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Search Committees

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Abstract

A committee decides by unanimity whether to accept the current alternative, or to continue costly search. Alternatives are described by several distinct attributes. Each committee member privately assesses the quality of one attribute (her "specialty"). Preferences are heterogeneous and interdependent: each specialist values all attributes, but puts a higher weight on her specialty (partisanship). We study how acceptance standards, members' welfare and expected search duration vary with the amount of conflict within the committee. We compare decisions made by specialist committees to decisions made by committees of generalists who can each assess all information available, and to one-person decision making.

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1 Introduction

Leonardo da Vinci was a painter, sculptor, architect, musician, mathematician, engineer, inventor, anatomist, geologist, cartographer and botanist. Isaac Newton was (merely) a physicist, mathematician, astronomer, natural philosopher, alchemist, master of the mint and theologian. Benjamin Franklin was an author, printer, political theorist, postmaster, scientist, inventor, statesman, and diplomat.

These giants had probably little use for committees. But most complex decisions in modern public or private organizations are taken by committees: Parliamentary committees prepare and often control legislative outcomes through their superior information; Hiring decisions for high-profile jobs that require multiple skills are made or prepared by search committees, e.g., for a CEO, public administrator or university professor; Investment decisions about various available projects are made by a board of directors, or by a partnership of venture capitalists; Funding decisions for (possibly interdisciplinary) research proposals are made by ad-hoc expert committees assembled within science agencies; Technological standards are set by committees where various experts represent firms within an industry, various industries on the producer or consumer side, or several countries; Monetary policy is set by a board instead of a sole governor in practically all important central banks.

Another ubiquitous feature of modern industrialized societies is the compartmentalization of knowledge. Just to give an example, medical specialization only started around 1830 in the great Paris hospitals, while the past half century has seen a tremendous increase in the number of medical specialties, along with the near disappearance of the general practitioner.¹

The causality relation between the above two phenomena seems obvious enough: The trend towards more specialization – which implies that no single individual has access to the entire necessary information – creates the necessity of delegating complex choices among multi-dimensional alternatives to committees of "specialists", each possessing information about some partial aspect of the problem at hand.²

But the possession of information that is not easily accessible to others creates incentives for strategic manipulation. The lack of efficient information transmission among specialized scientists was metaphorically bemoaned by Robert Oppenheimer in 1954: "The specialization of science is an inevitable accompaniment of progress;

¹For a history of specialization in the medical science see Weisz [2006].

 $^{^{2}}$ A lively debate takes place in the medical literature about the merits of specialists versus generalists. For example, Lowe et al [2000] study admission decisions for cardiac patients performed by doctors with different trainings. The decision problem is multi-dimensional since many of these patients suffer from *comorbidity* – the presence of several serious other conditions.

yet it is full of dangers, and it is cruelly wasteful since so much that is beautiful and enlighted is cut off from most of the world."

The danger perceived by Oppenheimer would be less severe if the experts/specialists in a committee would all rank the various feasible alternatives in the same way. But it is often the case that specialization also leads to a form of "bias" or "partisanship" – the view that one own's information/speciality is more important than others. For example, Hardy [1940], p.66 advises that: "It is one of the first duties of a professor, for example, in any subject, to exaggerate a little both the importance of his subject and his own importance in it." Surely all learned readers of this article can offer some empirical evidence to the effect that Hardy's advice is widely followed.

Differences in the weighting of various attributes may be intrinsic or psychological ("Anyone who defends his subject will find that he is defending himself," Hardy [1940], p. 144), or due to the fact that the decision makers are accountable to different constituencies and are better informed about the effects of the decision on their own constituency. One important example of the second type is offered by the monetary policy board of the European Central Bank. Gruener and Kiel [2004] argue that national central banks care about a policy that accommodates macroeconomic shocks in their own country, but, due to demand spillover effects, shocks in one country affect the desired policy in other participating countries. Moreover, they argue that national central bankers have some private information about their own national macroeconomic conditions (e.g., Greece during the last banking crisis). Another example is offered by international decisions about environmental policy where the nation states are interested in achieving less pollution in their own country, and presumably possess private information about the national amount of emissions, the costs to reduce emissions, or the economic consequences of a reduction. But it is obvious that the environmental situation in one member state is co-determined by the emissions in neighboring countries, and hence policy needs to be coordinated at international conferences (e.g., the Kyoto or Copenhagen conferences).

If strong enough, the degree of partisanship within a committee implies that each member will insist on a particularly high standard in his own speciality, leaving little room for trade-offs among the various attributes of each alternative. Such behavior leads to delay in reaching decisions. Here is, for example, what Farell and Saloner [1988] write in their influential study of standard setting committees:

"More than a hundred thousand people meet regularly in committees with the goal of reaching agreement on product and interface compatibility standards. The resources devoted to this formal standardization activity have roughly doubled in the last decade. But these committees too are imperfect coordinators. Often, by the time a committee is convened, participants have vested interests in incompatible positions, and the committee must resolve this conflict. Since the "consensus principle" which is generally accepted in voluntary standard setting, requires committees to seek a stronger consensus than a simple majority vote (though not necessarily unanimity), there may be a battle of wills in committee, while users wait. Such waiting is costly, whether simply because of delay or because eventually the participants can no longer wait, and the chance for coordination has been missed."

In order to study the interaction among specialization, private information, and partisanship in a dynamic framework where measuring delay is meaningful, we introduce a novel model whose main ingredients are:

- 1. A stream of alternatives (or projects, or candidates) is presented to a committee who has to decide whether to accept the current alternative, or to continue costly search (which can be seen as preserving a given status-quo). Thus, the environment is dynamic and the current alternative is compared to an endogenously determined continuation value rather than to a fixed, exogenously given outside option. This is a multi-person generalization of a classical one-person optimal stopping or search problem (see Chow, Robbins and Siegmund [1971] for a classical exposition).
- 2. Each alternative is described by a multi-dimensional bundle of several distinct attributes. Each committee member is able to privately assess the quality of one attribute (her "speciality"), but has only statistical knowledge about the distribution of other relevant attributes. Thus, the game our agents play is one with incomplete information.
- 3. Abstracting from informational issues (see also below), we follow here the approach of the so called "multi-attribute utility theory", a standard tool in decision analysis. The additive form of the utility function is the simplest, yet most widely used form: it states that the utility of an alternative is the weighted sum of the conditional utilities of the alternative's attributes, where the weights add up to one (For a classical exposition, see Keeney and Raiffa [1976]).
- 4. The members' preferences are interdependent (i.e., there are both private and common components) since the utility of each member is given by a convex combination of his own private signal and the private signals of other members.

We assume that committee members care most about the attribute about which they are also privately informed, but other cases can be also treated.

In our present technical treatment we focus on unanimity decisions in a committee with two members, but similar analyses for committees with more members who employ other decision rules (e.g., voting by majority) can be performed using the same tools. In particular, when alternatives are two-dimensional, a multi-person committee with a unanimity acceptance rule will be controlled by at most two members: those with the most stringent acceptance standard in each dimension, respectively.

Our main results study how acceptance standards, members' welfare and search duration vary with the amount of partisanship (or conflict), and compare the multiperson committee decision under specialization to two benchmarks: (i) committees without specialization, where all members are generalists and have access to all the available information (thus there is complete information); (ii) one-person committees (dictatorship).

It is important to point out that with extreme divergence of opinions, which corresponds here to the private values case (i.e., when each committee member puts all the weight on the attribute corresponding to her own speciality) it makes no difference whether the committee members have private information or not. The reason is that acceptance by member B (who votes based on information about her own speciality) conveys no additional information directly affecting member A's utility. Thus, whether A observes the attribute evaluated by B or not is inconsequential.

The situation dramatically changes when a member values attributes other than her speciality, i.e., when there is less conflict and values are interdependent. Then, under complete information, a committee member accepts candidates whose weighted combination of all observed attributes are above an optimal cutoff. In the dynamic search equilibrium, this cutoff is precisely equal to the continuation value obtained by not accepting the current candidate and continuing search.

Under incomplete information, behavior can be conditioned only on the single observed attribute (the member's speciality), and member A imprecisely infers from an acceptance by member B that the attribute monitored by B – which now directly matters for A – is of relatively high quality. The continuation value is now given by a convex combination of the acceptance cutoff in A's speciality and the inferred expected quality in the other dimension.

An interesting consequence of this difference is that increased partial partial partial or conflict leads to a more lenient acceptance rule under complete information, but to a more stringent rule under incomplete information! In particular, there are balanced but not exceptional candidates who are accepted by the specialized committee, but

rejected by the nonspecialized one. On the other hand, the specialized committee rejects candidates who are excellent in just one dimension that would be accepted by the generalist committee.

Although acceptance standards move in opposite directions, members' welfare in both settings behave similarly. Roughly speaking, welfare in committees increases with the covariance of the members' random utilities, where an increase in covariance can stem either from an increase in the variance of the underlying attributes (an effect that is shown to be beneficial in one-person decisions), or from a decrease in the degree of conflict within the committee. In particular, when conflict decreases the members' random utilities become *more associated*, where "more association" is a measure of positive dependence among random variables. Under specialization we can also show that the search duration increases as the degree of conflict increases.

It is obvious that a generalist dictator who possess information about all relevant attributes cannot gain by forming a committee where power has to be shared with others (unless the search costs can be passed to others). In contrast, when search costs are not too high, we show that a specialist dictator always prefers to share power and invite other specialists to the committee rather than take a partially uninformed decision by himself. Moreover, the invited specialist is better off by joining the committee rather than staying out and bearing the consequences of the dictator's decision. This Pareto improvement holds provided that there is a minimal congruence of interests among potential members, and is achieved in spite of a necessarily longer search duration in the committee problem. We also show that a specialist committee Pareto-dominates a generalist committee if the cost of assessing several dimensions of the decision problem is "convex enough" in the number of dimensions. Thus, our model yields intuitive explanations for the emergence of committees as a consequence of specialization.

