University of Toronto
Department of Economics

Working Paper 404

Labor Market Cycles and Unemployment Insurance Eligibility

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May 31, 2010

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Febrary 16, 2008

Abstract

If entitlement to UI benefits must be earned with employment, generous UI is an additional benefit to an employment relationship, so it promotes job creation. If individuals are risk neutral, UI is fairly priced, and the UI system prevents moral-hazard, the generosity of UI has no effect on unemployment. As with Ricardian Equivalence, this result should be useful to pinpoint the effects of UI to violation of its premises. In itself, the endogenous entitlement of UI benefits does not resolve if the Mortensen-Pissarides model is able to generate realistic cycles. However, it brings some insights into this debate: The widespread concern in the design of UI systems to minimize moral-hazard unemployment only makes sense if workers have sufficiently high values of leisure (80 percent of labor productivity in our baseline calculation for the United States). Also, the fact that the generosity of UI has potentially a small effect on unemployment reconciles a high response of unemployment to changes in labor productivity with a small response to changes in UI benefits.

JEL classification: E24 E32 J64

Keywords: Search, Matching, UI Eligibility, Business Cycles, Labor Markets

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1 Introduction

Most models of employment flows in the labor market assume that workers automatically qualify for unemployment insurance (UI) benefits while they are searching for a job. As pointed out by Mortensen (1977), Burdett (1979), and Hamermesh (1979), this simplistic view of how a UI system operates may lead to highly misleading conclusions about its impact on the labor market. To avoid this criticism, several papers taking into account more realistic features of the UI systems have emerged. However, because of the institutional complexities of actual UI systems, these models rely exclusively on numerical methods for their analyses, and, they either assume an exogenous distribution of real wages (Andolfatto and Gomme, 1996) or a non-standard mechanism for its determination (Brown and Ferrall, 2003). In this paper, we advance an analytically tractable version of the standard Mortensen-Pissarides search and matching model in which workers are not always entitled to UI benefits because such an entitlement must be earned with prior and not too distant employment, and it can be lost if workers quit their jobs voluntarily or refuse job offers.

If UI benefits are unconditionally received while searching for a job, they represent an opportunity cost of employment, and improve the bargaining position of workers while negotiating over wages with their employers. As a result, UI benefits reduce the expected profits of filling a vacancy, and hurt firms' incentives for job creation and therefore employment. In contrast, if UI benefits are conditional on prior employment, they are no longer an opportunity cost, but an indirect benefit of employment. True enough, once workers become eligible for UI their bargaining position improves and so their salaries rise. However, this is well anticipated by all involved, so UI benefits reinforce the bargaining position of firms dealing with workers who are not yet eligible for UI. Consequently, UI benefits promote the value of filling a vacancy and stimulate job creation. This is the entitlement effect stressed by Mortensen (1977), Burdett (1979), and Hamermesh (1979) but operating through a new channel. In those papers, the desire to earn UI entitlement reduces the reservation wages of workers searching for jobs, which in turn reduces unemployment. In our model, the entitlement effect operates through the bargaining positions of firms and workers. The UI benefits, making the employment match more attractive to workers, enable firms to appropriate a larger fraction of the match surplus, which translates into a stronger incentive to post vacancies.

Even if generous UI benefits encourage job creation, they may hurt employment when we take into account the financial costs of the UI system. When the UI program is funded by the UI contribution fees paid by employed workers, a generous UI system is also an expensive one, and the large fees needed to maintain it lower the workers' desire of being employed, and so the value of filling a vacancy. Therefore, an expensive UI system imposes a downward pressure on employment. Based on these two competing effects of UI benefits, we obtain the following analog to Ricardian Equivalence: If UI rules can prevent the moral hazard behavior of becoming or remaining unemployed, each employed worker is charged a fair unemployment insurance fee, and utilities are linear, then the generosity of UI benefits, the duration of these benefits, and the time it takes to become eligible for UI are all *irrelevant* to the determination of output, vacancies, and unemployment.

Like Ricardian Equivalence, this irrelevance result should be a useful benchmark to pinpoint the economic effects of a UI system as violations of its premises. That is, the economic relevance of a UI system must be found on the risk aversion of workers, the "unfair" pricing of UI services, and moral hazard. If workers are risk averse, UI provides the valuable service of smoothing consumption fluctuations in the presence of employment shocks. The Mortensen-Pissarides model typically abstracts from this purpose by assuming linear utilities, and we follow this tradition in this paper. If UI contributions, or equivalently taxes that ultimately fall on employed workers, do not match the expected present discounted value of the UI benefits to be received during unemployment spells, the entitlement effect of UI benefits does not offset the opportunity cost of financing these benefits. So, the UI system may either increase or decrease employment depending on if the insured workers in question are subsidized or not from other sources of government revenue. Finally, workers may alter the hazard of being or remaining unemployed by changing their search intensity, refusing job offers, or strategically quitting jobs once they are eligible for UI. In a balance between tractability and realism, the present contribution focuses on the third of these hazard; that is, the assumptions of the model allow for moral hazard quits, but abstract from the effect of UI on search intensity and the acceptance of job offers.

The possibility of moral hazard quits opens the door to an interesting form of multiple equilibria. For a generic set of parameters values, two types of equilibria coexist: A "good" equilibrium where workers do not quit once they are eligible for UI and a "bad" equilibrium where such quits occur. In the good equilibrium, few workers collect UI benefits and many are employed, so the UI contributions required to finance the UI program can be low, which in turn makes it undesirable to quit a job to collect UI benefits. In the bad equilibrium, many workers collect UI and few contribute to the UI system. Hence, UI contributions need to be high, which induces workers to quit as soon as they can collect UI. This multiplicity of equilibria is a reminder that fully rating UI contributions is not enough to curtail the moral hazards induced by a UI system.

When our model is confronted with data from Canada and the United States, it offers the following insights on the current debate about the appropriateness of the Mortensen-Pissarides model in explaining the cyclical fluctuations in the labor market. First, the eligibility rules of these two countries show a major concern to avoid moral hazard quits and such concern is only meaningful if the value of leisure is not too low. For example, in our baseline calibration with United States data, we calculate that for values of leisure below 80 percent of labor productivity, workers would never quit to collect UI even if they knew they would be able to collect the statutory UI replacement rate (40 percent) with probability one. Hence, even if the obvious political and social concern about unemployment implies that the value of leisure cannot be close to labor productivity (see Mortensen and Nagypál, 2007), the further concern to avoid moral hazard unemployment implies that the value of leisure cannot be too low either. Second, our calibrations of the model to cyclical data from Canada and the United States require similar values of leisure as a percentage of labor productivity (around 54 percent) even if the levels of generosity of UI systems in these two countries are quite different. Third, with sufficiently high values of leisure, our calibrations are able to generate realistically large labor market cycles in response to productivity shocks, even if unemployment responds little to correlated changes in taxes and UI benefits.

The rest of the paper is organized as follows. Section 2 describes our stochastic version of the Mortensen-Pissarides model with a UI system in which individuals need to earn their UI eligibility. Section 3 analyzes a deterministic version of the model. Section 4 fits the model to data on the labor market cycles in Canada and the United States. Finally, section 5 concludes.

2 The Model

Our model is a stochastic version of Pissarides (1985) search model. To simplify algebraic expressions, we use continuous time in the analysis of this and the following section, although a discrete time version of the same model will be employed in the numerical simulations of Section 4.

2.1 Basic Environment

In the economy, there is a continuum of measure one of workers, and a large measure of potential firms with free entry into the labor market. Both workers and firms are infinitely lived, risk neutral, maximize their expected utilities, and discount future utility flows at the common rate r. Production requires the cooperation of one worker and one firm. For this cooperation to take place, workers and firms must first enter the labor market and search for a suitable partner. Once a match has been formed, it produces a flow of output p until it breaks down. The productivity p, common to all matches in the economy, follows a Markov jump stochastic process with a constant arrival rate λ and takes values in a finite support $P \in R_{+}^{n}$. The surplus from a match is split between the two parties according to the generalized axioms of Nash. Finally, employment matches dissolve either exogenously as a result of separations which come at an arrival rate s, or endogenously if breaking the match is in the interest of one of the two parties.

The key feature we introduce to this standard environment is that workers do not always collect unemployment insurance benefits (UI) while they are searching for jobs. For workers to be eligible for UI, they must first be employed for a while, and benefits do not last forever. Furthermore, UI benefits are meant to be collected for workers who lose their jobs involuntarily, although, to be realistic, we allow some workers who quit to successfully pretend to have lost their jobs involuntarily.

To capture these features in a tractable way, the following assumptions are made. Newly employed workers are not eligible for UI, and eligibility is the outcome of a jump stochastic process with an arrival rate g. Eligible workers always collect UI if they suffer an exogenous separation from their jobs, but if they quit, they collect UI with probability $\pi \in [0,1]$. Finally, unemployed workers collecting UI lose eligibility either when they are offered a job or as a result of a jump stochastic process with an arrival rate d.

Unemployment insurance is provided by a government, which finances the UI system with a mandatory state dependent contribution fee τ_p collected from all employed workers. Since the government can borrow and save at the interest rate r, the UI program can run deficits or surpluses over time. Later on, we will allow for permanent deficits or surpluses by introducing general taxation and a public good.

All workers are identical in terms of preferences and abilities, and supply labor inelastically. The only difference across workers lies in the UI eligibility, which is indicated by the individual state variable i :

$$i = \begin{cases} 1, & \text{if the worker is eligible for UI, and} \\ 0, & \text{otherwise.} \end{cases}$$

Net of the UI contribution fee, employed workers earn a state dependent wage rate w_p^i, where the superscript i denotes the UI eligibility state, and the subscript p denotes the productivity state. The wage rate w_p^i depends on UI eligibility because UI benefits raise the opportunity cost of employment, so they improve the worker's bargaining position in the negotiations to split the match surplus. Unemployed workers receive a flow utility from leisure ℓ, and, if eligible for UI, they also receive UI benefits b. To avoid uninteresting possibilities, both ℓ and b are assumed to be positive, and ℓ is assumed smaller than the production in a match net of the UI contribution fee: $\ell < p - \tau_p$ for all $p \in P$. However, the total opportunity cost of employment for a worker who is entitled to collect UI, $\ell + b$, may surpass production net of the UI fee for some realizations of p, which raises the possibility of moral hazard quits.

All firms possess the same production technology and preferences. Each one of them chooses to either stay idle or be active in the labor market. An active firm searching for a worker posts a vacancy at a constant flow cost c, and an active firm paired up with a worker gains an output flow p and incurs a labor cost $w_p^i + \tau_p$. In addition to the flow costs of posting vacancies, we follow Mortensen and Nagypál (2007)[1] in assuming that there is a one time hiring and training cost k (training cost for short) when a worker and a firm meet. We assume that this cost is transferable, and split between the two parties by the same type of generalized Nash bargaining as in the wage negotiations. As a result, a firm and a worker end up incurring the respective costs k^f and k^w to start an employment relationship. This cost captures in a simplified fashion the fact that firms incur hiring and trainings costs when they recruit new employees, and workers typically suffer human capital losses when they undergo a spell of unemployment. Although most properties of our model do not depend on k being strictly positive, we believe that a successful numerical implementation of the model requires taking into account the full labor turnover costs.

The search frictions in the labor market are characterized by a constant returns to scale matching technology: $M(v,u)$. The function M maps vacancies posted v and unemployment u onto the number of successful matches formed. Let θ be the vacancy-unemployment ratio (v/u, also called market tightness). The constant returns to scale of M implies that the rate at which workers find jobs (finding rate) is just a function of $\theta: f(\theta) = M(v,u)/u = M(\theta,1)$. Likewise, the rate at which firms fill vacancies (filling rate) satisfies:

$$q(\theta) = \frac{M(v,u)}{v} = M\left(1, \theta^{-1}\right) = \frac{f(\theta)}{\theta}. \tag{1}$$

[1] The importance of training costs, or more generally turnover costs, for the dynamics of unemployment was earlier emphasized by Braun (2005), Nagypál (2005), Silva and Toledo (2005), and Yashiv (2005).

