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Explaining Educational Attainment across Countries and over Time

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Explaining Educational Attainment across Countries and over Time[†]

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Abstract

Consider the following facts. In 1950 the richest ten-percent of countries attained an average of 8.1 years of schooling whereas the poorest ten-percent of countries attained 1.3 years, a 6-fold difference. By 2005, the difference in schooling declined to 2-fold. The fact is that schooling has increased faster in poor than in rich countries even though the per-capita income gap has generally not decreased. What explains educational attainment across countries and their evolution over time? We develop a model of human capital accumulation that emphasizes productivity and life expectancy differences across countries and time. Calibrating the parameters of the model to reproduce historical data for the United States, we find that the model accounts for 95 percent of the difference in schooling levels between rich and poor countries in 1950 and 78 percent of the increase in schooling over time in poor countries. The model generates a faster increase in schooling in poor than in rich economies even when their income gap does not decrease. These results have important implications for educational policy.

Keywords: educational attainment, productivity, life expectancy, education policy, labor supply.
JEL codes: O1, O4, E24, J22, J24.

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1 Introduction

Human capital accumulation is believed to play a crucial role in understanding income differences across countries, although a precise assessment of this importance has been hindered by the lack of empirical measures of human capital.[1] A key component of human capital formation is investment in schooling. Cross-country data on schooling indicates that although educational attainment is substantially lower in poor than in rich countries, over time poor countries have increased schooling faster than rich countries. The convergence in schooling occurs despite the fact that the gap in income per capita across countries has generally not decreased. The reduction in the dispersion of schooling levels is a fact that is difficult to account for using existing explanations of schooling differences across countries. We develop a model of human capital accumulation that emphasizes differences in productivity and life expectancy to explain educational attainment across countries and over time. We find that the model accounts for 95 percent of the difference in schooling levels between rich and poor countries in 1950 and 78 percent of the increase in schooling over time in poor countries. The model generates a faster increase in schooling in poor than in rich economies even if their income gap does not decrease. These implications of the model are consistent with cross-country data and have important implications for educational policy.

We combine education data from Barro and Lee (2010) and income per capita data from the Conference Board (2010) to construct a panel dataset for 84 countries from 1950 to 2005. We emphasize three facts. First, there are large differences in schooling measures across countries, with average schooling being 8 years in rich countries and only 1 year in poor countries in 1950 (11 and 5 years in rich and poor countries in 2005). Second, average years of schooling have increased over time in all countries in our sample. Third, average years of

[1] Important exceptions include Hendricks (2002) and Schoellman (2010). See also recent quantitative work by Manuelli and Seshadri (2006) and Erosa, Koreshkova, and Restuccia (2010).

schooling have increased more in poor than in rich countries. Hence, dispersion in schooling levels has decreased overtime. This occurs despite the fact that dispersion in income levels has generally not decreased. What can explain schooling differences across countries and their evolution over time?

We develop a model of human capital accumulation to account for these facts. The basic features of the model are standard and build from Bils and Klenow (2000). There are two novel and noteworthy departures from the existing literature. First, the model features an income effect from non-homothetic preferences. Such preferences are central in theories of structural change and we argue are important in understanding the allocation of time between the production of goods and schooling across rich and poor countries.[2] Second, labor supply is endogenous. This feature is important because as the available data from the International Labor Organization show there are large cross-country differences in hours of work –average hours are lower in rich than in poor countries– thereby affecting schooling decisions. Broadly speaking, consider schooling as a time-allocation problem whereby a unit of time is allocated between the production of consumption goods, schooling, and leisure. Then, with non-homotetic preferences, increases in productivity lead to a reallocation of time away from the production of goods into schooling and leisure. Other things equal, abstracting from leisure over estimates the impact of increases in productivity on schooling (and hence over estimates the income elasticity of schooling). Other authors, such as Heckman (1976) and Blinder and Weiss (1976), have emphasized the importance of jointly modeling labor supply and human capital accumulation in bringing additional quantitative discipline to human capital models.

Our strategy to discipline the forces in the model is simple. We calibrate a benchmark

[2]For applications in development, see for instance Laitner (2000), Kongsamut, Rebelo, and Xie (2001), Gollin, Parente, and Rogerson (2002), and Duarte and Restuccia (2010). Other applications include the changing patterns of trade, e.g. Markusen (2010) and Fieler (2010); the study of broader transformations in an economy, e.g., Greenwood and Seshadri (2005); among others.

economy to fit a long time series for schooling and hours in the United States. The calibration puts restrictions on the strength of the income effect in the model. In the data for the United States there is substantial variation over time in hours of work, schooling, and income for our calibration strategy to provide identification of the income effect.[3] We then perform a cross-country quantitative experiment to assess the ability of the model in explaining schooling levels across countries and their evolution over time. In our quantitative experiment, we allow the levels of productivity and life expectancy to vary over time and across countries. We discipline these elements by reproducing the cross-country distribution of GDP per capita in 1950 and 2005 as well as the cross-country relationship between life expectancy and income in 1950 and 2005. The main result is that the model is consistent with the three facts emphasized earlier in the cross-country and time-series data. In particular, the model generates substantial dispersion in schooling levels across countries: in 1950, the model accounts for 95 percent of the difference in schooling between rich and poor countries. In addition, the model implies a faster increase in schooling for poor countries than for rich countries and, therefore, is consistent with the contraction in the distribution of schooling observed in the cross-country data. This contraction occurs in the model even though, as in the data, there is no reduction in income gaps across some groups of countries.

Our paper relates to a large literature in macroeconomics and development addressing the disparities in schooling levels across countries. The main focus of this literature is on differences in schooling at a point in time and, as a result, most of the existing frameworks are not designed to address the evolution of schooling over time.[4] Within this literature, the closest paper to ours is Bils and Klenow (2000) who also emphasize differences in productivity and

[3] Over the period we analyze, GDP per capita in the United States increased by a factor of 10 and average years of schooling increased by a factor of 2. The factor difference between the richest and poorest ten-percent of countries in 1950 is 16-fold in GDP per capita and 6-fold in average years of schooling.

[4] One important exception is the study of Manuelli and Seshadri (2009) that looks at the evolution of education for a subset of Asian and Latin American countries. They find that the model cannot account for the substantial increase in schooling in Latin American economies that precisely fail to catch up in income to the United States.

life expectancy. A key difference is that whereas Bils and Klenow (2000) mainly focus on the cross-sectional correlation of schooling and per-capita income growth found in the empirical literature –for instance Barro (1991), we focus on a broader dimension of the data, namely the level and time-series differences in schooling across countries. We also depart from Bils and Klenow (2000)'s framework by allowing for an income effect through non-homothetic preferences and for hours of work differences across countries. These departures are critical in understanding the convergence pattern in educational attainment across countries. Our paper relates to a recent literature in macroeconomics assessing the role of human capital in development, for instance Manuelli and Seshadri (2006) and Erosa, Koreshkova, and Restuccia (2010). The focus of this literature is on the amplification effect of human capital in explaining income gaps across countries. Our framework abstracts from features that generate amplification effects since this is not our focus. Incorporating amplification effects in the model would reduce the size of productivity gaps needed to reproduce income differences across countries in the quantitative experiments without altering our main findings. Another related paper is Cordoba and Ripoll (2010) that consider a model where fertility, mortality, credit constraints, and access to public education drive schooling differences across countries. We complement this work by emphasizing the time series dimension of the data. More importantly, we assess the contribution of productivity and life expectancy to schooling in a framework without frictions.

Our paper is also related to a broad literature that studies the impact of particular educational policies.[5] Our results highlight the importance of productivity (and life expectancy) in explaining a large portion of schooling differences across countries and, as a result, have novel and substantial implications for educational policy. The results emphasize the need to address the factors driving low productivity in poor countries. Even though there is room for other factors to be relevant such as credit constraints for investment in education, re-

[5]See, for instance, Duflo (2001).

strictions on school infrastructure, aid and policy, among others; our results stress the need for greater focus on the productivity problem in poor countries.

The paper is organized as follows. In the next section, we present the facts from a panel dataset of 84 countries from 1950 to 2005 for a measure of educational attainment and income. Section 3 presents the model. In Section 4 we calibrate the model. Section 5 performs a quantitative analysis of cross-country differences productivity and life expectancy in explaining the patterns in the panel data. We conclude in Section 6.