The paper is organized as follows. In the next subsection we review some related literature. In Section 2 we present the basic committee decision model. In Section 3 we focus on the complete information/generalist benchmark. In Subsection 3.1 we analyze a one-person decision problem. Proposition 1 shows that the dictator's acceptance cutoff and utility increase if his preferences become more biased towards one attribute. In Subsection 3.2 we analyze the committee problem under unanimity and complete information. We prove the existence of a unique stationary and symmetric equilibrium, and we show that both acceptance cutoff and utility go down when the members' preferences become more divergent, which corresponds to an increase in the degree of conflict within the committee (Proposition 2). Moreover, the cutoff under unanimity is always lower than the cutoff under dictatorship. The final result in this Section, Proposition 3, relates a decrease in conflict (and hence an increase in welfare) to a precise mathematical notion of positive dependence among random variables. The analysis in this Section uses insights from majorization theory. Section 4 is devoted to the incomplete information (or specialization) case. Proposition 4 looks at the one-person decision problem where the dictator is informed only about one attribute, and shows that the acceptance cutoff, utility and search duration all go up when the dictator's preference become more biased towards one attribute. Section 4.2 is devoted to the study of unanimity decisions under incomplete information. Proposition 5 proves existence and uniqueness of a stationary equilibrium in a general (not necessarily symmetric) setting if the attributes' distributions have a strict decreasing mean residual life (DMRL). Under the DMRL assumption, Propositions 6 analyzes the effects of more extreme preferences on acceptance cutoffs and search duration in possibly asymmetric committees. In particular, equilibrium search duration increases when at least one committee member becomes more biased. Proposition 7 focuses on symmetric settings: it proves the existence of an unique symmetric equilibrium, and shows that both the acceptance cutoff and expected search duration go up while utility goes down when the degree of conflict within the committee increases. It also gives an upper bound on the increase of the search duration. Propositions 8 and 9 study the effects of stochastic increases in the attributes' distributions within, and across committees, respectively. In Section 5 we first compare the performance of committees of specialists to that of committees of generalists. The analysis focuses on the ratio between the cost of assessing several dimensions of the decision problem and the cost of assessing a single dimension. Proposition 10 gives a simple bound on this ratio ensuring that a specialist committee Pareto-dominates a generalist one. Proposition 11 in Subsection 5.1 shows that forming a committee of specialized, partially informed members also offers a Pareto improvement when compared to the partially informed dictator case, despite the fact that the committee takes longer to reach a decision. Section 6 concludes. In Appendix A we prove a partial result about search duration under unanimity for a committee with generalists. Appendix B contains several technical proofs omitted from the main text.

1.1 Related Literature

Decision making in committees is the subject of much scholarly work. A large majority of the existing papers study static cases where the committee makes a decision just once. We refer the reader to the survey by Li and Suen [2009] for a discussion of some of the main topics addressed, and focus below on papers that incorporate committee decisions within a formal search model. A small literature, originating during the mid 70's-80's in Statistics/Operations Research, analyzes multi-person stopping games: a committee is presented with alternatives that arrive sequentially, and its members vote whether to accept the current alternative or to continue search. Each alternative is characterized by a set of attributes, and each committee member only cares about one attribute. This basic framework with two players where stopping requires unanimous consent has been first analyzed by Sakaguchi [1973]. Kurano, Yasuda and Nakagami [1980] and Yasuda, Nakagami and Kurano [1982] establish equilibrium existence for environments with more than two players and with more flexible voting rules, e.g. majority. Ferguson [2005] points out that the voting games analyzed by these authors typically have many non-trivial stationary equilibria, and offers conditions on the distribution of the alternatives' attributes ensuring the existence of a unique stationary equilibrium for the case of unanimous consent.

There is a more recent interest in collective search games in Economics. Wilson [2001] and Compte and Jehiel [2010a] take a bargaining perspective: they study environments where proposals are presented randomly and sequentially to a set of bargainers who can accept or not. These authors relate the bargaining outcome when players are very patient to the Nash Bargaining Solution. Compte and Jehiel also analyze who has more (if any) effect on the decision, how search duration is affected by the majority rule, and the impact of dimensionality on the size of the acceptance set. Albrecht, Anderson and Vroman [2010] derive the existence of a unique symmetric and stationary equilibrium for symmetric settings and general majority rules, and study how the committee size and voting rules affect the search outcome. Compte and Jehiel [2010b] compare majority rules with unanimity for large committees of very patient agents. Alpern and Gal [2009] and Alpern, Gal and Solan [2010] consider augmented voting games where the committee members can also veto candidates, but have a restricted number of vetoes.

Lizzeri and Yariv [2010] consider a committee that decides every period whether to continue deliberation (costly information gathering) or stop and make a final decision by voting. Under certain conditions, these authors show that voting rules are irrelevant while deliberation rules are critical for the determination of the duration and accuracy of final decisions. This theoretical finding is consistent with the experimental results documented in Goeree and Yariv [2010]. Strulovici [2010] studies a model of collective experimentation by voting, and shows that collective decision making often leads to an inefficient level of experimentation. In his model a time-dependent voting rule can restore efficiency.

Although various aspects of aggregation of private information is a significant

topic in the static literature on decision making in committees (see for example Gilligan and Krehbiel [1989], and Li, Rosen and Suen [2001]), all papers mentioned above (that discuss dynamic settings) conduct a complete information analysis. The combination of dynamic search, private information, multidimensional alternatives and interdependent values is a distinctive feature of the present paper.

Damiano, Li and Suen [2009] study the role of delay for information aggregation in a dynamic model of committee decision where committee members possess private information and have conflicting preferences. In their model members repeatedly vote on a decision – which is taken only once – until an agreement is reached.

Several static models of decision making in committees allow for interdependent values. For example, Gruener and Kiel [2004] consider direct revelation mechanisms in a static committee decision model with private information, interdependent values and a one-dimensional set of alternatives. Caillaud and Tirole [2007] focus on strategies for consensus building within a group and define a measure of *internal congruence* among committee members in order to capture the correlation among the members benefits. Mathematically, this is connected to our notion of degree of conflict or partisanship since both are related to measures of positive dependence among random variables.

We wish to mention here that several important tools in establishing both the uniqueness of stationary Bayes-Nash equilibria in the incomplete information case, and many of our comparative statics results revolve around the concept of *mean residual life* of a random variable, which is borrowed from *reliability* theory (see Shaked and Shanthikumar [2007], Chapter 2). Another important set of concepts and tools is borrowed from (stochastic) *majorization* theory (see the classical treatise by Marshall and Olkin [1979]).

2 The Model

We choose to present the model in a specific and familiar setting of a recruiting committee. As mentioned in the introduction, our analysis applies more broadly to other committee decision frameworks.

A hiring committee is in charge of filling an open position. Candidates are evaluated one at a time. In each period t the current candidate is evaluated on the basis of two attributes X_t and Y_t , where X_t and Y_t are non-negative random variables (say theoretical skills and empirical skills in an Economics department). These two attributes are drawn independently of each other, and independently across periods from commonly known distributions F and G, respectively. Both F and G have finite second moments and continuous densities, with a common support $[0,\overline{\theta}]$, where $\overline{\theta} \leq$ $+\infty.^3$

The committee consists of two members, A and B. Member A(B) is specialized in evaluating attribute X(Y), and privately observes the realization of this random variable.

The committee members view the value of a candidate in possibly different ways: each member is biased towards hiring a candidate that is strong in his own respective field of specialization. This is captured here by assuming that, net of search costs, the payoff for member A from hiring a candidate (x, y) is given by $\alpha x + (1 - \alpha) y$ with $\alpha \ge 1/2$. Net of search costs, the payoff of member B from hiring a candidate (x, y) is given by $\beta x + (1 - \beta) y$ with $\beta \le 1/2$. That is, member A puts relatively more weight on attribute x and member B puts relatively more weight on attribute y. Higher values of α and $1 - \beta$ represent here a higher degree of conflict within the committee, or more partisanship.

After each assessment, members simultaneously cast votes of "yes" or "no".⁴ A candidate is hired and search stops if both members vote "yes", otherwise search continues. In the latter case, member A incurs cost c_A and member B incurs a cost c_B , and the process repeats itself⁵. Once rejected, a candidate cannot be recalled. In the sequel we always focus on equilibria where search ends in finite time, in order to avoid the trivial equilibria where one agent never votes "yes".

3 Complete Information: Generalist Committees

We start our analysis with a discussion of the setting where the decision makers observe and can assess the current realizations of both attributes (but are uncertain about the future candidates). Thus, there is complete information, and agents are generalists. Roughly speaking, the analysis is more complex in the generalist case because the committee members need to condition their behavior on the sum (convolution) of two random variables corresponding to the two attributes instead of conditioning on the single attribute corresponding to their own speciality.

3.1 One-person Committees (Dictatorship)

Suppose first that member A alone controls the decision. We assume that this "dictator" incurs a search cost c_D and observes both realizations x_t and y_t . Letting $Z = Z_t = \alpha X_t + (1 - \alpha) Y_t$, we obtain an instance of the classical search problem first

³The assumption of common support is just for convenience of notation.

⁴Under the unanimity rule, simultaneity does not matter.

⁵These can be thought of as time costs, evaluation costs, etc....

analyzed in Chow and Robbins [1960], where the decision maker directly observes the realization of Z, which also gives the dictator's period utility in case of stopping. It is well known that the optimal policy is determined by a cutoff z_D that is constant over time: in any period, the dictator accepts the current candidate if and only if $Z \ge z_D$.⁶ Let v_D denote the continuation payoff of the dictator when he chooses the cutoff z_D optimally. We have then

$$v_D = -c_D + E[Z \mid Z \ge z_D] \Pr(Z \ge z_D) + [1 - \Pr(Z \ge z_D)] v_D$$

It also follows from optimality that $v_D = z_D$. Therefore, the equilibrium condition for the cutoff z_D is given by:

$$E[Z - z_D \mid Z \ge z_D] \Pr(Z \ge z_D) = c_D.$$
(1)

The left hand side of the above equation is decreasing in z_D , and hence the solution is unique. In order to analyze how the optimal cutoff/dictator's utility varies with the degree of bias α , we introduce a few concepts from majorization theory.

Definition 1 Let $a_{(1)} \leq a_{(2)} \leq ... \leq a_{(n)}$ and $b_{(1)} \leq b_{(2)} \leq ... \leq b_{(n)}$ denote the increasing arrangement of vector $\mathbf{a} = (a_1, a_2, ..., a_n)$ and $\mathbf{b} = (b_1, b_2, ..., b_n)$, respectively. The vector \mathbf{a} majorizes vector \mathbf{b} , denoted by $\mathbf{a} \succ \mathbf{b}$, if $\sum_{i=1}^n a_{(i)} = \sum_{i=1}^n b_{(i)}$ and

$$\sum_{i=1}^{j} a_{(i)} \le \sum_{i=1}^{j} b_{(i)} \text{ for all } j = 1, ..., n-1.$$

Definition 2 A function $f : \mathbb{R}^n \to \mathbb{R}$ is Schur-convex (concave) if for $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$,

$$\mathbf{a} \succ \mathbf{b} \Rightarrow f(\mathbf{a}) \ge (\le) f(\mathbf{b}).$$

A symmetric and convex (concave) function is Schur-convex (concave). Note (also for further reference) that for $\mathbf{a} = (1 - \alpha_1, \alpha_1)$ and $\mathbf{b} = (1 - \alpha_2, \alpha_2)$ where $\alpha_1 \ge \frac{1}{2}$ and $\alpha_2 \ge \frac{1}{2}$, the assertion $\mathbf{a} \succ \mathbf{b}$ is equivalent to $\alpha_1 \ge \alpha_2$. We also need the following result, which is a special case of a theorem due to Marshall and Proschan [1965] (see also Result B.2.c. in page 288 of Marshall and Olkin [1979]).

Theorem 1 (Marshall and Proschan [1965]) If $X_1, ..., X_n$ are exchangeable random variables, and if g is a continuous convex function, then the function

$$\phi(a_1, a_2, \dots a_n) = Eg(\sum a_i X_i)$$

is symmetric and convex, and hence Schur-convex.

