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Emission Tax or Standard? The Role of Productivity Dispersion

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Abstract

We use a general equilibrium model to address the question whether a regulatory emission standard or an emission tax yields higher societal welfare for any given target on aggregate emission. The key feature of the model is that the plants are heterogeneous in productivity and monopolistically competitive in the production of dirty-goods whose by-product is emission. We find that the standard yields higher welfare than the tax if and only if productivity dispersion is small and the monopoly power in the dirty-goods sector is strong. Thus, productivity dispersion should be important in the evaluation of market-based environmental policies relative to non-market policies.

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1 Introduction

The economics of climate change has attracted increasing attention in academic and public debates, especially after the publication of the Stern (2006) Review. There is wide disagreement on how stringent the emission target should be in order for the world economy to avoid a large loss that may be caused by climate change. In this paper, we address a different question: Given an emission target, is it more desirable to achieve it by imposing an emission tax or a (non-tradable) emission standard? The predominant view is that market-based instruments, such as emission taxes and tradable emission quotas, are more efficient than nontradable regulatory standards. We show that the answer is far from clear. In particular, we demonstrate that productivity dispersion in the economy and its interaction with abatement choices are important factors determining whether an emission tax or standard is more desirable for achieving an emission target.

Regulatory standards and market-based instruments are two common forms of environmental policy. A regulatory standard specifies the actions that a firm or individual must undertake to achieve environmental objectives. Firms and individuals cannot meet such a standard by trading with others in the market. In contrast, instruments like emission taxes and tradable emission quotas work though the price system. Traditionally, most environmental policies had been in the form of regulatory standards. Starting from the 1970s, however, opinions have shifted to favor market-based instruments. For example, the Stern Review (part IV, p310) states that "a common price signal is needed across countries and sectors to ensure that emission reductions are delivered in the most cost-effective way...... [Both] taxes and tradable quotas have the potential to deliver emission reductions efficiently." Reflecting this assessment, the main debate on policy instruments has now shifted onto which market instruments should be used and, in particular, onto the choice between emission taxes and tradable emission quotas.

Our view is that this shift is premature.[1] Most models that demonstrate the advantage of market instruments relative to regulatory standards have omitted productivity dispersion across firms or plants. There are good reasons why productivity dispersion should play a central role in the evaluation of environmental policies. First, productivity dispersion

[1] Freeman and Kolstad (2007) documents the past twenty years of experience in using market-based instruments, in comparison with command-and-control policies such as regulatory standards.

has direct implications on an economy's efficiency in production and, hence, on policy evaluation (see more discussions below). Second, productivity dispersion has been shown to be important for accounting for trade flows and for explaining how trade policies affect firms' trade decisions (e.g. Eaton and Kortum, 2002, and Melitz, 2003). It is natural to expect that productivity dispersion can play a similarly important role in determining how firms' production and abatement choices respond to environmental policies.

To uncover the importance of productivity dispersion, we introduce emission and environmental policies into a model of monopolistic competition that shares some elements with the Eaton-Kortum-Melitz model. There are two types of consumption goods: a clean good and a dirty-goods composite. The composite aggregates a continuum of varieties that are imperfect substitutes. Each variety is produced by one plant, and the sector of dirty goods is monopolistically competitive. The plants are heterogeneous in productivity. The production of a dirty good generates emission as a by-product, and aggregate emission reduces the households' utility. A plant's emission increases with its input and, hence, with its output. However, a more productive plant has a lower emission intensity in the sense that its emission-output ratio is lower. We consider two environmental policies. One is an emission-intensity standard that requires a plant's emission-output ratio not to exceed a given level, and the other is an emission tax imposed on each unit of emission.[2,3] As a response to the policies, a plant can put resources into emission abatement.

Given any arbitrary target on the aggregate level of emission, we compare the equilibria under the two policies. To isolate the role of productivity dispersion, we examine first the environment where the plants do not have access to an abatement technology and then the environment where the plants have such access. In each environment, we determine equilibrium quantities of goods produced and the aggregate level of abatement under each of the two policies. We rank the two policies according to the representative household's intertemporal utility (i.e., the welfare function).

[2] The regulatory standard examined here is a performance standard instead of a technology standard (see IPCC, 2007). A technology standard mandates specific pollution abatement technologies or production methods, such as specific CO_2 capture and storage methods on a power plant. A performance standard mandates specific environmental outcomes per unit of product (or input) such as a certain number of grams of CO_2 per kilowatt-hour of electricity generated.

[3] We do not examine tradable emission permits or quotas because they are equivalent to the emission tax in our model. In fact, the price of such tradable permits is equal to the tax rate.

In the presence of productivity dispersion, a critical difference between the tax and the standard is in how they affect the average productivity in the economy. The tax changes all plants' marginal costs by the same proportion since it is proportional to a plant's emission. As a result, the tax does not change the average productivity in the economy. In contrast, the standard imposes a more stringent constraint on the plants with low productivity than on the plants with high productivity, because high-productivity plants have a lower ratio of emission to output which allows them to meet the standard more easily. This uneven constraint induces productive resources to move from the plants with low productivity to the plants with high productivity. A natural conjecture is that this movement increases the average productivity in the economy, which enables the standard to achieve higher output and welfare than the tax does.

Surprisingly, this conjecture is not supported in the model. Instead, the opposite is true in the environment where the plants do not have access to an abatement technology. In this environment, the tax induces higher average productivity in the economy and a higher quantity of the dirty-goods composite than the standard does. In addition, more varieties of the dirty goods are produced in the economy under the tax, while the same quantity of the clean good is produced under the two policies. Thus, the tax is unambiguously better than the standard in terms of welfare when there is no abatement. This surprising result arises because the plants produce varieties that are not perfectly substitutable in the dirty-goods composite. A high-productivity plant has higher output but contributes less at the margin to the dirty-goods composite, because the marginal contribution of the output of each variety to the composite is diminishing. The average productivity in the economy is a weighted sum of the plants' productivity, where the weights for a plant are the variety's marginal contribution to the composite. This "value-weighted" marginal productivity must be equalized across all operating plants, as productive resources are perfectly mobile across the plants. When the standard induces resources to move from low-productivity plants to high-productivity plants, it increases output and, hence, reduces the value-weighted productivity in high-productivity plants. As a result, the average productivity is lower under the standard than under the tax.

Introducing an abatement technology changes the results significantly. In particular, when there is no productivity dispersion among the plants, the abatement technology enables the standard to induce a higher quantity of the dirty-goods composite and a lower quantity of the clean good than the tax does. When the monopoly power of each variety is sufficiently strong in the sense that the elasticity of substitution between the varieties is sufficiently low, the quantity of the dirty-goods composite is sufficiently higher under the standard than under the tax. In this case, the standard yields higher welfare than the tax. More generally, the standard induces a higher quantity of the dirty-goods composite and higher welfare than the tax if and only if productivity dispersion is sufficiently small and the monopoly power of each variety is sufficiently strong.

The reason why the abatement technology enables the standard to yield higher welfare than the tax is not that it reverses the ranking in the average productivity between the two policies; in fact, the tax still induces higher average productivity than the standard does. Rather, the reason is that with the abatement technology (and when productivity dispersion is small), the standard creates less upward pressure on the price index of the dirty-goods composite than the tax does which, in turn, induces higher demand for and higher output of the dirty goods. We will explain the underlying mechanism in subsection 4.3. There, we will also explain how the abatement choice interacts with productivity dispersion differently under the two policies.

As mentioned earlier, our model has similar elements to the recent trade literature that emphasizes productivity dispersion among plants, e.g., Eaton and Kortum (2002) and Melitz (2003). But our model is not a trade model - it has only one country. Environmental issues are clearly different from trade issues. Even when tariffs and quotas are introduced into the Eaton-Kortum-Melitz model, the analysis will differ significantly from ours. First, emission generates a negative externality to the society which does not have an apparent counterpart in the trade literature. Second, emission is a by-product of a firm's production process. An emission policy is imposed on this by-product rather than on the regular goods as are tariffs and trade quotas. Third, the producers can use an abatement technology to reduce emission without affecting the output of the regular goods, and different environmental policies affect the abatement choice differently. With all these differences, it can be misleading to compare our results with those in the trade literature on tariffs versus quotas (e.g., Young and Anderson, 1980).

More closely related to our paper are the applications to environmental issues of the price-versus-quantity literature originated from the seminal paper of Weitzman (1974).[4] See, for example, Pizer (2002), Hepburn (2006) and Mandell (2008). This literature addresses the question whether it is more desirable to control the price or the quantity of emission. The main result is that the answer depends on the slope of and the uncertainty in the marginal-cost curve of abatement relative to the marginal-benefit curve of emission reduction. If the marginal-cost curve is steeper than the marginal-benefit curve and is exposed to higher uncertainty, then it is more beneficial to control the price of emission and let the quantity of emission vary to reflect the large shift in the marginal cost. On the other hand, if the marginal-benefit curve is steeper and exposed to higher uncertainty, then it is more beneficial to control the quantity and let the price vary to reflect the large shift in the benefit curve.

The main issue in our paper is not price-versus-quantity. For both the tax and the standard, we fix the total quantity of emission at any arbitrary target. Moreover, the price-versus-quantity literature evaluates one group of market instruments (price instruments) against another group of market instruments (e.g., tradable quotas). In contrast, we evaluate a market instrument (the tax) against a nontradable instrument (the standard). Finally, the main player in our model is different from that in the price-versus-quantity literature. While uncertainty is the main character in this literature, we abstract from aggregate uncertainty and focus, instead, on productivity dispersion among the plants and its interaction with abatement choices.

We also abstract from many other factors that may be important for evaluating the relative merit of an emission tax and a standard. Examples of these factors include emission monitoring when producers have private information (see Montero, 2005) and practical difficulties in implementing an environmental policy (see Stern, 2006). Such abstraction is intended to focus on productivity dispersion, whose role is not well understood in the literature in the evaluation of environmental policies.

