University of Toronto
Department of Economics

Working Paper 398

Information Externalities and Intermediaries in Frictional Search Markets

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March 21, 2010

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March 12, 2010

Abstract

In frictional matching markets, buyers incur discrete inspection costs when assessing the suitability of goods on offer, and sellers incur discrete 'show' costs. This paper studies how intermediaries can help reduce these costs. Intermediaries, whose value derives from inventory, learning and memory, are shown to exist if goods are sufficiently heterogeneous. Intermediaries may either be firms that buy goods and hold inventory or brokers who search on behalf of their clients but do not buy or hold inventory. The parameter space, in terms of the ratio of inspection to show costs, naturally separates into two regions where firms exist versus where brokers exist.

*We thank Ettore Damiano, Jean Guillaume Forand, Robert McMillan, Shouyong Shi, Asher Wolinsky, Andriy Zapechelnyuk, and seminar participants at Peking University, Queen's University and University of Toronto for helpful comments and suggestions. The first author is grateful to the Alexander von Humboldt Foundation for financial support and to the Chair of Economic Theory II at University of Bonn for its hospitality where part of this work was completed. Both authors also thank SSHRC for financial support.

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1 Introduction

In frictional matching markets, heterogenous buyers and sellers engage in costly search for appropriate trading partners. As a stylized characterization, when a buyer and a seller meet, the buyer incurs a discrete inspection cost to determine his valuation of the good, and the seller likewise incurs a discrete 'show' cost. During the inspection process, the seller collects information about the buyer's preferences and the buyer finds out whether the good on offer is suitable. If the good turns out not to be appropriate, the information the buyer and the seller gathered during costly inspection adds no value to their future search,[1] although this information is often valued by other market participants. Therefore, from a social perspective, there is insufficient information generated and transmitted through private search, which implies that there is room for intermediaries to emerge and internalize this information externality.

Two types of intermediary, brokers and firms, are prevalent in frictional matching markets. Brokers, who work for a commission to connect buyers with sellers, actively seek clients to represent and search on their behalf, without directly buying or selling goods. In real estate, labor and securities markets, for instance, brokers are widespread and clearly play an important role in reducing search frictions when facilitating trades. Different from brokers, firms serving as intermediaries (henceforth just 'firms') buy goods directly from sellers to resell to buyers and profit from the spread between bid and ask prices. Although less visible to the public, such firms also help reduce search frictions, particularly in industries where producers have to find customers for their new products. For example, in the pharmaceutical industry, the most effective but costly form of marketing for a new prescription drug is so-called 'detailing,' which involves personal visits to a doctor by a sales representative (Gagnon and Lexchin 2007; Mizik and Jacobson 2004). Many new drugs and treatments are discovered in universities and small biotechnology firms, and these small producers typically find it too expensive to bring these drugs to market directly. Instead they form marketing alliances with, or license their patents to, or sell the company/product to pharmaceutical companies. The pharmaceutical companies then market the drug or treatment to doctors.

This paper studies how intermediaries, either as firms or brokers, can improve social

[1] This is the famous "no recall" theorem in the sequential search literature when the underlying distribution is known; it does not apply if the underlying distribution is unknown (DeGroot 1970).

welfare in search markets with heterogeneous market participants. Consider first firms that buy goods from sellers and resell them to buyers. For simplicity, suppose a firm can hold at most two goods in its inventory. After the firm inspects a good in the sellers' market, it learns the type of the good and which type of buyer the good will appeal to. After the firm obtains two different types of good from the sellers' market, it will go to the buyers' market to show the goods to buyers. When the firm incurs a show cost and a buyer incurs an inspection cost to see the first good on offer, the firm learns the buyer's type. If the buyer buys the first good, the transaction ends, while if the buyer rejects the first good, the firm learns about the buyer's preferences from the first showing to determine whether it is worthwhile to show additional goods from its inventory to the same buyer, thus economizing on its show cost and the buyer's inspection cost. Firms will exist and improve social welfare when there are many different types of goods in the market and the inspection cost exceeds the show cost.[2]

The above reasoning also applies to brokers.[3] Initially, these brokers have to search for buyers. After learning the preferences of her clients, the broker will incur inspection costs to search for goods on their behalf. When she finds a suitable good for one of her clients, the seller of the good will incur the relevant show cost and the client will incur the inspection cost to verify whether there is a match. The broker collects fees or commissions only when her client finds the good appropriate. Therefore, she will not recommend unsuitable goods to her clients, thus saving both inspection costs for her clients and show costs for the seller. Brokers exist and improve social welfare when there is sufficient heterogeneity in goods and the show cost exceeds the inspection cost. Thus there is an endogenous division in the parameter space relating to the existence of firms versus brokers.

We build on several large literatures. First, there is the literature on private search in frictional matching markets (see a recent comprehensive survey by Rogerson, Shimer and

[2]The explanation as to why the inspection cost must exceed the show cost is as follows. The spread between what a firm has to pay a seller and what it can charge a buyer has to cover the inspection and show costs of the firm for effecting that transaction. The buyer can avoid paying the spread by approaching sellers directly but will have to incur additional expected inspection costs. To a first approximation, the increase in the expected number of inspections is equal to the expected number of showings by the firm. So if the inspection cost is less than the show cost, it is not worthwhile for buyers to deal with the firm.

[3]In this paper, we will deal primarily with buyers' brokers. At the end of the paper, we will point out the difference between buyers' and sellers' brokers and how our analysis can apply to the sellers' brokers.

Wright 2005). Most of this literature deals with flow and delay costs of search rather than discrete inspection or show costs. A notable exception is Atakan (2006), though he does not study the role of intermediaries. The social inefficiency of private search is well known, and has generated a substantial literature on social learning.[4] Our work is complementary to this social learning literature.

The literature on search with intermediaries, starting with Rubinstein and Wolinsky (1987), focuses primarily on the role of intermediaries in reducing flow and delay costs.[5] In Rubinstein and Wolinsky, homogenous sellers with unit supply want to trade with homogenous buyers with unit demand. Intermediaries also carry one unit of the good but can meet customers faster than buyers can meet sellers directly. In a richer setting with heterogeneous goods and tastes, Johri and Leach (2001) show that intermediaries can improve match quality and reduce delay costs if they can carry two units. An important paper by Shevchenko (2004) endogenizes both the number of intermediaries and the size of inventory they carry and characterizes the equilibrium price distribution.