⁶Note that recall is never optimal in such a setting.

We are now ready to prove:

Proposition 1 Assume that F = G. Then the dictator's cutoff (or utility) z_D is increasing in α .

Proof. For a fixed z_D , let $g(t) = (t - z_D) \cdot \mathbf{1}_{\{t \ge z_D\}}$ where $\mathbf{1}_{\{\cdot\}}$ denotes the indicator function. Then g is clearly continuous and convex. The random variables X and Y are exchangeable since they are I.I.D. By the above Theorem, we obtain that

$$\phi(a_1, a_2) = Eg(a_1X + a_2Y) = E[(a_1X + a_2Y - z_D) \cdot \mathbf{1}_{\{a_1X + a_2Y \ge z_D\}}]$$

is Schur convex. Hence, by the remark before the Theorem, the function $\psi(\alpha) = Eg(\alpha X + (1 - \alpha)Y)$ is increasing in α for $\alpha \geq \frac{1}{2}$. The left hand side of equation (1), $E[Z - z_D | Z \geq z_D] \Pr(Z \geq z_D)$, viewed as a function of α is precisely equal to ψ . It is also clear that the same expression, viewed as a function of z_D , is decreasing. Therefore, the equilibrium cutoff z_D satisfying equation (1) must be increasing in α .

Remark 1 Marshall and Proschan's Theorem can also be invoked to show that $\sum a_i X_i$ is second-order stochastically dominated by $\sum b_i X_i$ whenever $\mathbf{a} \succ \mathbf{b}$.⁷ Thus, a change in preferences, corresponding to an increase in α , has here the same beneficial effect on utility as an increase in the variability of the candidate's attribute in a standard one-dimensional search model.

Another important issue is how a change in α affects the search duration, which is inversely related to the acceptance probability. An increase in α has two effects on the acceptance probability. First, an increase in α directly affects the acceptance probability for a fixed cutoff z_D . This effect is ambiguous and depends on the distribution. Second, as α increases, the optimal cutoff z_D increases, leading to a reduction of the acceptance probability. The overall effect of an increase in α on search duration is ambiguous, as illustrated in the following examples.

Example 1 Suppose that both F and G are uniform on the interval [0, 1]. The search cost c_D is assumed to be small, so that the equilibrium cutoff satisfies $z_D \ge \alpha$, and thus $\frac{z_D}{1-\alpha} \ge \frac{z_D}{\alpha} \ge 1$. Condition (1) becomes

$$c_D = \int_{[z_D - (1-\alpha)]/\alpha}^{1} \int_{(z_D - \alpha x)/(1-\alpha)}^{1} (\alpha x + (1-\alpha)y - z_D) \, dy \, dx$$

= $\frac{1}{6\alpha (1-\alpha)} (1-z_D)^3$

⁷The proof uses the same reasoning as above, but for an arbitrary convex function g instead of $g(t) = (t - z_D) \cdot \mathbf{1}_{\{t \ge z_D\}}$. This is Theorem 3.A.35 in Shaked and Shanthikumar [2007], but their proof is somewhat less transparent.

Therefore, the equilibrium cutoff is $z_D = 1 - \sqrt[3]{6\alpha (1-\alpha) c_D}$, which increases in α . The probability of acceptance

$$\int_{[z_D - (1-\alpha)]/\alpha}^1 \int_{(z_D - \alpha x)/(1-\alpha)}^1 dy dx = \sqrt[3]{\frac{9}{2\alpha (1-\alpha)}} \sqrt[3]{c_D^2}$$

is increasing in α , and thus the expected search duration decreases in α .

Example 2 Suppose $F(s) = G(s) = 1 - e^{-s}$ for $s \in [0, \infty)$. Equation (1) becomes

$$c_D = 1 - z_D - \int_0^{z_D/\alpha} \int_0^{(z_D - \alpha x)/(1 - \alpha)} (\alpha x + (1 - \alpha) y - z_D) e^{-y} dy e^{-x} dx$$
$$= \frac{\alpha^2}{2\alpha - 1} e^{-z_D/\alpha} - \frac{(1 - \alpha)^2}{2\alpha - 1} e^{-z_D/(1 - \alpha)}.$$

Fix $c_D = 0.01$ (note that this is quite small compared to the mean of the distribution). As α increases from 0.6 to 0.8, the optimal cutoff z_D increases from 3.1 to 3.7, while the search duration increases from 61 to 80.

3.2 Unanimity among Generalists

Suppose now that the decision is controlled by a committee with two members. At time t each member observes both realizations x_t and y_t of the current candidate/alternative's attributes, and a decision is taken by unanimity. Define a sequence of I.I.D. random variables $\{Z_t\}_{t=1}^{\infty}$ where $Z_t = (Z_{At}, Z_{Bt})$ is given by:

$$Z_{At} = \alpha X_t + (1 - \alpha) Y_t,$$

$$Z_{Bt} = \beta X_t + (1 - \beta) Y_t.$$

With this transformation, the setting is similar to the one where the committee members observe Z_{At} and Z_{Bt} directly (see Ferguson [2005]), except that we explicitly spell out the interdependence in the preferences. In principle, a member's voting strategy may depend on the whole history of the game. We focus however on stationary equilibria that employ cutoff strategies: each member votes "yes" if and only if his/her evaluation of the candidate's worth exceeds a cutoff that does not change over time. Since the realizations of both attributes are public, members A and B will impose cutoffs on Z_{At} and Z_{Bt} , respectively. If we denote by z_A and z_B the equilibrium cutoffs employed by members A and B, respectively, then member A(B) votes "yes" if and only if $z_{At} \ge z_A$ ($z_{Bt} \ge z_B$).

Let v_A denote the continuation value of member A when member A follows his/her optimal strategy given the equilibrium strategy of member B. The continuation value v_B of member B is defined analogously. As in the previous one-person case, it must hold that $v_A = z_A$ and $v_B = z_B$. It also follows from the definition of v_A that

$$v_{A} = -c_{A} + E [Z_{A} | Z_{A} \ge z_{A}, Z_{B} \ge z_{B}] \Pr (Z_{A} \ge z_{A}, Z_{B} \ge z_{B}) + [1 - \Pr (Z_{A} \ge z_{A}, Z_{B} \ge z_{B})] v_{A}.$$

By substituting z_A for v_A we obtain our first equilibrium condition:

$$E\left[Z_A - z_A \mid Z_A \ge z_A, Z_B \ge z_B\right] \Pr\left\{Z_A \ge z_A, Z_B \ge z_B\right\} = c_A.$$

$$(2)$$

Following the same procedure, we obtain our second equilibrium condition:

$$E[Z_B - z_B \mid Z_A \ge z_A, Z_B \ge z_B] \Pr\{Z_A \ge z_A, Z_B \ge z_B\} = c_B.$$
(3)

If (z_A, z_B) is an equilibrium with $\Pr \{Z_A \ge z_A, Z_B \ge z_B\} > 0$, then (z_A, z_B) must satisfy the above two equilibrium conditions. Equilibrium existence is established by Yasuda, Nakagami and Kurano [1982]. The equilibrium may not be unique, as illustrated by examples in Ferguson [2005]. But we show below that a unique symmetric equilibrium exists if the setting is symmetric, i.e., if F = G, $\alpha = 1 - \beta$ and $c_A = c_B$.

How do the recruiting standard and welfare vary with respect to α , the degree of conflict within the committee in this equilibrium? The answer may seem ambiguous since α has two opposite effects: on the one hand, an increase in α increases variability which is beneficial, as shown under dictatorship (see Remark 1); on the other hand, an increase in α shrinks the acceptance region. The key insight for the next result is to look instead at the sum of the members' utilities which equals $X_t + Y_t$ for any candidate, and is therefore independent of α . We show below that the acceptance cutoff always goes down when α increases, contrasting the result in the one-person decision problem.

Proposition 2 Suppose that F = G, $\alpha = 1 - \beta$, and $c_A = c_B = c$. Then there exists a unique symmetric equilibrium (z, z) that is characterized by

$$E[Z_A - z \mid Z_A \ge z, Z_B \ge z] \Pr(Z_A \ge z, Z_B \ge z) = c.$$
(4)

The equilibrium cutoff z (and thus each member's payoff) is decreasing in α . Moreover, $z \leq z_D$, the dictator's optimal cutoff under the same F, α , and c.

Proof. The equilibrium condition (4) is obtained from (2) by setting $z_A = z_B = z$. The existence and uniqueness follow from the observation that the left hand side of (4) is strictly decreasing in z. Since in the symmetric setting $X = X_t$ and $Y = Y_t$ are I.I.D., we obtain:

$$E \left[Z_A - z \mid Z_A \ge z, Z_B \ge z \right] \Pr \left(Z_A \ge z, Z_B \ge z \right)$$

$$= E \left[\left(\alpha X + (1 - \alpha) Y - z \right) \cdot \mathbf{1}_{\{\alpha X + (1 - \alpha) Y \ge z, (1 - \alpha) X + \alpha Y \ge z\}} \right]$$

$$= \frac{1}{2} E \left[\left(\alpha X + (1 - \alpha) Y - z \right) \cdot \mathbf{1}_{\{\alpha X + (1 - \alpha) Y \ge z, (1 - \alpha) X + \alpha Y \ge z\}} \right]$$

$$+ \frac{1}{2} E \left[\left(\alpha Y + (1 - \alpha) X - z \right) \cdot \mathbf{1}_{\{\alpha X + (1 - \alpha) Y \ge z, (1 - \alpha) X + \alpha Y \ge z\}} \right]$$

$$= \frac{1}{2} E \left[\left(X + Y - 2z \right) \cdot \mathbf{1}_{\{\alpha X + (1 - \alpha) Y \ge z, (1 - \alpha) X + \alpha Y \ge z\}} \right]$$

Thus, the equilibrium condition (4) can be re-written as:

$$E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\{\alpha X + (1-\alpha)Y \ge z, (1-\alpha)X + \alpha Y \ge z\}}\right] = c \tag{5}$$

Observe that function on the right hand side is Schur-concave in $(\alpha, 1 - \alpha)$ because the indicator function is Schur-concave, and because the function $\frac{1}{2}X + \frac{1}{2}Y - z$ does not depend on α , and is positive whenever the indicator function is not equal to zero. Thus, the right hand side decreases with α . Since it also obviously decreases with z, we obtain that the equilibrium cutoff decreases in α .

For the last assertion, note that $z_D(\frac{1}{2}) = z(\frac{1}{2})$, since the equilibrium conditions under unanimity and dictatorship coincide in this case:

$$E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\{\frac{1}{2}X + \frac{1}{2}Y \ge z\}}\right] = c$$

By Proposition 4 we know that $z_D(\alpha)$ increases in α . The result follows because $z(\alpha)$ decreases with α , as shown above.