The function M is assumed continuously differentiable, increasing in both arguments, and concave. Furthermore, it satisfies the terminal conditions: $M(1,0) = M(0,1) = 0$, and $M_1(0,1) = M_2(1,0) = \infty$. Therefore, workers find it easier to find jobs when vacancies are abundant relative to unemployment (in booms), while firms find it easier to fill their vacancies when the reverse is true (in recessions).

2.2 Bellman Equations

Workers may be in four possible states depending on whether they are employed or not and whether they are eligible for UI benefits or not. Analogously, firms paired with a worker may be in two possible states depending on the worker's UI eligibility state. Contingent on productivity being p, let the values of being an employed worker and an unemployed worker, respectively, be W_p^i and U_p^i, where superscript i denotes the worker's UI eligibility state. Similarly, let the values of a firm matched with a worker with UI eligibility state i be J_p^i. Finally, when the economy experiences a productivity change ($p \to p'$), let the expression $E_p X_{p'}^i$ denote the expected value of X (W, U, or J) conditional on p. Using this notation, the utility values W_p^i, U_p^i, and J_p^i for $i = 0, 1$ are recursively determined by the following Bellman equations.

The value of an unemployed worker who does not collect UI is the present discounted value of the utility from leisure plus the expected gains from transitions to employment, which comes with an arrival rate $f(\theta_p)$, or to a different productivity state, which comes with arrival rate λ. When a transition to employment happens, the worker incurs the training costs k^w :

$$rU_p^0 = \ell + f(\theta_p)(W_p^0 - U_p^0 - k^w) + \lambda(E_p U_{p'}^0 - U_p^0). \tag{2}$$

The analog equation for the value of an unemployed worker collecting UI includes the present discounted value of the utility from both leisure and UI benefits and the expected gains or losses from transitions to employment, UI ineligibility, and a different productivity state. These transitions come at the arrival rates $f(\theta_p)$, d, and λ, respectively:

$$rU_p^1 = \ell + b + f(\theta_p)(W_p^0 - U_p^1 - k^w) + d\left(U_p^0 - U_p^1\right) + \lambda(E_p U_{p'}^1 - U_p^1). \tag{3}$$

The value of an employed worker ineligible for UI is the present discounted value of wages plus the expected gains or losses associated with exogenously losing the job, becoming eligible for UI, and experiencing a productivity change. The arrival rates of these events are s, g, and λ, respectively:

$$rW_p^0 = w_p^0 + s\left(U_p^0 - W_p^0\right) + g\left(W_p^1 - W_p^0\right) + \lambda(E_p W_{p'}^0 - W_p^0), \text{ and} \tag{4}$$

A worker eligible for UI can choose to quit the job to collect UI with probability π instead of continuing with the match. The flow utilities attained with these two choices are respectively the first and second arguments of the max operator in the following equation:

$$rW_p^1 = \max\left\{(1-\pi)\, rU_p^0 + \pi r U_p^1,\ w_p^1 + s\left(U_p^1 - W_p^1\right) + \lambda(E_p W_{p'}^1 - W_p^1)\right\}. \tag{5}$$

Upon quitting, the worker gets the expected utility of being unemployed, otherwise the worker gets the wage plus the expected capital gains or losses associated with losing the job exogenously or experiencing a transition to a different productivity.

Because of free entry, the value of an unmatched firm is zero. The value of a firm employing a worker is the present discounted value of current profits plus the expected gains or losses associated with the worker becoming eligible for UI, the match exogenously dissolving, and productivity changing, which occur with arrival rates g, s, and λ. At any time, a firm can terminate the match, so the values of a matched firm cannot be negative. Given our assumptions, the value of a firm employing a worker ineligible for UI is always positive,[2] but the same cannot be assured if the worker is eligible for UI. Consequently,

$$rJ_p^0 = p - \tau_p - w_p^0 + g\left(J_p^1 - J_p^0\right) - sJ_p^0 + \lambda(E_p J_{p'}^0 - J_p^0). \tag{6}$$

[2]As proven in Proposition 1, $V_p^0 > 0$. Therefore, (11) implies $J_p^0 > 0$

$$rJ_p^1 = \max\left\{0,\ p - \tau_p - w_p^1 - sJ_p^1 + \lambda(E_p J_{p'}^1 - J_p^1)\right\}. \tag{7}$$

Unmatched firms do not post vacancies if there are no expected gains in filling them, so $\theta_p = 0$ if $J_p^0 - k^f \leq 0$. Otherwise, unmatched firms post vacancies until the flow costs of posting a vacancy is equal to the expected gains of filling it, which occurs with an arrival rate $q(\theta_p)$, so $c = q(\theta_p)\left(J_p^0 - k^f\right)$. Using (1) and $f(\theta_p) \geq 0$, these two relations can be summarized as follows:

$$c\theta_p = f(\theta_p)\max\left\{0, J_p^0 - k^f\right\} \tag{8}$$

Since we abstract from the possibility of workers carrying UI eligibility earned from past employment to a new job, firms get the same value from matching with a worker who is collecting UI as one who is not. Hence, it is consistent to assume that there is a single labor market where all workers and all firms interact. We leave the complexities derived from a dual labor market that differentiates workers depending on if they are eligible for UI or not to future research.

2.3 Nash Bargaining

The surplus of an employment match depends on the worker's entitlement to receive UI in case the match were dissolved. If the worker is not eligible for UI, the match surplus is defined as:

$$V_p^0 = W_p^0 - U_p^0 + J_p^0. \tag{9}$$

If the worker is eligible for UI, the match surplus depends on if the potential dissolution would be considered a quit or not by the UI agency. Because the agency imperfectly monitors why employment separations occur, we assume that if a match were to break down while bargaining, the worker would be able to collect UI with probability π. This is the same probability of collecting UI after a voluntary quit because a worker who quits can be considered as one who cannot successfully negotiate a suitable pay raise. This assumption implies that the worker's opportunity cost of employment is $(1-\pi)U_p^0 + \pi U_p^1$.

Consequently, the match surplus when a firm bargains with a worker eligible for UI is:

$$V_p^1 = W_p^1 - (1-\pi)\, U_p^0 - \pi U_p^1 + J_p^1. \tag{10}$$

The generalized Nash solution to the bargaining problem maximizes the weighted product of the match surpluses of the two parties: $\left(J_p^i\right)^{1-\beta}(V_p^i - J_p^i)^{\beta}$, where i takes values 1 or 0 depending on the UI eligibility state, and β denotes the worker's bargaining power. The solution to this problem leads to the familiar sharing rule:

$$J_p^i = (1-\beta)\, V_p^i, \quad \text{for } i = 0, 1. \tag{11}$$

Similarly, when a firm and a worker first meet, the surpluses of both parties must subtract the training costs required for employment relationship to commence, so generalized Nash bargaining implies:

$$J_p^i - k^f = (1-\beta)\left(V_p^i - k\right), \quad \text{for } i = 0, 1. \tag{12}$$

The combination of (11) and (12), together with $k^f + k^w = k$, results in the following split of the training costs:

$$k^f = (1-\beta)\, k, \text{ and } k^w = \beta k. \tag{13}$$

2.4 Equilibrium

A recursive stochastic equilibrium is a set of eleven functions θ_p, w_p^0, w_p^1, U_p^0, U_p^1, W_p^0, W_p^1, J_p^0, J_p^1, V_p^0, V_p^1 that satisfy the Bellman equations (2) to (7), the free entry condition (8), the match surplus definitions (9) and (10), and the Nash bargaining solutions (11) to (13). This system of equations can be reduced to the following four functional equations (see the Appendix):

$$c\theta_p = f(\theta_p)(1-\beta)\max\left\{0, V_p^0 - k\right\}, \tag{14}$$

$$\hat{U}_p = \frac{b + \lambda(E_p \hat{U}_{p'} - \hat{U}_p)}{r + d + f(\theta_p)}, \tag{15}$$

$$\hat{B}_p = \max \left\{ \frac{s\hat{U}_p + \lambda(E_p \hat{B}_{p'} - \hat{B}_p)}{r + s + g}, \pi \hat{U}_p - V_p^0 \right\}, \text{ and} \tag{16}$$

$$V_p^0 = \frac{p - \ell - \beta f(\theta_p)\left(V_p^0 - k\right) + g\hat{B}_p - \tau_p + \lambda(E_p V_{p'}^0 - V_p^0)}{r + s}. \tag{17}$$

Equation (14) is just the free entry condition (8) combined with the Nash bargaining rules (11) to (13). Equation (15) states that the value of UI eligibility for an unemployed worker, $\hat{U}_p \equiv U_p^1 - U_p^0$, is equal to the expected present discounted value of the UI benefits received by an eligible worker during a spell of unemployment. Equation (16) calculates the incremental value of achieving UI eligibility, which depends on if the match breaks down or not because of such eligibility. If the match survives UI eligibility (first term in 16), $\hat{B}_p$ is the difference between the expected present discounted values of the UI benefits to be received upon an exogenous separation of the current match if the worker is eligible for UI ($i = 1$) or not ($i = 0$), which are equal to:

$$B_p^1 = \frac{s\hat{U}_p + \lambda(E_p B_{p'}^1 - B_p^1)}{r + s}, \text{ and} \tag{18}$$

$$B_p^0 = \frac{g\left(B_p^1 - B_p^0\right) + \lambda(E_p B_{p'}^0 - B_p^0)}{r + s}. \tag{19}$$

If UI eligibility kills the employment match (second term in 16), then $\hat{B}_p$ is the expected value of UI eligibility for an unemployed worker minus the value of the match. Finally, equation (17) states that the value of the match between a firm and a worker ineligible for UI is the expected present discounted value of the benefits resulting from the match. These benefits include the labor productivity net of both the value of leisure and the workers' expected value of finding a new job if the match breaks down, $p - \ell - \beta f(\theta_p)\left(V_p^0 - k\right)$, the net benefits from the UI system, $g\hat{B}_p - \tau_p$, and the expected gains from a productivity change, $\lambda(E_p V_{p'}^0 - V_p^0)$. Notice that the UI contribution τ_p detracts from the value of the match exactly in the same way as the value of leisure does, but UI benefits have exactly the opposite effect. This implies that instead of reducing the value of a match, UI benefits make the match more attractive at least before workers become eligible to collect them.

The equilibrium functions θ_p, $\hat{U}_p$, $\hat{B}_p$, and V_p^0 solve (14) to (17), and the remaining functions that define an equilibrium follow recursively from these four. The following proposition establishes the existence and some basic properties of an equilibrium.

Proposition 1: *An equilibrium exists and has the following properties: $U_p^1 > U_p^0$ and $V_p^0 > 0$ for all $p \in P$. Furthermore, if $\pi \leq s/(r+s+g+\lambda)$, then $V_p^1 > 0$ for all $p \in P$.* (see the proof in the Appendix).

As one would expect, an unemployed worker benefits from being eligible for UI, and the match surplus is always positive if the worker is not eligible for UI. Also, if the probability of collecting UI is low when a worker eligible for UI quits a job, then the match surplus remains positive once eligibility is achieved. The following two propositions state additional properties of this equilibrium.