[4] We do not survey this literature here. For some examples, see Laffont (1977) for incorporating subjective uncertainty, Yohe (1978) for examining additional sources of uncertainty and informational difficulty within a regulated heirarchy, and Kelly (2005) for a general-equilibrium framework.

2 The Model Environment

Consider a one-period economy that is populated by a unit measure of households. Each household is endowed with one unit of resource that can be supplied as the input in productive plants and receives dividends from a diversified portfolio of the plants. A household's utility function is $u(c, Q, M)$, where c is consumption of a clean good, Q consumption of a composite of the dirty-goods varieties, and M the aggregate level of emission per household. Define the marginal rate of substitution between the dirty-goods composite and the clean good as

$$R(c, Q, M) = \frac{u_2(c, Q, M)}{u_1(c, Q, M)},$$

where the subscripts of u indicate partial derivatives.

Assumption 1 *(i)* $u_1 > 0$, $u_2 > 0$, $u_{11} < 0$ *and* $u_{22} < 0$; *(ii)* $u_3 < 0$; *(iii)* $R_3(c, Q, M) \leq 0$.

Part (i) of the assumption is standard. Part (ii) says that emission generates a negative externality on households. Part (iii) says that emission (weakly) increases a household's desire for the clean good relative to the dirty goods.

The clean good is homogeneous and its production does not generate emission. The technology for producing the clean good is wl_c, where l_c is the input and $w > 0$ is a constant. For the sake of simplicity, we lump all types of inputs into one so that the production function is linear in this input. There is perfect competition in the clean-good sector, and so the price of the resource in terms of the clean good is equal to the constant w. Throughout this paper, we will use the clean good as the numeraire.[5]

The dirty goods have varieties in a continuum, $[0, 1]$. Each variety is produced by at most one plant, and the dirty-goods sector is monopolistically competitive. At the beginning of the period, all plants are identical. A plant can choose at most one variety to produce. After the choice, the plant draws a productivity level x from the distribution (cdf) $G(.)$, with a support $[\underline{x}, \infty)$. We will refer to a plant with productivity x as plant x. Note that different plants that draw the same x produce different varieties. After the

[5] Including the clean good in the model is convenient because we can model all fixed costs and taxes in terms of the clean good, thereby simplifying the accounting in the model.

realization of x, a plant can choose whether or not to operate. As we will see, whether a plant chooses to be operative can depend on the environmental policy in place and, hence, the set of dirty goods produced in the economy may or may not be the same as the interval $[0, 1]$. If plant x operates, its output is

$$q(x) = e^x l(x),$$

where $l(x)$ is the plant's input. Because the plant is the only one that produces the variety, it does not take the price of the variety as given; instead, it takes as given the demand curve for the variety.

The dirty goods appear in a household's utility through the following composite:

$$Q = \left[\int_{i \in I} (q_i)^{\frac{\varepsilon - 1}{\varepsilon}} \ di \right]^{\frac{\varepsilon}{\varepsilon - 1}}, \quad \varepsilon > 1, \tag{1}$$

where q_i is the amount of consumption of variety i, the set I contains all the varieties of the dirty goods produced in the period, and ε is the elasticity of substitution between the varieties. The assumption $\varepsilon > 1$ is standard in the literature.

Production of a dirty good generates emission as a by-product. In the baseline model, the amount of emission of plant x obeys the following "emission process":

$$m(x) = b \ l(x), \quad b > 0. \tag{2}$$

In an extended model, we will introduce an abatement technology which a plant can use to reduce emission. Let us express the level of abatement, a, in terms of the input so that the cost of the abatement is wa. By choosing a level of abatement, $a(x)$, a plant x can change its emission level to

$$m(x) = b \ l(x) \left(1 + \frac{a(x)}{l(x)} \right)^{-\frac{1}{\gamma}}, \quad \gamma > 0. \tag{3}$$

The effectiveness of the abatement technology can be measured by $1/\gamma$. In the limit $\gamma \to 0$, even a tiny amount of abatement can eliminate the plant's emission; in the opposite limit $\gamma \to \infty$, abatement does not reduce emission.

This emission process captures two general and realistic features. First, in the absence of abatement (i.e., when $a(x) = 0$), a high-x plant has a lower emission-output ratio,

$m(x)/q(x)$, than a low-x plant although a high-x plant has a higher level of emission due to its higher output. The use of this feature will become apparent below when we describe the environmental policies. Second, the emission-input ratio, $m(x)/l(x)$, is strictly decreasing and convex in abatement. The decreasing feature is necessary for abatement to be useful, and convexity ensures that a plant's optimal level of abatement is unique. Although our main results can continue to hold with more general emission processes that have these features, the simple form above maintains tractability.[6]

We evaluate two environmental policies: (i) an emission tax τ that requires a plant to pay τ (in terms of the clean good) for every unit of emission, and (ii) an emission standard s that requires a plant's emission-output ratio not to exceed s. The revenue from the tax is rebated to the households through lump-sum transfers.[7] Note that the emission standard is on a plant's emission-output ratio, rather than on the plant's level of emission. This specification makes sense when the plants are heterogeneous in productivity. If an emission standard requires a plant's emission not to exceed a certain level, instead, then a more productive plant will be more constrained by the standard than a less productive plant is. Similarly, the presence of heterogeneous productivity is the reason why we assume that a plant's emission-output ratio is a decreasing function of x. If a plant's emission-output ratio is an increasing function of the plant's productivity, instead, then an emission standard puts an upper bound on the productivity level below which a plant can operate. Since an emission standard in this case prevents more productive plants from operating, it does not seem to be a good policy.

Another commonly debated policy is one that requires a plant to obtain an emission permit for each unit of emission. We do not examine this policy separately here because it is equivalent to the tax in our model, as stated below:

Remark 2 *When an emission permit is tradable in a competitive market, it is equivalent to an emission tax, with the price of the permit being equal to the tax rate.*

Moreover, an emission standard can be interpreted as a nontradable emission permit that is granted free to the plants whose emission-output ratio does not exceed the constant

[6]For example, as an alternative to (2), one can consider the specification $m(x) = m_0 q(x) + b\,[q(x)]^{\psi}$, where $m_0 > 0$, $b > 0$ and $\psi \in (0, 1)$.

[7]This assumption eliminates the need for government revenue as a potential difference between the tax and the standard. See Stern (2006, Part IV) for more discussions on this difference.

s. Thus, an emission tax differs from an emission standard in two ways. One is that the tax is a market instrument but the standard is nontradable. The other is that the tax directly affects a plant's marginal cost of production, but the standard affects the marginal cost only indirectly through the abatement choice and/or equilibrium effects.[8]

Note that uncertainty does not play an important role in this model. Although each individual plant's productivity is random, this random variable is realized before the plant makes the decisions. More importantly, there is no aggregate uncertainty in this economy, because there is a continuum of plants. Specifically, the supply curve of the dirty-goods composite, total input, total abatement, and total emission are all deterministic in this model. As said in the introduction, this abstraction from uncertainty sharpens our focus on the role of productivity dispersion in the evaluation of environmental policies.

3 Equilibrium and Policies without Abatement

In this section, we characterize the equilibrium and evaluate the two policies when an abatement technology does not exist, i.e., when the emission process obeys (2). The purpose is to isolate the role of productivity dispersion in policy evaluation. Section 4 will incorporate an abatement technology. In the current section, we unify the notation under the two policies by characterizing the equilibrium when both policies are present.

3.1 Household's decisions

A household chooses consumption of the clean good, c, and consumption of the dirty goods, $(q_i)_{i \in I}$, to maximize utility. It is convenient to index a dirty good by the plant's productivity level, x, rather than the index i. For each plant with a particular x, let $l(x)$ be its input, $q(x)$ its output, $p(x)$ the price of its product and $m(x)$ its emission. Because each plant's productivity is drawn randomly according to the distribution G, the density of plants with any particular x is $G'(x)$.[9] Let X be the set of productivity levels of operative

[8] Another policy is to issue a production permit (license) that is necessary for a plant to produce dirty goods and that is sold in the market at a competitive price. In contrast to an emission tax or standard, such production permits do not directly restrict a plant's emission, because a plant that obtains a production permit can produce as much as desirable. For this reason, a production permit cannot affect a plant's choice of the abatement technology.

[9] It is important to bear in mind that even if two plants draw the same x, they necessarily produce different varieties. The quantity $q(x)$ is not total output of all plants that draw the same x, but rather the

plants. The composite of the dirty goods in (1) can be expressed as

$$Q = \left[\int_{x \in X} [q(x)]^{\frac{\varepsilon-1}{\varepsilon}} dG(x) \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{4}$$

The household chooses c and $\{q(x)\}_{x \in X}$ to maximize $u(c, Q, M)$ subject to (4) and the following budget constraint:

$$c + \int_{x \in X} p(x) q(x) dG(x) \leq w + \int_{x \in X} \pi(x) dG(x) + T.$$

Here, $\pi(x)$ is the dividend from plant x and T the lump-sum transfer from the government. This maximization problem yields the following optimality conditions:

$$p(x) = P \left(\frac{q(x)}{Q} \right)^{-1/\varepsilon}, \tag{5}$$

$$\frac{u_2(c, Q, M)}{u_1(c, Q, M)} = P, \tag{6}$$

where P is the price of the composite Q:

$$P = \left[\int [p(x)]^{1-\varepsilon} dG(x) \right]^{\frac{1}{1-\varepsilon}}. \tag{7}$$

3.2 Plants' Decisions

Consider a plant x in the dirty-goods sector that chooses to produce. The plant's profit is:

$$\pi(x) = p(x) q(x) - w l(x) - \tau m(x).$$

The plant faces the demand curve for its product, given by (5). It chooses the input, $l(x)$, to maximize profit, taking as given the industry demand Q and the price index P. Substituting $p(x)$ from (5), $q(x)$ from the production function, and $m(x)$ from (2), we can express the plant's profit as

$$\pi(x) = P Q^{\frac{1}{\varepsilon}} \left[e^x l(x) \right]^{\frac{\varepsilon-1}{\varepsilon}} - (w + \tau b) l(x).$$

It is easy to verify that the plant's optimal input is

$$l(x) = Q \, e^{(\varepsilon-1)x} \left(\frac{(\varepsilon - 1) P}{\varepsilon (w + \tau b)} \right)^{\varepsilon}. \tag{8}$$

output of each of these plants. The same clarification applies to $l(x)$ and $m(x)$.