The intuition for the above result is as follows. For a fixed acceptance cutoff z, the acceptance area under unanimity is given by

$$\{(x,y) : \alpha x + (1-\alpha) \, y \ge z, (1-\alpha) \, x + \alpha y \ge z\}.$$

When α , the degree of conflict within the committee, increases from 1/2 to 1, the

acceptance region shrinks, as shown in the figure below:



Figure 1

As a result, a successful search takes more periods, which means that both members have to incur higher expected search costs if they keep the same standard. To counter this effect, both members lower their acceptance standard and settle on less desirable candidates, striking a balance between candidate quality and search costs.

Example 3 Suppose that both F and G are uniform on the interval [0,1], and that $\alpha = 1 - \beta$ and $c_A = c_B = c$. Assume also that the cost c is small enough so the symmetric equilibrium cutoff satisfies $z \ge 1 - \alpha$. The equilibrium condition (4) becomes

$$c = \int_{z}^{1} \int_{\frac{1}{\alpha}(z-(1-\alpha)y)}^{y} (\alpha x + (1-\alpha)y - z) \, dx \, dy$$

+
$$\int_{z}^{1} \int_{\frac{1}{\alpha}(z-(1-\alpha)x)}^{x} (\alpha x + (1-\alpha)y - z) \, dy \, dx$$

=
$$\frac{1}{6\alpha^{2}} (4\alpha - 1) (1-z)^{3}.$$

Therefore, the equilibrium cutoff is

$$z = 1 - \sqrt[3]{\frac{6\alpha^2 c}{4\alpha - 1}}.$$

The probability of acceptance given by

$$\int_{z}^{1} \int_{\frac{1}{\alpha}(z-(1-\alpha)y)}^{y} dx dy + \int_{z}^{1} \int_{\frac{1}{\alpha}(z-(1-\alpha)x)}^{x} dy dx = \frac{1}{\alpha} \left(\sqrt[3]{\frac{6\alpha^{2}c}{4\alpha-1}}\right)^{2}$$

is decreasing in α , which implies that the expected search duration increases in α in this example.

Remark 2 Recall that an increase in the degree of conflict α leads to a decrease in the second order stochastic dominance order of the random utility $\alpha X + (1-\alpha)Y$ (and hence to an increase in its variance), which was shown to be beneficial for dictators. But we also showed that the members' payoffs in committees decrease in α , which seems puzzling. But what is the benefit for member A of an increased variance if member B says "no" to the better candidates? What A really needs is that the expected value of B's utility, conditional on A's utility being high, is also high. Only then search stops in a committee. Note that the covariance of the members' random utilities

 $Cov[\alpha X + (1 - \alpha)Y, \alpha Y + (1 - \alpha)X] = 2\alpha(1 - \alpha)Var(X)$

is increasing in the variance of the underlying attributes, but is **decreasing** in the degree of conflict α on $[\frac{1}{2}, 1]$. Thus, our results show that the consensus effect is dominant in committees. This observation about the role of the covariance suggests a deeper mathematical connection: indeed, when conflict decreases, the members' random utilities become **more associated**, where "more association" is a well known measure of positive dependence among random variables, due to Schriever [1987].⁸ Incidentally, in his original paper, Schriever has proven the following:

Proposition 3 (Schriever [1987]): Consider a pair of random variables (X, Y), and let H_{α} denote the joint distribution function of the linear transform $T_{\alpha}(X,Y) = (\alpha X + (1-\alpha)Y, (1-\alpha)X + \alpha Y)$, where $\alpha \in [\frac{1}{2}, 1]$. Then $\alpha \geq \alpha'$ implies $H_{\alpha} \preceq_{assoc} H_{\alpha'}$.⁹

4 Incomplete Information: Specialist Committees

In this section we come back to our original model where the committee members are specialized and are able to privately assess the quality of the alternative/candidate in only one dimension. We start with a brief discussion of the very simple one-person problem.

⁸See also Chapter 9 in Shaked and Shanthikumar [2007] for the relations between this order and other notions such as *positive quadrant dependency*, or the *supermodular order*. In general, more positive dependence implies a higher covariance.

⁹The proof follows directly from the definition of the association order by observing that each T_{α} is monotonically increasing in each coordinate, and that the determinant of the Jacobian of $T_{\alpha'}(T_{\alpha}^{-1})$ is non-negative.

4.1 One-person Committees (Dictatorship)

The dictator (member A) incurs a search cost c_A and only observes the realization of X_t . Let x_D denote the cutoff employed by the dictator. Then we have the Bellman equation

$$v_{D} = -c_{A} + \max_{x_{D}} \left\{ E \left[\alpha X + (1 - \alpha) Y \mid X \ge x_{D} \right] \Pr \left(X \ge x_{D} \right) + \left[1 - \Pr \left(X \ge x_{D} \right) \right] v_{D} \right\}$$
$$= -c_{A} + \max_{x_{D}} \left\{ \int_{0}^{\overline{\theta}} \int_{x_{D}}^{\overline{\theta}} \left[\alpha x + (1 - \alpha) y \right] dF (x) dG (y) + F (x_{D}) v_{D} \right\}$$

The first-order condition for x_D implies that

$$v_D = \alpha x_D + (1 - \alpha) E[Y].$$

Note that here the equilibrium cutoff and the equilibrium utility do not coincide, a fact with numerous consequences in the multi-person committee decisions problem analyzed below. From the Bellman equation we obtain the following equilibrium condition for x_D :

$$\alpha E\left[X - x_D \mid X \ge x_D\right] \left[1 - F\left(x_D\right)\right] = c_A. \tag{6}$$

Proposition 4 There is a unique equilibrium acceptance cutoff x_D . This cutoff, the dictator's utility v_D and the expected search duration are all strictly increasing in α .

Proof. The implications follow immediately from the fact the left hand side of equation (6) is strictly increasing in α and strictly decreasing in x_D . Note that the acceptance probability is $1 - F(x_D)$, which is decreasing in α . Therefore, the expected search duration also increases in α .

4.2 Unanimity among Specialists

We now turn to committee decisions with specialized, privately informed members. Given a candidate with attributes (x_t, y_t) , member A privately observes x_t and member B privately observes y_t . Acceptance is by unanimity. We focus on stationary equilibria that employ cutoff strategies. Each member casts her vote based on her own information only. Specifically, if we let x^* and y^* denote the cutoffs used by member A and B, respectively, then member A votes "yes" if and only if $x_t \ge x^*$, and member B votes "yes" if and only if $y_t \ge y^*$.

4.2.1 Equilibrium Characterization

With some abuse of notation, let v_A denote the continuation value member A derives by following his optimal strategy given the equilibrium strategy of member B. The continuation value v_B of member B is defined similarly. Then the Bellman equation for member A becomes

$$v_{A} = -c_{A} + \max_{x^{*}} \left\{ \begin{array}{l} E\left[\alpha X + (1-\alpha)Y \mid X \ge x^{*}, Y \ge y^{*}\right] \Pr\left(X \ge x^{*}, y \ge y^{*}\right) \\ + \left[1 - \Pr\left(X \ge x^{*}, Y \ge y^{*}\right)\right] v_{A} \end{array} \right\}$$
$$= -c_{A} + \max_{x^{*}} \left\{ \begin{array}{l} \int_{x^{*}}^{\overline{\theta}} \int_{y^{*}}^{\overline{\theta}} \left[\alpha x + (1-\alpha)y\right] dG\left(y\right) dF\left(x\right) \\ + \left[1 - (1-F\left(x^{*}\right)\right)\left(1 - G\left(y^{*}\right)\right)\right] v_{A} \end{array} \right\}$$
(7)

By looking at the necessary first-order condition for x^* in the Bellman equations above, one immediately obtains the relations between cutoffs and utilities, as stated in the following Lemma.

Lemma 1 The relationships between the continuation values v_A, v_B and the optimal cutoff x^*, y^* are given by

$$v_A = \alpha x^* + (1 - \alpha) E[Y \mid Y \ge y^*],$$
 (8)

$$v_B = \beta E[X|X \ge x^*] + (1 - \beta) y^*.$$
 (9)

Intuitively, conditional on being pivotal (i.e., $Y \ge y^*$), member A is indifferent between accepting the marginal candidate with $X = x^*$ and continuing costly search. Therefore, the continuation value v_A must be equal to the expected payoff from hiring the marginal candidate, which is $\alpha x^* + (1 - \alpha) E[Y | Y \ge y^*]$. In contrast to the complete information setting, here the equilibrium cutoff and the equilibrium utility do not coincide: the opponent's cutoff y^* affects not only the probability of acceptance, but also the expected worth of the marginal candidate.

Since X and Y are independent, the Bellman equation can be re-written as

$$v_A = \alpha E[X \mid X \ge x^*] + (1 - \alpha) E[Y \mid Y \ge y^*] - \frac{c_A}{(1 - F(x^*))(1 - G(y^*))}.$$
 (10)

That is, the expected payoff for member A is equal to the expected value of the chosen alternative minus the expected search costs. Using the first-order condition (8), we obtain our first equilibrium condition:

$$\frac{1}{(1 - F(x^*))(1 - G(y^*))} = \frac{\alpha}{c_A} E[X - x^* | X \ge x^*].$$
(11)

The second equilibrium condition can be obtained similarly:

$$\frac{1}{(1 - F(x^*))(1 - G(y^*))} = \frac{1 - \beta}{c_B} E[Y - y^*|Y \ge y^*].$$
(12)

The two conditions (11) and (12) characterize the stationary equilibrium cutoffs (x^*, y^*) .

Remark 3 There is an important formal link between the analysis of complete information with private values and the analysis of incomplete information with interdependent values. In the complete information case with $\alpha = 1 - \beta = 1$, we have private values with $Z_A = X$ and $Z_B = Y$, and the equilibrium cutoffs (z_A, z_B) are given by

$$\frac{1}{c_A} E\left[X - z_A | X \ge z_A\right] = \frac{1}{c_B} E\left[Y - z_B | Y \ge z_B\right] = \frac{1}{\left[1 - F\left(z_A\right)\right] \left[1 - G\left(z_B\right)\right]}.$$

In the incomplete information case with any α and β , the equilibrium cutoffs (x^*, y^*) are given by

$$\frac{\alpha}{c_A} E\left[X - x^* | X \ge x^*\right] = \frac{1 - \beta}{c_B} E\left[Y - y^* | Y \ge y^*\right] = \frac{1}{\left[1 - F\left(x^*\right)\right] \left[1 - G\left(y^*\right)\right]}$$

If we define $\kappa_A = c_A/\alpha$ and $\kappa_B = c_B/(1-\beta)$ as the "pseudo-cost" in the incomplete information case, then the sets of cutoff equilibrium conditions coincide. Therefore, while the welfare analysis is different, properties of the acceptance cutoffs in a complete information framework with private values can be directly applied to a setting with incomplete information and interdependent values. Note that when $\alpha = 1$ and $\beta = 0$, the pseudo costs are equal to the usual costs. Therefore, in the private values setting, whether members' information is private or not does not affect the equilibrium.

Before discussing the existence and uniqueness of the equilibrium, we first introduce an important concept used in the theory of reliability.