All these papers assume that matching quality is freely revealed without inspection immediately after market participants meet. In contrast, market participants in our model have to incur a discrete inspection or show cost in order to find if there is a match. As a result, the role of inventory in our model is different from that explored by Johri and Leach, Shevchenko and others, and it closely interacts with learning and memory. After the firms shows a first good to the buyer, it knows whether the other good in its inventory is suitable or not for the buyer. When a broker finds out the preferences of her clients, she knows from inspecting a good whether there is a fit with any of her clients. Thus in our model, inventory *per se* does not automatically result in a cost advantage for intermediaries. It is the combination of inventory with learning and memory that allows the intermediaries to economize on show and inspection costs. Since both discrete and flow search costs are important forms of search frictions, our analysis is complementary to the existing literature which emphasizes flow and delay costs.

Another branch of the literature focuses on the role of intermediaries in mitigating bilateral contracting problems which arise from the search environment (e.g. Spulber 1996, 2009;

[4] Bikhchadani, Hirshleifer and Welch (1998) provide an excellent survey.

[5] In a related literature starting with Koyotaki and Wright (1993), monetary theorists have studied the role of money in facilitating trade in frictional matching models (e.g. Trejos and Wright 1995; Shi 1995).

Diamond 1984; Biglaiser 1993; Li 1998; Rust and Hall 2003). Our theory is also related to but different from the department store theory of a firm (e.g. Fischer and Harrington 1996; Fujita and Thisse 2002). Under a department store theory, a firm can minimize search (travelling) costs for buyers by locating many different types of good in one location. In our setup, the goods do not have to be in a single location. Firms and brokers in our model also play a more active role, due again to their use of learning and memory, than a department store.

The paper is organized as follows. We first present the benchmark case of search without intermediation. Next, we study the case of intermediation by firms. Finally, we study the case of intermediation with buyers' brokers, followed by some concluding remarks.

2 Search without intermediaries

Consider a market where buyers search for a durable good or a service provided by sellers. Each seller has one unit for sale, and each buyer wants to buy at most one unit. The environment considered here is standard and follows the sequential search literature. We will primarily focus on the limit solution when the discount rate r goes to zero.

Suppose there are n types of buyers and sellers, with an equal fraction of each type in the population. Consider a match between a randomly chosen buyer and a randomly chosen seller. If the buyer's type (preference) matches the type of the good for sale, the value of the good to the buyer is 1. Otherwise, the good has zero value to the buyer. The use value of the good to the seller is zero.

Before seeing a good, both the buyer and the seller do not know whether the buyer's preference matches the type of the good. They can find out the value of the match by inspectiing the good. Every time a buyer inspects a good, the seller has to pay a show cost c_s and the buyer has to pay an inspection cost c_b. These discrete costs are incurred only if a buyer agrees to inspect a good, which is different from the standard sequential search formulation that emphasizes flow costs when participants search. We use discrete costs per inspection to emphasize that if an intermediary wants to show multiple goods to a buyer, it has to pay multiple show costs and the buyer will also incur multiple inspection costs. In contrast, a flow cost specification will build in automatic economies of scale for an intermediary, which will obscure the issue considered here.

New buyers and new sellers, drawn uniformly from the n types, arrive at the market sequentially. Once they arrive at the market, they search for trading partners. When a seller meets with a buyer, if the seller has the type of good that the buyer wants to buy and they are able to negotiate a sale, both parties will leave the market permanently. We consider only steady states where the stocks of buyers and sellers do not vary over time. Let B be the stock of buyers and S the stock of sellers in the market. In each instant, $M(B,S)$ buyers will match with the same number of sellers. The matching function, $M(B,S)$, is increasing in both arguments and has constant return to scale. Let $\theta = B/S$ denote the market tightness.

The arrival rate of a match for a seller is

$$m(\theta) \equiv \frac{M(B,S)}{S} = M(\theta, 1),$$

and the arrival rate of a match for a buyer is

$$\frac{M(B,S)}{B} = \frac{S}{B} m(B/S, 1) = \frac{m(\theta)}{\theta}.$$

The length of a period is $\triangle$ and both parties discount the future with common discount rate r. Then when $\triangle$ is small, a buyer gets matched with a seller with probability $m(\theta)\triangle/\theta$, and a seller gets matched with a buyer with probability $m(\theta)\triangle$.

The continuation payoff U for a buyer being unmatched at the end of period $\triangle$, when $\triangle$ is small, is given by

$$U = \frac{1}{1+r\triangle}\left[\frac{m(\theta)}{\theta}\triangle\left(-c_b + \frac{1}{n}(1-t)\right) + \left(1 - \frac{1}{n}\frac{m(\theta)}{\theta}\triangle\right)U\right]. \tag{1}$$

In the above, c_b is incurred upon meeting a seller and t is the transaction price. The sale occurs only if the inspection shows that there is a match. Otherwise, the buyer returns to the market. The continuation payoff V for a seller being unmatched at the end of period $\triangle$, when $\triangle$ is small, is given by

$$V = \frac{1}{1+r\triangle}\left[m(\theta)\triangle\left(-c_s + \frac{1}{n}t\right) + \left(1 - \frac{1}{n}m(\theta)\triangle\right)V\right]. \tag{2}$$

In the above, c_s is incurred upon meeting a buyer. The sale occurs only if the inspection shows that there is a match. Otherwise, the seller returns to the market.

We assume that the transaction price t is determined by Nash bargaining with equal bargaining power. Moreover, to avoid the hold-up problem and a non-existence of equilibrium with trade when r approaches zero, we assume that the price t is negotiated before inspection,[6]

$$t = \arg\max_p \left[-c_b + \frac{1}{n}(1-p) + \frac{n-1}{n}U - U \right] \left[-c_s + \frac{1}{n}p + \frac{n-1}{n}V - V \right].$$

It follows that the transaction price is set at

$$t = \frac{1}{2}(1 + V - U + nc_s - nc_b). \tag{3}$$

We complete the model by imposing a free entry condition for producers. Let K be the cost to a producer of producing a good. Then free entry of producers implies $V = K$. In order for the market to exist, we assume throughout of the paper that

$$1 - nc_b - nc_s > K. \tag{4}$$

Definition 1 *A steady-state search equilibrium without intermediary is defined by the stocks of market participants (B, S), continuation payoffs (U, V), and market price t such that*

(a) steady-state conditions (1) and (2) hold;

(b) price t is determined by (3);

(c) free entry (market clearing) condition $V = K$ holds;

(d) buyers participate: $U \geq 0$.

