Cost benefit analysis vs. referenda

By Martin J. Osborne and Matthew A. Turner

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Martin J. Osborne
Department of Economics, University of Toronto

Matthew A. Turner
Department of Economics, University of Toronto

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ABSTRACT: We consider a planner who chooses between two possible public policies and ask whether a referendum or a cost benefit analysis leads to higher welfare. We find that a referendum leads to higher welfare than a cost benefit analyses in “common value” environments. Cost benefit analysis is better in “private value” environments.

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1. Introduction

Many collective decisions are made on the basis of either a referendum or a cost-benefit analysis. We study the efficacy of these two decision-making methods in an environment in which individuals’ preferences and information may differ.

Two factors determine which method is superior: the importance of preference differences relative to information differences, and the extent to which individuals behave strategically. The maintained assumption of most cost-benefit analyses is that each subject truthfully reveals her preferences. Under this assumption and the assumption that individuals vote strategically, cost-benefit analyses and referenda differ in two significant respects. First, a cost-benefit analysis is better at eliciting cardinal information about preferences than is a referendum. Second, strategic behavior may lead only well-informed individuals to vote in a referendum, whereas no such self-selection occurs in a cost-benefit analysis. These differences imply that the outcome of a cost-benefit analysis is superior when individuals have diverse preferences but similar information, whereas the outcome of a referendum is superior when individuals have similar preferences but different information.

We study environments in which the decision to be made is binary. We say that an environment has ‘private values’ if each individual knows her valuation for each decision and these valuations differ, and has ‘common values’ if the individuals’ valuations are the same but unknown. With this terminology, our main result may be stated simply and precisely: given truthful revelation by the subjects of a cost-benefit analysis and strategic behavior by voters, the outcome of a cost-benefit analysis is superior to that of a referendum in a private value environment, whereas the outcome of a referendum is superior to that of a cost-benefit analysis in a common value environment.

This result hinges on agents behaving differently in the two mechanisms: it requires strategic voting in referenda but truthful revelation of preferences in cost-benefit analyses. This assumption is easy to defend for a cost-benefit analysis based on revealed preference information such as housing prices or travel costs, but is more problematic for a cost-benefit analysis relying on stated preference methods. We find that if agents approach cost-benefit analyses with the same sophistication that they apply to referenda, the two mechanisms are equivalent.

Even without taking a position on the extent to which agents behave strategically when participating in a cost-benefit analysis, we can make a strong statement about the comparative advantages of cost-benefit analyses and referenda: a cost-benefit analysis is at least as good as a referendum in a ‘private value’ environment, whereas the converse is true in a ‘common value’ environment. For
reasons we discuss below, we are inclined to give some credence to the widely maintained assumption of truthful revelation in cost-benefit analyses, even in those relying on stated preference methods. In this case, cost benefit analysis is strictly preferred to referenda in private value environments, and conversely in common value environments.

2. Background: cost benefit analyses and referenda

Executive Order 12866 of the US government requires that “Each agency shall assess both the costs and the benefits of . . . regulation . . . ” (Clinton, 1993). Similarly, the US Office of Management and Budget is required to submit to congress “an estimate of the costs and benefits of Federal rules and paperwork . . . ” each year.\(^1\) Several major US regulations also mandate measurement of the benefits and costs of regulation. In addition, as of 1996, 10 states required an analysis of the benefits and costs of regulation (Hahn, 2000).

Given these directives, it is unsurprising that cost benefit analyses are often used to evaluate public policy and that these analyses appear to influence regulators in favor of policies that generate higher estimated benefits at lower estimated costs. For example, Cropper et al. (1992) provide evidence that the US Environmental Protection Agency’s decisions to regulate dangerous pesticides are influenced by estimates of the costs and benefits of the pesticide in question. Smith (2000) provides anecdotal evidence that cost benefit analysis has affected air quality regulation for the Grand Canyon. Viscusi (1996) argues that the US Department of Transportation began to pursue regulations with larger estimated net benefits after it incorporated cost benefit analysis into its decision process.

