University of Toronto
Department of Economics

Working Paper 358

Liquidity, Volume, and Price Behavior: The Impact of Order vs. Quote Based Trading

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May 11, 2009

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Abstract

We provide a three way theoretical comparison of dealer, limit order, and hybrid markets and analyze the impact that the organization of trading has on volume, liquidity, and price efficiency. We find, in particular, that trading volume is highest in the limit order market and lowest in the dealer market. Small order price impacts are lowest and large order price impacts are highest in limit order markets. Prices are most efficient in the hybrid market and least efficient in the dealer market, except when the level of informed trading is very high. Post-trade market transparency in a hybrid market hampers price efficiency for thinly traded securities. We further identify that traders behave as contrarians.

*We thank Bruno Biais, Ryan Davies, David Goldreich, Ingrid Lo, Rob McMillan, Christine Parlour, Jim Pesando, Avanidhar Subrahmanyam, and seminar participants at the CEA2008 meetings and the Karlsruhe Symposium on Finance, Banking and Insurance. Financial support from SSHRC and the Connaught Foundation is gratefully acknowledged.

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Intra-day financial market trading is organized using two major mechanisms: order driven and quote driven trading. Order driven markets typically employ a public limit order book. In quote driven markets, all trades are arranged by designated institutions that post quotes. The latter markets are commonly referred to as dealer markets. Many real world markets are hybrids, combining both organizational forms.[1]

The coexistence of competitive limit order and dealer markets and the differences in their trading outcomes have long been challenged by academic research. Madhavan (1992) shows that, with competitive liquidity provision, a quote driven system and a uniform price order driven system lead to identical outcomes. Glosten (1994) and Back and Baruch (2007) argue that a quote driven system that competes with a discriminatory limit order book in an anonymous market would mimic the limit order book.

Our paper builds on this line of research but serves a different purpose. We posit non-anonymity, in the sense that repeat order submissions are identified, and thus effectively take the coexistence of the mechanisms as given. Our goal is to describe the relative advantages and disadvantages of the three trading systems: a discriminatory limit order book, a dealer market, and a hybrid market.

Our major contribution is twofold. First, we provide an integrated theoretical framework that admits a three-way comparison. The differences in trading outcomes of the three trading mechanisms in our setting highlight, in particular, the significance of the discriminatory order book and post-trade market transparency. Second, we employ our framework to derive novel empirical predictions for the impact of the organization of trading on volume, liquidity, and price efficiency. In comparing competitive markets, we complement the literature on markets where liquidity providers have market power (e.g., Seppi (1997)), which we discuss below.

Limit order and dealer markets differ in many aspects. One defining feature of the systems is the level of market transparency enjoyed by the liquidity providers. Limit orders are posted prior to the liquidity demand realization, whereas dealers' quotes account for the order size. We show that this difference in the liquidity providers information both yields new empirical predictions and explains a large share of the previously noted heterogeneity among the trading outcomes.

The liquidity providers in the dealer market are able to determine the information

[1] See Harris (2003) for a comprehensive list. For example, on the NYSE and the Toronto Stock Exchange, traders can either send orders directly to the limit order book (the "downstairs" market) or arrange trades via floor brokers (NYSE) or "upstairs" dealers (Toronto); on Nasdaq, trades can be arranged on INET or through a dealer. Finally, a hybrid structure also arises when a limit order book competes with a dealer market. An example is Paris Bourse (a limit order market) and London Stock Exchange (a dealer market, until recently), which compete for the order flow in cross-listed stocks.

content of a transaction most accurately. The high level of transparency on the side of liquidity suppliers, however, lowers liquidity demanders' rents and causes them to trade less aggressively. The efficiency gain that stems from the dealers' informational advantage is thus muted by the lowest trading volume. We find that the liquidity demand reduction effect dominates, so that the lack of trading activity renders the dealer market to be least informationally efficient, except when the level of informed trading is very high. The hybrid market combines the limit order market's aggressive order submission with the dealer market's superior screening ability and yields the most efficient prices. We thus reject the commonly argued hypothesis that the presence of the upstairs dealer who absorbs a large fraction of uninformed order flow ("skims the cream") necessarily hurts the main (limit order book) exchange participants.

Our model has the following structure. Liquidity demanders trade either for reasons outside the model (e.g., to rebalance their portfolio), or they have private information about the security's fundamental value and optimally choose the size and direction of their trade (or abstain from trading). Liquidity is supplied by competitive, uninformed, and risk-neutral institutions, as in standard market microstructure models in the tradition of Kyle (1985) and Glosten and Milgrom (1985).[2]

In the dealer-market, the liquidity providers observe the order flow and then compete for it in a Bertrand fashion. The equilibrium price aggregates the information contained in the order flow. In the limit order market, liquidity providers post a schedule of buy and sell limit orders, each for the purchase or sale of a specific number of units. We assume a "discriminating" order book design, as in Glosten (1994), and the prices incorporate the information revealed when the respective limit order is "hit" by a market order of the same or larger size. The bid- and ask-prices in this setting are the "lower-tail" and "upper-tail" conditional expectations of the security value. One additional contribution of our paper is thus in formulating a model that tractably integrates both a limit order book and a dealer market in a multi-unit Glosten and Milgrom sequential trading setup.

Our setup builds on Easley and O'Hara (1987) who study a dealer market where an imperfectly informed trader, equipped with a signal of either high or low quality, submits a large or a small order and usually chooses a mixed strategy.[3] Our methodological innovation is that we employ private signals of a *continuum* of qualities. We are then able to focus on pure strategies and concisely characterize the equilibrium by the marginal buyers and sellers. These marginal traders, and hence a trader's choice of the order size,

[2]We further discuss the institutional features of dealer markets at the end of Section II.

[3]The setting is also related to Ozsoylev and Takayama (2008) who characterize an equilibrium of a dealer market where perfectly informed traders can choose among multiple size orders.

are endogenous to the market organization. This endogeneity and the fact that traders with more precise information prefer to submit larger orders are the key to our results.

Liquidity providers in dealer markets know the order size and are thus intrinsically better at pin-pointing the information content of a trade. As a result, when markets are operated in isolation, small trades receive better execution prices in the dealer market (that is, the bid-ask spread is smaller)[4] and large trades are cheaper in the limit order book. In a hybrid market, traders are additionally allowed to choose the segment to trade in. Consequently, in equilibrium, trading costs in a hybrid market for each order size must be the same across the two mechanisms. We show that to equalize the equilibrium trading costs, the dealer segment of a hybrid market must absorb most large trades, whereas the limit order segment will absorb most small trades. The total number of transactions in the limit order segment is larger than that in the dealer market segment. Further, large orders in the dealer market segment will have lower information content.

The results on the information content of trades are closely related to Seppi (1990) and Grossman (1992). Seppi studies the behavior of a single, possibly informed, large institution and finds, in particular, that repeated interactions among the exchange participants lead to routing of the uninformed trades to the off-exchange dealers. In Seppi, both the on exchange specialists and the off exchange dealers set prices according to a Kyle (1985)-style dealer market pricing rule. Grossman studies the relation of upstairs and downstairs markets, both of which employ uniform pricing rules. In his model some traders may leave a non-binding indication to trade with the upstairs dealers, which increases the effective liquidity in the upstairs segment. We complement this line of research by studying traders' self-selection into dealer and limit order segments, where the latter employs a discriminatory pricing rule.

Our analysis further shows that the behavior of traders in the dealer and hybrid markets is not stationary. For instance, as prices drop, unfavorably informed traders submit sell-orders less aggressively and favorably informed traders submit buy orders more aggressively, thus acting as *contrarians*. This result is supported by recent empirical findings (see Chordia, Roll, and Subrahmanyam (2002)) and we thus contribute to the literature by providing theoretical underpinnings for rational contrarian behavior.

In the second part of our paper, we compare market widths, price impacts, price efficiency, and trading volume in the three trading mechanisms: pure dealer markets, pure limit order books, and hybrid markets. In addition to the aforementioned results on efficiency and execution costs, we find that trading volume is the highest in the

[4] This property is known as the "small trade spread" and was previously shown, e.g., in Glosten (1994).

limit order market and the lowest in the dealer market. Finally, price impacts of small orders are stronger in dealer markets and those of large orders are stronger in limit order markets.

Our results on the price impacts of small trades may seem surprising at first: despite the smaller spreads, price impacts are larger in the dealer markets. To understand this, recall that small order prices in the limit order book account for the fact that the order might be large. Consequently, when a small order executes in a limit order book, the transaction price overshoots. The market participants will correct their expectations of the security once they know the order size, and the permanent price impact of the transaction will be smaller.

In the hybrid market this correction only occurs if there is post trade transparency in that market participants observe the segment that the order cleared in. The final result of our paper describes how a lack of post-trade transparency affects price efficiency. We find that noisier learning in an opaque market reduces market efficiency in frequently traded stocks. Interestingly, when trading activity is low, the effect of noisier learning is outweighed by that of larger price adjustments, and the opaque market is more efficient.

Other aspects of transparency have been studied in the literature. Pagano and Roell (1996) compare transparency of a uniform price auction with a dealer market system. Brown and Zhang (1997) combine a Kyle (1985)-style and a rational expectations style setup to study dealers' decisions to participate in a market, and to describe how dealers' decisions to supply liquidity affect the informational efficiency of prices. Boulatov and George (2008) analyze the effects of transparency in a uniform price market with informed and strategic liquidity provision.

Finally, market structures have also been compared in the literature that studies the strategic provision of liquidity.[5] Seppi (1997) studies a hybrid market, in which a monopolistic specialist competes with a pure limit order book, as is the case on the NYSE. He finds, in particular, that small orders receive better executions in the hybrid market whereas medium orders receive better executions in the pure limit order market. Parlour and Seppi (2003) extend Seppi (1997) by studying competition between these two exchanges. Buti (2007) builds on Seppi (1997) and adds relationship trading and price-quantity based screening by the specialist. We complement these studies by analyzing hybrid markets where *competitive* dealers compete with the limit order book, as is the case on many major equity markets.[6]

[5] Most papers on the strategic provision of liquidity are based on either Parlour (1998) or Foucault (1999); for an extensive up-to-date survey of the literature on limit order markets see Parlour and Seppi (2008).

[6] Examples are Nasdaq (dealer vs. INET), Toronto Stock Exchange (upstairs vs. downstairs), LSE

The remainder of this paper is organized as follows. Section I introduces the model. Section II derives the equilibria for the three trading mechanisms. Section III develops testable predictions for the hybrid market. Section IV compares the three mechanisms with respect to execution costs, volume, and price efficiency. Section V analyzes the impact of post-trade transparency in the hybrid market. Section VI discusses the results and possible policy implications. Appendix A derives properties of traders' information structures, Appendix B outlines the simulation procedures, and Appendix C contains all proofs. Most figures and tables are at the end of the manuscript, in particular, Tables III and IV, which summarize our findings and existent empirical support for our predictions.

I The Basic Setup

A General Market Organization

We consider a stylized model of security trading, in which informed and uninformed traders trade a single security by submitting market orders. At each discrete point in time there is exactly one trader who arrives at the market according to some random process. These individuals trade upon their arrival and only then. Short positions are filled at the true fundamental value.

Liquidity is supplied by uninformed, risk-neutral institutions that compete for order flow and earn zero expected profits. In the limit order market, the liquidity providers post a series of limit buy- and sell-orders. The former constitutes a series of ask-prices, the latter a series of bid-prices. Each price is for a single unit (i.e. a round lot). Traders post market orders after observing these prices.[7] In the dealer market, the trader posts his market order first and the liquidity providing dealers then compete in a Bertrand fashion for this order.[8] In a hybrid market, both systems coexist.

B Model Details

Security: There is a single risky security with a liquidation value V from a set of two potential values, $V \in \{0, 1\}$, with $\mathsf{Pr}(V=1) = 1/2$.

Traders: There is an infinitely large pool of traders out of which one is drawn at each point in time at random. Each trader is equipped with private information with

vs. Paris Bourse (for cross listed stocks), or Deutsche Börse (makler vs. XETRA).

[7]When referring to a liquidity provider in singular, we will use the female form and for liquidity demanders we will use the male form.

[8]At the end of Section II we will formally argue that our model can accommodate public dealer quotes.

probability $\mu > 0$; if not informed, a trader becomes a noise trader (probability $1 - \mu$). The informed traders are risk neutral and rational.

Noise traders have no information and trade randomly. These traders are not necessarily irrational, but they trade for reasons outside of this model, for example to obtain cash by liquidating a position.[9] To simplify the exposition, we assume that noise traders make trades of either direction and size with equal probability.

Trade Size: All trades are market orders for round lots. The order at time t is denoted by o_t where $o_t < 0$ indicates a sell-order and $o_t > 0$ is a buy-order. Traders can submit a large order, $|o_t| = 2$, a small order, $|o_t| = 1$, or abstain from trading, $o_t = 0$.

C Information

Public Information: The structure of the model and the prior distribution of fundamentals is common knowledge among all market participants. The identity of a trader and his signal are private information. The public information H_t at date $t > 1$ is the sequence of orders o_t and realized transaction prices at all dates prior to t: $H_t = ((o_1, \mathsf{p}_1), \ldots, (o_{t-1}, \mathsf{p}_{t-1}))$. H_1 refers to the initial history before trades occur.

Liquidity Providers' Information: In the dealer market, the liquidity providers know the public history H_t and the order o_t. In the limit order market, liquidity providers do not know which order will be posted at time t, and their information is only H_t.

Informed Traders' Information: We follow the sequential trading literature in the tradition of Glosten and Milgrom (1985) (henceforth: GM) and assume that traders receive a binary signal about the true liquidation value V. These signals are private, and they are independently distributed, conditional on the value V. Specifically, informed trader i is told "with chance q_i, the liquidation value is High/Low (h/l)" where

Pr(signal|true value)	$V = 0$	$V = 1$
signal $= l$	q_i	$1 - q_i$
signal $= h$	$1 - q_i$	q_i

This q_i is the *signal quality*. In contrast to most of the GM literature, we assume that these signals come in a *continuum* of qualities and that q_i is trader i's private information. The distribution of qualities is independent of the security's true value and can be understood as reflecting, for instance, the distribution of traders' talents to analyze securities. Figure 2 illustrates the distribution of noise and informed traders

[9] Assuming the presence of noise traders is common practice in the literature on micro-structure with asymmetric information to prevent "no-trade" outcomes à la Milgrom and Stokey (1982).