2 Facts

Data We construct a panel dataset of schooling and income as follows. We obtain average years of schooling for the population aged 25 to 29 from Barro and Lee (2010). We restrict the sample to the narrow age population to minimize the impact of demographics and other changes on schooling measures across time and space. It is also the definition that is best suited for the historical data on educational attainment we use for the United States and for the model we consider in Section 3. The schooling data is available for a large set of countries from 1950 to 2005 in 5 year intervals. We obtain gross domestic product (GDP) per capita from the Conference Board (2010), Total Economy Database. We restrict the time frame of this data from 1950 to 2005. To abstract from short-run fluctuations in real GDP we filter it using the Hodrick and Prescott (1997) filter with $\lambda = 100$ for yearly observations and keep the trend component of these time series. When we merge these two sets of data, we end up with a sample of 84 countries that have available data for schooling and GDP per capita from 1950 to 2005.[6]

[6] Our sample of 84 countries comprises a fairly representative set of the world's income distribution. For instance, the factor difference in GDP per capita between the richest and poorest five-percent of countries is 25-fold which is comparable to many previous studies.

Facts We emphasize three sets of facts that arise from analyzing these data. First, schooling differences across countries are large at any point in time between 1950 to 2005. Second, schooling increases over time in all countries in our sample. Third, schooling differences across countries are smaller in 2005 than they were in 1950. The reduction in schooling differences across countries is systematic and occurs despite the fact that the income gap between rich and poor countries has not generally decreased. We now document these facts in detail.

1. There are large differences in educational attainment across countries.

   Consider Table 1 which decomposes our sample into ten groups of countries according to the 1950 distribution of GDP per capita –i.e., the countries in each decile are the same in 1950 and in 2005. For each decile, the table reports the average GDP per capita relative to the United States and the average years of schooling. In 1950 there is a 6-fold difference in schooling between the richest and poorest decile of the distribution. In 2005 there remains a noticeable 2-fold difference. These differences are not specific to the top and bottom decile and/or to the initial and end year of our sample. Figure 1 documents them across all countries, for selected years, and shows that there has been a significant dispersion of schooling across all levels of income and at all dates. To put the magnitude of cross-country differences in schooling in perspective, consider that in 1900 in the United States a 35-year old had completed about 7.4 years of schooling. Hence, a 25-29 year old in 2005 in the average poor country still had 2 years less of schooling than a 35-year old in the United Sates in 1900.[7]

2. Educational attainment increased over time in all countries.

   Schooling increased between 1950 and 2005 for every country in our sample. Table 1 conveys an aggregated view of this fact since average years of schooling increase for

[7]The figure 7.4 years of schooling for the average 35-year old is from Goldin and Katz (2008). Table 1 shows that the years of schooling for the average 25-29 year old in the average poor country is 5.07 in 2005.

each decile of the distribution. The increase in schooling between 1950 and 2005 is 37 percent for countries in the top decile and 299 percent for countries in the bottom decile. We note that the increase in educational attainment is positive for all deciles of the income distribution regardless of the initial income level or subsequent income growth relative to the United States. We expand on this fact next.

3. Differences in educational attainment across countries have been reduced substantially over time.

   Poor countries exhibit a tendency to increase their schooling faster than rich countries. In Table 1 this is evidenced by the tendency of the 2005-to-1950 ratio of schooling (last column) to decrease with relative income. This is a remarkable finding given that for some deciles, such as the second and the fourth, relative income did not change between 1950 and 2005. For deciles such as the third or the fifth relative income increased and for the tenth decile relative income decreased. Yet, each group of countries experienced a substantial increase in schooling. A more complete and systematic documentation of the decline in schooling dispersion across countries is to report for each year the cross-sectional elasticity of schooling to income levels, as in Figure 2.[8] This elasticity decreases systematically over time. For instance, whereas two countries that differ in income per capita by one percent have in average a 0.6% difference in schooling in 1950, their schooling difference is reduced by half to 0.3% in 2005. The same declining pattern is observed for the time-series elasticity, that is the elasticity of income levels to schooling for each country over time, although with only 12 observations per country the pattern has more noise.

In summary, even though there are still large differences in educational attainment across countries, we find that these differences have been systematically reduced with poor countries

[8] In each year, we regress the log of average years of schooling (for people 25-29 years old) on a constant and the log of GDP per capita, and report the slope coefficient in Figure 2.

increasing their educational attainment over time faster than rich countries. While we have reported these facts for individuals 25-29 years of age, we emphasize that the facts are robust to other age categories and for males and females. Moreover, the convergence pattern is also robust to a broader set of countries. The convergence pattern becomes even stronger if we consider all 146 countries in Barro and Lee (2010)'s data set. Table A.1 in the appendix reports average years of schooling for people 25-29 for countries by deciles of the schooling distribution in 1950 using the entire sample in Barro and Lee (2010). The countries with lowest schooling in 1950 (Decile 1) increased their educational attainment from 0.3 to 4.1 years whereas those countries with the highest schooling in 1950 (Decile 10) increased their schooling from 8.7 to 11.7 years. Hence, the factor difference in educational attainment between these groups of countries declined from a 31-fold in 1950 to less than 3-fold in 2005.

3 The Model

Time is continuous. The world comprises a set of closed economies and hence in what follows we focus on a single economy to describe the model. At every moment a generation of homogeneous individuals of size one is born and lives for an interval of time of length T_τ. The index τ denotes an individual's generation: the date at which the individual is of age 0. We use t to refer to calendar time. Individuals are endowed with one unit of productive time at each moment and no assets at birth. There is a worldwide rate of interest r at which borrowing and lending can occur without constraint. The payment to a unit of human capital-hour is denoted by z_t at moment t. We generically refer to z_t as productivity and assume it grows at a time invariant rate g.

Preferences

Preferences are defined over lifetime sequences of consumption and leisure time as well as over time spent in school. We abstract from life-cycle considerations by imposing that consumption and leisure time remain constant throughout the individual's life. Hence, the preferences of an individual of generation τ can be represented by

$$\int_0^{T_\tau} e^{-\rho u} \left[U(c) + \alpha V(\ell)\right] du + \beta W(s),$$

where ρ is the subjective rate of discount assumed to be equal to the rate of interest, c is consumption, ℓ leisure time, and s time spent in school.[9] The parameters α and β are positive.

We let

$$V(\ell) = \frac{\ell^{1-\sigma} - 1}{1 - \sigma},$$

where $\sigma > 0$ and

$$W(s) = \ln(s).$$

We also assume that

$$U(c) = \ln(c - \bar{c}),$$

where $\bar{c}$ introduces a non-homotheticity which has the standard interpretation of a subsistence level above which consumption must remain at every point in time. This feature

[9] We note that our assumption of constant consumption over the lifecycle is not too restrictive since with separable utility in consumption and leisure and $r = \rho$ an individual optimally chooses a constant path of consumption. However, these assumptions do not guarantee a constant path of leisure. We impose a constant leisure profile over the lifecycle for simplicity. In addition, we do not have detailed data on the lifecycle behavior of labor supply for generations dating back to the 19th century and the changes in lifecycle labor supply for recent cohorts are small in comparison with the variation in hours over time. Hence, we think there is little benefit to modeling labor supply over the lifecycle since our focus is on changes across countries and over time.

of preferences plays an important role in both the theoretical and quantitative properties of the model. In particular, when income is sufficiently large (alternatively when $\bar{c} = 0$), preferences display the standard property in modern macroeconomics where income and substitution effects of changes in productivity cancel each other and labor supply –as well as schooling– are constant. Hence, the long-run properties of this model are standard in macroeconomics. But when income is relatively low ($\bar{c} > 0$) both schooling and leisure are increasing in productivity.[10]

Human Capital Technology

Individuals can acquire human capital by spending time in school and purchasing educational services. The human capital technology follows Bils and Klenow (2000) and is described by

$$H(s,x) = x^{\gamma} h(s) \equiv x^{\gamma} \exp\left(\frac{\theta}{1-\psi} s^{1-\psi}\right),$$

where x represents purchases of educational services whose relative price is denoted by q. These services are purchased up front. Hence, x is more appropriately described as the present value of educational services. The parameter $\gamma \in (0,1)$ measures the elasticity of human capital to educational services. At an optimum γ is the share of lifetime income spent by an individual in educational services. The parameters $\theta > 0$ and $\psi > -1$ govern the importance of the time input in the production of human capital.

