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Inequality and Growth in a Knowledge Economy

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Abstract

We develop a two sector growth model to understand the relation between inequality and growth. Agents, who are endowed with different levels of knowledge, select either into a retail or a manufacturing sector. Agents in the manufacturing sector match to carry out production. A by-product of production is creation of ideas that spill over to the retail sector and improve productivity, thereby causing growth. Ideas are generated according to an idea production function that takes the knowledge of all the agents in a firm as arguments. We go on to study how an increase in the inequality of the knowledge distribution affects the growth rate. A change in the distribution not only affects the occupational choice of agents, but also the way agents match within the manufacturing sector. We show that if the idea generation function is sufficiently convex, an increase in inequality raises the growth rate of the economy.

1 Introduction

History credits James Watt with the invention of the steam engine, Thomas Alva Edison with the invention of the electric light bulb and Henry Ford with the invention of the assembly line. But these great inventors did not work alone; they had very able partners or teams assisting them in their endeavors. Watt had Matthew Boulton, Edison had Charles W. Batchelor while Ford had C. Harold Wills among others. Whether Watt or Edison or Ford would have been equally successful if they had worked on their own is hard to conjecture. It is true, however, that the above mentioned partners and many other individuals made significant contributions to the various innovations that have made Watt et al. household names.

In this paper, we develop a model to understand how the matching of agents within firms affects growth through its impact on idea creation. Agents, in our model, are distributed according to a fundamental knowledge distribution which we take as given. Agents choose to work in either of the two sectors - retail or manufacturing. For the latter, they also choose whether they want to be a worker or a manager. The sectors vary in three important respects. Firstly, the manufacturing sector is *more knowledge-intensive*. Secondly, agents match to carry out production in the manufacturing sector; managers choose their workers. And finally, manufacturing firms also create new ideas every period according to an exogenous idea production function. These ideas spill over to the retail sector in the following period, thereby raising the total factor productivity of the retail sector firms.

An equilibrium in our model is characterized by two knowledge thresholds, where the agents with knowledge below the lower threshold sort into the retail sector, while those above the higher threshold become managers in the manufacturing sector. The ones in between choose to be workers in manufacturing firms. The knowledge distribution not only determines the thresholds, but by affecting how agents match, also determines the effectiveness and volume of ideas that are created. We go on to study how an increase in the inequality of the knowledge distribution affects the growth rate. A change in the distribution not only affects the occupational choice of agents, but also the way they match within the manufacturing sector. The effect on the growth rate, of course, depends on how ideas spill over.

We define a very general spillover function, which nests some of the more popular spillover functions used in the literature. In the case of the canonical growth model, which we briefly review in Section 2, an increase in inequality is accompanied by a decline in the growth rate. If the growth rate is determined by the knowledge of the best agents, as in Murphy et al. (1991), an increase in inequality could lead to a higher growth rate. We show that whether an increase in inequality raises or lowers the growth rate depends crucially on the elasticity of idea production with respect to knowledge. A more unequal distribution has fewer managers (the *allocation effect*) but they are, on average, more knowledgeable (the *distribution effect*). We show that if the idea production function is sufficiently convex, the latter effect dominates the former, leading to a rise in the growth rate.

The model is built on the presumption that the quality of ideas generated by a firm depends on the knowledge of the firm's employees. In a related paper, Lucas Jr (2009) develops a model where the aggregate knowledge of an economy is simply a list of the knowledge of its members. Like our model, a fundamental assumption of Lucas Jr (2009) is that knowledge is embodied - *".....all knowledge resides in the head of some individual person.....".* Lucas Jr (2009) does not, however, consider team formation or the problem of allocating agents across sectors, as we do here. In our model, both the allocation, as well as, the composition of firms matters for growth. In that respect, our paper is similar to Murphy et al. (1991), who look at how allocating individuals with different levels of knowledge across different activities can affect growth. Murphy *et al* consider two main activities, production and rent-seeking, and consider the implications of allocating the most knowledgeable agents in either of these two activities.[1] They, however, do not consider team formation. Since growth in their model is driven by the productivity of the best production worker, inequality has no effect on growth as long as the most knowledgeable agent in the economy is in the production sector. This does not happen in our model since a change in inequality alters team formation, thereby changing the quality of ideas that are being generated.

The first generation of endogenous growth models predicted scale effects (See Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). An increase in the size of the population, by bringing about a proportionate increase in the number of researchers, raises the growth rate of the economy. This prediction, however, is not borne out by the data. Jones (1995) looks at the U.S. in the 20th century and finds that the growth rate has been quite stable over this period. He interprets this as an absence

[1] Murphy et al. (1991), however, do not have an explicit dynamic model where the production and rent-seeking sectors evolve endogenously. Ehrlich and Lui (1999) develop an endogenous growth model where growth results from the interaction of two activities - accumulating physical capital which promotes growth, and accumulating political capital, which promotes rent-seeking.

of *strong* scale effects proposed by Romer and others, and in stead, proposes a model with *weak* scale effects, where the growth rate of the economy depends on the *growth rate* of the population and not on its level (See Jones, 1999). Similarly, Kortum (1997) provides evidence that in the U.S., the steady increase in the number of researchers between the years 1910 and 1990 has not been accompanied by an increase in the number of patents granted. He goes on to present a model, where steady-state growth is driven by population growth, as in Jones (1999). This paper differs from the above-mentioned papers in two important respects. Idea generation, in our model, is a by-product of production and firms fail to capture the full value of their own ideas. We abstract from the problem where firms optimally choose their level of research, (the first difference) because we want to focus on the effect of the underlying knowledge distribution on idea generation (the second difference). We believe that ours is the first paper to explore this issue.

Economists have been studying the relation between inequality and growth for a long time. The traditional view was that inequality is good for growth. This could be either because inequality generates incentives (the poor have an incentive to become rich) or, because investment projects involve large indivisibilities or, because the rich have a higher propensity to save compared to the poor (See Aghion et al., 1999).[2] Recent studies have unambiguously shown that inequality retards growth.[3] Accordingly, theories have been forwarded that try to find a causal link between inequality and growth. These theories either emphasize the absence of well-functioning credit-markets (See, for example Banerjee and Newman, 1991; Galor and Zeira, 1993; Aghion and Bolton, 1997) or introduce a political process through which agents choose the level of re-distribution in the economy (See, for example Alesina and Rodrik, 1994; Persson and Tabellini, 1994). None of these papers consider the effect of inequality on innovation or creation of ideas.

The remainder of the paper is organized as follows. In section 2, we briefly review the canonical growth model and its treatment of inequality. In Section 3, we introduce heterogeneity and matching. The equilibrium is characterized in Section 4. We define inequality in Section 5, and examine how it affects growth in Section 6. Section 7 concludes.

[2]Kaldor (1955-56) believed that the propensity to save out of profit income is higher than the propensity to save out of wage income

[3]Alesina and Rodrik (1994), Perotti (1996) and Persson and Tabellini (1994) among others, find that inequality lowers the growth rate. The latter also find a positive effect of equality on growth.

2 A Canonical Growth Model

Let us consider a simple version of the endogenous growth model of Romer (1990). The analysis is based on Jones (2005). The economy produces a homogeneous good Y, let us call it GDP, according to the following technology :

$$Y_t = A_t^{\sigma} L_{Y,t}, \tag{1}$$

where A_t is the stock of ideas in the economy while $L_{Y,t}$ is the amount of labor employed in production at time t. Ideas are produced according to the following linear production function :

$$A_{t+1} = e^{\delta L_{A,t}} A_t \tag{2}$$

where $L_{A,t}$ is the amount of labor employed in research at time t. For simplicity, we assume that labor is the only factor of production. The total labor force is L and there is no population growth. Therefore, the resource constraint of the economy is

$$L_{A,t} + L_{Y,t} = L \tag{3}$$

Suppose a fraction s of the labor force is allocated towards research (such a choice can be derived from a micro-founded model where s would depend on preference and institutional parameters), i.e.

$$L_{A,t} = sL \text{ and } L_{Y,t} = (1 - s)L$$

Define the growth rate of per capita income $y_t (= Y_t/L)$ as g. Then we have

$$\begin{aligned} g &= log(y_{t+1}) - log(y_t) \\ &= \sigma(log(A_{t+1}) - log(A_t)) \\ &= \sigma\delta sL \end{aligned}$$

The growth rate of per capita output is simply the growth rate of ideas, augmented by σ which measures the dependence of output on ideas - how a country grows essentially depends on how ideas are generated in the country. g depends on the the *total number of workers* in the economy. This is the standard scale effect in these kinds of models. A bigger population implies more researchers, which translates into faster generation of ideas and hence, faster growth of output. A consequence of this is that two countries with the same size of the labor force will grow at the same rate, other things being the same.