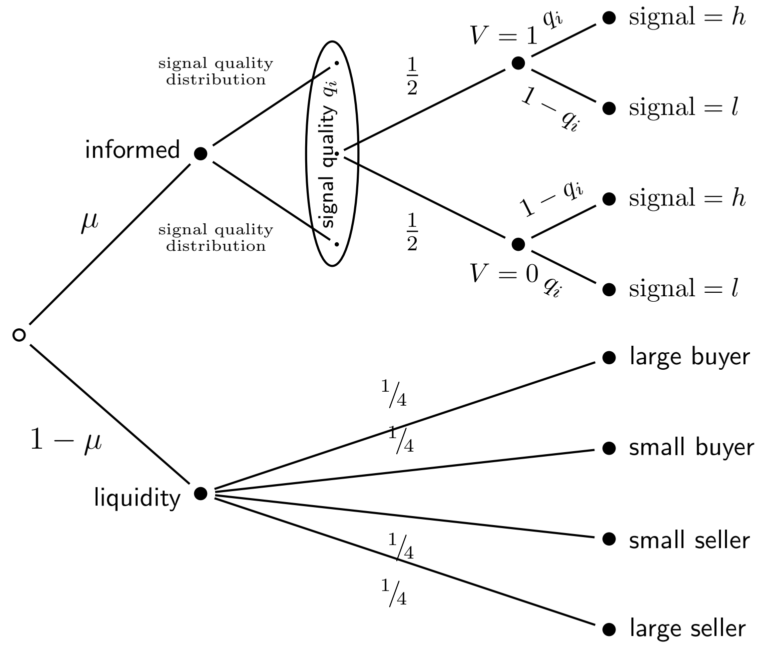

Figure 1: **Illustration of signals and noise.** This figure illustrates the mechanics of our signal distribution: first, it is determined whether a trader is informed (probability μ) or noise (probability $1-\mu$). If informed, the trader obtains a signal quality. Next, he receives the "correct" signal (h when $V = 1$ and l when $V = 0$) with probability q_i and the "wrong" signal with probability $1 - q_i$. (The draw of V is identical for all agents.) If the trader is noise, he will be a large and small buyer and seller with equal probabilities.

and the information structure.

In what follows, we will combine the binary signal (h or l) and its quality on $[1/2, 1]$ in a single variable on [0,1], namely, the trader's *private belief* that the security's liquidation value is high ($V = 1$). This belief is the trader's posterior on $V = 1$ after he learns his quality and sees his private signal but *before* he observes the public history. A trader's behavior given his private signal and its quality can then be equivalently described in terms of the trader's private belief. This approach allows us to characterize the equilibria in terms of a continuous scalar variable (as opposed to a vector) and thus simplifies the exposition.

The private belief is obtained by Bayes Rule and coincides with the signal quality if the signal is h, $\pi_i = \mathsf{Pr}(V = 1|h) = q_i/(q_i + (1 - q_i)) = q_i$. Likewise, $\pi_i = 1 - q_i$ if the signal is l. In what follows, we will denote the density of private beliefs by $f_1(\pi)$ when the fundamental is $V = 1$ and by $f_0(\pi)$ when $V = 0$. Appendix A fleshes out how these densities are obtained from the underlying distribution of qualities and it provides a numerical example.

II Dealer, Limit Order and Hybrid Markets

In what follows, we will focus on the buy side of the market; analogous results apply to the sell side. We will use L for the limit order market, D for the dealer market, and H for the hybrid market. When discussing findings for the hybrid market, we write HL for the limit order segment and HD for the dealer market segment.

The Trader's Decision. An informed trader receives his private signal, observes all past trades, and can trade upon arrival and only then. In the limit order market, he observes the posted prices, in the dealer market, he forms expectations about the price that he would be quoted, conditional on each order size. The trader chooses the order size to maximize his expected profits or abstains from trading if he expects to make negative trading profits.

Denote the total execution cost of a size o_t order by $C_t(o_t)$. To compress notation, we write the expectation of a trader with belief π after history H_t as $\mathsf{E}[V|H_t, \pi] =: \mathsf{E}_t\pi$. Then the payoff to submitting a buy order of size o_t for this trader is $o_t \cdot \mathsf{E}_t\pi - C_t(o_t)$.

A trader's expectation is increasing in the private belief. We thus focus on monotone decision rules, i.e. the higher the trader's belief is, the more he wants to buy. Specifically, we assume that traders use a "threshold" rule: they buy two units if their private belief π is at or above the time-t buy threshold π_t^2, $\pi \geq \pi_t^2$, they buy one unit if their belief is at or above π_t^1 but below π_t^2, $\pi \in [\pi_t^1, \pi_t^2)$, and they do not buy otherwise. To simplify the exposition, we will henceforth omit subscript t.

The marginal buyer of two units, π^2, is indifferent between buying one and two units. The marginal buyer of one unit, π^1, is indifferent between buying one unit and abstaining. Consequently, π^1 and π^2 solve respectively

$$1 \cdot \mathsf{E}\pi^1 = C(1), \quad 2 \cdot \mathsf{E}\pi^2 - C(2) = 1 \cdot \mathsf{E}\pi^2 - C(1). \tag{1}$$

Price Setting: Limit Order Book. The liquidity providers anticipate the traders' behavior given the marginal buyers $\pi_{\mathsf{L}}^1, \pi_{\mathsf{L}}^2$. The limit orders account for the information content of the market orders that they would be executed against. The limit order prices are the ask prices. The price $\mathsf{ask}_{\mathsf{L}}^1$ is the price for the first unit sold by liquidity providers, and it accounts for the fact that this unit is purchased by demanders of order size $o \geq 1$. The price $\mathsf{ask}_{\mathsf{L}}^2$ is the price for the second unit, and it accounts for the fact that the trade size is $o = 2$. As liquidity providers earn zero expected profits, it must hold that[10]

$$\begin{aligned}
\mathsf{ask}_\mathsf{L}^1 &= \mathsf{E}[V|\text{ trader buys } o \geq 1 \text{ units at } \{\mathsf{ask}_\mathsf{L}^1, \mathsf{ask}_\mathsf{L}^2\}, H_t], \\
\mathsf{ask}_\mathsf{L}^2 &= \mathsf{E}[V|\text{ trader buys } o = 2 \text{ units at } \{\mathsf{ask}_\mathsf{L}^1, \mathsf{ask}_\mathsf{L}^2\}, H_t].
\end{aligned}$$

Price Setting: Dealer Market. Since the liquidity demanders submit their orders before prices are posted, the information available to a dealer includes the size of the order. This implies that traders pay a uniform price for each unit that they buy.[11] Although traders do not know the price of their transaction before posting a market order, given the marginal buyers $\pi_\mathsf{D}^1, \pi_\mathsf{D}^2$, they can perfectly anticipate the quote. In light of this, we will henceforth refer to prices for buy orders as ask-prices. Then

$$\begin{aligned}
\mathsf{ask}_\mathsf{D}^1 &= \mathsf{E}[V|\text{ trader buys } o = 1 \text{ unit at } \{\mathsf{ask}_\mathsf{D}^1\}, H_t], \\
\mathsf{ask}_\mathsf{D}^2 &= \mathsf{E}[V|\text{ trader buys } o = 2 \text{ units at } \{\mathsf{ask}_\mathsf{D}^2\}, H_t].
\end{aligned}$$

Finding the Equilibrium. In the limit order market, we have $C_\mathsf{L}(1) = \mathsf{ask}_\mathsf{L}^1$ and $C_\mathsf{L}(2) = \mathsf{ask}_\mathsf{L}^1 + \mathsf{ask}_\mathsf{L}^2$. Then the marginal buyers' indifference conditions (1) can then be rewritten as

$$\mathsf{ask}_\mathsf{L}^1 = \mathsf{E}\pi_\mathsf{L}^1, \quad \text{and} \quad \mathsf{ask}_\mathsf{L}^2 = \mathsf{E}\pi_\mathsf{L}^2. \tag{2}$$

In the dealer market, we have $C_\mathsf{D}(1) = \mathsf{ask}_\mathsf{D}^1$ and $C_\mathsf{D}(2) = 2\mathsf{ask}_\mathsf{D}^2$, and conditions (1) can be rewritten as

$$\mathsf{ask}_\mathsf{D}^1 = \mathsf{E}\pi_\mathsf{D}^1, \quad \text{and} \quad \mathsf{ask}_\mathsf{D}^2 = \frac{1}{2}\,(\mathsf{E}\pi_\mathsf{D}^2 + \mathsf{ask}_\mathsf{D}^1). \tag{3}$$

In the limit order market, the first unit is bought by traders who demand one or two units. In the dealer market, the single unit is purchased only by traders who demand just one unit. Let $\beta_{v,m}^o$ denote the probability that there is a buy of $o \in \{1, 2\}$ units when the value of the security is $v \in \{0, 1\}$ in market $m \in \{\mathsf{L}, \mathsf{D}\}$,

$$\begin{aligned}
\beta_{v,\mathsf{L}}^1 &= 2\lambda + \mu(1 - F_v(\pi_\mathsf{L}^1)), & \beta_{v,\mathsf{L}}^2 &= \lambda + \mu(1 - F_v(\pi_\mathsf{L}^2)), \\
\beta_{v,\mathsf{D}}^1 &= \lambda + \mu(F_v(\pi_\mathsf{D}^2) - F_v(\pi_\mathsf{D}^1)), & \beta_{v,\mathsf{D}}^2 &= \lambda + \mu(1 - F_v(\pi_\mathsf{D}^2)).
\end{aligned}$$

The probability that a given trader is informed is independent of other traders' identities and the security's liquidation value. As private beliefs are independent conditional on the security's value, so are traders' actions. Suppressing indices L and D, the ask prices for unit $o \in \{1, 2\}$ and the expectation of the marginal trader π^o can be written explicitly

[10] This pricing rule is analogous to the one in Glosten (1994).

[11] This pricing rule is identical to the one used in Easley and O'Hara (1987); a more recent contribution is Ozsoylev and Takayama (2008).

as

$$\mathsf{ask}^o = \frac{\beta_1^o p}{\beta_1^o p + \beta_0^o (1-p)}, \quad \mathsf{E}\pi^o = \frac{\pi^o p}{\pi^o p + (1-\pi^o)(1-p)}. \tag{4}$$

Hybrid Markets. Most real world equity markets operate with a hybrid structure where traders have the choice of either arranging their trade with a dealer or posting it to the limit order book.[12] We now consider a trader's choice of order size and trading venue. We focus on the arguably most realistic scenario where both small and large quantities are traded in both market segments.[13]

In the isolated markets, the equilibrium description contained, loosely, information about *who* traded *how many* units. In hybrid markets, the equilibrium description must additionally include information about *where* someone trades. Suppose now that all order sizes are traded in each segment. Then the execution costs for orders of the same size must coincide across the two market segments because otherwise traders would switch to the cheaper segment. Put differently, in equilibrium traders must now self-select into market segments in such a way that trading costs coincide.

This cost equalization implies, in particular, that the marginal informed traders coincide in both market segments. Prices will then depend on the proportion of informed to noise traders. In equilibrium, loosely, the proportion must be such that it ensures equal costs across market segments. There are several ways to model the implied self-selection. We approach it by assuming that informed traders trade in each segment with equal chance, and we then study the self-selection of noise traders that yields the equilibrium.[14]

Specifically, let λ_m^o denote the mass of noise traders that submit buy orders of size $o \in \{1, 2\}$ to market segment $m \in \{\mathsf{HL}, \mathsf{HD}\}$. We then express $\beta_{v,m}^o$, the probability of a buy for fundamental $v \in \{0, 1\}$, as

$$\begin{aligned} \beta_{v,\mathsf{HL}}^1 &= \lambda_{\mathsf{HL}}^1 + \lambda_{\mathsf{HL}}^2 + \mu(1 - F_v(\pi_{\mathsf{H}}^1))/2, \quad & \beta_{v,\mathsf{HL}}^2 &= \lambda_{\mathsf{HL}}^2 + \mu(1 - F_v(\pi_{\mathsf{H}}^2))/2, \\ \beta_{v,\mathsf{HD}}^1 &= \lambda_{\mathsf{HD}}^1 + \mu(F_v(\pi_{\mathsf{H}}^2) - F_v(\pi_{\mathsf{H}}^1))/2, \quad & \beta_{v,\mathsf{HD}}^2 &= \lambda_{\mathsf{HD}}^2 + \mu(1 - F_v(\pi_{\mathsf{H}}^2))/2. \end{aligned}$$

[12] For instance, on the Toronto Stock Exchange traders can approach an upstairs dealer or they can send their order directly to the consolidated limit order book. Some systems are more complex: for instance, on NYSE, a market order that arrives at the specialist's desk could be filled with the current book, with the specialist or with floor brokers who opt to participate, or the specialist can auction the order to floorbrokers. On Nasdaq, small orders are routed to dealers according to a set of rules. We abstract from these institutional subtleties, and focus on the main distinction between the two general systems.

[13] In Grossman (1992) there are corner solutions in which *all* order flow gravitates towards a specific trading mechanism while the other disappears. Our model also admits corner solutions but they have a different flavour: each mechanism attracts a unique order size and both remain in operation. The existence result for corner solutions has been omitted to save space.

[14] In Section VI we discuss extensions of our model.

Ask prices and traders' expectations are expressed in the same way as in (4). We maintain the assumption that liquidity traders demand each quantity with equal probability, thus $\lambda^o_{\mathsf{HD}} = \lambda - \lambda^o_{\mathsf{HL}}$ and we will use only $\lambda^o = \lambda^o_{\mathsf{HL}}$. We then find the equilibrium by determining (a) the marginal buyers who are indifferent between trading one and two units (π^2_{H}) and one and no units (π^1_{H}) and (b) the noise masses λ^1, λ^2 that ensure equal costs. This gives rise to the following four equations that $\pi^1_{\mathsf{H}}, \pi^2_{\mathsf{H}}, \lambda^1, \lambda^2$ solve in equilibrium

$$C_{\mathsf{HL}}(1) = C_{\mathsf{HD}}(1),\ C_{\mathsf{HL}}(2) = C_{\mathsf{HD}}(2),\ \mathsf{E}\pi^1 = C(1),\ 2 \cdot \mathsf{E}\pi^2 - C(2) = 1 \cdot \mathsf{E}\pi^2 - C(1). \quad (5)$$

Costs $C_{\mathsf{HL}}(1), C_{\mathsf{HD}}(1)$ are the ask-prices, $C_{\mathsf{HD}}(2) = 2 \cdot \mathsf{ask}^2_{\mathsf{HD}}$, and $C_{\mathsf{HL}}(2) = \mathsf{ask}^1_{\mathsf{HL}} + \mathsf{ask}^2_{\mathsf{HL}}$.