Definition 3 The mean residual life (MRL) of a random variable $X \in [0, \overline{\theta}]$ is defined as

$$m(x) = \begin{cases} E[X - x|X \ge x] & \text{if } x < \overline{\theta} \\ 0 & \text{if } x = \overline{\theta} \end{cases}$$

If we let X denote the life-time of a component, then m(x) measures the expected remaining life of a component that has survived until time x. The MRL function is closely related to the more familiar hazard rate (or failure rate) $\lambda(x) = f(x) / [1 - F(x)]$ which measures the instantaneous failure probability conditional on survival up to time x.¹⁰ Both measures are conditional concepts (and uniquely determine the underlying distribution), but they are conceptually different: the hazard rate

¹⁰The mathematical relation between the two is $m(x) = \int_x^{\overline{\theta}} \exp\left\{-\int_x^t \lambda(u) \, du\right\} dt$ for $x < \overline{\theta}$.

 $\lambda(x)$ only takes into account the instantaneous present, while the mean residual life m(x) takes into account the complete future (see Guess and Proschan [1988]). The exponential distribution is the only distribution that has a constant mean residual life, and it is also the only distribution that has a constant hazard rate.

- **Definition 4** 1. A random variable X satisfies the (strict) DMRL (decreasing mean residual life) property if m(x) is (strictly) decreasing in x.
 - 2. A random variable X satisfies the IFR (increasing failure rate) property if $\lambda(x)$ is increasing in x.

The IFR assumption is commonly made in the economics literature. DMRL is a weaker property, and it is implied by IFR. We are now ready to state the first main result of this section.

Proposition 5 Suppose that the random variables X and Y satisfy the strict DMRL property. Then the unanimity game among specialists has a unique cutoff equilibrium.

Proof. See Appendix B. ■

The assumption of *strict* DMRL is critical for the uniqueness result. If this assumption fails, then multiple equilibria are possible, as illustrated in the following example.

Example 4 Suppose $F(s) = G(s) = 1 - e^{-s}$ for $s \in [0, \infty)$, $\alpha = 1 - \beta$ and $c_A = c_B = c$. Then both X and Y have a constant MRL equal to 1. The two equilibrium conditions reduce to

$$x^* + y^* = \ln \alpha - \ln c.$$

Any two non-negative numbers (x^*, y^*) satisfying the above condition form an equilibrium. There is a unique symmetric equilibrium given by

$$x^* = y^* = \frac{1}{2}(\ln \alpha - \ln c)$$

4.2.2 Comparative Statics

How do the equilibrium acceptance cutoffs vary with respect to the degree of conflict within the committee when its members are specialized and have private information? Using the DMRL condition we can show for any (possibly asymmetric) setting that when just one member becomes more extreme, he raises his own acceptance standard, while the other member responds by lowering her standard. Intuitively, a more extreme position should also lead to a longer search duration. **Proposition 6** Suppose that both X and Y satisfy the strict DMRL condition, and consider the unique equilibrium for given c_A, c_B and α . Keeping β constant, member A's equilibrium cutoff x^* and the expected search duration increase in α , while member B's equilibrium cutoff y^* decreases in α .

Proof. Suppose $\alpha < \alpha'$ and let (x^*, y^*) and (x', y') denote the cutoffs corresponding to α and α' , respectively. We need to show $x' > x^*$ and $y' < y^*$. Suppose first $x' \leq x^*$ and $y' < y^*$. Then it follows from the equilibrium condition and the DMRL property that

$$\frac{1}{(1 - F(x^*))(1 - G(y^*))} = \frac{\alpha}{c_A} E[X - x^* | X \ge x^*]$$

$$< \frac{\alpha'}{c_A} E[X - x^* | X \ge x^*]$$

$$\leq \frac{\alpha'}{c_A} E[X - x' | X \ge x']$$

$$= \frac{1}{(1 - F(x'))(1 - G(y'))},$$

which is impossible. Next suppose $x' \leq x^*$ and $y' \geq y^*$. Again, we have

$$\frac{\alpha'}{c_A} E[X - x'|X \ge x'] > \frac{\alpha}{c_A} E[X - x'|X \ge x']$$
$$\ge \frac{\alpha}{c_A} E[X - x^*|X \ge x^*]$$
$$= \frac{1 - \beta}{c_B} E[Y - y^*|Y \ge y^*]$$
$$\ge \frac{1 - \beta}{c_B} E[Y - y'|Y \ge y']$$

which violates the equilibrium conditions for (x', y'). Finally, suppose $x' > x^*$ and $y' \ge y^*$. Then we have

$$\frac{1}{(1 - F(x^*))(1 - G(y^*))} = \frac{1 - \beta}{c_B} E[Y - y^*|Y \ge y^*]$$
$$\ge \frac{1 - \beta}{c_B} E[Y - y'|Y \ge y']$$
$$= \frac{1}{(1 - F(x'))(1 - G(y'))}$$

a contradiction of $x' > x^*$ and $y' > y^*$. Therefore, we must have $x' > x^*$ and $y' < y^*$.

The equilibrium expected search duration is given by

$$\frac{1}{\Pr\{X \ge x^*, Y \ge y^*\}} = \frac{\alpha}{c_A} E\left[X - x^* | X \ge x^*\right] = \frac{1 - \beta}{c_B} E\left[Y - y^* | Y \ge y^*\right].$$

It is not a priori clear from the equilibrium condition for x^* (the first equality) whether search duration increases or not. But, we also know that y^* decreases in α . Therefore, from the second equality we obtain that the expected search duration necessarily increases if X and Y have the DMRL property.

We next look at a symmetric setting. Although the members' utilities decrease when there is more conflict – as was also the case under complete information/no specialization – the equilibrium acceptance cutoff goes up with the degree of conflict, in sharp contrast to the unanimity decision in the complete information/no specialization case!

Proposition 7 Suppose that F = G, $\alpha = 1 - \beta$, and $c_A = c_B = c$. Then there exists a unique symmetric cutoff equilibrium $(x^*, x^*)^{11}$. Both the cutoff x^* and expected search duration S are strictly increasing in α , but the members' utilities are strictly decreasing in α . Moreover, if X and Y have the DMRL property, then for any $\alpha, \alpha' \in [\frac{1}{2}, 1], \alpha \leq \alpha'$ it holds that

$$\frac{1}{2} \le \frac{\alpha}{\alpha'} \le \frac{S(\alpha)}{S(\alpha')} \le 1$$

Proof. The uniqueness of the symmetric equilibrium follows by the same argument as that in the case of generalist committees (see Proposition 2). Since in the symmetric setting X and Y are I.I.D., we obtain:

$$\alpha E \left[X - x^* \mid X \ge x^*, Y \ge x^* \right] \Pr\left(X \ge x^*, Y \ge x^* \right)$$

= $\alpha E \left[(X - x^*) \cdot \mathbf{1}_{\{X \ge x^*, Y \ge x^*\}} \right]$
= $\frac{1}{2} \alpha E \left[(X - x^*) \cdot \mathbf{1}_{\{X \ge x^*, Y \ge x^*\}} \right] + \frac{1}{2} \alpha E \left[(Y - x^*) \cdot \mathbf{1}_{\{X \ge x^*, Y \ge x^*\}} \right]$
= $\alpha E \left[\left(\frac{1}{2} X + \frac{1}{2} Y - x^* \right) \cdot \mathbf{1}_{\{X \ge x^*, Y \ge x^*\}} \right]$

Therefore, in the symmetric case the equilibrium condition 11 can be re-written as:

$$\alpha E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - x^*\right) \cdot \mathbf{1}_{\{X \ge x^*, Y \ge x^*\}}\right] = c \tag{13}$$

Observe that function on the left hand side is strictly increasing in α because the function $\frac{1}{2}X + \frac{1}{2}Y - x^*$ is positive whenever the indicator function is not equal to zero. Since the left hand side strictly decreases in x^* , we obtain that the equilibrium cutoff must strictly increase in α . As a consequence, the expected search duration,

¹¹Recall that under the strict DMRL assumption this is also the overall unique equilibrium.

which is given by $1/[1 - F(x^*)]^2$ is also strictly increasing in α . For $\alpha \le \alpha'$, we obtain under DMRL that

$$S(\alpha) = \frac{\alpha}{c} E\left[X - x^*(\alpha)\right] | X \ge x^*(\alpha) \ge \frac{\alpha}{c} E\left[X - x^*(\alpha')\right] | X \ge x^*(\alpha') \ge \frac{\alpha}{\alpha'} S(\alpha') \ge \frac{1}{2} S(\alpha')$$

To prove the result about utilities, we adapt equation (10) to the symmetric setting and obtain

$$v_{A} = \alpha E [X|X \ge x^{*}] + (1 - \alpha) E [X|X \ge x^{*}] - \frac{c}{[1 - F(x^{*})]^{2}}$$

= $E [X|X \ge x^{*}] - \frac{c}{[1 - F(x^{*})]^{2}}$
= $\frac{[1 - F(x^{*})] \int_{x^{*}}^{\overline{\theta}} sf(s) ds - c}{[1 - F(x^{*})]^{2}}.$

Therefore, v_A can be written as $v_A(x^*)$, a function of x^* only. Since x^* strictly increases in α , in order to show that v_A is strictly decreasing in α , it is sufficient to show that $v_A(x^*)$ is strictly decreasing in x^* . Note that

$$\frac{dv_A(x^*)}{dx^*} = \frac{f(x^*)}{[1 - F(x^*)]^3} \left\{ [1 - F(x^*)] \int_{x^*}^{\overline{\theta}} sf(s) \, ds - x^* [1 - F(x^*)]^2 - 2c \right\}$$

$$= \frac{f(x^*)}{1 - F(x^*)} \left\{ E[X - x^*|X \ge x^*] - \frac{2c}{[1 - F(x^*)]^2} \right\}$$

$$= \frac{f(x^*)}{1 - F(x^*)} (1 - 2\alpha) E[X - x^*|X \ge x^*]$$

$$< 0$$

for all $\alpha > 1/2$. Therefore, members' utilities are strictly decreasing in α .

It is instructive to compare the equilibrium condition derived above (equation 13),

$$\alpha E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - x^*\right) \cdot \mathbf{1}_{\{X \ge x^*, Y \ge x^*\}}\right] = c,$$

with the equilibrium condition for the symmetric, complete information case (equation 5):

$$E\left[\left(\frac{1}{2}X+\frac{1}{2}Y-z\right)\cdot\mathbf{1}_{\{\alpha X+(1-\alpha)Y\geq z,(1-\alpha)X+\alpha Y\geq z\}}\right]=c.$$

In the game among specialists, raising the degree of conflict while keeping the acceptance cutoff fixed raises the stakes controlled by each member without affecting the acceptance area. Thus, committee members respond by raising the cutoff. In contrast, in the game among generalists raising the degree of conflict while keeping the acceptance cutoff fixed has no effect on the controlled stake, but decreases the acceptance area. Thus, committee members respond by lowering the acceptance cutoff.