It is clear that in the above model, inequality can affect growth rate only through s; all the other parameters are exogenous. *A priori* we do not have any idea of the direction of change in s following an increase in inequality. We do know, however, that an increase in inequality can have a positive impact on the growth rate only by increasing the number of researchers. In other words, if an increase in inequality leads to a decline in s, growth rate of the economy goes down because there are fewer researchers in the economy. Since researchers are homogeneous in terms of their research productivity, only the mass of researchers matters for growth.

3 A model with heterogeneous agents

In this section, we lay out the basic economic environment of the model with heterogeneous agents.

3.1 Preferences and Endowments

Consider a closed economy. The economy is populated by a continuum of agents who live for one period. Time is discrete. Each agent owns a certain amount of labor (same across all agents), which he supplies inelastically. Since the measure of agents does not play any role in our analysis, we normalize it to 1.

Agents are endowed with knowledge k which is distributed according to $\Phi(k)$ with support $[\underline{k}, \overline{k}]$. One can think of the knowledge distribution as being determined by the government's education policy. For our purpose, $\Phi(k)$ is exogenously given and is the same every period. Of course, over the long run, technological progress will affect $\Phi(k)$ by changing the occupational choice set of agents and thereby influencing their educational decisions. Here, we abstract from the endogenous evolution of the knowledge distribution.

3.2 Production and Occupation

The economy consists of two sectors - a retail sector and a manufacturing sector. The retail sector produces a final good Y_t which is used for consumption. The manufacturing sector produces differentiated intermediate inputs that are used in the production of the final good. Hence, we are following Romer (1990) in making the intermediate input a differentiated good. The alternative, following Ethier (1982) and Grossman and Helpman (1991), is to assume that agents have a love-for-variety utility function and consequently, the final good is differentiated. Although they produce identical results, we go with the differentiated intermediate inputs assumption because it seems more reasonable in the current context, as discussed below.

3.2.1 Retail sector

The final good Y_t is produced competitively using the following technology :

$$Y_t = A_t L_{Y,t}^{1-\alpha}\Big(\int\limits_{M_t} x_t(k)^{\alpha} d\Phi(k)\Big), \tag{4}$$

where A_t, as before, is the stock of ideas at time t, $L_{Y,t}$ is the labor employed in the retail sector, $x_t(k)$ is the amount of the differentiated input produced by a manager with knowledge k and M_t is the set of agents who choose to be managers at time t. The inputs could be goods that get used up in production or machines that depreciate fully. (See Acemoglu, 2008). Equation (4) can, alternatively, be written as

$$Y_t = A_t L_{Y,t}^{1-\alpha} X_t^{\alpha},$$

where

$$X_t = \Big(\int\limits_{M_t} x_t(k)^{\alpha} d\Phi(k)\Big)^{\frac{1}{\alpha}}.$$

The alternative form shows that the production technology in the retail sector exhibits constant-returns-to-scale in labor and X_t, which is a CES aggregator of the differentiated inputs $x_t(k)$.

Workers in the retail sector perform routine tasks. Hence, what matters for production is the mass of workers rather than their knowledge. As a result, all workers in this sector earn an uniform wage s_t. Furthermore, each manager is a monopolist and sets a price of $p_t(k)$ for his inputs. Every period, a typical retail firm hires labor and purchases inputs in the spot market, carries out production and makes payment to the factors which exhausts total output. Since the firm does not make any inter temporal allocation decisions, it simply maximizes instantaneous profits. The problem of a typical firm is

$$\max_{L_{Y,t},x_t(k)} A_t L_{Y,t}^{1-\alpha}(\int_{M_t} x_t(k)^\alpha d\Phi(k)) - s_t L_{Y,t} - \int_{M_t} p_t(k) x_t(k) d\Phi(k). \tag{5}$$

The first-order condition with respect to labor gives rise to the following expression for wage :

$$s_t = (1-\alpha) A_t L_{Y,t}^{-\alpha}(\int_{M_t} x_t(k)^\alpha d\Phi(k)). \tag{6}$$

More inputs raises the marginal productivity of the retail sector workers and thereby increases the wage. The first-order condition with respect to input $x_t(k)$ yields the following iso-elastic demand curves :

$$x_t(k) = (\alpha A_t / p_t(k))^{\frac{1}{1-\alpha}} L_{Y,t}^{1-\alpha}.$$

The demand curve can be inverted to obtain an expression for the price of the intermediate inputs :

$$p_t(k) = \alpha A_t L_{Y,t}^{1-\alpha} x_t(k)^{\alpha-1}. \tag{7}$$

Note that the only dependence of input price $p_t(k)$ on the aggregate knowledge distribution is through the allocation of labor across the two sectors. In particular, it does not depend on how much of the other inputs are being produced.

3.2.2 Manufacturing sector

Each intermediate firm consists of one manager/entrepreneur and one worker. Output produced by each firm in a given time period is

$$x_t(k_m) = k_m k_w^\beta, \quad \beta < 1 \tag{8}$$

where the manager and worker's knowledge are denoted by k_m and k_w respectively.[4] The production technology exhibits complementarity. $\beta < 1$ captures the fact that output is relatively less sensitive to the worker's knowledge. Therefore, unlike the retail sector, the worker's knowledge does matter for producing manufactured goods, but it is relatively *less important* compared to the manager's knowledge. This feature of technology is essential for getting cross-matching of agents in the economy (See Kremer and Maskin, 1996). The dependence of output on time arises from the possibility that a manager might be matched with different workers in different time periods. We also make the following assumption about the parameters:

Assumption 1 : $\underline{k} > \beta^{\frac{1}{\alpha}}\overline{k}$.

Assumption 1 is a statement about the range of the knowledge support with respect to β. The role of this condition - especially in ensuring the uniqueness and existence of an equilibrium - will become clear later on. A manager hires a worker by paying wage and is the residual claimant of total revenue. Although a manager is a monopolist in the product market, he acts like a price-taker in the labor market. Accordingly, a manager takes the wage schedule as given. The manager's profit is given by

$$\begin{aligned}\pi_t(k_m) &= p_t(k_m)x_t(k_m) - w_t(k_w) \\ &= \alpha A_t L_{Y,t}^{1-\alpha} k_m^{\alpha} k_w^{\alpha\beta} - w_t(k_w)\end{aligned} \tag{9}$$

where $w_t(k)$ is the wage schedule for workers in the manufacturing sector and we have replaced $p_t(k_m)$ with its expression from equation (7). The first-order condition for profit-maximization yields

$$w_t'(k_w) = \alpha^2 \beta A_t L_{Y,t}^{1-\alpha} k_m^{\alpha} k_w^{\alpha\beta-1}. \tag{10}$$

At the margin, the increase in revenue coming from hiring a slightly more knowledgeable worker should be equal to the increase in cost due to the higher wage that has to be paid to the worker. The envelope condition yields

$$\pi_t'(k_m) = \alpha^2 A_t L_{Y,t}^{1-\alpha} k_m^{\alpha-1} k_w^{\alpha\beta}. \tag{11}$$

[4] Instead of the quantity of output depending on the knowledge of the agents, we could assume that the quality of output is a function of knowledge. More knowledgeable agents produce better-quality inputs. As long as we define $x_t(k_m)$ as quality adjusted inputs however, the above analysis remains unchanged.