Proposition 2: *If workers are always denied benefits after quitting a job voluntarily ($\pi = 0$) and the UI system is fully funded by UI contribution fees (each worker is charged the expected present discounted value of expected UI benefits), then the level of UI benefits, the duration of these benefits, and the time it takes to become eligible for UI are irrelevant for the determination of output, vacancies, and unemployment. In particular, the introduction or elimination of a fully funded UI system with $\pi = 0$ has no effect on these variables.*

Proof: Since $\pi = 0$, moral hazard quits never occur. Consequently, the expected present discounted value of UI contributions from a newly employed worker is:

$$T_p = \frac{\tau_p + \lambda(E_p T_{p'} - T_p)}{r+s}. \tag{20}$$

For the UI system to be fully funded, T_p must be equal to B_p^0. Comparison of (19) and (20) implies that this equality holds if and only if $\tau_p = g\hat{B}_p$. So, (17) implies that V_p^0 is independent of b, d, and g. Therefore, neither θ_p, nor output, unemployment, or vacancies depends on these variables.

> **Proposition 3:** *As long as there are no moral hazard quits, the equilibrium paths of vacancies and unemployment are independent of the probability of collecting UI after quitting a job voluntarily.*

Proof: Workers have no incentive to quit after they become eligible for UI if and only if the first argument in the *max* operator in (16) does not fall short of the second one, and if this holds for all $p \in P$, π drops out from the system of equations (14) to (17), which determines θ_p and so output, unemployment, and vacancies.

Propositions 2 and 3 taken together provide a set of conditions that render a UI system irrelevant. Like other irrelevance results, such as Ricardian Equivalence, these propositions should be useful to pin point the economic effects of a UI system as violations from their stated premises. In this vein, the effects of a UI system have to be found in incorrectly pricing its insurance services, moral hazard, and risk aversion. More precisely, the adverse effects of UI program on output and employment have to be found either in the way it is financed, which may distort job creation, or in the rules for the provision of benefits, which may engender strategic behavior such as quitting once eligibility is achieved or not searching while benefits last. Also, with risk aversion, the benefits of reducing income uncertainty with UI benefits affect the willingness to work and save in ways that are beyond the scope of the present contribution.

3 Deterministic Equilibrium

To obtain sharp results, this section follows Shimer (2005) and Mortensen and Nagypál (2007) and analyzes the special case where p is deterministic. As argued by Mortensen and Nagypál (2007), the comparative statics analysis of this deterministic model provides a good approximation for the dynamics of the stochastic model if productivity shocks are rare $\lambda \to 0,$ or they occur frequently but their changes are small.

As we will see, the predictions of how the economy reacts to shocks, such as a rise in productivity or an increased generosity of UI benefits, depends crucially on the assumptions we make about the UI contribution fee τ. On one extreme, we can assume that τ is an endogenous variable that adjusts to maintain the UI system fully funded. On the other extreme, we can assume that τ is an exogenous parameter not affected by the shocks considered. For this second assumption to be logically consistent in a general equilibrium context, we need to extend the model and assume that τ includes both UI contributions and general taxes, and that the government provides a public good, which yields separate utility. With this extension, when τ is kept constant while other parameters change, we are implicitly assuming that the government adjusts the provision of the public good endogenously to balance its budget.

3.1 Exogenous τ

With the simplification that p is deterministic, the system of equations (14) to (17) that characterizes an equilibrium can be reduced to the crossing of the two schedules depicted in Figure 1. These schedules relate the value of a newly formed match V^0 with the vacancy-unemployment rate θ as follows. Schedule JC (job creation) represents the free entry condition (14). Its upward sloping shape captures that firms respond to a rise in the expected profits associated with a rise in V^0 by posting more vacancies until the filling rate becomes sufficiently low so that the value of posting a vacancy falls back to zero. Schedule MV (match value) is the representation of the mapping from θ to V^0 implied by the remaining equilibrium equations (15) to (17). Using (14), the absence of productivity shocks and the equilibrium properties that $V^0 > 0$, these equations simplify into:

$$\hat{U} = \frac{b}{r + d + f(\theta)}, \tag{21}$$

$$\hat{B} = \max\left\{\frac{s\hat{U}}{r + s + g}, \pi\hat{U} - V^0\right\}, \text{ and} \tag{22}$$

$$V^0 = \frac{p - \ell - \tau - \beta(1-\beta)^{-1} c\theta + g\hat{B}}{r + s}. \tag{23}$$

Equation (23) implies that there are two reasons why the value of a match V^0 falls with θ as represented in Figure 1. First, as indicated by $\beta(1-\beta)^{-1}c\theta$, workers find jobs easier if there are more vacancies posted, which pulls up the wage due to the improved bargaining power and lowers the match surplus. Second, as captured by $g\hat{B}$, the expected present discounted value of the UI benefits received by an eligible worker in (21) falls with the job finding rate and so with θ. As a result, the value of the jobs needed to gain this eligibility falls as well.

Figure 1 is useful to analyze the qualitative implications of the model. For example, an increase in training costs k shifts the JC schedule up (firms post less vacancies), so it leads to a rise in V^0 and a fall in θ. In contrast, an increase in labor productivity or a fall in the value of leisure shift the MV schedule up (matches become more valuable), so both V^0 and θ increase. A more generous provision of UI benefits (a rise in b or g, or a fall in d) also shifts the MV schedule up because the matches that would make workers eligible for UI benefits become more valuable. Meanwhile, a more expensive UI contribution (a rise in τ) has the opposite effect on the value of matches because it is equivalent to an increase in the value of leisure ℓ. Consequently, in contrast with models where workers do not need to accumulate the employment time to become eligible for UI, a more generous UI system and a more expensive one have competing effects on the vacancy-unemployment ratio θ. Therefore, our model is able to reconcile the sharp response of V^0, θ, v, and u to productivity improvements with a mild response of them to changes in b and τ if these changes tend to happen together.

Depending on whether firms post vacancies in equilibrium or not, and whether workers eligible for UI quit their jobs or not, we can distinguish four possible types of equilibria:

$$\begin{array}{lll}
\text{Normal:} & V^0 > k & \bar{V}^1 \geq 0, \\
\text{Strategic:} & V^0 > k & \bar{V}^1 \leq 0, \\
\text{Phase-out:} & V^0 \leq k & \bar{V}^1 \geq 0, \\
\text{Autarky:} & V^0 \leq k & \bar{V}^1 \leq 0,
\end{array}$$

where $\bar{V}^1$ is the value of continuing the match once UI eligibility is achieved: $\bar{V}^1 =$

$V^0 + [s/(r+s+g) - \pi]\hat{U}$ (Since dissolving the match is an option, the value of the match is $V^1 = \max\{0, \bar{V}^1\}$). Vacancies are posted in equilibrium if and only if the value of a newly formed match exceeds the training costs $(V^0 > k)$, and employed workers have no incentive to strategically quit a job if the value of continuing the match is not exceeded by the expected UI benefits received by quitting ($\bar{V}^1 \geq 0$). In the normal equilibrium, both of these inequalities hold, so new jobs are created and matches survive when workers become eligible for UI. In the strategic equilibrium, the first inequality holds but not the second. So, new employment matches are formed, but they break down as soon as workers become eligible for UI. Finally, in the phase-out equilibrium and the autarky equilibrium, no new jobs are created. If the value of initial employment is positive, the worker-firm pair maintains until an exogenous separation comes in the phase-out equilibrium, while workers quit as soon as they become eligible for UI in the autarky equilibrium.

For V^0 to exceed k, the MV schedule in Figure 1 must cross the JC schedule above k. Consequently, the parameters that shift the MV schedule up and the JC schedule down must be sufficiently large relative to those that have the opposite effects. In particular, we need that the match is sufficiently productive and/or UI benefits sufficiently generous relative to the cost of posting a vacancy, the value of leisure, UI contributions-taxes, and training costs. For a match to survive once the worker is eligible for UI ($\bar{V}^1 \geq 0$), the probability of collecting benefits or the present value of the benefits ($[s/(r+s+g) - \pi]\hat{U}$) must be sufficiently low relative to V^0.

Figure 2 depicts how k and π interact in the determination of the various types of equilibria (see the Appendix for its construction). As long as $\bar{V}^1 \geq 0$ (π is sufficiently low), π has no effect on V^0, so the $V^0 = k$ line is horizontal at the value $\bar{k}$ in which this equality is satisfied. Once π is sufficiently high for workers to quit upon receiving UI eligibility ($\bar{V}^1 \leq 0$), an increase in π makes an employment match more valuable, so the $V^0 = k$ line is upward sloping. As long as $V^0 \leq k$ (k is sufficiently large), k has no local effect on the value of a match, so the line ($\bar{V}^1 = 0$) is vertical at the probability $\bar{\pi}$ in which this equality is satisfied. However, once k is sufficiently low for new jobs to be created $(V^0 > k)$, a reduction in k (downward shift of JC line in Figure 1) reduces V^0

and increases θ. As more vacancies are created, the job finding rate $f(\theta)$ goes up, which reduces *the value of UI eligibility* $\hat{U}$ *and so* $\hat{B}$. Consequently, the reduction in k brings down both the value of continuing a match ($V^0 + \hat{B}$) and the value of quitting ($\pi\hat{U}$) once a worker is eligible for UI, so it has an ambiguous effect on the value of π needed to maintain the equality $\bar{V}^1 = 0$. This implies an ambiguous slope for the line $\bar{V}^1 = 0$ in the region where $V^0 > k$.[3]

Since V^0 is guaranteed to be positive and finite, and moral hazard quits are ruled out if $\pi < s/(r + s + g)$, on regions in Figure 2 where the normal and the phase-out equilibria exist are never empty. However, depending on how productive matches are and how generous the UI system is, moral hazard quits may not happen even if $\pi = 1$, in which case there is no strategic or autarky equilibria for all admissible values of π.

Increasing the net productivity of a match $(p - \ell - \tau)$ or the generosity of UI benefits (a rise in b or g, or a fall in d) makes a newly formed match more valuable, so it shifts up the $V^0 = k$ line in Figure 2. Likewise, even if the worker is eligible for UI, the value of the match increases with net productivity, so a rise in $(p - \ell - \tau)$ shifts the $\bar{V}^1 = 0$ line to the right. However, the generosity of UI benefits has an ambiguous effect on the location of this line, because a more generous UI system raises both the value of continuing the match and the value of quitting. That is, paradoxically, increasing the generosity of the UI system may prevent quits in some regions of the parameter space. In the region where $V^0 \leq k$, the second effect is always dominant, so if b or g rise or d falls, $\bar{V}^1$ goes down and the line $\bar{V}^1 = 0$ line shifts left. However, we cannot be certain that a shift in the same direction occurs in the region where $V^0 > k$.

To study the quantitative predictions of the model, we can apply the standard comparative statics methodology to the equilibrium system of equations (14) and (21) to (23). Of particular interest is the elasticity of the finding rate with respect to labour productivity because it gives a good indication of the amplitude of the labor market cycles generated by productivity shocks (see Mortensen and Nagypál, 2007). As long as $\theta > 0$,

[3] As depicted in Figure 2, in a neighborhood of $(\bar{k}, \bar{\pi})$ the slope of this frontier must be negative (see Appendix).

this elasticity is the following (see the Appendix):

$$\frac{df}{dp}\frac{p}{f} = \begin{cases} \dfrac{p}{p-z}\left[\dfrac{(1-\eta)(r+s)+\beta f}{\eta(r+s+\beta f)} + \dfrac{f}{r+d+f}\dfrac{g\hat{B}}{p-z}\right]^{-1} & \text{if} \quad \bar{V}^1 > 0, \text{ and} \\ \dfrac{p}{p-z}\left[\dfrac{(1-\eta)(r+s+g)+\beta f}{\eta(r+s+g+\beta f)} + \dfrac{f}{r+d+f}\dfrac{g\pi\hat{U}}{p-z}\right]^{-1} & \text{if} \quad \bar{V}^1 < 0. \end{cases} \tag{24}$$

where η is the elasticity of f with respect to θ, and

$$z = \begin{cases} \ell + \tau + (r+s)k - g\hat{B} & \text{if} \quad \bar{V}^1 > 0, \text{ and} \\ \ell + \tau + (r+s+g)k - g\pi\hat{U} & \text{if} \quad \bar{V}^1 < 0. \end{cases} \tag{25}$$

In the absence of training costs and UI, the second term inside the square brackets in (24) drops and $z = \ell$, so the finding rate responds to changes in productivity as derived in Mortensen and Nagypál (2007). As Shimer (2005) pointed out, for reasonable parameter values and a low value of leisure, this response is too small to generate the pronounced cycles in the United States labor market. A high value of leisure, by making the profits margin $p - z$ small, leads to a sufficiently large elasticity of f with respect to p to rationalize the observed responses over the business cycle. For example, Hagedorn and Manovskii (2007) found that when ℓ equals 0.97, model is able to generate the variance of θ observed in the United States business cycles. As long as the equilibrium remains normal or strategic, (25) implies that the profit margin falls with τ and k, so a small profit margin may be compatible with a relatively small value of ℓ if UI contributions, taxes, and training costs are large. However, the effect of UI benefits on the profit margin works in the opposite directions. Positive UI benefits also add the second terms inside the square in (24), which further decreases the elasticity of f with respect to p.