The plant's output is $q(x) = e^x l(x)$ and its emission is $m(x) = bl(x)$. We can calculate the price of the plant's product from (5) as

$$p(x) = \frac{\varepsilon}{\varepsilon - 1}(w + \tau b)e^{-x}. \tag{9}$$

The term $(w+\tau b)e^{-x}$ is the plant's marginal cost adjusted for the plant's productivity and the emission tax, which we will refer to as the plant's *effective* marginal cost. The above result reveals that the price of a plant's product is a constant markup $\frac{1}{\varepsilon-1}$ of the plant's effective marginal cost. Moreover, the plant's maximized profit is:

$$\pi(x) = \frac{P^{\varepsilon}Q}{\varepsilon} e^{(\varepsilon-1)x} \left(\frac{\varepsilon - 1}{\varepsilon(w + \tau b)}\right)^{\varepsilon-1}. \tag{10}$$

A plant's decision on whether to operate follows a cutoff rule on productivity; that is, a plant-x operates if and only if $x \geq x_0$. The cutoff x_0 depends on the policy. Under the emission tax, since $\pi(x) > 0$ for all x, all plants choose to operate and so $x_0 = \underline{x}$. Under the emission standard, a plant x can operate only if $s \geq m(x)/q(x) = be^{-x}$. That is, $x_0 = \ln(b/s)$ under the emission standard. We summarize these two cases as

$$x_0 = \begin{cases} \ln(b/s), & \text{with the emission standard} \\ \underline{x}, & \text{without the tax.} \end{cases} \tag{11}$$

The set of productivity levels observed in the economy is $X = [x_0, \infty)$.

3.3 Aggregation and equilibrium

Let us denote total input in the production of dirty goods as $L = \int_{x_0}^{\infty} l(x)dG(x)$ and the average productivity in the dirty-goods sector as $\phi = Q/L$. Substituting $l(x)$ from (8) to compute L and $q(x) = e^x l(x)$ into (4) to compute Q, we can express the average productivity in the dirty-goods sector as $\phi = \phi(x_0)$ where

$$\phi(x_0) = \left[\int_{x_0}^{\infty} e^{(\varepsilon-1)x} dG(x)\right]^{\frac{1}{\varepsilon-1}}. \tag{12}$$

The average of the plants' effective marginal costs of production is $(w+\tau b)/\phi$. Substituting (9) into (7) reveals that the price level of the dirty-goods composite is a constant markup over this average effective cost. That is, $P = P(\tau, \phi)$ where

$$P(\tau, \phi) \equiv \frac{\varepsilon}{\varepsilon - 1}(\frac{w + \tau b}{\phi}). \tag{13}$$

Let us refer to $(\tau, s, \bar{M}, T)$ as the policies, where $\bar{M}$ is the target level of aggregate emission. An *equilibrium* under the policies $(\tau, s, \bar{M}, T)$ consists of the set $X = [x_0, \infty)$, the functions $(l(x), q(x), p(x))_{x \in X}$, the aggregate levels (c, Q, P, L, M) that satisfy the following requirements: (i) Given the functions $(p(x))_{x \in X}$, a household's demand for the clean good, c, and the demand for each dirty good, $q(x)$, satisfy (5) and (6); (ii) Given (P, Q) and the demand function (5), a plant operates if and only if $x \geq x_0$, where x_0 satisfies (11), and if a plant operates, its choices of input and output satisfy (8) and $q(x) = e^x l(x)$; (iii) The levels of (P, L, M) satisfy (13), $L = Q/\phi$ and $M = bL$, where ϕ is given by (12); (iv) The resource market clears, i.e., $l_c = 1 - L$; (v) The market of the clean good clears, i.e., $c = wl_c$, and the markets of the dirty goods clear; (vi) The policy τ or s ensures aggregate emission not to exceed the target $\bar{M}$, i.e., $M \leq \bar{M}$, and the transfer T satisfies $T = \tau M$ under the tax and $T = 0$ under the standard.

We examine the non-trivial case where the emission target $\bar{M}$ is binding, i.e., where the economy would produce $M > \bar{M}$ if there were no policy. In this case, $M = \bar{M}$ in the equilibrium. An equilibrium can be determined as follows. Part (iii) above gives $L = \bar{M}/b$, $Q = \phi \bar{M}/b$ and $P = P(\tau, \phi)$, while parts (iv) and (v) give $c = w(1 - \frac{\bar{M}}{b})$. With these results, (6) required by part (i) becomes

$$R\left(w - w\frac{\bar{M}}{b}, \phi\frac{\bar{M}}{b}, \bar{M}\right) = P(\tau, \phi), \tag{14}$$

where $\phi = \phi(x_0)$. Equation (14) determines the policy level that is needed to implement the emission target $\bar{M}$. Note that the emission standard enters the above equation through $\phi(x_0)$, because x_0 is a function of s. For any emission target $\bar{M}$, let $\tau(\bar{M})$ denote the tax, and $s(\bar{M})$ the standard, that achieves the target.

Lemma 3 *Suppose that the abatement technology does not exist. Assume that* $[QR(c, Q, M)]$ *is a strictly increasing function of* Q *for any given* (c, M) *and that* $\lim_{Q \to 0} (QR) = 0$. *The target* $\bar{M}$ *is binding if and only if* $\bar{M} < M_{\max}$, *where* $M_{\max}$ *is defined by*

$$R\left(w - w\frac{M_{\max}}{b}, \phi(\underline{x})\frac{M_{\max}}{b}, M_{\max}\right) = P(0, \phi(\underline{x})). \tag{15}$$

Given any binding target $\bar{M}$, *there is a unique equilibrium under each policy. Moreover,* $\tau'(\bar{M}) < 0$ *and* $s'(\bar{M}) > 0$.

The proof of this lemma appears in Appendix A. In addition to existence and uniqueness of an equilibrium, the lemma states the intuitive feature that the tax can be lower and the standard can be looser when the emission target is higher.

3.4 Comparison between the two policies

Let us add the subscript τ to (x_0, L, P, Q, ϕ, u) under the tax and the subscript s to the variables under the standard. The following proposition compares the equilibrium under the tax with the equilibrium under the standard (see Appendix A for a proof):

Proposition 4 *Assume that an abatement technology does not exist. Given any binding emission target $\bar{M}$, the following results hold: (i) $L_\tau = L_s$ and $c_\tau = c_s$; (ii) $x_{0\tau} < x_{0s}$, $Q_\tau > Q_s$, $\phi_\tau > \phi_s$ and $P_\tau < P_s$; (iii) $u_\tau > u_s$.*

Result (i) is not surprising. Since a plant's emission is proportional to the plant's input, total emission is proportional to total input in the dirty-goods sector. Given the same emission target, the emission process (2) implies that total input in the dirty-goods sector must be the same under the two policies. As a result, total input in the clean-good sector and, hence, consumption of the clean good must also be the same under the two policies. Result (ii) states that, relative to the standard, the tax induces a larger set of varieties of the dirty goods to be produced, a higher level of consumption of the dirty-goods composite, a higher average level of productivity and a lower price of the dirty-goods composite. We will explain this result below. Since the tax generates higher consumption of the dirty-goods composite and the same level of consumption of the clean good, the tax dominates the standard in welfare, as stated in Result (iii).

The outcome $x_{0\tau} < x_{0s}$ in Result (ii) means that more varieties of the dirty goods are produced under the tax than under the standard. This outcome arises from the difference in how the two policies affect a plant's marginal cost and emission. The tax increases every plant's effective marginal cost of production, thereby reducing every plant's input, output and emission. However, since each plant charges a price that is a constant markup over the effective marginal cost, the increase in the marginal cost does not eliminate the plant's profit. Instead, all plants continue to operate with positive profit under the tax – they simply reduce the input and output. In contrast, the standard does not affect a plant's effective marginal cost and emission. To meet the emission target at the aggregate level, some plants must shut down, and these are the plants with low levels of x. As a result, fewer varieties are produced under the standard than under the tax.

The two policies have different effects on the average productivity. The tax does not change the average productivity, which is clear from (12). Put differently, the tax reduces total input in the dirty-goods sector and output of the dirty-goods composite in the same proportion. In contrast, the standard shuts down low-x plants. Because total input in the dirty-goods sector is the same under the two policies (for any given emission target), the input that is released from low-x plants is shifted to high-x plants. A priori, one would guess that this shift should increase the average productivity. Surprisingly, the opposite is true, as (12) clearly shows that the average productivity falls as x_0 increases.

How can it be the case that shutting down low-x plants and moving the input from these plants to high-x plants reduces the average productivity? The key to answering this question is the fact that different varieties of the dirty goods are not perfect substitutes in the composite, Q. Measured in terms of the contribution to Q, the marginal productivity of the input is the same for all x. To verify this statement, note that the sum of the input in all plants with x is $G'(x)l(x)$. The marginal contribution of this input to Q is

$$\frac{\partial Q}{G'(x)\partial[l(x)]} = e^x \left(\frac{q(x)}{Q}\right)^{-1/\varepsilon} = \frac{\varepsilon}{\varepsilon - 1}\frac{w + \tau b}{P} = \phi(x_0).$$

The first equality follows from (4), the second from (5) and (9), and the last from (13). The driving force for such equalization of the "value-weighted" marginal productivity is perfect mobility of the resource across the plants. Although a low-x variety has a lower productivity than a high-x variety, a smaller amount of a low-x variety is produced. Since consumers value all varieties and the marginal utility of a variety is diminishing, a low-x variety generates a higher marginal utility (and hence a higher price) than a high-x variety. More precisely, in terms of the contribution to Q, productivity e^x is weighted by $[q(x)/Q]^{-1/\varepsilon}$. A low-$x$ variety has a lower relative quantity $q(x)/Q$ in the composite and hence a larger weight than a high-x variety. Weighted by this marginal utility, productivity is the same in all varieties.