This evidence suggests that many important allocation decisions are made, roughly, according to a ‘cost benefit decision rule’ that operates in two steps: measure the costs and benefits of a proposed action and then choose the action if and only if the net benefit is positive. Indeed, this stylized decision rule is broadly consistent with the injunction of Executive Order 12866 to “propose or adopt a regulation only upon a reasoned determination that the benefits of the intended regulation justify its costs” (Clinton, 1993), with similar mandates present in many state laws (Hahn, 2000), and with the exhortations of professional economists (e.g. Arrow et al. 1996).\(^2\)

Estimating the costs and benefits of public policies is not trivial. But economists have responded to the problem with considerable ingenuity, and many

\(^{1}\) FY2001 Treasury and Government Appropriations Act, §624.

\(^{2}\) Note that each of these sources also allows for the possibility that factors other than the costs and benefits of a policy, e.g., distributional implications, should influence the chosen policy.
techniques are now available. Of these, the three most common are stated preference methods, travel cost methods, and hedonic analysis. We develop a stylized description of stated preference cost benefit analysis and discuss later the extent to which our intuition applies to the travel cost and hedonic methods.

The stated preference method draws a sample from the affected population and asks each respondent to reveal information about the benefit they would derive from a particular policy. The concept is simple, but the method often involves sophisticated survey techniques. For example, stated preference surveys often describe policies and their consequences in detail, elicit demographic information, debrief respondents to assess their understanding of the questions (Hanemann 1994, Arrow et al. 1993), and even allow for the possibility that agents’ responses violate the axioms of revealed preference (Settle et al. 2003, Cherry et al. 2003).

Referenda are another important mechanism for collective decision-making. Aside from their pervasive use in choosing government officials, they are also widely used to resolve questions that might otherwise be left to regulators. For example, in California alone, 2004 saw some 16 state level referendum measures on topics ranging from health care to gambling to criminal law (State of California, November, 2004). All of these decisions could have been made on the basis of cost benefit analyses.

Given the prevalence and apparent interchangeability of cost benefit analysis and referenda, it is natural to ask when one mechanism should be preferred to another. We seek to answer this question.

3. Model: A simple public policy decision

Consider a planner who faces an environment in which there are $n$ agents (where $n \geq 3$), two states of the world, two possible policies, and each agent is one of four possible types:

States: $Z = \{0, 1\}$

Policies: $X = \{0, 1\}$

Agent types: $T = \{0, 1, i, u\}$.

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3Lupia (1994) reports that in 2004 more than 70% of the US population lived in jurisdictions where referenda were used in this way. He also reports widespread use of referenda in Europe.

4Choosing government officials by cost benefit analysis may seem odd, but is almost certainly practical. If we identify a candidate for office with a bundle of policies and attributes, the choice problem could be resolved by a cost benefit analysis.

5Diamond and Hausman (1994) call for a related calculation: a comparison of decisions made with and without cost benefit analysis.
Agents of types 0 and 1 are partisans who respectively prefer policies 0 and 1 in both states of the world. Agents of type \(i\) and \(u\) are independents who prefer the policy to match the state. Agents of type \(i\) know the state (are informed) whereas agents of type \(u\) do not (are uninformed); uninformed agents learn the state only after a policy is chosen. Denote the fraction of each type \(t\) in the population by \(p_t\).

We assume that the players' preferences are represented by the following payoff functions, with \(\pi_0 \geq 0\) and \(\pi_1 \geq 0\).