[10] The non-homothetic feature of preferences is common to a broad literature that emphasizes the shift in economic activity from agriculture to manufacturing and services such as Gollin, Parente, and Rogerson (2002) and Duarte and Restuccia (2010); models of the allocation of hours such as Rogerson (2008), models of the dynamics of saving rates such as Christiano (1989), among many other applications. See Atkeson and Ogaki (1996) for empirical evidence from micro and macro data.

Optimization

The optimization problem for an individual of generation τ is

$$\max_{c,\ell,x,s}\left\{\int_0^{T_\tau} e^{-\rho u}\left(U(c)+\alpha V(\ell)\right)du+\beta W(s):\right.$$
$$\left.\int_0^{T_\tau} e^{-ru}c du + x = z_\tau \int_s^{T_\tau} e^{(g-r)u}(1-\ell)H(s,x)du\right\}. \quad (1)$$

Our assumptions that consumption and leisure are constant throughout the life cycle and that the rate of interest equals the subjective discount rate imply that the optimization problem can also be written more compactly as:

$$\max_{c,\ell,x,s}\left\{a_\tau\left[U(c)+\alpha V(\ell)\right]+\beta W(s): a_\tau c + x = z_\tau(1-\ell)H(s,x)d_\tau(s)\right\},$$

where $a_\tau = \int_0^{T_\tau} e^{-\rho u}du$ and $d_\tau(s) = \int_s^{T_\tau} e^{(g-r)u}du$ are discount terms. Note that the discount term for education includes the foregone labor income of s years of schooling.

The first order condition for schooling after substituting for x is given by

$$a_\tau U'(c)c\left(\frac{h'(s)}{h(s)}+\frac{d_\tau'(s)}{d_\tau(s)}\right)+(1-\gamma)\beta W'(s)=0. \quad (2)$$

We make two remarks about this equation. First, when individuals derive no utility from schooling (i.e., $\beta = 0$), the optimal level of schooling s is determined by setting the term in parenthesis in (2) to zero. In this case, the optimal level of schooling maximizes lifetime income. To see this, note that $z_\tau(1-\ell)x^\gamma h(s)d_\tau(s)$ is lifetime income and its derivative with respect to schooling relative to lifetime income gives exactly the term in parenthesis. An increase in s raises lifetime income through human capital accumulation $h'(s)/h(s)$ and reduces lifetime income by the foregone time working $d_\tau'(s)/d_\tau(s)$. With $\beta = 0$ schooling is independent of productivity and may differ across generations only through changes in the function d_τ such as changes in life expectancy T. Second, when individuals derive utility from schooling ($\beta > 0$) the term in parenthesis is negative: the individual chooses more schooling than needed to maximize lifetime income. Third, productivity appears in the equation for optimal schooling only through consumption, that is through the intertemporal budget constraint in (1). The term $U'(c)c$ is critical, therefore, in driving the effect of productivity on the schooling choice. Given the functional form of U this term is

$$U'(c)c = \frac{c}{c - \bar{c}}.$$

To illustrate the properties of this function, consider an increasing path of income and consumption. At low levels of income consumption is close to $\bar{c}$ and the term $U'(c)c$ is large. Increases in income, reduce the value of $U'(c)c$ and since the term in parenthesis in (2) is negative the left-hand side of (2) increases so that optimal schooling increases. As income rises the term $U'(c)c$ asymptotes 1 and s becomes invariant to changes in income. Hence, qualitatively the model delivers the observed pattern in the data that schooling increases faster for poor than for rich countries.

The first order condition for leisure is

$$U'(c)c - (1 - \gamma)\alpha V'(\ell)(1 - \ell) = 0. \tag{3}$$

Given the functional form for V, the term $V'(\ell)(1 - \ell)$ is decreasing in ℓ. Hence, as income grows and $U'(c)c$ decreases towards 1, leisure time increases. Asymptotically, leisure time is constant. Note that our choice of $U(c)$ implies that this asymptotic value of ℓ is in the interior of $(0, 1)$.[11]

[11] Functions such as $U(c) = [(c - \bar{c})^{1-\eta} - 1]/(1 - \eta)$, where $\eta > 1$ imply that $U'(c)c \to 0$ as $c \to \infty$. In this case the asymptotic long-run value for leisure time is one.

We define y_τ as the period income of an individual of generation τ at age 35:

$$y_\tau = z_\tau e^{35g}(1 - \ell_\tau)H(s_\tau, x_\tau).$$

Since we use this measure in our quantitative analysis, we emphasize how increases in productivity affect income. First, an increase in productivity raises income through three channels: a direct effect through z_τ; an indirect effect through increases in schooling s; and another indirect effect through increases in expenditures in education x and therefore human capital. Second, an increase in productivity induces an increase in leisure time and therefore reduce labor income. The increase in leisure hinders the incentive to acquire education.

4 Calibration

We calibrate a benchmark economy to the time-series data for the United States. Although the emphasis of our quantitative exercise is on the cross-country implications from 1950 to 2005, we calibrate the model using the longest possible time series of the variables of interest for the United States. The motivation for this strategy is simple. Since the key channel in the model is the strength of the income effect on schooling and hours of work, the time path of these variables in the United States provides quantitative discipline. The long time series allows for better identification of the parameters of the model. In particular, as is well documented for the United States over time, there is a long-run increase in schooling followed by a slowdown toward the end of the 20th century. Similarly, there is a long-run decline in weekly hours followed by a slowdown. Our calibration procedure exploits these changing trends to discipline the strength of the income effect that is central to the quantitative implications of the model across countries.

For the United States, the schooling data that we use is provided by Claudia Goldin and Larry Katz and serve as the basis of Figures 1.4, 1.5 and 1.6 in their book.[12] The data give years of schooling by birth cohort, completed at age 35 for white people starting in 1876 until 1975. We HP-filtered the time series and used, for calibration purposes, cohorts from 1880 to 1965. These cohorts of people are of age 35 in years 1915 through 2000. The trend in schooling shows that a 35-year-old person in 1915 had completed about 8 years of schooling while the same-age person in 2000 had completed close to 14 years.[13]

The hours data that we use are built from various sources. For the period 1830 to 1880 we use data from Whaples (1990, Table 2.1), for the period 1890 to 1940 we use data from Kendrick (1961, Table A-IX), and for the period 1950 to 2000 we use data reported by McGrattan and Rogerson (2004, Table 1).[14] We HP-filtered the data and linearly interpolate between census dates to build a time series of hours from 1830 through 2000. The trend shows a decline from close to 72 hours per week in 1830 to 40 hours in 2000. Importantly, the rate of this decline is non-constant. There is a moderate decline in hours from 1830 to about 1910, followed by a sharp decline until about 1980, and a substantial flattening after 1980. The fact that the workweek declined significantly in the United States has been recognized elsewhere. Rogerson (2006), for example, uses data from Whaples (1990) and proposes to rationalize the decline in the workweek using non-homothetic preferences similar to our specification. Maddison (1987, Table A-9) and Huberman and Minns (2007, Table 1) show patterns of hours over time for countries such as Belgium, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Spain, Sweden, Switzerland and the U.K. between 1870 and 1990 that are similar to the pattern in the United States.

[12]See Goldin and Katz (2008, Figures 1.4–1.6).

[13]We verified that the Goldin and Katz data used for calibration is consistent with the Barro and Lee data for the United States for the period 1950 to 2000.

[14]The Whaples data are weekly hours worked collected from two surveys of manufacturing hours taken by the federal government in the context of the 1880 Census. The Kendrick data are average weekly hours in the private non-farm sector, finally the McGrattan and Rogerson data are average weekly hours worked for all workers.

The hours data are available in calendar time while the model predicts hours by generation. We choose to associate the 1830 hours data with the 1795 generation from the model, i.e. the generation that is 35 years old in 1830. That is, when we compare the model's predictions to the U.S. data we compare the hours chosen by the 1795 generation in the model with the 1830 data on hours. We associate subsequent data points and generations in the same way, until the 1965 generation which corresponds to individuals reaching age 35 in 2000. Thus, our calibration procedure implies that we compute decisions for 171 generations, starting from the 1795 generation and ending with the 1965 generation.