When low-x plants shut down under the emission standard and the resource is reallocated to high-x plants, output of the remaining plants increases. This higher quantity reduces the marginal contribution of each remaining variety to Q because such marginal contribution is diminishing. In the new equilibrium, the marginal contribution of the input is equalized at a lower level across the remaining plants. We can see this loss clearly by rewriting the average productivity equivalently as

$$\frac{1}{L}\int_{x_0}^{\infty}\frac{\partial Q}{G'(x)\partial l(x)}[l(x)G'(x)]dx = \frac{\partial Q}{G'(x)\partial l(x)}.$$

As the marginal productivity of each remaining variety (i.e., the right-hand side) falls, so does the average productivity.

Now it is easy to understand the results $P_\tau < P_s$ and $Q_\tau > Q_s$. The price level is a constant markup of the average effective marginal cost, $(w + \tau b)/\phi$. Since the average productivity is higher under the tax than under the standard, the average effective marginal cost is lower and, hence, the price level is lower under the tax. Moreover, higher productivity under the tax directly translates into higher output of the composite, because total input in the dirty-goods sector is the same under the two policies.

Note that productivity dispersion is necessary for a non-trivial comparison between the two policies in this model. If there is no dispersion in productivity across the plants, then all plants have to shut down under a binding standard while all plants continue to operate under the tax. In this case, the tax is evidently better than the standard. Introducing the abatement choice can avoid this trivial comparison, as shown in the next section.

Finally, let us make a remark on the optimal policy. Because emission generates a negative externality on the households, there might be a trade-off between the emission target, $\bar{M}$, and consumption, Q. In this case, the optimal emission target is determinate. For whatever target that is optimal under the standard, there exists a tax rate that achieves the same target and higher welfare than the standard (see Proposition 4). Thus, welfare is higher under the tax than under the standard even when the emission target is set to the optimal level under each policy.

4 Equilibrium and Policy Analysis with Abatement

In this section, we characterize the equilibrium and evaluate the two environmental policies when the plants has access to the abatement technology (3). We demonstrate that the emission standard can sometimes achieve higher welfare than the tax by affecting the plants' abatement choice differently from the tax. The analysis also uncovers how the relative advantage of the two policies depends on several key features of the economy such as productivity dispersion, the elasticity of substitution between the varieties of the dirty goods and the effectiveness of abatement.

4.1 Equilibrium characterization under each policy

A household's decisions are the same as those in subsection 3.1. A plant's decisions need to be modified. Consider the tax first. With the input into production, $l(x)$, and the input in abatement, $a(x)$, profit of a plant x is

$$\pi(x) = PQ^{\frac{1}{\varepsilon}}[e^x l(x)]^{1-\frac{1}{\varepsilon}} - wl(x) - \tau b\left(1+\frac{a(x)}{l(x)}\right)^{-\frac{1}{\gamma}} l(x) - wa(x),$$

where we have substituted the demand function for the plant's product, (5), and the emission process, (3). The plant chooses the abatement level, $a(x)$, and the input, $l(x)$, to maximize profit. It is easy to verify that the plant's optimal choices are[10]

$$a(x) = l(x)\left[\left(\frac{\tau b}{\gamma w}\right)^{\frac{\gamma}{\gamma+1}} - 1\right], \tag{16}$$

$$l(x) = Q\ e^{(\varepsilon-1)x}\left[\frac{(\varepsilon-1)P}{\varepsilon k}\right]^{\varepsilon}, \tag{17}$$

where

$$k \equiv w(\gamma+1)\left(\frac{\tau b}{\gamma w}\right)^{\frac{\gamma}{\gamma+1}}. \tag{18}$$

By comparing (17) with its counterpart without the abatement choice, (8), we can interpret the constant k as the plant's marginal cost of production which incorporates the price of the input, the marginal cost of abatement and the tax on emission. The optimal choices above induce the following levels of output, price, and emission under the tax:

$$q(x) = Q\ e^{\varepsilon x}\left[\frac{(\varepsilon-1)P}{\varepsilon k}\right]^{\varepsilon}, \quad p(x) = \frac{\varepsilon k e^{-x}}{\varepsilon - 1}, \quad m(x) = b\left(\frac{\tau b}{\gamma w}\right)^{\frac{-1}{\gamma+1}} l(x). \tag{19}$$

[10] To avoid unnecessary complications, we allow the choice a to be negative as well as positive, provided $1 + \frac{a}{l} \geq 0$. The interpretation of a choice $a < 0$ is that the plant uses a production technology that produces more emission than the production technology in the baseline model.

As in the baseline model, the price of a plant's product is a constant markup over the plant's effective marginal cost of production, which is ke^{-x} in the current environment. Moreover, $\pi(x) > 0$ for all x, and so all plants operate under the tax. That is, the set of productivity levels observed in the economy is $X = [\underline{x}, \infty)$.

Under the emission standard, profit of a plant x is

$$\pi(x) = PQ^{\frac{1}{\varepsilon}}[e^x l(x)]^{1-\frac{1}{\varepsilon}} - wl(x) - wa(x).$$

The plant must meet the emission standard, i.e., $m(x)/q(x) \leq s$. With the emission process, (3), we can rewrite this constraint as $a(x) \geq l(x) \left[\left(\frac{b}{s}\right)^\gamma e^{-\gamma x} - 1\right]$. Solving the constrained maximization problem, we obtain the following optimal choices of a and l:

$$a(x) = l(x) \left[\left(\frac{b}{s}\right)^\gamma e^{-\gamma x} - 1\right], \tag{20}$$

$$l(x) = Q\ e^{[\varepsilon(\gamma+1)-1]x} \left[\frac{(\varepsilon-1)P}{\varepsilon w}\left(\frac{s}{b}\right)^\gamma\right]^\varepsilon. \tag{21}$$

Under the standard, plant-x's output, price and emission are

$$q(x) = Q\ e^{\varepsilon(\gamma+1)x} \left[\frac{(\varepsilon-1)P}{\varepsilon w}\left(\frac{s}{b}\right)^\gamma\right]^\varepsilon, \tag{22}$$

$$p(x) = \frac{\varepsilon e^{-(\gamma+1)x}}{\varepsilon - 1} w \left(\frac{b}{s}\right)^\gamma, \quad m(x) = sq(x). \tag{23}$$

These expressions reveal that a plant's marginal cost is $w\left(\frac{b}{s}\right)^\gamma e^{-\gamma x}$, which includes the price of the input and the marginal cost of abatement. The effective marginal cost is $w\left(\frac{b}{s}\right)^\gamma e^{-(\gamma+1)x}$.

In contrast to the model without abatement, the model with abatement implies that every plant meets the standard. To meet the standard, a plant can spend enough in abatement and cut production sufficiently. Since all varieties are produced under both policies, we will omit the notation for the interval over which the integrals are computed in this section, which is $[\underline{x}, \infty)$.

We can compute the aggregate input in production, L, and the average productivity, ϕ, similarly to the baseline model. Also, compute the aggregate level of abatement as $A = \int a(x)dG(x)$. An equilibrium can be defined by incorporating abatement into the equilibrium definition in the baseline model. The following lemma determines the equilibrium (see Appendix B for a proof):

Lemma 5 *Assume that the abatement technology (3) is available. Under either policy, there is a unique equilibrium, where $L = Q/\phi$ and Q is determined by (6). Under the emission tax, $\phi = \phi(\underline{x})$, where the function $\phi(.)$ is given by (12), and other aggregate quantities and prices are as follows:*

$$\tau = \frac{\gamma w}{b}\left(\frac{bL}{M}\right)^{\gamma+1}, \quad P = \frac{\varepsilon w(\gamma+1)}{(\varepsilon-1)\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}, \tag{24}$$

$$A = L\left[\left(\frac{bL}{M}\right)^{\gamma} - 1\right], \quad c = w - \frac{wQ}{\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}. \tag{25}$$

Under the emission standard, the aggregate quantities and prices are:

$$\phi = \left[\int e^{(\gamma+1)(\varepsilon-1)x}dG(x)\right]^{\frac{\varepsilon}{\varepsilon-1}} \Big/ \int e^{[\varepsilon(\gamma+1)-1]x}dG(x)\,, \tag{26}$$

$$s = \frac{M}{L}\frac{\int e^{[\varepsilon(\gamma+1)-1]x}dG(x)}{\int e^{\varepsilon(\gamma+1)x}dG(x)}, \quad P = \frac{\varepsilon w\lambda}{(\varepsilon-1)\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}, \tag{27}$$

$$A = L\left[\left(\frac{bL}{M}\right)^{\gamma}\lambda - 1\right], \quad c = w - \frac{wQ}{\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}\lambda, \tag{28}$$

where

$$\lambda = \frac{\left[\int e^{(\varepsilon-1)(\gamma+1)x}dG(x)\right]\left[\int e^{\varepsilon(\gamma+1)x}dG(x)\right]^{\gamma}}{\left[\int e^{[\varepsilon(\gamma+1)-1]x}dG(x)\right]^{\gamma+1}}. \tag{29}$$

4.2 Comparing the two policies

With the abatement choice, aggregate quantities depend on the distribution of productivity in a complicated way as shown in Lemma 5. To gain insights into the model, let us assume a specific distribution function of x and a specific utility function in this subsection. We will consider more general forms of these functions in section 5. The specific distribution function (cdf) of x is exponential:

$$G(x) = 1 - e^{-(x-\underline{x})/\delta}, \quad \delta \in (0, \bar{\delta}), \tag{30}$$

where $\bar{\delta} = \min\{\frac{1}{2}, \frac{1}{\varepsilon(\gamma+1)}\}$. This distribution implies that productivity $z = e^x$ is distributed according to the Pareto distribution: $G_z(z) = 1 - \left(\frac{e^{\underline{x}}}{z}\right)^{1/\delta}$. The restriction $\delta < 1/[\varepsilon(\gamma+1)]$ is required for λ to be finite, while the restriction $\delta < 1/2$ is required for the variance of e^x to be finite. Under the restriction $\delta < 1/2$, the mean of e^x is $\frac{e^{\underline{x}}}{1-\delta}$ and the variance is

$\frac{\delta^2 e^{2\underline{x}}}{(1-2\delta)(1-\delta)^2}$. If we fix the mean of e^x at any arbitrary level $\bar{z} > 0$ by setting $\underline{x} = \ln[(1-\delta)\bar{z}]$, then the variance of e^x is $\frac{(\delta\bar{z})^2}{1-2\delta}$. Since this variance is increasing in δ, we refer to δ as the dispersion of productivity among the plants.