- **Type 0 (0-partisans):**
  \[
  \begin{cases}
  -\pi_0 & \text{if } x = 1 \\
  0 & \text{if } x = 0 \\
  \end{cases}
  \]
  for \(z = 0, 1\)

- **Type 1 (1-partisans):**
  \[
  \begin{cases}
  0 & \text{if } x = 1 \\
  -\pi_1 & \text{if } x = 0 \\
  \end{cases}
  \]
  for \(z = 0, 1\)

- **Types \(i\) and \(u\) (independents):**
  \[
  \begin{cases}
  0 & \text{if } x = z \\
  -1 & \text{if } x \neq z. \\
  \end{cases}
  \]

The probability of state 0 is \(\alpha\), with \(\alpha < \frac{1}{2}\), and this probability is common knowledge for the agents and planner. All agents are expected payoff maximizers.

Given these payoffs, the values of moving from policy 0 to policy 1 for each type of agent are

- 0-partisans: \(-\pi_0\)
- 1-partisans: \(\pi_1\)
- Informed independents: \(\begin{cases} 1 & \text{in state } 1 \\ -1 & \text{in state } 0 \end{cases}\)
- Uninformed independents: \(1 - 2\alpha\)

Note that \(1 - 2\alpha > 0\) by our assumption that \(\alpha < \frac{1}{2}\).

As a benchmark, we measure welfare by summing the payoffs of all agents. The per capita welfare gain associated with moving from policy 0 to policy 1 is

\[
w_0 \equiv -\pi_0 p_0 + \pi_1 p_1 - p_i - p_u \quad \text{in state } 0
\]

and

\[
w_1 \equiv -\pi_0 p_0 + \pi_1 p_1 + p_i + p_u \quad \text{in state } 1.
\]

In state \(z\) the first best action is thus to change from policy 0 to policy 1 if \(w_z > 0\).

We restrict the formal analysis to the two polar cases. In the pure private values case, all agents are partisans \((p_i = p_u = 0)\); in the pure common values case,
all agents are independents ($p_0 = p_1 = 0$). That is, in the case of pure private values, agents disagree about the best policy because their tastes differ. In the common values case, agents are in perfect agreement about the best policy conditional on the state, but some do not know the state.

Many social choices involve common values. For example, most people agree that high incomes, a pleasant environment, high security, and a steady supply of Chilean Sea Bass are desirable goals. They disagree about how to achieve these goals, at least in part because they are uncertain of the “state of the world”. In the more mundane decision-making environments of choosing a restaurant or a vacation destination, the purpose of guidebooks is to provide information about the state of the world, on the premise that people can benefit by relying on the author’s assessment of an unfamiliar good.

In a common value environment as we have formulated it, a simple method for selecting the first-best policy exists: delegate the decision to an informed agent. The difficulty with this method is of course that in most environments some partisans are present, and they have no incentive to reveal themselves, making it impossible for a planner to reliably identify an informed agent to whom to delegate the decision.\(^6\)

4. Cost benefit analysis

As we have mentioned, our model of cost benefit analysis captures the stated preference method. Estimates derived from stated preference methods generally rest on the assumption that the sample of respondents is representative of the population (at least on the basis of unobservable characteristics). In fact, since a high non-response rate creates the possibility of sampling bias, Arrow et al. (1993) give a high non-response rate as a reason for discounting the conclusions of a stated preference survey. Stated preference analyses usually also posit that survey respondents are truthful. The exceptions to this assumption seem to prove the rule. For example, Carson et al. (2000) contemplate the possibility of strategic responses to stated preference questions, but their objective is to formulate questions to which strategic responses will result in truthful revelation.

Like many aspects of stated preference methods, the extent to which agents respond to survey questions strategically is contentious. In order to avoid taking a position on this issue, we conduct our analysis of cost benefit analysis under both of the competing assumptions: that agents naïvely reveal their private in-

\(^6\)This problem of partisans masquerading as informed independents is essentially the same problem as the studied in Banerjee and Somanathan (2001).
formation, and that agents report their values strategically.