To calibrate the model lifespan T we note that its empirical counterpart is not life expectancy per se but rather the sum of years spent in school and on the labor market for a generation. Hazan (2009) reports market years for cohorts born in $1840, 1850, \ldots, 1930$. We combine this data with Goldin and Katz's figures for years of schooling achieved by these generations to obtain a measure of T_τ for cohorts born in $1840, 1850, \ldots, 1930$. We then estimate a linear time trend for T_τ:

$$T_\tau = a_T + b_T \tau, \tag{4}$$

and use it to compute T_τ for all cohorts of the model.[15]

We now discuss the specification of the non-homothetic preference term $\bar{c}$. This term is critical for the calibration of hours over time and as a result for the implications of the model for poor countries. While it is critical in our model to have non-homothetic preferences ($\bar{c} > 0$) for schooling to vary over time, regardless of the specification, the parameters of the human capital technology allow for an excellent fit of the schooling data. What we find is that our model with constant $\bar{c}$ implies an income elasticity of hours that falls exponentially

[15] We find $\hat{a}_T = -274.7819$ and $\hat{b}_T = 0.1692$. The standard error for the intercept and slope terms are 16.16 and 0.008. The linear trend fits the data very well with $r^2 = 0.99$.

as income rises. With constant income growth this implies that hours fall exponentially over time. This pattern of hours in the model can fit the U.S. hours data from 1910 onwards but not the pattern before 1910. Since the United States was poorer before 1910 than nowadays, fitting the income elasticity of hours at low levels of income has implications for the model's predictions about poor countries. Hence, allowing $\bar{c}$ to vary over time is critical to reproducing the historical time series of hours of work in the United States and to analyzing poor economies. In particular, for the model to fit the hours data in the United States before 1910, $\bar{c}$ must be lower before 1910. This implies that the model generates a lower income elasticity of hours (and therefore schooling) than with a constant $\bar{c}$. To allow for a flexible specification of $\bar{c}$ over time that we can estimate with data while still retaining the long-run implications of the model with a constant $\bar{c}$, we specify a transformed version of the logistic function as follows:

$$\bar{c}(z) = \frac{\mu}{1 + \exp(-z + \omega)},$$

where μ and ω are parameters to be determined. With constant productivity growth, $\bar{c}(z)$ is asymptotically constant and hence the long-run properties of the model remain as discussed previously.[16] While this feature of the model is only motivated by the behavior of the hours data, we note that there is an empirically plausible structural interpretation of time-varying $\bar{c}$ that can be derived from a model with household production. Intuitively, as home hours are substituted with market hours, more market goods are used to provide minimum consumption. Substitution of hours depends on preference and technology parameters such as the rates of growth of market and home productivity. To the extent that we cannot empirically pin down these growth rates, we opted for a reduced-form approach. We explore a quantitative version of such home production model in the Appendix.

[16] In Section 5.3, we explicitly estimate and explore the implications of the model assuming a constant $\bar{c}$ over time.

We now describe the details of the calibration procedure. We start the calibration of the model by normalizing the productivity parameter $z_{1795} = 1$. We set the discount factor to 4 percent, i.e., $\rho = 0.04$ and, following Bils and Klenow (2000), we choose $\gamma = 0.1$. We pick the rest of the parameters in order to minimize a measure of distance between the model's predictions and relevant U.S. data. Specifically, let λ be the vector of parameters to calibrate:

$$\lambda = (\alpha, \beta, \mu, \omega, \sigma, \psi, \theta, g)',$$

and let $\hat{s}_t(\lambda)$ and $\hat{n}_t(\lambda)$ represent optimal schooling and work time of generation τ. Let $\mathbf{s}_\tau$ and $\mathbf{n}_\tau$ be their empirical counterpart: $\mathbf{s}_\tau$ is years of schooling for generation τ in the U.S. data and $\mathbf{n}_\tau$ is the workweek at date $\tau + 35$ in the U.S. data. The mapping of hours between the model and the data is done by assuming that there are 112 hours of discretionary time per week.[17] Hence, a 40-hour workweek corresponds to 40/112 units of work time in the model. Finally, our calibration procedure also targets a growth rate in income per capita of 2 percent per year. Thus, we find λ by solving the following minimization problem:

$$\min_{\lambda} \left\{ \sum_{\tau=1880}^{1965} \left(\frac{\hat{s}_\tau(\lambda)}{\mathbf{s}_\tau} - 1 \right)^2 + \sum_{\tau=1795}^{1965} \left(\frac{\hat{n}_\tau(\lambda)}{\mathbf{n}_\tau/112} - 1 \right)^2 + \left(\frac{\hat{y}_{1965}(\lambda)/\hat{y}_{1795}(\lambda)}{\exp(0.02 \times 171)} - 1 \right)^2 \right\},$$

Table 2 shows the values of the calibrated parameters. Figures 3 and 4 show the excellent fit of the model to the U.S. data on schooling and hours. Note how the time series of hours implied by the model fits the changing pattern of the rate of change in actual hours. The calibrated function of $\bar{c}(z)$ permits this fit. We show that when $\bar{c}(z)$ is constant, the best fit the model can produce for the time series of hours is a strictly convex pattern that fails to fit the changing pattern in hours over time prior to 1910.

Given our calibration strategy, it should not be surprising that our model implies, as in the

[17] Assuming that a person needs 8 hours for sleep and other necessities, there are $(24 - 8) \times 7 = 112$ hours of discretionary time in a week.

data reported by Hazan (2009), that years spent on the labor market increases while total lifetime hours decrease across generations. Nevertheless, for the sake of comparison with the data we report the implications of the model for the 1840 and 1930 cohorts. We find that time in the labor market in the model is 31 years for the 1840 cohort (34 years in the data) whereas for the 1930 cohort it is 40 years (40 years in the data). In terms of lifetime hours the model implies 103,830 hours for the 1840 cohort and 85,780 hours for the 1930 cohort. The data for these cohorts is 103,324 and 77,502.[18]

The calibrated values of μ and ω imply that $\bar{c}(z)$ reaches its long-term value given by μ around 1910. We compute the ratio of μ to the income of the last generation in the benchmark economy and find it to be 2%. It is not obvious how to compare this value with data. One possibility is to relate it to the final expenditures on food relative to GDP.[19] For the United States, the expenditure share of food is 5.2% in 1996. Another possibility is to compare the incomes of countries far back in time. Maddison (2009) reports that GDP per capita in Western Europe between 1 and 1500 was between 450 and 771 at constant 1990 dollars, representing a range of 2 to 4.5 percent of the 1970 GDP per capita. Since measured income is likely to be lower due to non market production, we conclude that the value of μ relative to income of 2% is reasonable in light of the related evidence.

5 Cross-Country Experiments

We conduct quantitative experiments using the calibrated model to assess the importance of productivity and life expectancy in explaining educational attainment across countries and over time.

[18]We compute market years of generation τ as $T_\tau - s_\tau$ and lifetime hours as $(T_\tau - s_\tau)(1 - \ell_\tau) \times 52$.

[19]The data is for the 1996 Benchmark study of the International Comparison Program.

5.1 Baseline Experiment

We use the calibration of the benchmark economy and assume that countries are identical except in terms of productivity and life expectancy. In particular, we assume that countries differ in the initial level and growth of productivity z and g; and in the level and rate of change in life expectancy. We discipline our choice of life expectancy across countries by estimating two cross-sectional relationships between life expectancy and GDP per capita for 1950 and 2005. We then search for 10 combinations of z and g that match the relative income gaps in 1950 and 2005, as described in Table 1, while imposing that life expectancy in 1950 and 2005 be as described by the estimated cross-sectional relationships. A detailed description of this procedure is in the appendix. Table 3 displays the results of our baseline experiments. The implied values of z, g, and T for the first and last generations for each economy are reported in the first four columns. Before we describe the results in detail, we emphasize that even though our framework abstracts from amplification income effects such as those emphasized in Manuelli and Seshadri (2006) and Erosa, Koreshkova, and Restuccia (2010), the results from our quantitative experiments would not be affected. Amplification effects would reduce the size of the productivity gaps needed to reproduce the calibrated income differences across countries leaving the impact of income on schooling the same.