The utility function is assumed as

$$u(c, Q, M) = U(c + v(M)Q), \text{ with } U' > 0,\ U'' < 0,\ v > 0,\ v' < 0. \tag{31}$$

With this utility function, the marginal rate of substitution between the dirty-goods composite and the clean good is $u_2/u_1 = v(M)$. For any given emission target M, the equilibrium price of the dirty-goods composite is a constant $P = v(M)$. With (31), we can solve equilibrium Q explicitly from (6) as

$$Q = \begin{cases} \frac{M}{b}\left[\frac{(\varepsilon-1)v(M)}{\varepsilon w}\frac{(\phi_\tau)^{\gamma+1}}{\gamma+1}\right]^{1/\gamma}, & \text{with the tax} \\ \frac{M}{b}\left[\frac{(\varepsilon-1)v(M)}{\varepsilon w}\frac{(\phi_s)^{\gamma+1}}{\lambda}\right]^{1/\gamma}, & \text{with the standard.} \end{cases} \tag{32}$$

For any given emission target, the equilibrium value of Q differs under the two policies in two aspects. One is the difference in the average productivity. The other is that there is a constant $(\gamma + 1)$ under the tax, while the corresponding constant is λ under the standard.

Add the subscript τ to the variables under the tax and the subscript s to the variables under the standard. The following proposition contains the central results of the paper (see Appendix C for a proof):

Proposition 6 *Assume that the abatement technology is available as described by (3) and that productivity is distributed according to (30). Then, $\phi_s < \phi_\tau$. Define*

$$\varepsilon_0 = \frac{1}{\gamma}\left[(\gamma+1)^{(\gamma+1)/\gamma} - 1\right] \qquad (> 1).$$

With the utility function (31), $P_\tau = P_s = v(M)$, and the following results hold: (i) $\left(\frac{Q}{L+A}\right)_s < \left(\frac{Q}{L+A}\right)_\tau$; (ii) If $\varepsilon \geq \varepsilon_0$, then $u_s < u_\tau$ for all δ and, if $\varepsilon < \varepsilon_0$, then there exists $\delta_0 \in (0, \bar{\delta})$ such that $u_s > u_\tau$ iff $\delta < \delta_0$; (iii) $u_s > u_\tau \Longrightarrow Q_s > Q_\tau \Longrightarrow L_s > L_\tau \Longrightarrow A_s > A_\tau$, and $L_s > L_\tau \Longrightarrow c_s < c_\tau$; (iv) The emission target that maximizes utility is the same under the two policies and is given by $M^* = \arg\max M\,[v(M)]^{(1+\gamma)/\gamma}$.

The abatement choice does not change the ranking of the two policies in the average productivity. Measured as ϕ, the average productivity is still higher under the tax than under the standard. Although this difference is similar to that in the baseline model where abatement is not available, the mechanism is not as extreme as in the baseline model. Rather than forcing low-x plants to shut down as in the baseline model, the standard only reduces low-x plants' input and shifts it to high-x plants. To see why this shift occurs, note that in order to meet the emission standard, a low-x plant must spend more in abatement in proportion to its production. That is, the ratio of the input in abatement to production, $a(x)/l(x)$, is a decreasing function of x under the standard. A high-x plant employes more resources in production than a low-x plant, above and beyond what the difference in x alone calls for. Because the marginal contribution of each variety to the composite Q is diminishing, this shift of the input from low-output plants to high-output plants reduces the average contribution of the input to the composite. In contrast, the tax does not induce this shift of the input because the ratio of abatement to the input in production is constant across the plants under the tax. Thus, the average productivity is lower under the standard. Note that this result does not rely on the particular utility function (31).

The measure ϕ counts only the input in production of the dirty goods but not the input in abatement. Since total input in the dirty-goods sector is $(L + A)$, the effective productivity in the dirty-goods sector is $Q/(L + A)$. Part (i) of Proposition 7 says that, when the utility function has the form in (31), the effective productivity is also higher under the tax than under the standard.[11]

The abatement choice significantly modifies the ranking of the two policies in other aspects, including the ranking in welfare. Part (ii) of Proposition 7 says that the standard can yield higher welfare than the tax, which is not possible in the baseline model where the abatement choice is not available. Specifically, the standard yields higher welfare than the tax if productivity dispersion is sufficiently small and if the monopoly power of each variety is sufficiently strong in the sense that ε is small. Part (iii) provides a list of comparisons between the two policies. In particular, for the standard to yield higher welfare than the tax, the standard must induce a higher level of the dirty-goods composite, a higher input into production of dirty goods, a higher level of abatement, and lower consumption of the clean good. Part (iv) says that the optimal emission target is the same under the two policies. An implication is that the same ranking described in parts (i) - (iii) is valid if the

[11] As in the baseline model, one can compute the effective marginal productivity of the input in each plant x as $\frac{1}{G'(x)}\partial Q/\partial[l(x) + a(x)]$ and verify that it is equal to $Q/(L + A)$.

emission target is chosen optimally under each policy.

4.3 Why is the standard possibly better than the tax?

To answer this question, we examine first how the abatement choice changes the equilibrium when there is no dispersion in productivity and then how the abatement choice interacts with productivity dispersion.

4.3.1 The case with no dispersion in productivity

Consider an economy with no dispersion in productivity and let x be the level of productivity of all plants. In this economy, allowing for the abatement choice is necessary for the comparison between the two policies to be non-trivial. If abatement is not available in this economy, then a binding standard induces all plants to shut down and, hence, it is clearly inferior to the tax. With the abatement choice in this economy, the two policies are ranked as in the following proposition (see Appendix B for a proof):

Proposition 7 *When there is no productivity dispersion, the equilibrium with the abatement choice yields $\phi_s = \phi_\tau$, $\lambda = 1$, $Q_s > Q_\tau$, $L_s > L_\tau$, $A_s > A_\tau$ and $c_s < c_\tau$ for any given emission target M. Moreover, a sufficient condition for $u_s > u_\tau$ is $\varepsilon \leq 1 + \frac{1}{\gamma}$. On the other hand, if ε is sufficiently large, then $u_s < u_\tau$.*

The abatement choice enables all plants to operate under the standard, thus eliminating a main disadvantage of the standard relative to the tax. When there is no dispersion in productivity, the average productivity is clearly the same under both policies. Moreover, the standard induces higher input in and output of the dirty-goods production, and higher abatement. If the monopoly power of each variety is sufficiently strong in the sense that ε is small, then the standard generates higher welfare than the tax. Therefore, the abatement choice significantly alters the welfare ranking between the two policies.[12]

The standard can generate higher welfare than the tax because with abatement and homogeneous productivity, the standard creates less upward pressure on the price level of the dirty-goods composite than the tax does. We will explain this result below. Given this result on the price, it is clear that the household wants to consume more dirty goods

[12] Note that the results in Proposition 7 hold for general utility functions, not just for that in (31).

and less clean good under the standard than under the tax. To satisfy the demand, more resource is employed in the production of the dirty goods under the standard. Since higher output of the dirty goods generates higher emission, abatement is also higher under the standard in order for the dirty-goods sector to meet the standard. Since total input in the dirty-goods sector is higher under the standard, the input and output in the clean-good sector are lower than under the tax. The welfare ranking between the two policies depends on the relative change in the two types of consumption. When the monopoly power in the dirty-goods sector is sufficiently strong, the markup of price on the marginal cost is high, and a small difference in the marginal cost can translate into a large difference in price. In this case, the increase in the dirty-goods composite under the standard relative to the tax is sufficiently large to outweigh the decrease in the clean good, and so the standard induces higher utility than the tax.

We now explain how the two policies affect the price. When there is no dispersion in productivity, the price index of the dirty-goods composite mimics each individual plant's effective marginal cost of the input in production, as the former is a constant markup of the latter. The effective marginal cost of the input in production consists of the direct and the indirect cost. The direct marginal cost is the price of the input, which is the same (w) under the two policies. The indirect cost occurs in different places under the two policies. Under the standard, the indirect cost occurs through abatement. Because the plant's optimal choice of abatement is such that a plant just meets the standard, this choice depends on the input in production. By increasing emission, a higher input in production calls for an increase in abatement in order to meet the standard, which increases the effective marginal cost of production. Under the tax, in contrast, the indirect cost comes from the tax on emission: by increasing emission, an increase in the input in production increases the tax payment.[13] The indirect cost is lower under the standard than under the tax because for any given level of emission $m(x)$, an increase in the input $l(x)$ has the mitigating effect of relaxing the constraint imposed by the standard, $m(x) \leq s\ l(x)$. Thus, for the same emission level and the same input in production, a plant's effective marginal cost of

[13] A higher input in production also induces higher abatement. However, this increase in abatement does not increase the marginal cost of the plant under the tax, in contrast to the case under the standard. The reason is that higher abatement also reduces emission and, hence, lowers the tax payment. When the level of abatement is chosen optimally, these two effects of a higher input in production on the effective marginal cost through abatement cancel out – an implication of the envelope theorem.

production is lower and the upward pressure on the price of the dirty-goods composite is lower under the standard than under the tax.