Since p, ℓ, and τ only enter the equilibrium system of equations (14) and (21) to (23) in the determination of the profits margin, an increase in ℓ or τ affects f in the same way as a reduction in p of the same magnitude does. Similarly, UI benefits enter the determination of an equilibrium through the term $g\hat{B}$. Therefore, differentiation of (21) to (23) implies:

$$\frac{df}{db} = \begin{cases} \dfrac{df}{dp}\dfrac{g}{r+d+f}\dfrac{s}{r+s+g} & \text{if} \quad \bar{V}^1 > 0, \text{ and} \\ \dfrac{df}{dp}\dfrac{g}{r+d+f}\pi & \text{if} \quad \bar{V}^1 < 0. \end{cases} \tag{26}$$

As pointed out in the qualitative analysis of Figure 1, a rise in b unambiguously increases θ and so the job finding rate. As long as the increase in b does not trigger moral hazard quits, this implies that, paradoxically, a more generous UI system reduces unemployment. However, a move towards a more generous UI system may trigger a shift from the normal to the strategic equilibrium in which case the level of steady state unemployment will experience a discontinuous jump. Indeed, for the stock of unemployment to be constant, the flows into unemployment must equal the flows out of unemployment. Denoting u^{ss} as the steady state unemployment, we have:

$$\begin{aligned} (1-u^{ss})\, s = u^{ss} f & \qquad \text{in the normal equilibrium, and} \\ (1-u^{ss})\,(s+g) = u^{ss} f & \qquad \text{in the strategic equilibrium.} \end{aligned} \tag{27}$$

Solving for u^{ss} from these equations yields:

$$\begin{aligned} u^{ss} = \frac{s}{s+f} & \qquad \text{in the normal equilibrium, and} \\ u^{ss} = \frac{s+g}{s+g+f} & \qquad \text{in the strategic equilibrium.} \end{aligned} \tag{28}$$

Consequently, the effect of UI generosity on unemployment is non-monotonic. Even though a rise in b reduces u^{ss} through increases in f, it may also trigger moral hazard quits in which case the effective separation rate is $s+g$ instead of s. The discontinuous jump in u^{ss} predicted in this model is an artifact of workers being homogenous. With heterogeneity, a rise in b could increase or reduce u^{ss} depending on how many moral hazard quits it triggers.

3.2 Fully Funded UI System

If the UI system is fully funded, then using an argument analogous to the one in the proof of Proposition 2, the value of τ has to adjust to satisfy:

$$\tau = \begin{cases} g\hat{B} & \text{if} \quad \bar{V}^1 \geq 0, \text{ and} \\ g\pi\hat{U} & \text{if} \quad \bar{V}^1 \leq 0. \end{cases} \tag{29}$$

Therefore, the value of a new match simplifies into

$$V^0 = \begin{cases} \dfrac{p - \ell - \beta(1-\beta)^{-1} c\theta}{r+s} & \text{if} \quad \bar{V}^1 \geq 0, \text{ and} \\ \dfrac{p - \ell - \beta(1-\beta)^{-1} c\theta}{r+s+g} & \text{if} \quad \bar{V}^1 \leq 0. \end{cases} \tag{30}$$

As depicted in Figure 3, the endogenous adjustment of τ to maintain the UI system fully funded induces two different MV schedules depending on whether moral hazard quits occur ($\mathrm{MV^N}$) or not ($\mathrm{MV^S}$). The schedule $\mathrm{MV^N}$ lies above $\mathrm{MV^S}$ because moral hazard quits are costly to the UI system, so high UI contributions need to be imposed. Consequently, both the equilibrium value of a new match and the vacancy-unemployment ratio are higher in a normal equilibrium relative to their counterparts in a strategic equilibrium with the same parameter values: $\left(V^{0N}, \theta^N\right) > \left(V^{0S}, \theta^S\right)$. Using (21) and (22), the conditions for these equilibria to be consistent with the incentive to quit or not are:

$$\left(\pi - \frac{s}{r+s+g}\right) b \leq V^{0N}\left[r + d + f\left(\theta^N\right)\right] \quad \text{implies no incentive to quit, and} \tag{31}$$

$$\left(\pi - \frac{s}{r+s+g}\right) b \geq V^{0S}\left[r + d + f\left(\theta^S\right)\right] \quad \text{implies no incentive to continue.} \tag{32}$$

Notice that b, π, and d have no effect on neither $\left(V^{0N}, \theta^N\right)$ nor $\left(V^{0S}, \theta^S\right)$, and g has no effect on $\left(V^{0N}, \theta^N\right)$ and is inversely related to V^{0S} and θ^S. Therefore, if the UI system is very generous (b, g, and π are high, and d is low), then (32) is satisfied and (31) is violated, so moral hazard quits occur. On the other extreme, if the UI system is very stingy (b, g, and π are low, and d is high), then (31) is satisfied and (32) is violated, so moral hazard quits do not occur. Finally, since $\left(V^{0N}, \theta^{N}\right) > \left(V^{0S}, \theta^{S}\right)$, there is a generic set of intermediate UI systems such that both (32) and (31) are satisfied. In which case, two different equilibria coexist. In one of them, workers quit once they become eligible for UI, and in the other they do not.

The intuition for the generic multiplicity of equilibria when τ endogenously adjust to maintain the UI system fully financed is the following. In the "good" equilibrium, UI contributions are relatively low because workers eligible for UI do not quit, and workers have no incentive of quitting because the UI contributions they have to pay if they remain employed are low. Vice versa, in the "bad" equilibrium, UI contributions need to be large to finance the expensive UI payments since all workers quit upon earning UI eligibility, and these workers have incentives to quit because if they remain employed, they are burdened with large UI contributions.

Figure 4 illustrates the regions of the coexistence of the various types of equilibria for some intermediate values of π. For low values of k, new jobs are created, so the normal equilibrium may coexist with the strategic equilibrium. For high values of k, no new jobs are created, so the phase-out equilibrium may coexist with the autarky equilibrium. Finally, for intermediate values of k, the normal equilibrium may coexist with the autarky equilibrium. In this case, the high UI contributions need to finance the expensive UI system not only give workers incentives to quit once they are eligible for UI, but they also shut down the creation of new jobs.

4 Labor Cycles in Canada and the United States

This section calibrates the model to data from Canada and the United States allowing for the value of leisure in these two countries to be as high as needed to generate a realistic large volatility in the unemployment-vacancy ratio. In particular, we examine if the similar labor market cycles experienced in the two countries can be generated with reasonably similar values of leisure. This proves to be an insurmountable challenge for the standard version of the Mortensen-Pissarides model where entitlement to UI does not need to be earned (see Zhang, 2008). The reason for this difficulty is that Canada has both higher taxes and UI benefits than the United States, which is inconsistent in those models with the similar amplitude of the cycles experienced by unemployment and vacancies. In the present model, taxes and UI benefits affect the opportunity cost of employment in opposite directions, so there is hope that this challenge can be met. This section also examines if the calibrated values of leisure are neither too high for unemployment not to be a major social concern nor too low to make the UI rules trying to prevent moral hazard behavior nonsensical. Finally, we enquire about the response of unemployment to increases in UI benefits and taxes.

The numerical simulations in the calibration use a discrete time version of the model analyzed in Sections 2 and 3 with the following specializations. The matching function is assumed to be Cobb-Douglas: $M(v,u) = \mu u^{1-\eta} v^{\eta}$, where η is the elasticity of the finding rate with respect to the vacancy-unemployment ratio: $f(\theta) = \mu\theta^{\eta}$. Also, consistent with Shimer (2005), labor productivity is assumed to follow a stochastic process that satisfies: $p = \ell + \tau + e^{y}(p^{*} - \ell - \tau)$, where p^{*} is normalized to one, and y is a zero mean random variable that follows an eleven-state symmetric Markov process in which transitions only occur between contiguous states. As detailed in the Appendix, the transition matrix governing this process is fully determined by two parameters: the step size of a transition, Δ, and the probability that a transition occurs, λ.

The model period in the simulations is chosen to be one month, so the real interest rate is set to the conventional monthly rate of 0.4 percent. Even if the model period is one month, consecutive periods are aggregated to construct quarterly series to match empirical moments at that frequency. The calibration targets, summarized in Table 1, aim to replicate the main rates, the labor market flows and, in a stylized way, the key features of the taxation and UI systems in the two countries. The data sources and methodological details in calculating these targets can be found in the Appendix.

Table 1		
Calibration Targets	U.S.	Canada
Monthly real interest rate (r)	0.004	0.004
Average monthly finding rate (f)	0.452	0.309
Average monthly unemployment rate (u)	0.0567	0.0778
Elasticity of finding rate with respect to θ (η)	0.54	0.54
Average vacancy-unemployment ratio θ	1	1
Standard deviation of θ (*quarterly in logs*)	0.151/0.382	0.191/0.367
Standard deviation of of labor productivity (*quarterly in logs*)	0.020	0.021
Autocorrelation of labor productivity (*quarterly in logs*)	0.878	0.876
Average weeks of employment needed for UI eligibility ($1/g$)	20	15
Average weeks before UI benefits expire ($1/d$)	24	33
Average actual UI benefits replacement rate $\left(bu^1/wu\right)$	0.111	0.265
Average tax rate inclusive of UI contributions (τ)	0.30	0.35
Ratio of training costs to quarterly wage rate (k/w)	0.55/0	0.37/0
Standard deviation of real wage w (*quarterly in logs*)	free/0.012	free/0.016

From the labor market, the calibrations aim to replicate the standard deviation and autocorrelation of detrended labor productivity, the average monthly finding and unemployment rates, the standard deviation of the vacancy-unemployment ratio θ, and the elasticity of the finding rate with respect to the market tightness θ. Detrended labor productivity and the finding rates were calculated using the same methodologies as in Shimer (2005). The average unemployment rates are directly calculated using standard data from both countries over the sample periods 1951-2003 for the United States, and 1962-2003 for Canada. The average vacancy-unemployment ratio θ is normalized to be one, which implicitly defines the units in which vacancies are measured and sets the value of μ to be the average monthly finding rate. The standard deviation of θ is used as the gauge of the amplitude of the cyclical fluctuations in the labor market. In the baseline calibration, we follow Mortensen and Nagypál (2007) and target the standard deviation of θ conditional on p, recognizing in this way that productivity shocks are not the only source of cyclical variations. However, to check how much our results depend on this choice, we also report calibrations using the unconditional standard deviation of θ as the target. Finally, the elasticity of the finding rate with respect to the market tightness η is estimated using the method proposed by Mortensen and Nagypál (2007), which uses the law of motion of unemployment at the steady state and sets $\eta = 0.54$ in both countries (see Appendix for details).