It is clear that the slopes of the wage and profit schedules are positive. Totally differentiating the FOC, we have

$$w_t''(k_w)dk_w = \alpha^3\beta A_t L_{Y,t}^{1-\alpha} k_m^{\alpha-1} k_w^{\alpha\beta-1} dk_m + \alpha^2\beta(\alpha\beta-1) A_t L_{Y,t}^{1-\alpha} k_m^{\alpha} k_w^{\alpha\beta-1} dk_w.$$

Re-arranging,

$$\frac{dk_m}{dk_w} = \frac{w_t''(k_w) - \alpha^2\beta(\alpha\beta-1) A_t L_{Y,t}^{1-\alpha} k_m^{\alpha} k_w^{\alpha\beta-1}}{\alpha^3\beta A_t L_{Y,t}^{1-\alpha} k_m^{\alpha-1} k_w^{\alpha\beta-1}}.$$

The numerator of the above RHS expression is positive (from the second-order condition), and so is the denominator. Therefore,

$$\frac{dk_m}{dk_w} = m_t'(k) > 0,$$

where $m_t(k) : [\underline{k}, \overline{k}] \to [\underline{k}, \overline{k}]$ is the equilibrium matching function. In equilibrium, we have positive assortative matching (PAM) in the manufacturing sector. Note that this result arises out of the optimization behavior of managers and does not, in any way, depend on the shape of the knowledge distribution.

3.2.3 Occupational choice

By now, it should be clear that in each period, the agents face the following occupational choice :

- Being a worker in the retail sector
- Being a worker in the manufacturing sector
- Being a manager in the manufacturing sector

An agent chooses the occupation which generates the highest income, i.e. the problem facing an agent with knowledge k is

$$max\{s_t, w_t(k), \pi_t(k)\}. \tag{12}$$

Labor market clearing in each period requires that

$$L_{Y,t} + \int\limits_{W_t} d\Phi(k)) + \int\limits_{M_t} d\Phi(k)) = 1. \tag{13}$$

where W_t denotes the set of workers in the manufacturing sector. Since manufacturing firms are of size two, the above equation can also be written as

$$L_{Y,t} + 2\int\limits_{M_t} d\Phi(k)) = 1.$$

3.3 Evolution of Ideas

Every period, new ideas are generated within each manufacturing firm. Creation of ideas is a by-product of manufacturing. We make the following two assumptions. *Firstly*, new ideas add to the stock of existing ideas. *Secondly*, firms fail to internalize the benefit of developing ideas. The first assumption rules out obsolescence of ideas. The second is a consequence of the non-rival and non-excludable nature of ideas. Of course, a firm developing an idea can, at times, use patents to exclude others from using it. Here, we implicitly assume that the nature of the ideas is such that they can not be patented. As a result, new ideas spillover to the rest of the economy. In particular, the new ideas are absorbed by firms in the retail sector which in turn increases TFP in final good production, as is clear from equation (4).

We continue to assume that generation of ideas follows a log-linear technology, as in equation (2), with the difference being that $L_{A,t}$ is now replaced by a function that depends on the entire knowledge distribution through the matching of workers with managers. As the distribution changes, the matches change too, thereby affecting the amount of ideas that spillover. This, in turn, affects growth. Let A_t have the following law of motion

$$A_{t+1} = e^{\lambda_t} A_t, \tag{14}$$

where $\lambda_t : \Phi(k) \to \Re_+$ captures the exact form of the spillover. At this point, we do not make any further assumptions about λ_t and defer our discussion of λ_t till Section (**??**).

4 Equilibrium

Now that the basic structure of the model has been laid out, we are in a position to define an equilibrium.

Definition 1. *An equilibrium in this economy consists of the following objects :*
(i) allocation of agents to different occupations,
(ii) time paths of retail sector wages and employment, $\{s_t, L_{Y,t}\}_{t=0}^{\infty}$*,*
(iii) time paths of prices and quantities of differentiated inputs, $\{x_t(k), p_t(k)\}_{t=0}^{\infty}\big|_{k \in M_t}$*,*
(iv) time paths of manufacturing sector wage schedule, $\{w_t(k)\}_{t=0}^{\infty}\big|_{k \in W_t}$ *and profit schedule,* $\{\pi_t(k)\}_{t=0}^{\infty}\big|_{k \in M_t}$*,*
(v) time path of the matching function in the manufacturing sector, $\{m_t(k)\}_{t=0}^{\infty}$*,*
(vi) law of motion for ideas $\{A_t\}_{t=0}^{\infty}$*,*
such that (a) agents maximize equation (12), (b) firms maximize equation (9), (c) labor market clears and (d) ideas evolve according to equation (14).

4.1 Characterization of equilibrium

In the previous section, we had established that the manufacturing sector exhibits PAM. The implication is that if we have an interval $[k_1, k_2]$ of workers, then the corresponding managers will lie in the interval $[m_t(k_1), m_t(k_2)]$, where $m_t(k_1) > k_2$. There are numerous allocations (infinitely many) that are consistent with PAM. However, not all of them are consistent with equilibrium, as shown in the following lemmas.

Lemma 1. *In equilibrium, the sets of workers in the retail sector are connected and is of the form* $[\underline{k}, k^*]$*, where* k^* *is endogenous.*

Proof In the appendix.

Lemma 2. *In equilibrium, the sets of workers and managers in the manufacturing sector are connected.*

Proof See Dasgupta (2008).

Lemma 1 says that the least knowledgeable agents select into the retail sector. The less knowledgeable agents have a comparative advantage in the retail sector which is the relatively less knowledge-intensive sector. The intuition behind Lemma 2, on the other hand, is a bit more subtle. There are two opposing forces that determine matching - complementarity, which encourages agents with similar knowledge levels to match together and asymmetry, which goes in the opposite direction (See Kremer and Maskin, 1996). Assumption 1 is a statement about the relative strengths of these two forces. β captures the degree of asymmetry between the tasks of the worker and the manager (A smaller β implies more asymmetry). Assumption 1 says that the degree of asymmetry must be large enough. For example, if $\beta = 1$, the two tasks are perfectly symmetric, and Assumption 1 is violated. In this case, the equilibrium outcome is self-matching.

Lemmas 1 and 2 together imply that the equilibrium allocation is characterized by two thresholds, k_t^* and k_t^{**}, such that agents $\in [\underline{k}, k_t^*]$ are in retail, agents $\in [k_t^*, k_t^{**}]$ are workers in manufacturing and agents $\in [k_t^{**}, \overline{k}]$ are managers in manufacturing. Figure 1 shows a possible equilibrium allocation.

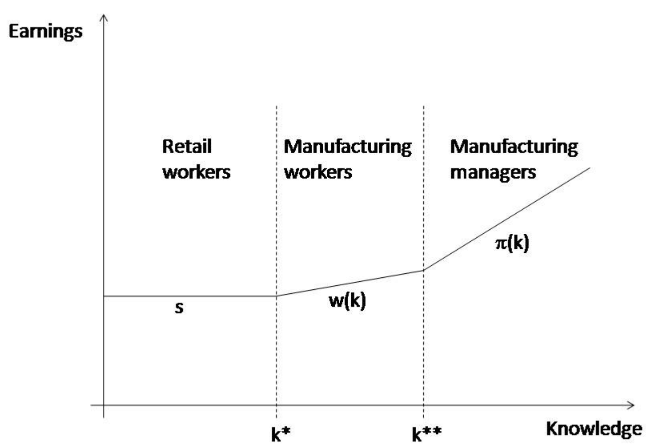

Figure 1: *An equilibrium allocation of agents*

As shown in the figure above, the agent with knowledge k_t^* must be indifferent between being a worker in the retail or the manufacturing sector, i.e. k_t^* must solve

$$s_t = w_t(k_t^*). \tag{15}$$

One-to-one matching in the manufacturing sector implies that k_t^{**} must solve the following equation :

$$\frac{1}{1-\Phi(k_t^*)}\int\limits_{k_t^*}^{k_t^{**}}\phi(k)dk=\frac{1}{2},$$

i.e., k_t^{**} is the median of the truncated distribution lying between k_t^* and $\overline{k}$. Denote $k_t^{**}=\eta(k_t^*)$. Clearly, $\frac{\partial\eta(k_t^*)}{\partial k_t^*}>0$. Henceforth, we drop the argument and write $\eta(k_t^*)$ simply as η_t.