The two functional equations (1) and (2) can be rearranged into

$$\begin{aligned} rU &= \frac{m(\theta)}{\theta}\left[-c_b + \frac{1}{n}(1 - t - U) \right], \\ rV &= m(\theta)\left[-c_s + \frac{1}{n}(t - V) \right]. \end{aligned}$$

Substituting t into the functional equations, we can solve U and V as

$$\begin{aligned} U &= \frac{m(\theta)}{m(\theta) + \theta m(\theta) + 2n\theta r}(1 - nc_b - nc_s), \\ V &= \frac{\theta m(\theta)}{m(\theta) + \theta m(\theta) + 2n\theta r}(1 - nc_b - nc_s). \end{aligned}$$

Letting $r \to 0$, we have

$$U = \frac{1}{1+\theta}(1 - nc_b - nc_s), \tag{5}$$

$$V = \frac{\theta}{1+\theta}(1 - nc_b - nc_s). \tag{6}$$

[6]Both the hold-up problem in this class of models (see Spulber 2009) and the non-existence problem of the equilibrium with trade as r approaches zero are well known (e.g. Camera 2001). Dealing with these problems detracts from our concern here.

It is easy to see that $U \geq 0$ whenever condition (4) holds.

The ex-ante social welfare for buyers and sellers are:

$$U + V = 1 - nc_b - nc_s. \tag{7}$$

Notice that when $r \to 0$ there is no time cost. Thus, the above result is expected since the expected number of inspections before a successful match is n. Furthermore, the net social surplus is shared according to market tightness θ: the seller gets a larger share if the market condition is more favorable to the seller (i.e. a higher θ). The market tightness θ is recovered from equation (6) and the free entry condition $V = K$.

3 Search with firms

Again suppose there are n types of goods and buyers with equal proportion. We add firms which buy goods from sellers and sell them to buyers. There are two physically distinct markets where trade occurs. In the sellers' market, firms buy goods from sellers. In the buyers' market, firms sell goods to buyers. Buyers, sellers and firms can visit either market at any time. Each participant can visit only one market at a time. We say a firm completes a "transaction" when it successfully buys a good from a seller in the sellers' market and then successfully sells to a buyer in the buyers' market.

Assume that a firm can carry at most two goods in its inventory. We assume for now that a firm wants to have two different types of goods in its inventory before going to the buyers' market. Upon selling one good, it will return to the sellers' market to obtain a good which is different from the type which is remaining in its inventory. After it buys another good such that it again has two different types of goods in its inventory, the firm will return to the buyers' market and so on.

We also assume for now that (i) buyers and sellers will not trade directly, (ii) sellers will not pretend to be firms with two goods, and (iii) buyers will not pretend to be firms with one good. At the end of this section, we will specify conditions under which, in the search equilibrium with firms, the incentive conditions (i)-(iii) for buyers and sellers hold and firms indeed will not go to the buyers' market unless they have two different types of goods in their inventory.

3.1 Matching process

We use the following notation. As before, θ defines the market tightness of the overall market $\theta = B/S$. Now we have two separate submarkets. Let F_s denote the number of firms with one good, and F_b denote the number of firms with two goods. Define θ_s the market tightness of the sellers' market where firms pick up goods: $\theta_s = F_s/S$. Similarly, define θ_b the market tightness of the buyers' market where firms sell goods to buyers: $\theta_b = B/F_b$.

We assume that the matching technology between firms and sellers (or buyers) is the same as the one in the market without intermediary. In the sellers' market, the arrival rate of a match for a seller is given by $m(\theta_s) \equiv M(F_s, S)/S = M(\theta_s, 1)$, and the arrival rate of a match for a firm is $M(F_s, S)/F_s = m(\theta_s)/\theta_s$. Similarly, in the buyers' market, the arrival rate of a match for a firm is $m(\theta_b)$, and the arrival rate of a match for a buyer is $m(\theta_b)/\theta_b$. To simplify notation, in what follows we write $m_s = m(\theta_s)$ and $m_b = m(\theta_b)$.

3.2 Equilibrium payoffs and welfare

We derive the continuation value for a seller, a firm with one good, a firm with two goods, and a buyer. Since firms in the sellers' market already have one good in inventory, a seller who meets a firm will be able to sell her good to the firm with probability $\frac{n-1}{n}$ because the firm wants to buy a different type of good compared to what it already owns. As before, let V denote the continuation payoff for a seller being in the market unmatched at the end of period $\triangle$:

$$V = \frac{1}{1+r\triangle}\left[m_s\triangle\left(-c_s + \frac{n-1}{n}t_s\right) + \left(1 - m_s\triangle\frac{n-1}{n}\right)V\right], \tag{8}$$

where t_s is the price paid by the firm to the seller.

A firm with two goods enters the buyers' market, meets with a buyer and shows one of its good. We assume that the firm learns the exact type of the buyer after inspection. If there is a match between the buyer and the first good, the buyer and the seller will negotiate a price and consummate the trade. If the initial inspection results in a mismatch, the firm has to decide whether it wants to show the second good in its inventory to the buyer. The firm has to incur another show cost if it has to show the buyer the second good. Similarly, the buyer will have to pay another inspection cost if he wants to see the second good. If the firm does not show the second good, both parties separate and go on their own ways.

Because the firm learns the exact type of the buyer from the first inspection, it will not show the second good to the buyer unless it knows that the second good is a match with the buyer's type. A more realistic assumption is that the firm gradually narrows down the set of the buyer's possible types after the first several goods were rejected. In section 5, we will briefly discuss the robustness of our results to alternative learning technologies.

Let W_1, W_2 denote the continuation payoff of a firm with a good and a firm with two goods being in the market unmatched, respectively:

$$W_1 = \frac{1}{1+r\triangle}\left[\frac{m_s}{\theta_s}\triangle\left(-c_b+\frac{n-1}{n}\left(W_2-t_s\right)\right)+\left(1-\frac{m_s}{\theta_s}\triangle\frac{n-1}{n}\right)W_1\right], \tag{9}$$

$$W_2 = \frac{1}{1+r\triangle}\left[m_b\triangle\left(-c_s+\frac{2}{n}\left(t_b+W_1-\frac{1}{2}c_s\right)\right)+\left(1-m_b\triangle\frac{2}{n}\right)W_2\right], \tag{10}$$

where t_b is the price the buyer pays to the firm. In W_2, one show cost is incurred upon meeting a buyer when the buyer sees the first good. The second show cost is incurred only if there is a match with the buyer with the second good in the firm's inventory.