From the UI programs, the calibrations aim to be consistent with the average time it takes for a worker to gain UI eligibility, the average duration of UI benefits, and the average actual replacement rates of UI benefits in the two countries. In the United States, UI eligibility takes around 20 weeks of work and the maximum duration of benefits is around 24 weeks[4]. In Canada, both the time needed for eligibility and the maximum duration of benefits have changed over time and currently depend on the unemployment rate in the region of residence. The targets used in the calibration, 15 weeks to gain eligibility and 33 weeks for the maximum duration of benefits, are representative figures over the sample period[5][6]. The average actual replacement rate of UI benefits is defined as the ratio of the average weekly UI benefits paid to unemployed workers over the average weekly insurable earnings paid to employed workers. As explained in the Appendix, these rates are obtained as the product of two ratios. The first ratio is the average weekly UI benefits paid to UI recipients over the average weekly insurable earnings paid to employed workers (b/w). The second ratio is the fraction of unemployed workers receiving UI benefits (u^1/u). Finally, the parameter τ is interpreted not just as UI contributions but also as a general tax, so the government is using a large fraction of τ to finance a public

[4] Card and Riddell (1992) documents that in most states in 1989, UI eligibility requires 20 weeks of work, or the earnings equivalent of 20 weeks of full-time work at the minimum wage, and the maximum duration of benefits lasted around 24 weeks. Similarly, Osberg and Phipps (1995) compares the UI eligibility requirements across states and finds that Texas (relatively less generous state) and New York (relatively more generous state) both set 20 weeks as the minimum employment weeks to qualify in 1992.

[5] As to the entitlement UI weeks in Canada, under the UI Act of 1971, regular UI eligibility required a minimum of 8 employment weeks during the base year. In 1977, the minimum employment weeks was replaced by variable entrance requirement (VER) and increased to 10-14 weeks in 1977, then to 10-20 weeks in 1990. Effective in 1997, the VER based on employment weeks was replaced by an entrance requirement based on hours of work. The minimum hours for regular UI benefits ranged from 420- 700 hours. We link these two VERs by converting hours of work to full-work weeks. For example, 420 hours is equivalent to 10.5 weeks of full-time work.

[6] With respect to the maximum duration of benefits, as reported in Table 4 in “EI Reform and Multiple Job-Holding - November 2001” from Human Resource and Social Development Canada, it was 32.8 weeks in 1995, and 32.9 weeks in 1997.

The legislations regarding the UI duration are rather complicated. For example, Canada has a five-stage benefits structure. The UI duration (after 1989) depends on the previous weeks of work and the prevailing unemployment rate in the region of residence. These complexities make it impossible to calculate an average over the sample period.

good, which yields separable utility to the constituents of the economy[7].

The final parameter that characterizes the UI programs in our model is the probability of collecting benefits after a voluntary quit, π. Because of Proposition 3, this probability is irrelevant in the determination of output, unemployment, and vacancies as long as moral hazard quits do not occur in equilibrium. In our model with homogeneous workers, if moral hazard quits occurred for some realizations of p, it would generate the strongly counterfactual prediction that occasionally all employed workers eligible for UI would quit at the same time. To avoid this prediction, we set π to the maximum probability that prevents moral hazard quits for all realizations of p. Because of Proposition 3, all probabilities lower than this maximum have identical predictions for output, unemployment, and vacancies.[8]

In the baseline calibration, the costs of training a worker in the United States are targeted to match the costs reported in the 1982 Employer Opportunity Pilot Project as fraction of the quarterly wage rate: $(k/w)_{US} = 0.55$. This is the same total cost used in Silva and Toledo (2007). In Canada, Goldenberg (2006) estimates that training costs as a fraction of wages are around two-thirds of those in the United States, which implies $(k/w)_{CA} = 0.37$. To check how much our results depend on the presence of these training costs, we also report calibrations without them. Finally, to determine the bargaining weight of workers, our baseline calibration uses the Hosios rule: $\beta = 1 - \eta$. To check the robustness of this choice, we also conduct a calibration where β is chosen to match the standard deviation of the real wage conditional on p.

The values of $\{r,\ \mu,\ \eta,\ \beta,\ \tau,\ g,\ d\}$ follow directly from the stated targets in Table 1. The values of the remaining parameters $\{s,\ b,\ c,\ \Delta,\ \lambda,\ k,\ l\}$ are obtained with the following iterative procedure. First, an initial guess about the values of these parameters is formed. Using this guess the model is simulated for a long horizon (24,000 months), and the initial guess is then revised. This process continues until the predictions of the

[7] See Annex 4 (Tax Relief: Issues and Options) of "The Economic and Fiscal Update 1999" by the Department of Finance Canada.
Website is http://www.fin.gc.ca/update99/annex_4e.html (downloaded in Jan 2008)

[8] The probability π does affect the real wage, and so its standard deviation. However, this effect turns out to be fairly small in our simulations.

model match the targets of Table 1. In this procedure, the probability of collecting UI after a voluntary quit π is set tentatively to 1. If with this probability moral-hazard quits occur for some values of p ($\bar{V}_p^1 < 0$ for all $p \in P$), then it is revised downward to the maximum value that prevents moral hazard for all possible values of p.

TABLE 2
Calibration Results

	Model 1		Model 2		Model 3		Model 4	
	US	CA	US	CA	US	CA	US	CA
Calibration targets								
std(θ)	0.151	0.191	0.382	0.367	0.151	0.191	0.151	0.191
std(w)	free	free	free	free	free	free	0.012	0.016
k/w	0.55	0.37	0.55	0.37	0	0	0.55	0.37
Parameter Values								
β	0.46	0.46	0.46	0.46	0.46	0.46	0.104	0.363
k	1.594	1.085	1.607	1.092	0	0	1.516	1.078
c	0.132	0.097	0.053	0.047	0.131	0.096	0.612	0.137
s	0.027	0.026	0.026	0.025	0.027	0.026	0.027	0.026
Δ	0.076	0.124	0.14	0.233	0.107	0.178	0.082	0.126
λ	0.3	0.302	0.295	0.29	0.3	0.297	0.3	0.302
b	0.169	0.403	0.171	0.409	0.174	0.41	0.161	0.401
π	1	1	1	1	1	0.48	1	1
l	0.53	0.542	0.609	0.596	0.58	0.576	0.543	0.543
Implications								
Average $\left(W^0 - U^0\right)/w^0$	1.05	0.91	0.89	0.75	0.26	0.28	0.36	0.79
Average $\left(W^1 - U^1\right)/w^1$	0.90	0.38	0.74	0.23	0.13	0.09	0.32	0.34
Critical l (quit to get UI)	0.80	0.57	0.73	0.56	0.54	0.45	0.70	0.58
Semi-elasticity of u w.r.t b	-0.16	-0.28	-0.45	-0.57	-0.16	-0.28	-0.20	-0.29
Semi-elasticity of u w.r.t τ	4.48	5.51	12.89	11.44	4.50	5.59	5.85	5.71

Table 2 displays the calibration results. The upper part of the table describes the specific targets for each particular calibration. Model 1 is our baseline calibration that targets the standard deviation of θ conditional on p, uses the Hosios rule to determine the bargaining weight β, and incorporates positive training costs. Model 2 deviates from model 1 by targeting the unconditional standard deviation of θ. Model 3 removes the training costs from the model. Finally, model 4 departs from the Hosios rule and calibrates

β by targeting the conditional standard deviation of the real wage. The middle part of the table reports the parameter values that fit the model to the observed target values for the two countries. Finally, the lower part of the table shows some of the implications for each model.

All the models are successfully calibrated to our intended targets with similar values of leisure for the United States and Canada. In the baseline Model 1, these values of leisure are respectively 53 and 54.2 percent of average labor productivity. The major reason that these values are so much lower than those in Hagedorn and Manovskii (2007) is that taxes, which are assumed to be 30 percent in the United States and 35 percent in Canada, are a separate variable in our model. Also, as demonstrated by the calibrations of Models 2 to 4, the value of leisure increases if the amplitude of the business cycle is targeted to match the unconditional standard deviation of θ, training costs are removed, or the standard deviation of real wages is targeted instead of imposing the Hosios rule. However, as long as the same model is used in both countries, the values of leisure in the United States and Canada are fairly similar.

All our calibrations are consistent with the widespread social concern about unemployment. For each extra worker unemployed, society as a whole loses the gap between labor productivity and the value of leisure $(p-l)$, which in our calibrations is on average between 40 and 47 percent of labor productivity. In addition, for each extra unemployed worker, the cost of posting vacancies increases by $c\theta$ (which averages c), and the cost of training workers increases by fk. As Tables 1 and 2 imply, these turnover costs vary widely across our simulations, but in all cases they are important. For example, in the Model 1 calibration to data from the United States, for each extra unemployed worker, the average extra cost of posting vacancies increases by 13 percent of labor productivity, and the average extra cost of training workers increases by a further 72 percent of labor productivity. Therefore, in this particular instance, the overall social costs of having an extra worker unemployed adds up to 132 percent of the output the worker would produce if employed.

The costs of unemployment as born by workers are reported in the first two lines of the lower part of Table 2. The first one of these lines reports how many months of pay a worker who is not eligible for UI would be willing to sacrifice to avoid a spell of unemployment. The second line reports the analog number of months for an eligible worker. In the baseline Model 1, an American worker is willing to sacrifice 1.05 months of pay to avoid becoming unemployed before earning UI eligibility and 0.9 months of pay after this eligibility is earned. The analog figures for a Canadian worker are 0.91 months of pay before earning UI eligibility and 0.38 months of pay after eligibility is earned. These costs are substantial but not catastrophic; in particular if the worker falls into the soft safety net of the Canadian UI system. In judging these figures, one must take into account that on average a spell of unemployment lasts only 2.2 months in the United States and 3.2 months in Canada. Also, while unemployed, the worker avoids payroll and income taxes and assigns a significant value to the extra leisure. The main reason that the cost of becoming unemployed remains substantial is that upon losing a job the worker incurs a significant loss in human capital, paid in the model through the one-time training costs. As Table 2 reflects, the costs of losing a job are much lower in Model 3 where there are no training costs. They are also lower in Model 4 where the worker appropriates a much smaller fraction of the employment-match surplus.

The design of the UI systems in Canada and the United States shows a clear concern to avoid workers choosing to become or remain unemployed to collect UI. For example, both systems require a long prior employment period to gain UI entitlement and terminate UI when unemployed workers experience a long unemployment spell, which is when they need UI support the most. Because of the homogeneity of workers, our simulations rule out moral-hazard quits to avoid the untenable prediction that all workers entitled to collect UI quit for some realizations of p. Allowing for sufficient heterogeneity to induce moral-hazard quits for a small but significant fraction of employed workers is beyond the scope of the present contribution. However, Table 2 (third line of the lower part) reports the minimum critical value of leisure that a deviant worker would need to have to quit once UI is earned assuming that the worker expects to collect the UI statutory replacement rate (0.4 for the United States and 0.55 for Canada) with certainty[9]. In the baseline Model 1, American workers are only willing to quit to collect UI if their value of leisure is no less than 80 percent of their labor productivity. Therefore, the concern built in the UI system to avoid moral-hazard unemployment only makes sense if some American workers have values of leisure not too distant from this level. In the alternative models, these critical values of leisure are lower, but except for the model without training costs these critical values are at least 70 percent of labor productivity. In Canada, with a more generous UI system, the analog critical values are much lower, which is consistent with the perception that "abusing" the UI system by some groups of the labor force is a common occurrence in this country.