The labor market clearing condition in manufacturing is given by

$$\int\limits_{k_t^*}^{k}\phi(s)ds=\int\limits_{\eta_t}^{m_t(k)}\phi(s)ds\ \ \forall k_t^*\leq k\leq\eta_t. \tag{16}$$

The LHS denotes the supply of workers in the interval $[k_t^*,k]$ for some k. The RHS denotes the demand for workers coming from managers in the corresponding interval $[\eta_t,m_t(k)]$. PAM implies that these two measures must be equal for *every* k. It is not enough for the above equation to be satisfied only for $k=\eta_t$, as would be the case in a standard Lucas span-of-control model without matching.

Differentiating the above equation with respect to k, we have

$$m_t'(k)=\frac{\phi(k)}{\phi(m_t(k))}. \tag{17}$$

The above differential equation is intuitive. The numerator measures the density of the workers with knowledge k, while the denominator measures the density of the corresponding managers. Loosely speaking, if the numerator is small relative to the denominator, the workers are more dispersed relative to the managers. Then measure consistency would require that workers with very different levels of knowledge be matched with managers with very similar levels of knowledge. Consequently, $m_t(k)$ increases slowly, i.e., the slope of the matching function is small.

For a given k_t^*, we can use the terminal conditions (i) $m_t(k_t^*)=k_t^{**}$ and (ii) $m_t(k_t^{**})=\overline{k}$ to solve equation (17) and obtain the matching function $m_t(k)$. Replacing the equilibrium matching function $m_t(k)$ in equation (10) yields

$$w_t'(k)=\alpha^2\beta A_t L_{Y,t}^{1-\alpha}m_t(k)^{\alpha}k^{\alpha\beta-1}.$$

The above equation gives the slope of the *equilibrium* wage schedule. Integrating, we have

$$w_t(\eta_t) - w_t(k_t^*) = \alpha^2 \beta A_t L_{Y,t}^{1-\alpha} \int\limits_{k_t^*}^{\eta_t} m_t(k)^\alpha k^{\alpha\beta-1} dk. \qquad (18)$$

Furthermore, the agent with knowledge η_t must be indifferent between being a worker or a manager in the manufacturing sector, i.e. $w_t(\eta_t) = \pi_t(\eta_t)$. Replacing $\pi_t(\eta_t)$ from equation (9) yields

$$w_t(\eta_t) = \alpha A_t L_{Y,t}^{1-\alpha} \eta_t^\alpha k_t^{*\alpha\beta} - w_t(k_t^*). \qquad (19)$$

Combining equations (18) and (19),

$$w_t(k_t^*) = \frac{\alpha A_t}{2} L_{Y,t}^{1-\alpha} [\eta_t^\alpha k_t^{*\alpha\beta} - \alpha\beta \int\limits_{k_t^*}^{\eta_t} m_t(k)^\alpha k^{\alpha\beta-1} dk]. \qquad (20)$$

Greater employment in the retail sector affects $w_t(k_t^*)$ in two ways. Firstly, a higher $L_{Y,t}$ implies fewer agents in the manufacturing sector and accordingly, fewer potential workers. Secondly, a higher $L_{Y,t}$ raises the demand for intermediate inputs, thereby making the occupation of a manager relatively more attractive and reducing the supply of workers. This tends to shift up the entire wage schedule, resulting in a higher $w_t(k_t^*)$.

Solving for the equilibrium requires solving a fixed-point problem. For a particular value of A_t, the equilibrium can be solved by carrying out the following iterations :

1. Guess a value for k_t^*, which in turn determines $L_{Y,t}$ ($\because L_{Y,t} = \Phi(k_t^*)$).
2. Solve for $m_t(k)$ (The manufacturing sector has a block recursive structure which allows us to solve for the matching function without any information on prices and earnings. See Dasgupta (2008) for more on this).
3. Use equation (6) to solve for s_t.
4. Use equation (20) to solve for $w_t(k_t^*)$.

If s_t and $w_t(k_t^*)$ thus computed are equal, then we are done. Otherwise, we just guess a different value for k_t^* and repeat steps 2-4. The following proposition shows that such iterations necessarily converge.

Proposition 1. *An equilibrium exists with thresholds k_t^* and η_t such that $s_t = w_t(k_t^*)$ and $w_t(\eta_t) = \pi_t(\eta_t)$.*

Proof In the appendix.

Finally, the growth rate of the economy is given by

$$g = ln(Y_{t+1}) - ln(Y_t). \tag{21}$$

This completes our characterization of the equilibrium.

4.2 Balanced Growth Path

Along a Balanced Growth Path (BGP), the growth rate of income is constant. As discussed in the previous section, given A_t, k_t^* is a sufficient statistic for the equilibrium. But k_t^* solves equation (15). Notice that A_t appears in the same form on both sides of this equation. Consequently, at any point in time, k_t^* is *independent* of the level of ideas in the economy. Intuitively, a higher A_t not only raises the retail sector wages, but by raising the demand for intermediate inputs, also raises manufacturing wages. Since A_t enters the production function of Y_t multiplicatively, the increase in the two wages cancel each other out, keeping the threshold unchanged. A constant threshold implies that all the variables, including λ_t, are also constant. Let this value of λ_t be given by λ^*. The growth rate is then given by equation (21),

$$\begin{aligned} g &= ln(A_{t+1}) - ln(A_t) \\ &= \lambda^* \end{aligned}$$

The economy is always on the BGP, growing at the rate λ^*. For the rest of the paper, we drop the time subscripts on all the variables. A change in the distribution affects the growth rate by changing λ^*. In the next section, we explore how a change in inequality might impact the growth rate.

5 Inequality

How does a change in inequality in the knowledge distribution affect growth? A change in $\Phi(k)$ affects production and idea generation by changing how agents match. As the nature of ideas spilling over to the retail sector change, the incentives of the agents change too. This, in turn, affects the matches. To analyze the effect of inequality on growth, we need to precisely define what we mean by an increase in inequality.

Consider two distributions $\Phi_A(k)$ and $\Phi_B(k)$, which differ only in terms of the degree of inequality. Let us assume that $\Phi_A(k)$ is more unequal than $\Phi_B(k)$ in the sense of second-order stochastic dominance. Since we want to compare countries at the same level of development, a mean-preserving spread seems like a more reasonable assumption rather than some arbitrary difference in the two distributions. The only restriction we impose on the shape of $\Phi_A(k)$ is that the distribution is downward-sloping, at least in the upper tail. If we use earnings as proxy for knowledge, evidence would seem to suggest that the knowledge distribution is indeed downward-sloping in the upper tail. For analytical tractability, we consider marginal changes in inequality. In particular, we choose t_1, t_2, s_1, s_2 as shown in the following diagram.

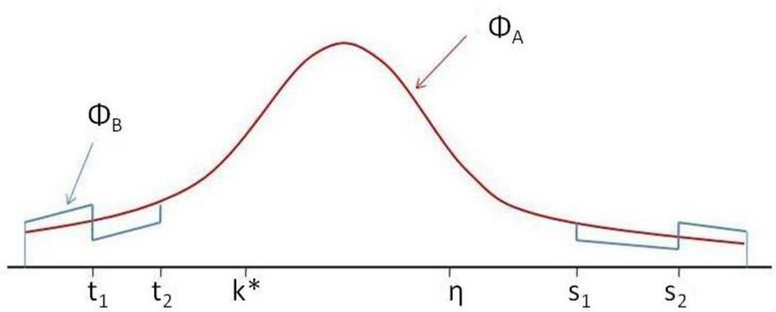

Figure 2: *Second-order stochastic dominance*

As indicated in Figure (2), the knowledge distribution in country B, $\Phi_B(k)$, is created by making the following changes to $\Phi_A(k)$:

$$\phi_B(k) = \begin{cases} \phi_A(k) + \epsilon & \text{for } k \in [\underline{k}, t_1] \\ \phi_A(k) - \epsilon & \text{for } k \in [t_1, t_2] \\ \phi_A(k) - \epsilon & \text{for } k \in [s_1, s_2] \\ \phi_A(k) + \epsilon & \text{for } k \in [s_2, \overline{k}] \end{cases} \tag{22}$$

$\Phi_B(k)$ is more unequal; it is a mean-preserving spread of $\Phi_A(k)$. The densities are modified in a way so as to keep the two distributions identical over most of their support.