We assume that the analyst conducting a cost benefit analysis is able to elicit each individual’s precise valuation of the policy change in question. While this assumption is arguably consistent with travel cost or hedonic methods, it appears to be at variance with accepted best practice in stated preference methods. The state of the art in stated preference methods calls for the elicitation of bounds for an individual’s valuation (Hanemann, 1994). The assumption that the analyst conducting the cost benefit analysis is able to elicit each individual’s precise valuation of the policy change assumption is therefore probably too favorable to stated preference methods.

In the context of our model, a cost benefit analysis of policy 1 begins by drawing a sample of \( k \) agents at random from the population of \( n \) affected agents. In practice, \( k \) is less than \( n \), so that the cost benefit analysis is subject to sampling error. Since referenda and cost benefit analyses generally draw different sized samples, the issue of sampling error is important when comparing the two decision-making methods. With this said, an analysis of sampling errors is a distraction from the intuition we are trying to develop, so we abstract from it by assuming \( k = n \).

In summary, we assume that a cost benefit analysis proceeds as follows. First, for each affected agent \( i \in \{1, \ldots, n\} \), the planner elicits a report \( r_i \) of the value that the agent places on the change from policy 0 to policy 1. Second, the planner sums all received reports. Finally, if this sum is positive the planner adopts policy 1, and if it is negative the planner adopts policy 0. If the sum of received reports is zero, the planner chooses a policy by tossing a fair coin.\(^7\)

4.1 Cost benefit analysis with naïve reporting

When agents report naïvely, the planner elicits the sum of the agents’ expected values of a change from policy 0 to policy 1, given their information. Uninformed independents believe that the state is 0 with probability \( \alpha \), in which case their value of the policy change is \(-1\), and that the state is 1 with probability \( 1 - \alpha \), in which case their value of the policy change is 1. Hence they report an expected value of \( 1 - 2\alpha \). Informed independents report \(-1\) in state 0 and \(1\) in state 1; 0-partisans report \(-\pi_0\) and 1-partisans report \(\pi_1\) independent of the state. Thus the per capita reported value of the change from policy 0 to policy 1 is

\[
v_0 \equiv -\pi_0p_0 + \pi_1p_1 - p_i + (1 - 2\alpha)p_u \quad \text{in state 0}
\]

\(^7\)It is probably more sensible to imagine the policy maker choosing with a toss of a fair coin in private value environments, and acting on the common prior to choose 1 in common value environments. The present assumption eases exposition by keeping the two types of environment symmetric.
and
\[ v_1 \equiv -\pi_0 p_0 + \pi_1 p_1 + p_i + (1 - 2\alpha) p_u \quad \text{in state 1}. \]

For a pure common value problem \((p_0 = p_1 = 0)\), \(v_1\) is positive like \(w_1\) (given \(\alpha < \frac{1}{2}\)), so that in state 1 (the more likely state) cost benefit analysis yields the correct decision. In state 1, informed agents report positive signals and uninformed agents report the expected value of moving from policy 0 to 1. Since both types of agents submit qualitatively correct reports, the sum of these reports is also qualitatively correct and leads to a correct policy choice. However, \(v_0\), unlike \(w_0\), may be positive, so that in state 0 cost benefit analysis may yield the wrong decision. This occurs if \(p_i < (1 - 2\alpha) p_u\). In state 0, informed agents make qualitatively correct negative reports, but uninformed agents make the same positive reports as they do in state 1, and these reports are now qualitatively incorrect. Thus if uninformed agents are sufficiently numerous, the sum of reports is positive and the cost benefit decision rule incorrectly chooses policy 1.

For a pure private value problem \((p_i = p_u = 0)\), we have \(v_z = w_z\), the true per capita welfare gain (see (1) and (2)). In this case, cost benefit analysis simply collects the information necessary to make the correct welfare calculation and thus always selects the welfare-maximizing policy.

The following result summarizes this discussion.

**Proposition 1** Consider cost benefit analysis with naïve reporting.

(a) (Pure common values) Suppose that \(p_0 = p_1 = 0\). The correct (welfare-maximizing) policy is chosen in state 1 (the more likely state) and is chosen in state 0 if \(p_i > (1 - 2\alpha) p_u\). The wrong policy is chosen in state 0 if \(p_i < (1 - 2\alpha) p_u\).