To support the above explanation, recall that a plant's effective marginal cost of production is $w(\gamma+1)(\frac{\tau b}{\gamma w})^{\frac{\gamma}{\gamma+1}}e^{-x}$ under the tax and $w\left(\frac{b}{s}\right)^{\gamma}e^{-(\gamma+1)x}$ under the standard. When there is no dispersion in productivity, each plant's input, abatement, and emission are equal to their industry average. That is, $l(x) = L$, $a(x) = A$ and $m(x) = M$. From the equation $m(x) = M$, we can solve the policy level (s or τ) that is required to meet the target M. With this policy level, the effective marginal cost of production is $w(\gamma+1)(\frac{bL}{M})^{\gamma}e^{-x}$ under the tax and $w(\frac{bL}{M})^{\gamma}e^{-x}$ under the standard. Clearly, the former is higher than the latter by a factor $(\gamma+1)$. Since the price level is $\varepsilon/(\varepsilon-1)$ times the effective marginal cost, it is also higher under the tax than under the standard by a factor $(\gamma+1)$.[14]

4.3.2 The interaction between productivity dispersion and abatement

We have explained that with homogeneous plants, the standard creates less upward pressure on the price index of the dirty-goods composite. Productivity dispersion changes this result by increasing the upward pressure on the price index exerted by the standard. First of all, with productivity dispersion, the average productivity in the dirty-goods sector is lower under the standard than under the tax, as we explained earlier. A lower average productivity adds upward pressure on the price. Moreover, the standard induces such abatement choices that push up the marginal cost by more at plants with high prices than at plants with low prices. This change in the distribution of relative prices also adds upward pressure on the price index under the standard, and is reflected by the factor $\lambda > 1$ in the pricing formula under the standard, (27).

Let us explain why the standard induces abatement choices that change the distribution of relative prices, but the tax does not. In order to meet the standard, low-x plants must spend a higher proportion of their resources in abatement relative to their input in production than high-x plants do (see the explanation immediately after Proposition 6). Thus, the standard increases low-x plants' marginal costs and prices by more than high-x plants'. Under monopolistic competition, low-x plants produce less and charge higher

[14]It can be verified that, with homogeneous productivity, the effective marginal cost of production is higher under the tax than under the standard even for a general emission process, $m(x) = \mu(l(x), a(x))$, where $\mu_1 > 0$ and $\mu_2 < 0$.

prices than high-x plants do even when the standard is not imposed. Imposing the standard increases high prices by a larger proposition than low prices, thus tilting the distribution of relative prices toward high prices. In contrast, the tax induces all plants to spend the same proportion in abatement relative to their input in production. Thus, prices of all varieties increase by the same proportion under the tax, leaving the relative price between any two plants unchanged.

The standard can create higher or lower upward pressure on the price index, depending on the extent of productivity dispersion. When productivity dispersion is sufficiently small, i.e., when δ is small, the two new forces above on the price index created by the standard are small. In this case, the economy is close to the one with homogeneous plants, in which the standard has lower upward pressure on the price index than the tax. When productivity dispersion is sufficiently high, the two new forces above on the price index dominate the force in the economy with homogeneous plants. In this case, the standard has higher upward pressure on the price index than the tax.

Differences between the two policies' pressure on the price index translate into differences in the quantity of the dirty-goods composite which, in turn, affect the welfare ranking of the two policies. With the utility function (31), the equilibrium price is fixed as $p = v(M)$ by the target on aggregate emission. Any upward pressure on the price index must be absorbed by a fall in output of the dirty-goods composite in order to restore the equilibrium. Thus, when productivity dispersion is small, the standard induces a higher quantity of the dirty-goods composite than the tax does. If the monopoly power is also sufficiently high, i.e., if ε is sufficiently small, this higher quantity of dirty goods dominates the lower quantity of the clean good, and so welfare is higher under the standard than under the tax. On the other hand, if productivity dispersion is sufficiently high, the standard induces a much lower quantity of the dirty goods than the tax does. In this case, the tax dominates the standard in welfare for all $\varepsilon \geq 1$. This welfare ranking is stated as part (ii) of Proposition 6.[15]

[15] Note that the effectiveness of the abatement technology, $1/\gamma$, also plays a role since the critical levels ε_0 and δ_0 in Proposition 6 depend on γ. In the limit $\gamma \to 0$, even a tiny amount of input in abatement can reduce emission to zero. In this case, the two policies are equivalent. In the opposite limit $\gamma \to \infty$, the abatement is not effective at all, and the model approaches the baseline model where the tax induces higher welfare than the standard.

Let us turn to the remaining parts of Proposition 6. Part (iii) is easy to understand. First, for utility to be higher under the standard than under the tax, the standard must generate less pressure on the price index of the dirty-goods composite, in which case the composite decreases by less under the standard than under the tax. Second, since productivity is lower under the standard than under the tax, the input in the dirty-goods sector must be higher under the standard in order to produce a higher composite of the dirty goods than under the tax. Third, with (3), abatement is proportional to the input in production. Thus, a higher input in the production of the dirty-goods composite also calls for a higher level of abatement. Finally, when the input in the dirty-goods sector is higher under the standard, the input in the clean-good sector is lower, which contributes to lower consumption of the clean good under the standard than under the tax.

Finally, part (iv) of Proposition 6 describes the optimal emission target, i.e., the target that maximizes the representative household's utility. With (31), the optimal level of c is proportional to vQ. Since vQ depends on M only through the term $Mv^{(1+\gamma)/\gamma}$, so does the utility level. The optimal target maximizes this term regardless of which policy is used to implement the target. This result implies that parts (i) - (iii) of Proposition 6 continue to hold when the emission target is set to the optimal level under each policy.

5 Robustness of the Results

For Proposition 6, we used the particular distribution function of productivity, (30), and the utility function, (31). These functional forms enabled us to obtain explicit solutions of equilibrium quantities of goods, input and abatement. However, the results hold more generally. We illustrate this robustness in this section.

5.1 The utility function

Consider first the following generalization of the utility function:

$$u(c, Q, M) = \left\{\alpha c^{\rho} + (1-\alpha)\left[v(M)Q\right]^{\rho}\right\}^{1/\rho}, \ \alpha, \rho \in [0,1], \ v > 0, \text{ and } v' < 0.$$

The restriction $\rho \geq 0$ is imposed to satisfy Assumption 1. The case $\rho = 1$ corresponds to (31), where the clean good and the dirty-goods composite are perfect substitutes. The case $\rho = 0$ is also analytically tractable and the results are the same as in Proposition 6 after a modification of ε_0. For any $\rho \in [0,1)$, the relative price of the dirty-goods composite to the clean good is endogenous, in contrast to the case $\rho = 1$ where the price is fixed by the emission target. We use numerical examples to illustrate the results for $\rho \in (0,1)$. Let $v(M) = M^{-\kappa}$ and fix some of the parameters as follows:

$$\alpha = 0.6,\ \gamma = 0.2,\ b = 3,\ w = 1,\ \underline{x} = \ln(1-\delta),\ \kappa = 0.5$$

The chosen value $\underline{x}$ implies that the mean of e^x is 1 and the variance is $\frac{\delta^2}{1-2\delta}$. We explore different values of $(\delta, \varepsilon, \rho)$. For each $\rho \in [0,1]$, we find the region of (δ, ε) in which the standard yields higher welfare than the tax.

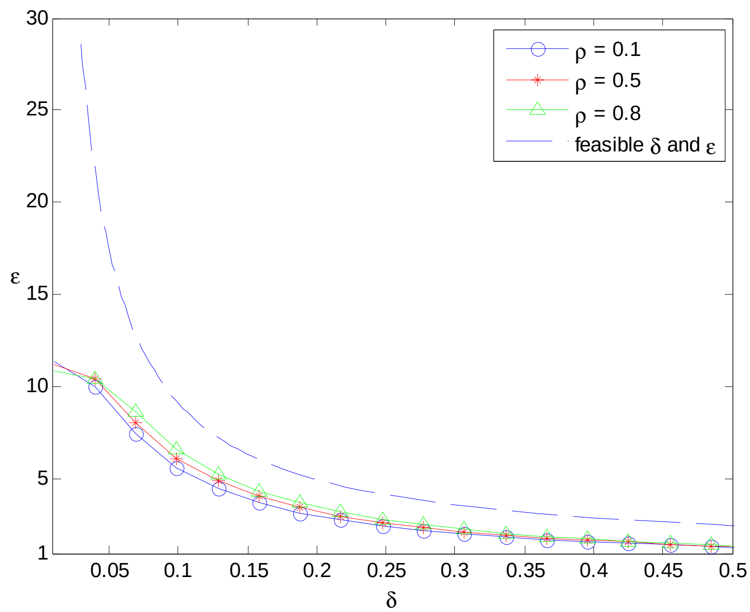


Figure 1. The region of (δ, ε) in which the standard dominates the tax

Figure 1 depicts two sets of curves in the (δ, ε) plane. One is the curve below which the values of (δ, ε) are feasible. The other is the curve along which the standard and the tax yield the same level of welfare, and this curve is drawn for three values of ρ, 0.1, 0.5 and 0.8. The standard yields higher welfare than the tax if and only if the values of (δ, ε) lie below this curve. For all three values of ρ, the standard dominates the tax in welfare if and only if δ and ε are small. This result is consistent with Proposition 6. In addition, when the elasticity of substitution between the clean good and the dirty-goods composite increases, i.e., when ρ increases, the curve moves slightly outward for middle values of δ, but inward for very small values of δ. That is, the more substitutable the clean good and dirty goods are, the more likely the standard dominates if there is some market power (and if δ is large enough).