Theorem 1 (Existence in Limit Order, Dealer, and Hybrid Markets)
For any prior $p \in (0,1)$

(a) [Limit Order Market] *there exists a unique symmetric equilibrium with marginal beliefs* $1/2 < \pi^1_{\mathsf{L}} < \pi^2_{\mathsf{L}} < 1$ *that solve the equations in (2);*

(b) [Dealer Market] *there exists a unique symmetric equilibrium with marginal beliefs* $1/2 < \pi^1_{\mathsf{D}} < \pi^2_{\mathsf{D}} < 1$ *that solve the equations in (3);*

(c) [Hybrid Market] *there exists a unique symmetric equilibrium with marginal beliefs* $1/2 < \pi^1_{\mathsf{H}} < \pi^2_{\mathsf{H}} < 1$ *and noise levels* λ^1, λ^2 *that solve the equations in (5).*

(d) [Monotonicity] *The decision rules in* $(a) - (c)$ *are monotone: traders with private beliefs* $\pi < \pi^1$ *do not buy, traders with private beliefs* $\pi \in [\pi^1, \pi^2)$ *buy one unit, and traders with private beliefs* $\pi \in [\pi^2, 1]$ *buy two units.*

Public Dealer Market Quotes. Our treatment of price formation in the dealer market is stylized: people submit their market orders without knowing the price and there are no standing quotes from dealers. In real markets dealers do publicly quote bid and offer prices. Moreover, in many markets, dealers are required to trade a guaranteed minimum number of units at these quotes (for instance, for most stocks a Nasdaq dealer's quote "must be good" for 1,000 shares). On some exchanges, e.g. the TSX, the upstairs dealers are required to trade at the best bid or offer (BBO) that are currently on the book, unless the size of the trade is very large. Exchanges that use small-order routing systems (i.e. small orders are given to dealers according to a pre-determined set of rules) require dealers to "improve prices" to at least match the BBO.

These institutional details are compatible with our setup. First, in the equilibrium of our model traders can perfectly anticipate the price that they will be quoted if they approach a dealer. Second, our model can be rewritten to accommodate (a) public quotes in the dealer market and (b) minimum fill sizes for these quotes. In this rewritten version of the model, the quoted ask price would be $\mathsf{ask}^2_{\mathsf{D}}$, the price for a large order.

When facing a small order, the dealer would then improve the price to $\mathsf{ask}_\mathsf{D}^1$. This alternative setup would also satisfy the BBO rule for hybrid markets. We chose our current formulation to simplify the exposition.

III Testable Predictions for the Hybrid Market

The hybrid market equilibrium is determined by the marginal traders and by the fractions of noise traders in each market segment, which in turn affect the informativeness of trades.

Suppose a small trade arrives in the dealer market segment. Then the dealer knows that the trade stems from an informed trader with belief $\pi \in [\pi_\mathsf{H}^1, \pi_\mathsf{H}^2)$ or from a noise trader. The first unit in the limit order segment, on the other hand, is hit by orders from informed traders with beliefs $\pi \in [\pi_\mathsf{H}^1, 1]$. Ceteris paribus, this should make the single unit trade in the limit order segment more informative and thus more expensive. To have equal costs, intuitively, there must be more noise in the limit order segment. The reverse applies to large orders. The following result confirms this intuition.

Proposition 1 (Trade Informativeness in the Hybrid Market Equilibrium) *The ratio of noise to informed traders of small size orders is larger in the limit order segment, and the ratio of noise to informed traders of large size orders is larger in the dealer segment.*

Proposition 1 provides a theoretical basis for the empirical finding that upstairs markets —which loosely correspond to the dealer market segment— are better at identifying uninformed trades. Our result shows that the co-existence of the two major trading mechanisms *necessarily* implies that more uninformed traders seek to trade large quantities with dealers.

The information content of trades implied by Proposition 1 is similar to that in Seppi (1990), where a large, possibly informed trader has the choice between trading anonymously on the exchange or non-anonymously off the exchange. If he chooses the latter option, he may be punished, due to repeated interactions, for "bagging the street". This threat drives traders' self selection. Notably, the pricing mechanisms on and off the exchange in Seppi (1990) follow a Kyle (1985)-style dealer market pricing rule. We thus complement Seppi (1990) by studying traders' self-selection into dealer and limit order segments.

Since the mass of informed traders in either market segment is the same, Proposition 1 implies that there are more small noise trades in the limit order segment and more large noise trades in the dealer segment. We can further show that there are in total more noise traders that trade either quantity in the limit order segment.

Proposition 2 (Transactions by Market Segment)
(a) *There are more large transactions in the dealer segment than in the limit order segment; the reverse holds for small transactions.*
(b) *The limit order segment attracts more transactions than the dealer segment.*

IV Comparison of the Three Trading Mechanisms

We will now analyze how the different market mechanisms affect spreads, execution costs, and volume and how efficient the systems are relative to each other. Our goal is to generate testable predictions for these major observable variables.

A Spreads, Market Width, and Price Impacts.

We will first compare the trading mechanisms with respect to market liquidity, namely market width and price impacts.

Market width (sometimes also referred to as market breadth) is the cost of doing a trade of a given size. It is the dual of market *depth*, which measures the size of a trade that can be arranged at a given cost.[15] For small trades, the width is associated with the bid-ask-spread. When people trade for informational reasons, a larger width indicates higher adverse selection costs and thus a lower willingness to provide liquidity. We measure width for order size o by the dollar cost of a buy transaction, $C(o)$.

The *price impact* reflects how the market assesses the information content of a trade. If the current transaction price and the public expectation coincide, then the price impact is the difference between the current and the past transaction price. If they differ, then the price impact reflects the permanent effect of a trade on prices.[16] We quantify the price impact by $\Delta \mathsf{p}^i = \mathsf{E}[V|H_t, \text{transaction of size } i \text{ at time } t] - \mathsf{E}[V|H_t]$.

[15]See Harris (2003), pp. 398-399, for the definitions of depth and width and for an extensive discussion.

[16]Loosely, permanent effects are associated with information transmission, temporary effects are associated with uninformative inventory re-balancing. Empirically the price impact is measured as the difference between the effective spread and the realized spread, where the *effective spread* is the transaction price at time t minus the midpoint of the bid-ask-spread at t and the realized spread is the transaction price at t minus the midpoint of the bid-ask-spread x minutes later at $t + x$ (see, for instance, Bessembinder (2003)).

Proposition 3 (Liquidity Measures)
(a) *Market width for small trades is ordered as follows:* $C_{\mathsf{D}}(1) < C_{\mathsf{H}}(1) < C_{\mathsf{L}}(1)$.
Market width for large trades is ordered as follows: $C_{\mathsf{L}}(2) < C_{\mathsf{H}}(2) < C_{\mathsf{D}}(2)$.
(b) *Price impacts of small trades are ordered as follows* $\Delta \mathsf{p}^1_{\mathsf{L}}, \Delta \mathsf{p}^1_{\mathsf{HL}} < \Delta \mathsf{p}^1_{\mathsf{D}} < \Delta \mathsf{p}^1_{\mathsf{HD}}$.
Price impacts of large trades are ordered as follows $\Delta \mathsf{p}^2_{\mathsf{HD}} < \Delta \mathsf{p}^2_{\mathsf{D}} < \Delta \mathsf{p}^2_{\mathsf{L}} < \Delta \mathsf{p}^2_{\mathsf{HL}}$.

The intuition underlying these results is that liquidity providers in dealer markets are intrinsically better at pin-pointing the information content of a transaction because they know the order size before setting the price. While this lowers traders' information rents, those with lower quality information are better off being identified, as is demonstrated by the lower spreads in the dealer market. For in the limit order market, very well informed traders hide among the less well informed ones and thus earn a rent at their expense.[17]

In GM models with single unit trades, the price impact is the change in the transaction price. In our model, this remains true in the dealer market and the dealer segment of the hybrid market. In the limit order market and the limit order segment of the hybrid market, however, the public expectation of the security value coincides with the transaction price only after large orders, but not after small orders.

To understand this point, observe that in limit order markets, the ask price for the single unit trade accounts for the fact that all informed traders with belief $\pi \in [\pi^1, 1]$ buy this unit. Yet *after* it is revealed that an arriving trader bought only one unit, it is known that this trade was performed by an informed trader with belief $\pi \in [\pi^1, \pi^2]$ (or by a noise trader). As a consequence, the price impact of small orders in the limit order segment is intuitively smaller than that implied by the transaction price. Hence the displayed order of price impacts. Finally, numerical simulations reveal that $\Delta \mathsf{p}^1_{\mathsf{HL}}$ and $\Delta \mathsf{p}^1_{\mathsf{L}}$ cannot be ordered.

B Dynamic Behavior

In the limit order market, the history of trades does not affect a trader's decision to buy or sell. In the dealer market and hybrid market, the behavior is history dependent.

Proposition 4 (Behavioral Dynamics and Contrarianism)
As p *traverses from 0 to 1,*
(a) *trading behavior does not change in the limit order market,*
(b) *all trading thresholds increase in the dealer market, and*
(c) *all trading thresholds increase in the hybrid market.*

[17] The result that small trades are cheaper on dealer markets has been previously noted by, for instance, Glosten (1994) or Seppi (1997).

Part (a) is implied by expressions (2) and (4). In the dealer and hybrid markets, however, the marginal types change with the prior p. If the prior favours a trader's opinion, two effects occur. First, the trader feels more encouraged by the public opinion. Second, the marginal value of his information declines. Traders "herd" when the first effect dominates so that they need lower quality signals to trade. Traders "act as contrarians" when the second effects dominates so that they need higher quality signals. Proposition 4 shows the latter for the dealer and hybrid markets, in line with empirical observations (see Chordia, Roll, and Subrahmanyam (2002)).

C Volume

The results in the remainder of this section are based on simulations of the trading process for the case of a uniformly distributed quality (see the example in Appendix A).

Although our model describes single trader arrivals for each period, we can proxy volume in market $m \in \{\mathsf{L}, \mathsf{D}, \mathsf{H}\}$ at prior $p = \mathsf{Pr}(V = 1)$ by the expected number of units that will be traded upon the arrival of a trader,

$$\mathsf{vol}_m = \sum_{v=0,1} \sum_{o=-2,-1,1,2} |o| \cdot \mathsf{Pr}(o|V = v, m, p) \cdot \mathsf{Pr}(V = v).$$

We compute vol numerically on a fine grid for the feasible parameters (the prior p and the amount of informed trading μ) and find the following.

Numerical Observation 1 (Volume) *For any prior p and any level of informed trading μ, volume is ordered as follows:* $\mathsf{vol}_\mathsf{L} > \mathsf{vol}_\mathsf{H} > \mathsf{vol}_\mathsf{D}$.

Figure 3 illustrates this numerical observation. The intuition for the finding stems from the behavior of the traders who submit large orders. In the dealer market, large order traders are identified, whereas in the limit order market they hide among the small order traders. As a consequence, the marginal buyer of the large quantity has the highest belief in the dealer market and the lowest belief in the limit order market (we show this formally in the proof of the existence theorem). Traders thus submit large orders least aggressively in the dealer market and most aggressively in the limit order market, which leads to the lowest and highest volumes respectively.

D Price Efficiency

Price efficiency measures the closeness of a price to the fundamental value of a security. We analyze it in two ways. First, we compute the expected price impact for each trading mechanism. Second, we simulate sequences of trades that mimic the arrival of traders during a specific trading day. The final price after a sequence can be interpreted as the closing price. We then base our analysis of price efficiency on the properties of the expected price impacts and of the closing prices.

We perform our analysis of closeness for the fundamental value $V = 1$.[18] Then the higher the public expectation, the closer it is to the true value. Thus higher price impacts and closing prices are associated with a more efficient market mechanism.

We measure the price impact by the change in the public expectation. We thus compute numerically for all priors $p \in (0, 1)$, levels of informed trading $\mu \in (0, 1)$ and market mechanisms $m \in \{\mathsf{L}, \mathsf{D}, \mathsf{H}\}$, using $\mathsf{E}[V|p] = p$,

$$\mathsf{E}[\Delta \mathsf{p}_m | V = 1, p] = \sum_{o \in \{-2,-1,0,1,2\}} \mathsf{Pr}(o|V = 1, m, p) \cdot \mathsf{E}[V|o, m, p] - p. \tag{6}$$

To simplify the exposition, we use $\mathsf{E}\Delta\mathsf{p}_m$ for the expected price impact.

We obtain the simulated closing prices as follows. For each level of informed trading $\mu \in \{.1, \ldots, .9\}$ we simulated 500,000 trading days with entry rates $\rho \in \{10, \ldots, 50\}$. For each day, we generated random realizations for the number of traders (a Poisson arrival; parameter ρ) and their entry order, traders' identities (noise vs. informed; parameter μ), trading decisions for noise traders, and beliefs for informed traders (fixing the fundamental value to $V = 1$). The public expectation that obtains after these traders have acted is the closing price for that day.[19] The signal quality distribution is assumed to be uniform.

We assess closeness of closing prices to the fundamental value in two ways. First, we compare the average closing prices. Second, we compare the empirical distributions of closing prices to see if one mechanism systematically yields higher and thus more efficient prices. "Systematically higher" in a distributional sense obtains if the empirical distributions of prices can be ranked in the sense of first order stochastic dominance. By definition, distribution F_x first order stochastically dominates distribution F_y if $F_x(\mathsf{p}) \leq F_y(\mathsf{p})$ for all closing prices p. We will thus compare the differences of empirical distributions.

To ensure that the distribution of closing prices is reasonably smooth, we focus on the

[18]To measure closeness to the true fundamental, we need to fix this value. The analysis for the case of $V = 0$ is symmetric.

[19]As we discussed after Proposition 3 this public expectation may differ from the last transaction price. To simplify the exposition, we will refer to the last value of the public expectation as a "closing price".

case $\rho = 50$ when analyzing properties of the empirical distributions of prices.[20] Further parametric and procedural details of the simulation process are outlined in Appendix B. We write $F_m, \bar{\mathsf{p}}_m$, and σ_m for the empirical distribution, average, and standard deviation of closing prices in market $m \in \{\mathsf{L}, \mathsf{D}, \mathsf{H}\}$.

In presenting our results we are loose in listing them for "low", "middle" and "high" μ. These regions of μ differ slightly for different ρ.[21] Table I displays the signs of the differences in averages and illustrates the observation; Figure 5 plots the differences of expected price impacts. Note that an ordering in the sense of first order stochastic dominance implies the same ordering for the average prices; we report both measures for completeness.