(b) (Pure private values) Suppose that \(p_i = p_u = 0\). The correct (welfare-maximizing) policy is always chosen.

4.2 Cost benefit analysis with strategic reporting

4.2.1 Cost benefit analysis as a game Suppose that each agent acts strategically when reporting her benefit from switching from policy 0 to policy 1. Allow each agent either to report some number from the interval \([-B, B]\), where \(B\) is a large positive number, or not to submit a report. That is, let the set of actions for each agent be \(\{\phi\} \cup [-B, B]\), where \(\phi\) represents nonparticipation. By bounding the set of permissible reports we are implicitly assuming that the survey administrator has a prior about the range of possible values and either rejects or truncates
responses that are unreasonably large or unreasonably small. Since nonparticipation is equivalent to reporting zero, we can restrict agents to reports in \([-B, B]\).

These assumptions define a simple Bayesian game with two states; each informed independent knows the state, whereas each uninformed independent does not. (Partisans’ payoffs are independent of the state, so it does not matter whether they know the state.) A strategy \(r_j\) for an informed independent \(j\) is a function that associates reports \(r_j(0)\) and \(r_j(1)\) with the two states. A strategy \(r_j\) for any other agent \(j\) is simply a report.

This game has many Nash equilibria. To fix ideas, consider the pure common value game. This game has a Nash equilibrium in which all agents report \(B\) regardless of their types and another equilibrium in which all agents report \(-B\) regardless of their types. In each case, no deviation by any agent affects the outcome (given that there are at least three agents).

These equilibria are unappealing because in each equilibrium each agent has actions different from her equilibrium action that are equally attractive if the other agents adhere to their equilibrium actions and more attractive if some agents deviate from their equilibrium actions. We focus on deviations to nonparticipation. Suppose that when choosing an action, each agent considers the possibility that each of the other agents may exogenously be prevented from participating (e.g. because a phone rings, a child cries, or a doorbell breaks). Specifically, suppose that each agent assumes that every other agent will be prevented from participating with small independent probability. Then, when choosing a strategy, each agent first limits herself to strategies that are optimal when all the other agents adhere to their strategies, and then, within this set, chooses on the basis of the performance of the strategies when some agents do not participate. Because the probability of a small number of nonparticipants is much higher than the probability of a large number, each agent gives most weight in her strategy choice to situations in which the number of nonparticipants is small. But if two of her strategies perform equally well when the number of nonparticipants is small, she then compares the strategies in the case that the number of nonparticipants is large.

In the equilibrium in which each type of each agent chooses \(B\), an informed agent who knows the state is 0 is better off switching to \(-B\) if enough of the other agents are prevented from participating (in which case her switch changes the policy to 0 in state 0), and is never worse off switching to \(-B\). The action \(B\) for an uninformed agent is not unambiguously dominated, but the action 0 is better if

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8One of the functions of the ‘closed-ended questions’ discussed in Hanemann (1994) is to impose such bounds.

9If no other agent participates, \(B\) (which yields policy 1 in both states) is better than 0 (which yields each policy with probability \(\frac{1}{2}\) in each state).
the number of nonparticipants is just small enough to cause the switch from $B$ to 0 to make a difference, and is thus inconsistent with equilibrium when each agent is prevented from participating with a small enough probability.

Precisely, we define an equilibrium as follows.\footnote{This definition is similar in spirit to the definition of a trembling hand perfect equilibrium of the strategic game in which each type of each player in the Bayesian game is a different player. It differs in that only perturbations to nonparticipation, rather than arbitrary perturbations, are considered, and the probability $\varepsilon$ is assumed to be the same for all agents. It is related also to the assumption of Feddersen and Pesendorfer (1996) that the number of agents is random.} Define the $\varepsilon$-perturbation of a strategy profile $r$ to be the strategy profile in which each player $j$ chooses $r_j$ with probability $1 - \varepsilon$ and 0 (nonparticipation) with probability $\varepsilon$.