There are two sets of results that we emphasize from Table 3: the cross-sectional implications of the model relative to the data in 1950 and the time-series behavior across countries relative to the data. We start with the cross-sectional implications in 1950. We find that the model accounts for 95 percent of the difference in schooling between countries in the 1st decile and the United States. To understand how we obtain this statistic, note that for countries in the poorest decile of income in 1950, the model implies 1.7 years of schooling whereas the data is 1.27 years (see Table 1). In 1950 the United States has 10.45 years of schooling which is closely reproduced by our calibrated benchmark economy. Hence, the model accounts for $(10.5 - 1.7)/(10.5 - 1.27) = 95\%$ of the difference. The model accounts for a lower percentage of schooling differences for countries in higher deciles of income: 89% for the 5th decile and 33% for the 10th decile. Therefore, there is a systematic tendency for the model to account for lower fractions of the schooling data as we consider richer countries. This is because the mechanisms emphasized in our theory (non-homotheticy in preferences and life expectancy) vanish at high levels of income to eventually play no role. For rich countries, factors other than income levels have first-order importance in the determination of schooling, e.g., public policy towards education, labor market institutions that compress wages, among many others. In poor countries, however, increases in productivity and income allow individuals to move farther away from subsistence consumption having a first-order effect on the allocation of time in schooling.

We now turn to the time-series implications of schooling across countries. We find that the model accounts for 78 percent of the increase in schooling in poor countries. We compute this statistic as follows. For the economy in the 1st decile, schooling increases from 1.7 years in 1950 to 5.1 in 2005, a $\ln(5.1/1.7)/55 = 1.99\%$ annual increase. It compares with 2.56% in the data. Thus, for this economy, the model accounts for $1.99/2.56 = 78\%$ of the increase in schooling. Similarly, for countries in the 5th and 10th deciles the model accounts for 82 and 125% of the increase in schooling. We note from Table 3 that schooling increases in all economies in our model, an implication that is consistent with the data. We also note the tendency for poor economies to increase their schooling faster than richer economies even though they are not necessarily catching up in relative income since the evolution of relative income exactly matches the data. As a consequence of this faster increase in schooling in poor economies, schooling differences in the model are smaller in 2005 than in 1950. This implication of the model is also consistent with the data. In particular, it is consistent with the decline in the income elasticity of schooling across countries over time displayed in Figure 2. Using Table 3, we compute an approximation of this elasticity with the ratio of relative changes in schooling and income between deciles of the distributions. Comparing the 1st and 10th decile in Table 3, we find an elasticity of 0.62 (i.e., $\ln(9.7/1.7)/\ln(0.83/0.05)$) in 1950 versus 0.41 in 2005. Again, the decrease of this elasticity is evidence of the reduced dispersion in schooling across countries in 2005 and, therefore, of the faster increase in schooling in poorer countries. Comparing the 1st and 5th deciles in Table 3 yields elasticities of 0.78 and 0.43 in 1950 and 2005. Comparing the 5th and 10th deciles yields 0.53 and 0.32. In the data of Figure 2 this elasticity decreases from 0.6 to 0.27.

In terms of hours there is limited data that can be brought to bear on the implications of the model. Nevertheless, we use the available hours data from the Conference Board (2010). They report yearly hours per worker and we plot the data against GDP per capita in Figure 5. We note that not only hours of work decline with income in 1950 and 2005, but also hours decrease as income rises for each country and hours fall faster for the poor than the rich countries. In the data in 1950, hours of work in poor countries relative to rich is about 1.4, while the same ratio drops to 1.2 in 2005. In the model, the ratio of hours in the poorest economy relative to the benchmark is 2.4 in 1950 and drops to 1.7 in 2005. The ratio of hours between an economy in the 5th decile and the benchmark is 1.6 in 1950 and drops to 1.3 in 2005. While the comparison here is very crude since hours data is missing for the poorest countries, the rough comparison suggests that the hours implications of the model are broadly in line with the data in terms of both the magnitude of hours differences in 1950 and the faster decline in hours over time in poor countries.

5.2 Equal Growth Rates across Countries

To illustrate the importance of differences in productivity growth and changes in life expectancy across countries, we conduct two additional experiments. First, we conduct an experiment similar to the baseline except that we assume that the rate of growth of productivity, g, is the same in all countries at the value in the benchmark economy. In a second experiment we assume that in addition the change in life expectancy over time is also the same across countries at the value in the benchmark economy. Essentially, we show with these experiments that the implications of the model for the cross-country differences in schooling in 1950 are not substantially affected by the differential growth components. We also show that even when abstracting from differences in productivity growth and changes in life expectancy, the model still accounts for a substantial portion of the changes in schooling over time across countries.

In the first experiment, we follow the baseline except that we assume equal productivity growth g across countries. Results are reported in Table 4. In the 1950 cross-section the model accounts for 92 percent of the difference in schooling between countries in the 1st decile and the United States. This compares to 95 percent in the baseline. At the 5th and 9th deciles the numbers are 88 and 34 percent (89 and 33 in the baseline). Turning to the time-series implications, the model with equal g across economies implies that in the 1st decile, the model accounts for 68 percent of the increase in schooling versus 78% in the baseline experiment. In the 5th decile the number is 78% (82% in the baseline). Just as in the baseline, the model with equal g predicts a narrowing of the schooling gap relative to income as observed in the data. As discussed previously, we compute an approximation of the cross-country income elasticity of schooling in 1950 and 2005 and show that the implied elasticity is lower in 2005 than in 1950. Comparing the 1st and 10th decile, we find that the income elasticity of schooling falls from 0.58 to 0.36. The corresponding figures, when we compare the 1st and 5th decile are 0.67 and 0.52 and when we compare the 5th and 10th they are 0.52 and 0.23.

In the second experiment, we follow the previous experiment and assume in addition that the change in life expectancy for each country is the same as in the benchmark economy. That is, relative to the previous experiment, life expectancy in 1950 is unchanged but the life expectancy in 2005 is 9.3 years above that of 1950 for all countries, which is the years increase in life expectancy in the benchmark economy between 1950 and 2005. The results of this experiment are reported in Table 5. We first note that the cross-sectional implications in 1950 of the model in this experiment and the previous experiment are identical.[20] In terms of the time series, abstracting from differences in rate of change of T reduces the ability of the model to account for the increase in schooling. Nevertheless, even when abstracting from differences in productivity growth and changes in life expectancy, the model accounts 50% of the growth rate of schooling in the 1st decile. At the 5th decile the corresponding figure is 50%. The model also implies a faster growth in schooling in poor than in rich countries.

5.3 Importance of $\bar{c}(z)$

We assess the quantitative importance of the assumption that $\bar{c}$ is time-varying. We conduct an experiment where we assume a constant $\bar{c}$. We calibrate the benchmark economy to U.S. data as in Section 4. The list of parameters to be determined by the minimization program is now $(\alpha, \beta, \sigma, \bar{c}, \psi, \theta, g)'$. Figure 6 shows the fit of this version of the model against the U.S. data. While this version of the model fits the time path for schooling as well as in the baseline, the implied time path of hours exhibits a convex shape that fails to fit the time series of hours, particularly in the earlier period. A consequence of this convex shape is that hours increase fast as productivity declines, potentially leading to unreasonable predictions about hours of work for poor countries.

[20]This is due to the fact that, by design, the experiment implies that the level of life expectancy and income are the same in 1950. The only difference between the two experiments is the level of life expectancy in 2005.

We conduct the same cross-country experiment as described in Section 5.1 using the calibrated parameters of the benchmark economy with a constant $\bar{c}$. Table 6 reports the results. The model generates such large effects on hours and schooling for poor countries that it is not possible to compute economies corresponding to levels of income below that of decile 8, i.e. an economy that is 37 percent of the income per-capita in the benchmark economy in 1950. The results for the economy in the 9th decile (an economy with 59% relative income in 1950) show that hours and schooling are magnified in the model with constant $\bar{c}$ compared to the baseline: hours of work for this economy are 62.6 versus 54.6 in the baseline and schooling is 6.7 years versus 7.9 years in the baseline. Thus, even at this level of relative income the labor supply elasticity is very high (as suggested by Figure 6) and the predictions for schooling are magnified as a result. The implications of the larger response of hours and schooling to productivity changes in this version of the model are even more striking when comparing a poorer economy. The poorest economy we are able to compute in this version of the model is an economy whose relative income in 1950 is 55 percent of the benchmark economy.[21] Note this economy is much closer to the economy in decile 9 than decile 8. Hours of work in this economy are 111.5 (out of a total 112 hours) compared to 62.9-54.6 hours in the 8th and 9th decile economies in the baseline and schooling is 3.3 years compared to 6.2-7.9 years in the 8th and 9th decile economies in the baseline.