Let U denote the continuation payoff of a buyer being unmatched in the market:

$$U = \frac{1}{1+r\triangle}\left[\frac{m_b}{\theta_b}\triangle\left(-c_b+\frac{2}{n}\left(1-t_b-\frac{1}{2}c_b\right)\right)+\left(1-\frac{m_b}{\theta_b}\triangle\frac{2}{n}\right)U\right]. \tag{11}$$

Similarly, one inspection cost is incurred upon meeting a firm when the buyer sees the first good. The second inspection cost is incurred only if there is a match with the firm with the second good in the the firm's inventory.

The price t_s between a seller and a firm with one good is determined by Nash bargaining before inspection:

$$t_s = \arg\max_p \left(-c_b+\frac{n-1}{n}(W_2-p)+\frac{1}{n}W_1-W_1\right)\left(-c_s+\frac{n-1}{n}p+\frac{1}{n}V-V\right).$$

Thus, the price a firm pays to a seller is

$$t_s = \frac{1}{2}\left(W_2-W_1+V+\frac{n}{n-1}\left(c_s-c_b\right)\right). \tag{12}$$

Similarly, Nash bargaining before inspection also help set the price t_b between a buyer and a firm with two goods:

$$t_b = \arg\max_p \left(-c_b+\frac{2}{n}\left(1-p-\frac{1}{2}c_b\right)+\frac{n-2}{n}U-U\right)\left(-c_s+\frac{2}{n}\left(p+W_1-\frac{1}{2}c_s\right)+\frac{n-2}{n}W_2-W_2\right).$$

It follows that a buyer needs to pay a price

$$t_b = \frac{1}{2}\left(1+W_2-W_1-U+\frac{1}{2}\left(n+1\right)\left(c_s-c_b\right)\right). \tag{13}$$

In the steady state, the number of firms picking up goods successfully must be equal to the number of firms selling goods successfully:

$$F_s \frac{n-1}{n} \frac{m(\theta_s)}{\theta_s} = F_b \frac{2}{n} m(\theta_b). \tag{14}$$

Finally, we impose free entry conditions to complete the model. First, with free entry of producers, sellers must get the same reservation utility as producers, that is, $V = K$. Second, since a firm with one good can always choose to be a seller, and a seller with one good can choose to be a firm, we can write the free entry conditions as:

$$W_1 = V = K. \tag{15}$$

Definition 2 *A steady-state search equilibrium with firms is defined by the stocks of market participants* (B, S, F_b, F_s)*, continuation payoffs* (U, W_1, W_2, V)*, and market prices* (t_b, t_s) *such that*

(a) steady-state conditions (8), (9), (10), (11) and (14) hold;

(b) prices (t_b, t_s) *are determined by (12) and (13);*

(c) free entry (market clearing) condition (15) holds;

(d) buyers and firms participate: $U, W_1, W_2 \geq 0$.

(e) the following incentive conditions hold: (i) buyers and sellers do not directly trade, (ii) firms do not carry two identical goods, and (iii) firms with one good, firms with two goods, buyers and sellers will not pretend to be one another.

By inserting the transaction prices t_s and t_b into the four functional equations, we can simplify the four functional equations into

$$rV = m_s \left(\frac{n-1}{2n} (W_2 - W_1 - V) - \frac{1}{2} (c_s + c_b) \right), \tag{16}$$

$$rW_1 = \frac{m_s}{\theta_s} \left(\frac{n-1}{2n} (W_2 - W_1 - V) - \frac{1}{2} (c_s + c_b) \right), \tag{17}$$

$$rW_2 = m_b \left(\frac{1}{n} (1 - W_2 + W_1 - U) - \frac{n+1}{2n} (c_s + c_b) \right), \tag{18}$$

$$rU = \frac{m_b}{\theta_b} \left(\frac{1}{n} (1 - W_2 + W_1 - U) - \frac{n+1}{2n} (c_s + c_b) \right). \tag{19}$$

The first two equations imply $V = \theta_s W_1$, while the last two equations give us $U = W_2/\theta_b$.

Substituting $V = \theta_s W_1$ and $U = W_2/\theta_b$ into equation (17) and (18), we derive the following limit solution ($r \to 0$):

$$W_1 = \frac{\left(1 - \frac{n+1}{2}\left(c_s + c_b\right)\right)\theta_b - \frac{n}{n-1}\left(\theta_b + 1\right)\left(c_s + c_b\right)}{1 + \theta_s + \theta_s\theta_b}, \tag{20}$$

$$W_2 = \frac{\left(1 - \frac{n+1}{2}\left(c_s + c_b\right)\right)\left(\theta_s + 1\right)\theta_b - \frac{n}{n-1}\theta_b\left(c_s + c_b\right)}{1 + \theta_s + \theta_s\theta_b}. \tag{21}$$

It follows from $V = \theta_s W_1$ and $U = W_2/\theta_b$ that

$$U = \frac{\left(1 - \frac{n+1}{2}\left(c_s + c_b\right)\right)\left(\theta_s + 1\right) - \frac{n}{n-1}\left(c_s + c_b\right)}{1 + \theta_s + \theta_s\theta_b}, \tag{22}$$

$$V = \frac{\left(1 - \frac{n+1}{2}\left(c_s + c_b\right)\right)\theta_s\theta_b - \frac{n}{n-1}\theta_s\left(\theta_b + 1\right)\left(c_s + c_b\right)}{1 + \theta_s + \theta_s\theta_b}. \tag{23}$$

In the search market with firms, the expected total payoff for a pair of seller and buyer is given by

$$U + V = 1 - \frac{n+1}{2}\left(c_b + c_s\right) - \frac{n}{n-1}\left(c_b + c_s\right). \tag{24}$$

The term $\frac{n+1}{2}\left(c_b + c_s\right)$ is the expected social cost of completing a trade in the sellers' market, and $\frac{n}{n-1}\left(c_b + c_s\right)$ is the expected social cost of completing a trade in the buyers' market. So the above expression means that the buyers and sellers obtain all the surplus from participating in these markets. Recall that each transaction consists of a purchase and a sale, so firms earn zero profit from each transaction (see a formal proof of this result in the Appendix). This zero profit per transaction result is expected because firms are infinitely lived. If the profit per transaction is positive at $r = 0$, firms will make infinite total profits.