The effect of higher inequality is asymmetric across the two sectors. Since knowledge does not matter for production in the retail sector, there is no direct effect. The manufacturing sector, on the other hand, is the knowledge-intensive sector. A higher inequality, as defined above, implies that there are relatively fewer middle managers under B (middle managers are the managers who run mid-sized firms, where size is measured in terms of output). Since middle managers are scarce under B, their earnings are higher relative to the more knowledgeable managers and manufacturing workers. When inequality goes up, the manufacturing workers are made relatively worse-off, forcing the least knowledgeable among them to switch to the retail sector. The following proposition shows the final effect of a change in knowledge inequality on labor allocation across the different occupations.

Proposition 2. *A rise in inequality increases the threshold between the retail and the manufacturing sector, k^*.*

Proof In the appendix.

A corollary of the above proposition is that η rises as inequality increases. The increase in inequality raises the average knowledge of the managers. PAM implies an increase in demand for the more knowledgeable workers resulting in their wages going up. Therefore, the opportunity cost of the least knowledgeable managers goes up, with some of them switching occupation.

Corollary 1. *A rise in inequality increases the knowledge of the best manufacturing worker, η.*

6 Effect of an increase in Inequality on Growth

Recall from Section 4, that the steady-state growth rate is given by λ^*, which is an aggregation of ideas. We define λ_i^* as follows :

$$\lambda_i^* = \int_{\eta_i}^{\overline{k}} \omega_i(k) I(k, m_i^{-1}(k)) \phi_i(k) dk.$$

where $i = \{A, B\}$. Ideas are generated by every manufacturing firm in the economy. $I(k, m_i^{-1}(k))$ is the idea-generation function, while $\omega_i(k)$ is a weight attached to the idea produced by each firm. Every idea has two attributes - "effectiveness" and "relevance". Since ideas have no well-defined units, we interpret $I(k, m_i^{-1}(k))$ as measuring the effectiveness of ideas. The effectiveness of an idea measures the contribution of the idea to total factor productivity, *if there were no other ideas*. There are, of course, lots of ideas in the economy. The relevance of an idea captures the relative importance of the idea with respect to other ideas in the economy. We elaborate below.

The idea generated within a firm could depend not only on the manager's knowledge but could potentially depend on the worker's knowledge too. Since we are interested in how the manager's and the worker's knowledge affect λ_i^* separately, we assume that k and $m_i^{-1}(k)$ enter $I(k, m_i^{-1}(k))$ multiplicatively. Therefore,

$$\lambda_i^* = \int_{\eta_i}^{\overline{k}} \omega_i(k) a(k) b(m_i^{-1}(k)) \phi_i(k) dk, \tag{23}$$

where $a'(k) > 0$, $b'(m_i^{-1}(k)) \geq 0$. Notice that if $a', b' > 0$, then the idea generation function exhibits complementarity. In this case, team formation is going to determine the effectiveness of ideas generated. To isolate the effects of this complementarity from other distributional effects of a change in inequality, first we consider the case where the dependence of ideas on $m^{-1}(k)$ is suppressed.

6.1 Case where $b(m_i^{-1}(k)) = \overline{b}$

By setting $b(m_i^{-1}(k))$ equal to a constant, we make idea generation depend only on the manager's knowledge. Let us set $\overline{b} = 1$. An increase in inequality, as defined in the Section 5.1, affects λ^* through possible changes in η, ω and ϕ. Recall that $Phi_B(k)$ is the more unequal distribution. From Corollary (1), we know that η increases with a rise in inequality. This allows us to write,

$$\begin{aligned} \lambda_A^* &= \int_{\eta_A}^{\overline{k}} \omega_A(k)) a(k) \phi_A(k) dk \\ &= \underbrace{\int_{\eta_A}^{\eta_B} \omega_A(k) a(k) \phi_A(k) dk}_{P} + \underbrace{\int_{\eta_B}^{\overline{k}} \omega_A(k) a(k) \phi_A(k) dk}_{Q} \end{aligned} \tag{24}$$

and

$$\lambda_B^* = \underbrace{\int_{\eta_B}^{\overline{k}} \omega_B(k)) a(k) \phi_B(k) dk}_{R} \tag{25}$$

The terms Q and R defined above capture the contribution of the managers with knowledge in $[\eta_B, \overline{k}]$. These are the managers who operate under both distributions. Furthermore, λ_A^* has an additional component, denoted by P in equation (24). P captures the contribution of managers who operate under Φ_A but switch occupation under Φ_B. Notice that the difference between Q and R arises due to a difference in the densities and the ωs. If all ideas are equally relevant, i.e., if $\omega_i(k) = 1$ for all k, the effect of inequality on growth depends only on the shape of $a(k)$. A particularly simple case is the one where $a(k)$ is equal to a constant.

Example 1 (Canonical growth model) : Setting $\omega_i(k)) = \overline{a}$ in equation (23), we have the following,

$$\begin{aligned}\lambda_i^* &= \overline{a}\int_{\eta_i}^{\overline{k}} \phi_i(k)dk \\ &= \overline{a}(1 - \Phi_i(\eta_i))\end{aligned}$$

The growth rate depends only on the fraction of managers in the economy (or the mass of managers since we have set population to 1). According to equation (22), $\Phi_A(\eta_A) = \Phi_B(\eta_A)$. Corollary (1) then implies that $\Phi_A(\eta_A) < \Phi_B(\eta_B)$. Therefore, the more unequal distribution has fewer managers and consequently, lower growth. [5] This result is similar to the one obtained in Section 2, where growth rate was a function of the size of the population.

This result might seem surprising, given that the average knowledge of the managers in the unequal distribution is higher. In this particular case, however, knowledge of the managers does not really matter for idea generation because more ideas get equal weight.[6] Although a more unequal knowledge distribution has managers who are, on average, more knowledgeable, the more knowledgeable managers actually drive out the less knowledgeable managers (in this model, the latter become workers) with the

[5] An alternative way to get the same conclusion is to assume that new ideas arrive in firms at the Poisson rate p. Ideas have the same effectiveness and hence, carry the same weight. This has been a standard assumption in the existing literature with homogeneous firms. What matters, then, is the total number of ideas. The Law of Large Numbers implies that in any period, the mass of firms which generate new ideas is pM, where M is the steady-state mass of firms (or managers). Therefore, the growth rate must be equal to pM.

[6] A possible interpretation is that each firm generates an idea that is very different from other ideas. For example, one idea could be about more efficient organization of the workspace while another could be about streamlining supply chains. A third could be about the optimal location of stores. Each of these ideas could be generated by different firms. Furthermore, each of these ideas could have similar impact on the productivity of other firms in the economy.

result that there are fewer managers in equilibrium.

Ideas may not, however, be equally relevant. Suppose managers with similar levels of knowledge also produce similar ideas.[7] In this case, how relevant an idea is could depend on the distribution of knowledge. This is due to the "fishing in the pond" effect. If we believe that there is a set of ideas that is given and invention or innovation amounts to discovery of such ideas, then generation of ideas independently by many firms would involve lots of duplications. In other words, having many firms run by very knowledgeable managers may not necessarily lead to high growth because the firms could generate ideas that are very similar.[8] Hence, ideas generated by managers who are more abundant would be less relevant. The following example illustrates our point.

Example 2 (Jacobs' view of spillovers) : Setting $\omega_i(k)) = \frac{1}{\phi_i(k)}$ in equation (23), we have the following,

$$\lambda_i^* = \int_{\eta_i}^{\overline{k}} a(k)dk.$$

The growth rate depends on the un-weighted sum of ideas generated in the economy. The distribution of managers does not matter. Any change in densities, brought about by a change in inequality, is nullified by the changing weights. The justification is as above. Firms in a particular industry generate ideas that are similar in nature. Naturally, the more firms there are in an industry, the more duplication there will be. Then one can interpret the effective ideas generated in an industry as $\frac{a(k)}{\phi_i(k)}$. In this case, $\lambda_B^* - \lambda_A^* = -\int_{\eta_A}^{\eta_B} a(k)dk$. This expression will be negative as long as $a(k) \geq 0$ for $k \in [\eta_A, \eta_B]$. Without loss of generality, let us assume that $a(k) = 1\ \forall k$. Then $\lambda^* = \overline{k} - \eta = \mu[\eta, \overline{k}]$ where μ is the Lebesgue measure. An increase in inequality, by reducing $\mu[\eta, \overline{k}]$, reduces the growth rate.