**Definition 1** A strategy profile $r$ is an equilibrium if there exists $\varepsilon > 0$ such that for all $\varepsilon < \varepsilon$, the strategy $r_j$ of each agent $j$ is a best response to the $\varepsilon$-perturbation of $r$.

The Nash equilibrium that we consider above in which every agent reports $B$ is not an equilibrium in this sense. When all agents report $B$, a deviation by an agent affects the outcome only if the number of other participants is zero or one; in both cases, an informed independent is better off deviating to $-B$ in state 0, changing the outcome in state 0 either from policy 1 to a $\frac{1}{2} - \frac{1}{2}$ mixture of policies 0 and 1 or from policy 1 to policy 0. Thus for any small positive probability that each agent involuntarily does not participate, the deviation is profitable, so that the strategy profile is not an equilibrium.

### 4.2.2 Equilibrium in the cost benefit analysis game

**Common values** To understand equilibrium in the case of pure common values, it is useful to recall the intuition developed in Feddersen and Pesendorfer (1996) for equilibria of referenda in common value environments. Consider a referendum with only two agents, both independents, one informed and one not. For the informed agent, voting for the policy that matches the state weakly dominates her other actions (abstain and vote for the policy different from the state). Given that the informed agent votes for the policy that matches the state, the uninformed agent’s vote affects the outcome only when it differs from that of the informed voter. In this case, the uninformed voter changes the outcome from the policy that matches the state to a tie between the policies, so that the uninformed agent’s vote is unambiguously detrimental. Therefore the uninformed voter is best off abstaining. It follows that the outcome of the referendum is determined by the informed voter and coincides with outcome that would occur if both agents were informed and voted sincerely.
The intuition behind equilibrium behavior in the common value cost-benefit analysis game is similar. Uninformed independents want to choose reports that are not pivotal, and thus choose small reports. Informed agents want to influence the collective decision so that the policy matches the state, and thus choose reports that are large relative to those of the uninformed agents. In the resulting equilibria, the correct policy is selected in both states.

*Private values* In a pure private value problem, a report of \(-B\) weakly dominates all other reports for a 0-partisan and a report of \(B\) weakly dominates all other reports for a 1-partisan. Hence in every equilibrium every 0-partisan reports \(-B\) and every 1-partisan reports \(B\).\(^{11}\) It follows that the policy favored by the larger group of partisans is always chosen in equilibrium. This leads to an incorrect policy choice when a minority places a high enough value on one policy and the majority places a small enough value on the other policy.

These results are stated formally in the following proposition, which is proved in the appendix.

**Proposition 2** Consider cost-benefit analysis with strategic participation and reporting.

(a) (Pure common values) Suppose that \(p_0 = p_1 = 0\) and that at least one agent is informed.

(i) Every strategy profile in which each informed agent reports \(-B\) in state 0 and \(B\) in state 1 and the report of each uninformed agent lies in \((0, B/|U|)\) is an equilibrium, where \(U\) is the set of uninformed agents.

(ii) In every equilibrium the correct (welfare-maximizing) policy is chosen in each state.

(b) (Pure private values) Suppose that \(p_1 = p_u = 0\) and that there is at least one partisan of each type.

(i) The game has a unique equilibrium, in which every 0-partisan reports \(-B\) and every 1-partisan reports \(B\), and the policy chosen is the one favored by the majority of agents.

(ii) The policy selected in the unique equilibrium differs from the correct (welfare-maximizing) policy if \(\frac{1}{2} < p_1 < \frac{\pi_0}{(\pi_0 + \pi_1)}\) or if \(\frac{\pi_0}{(\pi_0 + \pi_1)} < p_1 < \frac{1}{2}\).