5.2 The distribution function of productivity

Now let us return to the utility function (31) but consider a different distribution of productivity. The density function of the distribution of productivity z $(= e^x)$ is given by

$$G_z'(z) = \begin{cases} \frac{\beta}{|\delta|}\left(z - \underline{z}\right)^{1/\delta - 1} e^{-\beta(z-\underline{z})^{1/\delta}} dz, & \text{if } z > \underline{z} \\ 0, & \text{otherwise.} \end{cases} \tag{33}$$

This distribution is Weibull if $\delta > 0$ and Frechet if $\delta < 0$. We normalize $\underline{z} = 0$ to simplify the algebra. The mean of productivity z is $\beta^{-\delta}\Gamma(1+\delta)$, and the variance of z is $\beta^{-2\delta}\left[\Gamma(1+2\delta) - [\Gamma(1+\delta)]^2\right]$, where Γ is the gamma function. If we fix the mean of z at any arbitrary level $\bar{z}$, then the variance of z is $\bar{z}^2\left[\frac{\Gamma(1+2\delta)}{[\Gamma(1+\delta)]^2} - 1\right]$. Since this variance is increasing in $|\delta|$, we refer to $|\delta|$ as a measure of productivity dispersion among the plants. We restrict $\delta > -1/2$ for the variance to be bounded and restrict $\delta > -\frac{1}{\varepsilon(\gamma+1)}$ for λ (defined in (29)) to exist.

With the above distribution function, we can compute the average productivity under the two policies, respectively, as

$$\phi_\tau = \beta^{-\delta}\left[\Gamma\left(1+\delta\left(\varepsilon-1\right)\right)\right]^{\frac{1}{\varepsilon-1}}, \quad \phi_s = \frac{\beta^{-\delta}\left[\Gamma\left(1+\delta\left(\gamma+1\right)\left(\varepsilon-1\right)\right)\right]^{\frac{\varepsilon}{\varepsilon-1}}}{\Gamma\left(1+\delta\left(\varepsilon\left(\gamma+1\right)-1\right)\right)}.$$

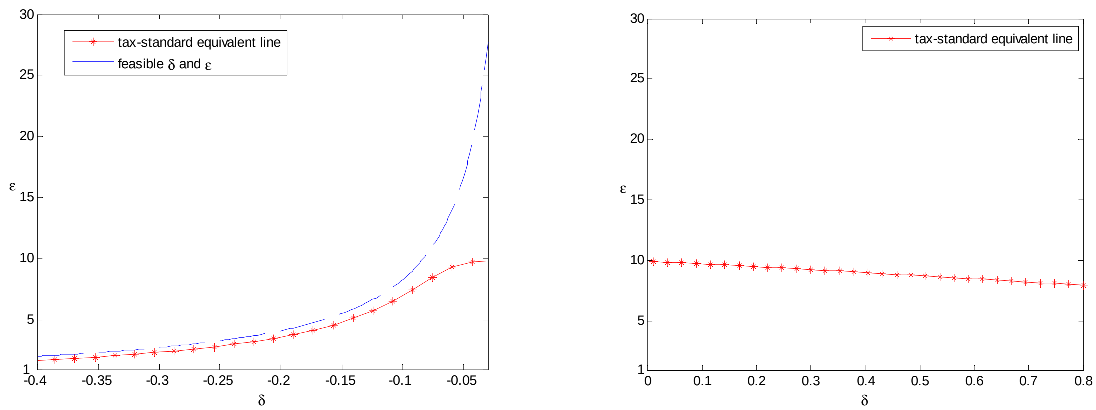
The term λ defined in (29) is

$$\lambda = \frac{\Gamma\left(1+\delta\left(\gamma+1\right)\left(\varepsilon-1\right)\right)\left[\Gamma\left(1+\delta\varepsilon\left(\gamma+1\right)\right)\right]^\gamma}{\left[\Gamma\left(1+\delta\left(\varepsilon\left(\gamma+1\right)-1\right)\right)\right]^{\gamma+1}}.$$

Using (31), welfare is higher under the standard than under the tax if and only if

$$\left(\frac{\phi_s}{\phi_\tau}\right)^{(\gamma+1)}\left(\frac{\gamma+1}{\varepsilon\gamma+1}\right)^\gamma \frac{\gamma+1}{\lambda} > 1. \tag{34}$$

The condition (34) depends only on four parameters: γ, δ, ε, and β. We take $\gamma = 0.2$ as in the previous subsection and choose β to normalize the mean of the productivity, $\beta^{-\delta}\Gamma(1+\delta)$, to 1. The numerical results are depicted in Figure 2, where the left panel is for the Frechet distribution and the right panel for the Weibull distribution. The curve labeled “tax-standard equivalent line” contains the combinations of δ and ε with which the two policies yield the same welfare. In the region below this line, the standard yields higher welfare than the tax. It is clear that this region is non-empty for both the Frechet and the Weibull distribution. Thus, the standard yields higher welfare than the tax if and only if productivity dispersion is small (i.e., if $|\delta|$ small) and the monopoly power is strong (i.e., if ε is small). Therefore, the main qualitative result in Proposition 6 also holds under the distribution function (33).



Frechet distribution Weibull distribution

Figure 2. The region of (δ, ε) in which the standard dominates the tax

6 Conclusion

When a society wants to control aggregate emission under a certain target level, is it more desirable to impose a tax or a regulatory standard on emission? To answer this question, we explore a model where plants are heterogeneous in productivity and monopolistically competitive in the production of a set of varieties of (dirty-) goods whose by-product is emission. The main result is that the standard yields higher welfare than the tax if and only if productivity dispersion is small and the monopoly power in the dirty-goods sector is strong. In the process of obtaining this result, we find that, if the plants have no access to an abatement technology, then the tax dominates the standard unambiguously. When the plants do have access to an abatement technology, there can be less price distortion under the standard than under the tax, in which case the standard can yield higher welfare.

Our results illustrate the importance of productivity dispersion and its interaction with abatement choices in designing environmental policies. They demonstrate when and why a non-market instrument such as the regulatory standard can be better than a market-based instrument, such as an emission tax, for achieving an emission target. Both the focus on productivity dispersion and the assumption of a fixed emission target deviate from the price-versus-quantity literature in environmental economics. The latter addresses the question whether it is more desirable to let the price or the aggregate quantity of emission to fluctuate when there is uncertainty in the marginal cost and benefit of emission reduction.

There are many directions in which one can explore further the importance of productivity dispersion for environmental policies. For example, with productivity dispersion, the two policies considered in this paper have different effects on plants' exit and entry. It is interesting to examine the dynamic effects of these policies. Another exercise is to calibrate a dynamic model and calculate the cost of reaching an environmental target. Li and Sun (2009) have made an attempt on such quantitative exercises.

Appendix

A Proofs of Lemma 3 and Proposition 4

We establish Lemma 3 first. Under the tax, (14) can be written as

$$R\left(w - w\frac{\bar{M}}{b}, \phi(\underline{x})\frac{\bar{M}}{b_0}, \bar{M}\right) = P(\tau, \phi(\underline{x})).$$

For any given $\bar{M}$, the left-hand side of the above equation is independent of τ and the right-hand side is an increasing function of τ. Thus, there exists a unique level of τ, denoted $\tau(\bar{M})$, that solves the above equation. With this level of the tax, other equilibrium variables are uniquely determined as in the main text. Moreover, the target $\bar{M}$ is binding if and only if $\tau(\bar{M}) > 0$. The latter requirement is equivalent to the condition that the left-hand side of the above equation is strictly greater than $P(0, \phi(\underline{x}))$. Since the left-hand side of the above equation is a strictly decreasing function of $\bar{M}$ (see Assumption 1), the target is binding if and only if $\bar{M} < M_{\max}$. It is easy to see that $\tau'(\bar{M}) < 0$ and $\tau(M_{\max}) = 0$.

Under the emission standard, $x_0 = \ln(b/s)$, and (14) can be written as

$$\phi\; R\left(w - w\frac{\bar{M}}{b}, \phi\frac{\bar{M}}{b}, \bar{M}\right) = \frac{\varepsilon}{\varepsilon - 1}w,$$

where $\phi = \phi(\ln(b/s))$. For any given $\bar{M}$, the assumption on qR imposed in Lemma 3 ensures that the left-hand side of the above equation is a strictly increasing function of ϕ and reaches 0 at $\phi = 0$. There is a unique level of ϕ, denoted $\phi_s(\bar{M})$, that solves the above equation. The implied standard can be calculated from $\phi_s(\bar{M}) = \phi(\ln(b/s(\bar{M})))$ and other equilibrium variables can be uniquely determined as in the main text. Moreover, the target $\bar{M}$ is binding iff $\ln(b/s(\bar{M})) > \underline{x}$, i.e., iff $\phi_s(\bar{M}) < \phi(\underline{x})$. This requirement is equivalent to $\bar{M} < M_{\max}$. Furthermore, it is easy to verify that $\phi_s'(\bar{M}) > 0$ and $\phi_s(M_{\max}) = \phi(\underline{x})$. Hence, $s'(\bar{M}) > 0$ and $s(M_{\max}) = be^{-\underline{x}}$. This completes the proof of Lemma 3.