Greater diversity in the manufacturing sector leads to the creation of more ideas. This is the view forwarded by urban planner Jane Jacobs (Jacobs, 1970). According to Jacobs, ideas are transmitted through the interaction of individuals and the more diverse the industries, the better the ideas.[9] Accordingly, a city like New York would grow

[7]This is not unreasonable because each level of knowledge corresponds to a differentiated product.

[8]An additional unit of an intermediate good has a positive marginal product, even if this unit is identical to other units. The same is not true for ideas.

[9]Of course, Jacobs' view of the spillover of ideas had a spatial dimension. Jacobs was thinking of diversity of industries concentrated in a geographical area. Although there is no space in our model, one

faster because of the diversity in the local industries compared to a city like Detroit, which is dominated by one industry.[10]

In the above example, ideas were similar within industries but different across industries. It is possible, however, that ideas overlap across industries too. Imagine that an idea consists of a set of mini-ideas. Then the aggregate idea in the economy is the union of all such sets. Now, if the idea set corresponding to manager k_1 is a subset of the idea set corresponding to manager k_2, then the idea of k_1 is completely irrelevant; the idea of k_2 is, in some sense, at least as good as that of k_1. An extreme case of such overlaps in ideas is considered below. **Example 3 (Murphy, Shleifer, Vishny (1990) :**

Consider the following functional form for $\omega_i(k)$:

$$\omega_i(k)) = \begin{cases} 0 & \forall k < \overline{k} \\ 1 & \text{if } k = \overline{k} \end{cases}$$

Given these weights, the growth rate of the economy is given by

$$\lambda_i^* = a(\overline{k})\phi_i(\overline{k})dk.$$

Although every manufacturing firm in the economy produces new ideas, only the best idea spills over to the retail sector. If we assume that all (or most) the ideas generated in the manufacturing sector pertain to making the production of the final good more cost-effective, then the idea that leads to the greatest reduction in cost are adopted by the retail sector; the *best practice* in period t becomes the standard in period $t+1$. If we believe that the most effective ideas are generated by the most productive firms in the economy (i.e. $a' > 0$), the knowledge of the managers of those firm becomes important for growth.

This is similar to Murphy et al. (1991), where the growth rate is determined by the knowledge of the most knowledgeable entrepreneur in the economy. In their model, if the parameters are such that the best agents become entrepreneurs, the growth rate would correspond to the knowledge of the most knowledgeable agent in the economy. In our model, as in Murphy et al. (1991), an increase in inequality does not change the

can think of the manufacturing industries as being located in one "industrial hub".

[10]This is in contrast to the MAR view (named after its proponents - Alfred Marshall, Kenneth Arrow and Paul Romer) which states that the more concentrated employment is in *one* industry, the greater are the knowledge spillovers.

identity of the best manager in the economy, because comparative advantage implies that the most knowledgeable agents always select into being managers. But, although the identity of the most knowledgeable manager remains unchanged, higher inequality implies that there are more of them. Consequently, a change in inequality raises the growth rate.

The examples presented above suggest that when the effectiveness of ideas depends only on the knowledge of the manager, the weight attached to the ideas will determine whether an increase in inequality causes the growth rate to fall (as in Example 2), or remain unchanged, or rise (Example 3). Going back to equations (24) and (25), we see that the increase in inequality affects growth through two effects - (i) a *distribution effect*, captured by $Q - R$ and (ii) an *allocation effect*, captured by P. When $b(m^{-1}(k)) = 1$, a change in the knowledge distribution has no impact on the effectiveness of ideas generated by any manager. The densities, on the other hand, change. In particular, conditional on being truncated at η_B, Φ_B first-order stochastic dominates Φ_A. Therefore, if the weights are held constant, we would have $Q - R < 0$, that is the distribution effect of an increase in inequality is positive.[11] At the same time, since a rise in inequality leads the least-knowledgeable managers to switch occupations, some ideas are lost. Therefore, the composition effect is always negative, provided that the weights attached to the ideas generated by the less knowledgeable managers are positive. Hence, the final impact will depend on which effect dominates. If the ωs are such that they nullify the effects of a change in the distribution, the composition effect dominates, as in Example 2.

If ω is constant across distributions, but non-decreasing in knowledge, the effect of $\omega(k)$ and $a(k)$ are not separately identifiable. Then we can simply define $\hat{a}(k) = \omega(k)a(k)$, where $\hat{a} \geq 0$. Observe, that the marginal increase in $R - Q$ will be greater, the more convex $\hat{a}$ is for $k \in [\eta_B, \overline{k}]$. At the same time, P will be small if $\hat{a}$ is small over the relevant range. Combining the two observations, we can infer that if $\hat{a}$ is sufficiently convex, the *distribution effect* dominates and the growth rate rises following an increase in inequality. An extreme form of this convexity is illustrated in Example 3. If, however, $\hat{a}$ is sufficiently flat, the *allocation effect* dominates. This is shown in Example 1, where $\hat{a} = 1$. We can summarize the above discussion in the following proposition :

[11] This follows from the definition of first-order stochastic dominance

Proposition 3. *When $b(m^{-1}(k)) = 1$, an increase in inequality leads to higher growth if $\omega(k)a(k)$ is increasing and sufficiently convex.*

A convex $\omega(k)a(k)$ function implies that ideas generated by more knowledgeable managers are either more effective, or more important, or both. Therefore, although an increase in inequality reduces the mass of managers, the loss of ideas is small compared to the improvement in ideas coming from managers who are, on average, more knowledgeable than before.

6.2 Complementarity in idea generation

Next we consider the case where $b(m_i^{-1}(k))$ is no longer a constant. For simplicity, let us assume that $\omega_i(k) = \omega(k)$, where $\omega(k)$ is non-decreasing. Hence, as in the previous subsection, we can define $\hat{a}(k) = \omega(k)a(k)$. In this case, a change in the distribution not only affects $\phi_i(k)$ but also $m_i^{-1}(k)$. In order to analyze the effect of a mean-preserving spread on matching, we state the following lemma.

Lemma 3. *With an increase in inequality, the match of the manager with knowledge η_B worsens, i.e. $k_B^* \leq m_A^{-1}(\eta_B)$.*

Proof In the appendix.

We know that $m_A^{-1}(\overline{k}) < m_B^{-1}(\overline{k})$. With an increase in inequality, the inverse matching function becomes steeper for lower values of k but eventually flattens out. Combined with Lemma (3), this implies that there is a set $E \subset [\eta_B, \overline{k}]$, such that $m_A^{-1}(k) > m_B^{-1}(k)$ for $k \in E$ and $m_A^{-1}(k) < m_B^{-1}(k)$ for $k \in E^c$. This is illustrated in Figure (3).