6 Conclusions

We developed a model of human capital accumulation to quantitatively assess the importance of productivity and life expectancy in explaining differences in educational attainment across countries and over time. We calibrated a benchmark economy to reproduce the historical

[21] We assume that this economy features the same growth rate in labor productivity as the average country in the 8th decile.

evolution of schooling and hours in the United States. We found that the model accounts for 95 percent of the difference in schooling between rich and poor countries in 1950. The model accounts for 78 percent of the increase in schooling levels over time in poor countries. The model generates a faster increase in schooling levels in poor than in rich countries. Hence, the model explains the convergence in cross-country schooling levels observed in the data even though the per-capita income gap between these countries has generally not decreased. Our results emphasize the importance of productivity (and life expectancy) in explaining the bulk of differences in educational attainment across countries and their evolution over time. These results have important implications for educational policy as they shift the focus of attention from frictions and market imperfections to the determinants of low productivity in poor countries. Nevertheless, we think that extending our framework to incorporate complementary factors such as credit market frictions and public education (such as school infrastructure) can yield additional insights. We leave these important extensions of the model for future work.

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A Schooling Data

Table A.1 reports average years of schooling for people 25 to 29 years of age for countries by deciles of the schooling distribution in 1950. The data is from Barro and Lee (2010) and includes the entire sample of 147 countries. Compared to the dispersion in schooling between rich and poor countries in our restricted sample in Table 1, the larger sample in Table A.1 shows that the pattern of convergence in schooling across countries over time is even stronger, with the schooling gap between countries in the tenth and first deciles being a factor of 31-fold in 1950 and less than 3-fold in 2005.

Table A.1: Average Years of Schooling across Countries

Decile	s_{50}	s_{05}	s_{05}/s_{50}
1	0.28	4.06	14.60
2	0.60	6.11	10.26
3	1.07	7.02	6.57
4	1.58	7.34	4.66
5	2.41	8.63	3.58
6	3.39	9.64	2.85
7	4.40	10.11	2.30
8	5.28	10.74	2.03
9	6.85	11.26	1.64
10	8.73	11.69	1.34
$R_{10/1}$	31.41	2.88	–

Note: s is average years of schooling of the 25-29 year old population. Numbers reported are the average of each decile. The countries in each decile are the same in each year and represent the 1950 distribution of schooling.

B Structural Model of Time Varying $\bar{c}$

We describe a structural interpretation of the time-varying $\bar{c}$ in our baseline model and show its empirical plausibility. Since the issue of time-varying $\bar{c}$ pertains to the model's ability to fit the labor supply data in the time series, we abstract from life-cycle and schooling decisions in what follows.

Suppose that an individual lives for one period and has preferences represented by

$$\ln(C(c_m, c_n) - \bar{m}) + \alpha \ln(\ell),$$

where c_m and c_n are consumption of a market and a non-market good. The variable ℓ represents leisure time and $\bar{m}$ is a constant. The function C aggregates the consumption of the market and home good. Assume that $C(c_m, c_n) = \phi c_m + (1-\phi)c_n$. The home good is produced with time, in line with the technology $c_n = z_n h^{\mu}$ where $\mu \in (0,1)$, h is the time devoted to home production and z_n is productivity in the home technology. The budget constraint of the individual is $c_m = z_m(1-\ell-h)$, where z_m stands in for market productivity. The individual's optimization problem is then

$$\max_{c_m,h,\ell} \left\{\ln(\phi c_m + (1-\phi) z_n h^{\mu} - \bar{m}) + \alpha \ln(\ell) : c_m + z_m(h+\ell) = z_m\right\}.$$

The solution for home hours is

$$h = \left(\frac{1-\phi}{\phi}\mu\frac{z_n}{z_m}\right)^{\frac{1}{1-\mu}},$$

and the solution for leisure time is

$$\ell = \frac{\alpha}{1+\alpha}\left(1 - \delta\left(\frac{z_n}{z_m}\right)^{\frac{1}{1-\mu}} - \frac{\bar{m}/\phi}{z_m}\right),$$

where

$$\delta = \left(\frac{1-\phi}{\phi}\mu\right)^{1/(1-\mu)} - \frac{1-\phi}{\phi}\left(\frac{1-\phi}{\phi}\mu\right)^{\mu/(1-\mu)}.$$

Define labor supply as $n = 1 - \ell - h$ and income as $y = z_m n$.

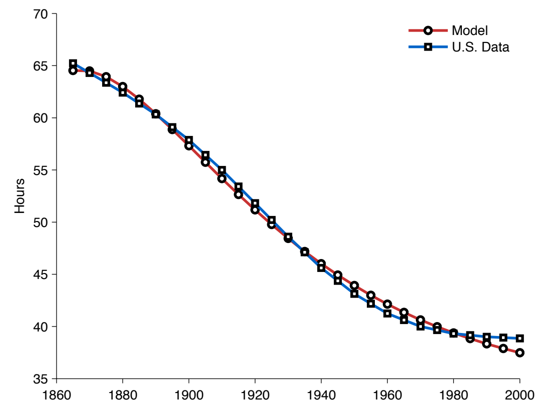
Note a few points. First, the utility derived from consumption can be written as $\ln(\phi c_m - \bar{c}(z_m, z_n))$ where $\bar{c}(z_m, z_n) = \bar{m} - (1-\phi) z_n h^\mu$ is analogous to the time-varying $\bar{c}$ in our baseline model. This object is a decreasing function of home hours. Second, if market productivity grows fast enough relative to home productivity, home hours are high and $\bar{c}$ low at low levels of income. Hence, the model can deliver the feature discussed in Section 4 that the time-varying $\bar{c}$ must be lower at low levels of income. Third, if market productivity grows faster than home productivity, leisure time converges to a constant between 0 and 1 in the long run. Fourth, we assumed perfect substitutability between home and market consumption for the sake of simplicity. Allowing for a CES aggregator of home and market consumption would enhance the model's ability to fit the time series of work hours.

We now investigate the ability of this model to fit the hours data. Let z_m and z_n grow at constant rates: $z_{m,\tau} = z_{m,1830} e^{g_m(\tau-1830)}$ and $z_{n,\tau} = z_{n,1830} e^{g_n(\tau-1830)}$. Normalize the level of market productivity $z_{m,1830} = 1$ and define the following vector of 7 parameters to be determined:

$$\lambda = (g_m, g_n, \alpha, \bar{m}, \phi, \mu, z_{n,1830})'$$

and define $\hat{n}_\tau(\lambda)$ and $\hat{y}_\tau(\lambda)$ as hours and income at date τ and let $\mathbf{n}_\tau$ be actual hours. In

Figure B.1: Work Hours – Model with Household Production and Constant Subsistence



the same spirit as in the baseline calibration, we find λ by solving

$$\min_{\lambda} \left\{ \sum_{\tau=1830}^{2000} \left(\frac{\hat{n}_\tau(\lambda)}{\mathbf{n}_\tau/112} - 1 \right)^2 + \left(\frac{\hat{y}_{2000}(\lambda)/\hat{y}_{1830}(\lambda)}{\exp(0.02 \times 171)} - 1 \right)^2 \right\}.$$

We find $\alpha = 2.3$, $\phi = 0.72$, $\bar{m} = 0.47$ and $\mu = 0.38$. The rate of growth of market and home productivity are $g_m = 0.024$ and $g_n = 0.004$. The behavior of market hours is represented in Figure B.1. The model predicts that home and market hours are declining over time and leisure time is increasing.

C Cross-Country Experiments

We describe in detail our strategy to restrict the four parameters varying across countries in the cross-country experiments in Section 5. These parameters are the level and growth rate of productivity (z, g) and the life expectancy of the 1915 generation (reaching 35 in 1950) and the 1970 generation (reaching 35 in 2005).