Now we prove Proposition 4 by verifying statements (i)-(iii) in the proposition. Statement (i) is evident, since $L = \bar{M}/b$ and $c = w(1 - L)$ under both policies. For statement (ii), the proof of Lemma 3 has already established $x_{0\tau} = \underline{x} < \ln(b/s) = x_{0s}$ for any binding target. Because $\phi(x_0)$ defined by (12) is a strictly decreasing function of x_0, then $\phi_\tau > \phi_s$. Since $Q = \phi\bar{M}/b$, then $Q_\tau > Q_s$. Recall that $P = R(c, Q, M)$ and that $R(c, Q, M)$ is a strictly decreasing function of Q. We have $P_\tau = R(c, Q_\tau, \bar{M}) < R(c, Q_s, \bar{M}) = P_s$. Finally, $u_\tau = u(c, Q_\tau, \bar{M}) > u(c, Q_s, \bar{M}) = u_s$. QED

B Proofs of Lemma 5 and Proposition 7

For Lemma 5, we only derive the formulas for the tax and prove that there exists a unique equilibrium. The derivation and the proof for the standard are similar and, hence, are omitted. Under the tax, we substitute $q(x)$ from (19) into (1) to compute Q and aggregate $l(x)$ from (17) to compute L, which yields $\phi = Q/L = \phi(\underline{x})$, where $\phi(.)$ is the function defined by (12). Aggregating $m(x)$ in (19), we have $M = b(\frac{\tau b}{\gamma w})^{\frac{-1}{\gamma+1}} L$. Inverting this result yields τ as the function of (L, M) in (24). Substituting $p(x)$ from (19) into (7), we obtain $P = \frac{\varepsilon k}{(\varepsilon-1)\phi}$, where k is a function of τ given by (18). Substituting τ, we obtain

$$k = w(\gamma + 1) \left(\frac{bL}{M}\right)^{\gamma}, \tag{35}$$

and, hence, P is as in (24). Aggregating $a(x)$ in (16) and substituting τ from (24), we obtain A as in (25). Since total input in the dirty-goods sector is $(L + A)$, consumption of the clean good is $c = w(1 - L - A)$. Substituting A, we obtain c as in (25). The quantity Q is determined by (6). Proving that there exists a unique equilibrium amounts to proving that there is unique solution for Q. Substituting P from (24), we have

$$\frac{u_2(c(Q), Q, M)}{u_1(c(Q), Q, M)} = \frac{\varepsilon w(\gamma + 1)}{(\varepsilon - 1)\phi} \left(\frac{bQ}{M\phi}\right)^{\gamma}, \tag{36}$$

where $c(Q)$ is given by (25). Under Assumption 1, the left-hand side of (36) is a strictly decreasing function of Q, while the right-hand side is a strictly increasing function of Q. With these features, it is easy to prove that there is a unique solution for Q to (36). This completes the proof of Lemma 5.

For Proposition 7, we assume that the measure of plants is one without loss of generality. When all plants have the same productivity, $l(x) = L$, $a(x) = A$, $q(x) = Q$, and $m(x) = M$. It is evident that $\phi_s = \phi_\tau = e^x$ and $\lambda = 1$. Substituting the pricing formulas in (24) and (27) into (6), we obtain:

$$\frac{u_2(c, Q, M)}{u_1(c, Q, M)} = \frac{\sigma \varepsilon w}{(\varepsilon - 1)\phi} \left(\frac{bQ}{M\phi}\right)^{\gamma},$$

where $\sigma = \gamma + 1$ under the tax and $\sigma = 1$ under the standard. Under both policies, $c = w - \frac{wQ}{\phi}\left(\frac{bQ}{M\phi}\right)^{\gamma}$ and $\phi = e^{x}$ in the above equation. Denote the solution to the above equation as $Q(\sigma)$. It is clear that $Q'(\sigma) < 0$. Thus, $Q_s > Q_\tau$. From (25), (28) and $L = Q/\phi$, it is clear that $L_s > L_\tau$, $A_s > A_\tau$, and $c_s < c_\tau$. Express the utility level as $\hat{u}(\sigma) \equiv u(c(\sigma), Q(\sigma), M)$. Then, $u_s > u_\tau$ iff $\hat{u}(1) > \hat{u}(\gamma + 1)$. It can be verified that $\hat{u}'(\sigma) < 0$ iff $\sigma > (\gamma+1)(\varepsilon-1)/\varepsilon$. If $1 \geq (\gamma+1)(\varepsilon-1)/\varepsilon$, i.e., if $\varepsilon \leq 1 + \frac{1}{\gamma}$, then $\hat{u}'(\sigma) < 0$ for all $\sigma > 1$. In this case, $u_\tau = \hat{u}(\gamma+1) < \hat{u}(1) = u_s$. On the other hand, if $\varepsilon \to \infty$, then $\hat{u}'(\sigma) > 0$ for all $\sigma < \gamma + 1$. In this case, $u_\tau = \hat{u}(\gamma+1) > \hat{u}(1) = u_s$. QED

C Proof of Proposition 6

Using (30), we compute the average productivity as

$$\phi = \begin{cases} [1 - \delta(\varepsilon - 1)]^{\frac{-1}{\varepsilon - 1}} e^{\underline{x}}, & \text{with the tax} \\ \frac{1 + \delta - \delta\varepsilon(\gamma+1)}{[1 - \delta(\varepsilon-1)(\gamma+1)]^{\frac{\varepsilon}{\varepsilon-1}}} e^{\underline{x}}, & \text{with the standard.} \end{cases} \tag{37}$$

The statement $\phi_s < \phi_\tau$ is true if and only if

$$\frac{\varepsilon}{\varepsilon - 1} \ln[1 - \delta(\varepsilon-1)(\gamma+1)] - \ln[1 + \delta - \delta\varepsilon(\gamma+1)] - \frac{1}{\varepsilon - 1}\ln[1 - \delta(\varepsilon - 1)] > 0.$$

Temporarily denote the left-hand side as $LHS(\delta)$. Note that $LHS(0) = 0$ and $LHS(\frac{1}{\varepsilon(\gamma+1)}) > 0$. Also, $LHS'(\delta)$ has the same sign as the following expression:

$$\frac{\delta\varepsilon\gamma(\gamma+1)}{[1 - \delta(\varepsilon-1)(\gamma+1)][1 + \delta - \delta\varepsilon(\gamma+1)]} + \frac{1}{[1 - \delta(\varepsilon - 1)]}.$$

Thus, $LHS'(\delta) > 0$. For all $\delta > 0$, $LHS(\delta) > LHS(0) = 0$.

With the utility function (31), it is clear that $P_s = P_\tau = v(M)$. For part (i) of the proposition, we can compute

$$\frac{Q}{L + A} = \begin{cases} \frac{\varepsilon w}{(\varepsilon - 1)v(M)}(\gamma + 1), & \text{with the tax} \\ \frac{\varepsilon w}{(\varepsilon-1)v(M)}, & \text{with the standard.} \end{cases}$$

It is evident that $Q/(L + A)$ is higher under the tax than under the standard.

For other parts of the proposition, we substitute G from (30) into (29) to compute

$$\lambda = \frac{[1 + \delta - \delta\varepsilon(\gamma+1)]^{\gamma+1}}{[1 - \delta\varepsilon(\gamma+1)]^{\gamma}[1 - \delta(\varepsilon-1)(\gamma+1)]} \; (> 1). \tag{38}$$

For part (ii), we use (32) to solve for L, A and c. Substituting (c, Q) and λ into (31), we know that $u_s > u_\tau$ iff $\frac{Q_s}{Q_\tau} > \frac{\varepsilon\gamma+1}{\gamma+1}$. Write the latter condition equivalently as follows:

$$\begin{aligned}&\tfrac{1}{\varepsilon-1}\ln\left[1-\delta(\varepsilon-1)\right]+\tfrac{\gamma}{\gamma+1}\ln\left[1-\delta\varepsilon(\gamma+1)\right]\\&-\tfrac{\varepsilon\gamma+1}{(\varepsilon-1)(\gamma+1)}\ln\left[1-\delta(\varepsilon-1)(\gamma+1)\right]+\ln(\gamma+1)-\tfrac{\gamma}{\gamma+1}\ln(\varepsilon\gamma+1)\quad>0.\end{aligned}$$

Temporarily denote the left-hand side as $LHS(\delta)$. It is clear that $LHS(\frac{1}{\varepsilon(\gamma+1)}) = -\infty$. Also, $LHS(0) > 0$ iff $\varepsilon < \varepsilon_0$, where ε_0 is defined in the proposition. Moreover, we can verify that

$$LHS'(\delta) = \frac{-\delta\gamma(\varepsilon\gamma+1)}{\left[1-\delta(\varepsilon-1)\right]\left[1-\delta(\varepsilon-1)(\gamma+1)\right]\left[1-\delta\varepsilon(\gamma+1)\right]} < 0.$$

If $\varepsilon \geq \varepsilon_0$, then $LHS(\delta) < LHS(0) \leq 0$, in which case $u_s < u_\tau$ for all $\delta \in (0, \bar{\delta})$. If $\varepsilon < \varepsilon_0$, then $LHS(0) > 0$, in which case there exists $\delta_0 \in (0, \bar{\delta})$ such that $u_s > u_\tau$ iff $\delta < \delta_0$.

For part (iii), recall that $u_s > u_\tau$ iff $\frac{Q_s}{Q_\tau} > \frac{\varepsilon\gamma+1}{\gamma+1}$. Because $\varepsilon > 1$, then $u_s > u_\tau$ implies $Q_s > Q_\tau$. Similarly, since $L = Q/\phi$ and $\phi_\tau > \phi_s$, then $Q_s > Q_\tau$ implies $L_s > L_\tau$. To compare the aggregate level of abatement and consumption of the clean good under the two polices, recall that $A = L\left[\left(\frac{bL}{M}\right)^\gamma - 1\right]$ and $c = w - wL\left(\frac{bL}{M}\right)^\gamma$ under the tax, while $A = L\left[\left(\frac{bL}{M}\right)^\gamma \lambda - 1\right]$ and $c = w - wL\left(\frac{bL}{M}\right)^\gamma \lambda$ under the standard. Since $\lambda > 1$, the inequality $L_s > L_\tau$ is sufficient for $A_s > A_\tau$ and $c_s < c_\tau$.

For part (iv), we can substitute equilibrium values of (c, Q) into the utility function to verify that $u(c + v(M)Q)$ depends on M entirely through the term $M[v(M)]^{(1+\gamma)/\gamma}$ and is increasing in this term. Then, part (iv) is evident. QED

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