\(^{11}\)Similar intuition is developed by Kalai and Kalai (2001) in a different context.
Comparing Propositions 1 and 2, we see that cost benefit analysis with truthful reporting always chooses correctly for private value problems and sometimes chooses incorrectly for common value problems, whereas cost benefit analysis with strategic reporting always chooses the correct policy for common value problems and sometimes chooses incorrectly for private value problems.

Another interesting implication of Proposition 2 is that when reporting is strategic, the individuals’ reports do not necessarily convey much information about the individuals’ preferences. When values are private, all individuals make extreme reports regardless of their true values for the policies. When values are common, the game has multiple equilibria, and in at least one equilibrium the reports of all informed agents are qualitatively correct but extreme and the reports of all uninformed agents are close to zero. We comment further on this point in Section 8.

5. Referenda

In a referendum, agents choose whether to vote and if so for which policy. The policy that receives the most votes is selected; in the event of a tie, the policy is selected by the toss of a fair coin. Formally, each agent’s set of actions is \{φ\} ∪ \{-1, 1\}, where the action 1 is a vote for policy 1 and the action -1 is a vote for policy 0. Policy 1 is selected if the sum of the actions is positive, policy 0 is selected if the sum is negative, and each policy is selected with probability \frac{1}{2} if the sum is zero. As in the case of cost benefit analysis, we can simplify the notation by letting the set of actions be \{-1, 0, 1\} and identifying the action 0 with nonparticipation. We use the same notion of equilibrium as in our study of cost benefit analysis.

The equilibria of the referendum game are qualitatively similar both to the equilibria of the benefit cost analysis game and to the equilibria of the model studied by Feddersen and Pesendorfer (1996). In the case of pure common values, uninformed independents are pivotal only in the event that they oppose informed agents, and thus optimally abstain. For the case of pure private values, all agents optimally vote sincerely. These conclusions are formalized in the following proposition, which is proved in the Appendix.

**Proposition 3** Consider a referendum with strategic participation and voting.

(a) (Pure common values) Suppose that \( p_0 = p_1 = 0 \) and that at least one agent is informed.

(i) The strategy profile in which every informed agent reports -1 in state 0 and 1 in state 1 and no uninformed agent votes is an equilibrium.
In every equilibrium the correct (welfare-maximizing) policy is chosen in each state.

(b) (Pure private values) Suppose that \( p_1 = p_u = 0 \) and that there is at least one partisan of each type.

(i) The game has a unique equilibrium, in which every 0-partisan reports \(-1\) and every 1-partisan reports \(1\), and the policy chosen is the one favored by the majority of agents.

(ii) The policy selected differs from the correct (welfare-maximizing) policy if \( \frac{1}{2} < p_1 < \frac{\pi_0}{\pi_0 + \pi_1} \) or if \( \frac{\pi_0}{\pi_0 + \pi_1} < p_1 < \frac{1}{2} \).

This result shows that referenda always arrive at the correct policy decision in common value environments and sometimes make incorrect decisions in private value environments. Incorrect decisions in private value environments result when, for example, many 0-partisans who prefer policy to 0 to policy 1 only weakly are able to outvote a few 1 partisans who strongly prefer policy 1 to policy 0.

The result is closely related to a result of Feddersen and Pesendorfer (1996). Feddersen and Pesendorfer define a referendum to “fully aggregate information” if it results in the same outcome as would result from sincere voting by an electorate in which all independents know the state of the world. They find that as the electorate becomes large, the probability that a referendum fully aggregates information approaches one, a result that occurs, in part, because uninformed agents have an incentive to abstain. Because of the different way in which we have formulated our model, our result holds for all population sizes, not only asymptotically.

That uninformed agents are less likely to vote than informed agents also has empirical support. For example, Wolfinger and Rosenstone (1980) find that more educated individuals are more likely to vote, and Milligan et al. (2004) find that in the US more educated citizens are more likely to vote (although the effect is less strong in the UK). Assuming that more educated agents are better informed, this evidence is consistent with better-informed agents’ being more likely to vote.