The empirical measure of T used for the benchmark economy that is best suited for the model is the sum of years spent in school and on the labor market for a generation. The same data is not readily available for the time period and set of countries that we analyze. Hence, our approach to calibrating T across countries and time is described in two steps:

1. We estimate an empirical relationship observed across countries between life expectancy at birth and income per capita as follows:

   $$\text{Life Expectancy} = \text{slope} \times \ln\left(\text{GDP per capita}\right) + \text{constant} + \text{error}.$$

   We estimate this relationship for two time periods. We start with life expectancy of the 1915 generation. There does not exists a wealth of data to estimate this relationship. Thus, we use the data from Preston (1975) pertaining to the 1930s.[22] The estimated relationship fits the data very well with an adjusted $r^2 = 0.82$. The estimated slope coefficient is 9.481. We estimate the same relationship for life expectancy of the 1970's generation using data from the World Bank Development Indicators and Penn World Tables. The fit of the data is also very good with an $r^2 = 0.52$ and an estimated slope coefficient of 7.1684. The assumed empirical relationship implies that the difference in life expectancy between any two economies at a point in time is given by the slope coefficient times the log of the factor difference in income per capita between the two economies:

   $$T_t - T_t^{us} = \text{slope}_t \times \ln\left(\frac{y_t}{y_t^{us}}\right).$$

   We use this relationship for 1915 and 1970 and the life expectancy for the benchmark economy to estimate the implied life expectancy for all economies in our cross-country experiments.

2. For each of the 10 economies we consider we search for an initial level and a growth

[22]Preston also offers data for the 1900s but this data contains only 10 countries. The 1930s data report life expectancy at birth and real income per capita for 38 countries.

rate of productivity, i.e. a pair (z, g), such that the model matches the income per capita of the economy relative to the benchmark economy as described in Table 1 for 1950 and 2005. Life expectancy of the generations reaching 35 in 1950 and 2005 are dictated by the relationships estimated in step 1.

Table 1: GDP per Capita and Schooling across Countries

	1950		2005		
Decile	y_{50}	s_{50}	y_{05}	s_{05}	s_{05}/s_{50}
1	0.05	1.27	0.06	5.07	3.99
2	0.07	1.65	0.07	6.79	4.12
3	0.10	2.82	0.18	8.47	3.00
4	0.13	1.88	0.13	8.01	4.26
5	0.18	3.55	0.31	10.29	2.90
6	0.21	3.34	0.24	9.16	2.74
7	0.24	4.23	0.33	10.36	2.45
8	0.37	5.30	0.57	10.56	1.99
9	0.59	6.73	0.71	11.74	1.74
10	0.83	8.11	0.78	11.08	1.37
$R_{10/1}$	16.60	6.39	13.00	2.19	-
$R_{9/1}$	11.80	5.30	11.83	2.32	-

Note: y is real GDP per capita relative to that of the United States and s is average years of schooling of the 25-29 year old population. Numbers reported are the average of each decile. The countries in each decile are the same in each year and represent the 1950 distribution of GDP per capita.

Table 2: Calibration

Preferences	$\rho = 0.04$, $\alpha = 3.39$, $\beta = 1.92$
	$\sigma = 0.0$, $\mu = 1.39$, $\omega = 1.96$
Technology	$\gamma = 0.1$
	$\theta = 0.04$, $\psi = 0.13$
Productivity	$g = 0.0199$, $z_{1795} = 1.0$
Demography	$T_\tau = 0.1692\tau - 274.7819$

Table 3: Model's Implications – Baseline Experiment

Calibrated Parameters				1950 Results			2005 Results		
z_{1795}	g (%)	T_{1915}	T_{1970}	Rel. y	s	$1-\ell$	Rel. y	s	$1-\ell$
0.01	2.77	21.7	35.9	0.05	1.7	112.6	0.06	5.1	66.0
0.05	2.18	24.0	39.2	0.07	2.4	82.6	0.07	6.0	63.6
0.02	3.15	27.4	42.5	0.10	2.9	81.4	0.18	7.2	58.6
0.12	2.06	29.9	45.1	0.13	3.7	68.8	0.13	8.1	56.7
0.03	3.1	33.0	49.8	0.18	4.3	68.2	0.31	10.3	48.3
0.10	2.44	34.4	48.9	0.21	4.7	65.7	0.24	10.0	48.3
0.07	2.78	35.7	50.6	0.24	5.0	65.3	0.33	10.8	45.8
0.07	3.03	39.8	53.4	0.37	6.2	62.9	0.57	12.3	41.7
0.23	2.54	44.2	55.9	0.59	7.9	54.6	0.71	13.5	39.4
0.91	1.93	47.4	57.4	0.83	9.7	45.2	0.78	14.3	38.3
1.00	1.99	49.2	58.5	1.00	10.5	43.6	1.00	14.9	37.7

Note: y is output per capita, s is average years of schooling, and $1-\ell$ is hours worked. In this experiment countries differ by the level and rate of growth of productivity (z and g) and life expectancy in 1950 and 2005.

Table 4: Model's Implications – The Effect of Equal Productivity Growth across Countries

Calibrated Parameters				1950 Results			2005 Results		
z_{1795}	g (%)	T_{1915}	T_{1970}	Rel. y	s	$1-\ell$	Rel. y	s	$1-\ell$
0.05	1.99	21.7	35.9	0.05	2.0	91.4	0.05	5.1	66.0
0.07	1.99	24.0	39.2	0.07	2.5	80.7	0.06	6.0	63.7
0.10	1.99	27.4	42.5	0.10	3.2	72.2	0.10	7.1	60.5
0.13	1.99	29.9	45.1	0.13	3.7	68.7	0.13	8.1	56.8
0.17	1.99	33.0	49.8	0.18	4.4	66.1	0.17	10.1	51.4
0.20	1.99	34.4	48.9	0.21	4.8	65.1	0.19	9.9	49.5
0.22	1.99	35.7	50.6	0.24	5.1	64.2	0.22	10.7	47.7
0.34	1.99	39.8	53.4	0.37	6.4	59.5	0.32	12.2	43.1
0.57	1.99	44.2	55.9	0.59	8.2	50.7	0.55	13.5	39.7
0.83	1.99	47.4	57.4	0.83	9.7	45.5	0.81	14.3	38.2
1.00	1.99	49.2	58.5	1.00	10.5	43.6	1.00	14.9	37.7

Note: y is output per capita, s is average years of schooling, and $1-\ell$ is hours worked. In this experiment countries differ by the level of productivity (z) and life expectancy in 1950 and 2005. Productivity growth (g) is assumed the same across countries as in the benchmark economy.

Table 5: Model's Implications – The Effect of Equal Productivity Growth and Change in Life Expectancy across Countries

Calibrated Parameters				1950 Results			2005 Results		
z_{1795}	g (%)	T_{1915}	T_{1970}	Rel. y	s	$1-\ell$	Rel. y	s	$1-\ell$
0.05	1.99	21.7	31.0	0.05	2.0	91.4	0.04	3.9	67.6
0.07	1.99	24.0	33.3	0.07	2.5	80.7	0.06	4.5	65.5
0.10	1.99	27.4	36.7	0.10	3.2	72.2	0.09	5.4	62.0
0.13	1.99	29.9	39.2	0.13	3.7	68.7	0.12	6.3	58.1
0.17	1.99	33.0	42.3	0.18	4.4	66.1	0.16	7.5	52.6
0.20	1.99	34.4	43.7	0.21	4.8	65.1	0.18	8.1	50.3
0.22	1.99	35.7	45.0	0.24	5.1	64.2	0.20	8.6	48.4
0.34	1.99	39.8	49.1	0.37	6.4	59.5	0.31	10.5	43.4
0.57	1.99	44.2	53.5	0.59	8.2	50.7	0.53	12.5	39.8
0.83	1.99	47.4	56.7	0.83	9.7	45.5	0.80	14.1	38.3
1.00	1.99	49.2	58.5	1.00	10.5	43.6	1.00	14.9	37.7

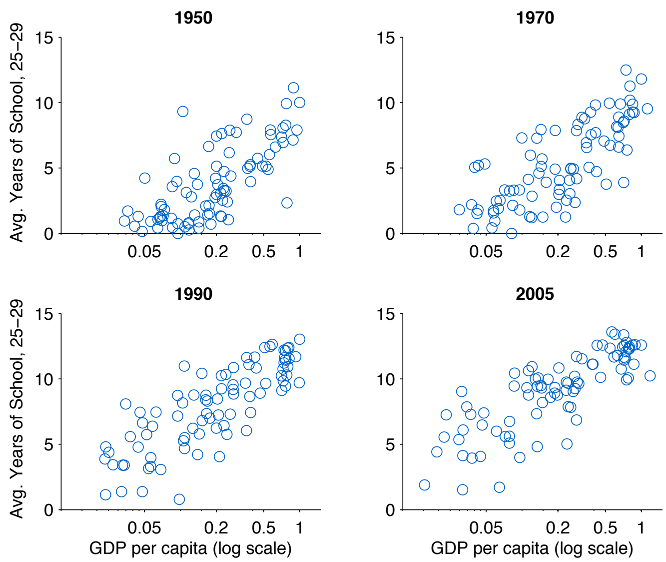
Note: y is output per capita, s is average years of schooling, and $1-\ell$ is hours worked. In this experiment countries differ by the level of productivity (z) and life expectancy in 1950 and 2005. Productivity growth (g) and the change in life expectancy between 1950 and 2005 are assumed the same across countries as in the benchmark economy.