Numerical Observation 2 (Price Efficiency)

(*a*) *For low* μ*:* $\mathsf{E}\Delta\mathsf{p}_\mathsf{D} < \mathsf{E}\Delta\mathsf{p}_\mathsf{L} < \mathsf{E}\Delta\mathsf{p}_\mathsf{H}$ *and* $\bar{\mathsf{p}}_\mathsf{D} < \bar{\mathsf{p}}_\mathsf{L} < \bar{\mathsf{p}}_\mathsf{H}$.

(*b*) *For medium* μ*:* $\mathsf{E}\Delta\mathsf{p}_\mathsf{D} < \mathsf{E}\Delta\mathsf{p}_\mathsf{L} < \mathsf{E}\Delta\mathsf{p}_\mathsf{H}$*,* $\bar{\mathsf{p}}_\mathsf{D} < \bar{\mathsf{p}}_\mathsf{L} < \bar{\mathsf{p}}_\mathsf{H}$*, and* F_H *fosd* F_L *fosd* F_D.

(*c*) *For high* μ*:* $\mathsf{E}\Delta\mathsf{p}_\mathsf{L} < \mathsf{E}\Delta\mathsf{p}_\mathsf{H} < \mathsf{E}\Delta\mathsf{p}_\mathsf{D}$*,* $\bar{\mathsf{p}}_\mathsf{L} < \bar{\mathsf{p}}_\mathsf{H} < \bar{\mathsf{p}}_\mathsf{D}$*, and* F_D *fosd* F_H *fosd* F_L.

The results on expected price impacts and price distributions are consistent: the hybrid market dominates the limit order market, which dominates the dealer market, except when μ is very large. We do not have conclusive results concerning the price distributions for low levels of μ, where prices are driven largely by noise.

An efficiency measure based on the average price alone could be criticized if higher average prices go along with higher price volatility. Indeed, this is what we observe for low μ where $\sigma_\mathsf{H} > \sigma_\mathsf{L} > \sigma_\mathsf{D}$. For medium and high levels of μ, however, this criticism does not apply. The first order stochastic dominance ordering implies the same ordering in the sense of second order stochastic dominance, and thus more efficient prices are also less dispersed.

Numerical Observation 2 argues, in particular, that the hybrid market is more efficient than the pure limit order market. The dealer segment thus serves an important role in enhancing market efficiency. To see the intuition for this, observe, first, that in the hybrid market most noise traders of large size orders submit them in the dealer market segment (Proposition 1). Second, relative to informed traders, noise traders are more likely to trade in the "wrong" direction. Finally, the price impact of a large order in the pure limit order book is smaller than that in the limit order segment and larger than that in the dealer market segment (Proposition 3). In other words, the limit order

[20] For $\rho = 50$ there are on average 50 traders per day, yielding decisions for about 25,000,000 traders.

[21] For instance, for $\rho = 50$, the "middle" μ is between .3 and .7. The general observation is that the region of "middle" μ increases in the average number of traders, ρ.

segment of the hybrid market attracts most of the "right" direction large trades and these have the highest price impact; the converse obtains for the dealer market segment. Together these effects imply the relation between hybrid and limit order markets.

V Transparency vs. Opacity in the Hybrid Market

Transparency is usually separated into pre-trade and post-trade transparency. The former reflects the information that trading parties possess before they either demand or supply liquidity, the latter reflects their information about past transactions.

Our model assumes that liquidity demanders have full pre-trade transparency, i.e. they can either see all available quotes (for the limit order book) or they can infer the prices that they will be quoted in equilibrium (for the dealer market). Liquidity suppliers face higher pre-trade transparency in the dealer market than in the limit order market, because they know the order size for which they supply liquidity.

Post-trade transparency obtains in the limit order market because anyone can observe when and how far an order "walks the book". We also assume full post-trade transparency for the dealer market in that all transactions are disclosed.

Our analysis of the hybrid market thus far has assumed the same level of post-trade transparency. In particular, we have assumed that after every transaction all market participants learn the size of a trade *and* the segment it cleared in. We will now investigate the impact of the information about the trading venue. In what follows we refer to the market where the venue is revealed as the *transparent* market and we refer to the market where the venue is not revealed as the *opaque* market.

In the opaque market, the price impact of any small trade is driven purely by the transaction price. In the transparent market, on the other hand, small trades in the limit order segment have smaller price impacts than those implied by the transaction prices. We analyze the effect of these different price impacts on price efficiency, using the measures described in the last section. Table II and Figure 6 illustrate the following results.

Numerical Observation 3 (Transparency vs. Opacity)

(a) *For a large enough entry rate ρ, the transparent market is more efficient in the sense that average closing prices there are higher and thus more efficient; for small entry rates it is the reverse.*

(b) *Prices in the transparent market are less volatile than those in the opaque market in the sense of the second order stochastic dominance.*

The expected price impacts, or changes in the public expectation, as defined in (6), are not ordered, but the displayed patterns (see Figure 7) are consistent with the above findings and provide an intuition for them. Recall that, fixing the fundamental to $V = 1$, larger price impacts are associated with higher efficiency. We observe that for low and medium levels of the prior p, the expected price impact in the opaque market is larger than that in the transparent market; the reverse holds for high levels of p. This switch explains why the opaque market is more efficient for low entry rates ρ (or, "thinly" traded stocks). For small numbers of trades (low ρ), the public expectation moves little so that the prior p remains close to $1/2$ where the opaque markets yields stronger movements of prices in the direction of the fundamental. Since on average, traders are more likely to be "right" than "wrong", when there are many trades (high entry rate ρ), the public expectation will be close to the fundamental, 1, most of the time. In this region, the transparent market yields larger price impacts and thus higher price efficiency.

The larger dispersion in the opaque market is explained by the fact that small trades there move prices more strongly in either direction.

VI Conclusion

This paper provides a three way theoretical comparison of dealer, limit order and hybrid markets. We analyze the impact of these trading mechanisms on price efficiency, volume, liquidity, and trade execution costs and generate several new empirical predictions.

The organization of trading, the regulations and the rules can differ dramatically among different exchanges. Yet almost all trading arrangements can be classified as one of the three mechanisms that this paper studies. Our model provides the benchmark differences that these mechanisms would display empirically, controlling for other institutional details.

In addition to generating empirical predictions, our paper has implications for empirical methodology. First, we identify that trading behavior in hybrid and dealer markets changes throughout the trading day. Estimations that use aggregate numbers of trades, as is common practice when estimating the probability of informed trading, must thus account for this possibility. Second, our results indicate that the price impacts of small trades in limit order markets are smaller than the changes in transaction prices. The difference between these two measures reflects the information conveyed by the total order size. Attributing this difference to inventory risk would overestimate such costs.

Finally, our findings have policy implications for professional market design. Historically, many equity markets have developed from pure dealer markets to hybrid markets where a dealer segment coexists with a limit order book. Our analysis highlights the advantages of such developments, such as increased price efficiency and trading volume.

We further argue that a hybrid market has advantages over a pure limit order market in that prices are more efficient and costs for small orders are lower. The efficiency gain stems from the stronger price impact of large trades in the limit order segment of the hybrid market. Extending our model to allow exogenous variations in relative trading volume in the market segments, we can show that this result requires sufficient volume in the limit order book. If traders were to exogenously gravitate towards the dealer segment so that there are only few transactions in the limit order segment, then the informational advantage of the hybrid market would be lost. Thus market designers and regulators may find it beneficial to guarantee that a sufficient order flow reaches the limit order book. Indeed, some exchanges, e.g. the Toronto Stock Exchange, require small orders to be routed to the limit order book.

A Appendix: Quality and Belief Distributions

Financial market microstructure models with binary signals and states typically employ a constant common signal quality $q \in [{}^1\!/_2, 1]$, with $\mathsf{Pr}(\text{signal} = h|V = 1) = \mathsf{Pr}(\text{signal} = l|V = 0) = q$. This parameterization is easy to interpret, as a trader who receives a high signal h will update his prior in favor of the high liquidation value, $V = 1$, and a trader who receives a low signal l will update his prior in favor of $V = 0$. We thus use the conventional description of traders' information, with qualities $q \in [{}^1\!/_2, 1]$, in the main text.

As discussed in the main text, to facilitate the analysis, we map a vector of a trader's signal and its quality into a scalar continuous variable on $[0, 1]$, namely, the trader's private belief. To derive the distributions of traders' private beliefs, it is mathematically convenient to normalize the signal quality so that its domain coincides with that of the private belief. We will denote the distribution function of this normalized quality on $[0, 1]$ by G and its density by g, whereas the distribution and density functions of original qualities on $[{}^1\!/_2, 1]$ will be denoted by $\tilde{G}$ and $\tilde{g}$ respectively.

The normalization proceeds as follows. Without loss of generality, we employ the density function g that is symmetric around ${}^1\!/_2$. For $q \in [0, {}^1\!/_2]$, we then have $g(q) = \tilde{g}(1 - q)/2$ and for $q \in [{}^1\!/_2, 1]$, we have $g(q) = \tilde{g}(q)/2$.

Under this specification, signal qualities q and $1 - q$ are equally useful for the individual: if someone receives signal h and has quality ${}^1\!/_4$, then this signal has "the opposite meaning", i.e. it has the same meaning as receiving signal l with quality ${}^3\!/_4$. Signal qualities are assumed to be independent across agents and independent of the fundamental value V.

Beliefs are derived by Bayes Rule, given signals and signal qualities. Specifically, if a trader is told that his signal quality is q and receives a high signal h then his belief is $q/[q + (1 - q)] = q$ (respectively $1 - q$ if he receives a low signal l), because the prior is $^1\!/_2$. The belief π is thus held by people who receive signal h and quality $q = \pi$ and by those who receive signal l and quality $q = 1 - \pi$. Consequently, the density of individuals with belief π is given by $f_1(\pi) = \pi[g(\pi) + g(1 - \pi)]$ when $V = 1$ and analogously by $f_0(\pi) = (1-\pi)[g(\pi)+g(1-\pi)]$ when $V = 0$. Smith and Sorensen (2008) prove the following property of private beliefs (Lemma 2 in their paper):

Lemma 1 (Symmetric beliefs, Smith and Sorensen (2008)) *With the above the signal quality structure, private belief distributions satisfy $F_1(\pi) = 1 - F_0(1 - \pi)$ for all $\pi \in (0, 1)$.*

Proof: Since $f_1(\pi) = \pi[g(\pi) + g(1 - \pi)]$ and $f_0(\pi) = (1 - \pi)[g(\pi) + g(1 - \pi)]$, we have $f_1(\pi) = f_0(1 - \pi)$. Then $F_1(\pi) = \int_0^\pi f_1(x)dx = \int_0^\pi f_0(1 - x)dx = \int_{1-\pi}^1 f_0(x)dx = 1 - F_0(1 - \pi)$. □ Belief densities obey the monotone likelihood ratio property as the following increases in π

$$\frac{f_1(\pi)}{f_0(\pi)} = \frac{\pi[g(\pi) + g(1 - \pi)]}{(1 - \pi)[g(\pi) + g(1 - \pi)]} = \frac{\pi}{1 - \pi}. \tag{7}$$

One can recover the distribution of qualities on $[^1\!/_2, 1]$, denoted by $\tilde{G}$, from G by combining qualities that yield the same beliefs for opposing signals (e.g $q = {}^1\!/_4$ and signal h is combined with $q = {}^3\!/_4$ and signal l). With symmetric g, $G(^1\!/_2) = {}^1\!/_2$, and

$$\tilde{G}(q) = \int_{\frac{1}{2}}^{q} g(s)ds + \int_{1-q}^{\frac{1}{2}} g(s)ds = 2\int_{\frac{1}{2}}^{q} g(s)ds = 2G(q) - 2G(^1\!/_2) = 2G(q) - 1.$$

An Example of private beliefs. Figure 2 depicts an example where the signal quality q is uniformly distributed. The uniform distribution implies that the density of individuals with signals of quality $q \in [^1\!/_2, 1]$ is $\tilde{g}(q) = 2q$. When $V = 1$, private beliefs $\pi \geq {}^1\!/_2$ are held by traders who receive signal h of quality $q = \pi$, private beliefs $\pi \leq {}^1\!/_2$ are held by traders who receive signal l of quality $q = 1 - \pi$. Thus, when $V = 1$, the density of private beliefs π for $\pi \in [^1\!/_2, 1]$ is given by $f_1(\pi) = \mathsf{Pr}(h|V = 1, q = \pi)\tilde{g}(q = \pi) = 2\pi$ and for $\pi \in [0, ^1\!/_2]$ it is given by $f_1(\pi) = \mathsf{Pr}(l|V = 1, q = 1-\pi)\tilde{g}(q = 1-\pi) = 2\pi$. Similarly, the density conditional on $V = 0$ is $f_0(\pi) = 2(1 - \pi)$. The distributions of private beliefs are then $F_1(\pi) = \pi^2$ and $F_0(\pi) = 2\pi - \pi^2$. Figure 2 also illustrates that

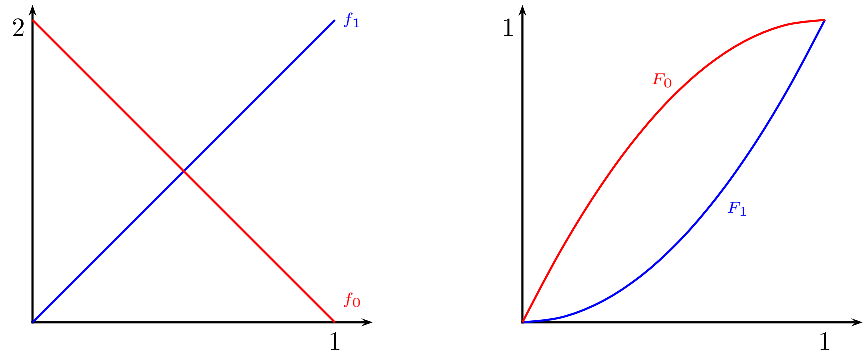

Figure 2: **Plots of belief densities and distributions.** *Left Panel:* The densities of beliefs for an example with uniformly distributed qualities. The densities for beliefs conditional on the true fundamental being 1 and 0 respectively are $f_1(\pi) = 2\pi$ and $f_0(\pi) = 2(1-\pi)$; *Right Panel:* The corresponding conditional distribution functions: $F_1(\pi) = \pi^2$ and $F_0(\pi) = 2\pi - \pi^2$.

signals are informative: recipients in favor of $V = 0$ are more likely to occur when $V = 0$ than when $V = 1$.