By comparing the social welfare with firms and without firms, we obtain the welfare improvement due to firms:

$$\triangle(U + V) = \frac{n^2 - 4n + 1}{2\left(n - 1\right)}(c_b + c_s). \tag{25}$$

Therefore, the introduction of firms improves social welfare as long as $n \geq 4$.

This result is expected. Firms have to incur inspection costs to buy goods. In order to recover these inspection and show costs, the degree of heterogeneity must be sufficiently large such that firms can economize on show and inspection costs of market participants. Moreover, when $n \geq 4$ and assumption (4) holds, $W_2 > W_1$ and $U \geq 0$ so that both firms and buyers will participate.

3.3 Incentive conditions

In order to fully characterize the equilibrium, we need to find conditions under which the following incentive conditions hold: (i) buyers and sellers will not trade directly, (ii) sellers will not directly go to the buyers' market, (iii) buyers will not directly go to the sellers' market, and (iv) firms will not go to the buyers' market unless they have two different types of goods in their inventory.

Recall that, when $n \geq 4$, the joint payoffs of buyers and sellers are higher in dealing with firms compared to trading directly. By trading directly either the seller or the buyer will be worse off and thus at least one of the two parties will refuse to trade directly. Therefore, incentive condition (i) is satisfied if $n \geq 4$. It remains to find conditions for (ii)-(iv).

Seller's incentives

A seller can pretend to be a firm and approach buyers in the buyers' market. If the buyer rejects the good on offer, the seller can say that the other good in its phantom inventory does not fit the buyer's type either.

When $r \to 0$, the bid-ask spread is equal to the firm's expected information cost for each transaction:

$$t_b - t_s = \frac{n}{n-1}c_b + \frac{n+1}{2}c_s.$$

When a seller goes to the buyers' market directly by pretending to be a firm, he pockets the bid ask spread but incurs an increase in expected show cost equal to

$$(n - \frac{n}{n-1})c_s = \frac{n^2 - 2n}{n-1}c_s.$$

Therefore, a seller will not pretend to be a firm if:

$$\frac{n}{n-1}c_b + \frac{n+1}{2}c_s \leq \frac{n^2 - 2n}{n-1}c_s$$

which is equivalent to

$$\frac{c_b}{c_s} \leq \frac{n^2 - 4n + 1}{2n}. \tag{26}$$

Buyer's incentives

A buyer can pretend to be a firm with one good and buy directly from a seller at price t_s in the following way. If the good fits the buyer's preference, a buyer pays t_s to the seller. If the good does not fit, the buyer tells the seller that the good coincides what he already has.

When $r \to 0$, the bid-ask spread is equal to the firm's expected information cost for each transaction:

$$t_b - t_s = \frac{n}{n-1}c_b + \frac{n+1}{2}c_s.$$

When a buyer goes to the sellers' market directly by pretending to be a firm, he saves on the bid ask spread but incurs an increase in expected inspection cost equal to

$$(n - \frac{n+1}{2})c_b = \frac{n-1}{2}c_b$$

Buyers will not enter the sellers' market directly if:

$$\frac{n}{n-1}c_b + \frac{n+1}{2}c_s \leq \frac{n-1}{2}c_b \Rightarrow \frac{c_b}{c_s} \geq \frac{n^2-1}{n^2-4n+1}. \tag{27}$$

In particular, when $c_s > c_b$, the show cost portion of the bid-ask spread, $\frac{n+1}{2}c_s$, exceeds the increase in expected inspection cost from going to the sellers' market directly. In this case, the buyer will not want to deal with firms and his incentive condition is violated.

Firm's incentives

First of all, we need to insure that a firm will not go to the buyers' market with two identical goods. We focus on the potential gain of the firm from each transaction by deviation. Recall that a transaction for a firm consists of a successful purchase from a seller and a successful sale to a buyer. If a firm cannot gain from deviation for one transaction, then it cannot gain by deviating for more than one transactions. Therefore, although in principle the firm can deviate for any number of transactions, it is sufficient to show that the firm cannot gain from deviating for one transaction.

Suppose a firm with one good meets a seller and finds out that the seller's good coincides with the one it already has. If it rejects the seller and continues to search sellers, its expected cost of completing a transaction is

$$c_b + \frac{n}{n-1}c_b + \frac{n+1}{2}c_s.$$

The first term is the inspection cost just incurred, the second term is the expected inspection cost of picking up a good which is different from what it already has, and the third term is the show cost of selling a good to a buyer.

If instead the firm buys the good from the seller and thus goes to the buyers' market with two identical goods, its expected cost of completing a transaction is $c_b + nc_s$. Given that the firm's deviation cannot change market prices, the firm will not pick up duplicated goods if

$$c_b + \frac{n}{n-1}c_b + \frac{n+1}{2}c_s \leq c_b + nc_s,$$

which is equivalent to

$$\frac{c_b}{c_s} \leq \frac{n^2 - 2n + 1}{2n}. \tag{28}$$

It remains to show a firm with one good will not immediately go to the buyers' market to look for a buyer. That is, when it sells one good, it returns to the sellers' market to pick up another good. Again, since a firm's deviation cannot affect market prices, we only need to compare expected cost to complete a transaction. If a firm goes to the buyers' market only after it picks up two different goods, its expected cost to complete a transaction is

$$\frac{n}{n-1}c_b + \frac{n+1}{2}c_s.$$

If a firm with one good goes to the buyers' market directly, its expected cost to complete a transaction is $nc_s + c_b$. Therefore, a firm with one good will not pretend to be a firm with two goods and search buyers directly if

$$\frac{n}{n-1}c_b + \frac{n+1}{2}c_s \leq nc_s + c_b.$$

which reduces to

$$\frac{c_b}{c_s} \leq \frac{n^2 - 2n + 1}{2}. \tag{29}$$

3.4 Summary

It is easy to see that the firm's incentive conditions (28) and (29) are implied by the seller's incentive condition (26) for all $n \geq 2$. Therefore, all parties' incentive conditions are satisfied if $n \geq 4$ and

$$\frac{n^2 - 1}{n^2 - 4n + 1} \leq \frac{c_b}{c_s} \leq \frac{n^2 - 4n + 1}{2n}. \tag{30}$$

Notice that the set of cost ratio c_b/c_s that satisfies above constraints is non-empty as long as $n \geq 8$. Moreover, as n is large, the seller's IC constraint (26) is always satisfied, while the buyer's IC constraint is also satisfied if c_b is relatively higher than c_s. The following proposition summarizes the main result of this section.