Alternatively, we can get the intuition for the specialization case by focusing on the search cost c. An important observation stemming from Remark 3 is that with incomplete information the degrees of conflict (α or β) and the search costs (c_A or c_B) affect equilibrium play only through the "pseudo-costs", κ_A and κ_B . Moreover, a higher α has exactly the same effect on the equilibrium cutoffs as a lower search cost c_A . It is intuitive that a decrease in search cost c leads to an increase in the equilibrium cutoffs, so we can again conclude that the equilibrium cutoffs must be increasing in α .

Analogous to the complete information case, individual payoffs decrease when the degree of conflict increases. As we pointed out earlier, with incomplete information the continuation payoffs are different from equilibrium cutoffs. This observation holds the key to understand why members' utilities decrease although the equilibrium cutoff increases in α . In particular, member A's payoff is given by $v_A = \alpha x^* + (1 - \alpha) E[Y|Y \ge y^*]$. In the symmetric setting, it holds that $x^* = y^*$ and $x^* < E[Y|Y \ge y^*]$. Therefore, an increase in α has two effects on v_A : a higher α also shifts weight from the larger term $E[Y|Y \ge y^*]$ to the smaller term x^* and thus lowers v_A . It turns out the second effect dominates, and thus v_A is decreasing in α .

Finally, we investigate how the equilibrium varies when we change the distribution of the candidate's attributes. We first need to introduce several stochastic orders, which, intuitively, should be connected to the mean residual life, or to its close cousin, the hazard rate.

- **Definition 5** 1. Let m and l denote the mean residual life function of random variables X and Y, respectively. Then X is said to be smaller than Y in the mean residual life order, denoted by $X \leq_{MRL} Y$, if $m(t) \leq l(t)$ for all $t \in [0, \overline{\theta}]$.
 - 2. Let r and q denote the hazard rate function of random variables X and Y, respectively. Then X is said to be smaller than Y in the hazard rate order, denoted by $X \leq_{HR} Y$, if $r(t) \geq q(t)$ for all $t \in [0, \overline{\theta}]$.

The MRL order is independent of the usual stochastic order (denoted by \leq_{ST}), and neither implies the other. The hazard rate order \leq_{HR} implies both \leq_{MRL} and \leq_{ST} , and $\widetilde{X} \leq_{HR} X$ if and only if $[\widetilde{X} | \widetilde{X} \geq t] \leq_{ST} [X | X \geq t]$ for all $t \in [0, \overline{\theta}]$. Moreover, if $\widetilde{X} \leq_{MRL} X$ and $E\widetilde{X} = EX$, then \widetilde{X} second-order stochastically dominates X, and hence \widetilde{X} has a lower variance than X (see Shaked and Shanthikumar [2007] for all these results). Our first result shows that, within a given committee, the member who observes a stochastically higher attribute in the MRL sense imposes a higher equilibrium acceptance standard, and is better off.

Proposition 8 Suppose that both X and Y satisfy the strict DMRL condition, $\beta = 1 - \alpha$, and $c_A = c_B = c$. If $X \leq_{MRL} Y$, then in the unique equilibrium it holds that $x^* \leq y^*$ and $v_A \leq v_B$.¹²

Proof. See Appendix B.

Our second result looks across committees, and shows the members' acceptance standards and utilities go up given a stochastic improvement in the privately observed attributes. Here we need the improvement to be in the sense of the stronger hazard rate order.

Proposition 9 Consider a symmetric committee C_1 where attributes are governed by I.I.D random variables X and Y, and another symmetric committee C_2 where attributes are governed by I.I.D. random variables \widetilde{X} and \widetilde{Y} . Suppose that X satisfies the DMRL property, and that $\widetilde{X} \leq_{HR} X$. Then, for any $\alpha = 1 - \beta$ and c, the acceptance cutoff and the members' utilities in the respective unique symmetric equilibrium are higher in committee C_1 than in C_2 .¹³

Proof. See Appendix B.

5 The Relative Performance of Specialist Committees

In this section we first compare the performance of committees of specialists to that of committees of generalists. The analysis focuses on the cost of assessing several dimensions of the decision problem. We next discuss the incentives of a specialist "dictator" (who is only informed about one dimension of the problem) to involve in the decision making process a specialist on a different dimension: there is a trade-off between sharing power and gaining valuable information.

¹²Thomas Watson, the founder of IBM is said to have advised: "If you want to be more successful, increase your failure rate." Our result shows that increasing mean residual life is sufficient, at least in committee interactions.

¹³Recall that these are the overall unique equilibria if X and \widetilde{X} satisfy the strict DMRL condition.

5.1 Specialist Committees versus Generalist Committees

We focus here on the symmetric setting with F = G, $\alpha = 1 - \beta$ and $c_A = c_B$ and compare the performances of specialist versus generalist committees. Their relative performance certainly depends on ratio of respective search costs. Let c_g denote the search cost incurred by a generalist to evaluate **both** attributes, and let c_s denote the search cost incurred by each specialist to evaluate the **single** attribute in her specialty. We can rewrite the equilibrium condition for a generalist committee (equation 5) as

$$E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\{\alpha X + (1-\alpha)Y \ge z, (1-\alpha)X + \alpha Y \ge z\}}\right] = c_g, \tag{14}$$

and the equilibrium condition (equation 13) for a specialist committee as

$$\alpha E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - x^*\right) \cdot \mathbf{1}_{\{X \ge x^*, Y \ge x^*\}}\right] = c_s.$$

$$(15)$$

Let us first consider the benchmark case where a generalist enjoys a very strong return to scope in knowledge, so that $c_g = c_s$. When $\alpha = 1$ the above two equilibrium conditions coincide and we have $z = x^*$. As shown in Propositions 2 and 7, the optimal cutoffs move in opposite directions for lower degrees of conflict. This immediately implies that for any α , the equilibrium cutoff z under complete information is always higher than the equilibrium cutoff x^* under incomplete information. It is intuitive that a specialized committee rejects candidates who are excellent in just one dimension (candidates in area A and B) that would be accepted by generalists. But, we also obtain that there are always balanced candidates with attributes above and close to (x^*, x^*) (candidates in area C in the figure) who are accepted by the specialized committee while being rejected by the generalist one. The acceptance areas in these two cases are illustrated in the following figure:



Figure 2

In practice, however, a generalist who is able to assess several dimensions of a complex problem may have less precise information about each than a specialist. Alternatively, in order to obtain the same quality of information as several specialists (one for each dimension), a generalist may face a cost function that is convex in the number of assessed dimensions, i.e. the cost incurred by a generalist for assessing an additional dimension is higher than the cost incurred by a specialist who only assesses that particular dimension. Under a simple condition on the distribution of attributes, our next Proposition shows that a specialist committee outperforms a generalist committee if the generalist cost is convex enough, i.e. if the cost ratio c_g/c_s is high enough¹⁴.

Proposition 10 Suppose that F = G and that F is $convex^{15}$. Consider a generalist committee where each member faces $cost c_g$, and a specialist committee where each member faces $cost c_s$ such that $c_g/c_s \ge 4$. Then for any $\alpha = 1 - \beta \in [1/2, 1]$, hiring standards and members' utilities are higher in the specialist committee than in the generalist committee.

Proof. From Lemma 2 in Appendix B we obtain that

$$E\left[\left(\frac{1}{2}X+\frac{1}{2}Y-z\right)\cdot\mathbf{1}_{\left\{\frac{1}{2}X+\frac{1}{2}Y\geq z\right\}\setminus\left\{X\geq z,Y\geq z\right\}}\right] < E\left[\left(\frac{1}{2}X+\frac{1}{2}Y-z\right)\cdot\mathbf{1}_{\left\{X\geq z,Y\geq z\right\}}\right]$$

which implies

$$E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\left\{\frac{1}{2}X + \frac{1}{2}Y \ge z\right\}}\right] < 2E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\left\{X \ge z, Y \ge z\right\}}\right].$$

For $\alpha = 1/2$, it follows from equilibrium conditions (14) and (15) and from the last inequality above that

$$\frac{1}{2}E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - x^*\right) \cdot \mathbf{1}_{\{X \ge x^*, Y \ge x^*\}}\right]$$

$$= c_s \le \frac{1}{4}c_g$$

$$= \frac{1}{4}E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\left\{\frac{1}{2}X + \frac{1}{2}Y \ge z\right\}}\right]$$

$$< \frac{1}{2}E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\{X \ge z, Y \ge z\}}\right]$$

¹⁴Related conditions on the cost ratio are also sufficient for a specialist committee to dominate a generalist dictator.

¹⁵We conjecture that a similar result is true for the larger class of distributions who have an increasing hazard rate.

Because $E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\{X \ge z, Y \ge z\}}\right]$ is strictly decreasing in z, we have $x^*\left(\frac{1}{2}\right) > z\left(\frac{1}{2}\right)$ where x^* are the optimal cutoffs in the specialist and generalist committees, respectively.

We know from Propositions 2 and 7 that $x^*(\alpha)$ is increasing in α and that $z(\alpha)$ is decreasing in α . Therefore, $x^*\left(\frac{1}{2}\right) > z\left(\frac{1}{2}\right)$ implies $x^*(\alpha) > z(\alpha)$ for all $\alpha \in [1/2, 1]$. Furthermore, a member's utility in the specialist committee is given by $\alpha x^*(\alpha) + (1-\alpha) E[Y|Y \ge x^*(\alpha)] > x^*(\alpha) > z(\alpha)$ for all α . Therefore, a member's utility is higher in a specialist committee than in a generalist committee.

The above condition $c_g/c_s \ge 4$ is a lower bound that works for many distributions. For specific distributions sharper results can be obtained. The next example shows that $c_g/c_s \ge 2$ (i.e., a cost function that is linear in the number of assessed dimensions) is sufficient for the specialist committee to dominate the generalist committee if the distribution of attributes is uniform.

Example 5 (Uniform) Assume that both F and G are uniform on [0,1]. Suppose that $c_g = 2c_s$. Then the equilibrium cutoffs are given by

$$z(\alpha) = 1 - \sqrt[3]{\frac{6\alpha^2 c_g}{4\alpha - 1}} = 1 - \sqrt[3]{\frac{12\alpha^2 c_s}{4\alpha - 1}}$$
$$x^*(\alpha) = 1 - \sqrt[3]{\frac{2c_s}{\alpha}}$$

A member's utility in a specialist committee is given by

$$v(\alpha) = \alpha x^* + (1 - \alpha) E[Y|Y \ge x^*] = 1 - (1 + \alpha) \sqrt[3]{\frac{c_s}{4\alpha}}$$

while a member's utility in a generalist committee coincides with the cutoff $z(\alpha)$. It is easy to verify that $v(\alpha) > z(\alpha)$ for all $\alpha \in [1/2, 1]$.

5.2 The Emergence of Committees and Their Management

Whenever there are conflicts of interests, a completely informed, generalist dictator (say member A) obviously stands to lose if he invites member B to form a committee and share the power of choosing the suitable course of action. Thus, whenever decision power is asymmetrically distributed, a lack of specialization suggests that most decisions will be made by the authority person who is in power.