The last two lines of Table 2 report the semi-elasticities of unemployment with respect to UI benefits and taxes; that is, they report the average percentage increase in unemployment if b or τ increases by 1 percent of labor productivity. As Proposition 2 implies, if b and τ increase in tandem to keep the bottom line of the UI finances untouched, unemployment would not be affected. However, if only one of these variables changes, it does have an effect on unemployment. If b increases, unemployment falls because the jobs needed to earn UI eligibility become more attractive, so the employment-match surplus, and therefore, the number of vacancies posted by firms increase. The bulk of empirical evidence contradicts this negative response. One could realign the model with reality by allowing the increase in b to induce moral-hazard quits. Unfortunately, to accomplish this without falling in the absurd prediction that once all workers eligible for UI quit, it would require to introduce worker or match heterogeneity which is beyond the scope of the present contribution. We leave this interesting extension to future work. If τ increases by 1 percent of labor productivity, unemployment increases. In the Model 1 calibration, this increase is 4.5 percent in the United States and 5.5 percent in Canada. These semi-elasticities are higher than the values typically found in the empirical litera-

[9] In Canada, according to the 1955 Employment Insurance Acts and the subsequent amendments, the statutory UI replacement rate averaged, over the period 1962-2003, 55 (60 for claimants with dependents) percent of the average yearly insurable earnings in the qualifying period. The value of 0.4 in the United States is from Shimer (2005).

ture (see Costain and Reiter, 2005), which finds values around 2. However, the empirical estimates are not uncontroversial because the response of unemployment to increases in τ depends crucially on how the extra money collected from taxes is used. If it is used, at least in part, to raise UI benefits, then the overall responses of unemployment would be predicted to be smaller than the elasticities reported in Table 2. Theoretically, it is easy to perfectly control the increases in τ while keeping b constant, but in empirical work it is difficult to identify increases in τ which are uncorrelated with variables that might affect the attractiveness of finding or keeping jobs.

5 Conclusion

Once workers have to earn their entitlement to UI benefits with prior employment, a generous UI system is an additional benefit to an employment relationship and as such promotes job creation. This positive entitlement effect counteracts the negative effect derived from the high cost of financing a generous UI system. If individuals are risk neutral, the UI system is fairly priced, and the rules of the UI system prevent moral-hazard unemployment, then the presence and/or generosity of the UI system have no effect on output, unemployment, and vacancies. As with Ricardian Equivalence, this irrelevance result should be useful to pinpoint the effects of a UI system to violation of its premises. That is, the economic effects of a UI system arise from three sources: The insurance it provides to smooth the income fluctuations experienced by risk-averse workers. The potential financial unbalance if the UI provisions for a segment of the labor force are subsidized or taxed. And, the potential moral-hazard effects on search behavior, acceptance of job offers, and quit decisions.

In itself, the endogenous entitlement to UI benefits as modelled in this paper does not resolve the current debate about the suitability of the Mortensen-Pissarides model in generating realistic labor market cycles. However, it does bring some insights into the debate. The obvious concern to prevent moral-hazard quits in the design of the UI systems is meaningful only if workers have a sufficiently high value of leisure. Also, since the generosity of a UI system has an ambiguous and potentially small effect on unemployment, one can reconcile a high response of unemployment to changes in labor productivity with a small response of unemployment to changes in UI benefits. In particular, this is important to resolve why Canada and the United States have similar labor cycles even though taxes and UI benefits are considerably higher in Canada.