Proposition 1 *A search equilibrium with firms exists and improves social welfare if condition (30) holds.*

4 Search with brokers

In the previous section, firms as intermediaries buy goods from sellers and sell them to buyers. The discussion following (27) shows that the search equilibrium with firms may not exist if $c_s > c_b$. Can other types of intermediaries exist if $c_s > c_b$? This section studies the role of brokers who represent buyers in their search for goods.

Again suppose there are n types of goods and buyers with equal proportion. New buyers and sellers, drawn uniformly from the n types, arrive at the market sequentially. Instead of firms, consider a buyer's broker (henceforth broker) who first contacts buyers in the buyers' market. When a broker meets a buyer, the broker incurs a show cost c_s, the buyer pays a inspection cost c_b, and the broker finds out the buyer's preference. If they agree on the commission ϕ and representation, then the broker will represent the buyer in the sellers' market and receive ϕ. Once a broker meets with a seller, the seller pays a show cost c_s to show his good and the broker pays c_b to inspect the good. After the broker finds a suitable good for her client to buy, the buyer will inspect the good and negotiate to buy the good from the seller. The buyer and seller leave the market permanently after trade. Assume that a broker can represent at most two buyers.

Similar to the case of firms, we assume for now that (i) buyers and sellers will not trade directly, (ii) sellers will not pretend to be brokers with one client, (iii) buyers will not pretend to be brokers with two clients, and (iv) brokers will not go to the sellers' market unless they have already contracted with two different buyers. At the end of this section, we will specify conditions under which the incentive conditions (i)-(iv) hold.

Our main result in this section is that for n sufficiently large, an equilibrium with brokers will exist if $c_s > c_b$. Thus there is a natural separation of the parameter space where firms which buy inventory exist versus where brokers who search on behalf of their clients exist.

4.1 Matching Process

Let A_s denote the number of brokers with two buyers, and A_b denote the number of brokers with one buyer. Define θ_s the market tightness of the sellers' market where brokers try to find the right sellers to their clients (buyers): $\theta_s = A_s/S$. Similarly, define θ_b the market tightness of the buyers' market where brokers find buyers: $\theta_b = B/A_b$.

We assume that the matching technology between brokers and sellers (or buyers) is the same as the one in markets without intermediary. Therefore, with abuse of notation, in the sellers' market, we denote the arrival rate of a match for a seller by $m(\theta_s) \equiv M(\theta_s, 1)$, and the arrival rate of a match for a broker by $m(\theta_s)/\theta_s$. Similarly, in the buyers' market, the arrival rate of a match for a broker is $m(\theta_b)$ and the arrival rate of a match for a buyer is $m(\theta_b)/\theta_b$.

Similar to the case of firms, in the steady state, the number of brokers picking up buyers successfully must be equal to the number of brokers who find the right good for one of their clients:

$$A_s \frac{2}{n} \frac{m(\theta_s)}{\theta_s} = A_b \frac{n-1}{n} m(\theta_b). \tag{31}$$

As before, in what follows we write $m_s = m(\theta_s)$ and $m_b = m(\theta_b)$ to simplify notation.

4.2 Continuation Payoffs and Welfare

Let U be the continuation value of a buyer without an broker at the end of period $\triangle$, U_a be the continuation value of a buyer contracted with a broker who has another client, and U_b be the continuation value of a buyer who has to return to the buyers' market with his broker without having bought a good so that his broker can pick up another buyer. Then we have

$$\begin{aligned} U &= \frac{1}{1+r\triangle}\left[\frac{m_b}{\theta_b}\triangle\left(-c_b + \frac{n-1}{n}(U_a - \phi)\right) + \left(1 - \frac{m_b}{\theta_b}\triangle\frac{n-1}{n}\right)U\right], \\ U_a &= \frac{1}{1+r\triangle}\left[\frac{m_s}{\theta_s}\triangle\left(\frac{1}{n}(1-t-c_b) + \frac{1}{n}U_b\right) + \left(1 - \frac{m_s}{\theta_s}\triangle\frac{2}{n}\right)U_a\right], \\ U_b &= \frac{1}{1+r\triangle}\left[m_b\triangle\left(\frac{n-1}{n}U_a\right) + \left(1 - m_b\triangle\frac{n-1}{n}\right)U_b\right], \end{aligned}$$

where ϕ is the commission a buyer has to pay to his broker, and t is the price of the good charged to a buyer. Notice that when the broker finds an appropriate good for one of her two buyers, this buyer still has to inspect the good in order to verify that his preference matches the type of the good. That is why there is a c_b in the equation of U_a.

With abuse of notation, let W_1 and W_2 be the continuation value of a broker with one buyer and a broker with two buyers at the end of period $\triangle$, respectively:

$$\begin{aligned} W_1 &= \frac{1}{1+r\triangle}\left[m_b\triangle\left(-c_s + \frac{n-1}{n}(W_2 + \phi)\right) + \left(1 - m_b\triangle\frac{n-1}{n}\right)W_1\right], \\ W_2 &= \frac{1}{1+r\triangle}\left[\frac{m_s}{\theta_s}\triangle\left(-c_b + \frac{2}{n}W_1\right) + \left(1 - \frac{m_s}{\theta_s}\triangle\frac{2}{n}\right)W_2\right]. \end{aligned}$$

Finally, let V be the continuation value of a seller being unmatched at the end of period $\triangle$:

$$V = \frac{1}{1+r\triangle}\left[m_s\triangle\left(-c_s + \frac{2}{n}(t-c_s)\right) + \left(1 - m_s\triangle\frac{2}{n}\right)V\right].$$

Notice that, after the broker finds out the seller's good is a good match with one of broker's clients (with probability $2/n$), the seller still needs to incur one more show cost c_s when showing it to the client.

The commission ϕ between the buyer and the broker is established by Nash bargaining before inspection:

$$\phi = \arg\max_p \left(-c_s + \frac{n-1}{n}(p+W_2) + \frac{1}{n}W_1 - W_1\right)\left(-c_b + \frac{n-1}{n}(-p+U_a) + \frac{1}{n}U - U\right)$$

The transaction price t between a seller and a buyer is determined by Nash bargaining before the broker inspects the good:

$$t = \arg\max_p \left(-c_s + \frac{2}{n}(p-c_s) + \frac{n-2}{n}V - V\right)\left(\frac{1}{n}(1-p-c_b) + \frac{1}{n}U_b + \frac{n-2}{n}U_a - U_a\right)$$

To complete the model, we impose the stationary condition (31) and free entry condition for sellers and brokers. The seller's free entry condition is the same as before: $V = K$. For the brokers, let L denote the broker's outside option. The broker without a client can go to the buyers' market to pick up a buyer by incurring cost c_s. But the continuation value for a broker with one client is W_1. Therefore, when $r \to 0$, we can write the free entry condition for the broker as: $W_1 = L + c_s - \phi$. Then the search equilibrium with brokers can be defined analogously to the one with firms.