Table 6: Model's Implications with $\bar{c}$ Constant – Baseline Experiment

Calibrated Parameters				1950 Results			2005 Results		
z_{1795}	g (%)	T_{1915}	T_{1970}	Rel. y	s	$1-\ell$	Rel. y	s	$1-\ell$
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-
0.20	1.64	44.4	55.9	0.59	6.7	62.6	0.71	12.9	44.0
0.94	0.96	47.4	57.4	0.83	9.4	46.7	0.78	14.3	40.4
1.00	1.05	49.2	58.5	1.00	10.4	44.4	1.00	15.1	38.9

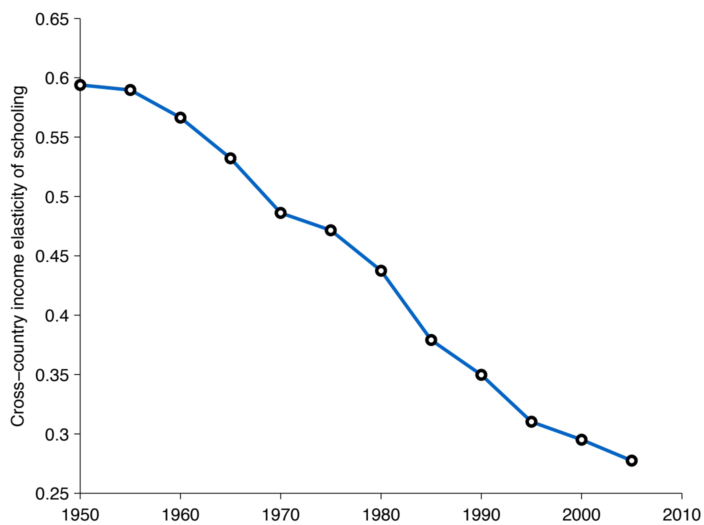
Note: y is output per capita, s is average years of schooling, and $1-\ell$ is hours worked. In this experiment countries differ by the level and rate of growth of productivity (z and g) and life expectancy in 1950 and 2005.

Figure 1: Average Years of Schooling Population 25 to 29 – Selected Years



Note: The source of data is Barro and Lee (2010) for schooling and the Conference Board (2010), Total Economy Database for GDP per capita. The horizontal axis measures GDP per capita relative to the United States. The vertical axis measures average years of schooling for the 25-29 population.

Figure 2: Income Elasticity of Schooling across Countries



Note: For each year, we regress log average years of schooling on a constant and log real GDP per capita across countries in our sample. The slope coefficient is plotted for each year.

Figure 3: Years of School Completed at age 35 – Model and U.S. Data

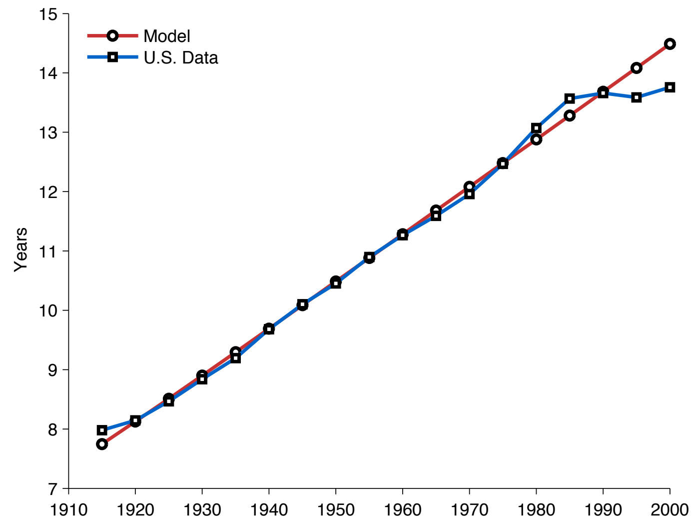


Figure 4: Work Hours – Model and U.S. Data

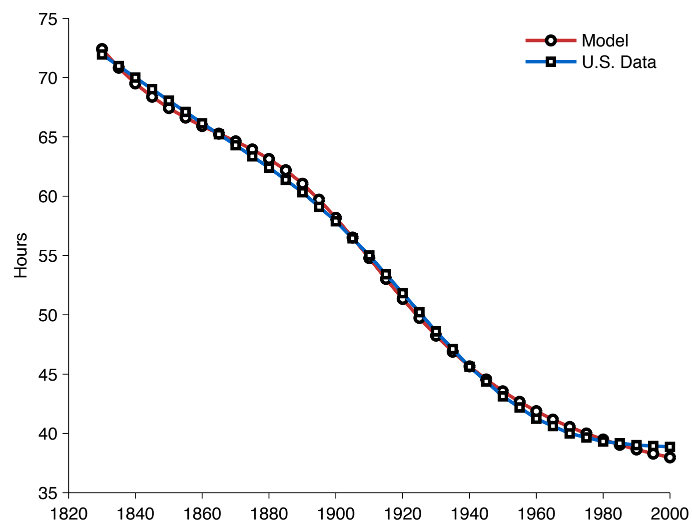
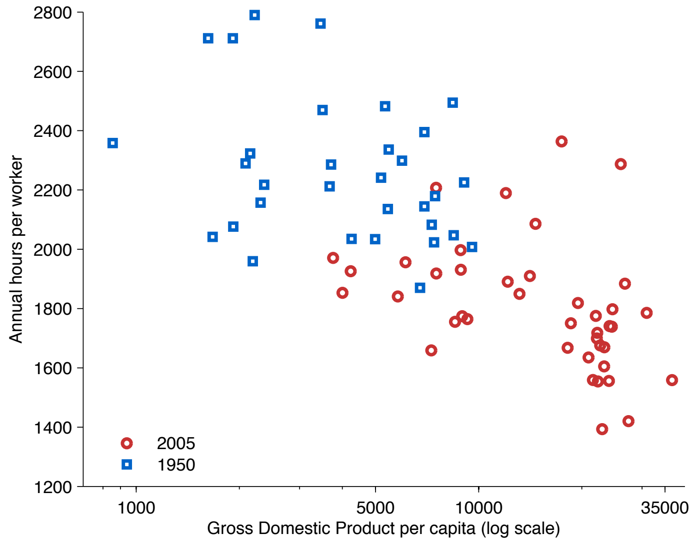
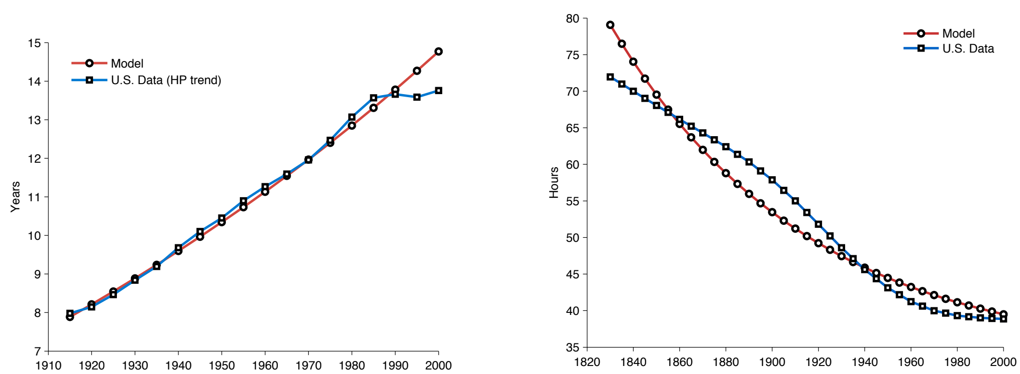


Figure 5: Work Hours across Countries



Note: Average annual hours per worker from the Conference Board (2010), Total Economy Database.

Figure 6: Model with Constant $\bar{c}$ and U.S. Data



Schooling Hours