6 Appendix

6.1 Figures

Figure 1

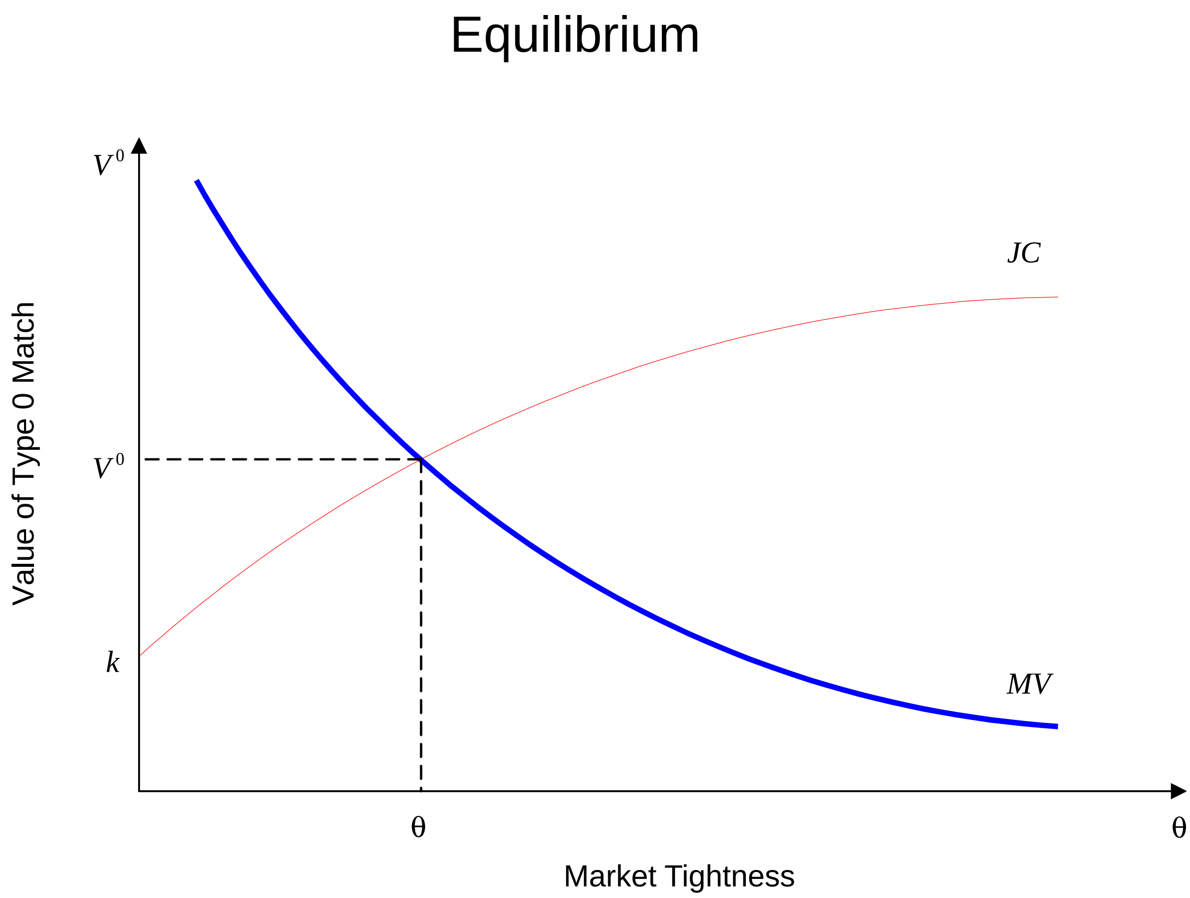


Figure 2

Types of Equilibrium

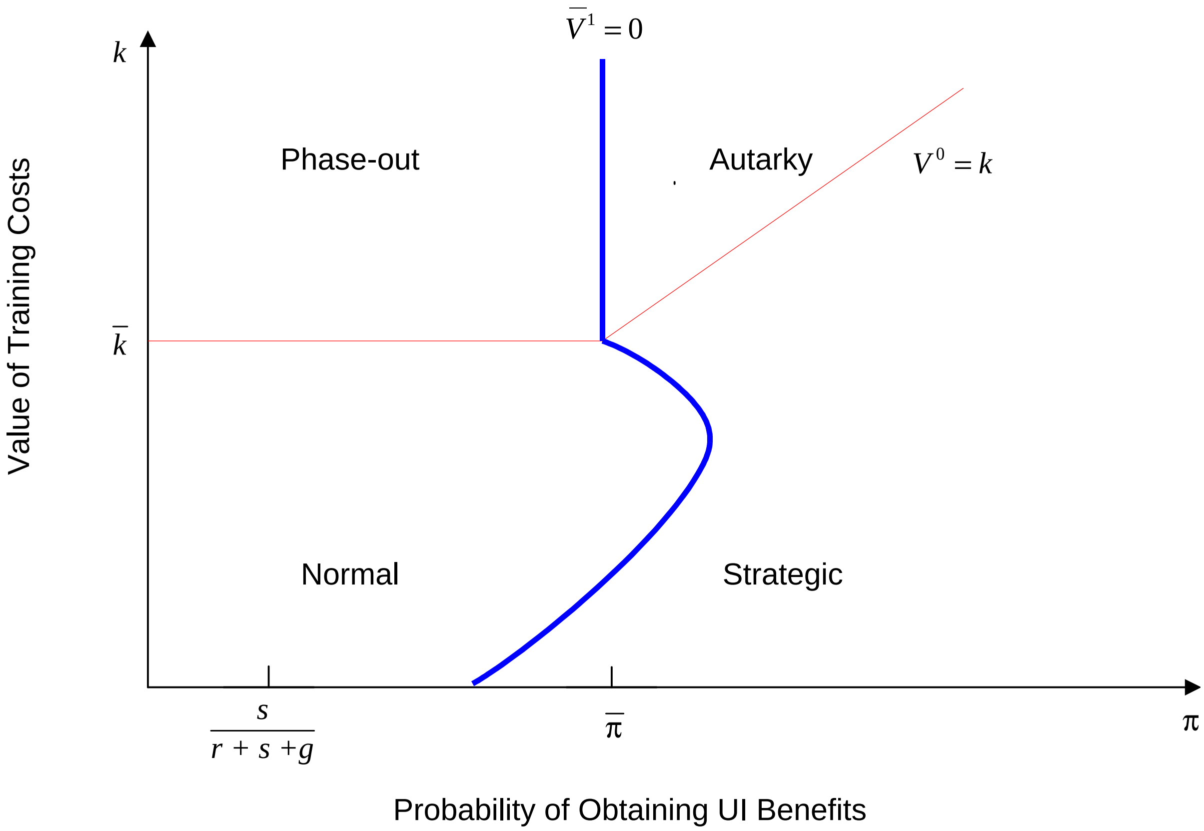


Figure 3

Equilibrium with Fully Funded UI

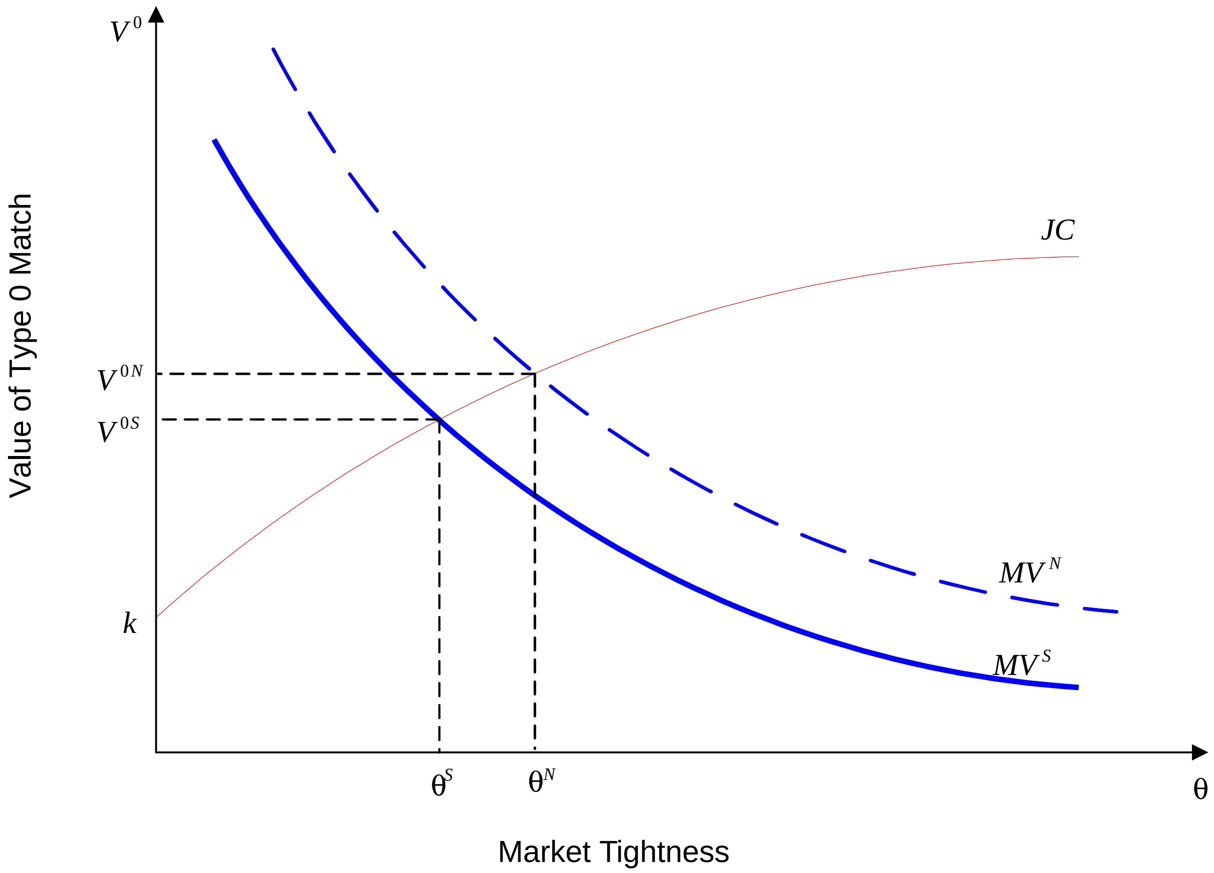
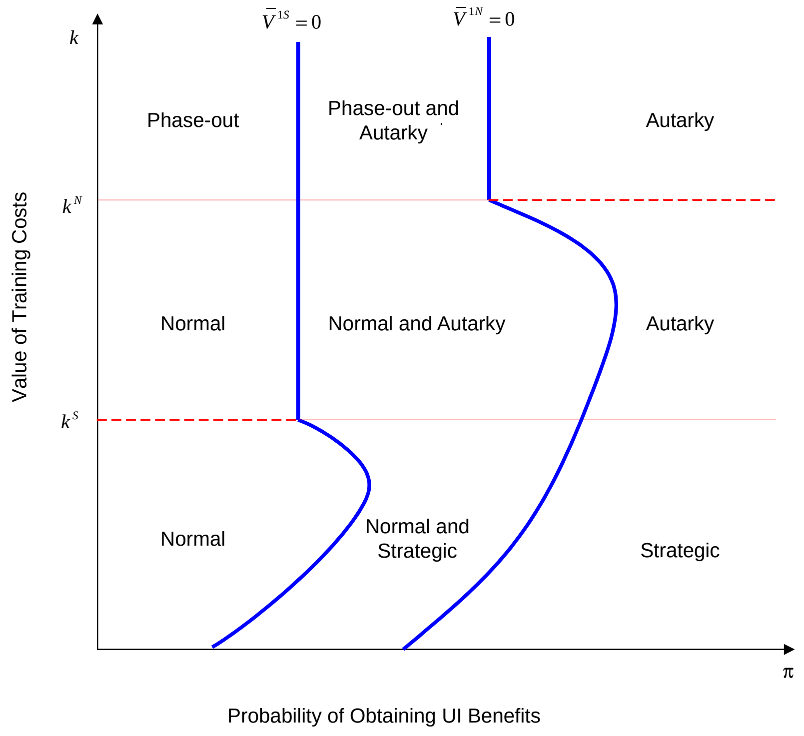


Figure 4

Types of Equilibrium with Fully Funded UI



6.2 Proofs in Section 2

6.2.1 Derivation of the System (14) to (17)

Substitute the value functions U_p^0, U_p^1, W_p^0, W_p^1, J_p^0 and J_p^1 from (2) to (7) into (9) and the definition of $\hat{U}$. Use (11) to (13) to simplify terms and obtain (15) and the following equation:

$$(r+s)\,V_p^0 = p-\ell-\beta f(\theta_p)\left(V_p^0-k\right)+g\left(W_p^1+J_p^1-J_p^0-W_p^0\right)-\tau_p+\lambda(E_pV_{p'}^0-V_p^0). \quad (33)$$

Define $\hat{B}_p = W_p^1+J_p^1-J_p^0-W_p^0$ to obtain (17). Finally, to obtain (16), substitute (4) to (7) into the definition of $\hat{B}_p$ and simplify using that because of Nash bargaining a worker is willing to quit a job if and only if the employing firm is also willing to terminate the match.

Proof Proposition 1 For this proof, it is convenient to rewrite the system of equations characterizing an equilibrium as follows. Define $\theta\left(V^0\right)$ to be the real function that satisfies: $c\theta = f\left(\theta\right)(1-\beta)\max\left\{0, V^0-k\right\}$. Also, define

$$\Gamma\left(V^0\right) = V^0 + \frac{\beta c\theta\left(V^0\right)}{(1-\beta)\left(r+s+\lambda\right)}.$$

The assumed properties of the matching function imply that $\theta/f\left(\theta\right)$ is a strictly increasing function of θ such that $\lim_{\theta\to 0}\left[\theta/f\left(\theta\right)\right]=0$, so $\theta\left(V^0\right)$ is well defined, continuous and increasing, and $\theta\left(0\right)=0$. Therefore, $\Gamma\left(V^0\right)$ has the same properties, and $\Gamma\left(V^0\right)$ is positive if and only if V^0 is positive. Using the above definitions, $z_p = \ell+\tau_p$ and (14), the system of equations (15) to (17) can be rewritten as:

$$\hat{U}_p = \frac{b+\lambda E_p\hat{U}_{p'}}{r+d+f\left(\theta\left(V_p^0\right)\right)+\lambda}, \quad (34)$$

$$\hat{B}_p = \max\left\{\pi\hat{U}_p - V_p^0, \frac{s\hat{U}_p+\lambda E_p\hat{B}_{p'}}{r+s+g+\lambda}\right\}, \text{ and} \quad (35)$$

$$\Gamma\left(V_p^0\right) = \frac{p - z_p + g\hat{B}_p + \lambda E_p V_{p'}^0}{r + s + \lambda}. \tag{36}$$

For a set of functions $\{V_p^0, \hat{B}_p, \hat{U}_p\}$, let $x \in R_+^{3n}$ be the vector $(V_1^0, ..., V_n^0, \hat{B}_1, ...\hat{B}_n, \hat{U}_1, ..., \hat{U}_n)$, and $F(x) \in R_+^{3n}$ be the values of $\{V_p^0, \hat{B}_p, \hat{U}_p\}_{p \in P}$ on the right-hand-side of (34) to (36) when the left-hand-side of these equations is evaluated at x. Define X as the subset of R_+^{3n} that satisfies the following bounds: $x_i \in [0, p_i - z_i + b/(r+d)]$ for $i = 1$ to n, and $x_i \in [0, b/(r+d)]$ for $i = n+1$ to $3n$. The set X is non-empty, closed, bounded, and convex. Also, using $V^0 \leq \Gamma(V^0)$ and $p > z_p$ for all $p \in P$, one can easily check that F maps X onto itself. Consequently, Brower's fixed point theorem implies that F has a fixed point in X.

Given the bounds from the previous paragraph, equations (34) and (35) imply that $\hat{U}_p$, $\hat{B}_p > 0$ if $b > 0$. Similarly, since $\Gamma(V^0)$ is positive if and only if V^0 is positive, (36) implies that $V_p^0 > 0$. Finally, since $V_p^1 > 0$ is equivalent to $\hat{B}_p > \pi\hat{U}_p - V_p^0$, and $\hat{B}_p > s\hat{U}_p/(r+s+g+\lambda)$ because $E_p\hat{B}_{p'} > 0$. V_p^1 can be guaranteed to be positive if $\pi \leq s/(r+s+g+\lambda)$.

6.2.2 Comparative Statics Derivations

Combining (14) to (17), the following equations determine θ:

$$\begin{cases} \dfrac{c\theta}{1-\beta}\left(\dfrac{r+s}{f(\theta)} + \beta\right) = p - \ell - \tau - (r+s)k + \dfrac{sg}{r+s+g}\dfrac{b}{r+d+f(\theta)} & \text{if} \quad \bar{V}^1 \geq 0, \text{ and} \\ \dfrac{c\theta}{1-\beta}\left(\dfrac{r+s+g}{f(\theta)} + \beta\right) = p - \ell - \tau - (r+s+g)k + \dfrac{\pi g b}{r+d+f(\theta)} & \text{if} \quad \bar{V}^1 \leq 0. \end{cases} \tag{37}$$

Applying the Implicit Function Theorem,

$$\frac{\partial\theta}{\partial p} = \begin{cases} \left[\dfrac{c}{1-\beta}\left[\dfrac{1-\eta}{f}(r+s) + \beta\right] + \dfrac{s}{r+s+g}\dfrac{bgf'}{(r+d+f)^2}\right]^{-1} & \text{if} \quad \bar{V}^1 \geq 0, \text{ and} \\ \left[\dfrac{c}{1-\beta}\left[\dfrac{1-\eta}{f}(r+s+g) + \beta\right] + \dfrac{\pi g b f'}{(r+d+f)^2}\right]^{-1} & \text{if} \quad \bar{V}^1 \leq 0. \end{cases} \tag{38}$$

Therefore, using $\partial f/\partial p = f'(\partial\theta/dp)$, and the definitions of $\hat{U}$ in (21), $\hat{B}$ in (22), and η, we obtain

$$\frac{\partial f}{\partial p}\frac{p}{f} = \begin{cases} p\left[\frac{c\theta}{1-\beta}\left(\frac{r+s}{f}+\beta\right)\frac{(1-\eta)(r+s)+\beta f}{\eta(r+s+\beta f)} + \frac{fg\hat{B}}{r+d+f}\right]^{-1} & \text{if} \quad \bar{V}^1 \geq 0, \text{ and} \\ p\left[\frac{c\theta}{1-\beta}\left(\frac{r+s+g}{f}+\beta\right)\frac{(1-\eta)(r+s+g)+\beta f}{\eta(r+s+g+\beta f)} + \frac{fg\pi\hat{U}}{r+d+f}\right]^{-1} & \text{if} \quad \bar{V}^1 \leq 0. \end{cases} \tag{39}$$

Finally, using (37), (21), (22) and $\hat{z}$ in (25), and simplifying yields (24).

If the UI system is fully funded, equation (37) simplifies into:

$$\begin{cases} \frac{c\theta}{1-\beta}\left(\frac{r+s}{f(\theta)}+\beta\right) = p-\ell-(r+s)k & \text{if} \quad \bar{V}^1 \geq 0, \text{ and} \\ \frac{c\theta}{1-\beta}\left(\frac{r+s+g}{f(\theta)}+\beta\right) = p-\ell-(r+s+g)k & \text{if} \quad \bar{V}^1 \leq 0. \end{cases} \tag{40}$$

Therefore, the second terms inside the square brackets in (39) and the effect of UI in the definition of z drop out.