As we remarked in the introduction, modern technological societies seem to pursue an inexorable path towards more specialization. It is intuitive, and we show it below, that under specialization, a potential dictator A who is well informed only about one dimension of the problem at hand stands to gain by forming a committee with another member B who is informed about another dimension. Moreover, informed members who were excluded from decision making also gain by affecting the decision within a committee, even when the extra search cost is taken into account. In spite of the fact that the expected search duration in a committee is higher than under dictatorship, this conclusion holds even for large degrees of conflict between A and Bif the search costs are sufficiently small.

In other words, a late informed decision is better than an early uninformed one. Thus, the trend to more specialization offers a natural explanation for the observed increased frequency of decisions by committees, and for the often bemoaned increase in delay of reaching those decisions.

To make the above argument precise, consider then the specialized dictator's problem: member A is the dictator who incurs a search cost c_A and observes the realizations of X only. If member A dictates the decision, his continuation payoff v_A^D is given by

$$v_A^D = \alpha x_D + (1 - \alpha) E[X]$$

where the acceptance cutoff x_D is determined by

$$\alpha E [X - x_D | X \ge x_D] [1 - F (x_D)] = c_A.$$
(16)

Consider now for simplicity the symmetric setting where $F = G, \alpha = 1 - \beta$ and $c_A = c_B = c$. Member B's payoff is given by:

$$v_B = \alpha E[X] + (1 - \alpha) E[X|X \ge x_D]$$

The above expression assumes that member B can free ride on A's decision without paying any search cost. Our conclusion about the value of forming committees under specialization gains extra support if B also incurs some extra cost (of waiting, say) while being outside the committee.

If A invites member B to join a committee that employs the unanimity rule, then their payoffs in this symmetric setting are given by

$$v_A^U = v_B^U = \alpha x^* + (1 - \alpha) E[X|X \ge x^*],$$

where the cutoff x^* is determined by

$$\alpha E \left[X - x^* | X \ge x^* \right] \left[1 - F \left(x^* \right) \right]^2 = c.$$
(17)

Proposition 11 Suppose that F = G, $\alpha = 1 - \beta \in (1/2, 1)$, $c_A = c_B = c$. Assume that F has strict DMRL and bounded support with upper bound $\overline{\theta} < \infty$. Then the acceptance standard goes down while the expected search duration goes up in the transition from specialized dictatorship to specialized unanimity. As long as the search cost c is sufficiently small, both members gain by forming a committee. **Proof.** The fact that $x_D > x^*$ follows immediately from the two equilibrium conditions (16 and 17), the DMRL property, and the fact that $\frac{1}{1-F(x)} \leq \frac{1}{(1-F(x))^2}$. Concerning search duration we have

$$\frac{1}{1 - F(x_D)} = \frac{\alpha}{c} E\left[X - x_D | X \ge x_D\right] < \frac{\alpha}{c} E\left[X - x^* | X \ge x^*\right] = \frac{1}{\left[1 - F(x^*)\right]^2}$$

where the inequality follows from our DMRL assumption and $x_D > x^*$.

For the second part, observe that the difference $(x_D - x^*)$ tends to zero as c goes to zero since both tend to the upper boundary of the attributes' support. As c tends to zero, the dictator's expected gain from forming a committee is

$$\lim_{c \to 0} (v_A^U - v_A^D) = \lim_{c \to 0} \left[(1 - \alpha) \left(E \left[X | X \ge x^* \right] - E \left[X \right] \right) - \alpha \left(x_D - x^* \right) \right]$$

= $(1 - \alpha) \lim_{c \to 0} \left(E \left[X | X \ge x^* \right] - E \left[X \right] \right)$
= $(1 - \alpha) \left(\overline{\theta} - E[X] \right) > 0$

Similarly, member's B expected gain from joining the committee is

$$\lim_{c \to 0} (v_B - v_B^D) = \lim_{c \to 0} (\alpha x^* + (1 - \alpha) E[X|X \ge x^*] - \alpha E[X] - (1 - \alpha) E[X|X \ge x_D])$$

=
$$\lim_{c \to 0} (\alpha (x^* - E[X]) + (1 - \alpha) (E[X|X \ge x^*] - E[X|X \ge x_D]))$$

=
$$\alpha (\bar{\theta} - E[X]) > 0$$

Therefore, as $c \to 0$, both members gain from forming a committee. It is clear by the above expressions, and by continuity, that both benefits are positive for any sufficiently small c.

The above proof also offers a glimpse into the distribution of gains from forming a committee. When the degree of conflict is small (α close to $\frac{1}{2}$), the gains are more evenly divided, whereas member B stands to gain more when the degree of conflict is relatively high (α close to 1). This is intuitive since a dictator that is informed about the only dimension that is of interest to him (i.e., private values) has obviously nothing to gain by forming a committee, while with private values member B gains control of the dimension that is of interest to him by joining the committee, whereas he had none before.

In this context, it is interesting to note that a more flexible communication structure within a committee allows both an increase in the dictator's payoff and sometimes a Pareto-improvement over unanimity. Consider the above setting, and suppose that dictator A can consult with member B before making a decision, without giving Bveto power. For simplicity, let us assume that member B can send either a "yes" or a "no" message to member A, and let y_B denote member B's cutoff for sending the message "yes". Member A then uses two cutoffs: x_A^1 when member B says "yes" and x_A^0 when member B says "no". Member A can implement the outcome of unanimity by setting $x_A^1 = x^*$ (his equilibrium cutoff under unanimity) and $x_A^0 = \overline{\theta}$ (the upper bound of the attribute's support), because the best response for member B is then to set $y_B = y^*$, his own equilibrium cutoff under unanimity. Therefore, by optimizing the two cutoffs x_A^1 and x_A^0 , the dictator A can do even better.¹⁶ Depending on the parameters, even member B can be made better off.

We now offer two last comments on committee management. In symmetric settings, we have shown above that members' utilities decrease in α , the degree of conflict, with both complete or incomplete information. Without symmetry, it is not necessarily true that a dictator is always better off by inviting a more moderate member. The reason is that a member with more extreme preferences is more motivated to maintain a high acceptance standard despite a high search cost. This can sometimes be beneficial through the higher quality of the taken decision. Here is an example under specialization.

Example 6 Suppose that the dictator A with preference $\alpha x + (1 - \alpha) y$, $\alpha \ge 1/2$, invites member B with preference $\beta x + (1 - \beta) y$, $\beta \le 1/2$, to join the committee. Suppose that both F and G are uniform on [0,1], and that $c_B = 8c_A = 8c$. The equilibrium cutoffs x^* and y^* are given by

$$x^* = 1 - \frac{1}{2\alpha} \sqrt[3]{2\alpha (1 - \beta) c}, \quad y^* = 1 - \frac{4}{1 - \beta} \sqrt[3]{2\alpha (1 - \beta) c}$$

Member A's payoff is

$$v_A = 1 - \left(\frac{1}{2} + \frac{2(1-\alpha)}{1-\beta}\right) \sqrt[3]{2\alpha(1-\beta)c},$$

which is strictly decreasing in β , as long as $8(1 - \alpha) > (1 - \beta)$. Note that $\beta \leq 1/2$, and that member B is more extreme when β is lower. Therefore, as long as the dictator's preference is not too extreme, he is better off by inviting a more extreme member B.

Similarly, it is not always better to invite agents with low search cost to join the committee. Suppose member A has the option to choose his committee colleague among several candidates. Suppose also that all candidates to join him have the same preference with $\beta = 1 - \alpha$, but have different search cost c_B . Should A choose a colleague with a high or low search cost? A general rule is that the cost has to

¹⁶This seems to be the modus operandi of most scientific journals: experts are consulted but the decision is taken by an editor.

be moderate. If it is too high, then member B may set a too low standard. If it is too low, then member B may set a too high standard and hold member A up. A precise answer to this question is distribution-specific. For example, in the exponential distribution case, it is never optimal for member A to choose anyone with a higher cost than himself, because then member B will become the dictator. It is also not useful for A to choose a member B with a cost lower than himself, because then the choice of member B does not matter, since A is the dictator. Thus, if it is optimal for a dictator to form a committee, he should choose someone with the same search cost as his own.

6 Concluding Remarks

We have introduced a fairly rich model for the analysis of committee search conducted by generalists or by specialized, privately informed members with heterogenous preferences defined on a multi-dimensional alternative space. The model generates a wealth of implications that could, in principle, be tested in the field or in the laboratory. We have also provided an array of helpful technical tools that seem well suited for the problem at hand.

The following table summarizes our main findings about the various effects of increases in the degree of conflict within committees (for unanimity the implications shown are for symmetric settings).

	acceptance standard	individual payoff	search duration
unanimity with complete info.	\searrow	\searrow	$?^{17}$
unanimity with incomplete info.	7	<u>\</u>	7
dictatorship with complete info.	7	7	ambiguous
dictatorship with incomplete info.	7	7	7

We see several avenues for future research: 1) Consider committees with more members (some specialized, some generalists), and the interplay between the number of members and the dimension of the space of alternatives; 2) Consider different aggregation rules, e.g., decisions by qualified majority; 3) Endogenize the choice of information acquisition/specialization.

¹⁷We suspect that search duration may be increasing in α under stronger conditions than DMRL, but we have not been able to prove this assertion so far. In Appendix A we prove a partial result in this direction.

7 Appendix A

The analysis of the expected equilibrium search duration in the complete information, unanimity case is somewhat complex. We prove the following partial result:

Proposition 12 Consider the symmetric, complete information case under unanimity, where F = G, $\alpha = 1 - \beta$, and $c_A = c_B = c$. Assume that X, Y have IFR, and assume that $\frac{1}{2}X + \frac{1}{2}Y \leq_{MRL} X$. Then the expected search duration for $\alpha = \frac{1}{2}$ is less than the expected search duration for $\alpha = 1$. Moreover, if the dictator's expected search duration increases in α , the expected search duration under unanimity cannot decrease.¹⁸

Proof. Let $z(\frac{1}{2}) \ge z(1)$ be the optimal cutoffs under unanimity for $\alpha = \frac{1}{2}$ and $\alpha = 1$ respectively. The inequality follows by Proposition 2. For $\alpha = \frac{1}{2}$ the equilibrium condition is:

$$E\left[\frac{1}{2}X + \frac{1}{2}Y - z(\frac{1}{2}) \mid \frac{1}{2}X + \frac{1}{2}Y \ge z(\frac{1}{2})\right] \Pr\left\{\frac{1}{2}X + \frac{1}{2}Y \ge z(\frac{1}{2})\right\} = c.$$

For $\alpha = 1$, the equilibrium condition is

$$E[X - z(1) \mid X \ge z(1)] (\Pr\{X \ge z(1)\})^2 = c.$$

We obtain then for search durations that:

$$\begin{aligned} \frac{c}{\Pr\left\{\frac{1}{2}X + \frac{1}{2}Y \ge z(\frac{1}{2})\right\}} &= E\left[\frac{1}{2}X + \frac{1}{2}Y - z(\frac{1}{2}) \mid \frac{1}{2}X + \frac{1}{2}Y \ge z(\frac{1}{2})\right] \\ &\leq E\left[\frac{1}{2}X + \frac{1}{2}Y - z(1) \mid \frac{1}{2}X + \frac{1}{2}Y \ge z(1)\right] \\ &\leq E\left[X - z(1) \mid X \ge z(1)\right] \\ &= \frac{c}{(\Pr\left\{X \ge z(1)\right\})^2} \end{aligned}$$

The first inequality follows because $\frac{1}{2}X + \frac{1}{2}Y$ has the DMRL property, as shown in Corollary 2.A.24 in Shaked and Shanthikumar [2007] – this is the reason why we need the stronger IFR condition on the random variables X, Y. The second inequality holds because we assumed $\frac{1}{2}X + \frac{1}{2}Y \leq_{MRL} X$.