B Appendix: Simulation Procedure for Price Efficiency

We employed the following data generation procedure. We obtained 500,000 observations of trading days for each of the Poisson arrival rates $\rho \in \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$ and levels of informed trading $\mu \in \{.1, .2, .3, .4, .5, .6, .7, .8, .9\}$. The Poisson arrival rate ρ implies that, on average, ρ traders arrive on any given day (some may choose not to trade). Fixing the fundamental to $V = 1$, higher prices are closer to the true fundamental and thus more efficient. To capture the effect of the entry rate ρ for transparent vs. opaque hybrid markets, we also ran these simulations for low entry rates $\rho \in \{2, 3, \ldots, 15\}$ for $\mu \in \{.2, .5, .8\}$.

For each series, we first drew the number of traders for the session and performed the random allocation of traders into noise and informed and their entry order. Signals for informed traders and trading roles for noise traders were assigned as depicted in Figure 1, conditional on the fundamental being $V = 1$. These traders then acted in sequence, and we determined the informed traders' optimal decisions and (for the hybrid market) the noise traders' choice of the trading venue, based on the preceding history, as described in Section II. We let the same sequence of traders act for each of the four trading rules (limit order market, dealer marker, hybrid market, and opaque hybrid market); note that the same informed trader may take different decisions in different markets. We then recorded the public expectation at the end of each sequence of traders for each of the four trading rules.

Our random number generation employs the Mersenne Twister algorithm (Matsumoto and Nishimura (1998)). This algorithm greatly reduces the correlation of successive values that arises with most other pseudo-random number generators.

C Appendix: Omitted Proofs

C.1 Some General Results and Notation

We will first introduce some notation and establish basic results that facilitate the analysis and proofs of our main results.

C.1.1 General notation for all proofs

In what follows, we will use function $a_t(\Lambda, \underline{\pi}, \overline{\pi})$ to denote the time-t liquidity provider's expectation of the security value conditional on a buy order that stems from either a noise trader drawn from a mass of size Λ, or from an informed trader drawn from a mass of size μ and equipped with a private belief between $\underline{\pi}$ and $\overline{\pi}$. Conditional on the true value being $V = v$, the probability of such an order is $\beta_v(\Lambda, \underline{\pi}, \overline{\pi}) = \Lambda + \mu(F_v(\overline{\pi}) - F_v(\underline{\pi}))$. Then, using Bayes Rule and rearranging,

$$a_t(\Lambda, \underline{\pi}, \overline{\pi}) = \left[1 + \frac{1-p_t}{p_t} \frac{\beta_0(\Lambda, \underline{\pi}, \overline{\pi})}{\beta_1(\Lambda, \underline{\pi}, \overline{\pi})}\right]^{-1} \tag{8}$$

This specification allows us to compactly express all equilibrium ask prices. For instance, the equilibrium ask price for the small size order in the pure limit order market can be written as $\mathsf{ask}^1_{\mathsf{L}} = a_t(2\lambda, \pi^1_{\mathsf{L}}, 1)$. In the hybrid market, the mass of informed traders in each segment is $\mu/2$, and the probability of, say, a small buy order in a limit order segment is $\lambda_1 + \lambda_2 + (\mu/2) \cdot (F_v(\pi^2_{\mathsf{H}}) - F_v(\pi^1_{\mathsf{H}}))$. Renormalizing, we can write the equilibrium ask price for this order as $\mathsf{ask}^1_{\mathsf{HL}} = a_t(2\lambda^1 + 2\lambda^2, \pi^1_{\mathsf{H}}, 1)$. All other prices are similar.

Further, we will use function $\pi^*_t(\Lambda, \overline{\pi})$ to denote π that solves

$$\mathsf{E}_t \pi = a_t(\Lambda, \pi, \overline{\pi}). \tag{9}$$

Function $\pi^*_t(\Lambda, \overline{\pi})$ will be useful in compactly expressing the equilibrium thresholds, and we study its properties in more detail in the next subsection. In what follows, we will omit the subscript t whenever the usage is clear from the context.

C.1.2 Preliminary Properties

In what follows, it will often be mathematically convenient to express the private belief distributions F_1, F_0 in terms of the underlying quality distribution function G:

$$F_1(\pi) = 2\int_0^{\pi} s \cdot g(s)\; ds,\;\; F_0(\pi) = 2\int_0^{\pi} (1-s) \cdot g(s)\; ds \;\Rightarrow\; F_1(\pi) + F_0(\pi) = 2G(\pi), \quad (10)$$

and by partial integration,

$$F_1(\pi) = 2\pi G(\pi) - 2\int_0^{\pi} G(s)\; ds. \quad (11)$$

Lemma 2 (Properties of the equilibrium thresholds)

(a) *For every $(\Lambda, \overline{\pi})$ such that $0 < \Lambda < 1$ and $1/2 < \overline{\pi} < 1$, there exists a unique $\underline{\pi} \in (.5, \overline{\pi})$ that solves equation (9). This solution is independent of H_t: $\pi_t^*(\Lambda, \overline{\pi}) = \pi^*(\Lambda, \overline{\pi})$.*

(b) *$\pi^*(\Lambda, \overline{\pi})$ decreases in Λ and increases in $\overline{\pi}$: $\partial\pi^*/\partial\Lambda < 0$ and $\partial\pi^*/\partial\overline{\pi} > 0$.*

(c) *For fixed $(\Lambda, \overline{\pi})$, $\pi = \pi^*(\Lambda, \overline{\pi})$ maximizes $a_t(\Lambda, \pi, \overline{\pi})$. Further, $a_t(\Lambda, \pi, \overline{\pi})$ increases in π for $\pi < \pi^*(\Lambda, \overline{\pi})$ and it decreases in π for $\pi > \pi^*(\Lambda, \overline{\pi})$.*

Proof of (a): Equation (9) can be rewritten as

$$\frac{\pi}{1-\pi} = \frac{\Lambda + \mu(F_1(\overline{\pi}) - F_1(\pi))}{\Lambda + \mu(F_0(\overline{\pi}) - F_0(\pi))}, \quad (12)$$

thus the solution does not depend on the history H_t. Using (10) and (11), we rewrite (12) as

$$2\mu G(\overline{\pi})(\overline{\pi} - \pi) - 2\mu\int_{\pi}^{\overline{\pi}} G(s)\; ds - \Lambda(2\pi - 1) = 0. \quad (13)$$

Denote the left hand side of the above equation by $\delta(\Lambda, \pi, \overline{\pi})$. Then

(i) $\delta(\Lambda, \pi, \overline{\pi})$ strictly decreases in π for $\pi \leq \overline{\pi}$: $\partial\delta/\partial\pi = -2\Lambda - 2\mu(G(\overline{\pi}) - G(\pi)) < 0$;

(ii) at $\pi = 1/2$, $\delta(\Lambda, 1/2, \overline{\pi}) = 2\mu G(\overline{\pi})(\overline{\pi} - 1/2) - 2\mu\int_{1/2}^{\overline{\pi}} G(s)\; ds > 0$;

(iii) at $\pi = \overline{\pi}$, $\delta(\Lambda, \overline{\pi}, \overline{\pi}) = -\Lambda(2\overline{\pi} - 1) < 0$.

Steps (i) – (iii) imply existence and uniqueness of $\pi^*(\Lambda, \overline{\pi})$.

Proof of (b): Applying the Implicit Function Theorem and differentiating both sides of equation (13) with respect to Λ for a fixed $\overline{\pi}$, we obtain

$$\frac{\partial\pi^*}{\partial\Lambda} = -\frac{2\pi^*(\Lambda, \overline{\pi}) - 1}{2\mu(G(\overline{\pi}) - G(\pi^*(\Lambda, \overline{\pi}))) + 2\Lambda} < 0,$$

since $\pi^*(\Lambda, \overline{\pi}) \in (^1/_2, \overline{\pi})$ and G is increasing. Likewise, differentiating both sides of equation (13) with respect to $\overline{\pi}$ for a fixed Λ and using g to denote the density function of qualities, we obtain

$$\frac{\partial \pi^*}{\partial \overline{\pi}} = \frac{2\mu g(\overline{\pi})(\overline{\pi} - \pi^*(\Lambda, \overline{\pi}))}{2\mu(G(\overline{\pi}) - G(\pi^*(\Lambda, \overline{\pi}))) + 2\Lambda} > 0.$$

Proof of (c)**:** The first order condition for maximizing $a_t(\Lambda, \pi, \overline{\pi})$ in π can be written as

$$\frac{\beta_1(\Lambda, \pi, \overline{\pi})}{\beta_0(\Lambda, \pi, \overline{\pi})} = \frac{\partial\beta_1(\Lambda, \pi, \overline{\pi})/\partial\pi}{\partial\beta_0(\Lambda, \pi, \overline{\pi})/\partial\pi} \quad \Leftrightarrow \quad \frac{\beta_1(\Lambda, \pi, \overline{\pi})}{\beta_0(\Lambda, \pi, \overline{\pi})} = \frac{\pi}{1-\pi}, \tag{14}$$

where the last equality follows from equation (7). Observe that this last equality coincides with equation (12). Consequently, there exists a unique π that maximizes $a_t(\Lambda, \pi, \overline{\pi})$ and this $\pi = \pi^*(\Lambda, \overline{\pi})$.

By (8), $a_t(\Lambda, \pi, \overline{\pi})$ increases in π when $\beta_1(\Lambda, \pi, \overline{\pi})/\beta_0(\Lambda, \pi, \overline{\pi})$ increases in π. Using (7), (10), and (11), it can be shown that $(\partial/\partial\pi)(\beta_1(\Lambda, \pi, \overline{\pi})/\beta_0(\Lambda, \pi, \overline{\pi})) > 0$ when $\delta(\Lambda, \pi, \overline{\pi}) > 0$. The desired slopes then follow from part (a).

C.2 Existence in the Limit Order Market: Proof of Theorem 1 (a)

Applying Lemma 2, the equilibrium thresholds are $\pi^1_{\mathsf{L}} = \pi^*(2\lambda, 1)$ and $\pi^2_{\mathsf{L}} = \pi^*(\lambda, 1)$.

C.3 Existence in the Dealer Market: Proof of Theorem 1 (b)

By Lemma 2, we know that for every marginal trader of 2 units $\pi^2 \in (^1\!/_2, 1)$, there exists a unique marginal trader $\pi^1 = \pi^*(\lambda, \pi^2)$ who is indifferent between buying 1 unit and abstaining. Further, π^1 is increasing in π^2. What remains to be shown is that there exists a marginal trader π^2 who is indifferent between purchasing 2 units and 1 unit, so that π^2 solves

$$2a(\lambda, \pi^2, 1) - \mathsf{E}\pi^2 - \mathsf{E}\pi^*(\lambda, \pi^2) = 0 \tag{15}$$

Denote the left hand side of (15) by $\delta_{\mathsf{D}}(\lambda, \pi^2, p)$. Recall that $\pi^2_{\mathsf{L}} = \pi^*(\lambda, 1)$ is the equilibrium threshold for the large quantity in the limit order market. Then

(i) $\delta_{\mathsf{D}}(\lambda, \pi^2, p)$ strictly decreases in π^2 for $\pi^2 \in (\pi^2_{\mathsf{L}}, 1)$;
(ii) at $\pi^2 = \pi^2_{\mathsf{L}}$, $\delta_{\mathsf{D}}(\lambda, \pi^2, p) = (\mathsf{E}\pi^2_{\mathsf{L}} - \mathsf{E}\pi^*(\lambda, \pi^2_{\mathsf{L}}))/2 > 0$;
(iii) at $\pi^2 = 1$, $\delta_{\mathsf{D}}(\lambda, \pi^2, p) = -(1 - \mathsf{E}\pi^*(\lambda, 1))/2 < 0$,

where step (i) follows from Lemma 2 and the fact that $\mathsf{E}\pi$ strictly increases in π for $p \in (0, 1)$. Steps (i)–(iii) imply existence and uniqueness of $\pi^2 \in (\pi^2_{\mathsf{L}}, 1)$ that solves (15).

This π^2 is the equilibrium marginal buyer of large order, π^2_{D}, and the equilibrium marginal buyer of a small order is $\pi^1_{\mathsf{D}} = \pi^*(\lambda, \pi^2_{\mathsf{D}})$.

The proof of part (b) of Theorem 1 implies, in particular, the following lemma:

Lemma 3 (Relation of the equilibrium thresholds) *Equilibrium thresholds for large orders in the limit and the dealer markets, π^2_{L} and π^2_{D} respectively, satisfy $\pi^2_{\mathsf{L}} < \pi^2_{\mathsf{D}}$.*

C.4 Hybrid Market: Proof of Theorem 1 (c) and Proposition 1

As discussed in the main text, we construct the equilibrium in the hybrid market by adjusting the distribution of noise trading in such a manner that (a) marginal buyers π^1_{H} and π^2_{H} satisfy the indifference conditions, and (b) execution costs in the two segments coincide.

As in the main text, π^o_m denotes the equilibrium marginal buyer of $o = 1, 2$ units in markets $m = \mathsf{L}, \mathsf{D}, \mathsf{H}$; similarly for ask^o_m. We continue to use λ^o to denote the mass of noise traders that buy o units in the *limit order* segment of the hybrid market and λ to denote the mass of noise traders that buy either quantity in the isolated markets. The mass of noise traders that buy o units in the dealer market segment is then $\lambda - \lambda^o$.

We first assume that the marginal buyer π^2_{H} of a large order in a hybrid market satisfies $\pi^2_{\mathsf{H}} \geq \pi^2_{\mathsf{L}}$, construct the unique equilibrium in three steps and show that $\pi^2_{\mathsf{H}} \in (\pi^2_{\mathsf{L}}, \pi^2_{\mathsf{D}})$. The fourth step shows that there does not exist an interior hybrid market equilibrium with $\pi^2_{\mathsf{H}} < \pi^2_{\mathsf{L}}$. As before, we omit the time subscripts to simplify the exposition.

Step 1: $\forall \pi^2 \in [\pi^2_{\mathsf{L}}, 1]$ we find $\lambda^2(\pi^2)$ such that π^2 satisfies the equilibrium condition for the marginal buyer of a larger order in the limit order segment,[22]

$$\mathsf{E}\pi^2 = a(2\lambda^2(\pi^2), \pi^2, 1). \tag{16}$$

We show that a unique such $\lambda^2(\pi^2)$ exists, that $\lambda^2(\pi^2) \in [0, \frac{\lambda}{2}]$, and $\frac{d}{d\pi^2}\lambda^2(\pi^2) < 0$.