The above figure suggests that the *distribution effect* defined earlier is no longer unambiguously positive. Following a rise in inequality, some of the surviving managers have better matches, but there are some managers who are matched with less knowledgeable workers than before. Of course, how this affects idea generation in each firm and consequently in the aggregate will depend on $b(.)$. If b' is small (idea generation is not very sensitive to the knowledge of the worker), the *distribution effect* continues to be positive. In fact, as long as $b(.)$ is sufficiently convex, the *distribution effect* is not only positive, but it also dominates the negative *allocation effect*. The intuition is very similar to the one offered in the previous subsection. A convex $b(.)$ re-enforces the effect of a shift in the mass towards more knowledgeable managers. The improvement in

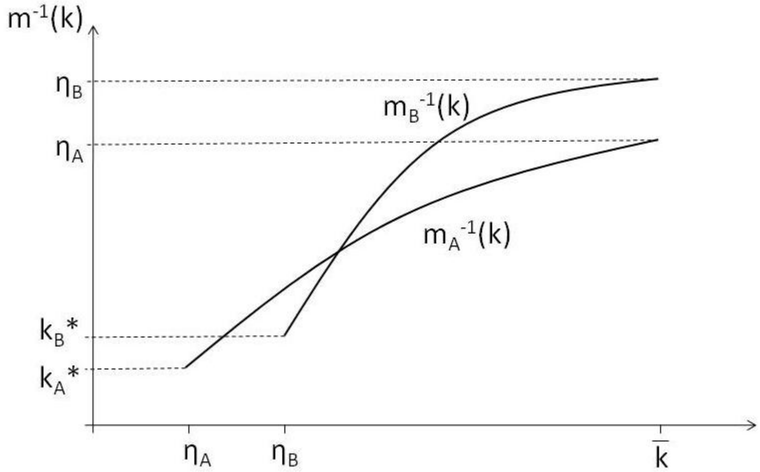

Figure 3: *The $m^{-1}(k)$ function under the two distributions*

ideas coming from the more knowledgeable managers is much bigger compared to the decrease in effectiveness of ideas arising from the worsening of some matches. This allows us to state the following proposition.

Proposition 4. *If $\omega(k)$ is independent across distributions, an increase in inequality will raise the growth rate if either (i) $\hat{a}(k)$ is sufficiently convex and $b(k)$ is not too concave, or (ii) $b(k)$ is sufficiently convex and $\hat{a}(k)$ is not too concave.*

6.3 Discussion

In our model, although an increase in inequality keeps the mean of the entire knowledge distribution unchanged, the mean knowledge of the managers goes up. If the production function for ideas increases sufficiently quickly with knowledge, fatter tails would lead to more (effective) ideas by the surviving managers. This is not only because marginal effectiveness of ideas goes up with the manager's knowledge, but also because the better managers are matched with better workers. For an unchanged knowledge support for the managers, this results in more effective ideas on the aggregate. But a change in inequality affects the allocation of agents across occupations, which in turn means fewer ideas. The effect on the growth rate depends on whether having relatively more knowledgeable managers can compensate for having fewer managers.

It should be noted, that the mechanism (through which inequality affects growth) that we have highlighted here has a fundamental difference with what has been proposed in the existing literature on growth and inequality. In the existing literature, inequality affects growth primarily through its impact on the poorer sections of the society. For example, in the presence of credit market imperfections, inequality could affect investment by limiting the opportunities of poor individuals. In other words, the lower tail of the income distribution would matter for growth. On the contrary, in our model, growth depends primarily on what is happening at the upper tail of the distribution. That is because, growth is driven by ideas which are created in the knowledge-intensive sector of the economy. The less knowledgeable agents (usually the poor) in the economy sort into the retail sector which benefits from the spillover but does not generate any. A change in the distribution of agents working in the retail sector has no direct impact on growth, because it does not change the way ideas are created.

7 Conclusion

Neo-Classical growth theory has come a long way since the seminal work of Robert Solow (Solow, 1956). Economists have studied the relation between the rate of growth of a country and a myriad phenomena, including innovation, migration, corruption and pollution, to name just a few. Although it has received some attention, the interaction between inequality and growth is far from being well-understood (See Helpman, 2004, Chapter 6 for a non-technical summary of the relevant issues).

The recent empirical literature on growth and inequality, surveyed well by Benabou (1996), finds a clear negative correlation between income inequality and GDP growth across countries. The direction of causality is, however, much less clear. In this paper, we take a position on causality and proceed to analyze the nature of this causality, being fully aware of the fact that there could be reverse causality. We take the distribution of knowledge in the population as given and go on to examine the implications of a mean-preserving spread of this distribution on the the growth rate of the economy. Since the knowledge distribution of a country maps into an income distribution, our model generates an indirect link between income distribution and the growth rate of a country. Of course, the knowledge distribution itself is, to a certain extent, determined by the wealth distribution of a country. This is even more true when credit markets are imperfect.[12] Our focus on the knowledge distribution, rather than the wealth distribution,

[12]Borrowing against future human capital is difficult due to the embodied nature of human capital REF

stems from our belief that the effectiveness of productive ideas ultimately depends on the knowledge of the agents engaged in the generation of those ideas. Then a change in the distribution affects the growth rate, both by changing the occupational choice of agents and the matching between agents across firms.

The process through which ideas are created is treated as a black box in our model. Since our aim was to understand the relation between knowledge inequality and the aggregate growth rate, we have abstracted from explicitly modeling the mechanism through which managers and workers get together in organizations to create new ideas. The matches, in our model, are determined solely on the basis of the production relation. A more complete treatment would require micro-founding the idea generation function. In such a model, the manager would be deciding not only how much output to produce but also how much to invest in research and development. The manager would be choosing how to allocate agents across tasks (production and idea-generation) in order to maximize some profit function. In this respect, the framework developed in Garicano (2000) could be useful. We leave the development of such a model for future work.

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Appendix

Proof of Lemma 1 : Suppose that, on the contrary, we have the following allocation: $[k_1, k_2] \in retail$, $[k_2, k_3] \in manufacturing$, $[k_3, k_4] \in retail$. Then both k_2 and k_3 must be earning $w_{Y,t}$, since both of them are indifferent between being in retail or manufacturing. If k_2 chooses to be a worker, then $w_t(k_2) = s_t$. Otherwise, $\pi_t(k_2) = s_t$. Either way, since $w_t'(k) > 0$ and $\pi_t'(k) > 0$, k_3 must be earning more than k_2 and consequently, more than s_t. A contradiction.

To show that the least knowledgeable agents are in retail, suppose that, on the contrary, $[\underline{k}, k_1] \in manufacturing$, $[k_1, k_2] \in retail$. Without loss of generality, suppose that k_1 is a worker in the manufacturing sector. Consider the agent with knowledge $k_1 + \epsilon$, where $\epsilon \longrightarrow 0$. Since he is in retail, he must be earning s_t. However, if he moves to sector I as a worker, he can earn $w_t(k_1 + \epsilon) > w_t(k_1) = s_t$, since $w_t'(k_1) > 0$. Therefore, this allocation can not be an equilibrium.

Proof of Proposition 1 : Define $\zeta(k_t^*) = w_t(k_t^*) - s_t$, where s_t is given by equation (6). Differentiating $w_t(k_t^*)$ with respect to k_t^*,

$$\begin{aligned}\frac{\partial w_t(k_t^*)}{\partial k_t^*} &= \frac{\alpha A_t}{2} L_{Y,t}^{1-\alpha}[\alpha \eta_t^{\alpha-1} \eta_t' k_t^{*\alpha\beta} + 2\alpha\beta \eta_t^{\alpha} k_t^{*\alpha\beta-1} - \alpha\beta \overline{k}^{\alpha} \eta_t^{\alpha\beta-1} \eta_t'] \\ &+ \frac{\alpha A_t}{2} \frac{\partial L_{Y,t}^{1-\alpha}}{\partial k_t^*}[\eta_t^{\alpha} k_t^{*\alpha\beta} - \alpha\beta \int_{k_t^*}^{\eta_t} m_t(k)^{\alpha} k^{\alpha\beta-1} dk]\end{aligned}$$

Now, $L_{Y,t} = \Phi(k_t^*)$. So, $\frac{\partial L_{Y,t}^{1-\alpha}}{\partial k_t^*} > 0$. Therefore, the above expression will be positive if $\alpha\beta(2\eta_t^{\alpha} k_t^{*\alpha\beta-1} - \overline{k}^{\alpha} \eta_t^{\alpha\beta-1} \eta_t') > 0$, i.e. if

$$2\eta_t^{\alpha(1-\beta)} k_t^{*\alpha\beta} > \beta \overline{k}^{\alpha}$$

Now, $2\eta_t^{\alpha(1-\beta)} k_t^{*\alpha\beta} > 2\underline{k}^{\alpha(1-\beta)} \underline{k}^{\alpha\beta} = 2\underline{k}^{\alpha}$. Hence, a sufficient condition for $\frac{\partial w_t(k_t^*)}{\partial k_t^*}$ to be positive is $2\underline{k} > \beta^{\frac{1}{\alpha}} \overline{k}$, which is ensured by Assumption 1. It is straightforward to show that $\frac{\partial s_t}{\partial k_t^*} < 0$. Therefore, $\frac{\partial \zeta(k_t^*)}{\partial k_t^*} > 0$. Furthermore, when $k_t^* = \underline{k}$, $L_{Y,t} = 0$. Therefore, $\zeta(k_t^*)|_{k_t^* = \underline{k}} = -\infty$. And when $k_t^* = \overline{k}$, $\zeta(k_t^*)|_{k_t^* = \overline{k}} = \frac{\alpha A_t}{2} \overline{k}^{\alpha(1+\beta)} > 0$. Applying the intermediate value theorem, $\exists k_t^*$ such that $\zeta(k_t^*) = 0$.