Following the same procedure as in the case of firms, we can obtain the following limit solution:

$$\begin{aligned}
V &= \frac{2\theta_s}{1+2\theta_s}\left(1 - c_b - \frac{n+2}{2}c_s\right) && (32)\\
U &= \frac{1}{1+2\theta_s}\left(1 - c_b - \frac{n+2}{2}c_s\right) - \frac{n}{n-1}(c_s+c_b) - \frac{n}{2}c_b && (33)\\
W_1 &= \theta_b U && (34)\\
W_2 &= W_1 - \frac{n}{2}c_b && (35)
\end{aligned}$$

Therefore, the total social welfare is

$$U + V = 1 - \frac{n+2}{2}(c_s+c_b) - \frac{n}{n-1}(c_s+c_b)$$

Again the broker's expected profit per transaction is zero (see Appendix for a formal proof).

Social welfare with brokers minus the social welfare without intermediary is

$$\triangle(U+V)=\frac{n^2-5n+2}{2\,(n-1)}(c_b+c_b),$$

which is positive for $n \geq 5$.

4.3 Incentive conditions

As in the case of firms, we need to check incentive compatibility conditions for sellers, buyers and brokers. Notice that when $n \geq 5$ the joint payoffs of buyers and sellers are higher in dealing with brokers compared to trading directly. Therefore, either the seller or the buyer will be worse off by trading directly. Therefore, we only need to worry about incentive conditions (ii)-(iv).

Seller's incentives

The seller can pretend to be a broker with one client and approach buyers directly. Once he meets a buyer, he incurs cost c_s to find out the type of the buyer. If the buyer's preference matches the seller's good, the seller will incur another show cost c_s to show his good to the buyer and collect both commission ϕ and price t.[7] If the buyer does not match the seller's good, the seller can claim that the buyer's type coincides with the type of client whom he has already contracted with.

We know that in the limit $r \to 0$ the broker earns zero profits per transaction:

$$\phi=\frac{n}{n-1}c_s+\frac{n}{2}c_b.$$

If the seller goes to the buyers' market directly, he saves commission ϕ but incurs more show cost. By dealing with brokers, the expected show cost incurred by the seller is $\left(\frac{n}{2}+1\right)c_s$, because the seller on average meets $n/2$ brokers in order to make a sale and in case of a match the seller has to incur another show cost to show the final buyer. If the seller goes to the buyers' market directly, his expected show cost is $(n+1)\,c_s$, because the seller on average meets n buyers in order to make a sale and in case of a sale the seller need to show

[7]Notice that after finding out that there is a match the buyer is happy to accept the seller's offer even though the buyer knows that the seller is not a broker.

the good one more time to the buyer. Therefore, a seller will not pretend to be a broker if

$$\frac{n}{n-1}c_s + \frac{n}{2}c_b \leq (n+1)\,c_s - \left(\frac{n}{2}+1\right)c_s,$$

which implies

$$\frac{c_b}{c_s} \leq \frac{n-3}{n-1}. \tag{36}$$

In particular, if $c_b > c_s$, the above condition cannot hold and the equilibrium with brokers does not exist.

Buyer's incentives

A buyer can pretend to be a broker with two buyers and buy a good directly from a seller at price t. If the good fits the buyer's preference, such a buyer pays t to the seller. If the good does not fit, the buyer tells the seller that the other buyer he represents does not like the good either.

If the buyer pretends to be a broker and goes to the sellers' market directly, he can pocket commission ϕ but has to incur more inspection cost in expectation. If he trades through broker, his expected inspection cost is $\left(\frac{n}{n-1}+1\right)c_b$, because on average the buyer needs to contact $n/\left(n-1\right)$ brokers and in case of a match he need to do a final inspection. If he goes to the sellers' market directly, his expected inspection cost is nc_b since on average he needs to contact n sellers in order to find the right good. Therefore, a buyer will not pretend to be a broker if

$$\phi = \frac{n}{n-1}c_s + \frac{n}{2}c_b \leq nc_b - \left(\frac{n}{n-1}+1\right)c_b,$$

which reduces to

$$\frac{c_b}{c_s} \geq \frac{2n}{n^2-5n+2}. \tag{37}$$

Broker's incentives

For the broker's incentives, we first find conditions under which a broker will not serve two buyers with the same preference. Suppose the broker with a buyer meets another buyer and finds out that this buyer's type coincides with the one he has already contracted with. If the broker rejects the buyer, her total expected cost to complete a transaction is

$$c_s + \frac{n}{n-1}c_s + \frac{n}{2}c_b.$$

If the broker deviates and accepts the buyer, then her total expected cost to complete a transaction is $c_s + nc_b$. Therefore, a broker will contract with buyers with different types only if

$$c_s + \frac{n}{n-1}c_s + \frac{n}{2}c_b \leq c_s + nc_b,$$

which is equivalent to

$$\frac{c_b}{c_s} \geq \frac{2}{n-1}. \tag{38}$$

Second, we also need to insure that a broker with one good will not immediately go to the sellers' market to look for a seller. If the broker goes to the seller's market only if she has two different buyers, the total cost to the broker of completing a transaction is $\frac{n}{n-1}c_s + \frac{n}{2}c_b$. But if the broker with one buyer goes to the seller's market directly, the total expected cost to the broker of completing a transaction is $c_s + nc_b$. Therefore, a broker with one buyer will not go to the seller's market directly if

$$\frac{n}{n-1}c_s + \frac{n}{2}c_b \leq c_s + nc_b,$$

which is equivalent to

$$\frac{c_b}{c_s} \geq \frac{2}{n(n-1)}. \tag{39}$$

4.4 Summary

Notice that the broker's incentive conditions (38) and (39) are implied by the seller's incentive condition (37) for all $n \geq 2$. Therefore, all parties' incentive conditions are satisfied if $n \geq 5$ and

$$\frac{2n}{n^2 - 5n + 2} \leq \frac{c_b}{c_s} \leq \frac{n-3}{n-1}. \tag{40}$$

The set of cost ratio c_b/c_s that satisfies above constraints is non-empty as long as $n \geq 8$. When n is large, the buyer's IC constraint (the first inequality) in (40) is always satisfied, while the buyer's IC constraint is also satisfied if c_b is relatively higher than c_s. To summarize our analysis with brokers, we have

Proposition 2 *A search equilibrium with brokers exists and improves social welfare if condition (40) holds.*

4.5 Firms vs. brokers

Our analysis demonstrates that the parameter space for firms to exist and for brokers to exist is naturally separated. In particular, search equilibrium with firms exist when inspection cost c_b is higher than show cost c_s, and search equilibrium with brokers exist when $c_b < c_s$.