Proof: By Lemma 2, for every $\Lambda^2 \in [0, \frac{\lambda}{2}]$, there exists a unique $\pi^*(2\Lambda^2, 1) \in [\pi^2_{\mathsf{L}}, 1]$ that solves $\mathsf{E}\pi = a(2\Lambda^2, \pi, 1)$ in π. Further, $\partial\pi^*/\partial\Lambda^2 < 0$, $\pi^*(0, 1) = 1$, and $\pi^*(\lambda, 1) = \pi^2_{\mathsf{L}}$. With such a 1:1 mapping, we then know that for every $\pi^2 \in [\pi^2_{\mathsf{L}}, 1]$ there exists a corresponding $\Lambda^2 \in [0, \frac{\lambda}{2}]$ that solves $\mathsf{E}\pi^2 = a(2\Lambda^2, \pi^2, 1)$, and that it strictly decreases in π^2. Further, $\Lambda^2 \in (0, \frac{\lambda}{2})$ when $\pi^2 \in (\pi_{\mathsf{L}}, 1)$. We then define $\lambda^2(\pi^2) = \Lambda^2$.

[22] Recall that the definition of $a(\Lambda, \underline{\pi}, \overline{\pi})$ assumes that noise traders are drawn from mass Λ and informed traders are drawn from mass μ. Each hybrid market segment attracts mass $\mu/2$ of informed traders. Expressing the ask prices in terms of function a thus yields a renormalizing factor 2 in front of the mass of noise traders. The same applies to $\pi^*(\Lambda, \overline{\pi})$.

Step 2: $\forall \pi^2 \in [\pi^2_{\mathsf{L}}, 1]$ and given $\lambda^2(\pi^2)$ we construct $\lambda^1(\pi^2)$ and $\pi^1(\pi^2)$ so that they satisfy, first, the equilibrium conditions for the marginal buyer of small size orders in each segment and, second, the cost equalization condition for small orders when the marginal buyer of a large order is assumed to be π^2:

$$\mathsf{E}\pi^1(\pi^2) = a(2\lambda^1(\pi^2) + 2\lambda^2(\pi^2), \pi^1(\pi^2), 1) = a(2\lambda - 2\lambda^1(\pi^2), \pi^1(\pi^2), \pi^2). \quad (17)$$

We show that unique such $\lambda^1(\pi^2), \pi^1(\pi^2)$ exist, $\lambda^1(\pi^2) \in (\frac{\lambda}{2}, \lambda)$, and $\frac{d}{d\pi^2}\pi^1 > 0$.

Proof: By Lemma 2, for every $\pi^2 \in [\pi^2_{\mathsf{L}}, 1]$ and $\Lambda^1 \in [\frac{\lambda}{2}, \lambda]$, there exist unique $\pi^*(2\Lambda^1 + 2\lambda^2(\pi^2), 1)$ and $\pi^*(2\lambda - 2\Lambda^1, \pi^2)$ that satisfy the equilibrium conditions for the marginal buyer of a small order in limit and dealer market segments respectively. What remains to be shown is that for every $\pi^2 \in [\pi^2_{\mathsf{L}}, 1]$ there exists a unique Λ^1 that equalizes these marginal buyers (and, consequently, the costs for small orders):

$$\pi^*(2\Lambda^1 + 2\lambda^2(\pi^2), 1) - \pi^*(2\lambda - 2\Lambda^1, \pi^2) = 0. \quad (18)$$

Denoting the left-hand side of (18) by $\delta_{\mathsf{H}}(\Lambda_1, \pi^2)$,

(i) for $\Lambda^1 \in [\frac{\lambda}{2}, \lambda]$, we have $\frac{\partial}{\partial \Lambda_1}\delta_{\mathsf{H}} < 0$;
(ii) at $\Lambda^1 = \frac{\lambda}{2}$, we have $\delta_{\mathsf{H}}(\Lambda_1, \pi^2) > 0$ since $\pi^*(\lambda, \pi^2) < \pi^*(\lambda + 2\lambda^2(\pi^2), 1)$;
(iii) at $\Lambda^1 = \lambda$, we have $\delta_{\mathsf{H}}(\Lambda_1, \pi^2) < 0$ since $\pi^*(2\lambda + 2\lambda^2(\pi^2), 1) < \pi^2 = \pi^*(0, \pi^2)$.

Parts (i) and (iii) follow by Lemma 2 and the definition of $\lambda^2(\pi^2)$. Part (ii) is implied by $\mathsf{E}\pi^*(\lambda, \pi^2) < a(\lambda + 2\lambda^2(\pi^2), \pi^*(\lambda, \pi^2), 1)$, since $\pi^*(\lambda + 2\lambda^2(\pi^2), 1)$ maximizes $a(\lambda + 2\lambda^2(\pi^2), \pi, 1)$ in π. Using $\mathsf{E}\pi^*(\lambda, \pi^2) = a(\lambda, \pi^*(\lambda, \pi^2), \pi^2)$ and the definition of $\lambda^2(\pi^2)$, after some algebraic simplification we can rewrite the latter inequality as $\pi^*(\lambda, \pi^2) < \pi^2$, which holds by Lemma 2.

Together $(i) - (iii)$ imply that there exists a unique $\Lambda^1 \in (\frac{\lambda}{2}, \lambda)$ that solves (18). We will henceforth denote this Λ^1 by $\lambda^1(\pi^2)$ and the corresponding threshold by $\pi^1(\pi^2)$.

To show that $\frac{d}{d\pi^2}\pi^1 > 0$, we proceed by contradiction. Suppose that there exist $\pi^2, \tilde{\pi}^2$ with $\pi^2 < \tilde{\pi}^2$, such that $\pi^1(\tilde{\pi}^2) \leq \pi^1(\pi^2)$. By Step 1, $\lambda^2(\tilde{\pi}^2) < \lambda^2(\pi^2)$. Then we must have $\lambda^1(\tilde{\pi}^2) > \lambda^1(\pi^2)$ in order for the marginal buyer of a small order in the limit order market segment to satisfy $\pi^*(2\lambda^1(\tilde{\pi^2}) + 2\lambda^2(\tilde{\pi}^2), 1) \leq \pi^*(2\lambda^1(\pi^2) + 2\lambda^2(\pi^2), 1)$. But $\lambda^1(\tilde{\pi}^2) > \lambda^1(\pi^2)$, together with $\pi^2(\tilde{\pi}^2) > \pi^2(\pi^2)$, implies the reverse inequality for this marginal buyer in the dealer market segment, $\pi^*(2\lambda - 2\lambda^1(\tilde{\pi}^2), \pi^2(\tilde{\pi}^2)) > \pi^*(2\lambda - 2\lambda^1(\pi^2), \pi^2(\pi^2))$, a contradiction.

Step 3: We show that there exists a unique $\pi^2 \in (\pi^2_{\mathsf{L}}, \pi^2_{\mathsf{D}})$ that satisfies the equilibrium condition for the marginal buyer of a large size order in the dealer market segment, given functions $\pi^1(\pi^2), \lambda^1(\pi^2)$ and $\lambda^2(\pi^2)$ defined above:

$$2a(2\lambda - 2\lambda^2(\pi^2), \pi^2, 1) - \mathsf{E}\pi^1(\pi^2) - \mathsf{E}\pi^2 = 0. \tag{19}$$

Proof: Denote the left hand side of equation (19) by $\delta_{\mathsf{D}}(\pi^2)$. Then
(i) at $\pi^2 = \pi^2_{\mathsf{L}}$, we have $\delta_{\mathsf{D}}(\pi^2_{\mathsf{L}}) > 0$, because $\lambda^2(\pi^2_{\mathsf{L}}) = \frac{\lambda}{2}$, and

$$2a(\lambda, \pi^2_{\mathsf{L}}, 1) - \mathsf{E}\pi^1(\pi^2_{\mathsf{L}}) - \mathsf{E}\pi^2_{\mathsf{L}} = 2\mathsf{ask}^2_{\mathsf{L}} - \mathsf{E}\pi^1(\pi^2_{\mathsf{L}}) - \mathsf{E}\pi^2_{\mathsf{L}} = \mathsf{ask}^2_{\mathsf{L}} - \mathsf{E}\pi^1(\pi^2_{\mathsf{L}}) > 0,$$

where the last equality is due to the equilibrium condition in the limit order market, and the inequality is due to $\pi^1(\pi^2_{\mathsf{L}}) = \pi^*(2\lambda - 2\lambda^1(\pi^2_{\mathsf{L}}), \pi^2_{\mathsf{L}}) < \pi^2_{\mathsf{L}}$ by Lemma 2 as $\lambda^1(\pi^2_{\mathsf{L}}) < \lambda$;
(ii) at $\pi^2 = \pi^2_{\mathsf{D}}$, we have $\delta_{\mathsf{D}}(\pi^2_{\mathsf{D}}) < 0$, because

$$2a(2\lambda - 2\lambda^2(\pi^2_{\mathsf{D}}), \pi^2_{\mathsf{D}}, 1) - \mathsf{E}\pi^1(\pi^2_{\mathsf{D}}) - \mathsf{E}\pi^2_{\mathsf{D}} < 2\mathsf{ask}^2_{\mathsf{D}} - \mathsf{E}\pi^1(\pi^2_{\mathsf{D}}) - \mathsf{E}\pi^2_{\mathsf{D}} < 2\mathsf{ask}^2_{\mathsf{D}} - \mathsf{E}\pi^1_{\mathsf{D}} - \mathsf{E}\pi^2_{\mathsf{D}} = 0,$$

where the first inequality follows since $\lambda^2(\pi^2_{\mathsf{D}}) < \frac{\lambda}{2}$ and function $a(\Lambda, \underline{\pi}, \overline{\pi})$ is decreasing in Λ. The second inequality is a consequence of Step 2, which showed, in particular, that $\lambda^1(\pi^2_{\mathsf{D}}) > \frac{\lambda}{2}$ and thus $\pi^1(\pi^2_{\mathsf{D}}) = \pi^*(2\lambda - 2\lambda^1(\pi^2_{\mathsf{D}}), \pi^2(\pi^2_{\mathsf{D}})) > \pi^*(\lambda, \pi^2_{\mathsf{D}}) = \pi^1_{\mathsf{D}}$.
(iii) To complete the proof of Step 3, we will now show that for $\pi^2 \in (\pi^2_{\mathsf{L}}, \pi^2_{\mathsf{D}})$, $\frac{d}{d\pi^2}\delta_{\mathsf{D}} < 0$. By Step 2, it suffices to show that $\frac{d}{d\pi^2}a(2\lambda - 2\lambda^2(\pi^2), \pi^2, 1) < 0$, which is equivalent to

$$\begin{aligned} -\tfrac{d\lambda^2}{d\pi^2} 2\mu \left[F_1(\pi^2) - F_0(\pi^2)\right] - [f_1(\pi^2)(2\lambda - 2\lambda^2(\pi^2) + \mu(1 - F_0(\pi^2))) \\ - f_0(\pi^2)(2\lambda - 2\lambda^2(\pi^2) + \mu(1 - F_1(\pi^2)))] < 0. \end{aligned}$$

Since $-\frac{d}{d\pi^2}\lambda^2(\pi^2) > 0$ and $F_1(\pi^2) - F_0(\pi^2) < 0$, it suffices to show that the last term in brackets is positive. Employing $f_1(\pi^2)/f_0(\pi^2) = \pi^2/(1 - \pi^2)$ and rearranging, we find that this term is positive if and only if $\mathsf{E}\pi^2 > a(2\lambda - 2\lambda^2(\pi^2), \pi^2, 1)$. The inequality holds because $\mathsf{E}\pi^2 = a(2\lambda^2(\pi^2), \pi^2, 1)$ by definition of $\lambda^2(\pi^2)$, $\lambda^2(\pi^2) < \lambda$ by Step 1, and function $a(\Lambda, \pi^2, 1)$ is decreasing in Λ.

Step 4: We show that π^2_{H} cannot be smaller than π^2_{L}.

Proof: Suppose that $\pi^2_{\mathsf{H}} < \pi^2_{\mathsf{L}}$. Since $\pi^2_{\mathsf{H}} = \pi^*(2\lambda^2, 1)$ and $\pi^2_{\mathsf{L}} = \pi^*(\lambda, 1)$, Lemma 2 implies that $\lambda^2 > \frac{\lambda}{2}$, and consequently $\mathsf{E}\pi^2_{\mathsf{H}} = a(2\lambda^2, \pi^2_{\mathsf{H}}, 1) < a(2\lambda - 2\lambda^2, \pi^2_{\mathsf{H}}, 1)$. Buyer π^2_{H} then earns negative profits in the dealer market segment and will not trade there, a contradiction.

Steps 2 and 3 show that $\lambda^1 > \frac{\lambda}{2} > \lambda^2$ and thus yield Proposition 1.

C.5 Hybrid Market Transactions: Proof of Proposition 2

The number of transactions in each segment is proportional to the total mass of traders. Since the mass of informed traders is the same in either segment, the number of transactions in the limit order segment is higher when the mass of noise traders there is larger, $\lambda^1 + \lambda^2 > 2\lambda - \lambda^1 - \lambda^2$. By Step 2 of the proof of Proposition 3, $\pi^*(2\lambda^1 + 2\lambda^2, 1) = \pi^1_{\mathsf{H}} < \pi^1_{\mathsf{L}} = \pi^*(2\lambda, 1)$. Lemma 2 then implies $\lambda^1 + \lambda^2 > \lambda$, which yields the desired inequality.