For the second part, note that the dictator's cutoff and the symmetric unanimity cutoff are the same for $\alpha = \frac{1}{2}$ under complete information. The same applies for the expected search duration, since the equilibrium conditions are then the same.

¹⁸Recall that our examples show that the behavior of the expected search duration under dictatorship with complete information is ambiguous.

By Proposition 2 we also know that the dictator's cutoff is always higher than the unanimity cutoff. Under DMLR (which is implied by the IFR condition) this yields that the search duration for $\alpha = 1$ is higher under unanimity than under dictatorship.

The condition $\frac{1}{2}X + \frac{1}{2}Y \leq_{MRL} X$ will be satisfied whenever the mean residual life function of a weighted convolution of random variables is monotone in some measure of the dispersion of the weights (e.g., Schur-convexity), but such results are not easy to establish for some general class of distributions. A small literature proves such results for particular distributions, e.g., Zhao and Balakrishnan [2009] who look at convolutions of exponentials.

8 Appendix B

Proof of Proposition 5. By the analogy exposed in Remark 3, it is enough to establish the existence and uniqueness of the solution for the system of two equations determining the optimal cutoffs in the complete information case with private values, i.e., $\alpha = 1 - \beta = 1$. This has been shown in Ferguson [2005]. For completeness, and in order to explain the role of DMRL, we reproduce the simple proof below.

Let us define

$$\overline{\rho} \equiv \max\left\{\frac{\alpha}{c_A} E\left[X\right], \frac{1-\beta}{c_B} E\left[Y\right]\right\}$$

and

$$\underline{\rho} = \min\left\{\frac{\alpha}{c_A}\lim_{x\to\overline{\theta}} E[X-x \mid X \ge x], \frac{1-\beta}{c_B}\lim_{y\to\overline{\theta}} E[Y-y \mid Y \ge y]\right\}.$$

Note that all threshold equilibria must satisfy the two equilibrium conditions (11) and (12). If $\overline{\rho} \leq 1$, then by the DMRL assumption, we must have $x^* \leq 0$ and $y^* \leq 0$. This means we have a corner solution where the committee accepts any candidate, which is indeed an equilibrium and essentially unique. From now on, we assume $\overline{\rho} > 1$. The two equilibrium conditions imply that

$$\frac{\alpha}{c_A} E[X - x^* \mid X \ge x^*] = \frac{1 - \beta}{c_B} E[Y - y^* \mid Y \ge y^*].$$

Since F and G have strict DMRL, we can find, for each $\xi \in (\underline{\rho}, \overline{\rho})$, a unique pair $(x^*(\xi), y^*(\xi))$ (one of them could be negative) such that

$$\frac{\alpha}{c_A} E[X - x^*(\xi) \mid X \ge x^*(\xi)] = \frac{1 - \beta}{c_B} E[Y - y^*(\xi) \mid Y \ge y^*(\xi)] = \xi.$$

As ξ increases, both $x^*(\xi)$ and $y^*(\xi)$ decrease strictly and continuously, until one or both of them reach the upper bound $\overline{\theta}$. At the same time, when ξ increases, and $x^{*}(\xi)$ and $y^{*}(\xi)$ decrease, the function

$$\frac{1}{P(\xi)} \equiv \frac{1}{(1 - F(x^*(\xi)))(1 - G(y^*(\xi)))}$$

decreases strictly and continuously. Note that when $\xi = \underline{\rho}$,

$$\frac{1}{P\left(\underline{\rho}\right)} = \frac{1}{\left(1 - F\left(\overline{\theta}\right)\right) \left(1 - G\left(\overline{\theta}\right)\right)} \to +\infty > \underline{\rho},$$

and when $\xi = \overline{\rho}$,

$$\frac{1}{P\left(\overline{\rho}\right)} = 1 < \overline{\rho}.$$

Therefore, there exists a unique value $\xi_0 \in (\underline{\rho}, \overline{\rho})$ such that $\xi_0 = 1/P(\xi_0)$. Since each ξ corresponds to an essentially unique pair of $(x^*(\xi), y^*(\xi))$, a cutoff equilibrium exists and is unique.

Proof of Proposition 8. We know that

$$\frac{1}{(1 - F(x^*))(1 - G(y^*))} = \frac{\alpha}{c} E[X - x^* \mid X \ge x^*] = \frac{\alpha}{c} E[Y - y^* \mid Y \ge y^*]$$

By $X \leq_{MRL} Y$ we obtain that

$$\forall x, \ E[X - x | X \ge x] \le E[Y - x | Y \ge x]$$

Together with DMRL, this implies $x^* \leq y^*$. On the one hand, from (8) and (9) we have

$$v_B - v_A = \alpha(y^* - x^*) + (1 - \alpha) \left(E[X \mid X \ge x^*] - E[Y \mid Y \ge y^*] \right)$$

On the other hand, from (10) we also have

$$v_B - v_A = (1 - \alpha)E[X \mid X \ge x^*] + \alpha E[Y \mid Y \ge y^*] - \frac{c}{(1 - F(x^*))(1 - G(y^*))} \\ - \left(\alpha E[X|X \ge x^*] + (1 - \alpha)E[Y|Y \ge y^*] - \frac{c}{(1 - F(x^*))(1 - G(y^*))}\right) \\ = (1 - 2\alpha)(E[X \mid X \ge x^*] - E[Y \mid Y \ge y^*])$$

From the two representations above, and from $y^* \ge x^*$ we obtain:

$$E[X \mid X \ge x^*] - E[Y \mid Y \ge y^*] = -(y^* - x^*) \le 0.$$

Because $\alpha \geq \frac{1}{2}$, we obtain

$$v_B - v_A = (1 - 2\alpha) \left(E[X \mid X \ge x^*] - E[Y \mid Y \ge y^*] \right) \ge 0$$

as desired. \blacksquare

Proof of Proposition 9. Recall that $\widetilde{X} \leq_{HR} X$ implies both $\widetilde{X} \leq_{MRL} X$ and $\widetilde{X} \leq_{ST} X$. Let $F(\widetilde{F})$ denote the distribution of X and Y (\widetilde{X} and \widetilde{Y}). We first show that $x^* \geq \widetilde{x}^*$. Suppose the opposite $(x^* < \widetilde{x}^*)$ is true. Then from the equilibrium conditions we have

$$\frac{1}{\left(1 - F\left(x^*\right)\right)^2} = \frac{\alpha}{c} E[X - x^* \mid X \ge x^*]$$
$$\geq \frac{\alpha}{c} E[X - \tilde{x}^* \mid X \ge \tilde{x}^*]$$
$$\geq \frac{\alpha}{c} E[\tilde{X} - \tilde{x}^* \mid \tilde{X} \ge \tilde{x}^*]$$
$$= \frac{1}{\left(1 - \tilde{F}\left(\tilde{x}^*\right)\right)^2}$$

The two inequalities follow from DMRL assumption of X and the assumption $\widetilde{X} \leq_{MRL} X$, respectively. Therefore, we must have $F(x^*) \geq \widetilde{F}(\widetilde{x}^*)$. Since $\widetilde{X} \leq_{ST} X$, we also have $F(x^*) \geq \widetilde{F}(\widetilde{x}^*) \geq F(\widetilde{x}^*)$, which implies that $x^* \geq \widetilde{x}^*$, a contradiction.

In equilibrium we also have

$$v_A = \alpha x^* + (1 - \alpha) E[X|X \ge x^*]$$

$$\ge \alpha \widetilde{x}^* + (1 - \alpha) E[X|X \ge \widetilde{x}^*]$$

$$\ge \alpha \widetilde{x}^* + (1 - \alpha) E[\widetilde{X}|\widetilde{X} \ge \widetilde{x}^*] = \widetilde{v}_A$$

The first inequality follows because v_A is increasing in x^* , while the second inequality follows by recalling that $\widetilde{X} \leq_{HR} X$ implies $[\widetilde{X} \mid \widetilde{X} \geq \widetilde{x}^*] \leq_{ST} [X \mid X \geq \widetilde{x}^*]$.

The following technical Lemma is used in the proof of Proposition 10.

Lemma 2 Suppose that F = G is convex. Then for all z,

$$\Phi(z) \equiv E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\{X \ge z, Y \ge z\}}\right] - E\left[\left(\frac{1}{2}X + \frac{1}{2}Y - z\right) \cdot \mathbf{1}_{\left\{\frac{1}{2}X + \frac{1}{2}Y \ge z\right\} \setminus \{X \ge z, Y \ge z\}}\right] > 0$$

Proof. Since F is convex we have

$$\left[\left(\frac{1}{2}x + \frac{1}{2}y - z\right)f(x)\right]' = \frac{1}{2}f(x) + \left(\frac{1}{2}x + \frac{1}{2}y - z\right)f'(x) > 0$$

for all $(x,y) \in \{(x,y) : \frac{1}{2}x + \frac{1}{2}y \ge z\}$. If $z \ge \overline{\theta}/2$, we can apply the symmetry of X

and Y to obtain

$$\begin{aligned} &\frac{1}{2}\Phi(z) \\ &= \int_{z}^{\overline{\theta}}\int_{z}^{y}\left(\frac{1}{2}x + \frac{1}{2}y - z\right)f(x)f(y)\,dxdy - \int_{z}^{\overline{\theta}}\int_{2z-y}^{z}\left(\frac{1}{2}x + \frac{1}{2}y - z\right)f(x)f(y)\,dxdy \\ &> \int_{z}^{\overline{\theta}}\left(\frac{1}{2}z + \frac{1}{2}y - z\right)(y-z)f(z)f(y)dy - \int_{z}^{\overline{\theta}}\left(\frac{1}{2}z + \frac{1}{2}y - z\right)f(z)(y-z)f(y)\,dy \\ &= 0 \end{aligned}$$

where the inequality follows from the monotonicity of $(\frac{1}{2}x + \frac{1}{2}y - z) f(x)$. The case of $z < \overline{\theta}/2$ can be proved analogously. Therefore, we have $\Phi(z) \ge 0$ for all z.

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