Proof of Proposition 2 : Suppose, on the contrary, the threshold between the retail and the manufacturing sector remains unchanged following a rise in inequality, i.e. $k_A^* = k_B^* = k^*$. Then, $\Phi_A(k) = \Phi_B(k)$. Since the density between k_A^* and η_A is the same for the two distributions, we must also have $\eta_A = \eta_B = \eta$. From equation (17), we have $m_B'(k^*) = \frac{\phi_B(k^*)}{\phi_B(\eta)} = \frac{\phi_A(k^*)}{\phi_A(\eta)} = m_A^*(k^*)$. In fact, this relation must hold for all $k \in [k^*, m_A^{-1}(s_1)]$, i.e.

$$m'_B(k) = m'_A(k) \quad \forall k \in [k^*, m_A^{-1}(s_1)]$$

It follows that,

$$m_B(k) = m_A(k) \quad \forall k \in [k^*, m_A^{-1}(s_1)] \tag{i}$$

We also know that, $\phi_B(s_1) < \phi_A(s_1)$. Therefore, at $m_B^{-1}(s_1) = m_A^{-1}(s_1)$,

$$m'_B(m_B^{-1}(s_1)) = \frac{\phi_B(m_B^{-1}(s_1))}{\phi_B(s_1)} > \frac{\phi_A(m_A^{-1}(s_1))}{\phi_A(s_1)} = m'_A(m_A^{-1}(s_1))$$

In fact, $\forall s \; s.t. \; m_B(k) \in [s_1, s_2]$

$$m'_B(k) = \frac{\phi_B(k)}{\phi_B(m_B(k))} > \frac{\phi_A(k)}{\phi_A(m_B(k))} > \frac{\phi_A(k)}{\phi_A(m_A(k))} = m'_A(k)$$

where the first inequality follows from definition and the second inequality follows from the assumption that density is decreasing in knowledge. It follows that

$$m_B(k) > m_A(k) \quad \forall k \in [m_B^{-1}(s_1), m_B^{-1}(s_2)] \tag{ii}$$

By assumption, $m_A(\eta) = m_B(\eta) = \overline{k}$. Furthermore, $m'_B(\eta) = \frac{\phi_B(\eta)}{\phi_B(\overline{k})} < \frac{\phi_A(\eta)}{\phi_A(\overline{k})} = m'_A(\eta)$. Therefore, in an $\epsilon - ball$ around η, $m_B(k) > m_A(k)$. In fact, using a similar logic as above, it can be shown that

$$m_B(k) > m_A(k) \quad \forall k \in [m_B^{-1}(s_2), \eta] \tag{iii}$$

Combining equations (i), (ii) and (iii), we see that $\exists S \in [k^*, \eta]$, s.t.

$$\begin{array}{ll} m_B(k) > m_A(k) & \forall k \in S \\ m_B(k) = m_A(k) & \text{otherwise} \end{array}$$

Using the result that matches weakly improve with inequality in equation (6), we see that $w(k^*)$ under Φ_B is less than that under Φ_A. We also know that $s = (1-\alpha)AL_Y^{-\alpha}(\int_\eta^{\overline{k}} x(k)^\alpha d\Phi(k))$. But,

$$\int_\eta^{\overline{k}} x(k)^\alpha d\Phi(k) = \int_{k^*}^{\eta} x(m(k))^\alpha d\Phi(k)$$

i.e. output produced can be attributed either to the worker or the manager. Since the difference between Φ_A and Φ_B occurs only in the tails, $\phi_A(k) = \phi_B(k)$. Therefore,

$$
\begin{aligned}
\int_{k^*}^{\eta} x(m_A(k))^\alpha d\Phi_A(k) &= \int_{k^*}^{\eta} m_A(k)^\alpha k^{\alpha\beta} d\Phi_A(k) \\
&= \int_{k^*}^{\eta} m_A(k)^\alpha k^{\alpha\beta} d\Phi_B(k) \\
&= \int_{k^*}^{m_A^{-1}(s_1)} m_A(k)^\alpha k^{\alpha\beta} d\Phi_B(k) + \int_{m_A^{-1}(s_1)}^{\eta} m_A(k)^\alpha k^{\alpha\beta} d\Phi_B(k)
\end{aligned}
$$

Now,

$$
\int_{k^*}^{m_A^{-1}(s_1)} m_A(k)^\alpha k^{\alpha\beta} d\Phi_B(k) = \int_{k^*}^{m_A^{-1}(s_1)} m_B(k)^\alpha k^{\alpha\beta} d\Phi_B(k)
$$

while

$$
\int_{m_A^{-1}(s_1)}^{\eta} m_A(k)^\alpha k^{\alpha\beta} d\Phi_B(k) < \int_{m_A^{-1}(s_1)}^{\eta} m_B(k)^\alpha k^{\alpha\beta} d\Phi_B(k)
$$

Hence,

$$
\begin{aligned}
\int_{k^*}^{\eta} x(m_A(k))^\alpha d\Phi_A(k) &< \int_{k^*}^{m_A^{-1}(s_1)} m_B(k)^\alpha k^{\alpha\beta} d\Phi_B(k) + \int_{m_A^{-1}(s_1)}^{\eta} m_B(k)^\alpha k^{\alpha\beta} d\Phi_B(k) \\
&= \int_{k^*}^{\eta} x(m_B(k))^\alpha d\Phi_B(k)
\end{aligned}
$$

It follows that s increases as inequality goes up. Thus, as we move from Φ_A to Φ_B, if we hold the threshold k^* constant, $w(k^*)$ goes down while s goes up. Therefore, at $k = k^*$, $w(k^*) < s$. This can not be an equilibrium. Hence, the thresholds must change. In particular, under Φ_B, there is an excess demand for workers in the retail sector if the threshold is at k_A^*. Hence, workers move from manufacturing to the retail sector, resulting in k^* moving to the right. The conditions on the parameters ensure that $s - w(k^*)$ falls as k^* rises. In equilibrium, $k_B^* > k_A^*$.

Proof of Lemma 3 : From Proposition (2), we know that $\eta_B > \eta_A$. By definition, $\Phi_A(\eta_B) - \Phi_A(\eta_A) = \Phi_A(m_A^{-1}(\eta_B)) - \Phi_A(k_A^*)$. Also, $\Phi_A(m_A^{-1}(\eta_B)) - \Phi_A(k_A^*) = \Phi_B(m_A^{-1}(\eta_B)) - \Phi_B(k_A^*)$. Let us consider the following two scenarios. In scenario one, s_1 in Figure (2) is greater than η_B. Then, we must have $\Phi_B(\eta_B) - \Phi_B(\eta_A) = \Phi_A(\eta_B) - \Phi_A(\eta_A)$. In scenario two, s_1 is less than η_B. In this case, $\Phi_B(\eta_B) - \Phi_B(\eta_A) < \Phi_A(\eta_B) - \Phi_A(\eta_A)$. Combining the two, we have $\Phi_B(\eta_B) - \Phi_B(\eta_A) \leq \Phi_A(\eta_B) - \Phi_A(\eta_A)$. Therefore, $\Phi_B(\eta_B) - \Phi_B(\eta_A) \leq \Phi_B(m_A^{-1}(\eta_B)) - \Phi_B(k_A^*)$. It follows that $m_B^{-1}(\eta_B) = k_B^* \leq m_A^{-1}(\eta_B)$.