C.6 Execution Costs: Proof of Proposition 3

The market maker is competitive and expects to break even. Thus it must hold that

$$(\mathsf{E}[V|o=1] - C(1))\mathsf{Pr}(o=1) + (2\mathsf{E}[V|o=2] - C(2))\mathsf{Pr}(o=2) = 0$$

Using the Law of Iterated Expectations and the indifference conditions for the marginal buyers, $\mathsf{E}\pi^1 = C(1)$ and $2\mathsf{E}\pi^2 - C(2) = \mathsf{E}\pi^2 - C(1)$, we can rewrite the above equation as

$$(\mathsf{E}[V|o \geq 1] - \mathsf{E}\pi^1)\mathsf{Pr}(o \geq 1) + (\mathsf{E}[V|o=2] - \mathsf{E}\pi^2)\mathsf{Pr}(o=2) = 0. \tag{20}$$

In all three markets, equation (20) implies the following relation

$$\begin{aligned} 0 \;=\; & p\beta_1(2\lambda, \pi^1, 1) - \mathsf{E}\pi^1 \cdot [p\beta_1(2\lambda, \pi^1, 1) + (1-p)\beta_0(2\lambda, \pi^1, 1)] \\ & + p\beta_1(\lambda, \pi^2, 1) - \mathsf{E}\pi^2 \cdot [p\beta_1(\lambda, \pi^2, 1) + (1-p)\beta_0(\lambda, \pi^2, 1)], \end{aligned} \tag{21}$$

where, as defined in Section C.1.1, $\beta_v(\Lambda, \underline{\pi}, \overline{\pi}) = \Lambda + \mu(F_v(\overline{\pi}) - F_v(\underline{\pi}))$. In the pure limit and dealer markets equation (21) is equivalent to (20). To see that (21) also holds in the hybrid market, observe first that (20) is satisfied in each segment of the hybrid market. Multiplying the segment-specific equations with the respective execution probabilities and applying the Law of Iterated Expectations yields (21).[23] Define $\mathsf{dn}(\pi) = \pi p + (1-\pi)(1-p)$. Equation (21) can then be rewritten as

$$\frac{\delta(2\lambda, \pi^1, 1)}{\mathsf{dn}(\pi^1)} + \frac{\delta(\lambda, \pi^2, 1)}{\mathsf{dn}(\pi^2)} = 0, \tag{22}$$

where $\delta(\Lambda, \underline{\pi}, \overline{\pi})$ is as defined in the proof of Lemma 2 and, in particular, $\delta(\Lambda, \pi, 1) = -(2\pi - 1)(\Lambda + \mu) + 2\mu \int_0^\pi G(s)ds$. G is the distribution function of qualities. In the

[23] Equation (21) can also be obtained by directly rearranging the equilibrium equations for each of the three markets. Here we provide the more intuitive derivation.

remainder, we will need the following derivative

$$\frac{d}{d\pi}\frac{\delta(\Lambda,\pi,1)}{\mathsf{dn}(\pi)} = -\frac{(1-p)\beta_0(\Lambda,\pi,1)+p\beta_1(\Lambda,\pi,1)}{\mathsf{dn}(\pi)^2} < 0, \tag{23}$$

Step 1: We show that for every $\pi^2 \in [\pi^2_{\mathsf{L}}, \pi^2_{\mathsf{D}}]$ there exists a unique $\pi^1(\pi^2) \in [\pi^1_{\mathsf{D}}, \pi^2)$ that solves (22).

The uniqueness of $\pi^1(\pi^2)$ implies that $\pi^1_{\mathsf{D}} = \pi^1(\pi^2_{\mathsf{D}})$, $\pi^1_{\mathsf{L}} = \pi^1(\pi^2_{\mathsf{L}})$, and $\pi^1_{\mathsf{H}} = \pi^1(\pi^2_{\mathsf{H}})$.

Proof: Denote the left hand side of (22) by $\xi(\pi^1,\pi^2)$. Then
(i) by (23), $\partial\xi/\partial\pi^1 < 0$;
(ii) at $\pi^1 = \pi^1_{\mathsf{D}}$, we have $\xi(\pi^1_{\mathsf{D}},\pi^2) \geq 0$ for $\pi^2 \leq \pi^2_{\mathsf{D}}$, as $\xi(\pi^1_{\mathsf{D}},\pi^2_{\mathsf{D}}) = 0$ and $\partial\xi/\partial\pi^2 < 0$;
(iii) at $\pi^1 = \pi^2$, we have $\xi(\pi^2,\pi^2) = -(2\pi^2-1)\lambda/\mathsf{dn}(\pi^2) < 0$.
Together $(i)-(iii)$ imply existence and uniqueness of the desired $\pi^1 \in [\pi^1_{\mathsf{D}}, \pi^2)$.

Step 2: Part (a): Market Width for Small Trades. We show that the execution costs are ordered as follows $C_{\mathsf{D}}(1) < C_{\mathsf{H}}(1) < C_{\mathsf{L}}(1)$.

Proof: By the proof of Theorem (existence), $\pi^2_{\mathsf{L}} < \pi^2_{\mathsf{H}} < \pi^2_{\mathsf{D}}$. Since in equilibrium, $C(1) = \mathsf{ask}^1 = \mathsf{E}\pi^1$ in all 3 markets, it suffices to show that $\pi^1(\pi^2)$ defined in Step 1 strictly decreases in π^2. Applying the Implicit Function Theorem to (22) and using (23),

$$\frac{d\pi^1}{d\pi^2} = -\frac{d}{d\pi^2}\frac{\delta(\lambda,\pi^2,1)}{\mathsf{dn}(\pi^2)} \cdot \left[\frac{d}{d\pi^1}\frac{\delta(2\lambda,\pi^1,1)}{\mathsf{dn}(\pi^1)}\right]^{-1} < 0. \tag{24}$$

Step 3: Part (a): Execution Costs for Large Trades. We show that the execution costs are ordered as follows $C_{\mathsf{L}}(2) < C_{\mathsf{H}}(2) < C_{\mathsf{D}}(2)$.

Proof: The equilibrium conditions for marginal buyers imply that the cost for a large order coincides with the sum of the marginal buyers expectations, $C(2) = \mathsf{E}\pi^1 + \mathsf{E}\pi^2$. It thus suffices to show that the latter sum increases in π^2,

$$\frac{d}{d\pi^2}[\mathsf{E}\pi^1 + \mathsf{E}\pi^2] = p(1-p)\left[\frac{1}{\mathsf{dn}(\pi^1)^2}\frac{d\pi^1}{d\pi^2} + \frac{1}{\mathsf{dn}(\pi^2)^2}\right] > 0.$$

Using (23) and (24), the above inequality is true if and only if

$$\frac{(1-p)\beta_0(\lambda,\pi^2,1)+p\beta_1(\lambda,\pi^2,1)}{(1-p)\beta_0(2\lambda,\pi^1,1)+p\beta_1(2\lambda,\pi^1,1)} < 1.$$

Rearranging, the latter is equivalent to $(1-p)\beta_0(\lambda,\pi^1,\pi^2) + p\beta_1(\lambda,\pi^1,\pi^2) > 0$, which holds for all $\pi^1 \leq \pi^2$.

Step 4: Part (b): Price Impacts of Small Trades. Price impacts in the dealer market and the dealer market segment of the hybrid market are determined by the respective transaction prices; the relation $\Delta\mathsf{p}^1_{\mathsf{D}} < \Delta\mathsf{p}^1_{\mathsf{H,D}}$ thus follows from Step 2. Here we show $\Delta\mathsf{p}^1_{\mathsf{L,H}} < \Delta\mathsf{p}^1_{\mathsf{D}}$ and $\Delta\mathsf{p}^1_{\mathsf{L}} < \Delta\mathsf{p}^1_{\mathsf{D}}$.

Proof: Price impacts in the limit order market reflect the change in the public expectation after the order size is revealed. The relation $\Delta\mathsf{p}^1_{\mathsf{L,H}} < \Delta\mathsf{p}^1_{\mathsf{D}}$ is thus equivalent to $a(2\lambda^1, \pi^1_{\mathsf{H}}, \pi^2_{\mathsf{H}}) < a(\lambda, \pi^1_{\mathsf{D}}, \pi^2_{\mathsf{D}})$. The latter holds by Lemma 2 since $\pi^2_{\mathsf{H}} < \pi^2_{\mathsf{D}}$ and $\lambda^1 > \frac{\lambda}{2}$:

$$a(2\lambda^1, \pi^1_{\mathsf{H}}, \pi^2_{\mathsf{H}}) \leq \mathsf{E}\pi^*(2\lambda^1, \pi^2_{\mathsf{H}}) < \mathsf{E}\pi^*(\lambda, \pi^2_{\mathsf{H}}) < \mathsf{E}\pi^*(\lambda, \pi^2_{\mathsf{D}}) = a(\lambda, \pi^1_{\mathsf{D}}, \pi^2_{\mathsf{D}}).$$

Likewise, $\Delta\mathsf{p}^1_{\mathsf{L}} < \Delta\mathsf{p}^1_{\mathsf{D}}$ holds since $a(\lambda, \pi^1_{\mathsf{L}}, \pi^2_{\mathsf{L}}) = \mathsf{E}\pi^*(\lambda, \pi^2_{\mathsf{L}}) < \mathsf{E}\pi^*(\lambda, \pi^2_{\mathsf{D}}) = a(\lambda, \pi^1_{\mathsf{D}}, \pi^2_{\mathsf{D}})$.

Step 5: Part (b): Price Impacts of Large Trades. To describe the price impacts of large trades, we show that $\mathsf{ask}^2_{\mathsf{D,H}} < \mathsf{ask}^2_{\mathsf{D}} < \mathsf{ask}^2_{\mathsf{L}} < \mathsf{ask}^2_{\mathsf{L,H}}$.

Proof: The first inequality follows from Step 3, since $2\mathsf{ask}^2_{\mathsf{D,H}} = C_{\mathsf{H}}(2) < C_{\mathsf{D}}(2) = 2\mathsf{ask}^2_{\mathsf{D}}$. The remaining inequalities follow by Lemma 2, since $\lambda^2 < \frac{\lambda}{2}$:

$$\mathsf{ask}^2_{\mathsf{D}} = a(\lambda, \pi^2_{\mathsf{D}}, 1) < a(\lambda, \pi^2_{\mathsf{L}}, 1) = \mathsf{ask}^2_{\mathsf{L}} = \mathsf{E}\pi^*(\lambda, 1) < \mathsf{E}\pi^*(2\lambda^2, 1) = \mathsf{ask}^2_{\mathsf{L,H}}.$$

C.7 Behavioral Dynamics: Proof of Proposition 4

We show only the proof for the buy-thresholds; the sell-thresholds are analogous.

Lemma 2 implies the result for the limit order market.

Next, trading thresholds in all markets must satisfy the market maker's zero expected profit condition, which can be rewritten as equation (22). In the proof of Proposition 3, we fixed p and viewed equation (22) as the relation between equilibrium thresholds π^1 and π^2 across different markets. Here, we use the same equation but view it as the relation between price p and equilibrium thresholds π^1, π^2 within a fixed market.

In dealer and hybrid markets, given the equilibrium marginal buyer of a large order, π^2, the equilibrium threshold for a marginal buyer of a small order, π^1, only depends on prior p through π^2. Further, π^1 is increasing in π^2 (this follows from the existence proofs, C.3 and C.4, Step 2). It thus suffices to show that π^2 is increasing in p.

In light of the above discussion, for a fixed market, the left-hand side of equation (22) can be viewed as a function of π^2 and p. Denoting this function by $\psi(\pi^2, p)$, we have

$\psi(\pi_p^2, p) = 0$ and (using (23) and $d\pi^1/d\pi^2 > 0$)[24]

$$\frac{\partial \psi}{\partial \pi^2} = \left[\frac{d}{d\pi^1}\frac{\delta(2\lambda, \pi^1, 1)}{\mathsf{dn}(\pi^1)}\right]\frac{d\pi^1}{d\pi^2} + \frac{d}{d\pi^2}\frac{\delta(\lambda, \pi^2, 1)}{\mathsf{dn}(\pi^2)} < 0,$$

where we use π_p^2 to denote the equilibrium threshold π^2 for prior p. It thus suffices to show that $\partial\psi/\partial p|_{\pi^2=\pi_p^2} > 0$. For, we then have $\psi(\pi_p^2, p+\epsilon) > \psi(\pi_p^2, p) = 0$ for $\epsilon > 0$, and thus $\pi_{p+\epsilon}^2$ that solves $\psi(\pi_{p+\epsilon}^2, p+\epsilon) = 0$ must be strictly above π_p^2.

To complete the proof we thus show that $\partial\psi/\partial p|_{\pi^2=\pi_p^2} > 0$. Denoting the equilibrium threshold π^1 for prior p by π_p^1, we have

$$\begin{aligned}\frac{\partial \psi}{\partial \pi^2}|_{\pi^2=\pi_p^2} &= \left[-\frac{\delta(2\lambda, \pi^1, 1)}{\mathsf{dn}(\pi^1)^2}(2\pi^1 - 1) - \frac{\delta(\lambda, \pi^2, 1)}{\mathsf{dn}(\pi^2)^2}(2\pi^2 - 1)\right]|_{\pi^2=\pi_p^2} \\ &= \frac{\delta(\lambda, \pi_p^2, 1)}{\mathsf{dn}(\pi_p^2)}\left[\frac{2\pi_p^1 - 1}{\mathsf{dn}(\pi_p^1)} - \frac{2\pi_p^2 - 1}{\mathsf{dn}(\pi_p^2)}\right] = \frac{\delta(\lambda, \pi_p^2, 1)}{\mathsf{dn}(\pi_p^2)}\frac{\pi_p^1 - \pi_p^2}{\mathsf{dn}(\pi_p^1)\cdot\mathsf{dn}(\pi_p^2)} > 0,\end{aligned}$$

where the second equality follows from (22), and the inequality follows as $\delta(\lambda, \pi_p^2, 1) < 0$. To see the latter, observe first, that $\delta(\lambda, \pi^2, 1)|_{\pi^2=\pi_{\mathsf{L}}^2} = 0$ and $\frac{\partial\delta(\lambda,\pi^2,1)}{\partial\pi^2} < 0$ by the proof of Lemma 2, and, second, that for $m \in \{\mathsf{D}, \mathsf{H}\}$, $\pi_m^2 > \pi_{\mathsf{L}}^2$ for any p.