6.2.3 Existence of Different Types of Equilibria.

If $V^0 \leq k$, then $\theta = f = 0$. Therefore, (21) to (23) imply that $V^0 \leq k$ is equivalent to

$$k \geq \begin{cases} \frac{p-\ell-\tau}{r+s} + \frac{g}{r+s}\frac{s}{r+s+g}\frac{b}{r+d} \equiv \bar{k} & \text{if} \quad \bar{V}^1 \geq 0, \text{ and} \\ \frac{p-\ell-\tau}{r+s+g} + \pi\frac{g}{r+s+g}\frac{b}{r+d} & \text{if} \quad \bar{V}^1 \leq 0. \end{cases} \tag{41}$$

Furthermore, if $V^0 \leq k$,using (21) to (23), the condition for workers not to have incentives to remain in their employment matches once they are eligible for UI simplifies with the help of (21) to (23) into:

$$\pi \geq \frac{(p-\ell-\tau)(r+d)}{b(r+s)} + \frac{s}{r+s}\frac{g}{r+s+g} \equiv \bar{\pi}. \tag{42}$$

If $V^0 > k$, for an equilibrium to be consistent with no strategic quits, it must satisfy: $\theta c = f(\theta)(1-\beta)(V^0-k)$, (21) to (23), and $\bar{V}^1 \geq 0$. Once the values for V^0, $\hat{B}$, and $\hat{U}$

are substituted into the definition $\bar{V}^1$, these conditions reduce to:

$$\frac{c\,(r+s)}{(1-\beta)}\frac{\theta}{f\,(\theta)}+\frac{\beta c\theta}{1-\beta}-\frac{gsb}{(r+s+g)\,[r+d+f\,(\theta)]} = p-\ell-\tau-k\,(r+s)\,, \text{ and} \tag{43}$$

$$\frac{s}{r+s+g}+\left[k+\frac{\theta c}{(1-\beta)\,f\,(\theta)}\right]\frac{r+d+f\,(\theta)}{b} \geq \pi \tag{44}$$

Therefore, the $V^0+\hat{B}=\pi\hat{U}$ line must satisfy (43) and (44) with equality, or, after simplifying, (43) and the following equation:

$$\pi=\frac{s}{r+s}\frac{g}{r+s+g}+\left(p-\ell-\tau-\frac{\beta c\theta}{1-\beta}\right)\frac{r+d+f\,(\theta)}{b}$$

Since $\theta/f(\theta)$ is increasing with θ, (43) defines θ as an implicit decreasing function of k. Therefore, in Figure 2, the line $\bar{V}^1=0$ has a negative slope in the region where $V^0>k$ if the expression inside the first parenthesis in (43) is positive, which must be true around the point $\left(\bar{k},\bar{\pi}\right)$ where $\theta=0$. In other regions, the slope of this line is ambiguous.

If the UI system is fully funded, (41) simplifies to

$$k \geq \begin{cases} \dfrac{p-\ell-\beta\,(1-\beta)^{-1}\,\theta^N}{r+s}=V^{0N} & \text{if} \quad V^{0N}+\hat{B}^N\geq\pi\hat{U}^N, \text{ and} \\ \dfrac{p-\ell-\beta\,(1-\beta)^{-1}\,\theta^S}{r+s+g}=V^{0S} & \text{if} \quad V^{0S}+\hat{B}^S\leq\pi\hat{U}^S. \end{cases} \tag{45}$$

While the conditions for strategic quits to occur or not simplify into (31) and (32).

6.3 Calibration Strategy and Methodology

6.3.1 The Stochastic Process of Productivity

The random variable y takes values in a finite-ordered (11 states) set of real numbers Y defined as follows:

$$Y=\{-5\Delta,-4\Delta,...,0,...,4\Delta,5\Delta\}.$$

where $\Delta>0$ is called the step size of a transition and measures the amplitude of changes in the cyclical component of log of labor productivity. At the beginning of each period (the numerical simulations use a discrete time version of the model), the value of y takes a new value y' with probability λ or stays unchanged with complementary probability. The new value y' moves up or down by one step Δ as follows:

$$y' = \begin{cases} y + \Delta, & \text{with prob } \frac{\lambda}{2}(1 - \frac{y}{5\Delta}), \quad \text{and} \\ y - \Delta, & \text{with prob } \frac{\lambda}{2}(1 + \frac{y}{5\Delta}). \end{cases}$$

Remark that the probability of moving up (down) is decreasing (increasing) in the current value of y, and it is zero at $y = 5\Delta$ ($y = -5\Delta$), so $y' \in Y$ for all $y \in Y$. The parameters Δ and λ are calibrated to match the standard deviation and the autocorrelation coefficient of productivity (quarterly in logs).

6.3.2 Estimation of η

Following Mortensen and Nagypál (2007), η is estimated using the law of motion of unemployment at the steady state: The flows out of unemployment (also the number of successful matches) equal the flows into unemployment. Therefore, we have that

$$m(u, v) = s(1 - u). \tag{46}$$

Using the Cobb-Douglas specification of m and taking logarithms on both sides of equation (46), we obtain $\ln \mu + \eta \ln v + (1 - \eta) \ln u = \ln s + \ln(1 - u)$. Taking derivatives, it follows that

$$\frac{\partial \ln v}{\partial \ln u} = -\frac{1}{\eta} \left(\frac{u}{1 - u} + 1 - \eta \right). \tag{47}$$

The empirical counterpart of $\partial \ln v / \partial \ln u$ is the slope of the Beveridge curve, which can be calculated regressing $\ln v$ on $\ln u$, that is, $\partial \ln v / \partial \ln u = \rho_{vu} \sigma_v / \sigma_u$. Combining this with (47) implies that

$$\rho_{vu} \frac{\sigma_v}{\sigma_u} = -\frac{1}{\eta} \left(\frac{u}{1 - u} + 1 - \eta \right) \tag{48}$$

Using the data moments in Shimer (2005) and Zhang (2008) (see Table 1 in these two papers) to calculate the left-hand side of (48) and the average unemployment rates over the sample period (7.78 percent over the period of 1962-2003 in Canada and 5.67 percent over the period of 1951-2003 in the United States), we obtain that the estimated value of η is 0.54 in both countries.

6.3.3 Actual UI Benefits Replacement Rate

The actual UI benefits replacement rate is measured as the ratio of the average weekly UI benefits paid to unemployed workers over the average weekly earnings paid to employed workers. To calculate this ratio, we multiply the fraction of unemployed workers who receive UI benefits times the ratio of the average weekly UI benefits paid to UI recipients over the average weekly earnings paid to employed workers (see Appendix 6.4.2 and for data sources):[10]

Actual UI Benefits Replacement Rate, 1972-2003

	Actual UI Replacement Rate =	$\frac{\text{UI recipients}}{\text{Unemployment}}$ ×	$\frac{\text{Average UI benefits of recipients}}{\text{Average earnings of workers}}$
Canada	0.265	0.653	0.406
U.S.	0.111	0.310	0.357

6.4 Data Sources

6.4.1 Variables in the labor market

1. Unemployment: Statistics Canada, CANSIM II, V2062814 over the period of 1976-2005; the Historical Labor Force Statistics, Catalogue, Vol.1971-1974 over the period of 1962-1974; and Statistics Canada, Labor Force, Catalogue 71-001, Vol. 1975 for the year 1975.

2. Vacancy: Statistics Canada, CANSIM II, V3687 (1981=100) over the period of 1962-1988 and V3759 (1996=100) over the period of 1981-2003.

3. Job-finding rate: The job-finding rate is computed by equation (1) in Shimer (2005). The data required are from Statistics Canada, CANSIM II, V2064893 and V3433878 over

[10] Hall (2005) argues that the replacement rate in the United States is around $10-15$ percent in recent years.

the period of 1976-2003; the Historical Labor Force Statistics, Catalogue 71-210, Vol. 1971-1974 over the period of 1962-1974; and Statistics Canada, Labor Force, Catalogue 71-001, Vol. 1975 for the year 1975.

4. Separation rate: The separation rate is constructed by equation (2) in Shimer (2005). The data required are from Statistics Canada, CANSIM II, V 2064890 over the period of 1976-2005; the Historical Labor Force Statistics, Catalogue 71-210, Vol. 1971-1974 over the period of 1962-1974; and Statistics Canada, Labor Force, Catalogue 71-001, Vol. 1975 for the year 1975.

5. Labor productivity: Labor productivity is measured as real output per worker in industries excluding agriculture and public sector, 1992=100. The data source for GDP is Statistics Canada, CANSIM II, V328916, V328932, V329123, V329126, V329144, V329155, V329156, V329157, V329170, V329217, V329218 over the period of 1961-1996, and V2035520, V2035521, V2035524, V2035541, V2035545, V2035549, V2035736, V2035737, V2035738, V2035758, V2035773, V2035783, V2035794 over the period of 1997-2003. The data source for employment is Statistics Canada, CANSIM II, V2057606, V2057607, V2057608, V2057609, V2057611, V2057612, V2057613, V2057614, V2057615 over the period of 1987-2003; Statistics Canada, Labor Force, Catalogue 71-001, Vol. 1960-1966 over the period of 1962-1966; and the Historical Labor Force Statistics, Catalogue, 71-201, Vol. 1971-1974 and Vol. 1986-1987 over the period of 1966-1986.

Data on the variables listed above in the United States is from Shimer (2005).

6. Unemployment rate: In Canada, the data required are from Statistics Canada, CANSIM II, V2062815 over the period of 1976-2003; Data over the period of 1962-1975 are calculated using the data on unemployment and employment from CANSIM II as mentioned above. In the United States, they are from the Bureau of Labor Statistics, Series LNS14000000 over the period of 1951-2003.

7. Real wage: Measured as the average nominal wage per worker in all industries divided by the implicit GDP deflator. The implicit GDP deflator is calculated as nominal GDP divided by real GDP. In Canada, the data required are from Statistics Canada, CANSIM II, V500266 and V1996471 for the nominal wage, V498943 for the real GDP

and V498086 for the nominal GDP over the period of 1962-2003. In the United States, the data are from the Bureau of Labor Statistics, Series PRS85006063 for the nominal compensation, PRS85006013 for employment, PRS85006043 for the real GDP and PRS85006053 for the nominal GDP.

6.4.2 Indicators of Generosity of Unemployment Insurance System

1. UI eligibility rate (u^1/u): Measured as the ratio of the monthly number of regular UI recipients to the monthly number of unemployment. In Canada, the data on the monthly regular UI recipients are from Statistics Canada, CANSIM II, V384652 and V2062814 over the period of 1976-2003. In the United States, this ratio is directly calculated from data in Table C.1 in Wayne Vroman (2004) over the period of 1967-2003.

2. UI benefits replacement rate (b/w): Measured as the ratio of the average weekly regular UI benefits paid to UI recipients to the average weekly earnings paid to employed workers on a gross basis. In Canada, the data required are from Statistics Canada, CANSIM II, V384494 for average weekly regular UI benefits, and V75249, V729405, V1597104 for the average weekly earnings over the period of 1972-2003. In the United States, this ratio is directly calculated from U.S. Department of Labor Employment and Training Administration (DLETA hereafter) annual report and financial data (Taxable and reimbursable claim, *Column 33*) over the period of 1972-2003. Download from the website http://workforcesecurity.doleta.gov/unemploy/hb394.asp in July 2007.

3. Minimum employment weeks to qualify for the regular UI benefit: In Canada, the data required are from Statistics Canada, Table 1 in Publication 11F0019MPE No.125 over the period of 1962-1994. After 1994, the minimum employment hours (or equivalently weeks) became depending on the regional unemployment rate. According to the relevant information from Human Resources and Skills Development Canada (see example from http://srv200.services.gc.ca/iiws/EIRegions/toronto.aspx?rates=1), matching the long run unemployment rate 7.78%, 15 weeks is picked to be the approximate estimate. In the United States, see detailed discussion in Card and Riddell (1992) and Osberg and Phipps (1995): Publication IN-AH-223E-11-95 from Human Resources and

Social Development Canada (HRSDC hereafter).

4. Entitlement weeks of regular UI benefit: In Canada, the data required are from Table 4 in “EI Reform and Multiple Job-Holding - November 2001” released by the HRSDC. The same estimate can be seen in Belzil (2001), "Unemployment Insurance and Subsequent Job Duration: Job Matching versus Unobserved Heterogeneity," *Journal of Applied Econometrics 16(5).* In the United States, the data are directly calculated from DLETA, annual report and financial data (Taxable and reimbursable claim, *Column 27*) over the period 1951-2003. Download from the website http://workforcesecurity.doleta.gov/unemploy/hb394.asp in July 2007.

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