To understand the underlying rationale for this separation, let us compare the bid-ask spread with firms and the commission with brokers:

$$\begin{aligned} t_b - t_s &= \frac{n}{n-1}c_b + \frac{n+1}{2}c_s, \\ \phi &= \frac{n}{n-1}c_s + \frac{n}{2}c_b. \end{aligned}$$

In the search equilibrium with firms, when n is large, the size of bid-ask spread is primarily determined by the size of show cost c_s. In contrast, in the search equilibrium with brokers, the size of commission is primarily determined by the size of inspection cost c_b. The main constraint for the existence of firms or brokers is that either buyers or sellers may not want to deal with intermediaries if the bid-ask spread or the commission fee is too high. Therefore, whether firms or brokers will exist depends on which intermediary can operate with a lower intermediation charges. As a result, intermediation through firms is less prone for deviation by market participants when $c_b > c_s$, whereas intermediation through brokers is less prone for deviation when $c_b < c_s$.

With minimal capacity two, the parameter space for the existence of firms and (buyer) brokers is disjoint. If intermediaries are allowed to hold three or more inventories, then these two types of intermediation may coexist (e.g. when $c_b = c_s$). In this case, the social welfare is higher in equilibrium with firms than the one in equilibrium with brokers. The difference in welfare is due to the nature of the first meeting between the buyer and the intermediary. A firm learns of a buyer's type when the firm shows the buyer its first good in inventory. If the good, which is chosen without knowledge of the buyer's type, is a fit, the buyer will buy the good without having to incur additional inspection cost. On the other hand, when a buyer's broker meets with her client to elicit his preferences, the buyer has to incur a meeting (inspection) cost without inspecting any good. Thus even if the buyer buys the first good which is shown to the buyer by the broker, the buyer has incurred a meeting cost and an inspection cost.

Case of sellers' brokers

When $c_b > c_s$, there is also a search equilibrium with sellers' brokers who first search for sellers to represent and then sell good to buyers on behalf of the sellers. The analysis is similar to that of buyers' brokers discussed above, and is available upon request. The welfare property of this equilibrium is similar to the welfare property of the search equilibrium with firms. Thus our model explains not only when firms or buyers' brokers exist; but also when buyers' or sellers' brokers exist.

5 Discussion

In order to starkly demonstrate the benefits of having inventory or multiple clients, we only allow intermediaries to hold limited inventory or clients. Endogenizing the size of the inventory or number of clients is needed. We have ignored the supply of effort by brokers. Nesting the moral hazard problem with brokers into the model is an important goal for future research.

We also assume that intermediaries learn perfectly the preference of the buyer after the first showing. If the learning technology is not perfect, the parameter space for the existence of intermediation is smaller. On the other hand, if we allow firms or brokers to hold more goods or represent more clients, the parameter space for the existence of intermediation is larger. An important future research question is then to investigate how the inventory size and the speed of learning jointly determines the possibility of intermediation.

A recent innovation in online retailing is the recommendation systems that online retailers have developed. For example, the Amazon.com recommendation system recommends a buyer books which have been purchased by other buyers with similar purchase histories as that buyer. From the buyer's purchase history, Amazon knows the buyer's preferences. However, it does not know which books among the thousands that it has fit the buyer's preferences. Amazon.com uses the purchases of the other buyers with similar purchase histories as the buyer to figure out what other books the buyer will like. Thus Amazon does not have to a priori categorize books to recommend to buyers. Our current model assumes that an intermediary can recognize whether a new good it inspects is suitable for a buyer with known preferences. The Amazon recommendation system shows that such knowledge is not necessary. Extending our model to deal with this case is another goal for future research.

6 Appendix

This Appendix shows the expected profit per transaction for intermediaries (either firms or brokers) is zero in the limit equilibrium as $r \to 0$.

First consider the case of firms. The expected total information cost a firm incurs in order to successfully complete a transaction is $\frac{n}{n-1}c_b + \frac{n+1}{2}c_s$. The bid-ask spread for the firm is

$$\begin{aligned} t_b - t_s &= \frac{1}{2}\left(1 + W_2 - W_1 - U + \frac{n+1}{2}(c_s - c_b)\right) \\ &\quad -\frac{1}{2}\left(W_2 - W_1 + V + \frac{n}{n-1}(c_s - c_b)\right) \\ &= \frac{1}{2}\left(1 + \frac{n+1}{2}(c_s - c_b) - \frac{n}{n-1}(c_s - c_b)\right) - \frac{1}{2}(U+V) \\ &= \frac{n}{n-1}c_b + \frac{n+1}{2}c_s \end{aligned}$$

The last equality follows from (24). Therefore, the price difference exactly covers the expected information cost incurred by a firm per successful transaction.

The broker's case is analogous. The expected total cost of making a sale for a broker is $\frac{n}{n-1}c_s + \frac{n}{2}c_b$. By inserting expression of U, W_1, W_2 and U_a, we obtain

$$\begin{aligned} \phi &= \frac{1}{2}\frac{n}{n-1}(c_s - c_b) + \frac{1}{2}(W_1 - W_2 + U_a - U) \\ &= \frac{1}{2}\frac{n}{n-1}(c_s - c_b) + \frac{1}{2}\left(\begin{array}{c} \frac{n}{2}c_b + \frac{1}{1+2\theta_s}\left(1 - c_b - \frac{n+2}{2}c_s\right) \\ -\frac{1}{1+2\theta_s}\left(1 - c_b - \frac{n+2}{2}c_s\right) + \frac{n}{n-1}(c_s + c_b) + \frac{n}{2}c_b \end{array}\right) \\ &= \frac{n}{n-1}c_s + \frac{n}{2}c_b. \end{aligned}$$

Therefore, a broker's expected profit per transaction is zero.

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