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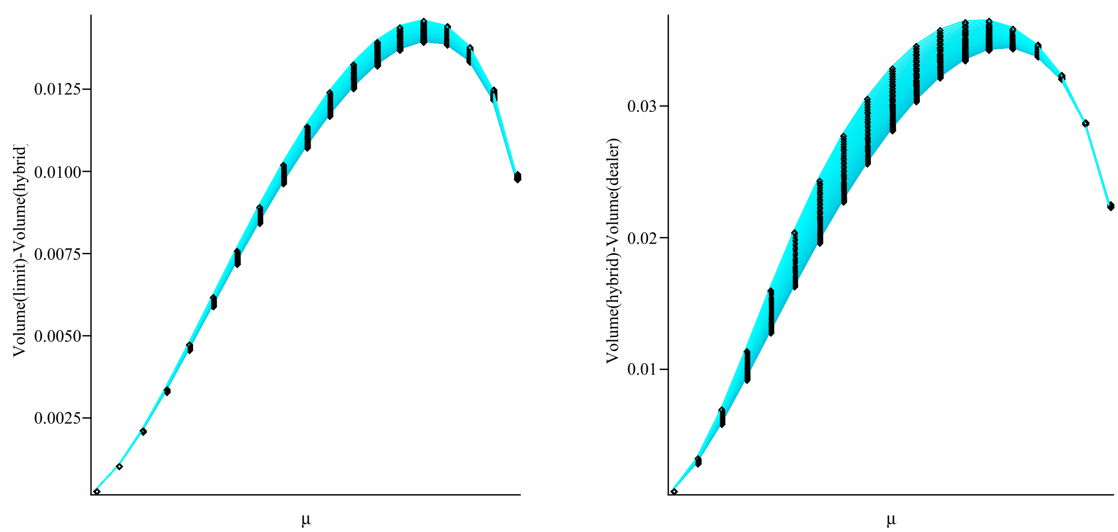
	Limit order Book minus Dealer Market									Limit order Book minus Hybrid Market									Hybrid Market minus Dealer Market								
	10	15	20	25	30	35	40	45	50	10	15	20	25	30	35	40	45	50	10	15	20	25	30	35	40	45	50
0.1	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.2	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.3	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.4	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.5	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.6	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.7	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.75	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.8	+	+	+	+	-	+	-	-	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.85	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.95	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
0.1	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.2	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.3	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-
0.4	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-
0.5	+	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
0.6	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-
0.7	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-
0.75	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-
0.8	-	-	-	-	+	-	-	+	-	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-
0.85	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-
0.9	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.95	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table I: **Differences of Averages and Standard Deviations for Price Distributions.** This table is based on the simulations of the closing prices. Rows denote the level of informed trading μ, columns denote the entry rate ρ. The top half of the table reports the sign of the difference of the average closing prices for a specific (ρ, μ)-combination between two markets that are named at the top of the table. The bottom half of the table reports the sign of the difference of the standard deviations of closing prices. As the underlying true value is $V = 1$, the higher a price is, the closer it is to the true value and the more efficient it is. Thus a positive difference of the average closing prices indicates that prices in the first named market are more efficient. A positive difference of the standard deviations of closing prices indicates that prices in the first named market are more dispersed.

	entry rate ρ						
	2	3	4	5	6	7	8
$\mu = 0.2$	-	-	-	+	+	+	+
0.5	-	-	-	-	+	+	+
0.8	-	-	-	-	-	-	-
	9	10	11	12	13	14	15
0.2	+	+	+	+	+	+	+
0.5	+	+	+	+	+	+	+
0.8	-	-	-	+	+	+	+

	level of informed trading μ								
	.1	.2	.3	.4	.5	.6	.7	.8	.9
$p = .1$	-	-	-	-	-	-	-	-	-
.2	-	-	-	-	-	-	-	-	-
.3	-	-	-	-	-	-	-	-	-
.4	-	-	-	-	-	-	-	-	-
.5	-	-	-	-	-	-	-	-	-
.6	+	+	+	+	-	-	-	-	-
.7	+	+	+	+	+	+	+	+	-
.8	+	+	+	+	+	+	+	+	+
.9	+	+	+	+	+	+	+	+	+

Table II: **Differences of Average Closing Prices and Price Impacts: Transparent vs. Opaque Hybrid Market.** The *left table* displays the sign of the difference of the average closing prices between the transparent and opaque markets as Table I. The *right table* displays the sign of the difference of expected price impacts between the transparent and opaque markets. A positive sign indicates that the transparent market is more efficient.



Limit Order minus Hybrid Market

Hybrid minus Dealer Market

Figure 3: **Volume in Limit Order, Dealer and Hybrid Markets.** Each panel plots the difference of volumes that arise under the two named market mechanisms. Volume here is a function of the prior, p, and the level of informed trading, μ, as defined in Subsection IV.C. The graph projects the volume difference such that μ is on the horizontal axis. Positive values indicate that the volume in the first named market exceeds that in the second.

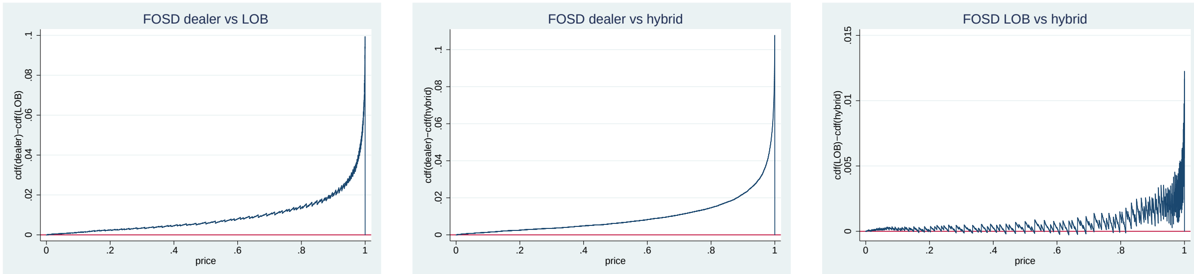
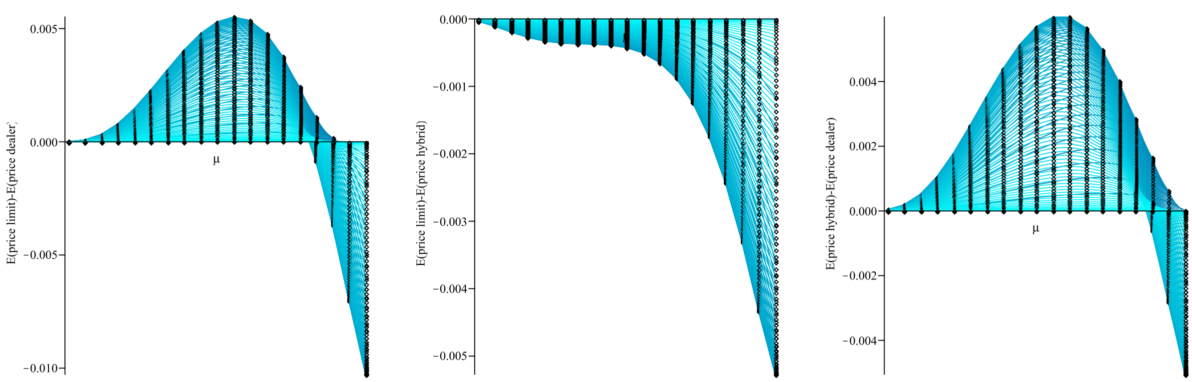
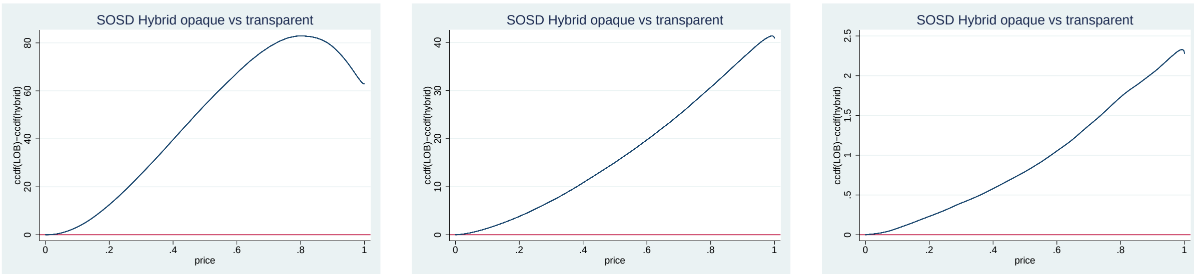


Figure 4: **First Order Stochastic Dominance of Closing Prices for $\mu = .5$.** The panels plot differences of empirical distributions $F_{\mathsf{D}} - F_{\mathsf{L}}$, $F_{\mathsf{D}} - F_{\mathsf{H}}$ and $F_{\mathsf{L}} - F_{\mathsf{H}}$. A graph that has only positive values indicates first order stochastic dominance. As $F_{\mathsf{D}} - F_{\mathsf{L}} > 0$ for all prices, the distribution of closing prices in the limit order market first order stochastically dominates that in the dealer market. Thus prices in the limit order market are systematically higher and more efficient. Similarly for $F_{\mathsf{D}} - F_{\mathsf{H}}$ and $F_{\mathsf{L}} - F_{\mathsf{H}}$.



Limit Order minus Dealer | Limit Order minus Hybrid | Hybrid minus Dealer

Figure 5: **The Difference of Price Impacts.** The three panels plot the difference of expected price impacts as defined in Subsection IV.D. This difference is a function of the probability of informed trading μ and the prior p. We display it here as a projection such that μ is on the horizontal axis. If for a given μ the graph is entirely above the horizontal axis, then for that μ, price movements in the first named market are stronger in the direction of the fundamental and this market is thus more efficient.



$\mu = .2$ $\mu = .5$ $\mu = .8$

Figure 6: **Second Order Stochastic Dominance of Closing Prices for Transparent vs. Opaque Hybrid Markets.** The panels plot differences of cumulations of the empirical distributions, $\int_0^{\mathsf{p}}[F_{\mathsf{opaque}}(s) - F_{\mathsf{transparent}}(s)]ds$, for $\mu = 0.2, 0.5, 0.8$. Since the values are always positive, the distribution of closing prices for the transparent mechanism second order stochastically dominates that for the opaque one. Prices under the transparent mechanism are therefore less dispersed.

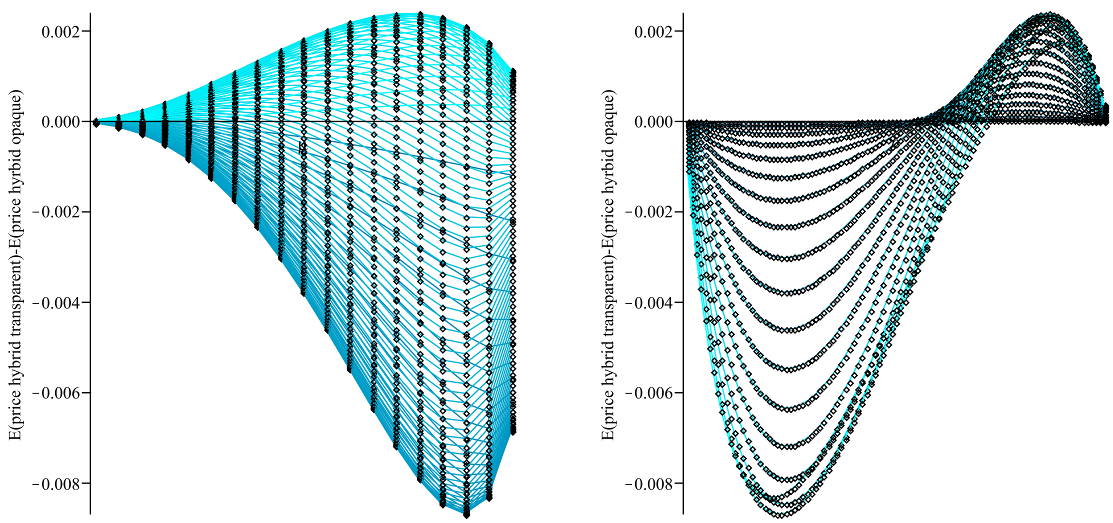


Figure 7: **The Difference of Price Impacts Transparent vs. Opaque Hybrid Market.** The panels are analogous to Figure 5 and plot the difference of the expected price impacts between the transparent and the opaque hybrid markets. The *left panel* plots a projection such that the probability of informed trading μ is on the horizontal axis. The *right panel* plots a projection such that the prior p is on the horizontal axis.

Result	Economic Variable	Order of Markets	Existing empirical evidence
Proposition 3 (a)	market width, small trades	dealer < hybrid < LOB	Nimalendran and Petrella (2003) (LOB vs. hybrid) lower execution costs (support for small trade part of our results)
Proposition 3 (a)	market width, large trades	LOB < hybrid < dealer	Domowitz (2002) (dealer vs. hybrid) Naik and Yadav (2004) spreads for large trades decrease dealer vs. hybrid
Proposition 3 (b)	price impact small trades price impact large trades	LOB & hybrid LOB < < dealer < hybrid dealer hybrid dealer < dealer < < LOB < hybrid LOB	Domowitz (2002) (dealer vs. hybrid) Smith, Turnbull, and White (2001) (hybrid only: Toronto) Booth, Lin, Martikainen, and Tse (2002) (hybrid only: Helsinki) Bessembinder and Venkataraman (2004) (hybrid only: Paris): upstairs (dealer) trades have lower price impact than downstairs (LOB) trades Madhavan and Cheng (1997) (NYSE): upstairs price impact lower for large trades
Proposition 4	behavior not time-invariant in dealer and hybrid markets, dealer and hybrid markets have contrarian tendency		contrarianism: Chordia, Roll, and Subrahmanyam (2002) Alevy, Haigh, and List (2007) (experimental)
Numerical Observation 1	volume	dealer < hybrid < LOB	
Numerical Observation 2	price informativeness (average closing prices)	small & medium μ: dealer$\prec$LOB$\prec$ hybrid large μ: LOB$\prec$hybrid$\prec$dealer	

Table III: **Testable Implications from the Three-way Comparison.** This table summarizes the main results and testable predictions that this paper generates.

Result	**Description**	**Existing empirical evidence**
Proposition 1	self selection of traders in hybrid markets: relatively more uninformed traders trade large quantities in the dealer market, relatively more uninformed traders trade small quantities in the LOB	Smith, Turnbull, and White (2001) (Toronto) Booth, Lin, Martikainen, and Tse (2002) (Helsinki) Bessembinder and Venkataraman (2004) (Paris): upstairs (dealer) trades have lower information content than downstairs (LOB) trades
Proposition 2	hybrid market: more small transactions in the LOB segment; more large transactions in the dealer segment; more transactions total in LOB segment	de Jong, Nijman, and Roell (1995): more small trades on Paris Bourse (LOB) more large trades on LSE (dealer) Viswanathan and Wang (2002) (NYSE): large orders filled by dealers, small orders filled in the LOB Booth, Lin, Martikainen, and Tse (2002) (Helsinki): upstairs trades are larger
Numerical Observation 3 (a)	hybrid market transparency vs. opaqueness: with many traders, prices are more efficient in the transparent market; with few traders, prices are more efficient in the opaque market	
Numerical Observation 3 (b)	hybrid market transparency vs. opaqueness: prices in the transparent market second order stochastically dominate those in the opaque one	

Table IV: **Testable Implications for the Hybrid Market.** This table summarizes the results and testable predications of our model that pertain to the hybrid market only.