (expected)</td>
<td>10.6</td>
<td>9.8</td>
<td>4.6</td>
</tr>
<tr>
<td>HS/Less HS (actual)</td>
<td>12.8</td>
<td>12.8</td>
<td>12.8</td>
</tr>
<tr>
<td>HS/Less HS (expected)</td>
<td>12.8</td>
<td>15.0</td>
<td>7.7</td>
</tr>
</tbody>
</table>
Figure 1: Educational Attainment in the U.S., 1940–2000

Source – IPUMS samples from the 1940–2000 Census. The education variable is “educ.” We define the less-than-high-school level as an educational achievement between nursery school and grade 11. The high-school-or-some-college level is from grade 12 to 3 years of college. The college level is defined as at least 4 years of college.
Figure 2: Relative Earnings of White Men in the U.S., 1940–2000

Source – IPUMS samples from the 1940–2000 Census. The education variable is “educ.” The earnings variable is “incwage,” that is wage and salary income for employed individuals. The education groups are defined as in Figure 1.
Figure 3: The Determination of Educational Attainment for Generation $\tau$

Panel a – Standard case where each level of schooling is chosen by some individuals

Panel b – Case where no individual chooses high-school
Figure 4: Relative Earnings Model vs. Data

Note – The model results are represented with solid lines. The U.S. data are represented by dotted lines.
Figure 5: Educational Attainment Model vs. Data

Note – The model results are represented with solid lines. The U.S. data are represented by dotted lines.