Lassen (2005) finds more direct evidence that voters are better informed than non-voters. He examines a natural experiment in Copenhagen in which city residents were asked to vote in a referendum to decentralize municipal service provision. He finds that residents who were better informed about the proposal, by virtue of living in arbitrarily selected pilot districts, were more likely to vote.

\footnote{See also Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1998), and Feddersen and Pesendorfer (1999).}
than other residents. Further, the effect is substantial: by one estimate, being informed increased the propensity to vote by 20 percentage points.

While strategic abstention by uninformed independents has not to our knowledge been observed in laboratory experiments, similar strategic behavior has been observed. In particular, Guarnaschelli et al. (2000) and Ladha et al. (2003) conducted experiments in which small groups of subjects (3 and 6) played a common value majority rule voting game. Both find evidence for strategic voting behavior of the sort predicted by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998). Wit (1999) also conducted an experimental analysis of a common value election (also with small groups) and finds that elections are quite effective at aggregating dispersed information.

In sum, our results, other theoretical results, empirical studies, and laboratory experiments all suggest that elections can be effective at aggregating information.

6. Cost benefit analysis vs. referenda

We can now compare cost benefit analysis with referenda. Under the assumption that the subjects of a cost benefit analysis behave naively, the result of the cost benefit decision rule is given by Proposition 1: in private value environments this institution always arrives at the correct decision whereas in some common value environments it makes incorrect decisions. The outcome of a referendum is described by Proposition 3: in common value environments referenda always choose correctly whereas in private value environments they may choose incorrectly. These results lead immediately to the conclusion that, if the subjects of a cost benefit analysis behave naively, then cost benefit analysis is strictly better than referenda for private value environments, whereas referenda are strictly better than cost benefit analysis for common value environments.

The conclusion is different if the subjects of a cost benefit analysis behave strategically. In this case, comparing Proposition 2 with Proposition 3, we see that the two decision rules lead to exactly the same decisions in both common and private value environments. When reporting strategically in a private value environment, all agents report the largest or smallest possible report. Thus the realized reports are identical, except for their names, with those realized in a referendum. The incentive for agents to strategically mis-report their private values in a cost benefit analysis prevents the analyst from learning any cardinal information about their preferences; only ordinal information, which is also revealed by a referendum, is obtained. When reporting strategically in a common value environment, informed and uninformed agents face similar incentives in refer-
enda and cost benefit analyses. Uninformed agents generally do not want to be pivotal, whereas informed agents do. The larger strategy sets in a cost benefit analysis generate more equilibria than exist for a referendum, but all these equilibria lead to perfect information aggregation.

7. Implications for travel cost and hedonic methods

Like stated preference methods, revealed preference methods (e.g. travel cost and hedonic methods) seek to estimate agents’ willingness to pay for a particular policy. Revealed preference methods are based on observed behavior, which is not plausibly chosen to influence the outcome of a cost benefit analysis. Therefore when comparing a revealed preference cost benefit analysis with a referendum, it is surely not correct to assume strategic reporting of the sort assumed in Proposition 2.

In the case of private value decisions it is probably reasonable to assume that a skillful revealed preference cost benefit analysis using the travel cost or hedonic method will lead to a decision that corresponds to the prediction of Proposition 1. The private value environment corresponds closely to the econometric environment in which these methods are intended to operate and the private values of our theory correspond closely to the preferences these methods are intended to estimate.

In the case of common value decisions it is less clear that we should expect a revealed preference cost benefit analysis to lead to decisions consistent with Proposition 1—that is, to sometimes fail to arrive at a correct decision. If cost benefit analyses based on revealed and stated preference data are to agree in common value environments, revealed preference data must reflect the decisions of uninformed agents who act on the basis of imperfect information that is sometimes incorrect. In some cases, the data plausibly do so. For example, individuals surely choose recreation destinations when they are uncertain about how they value these destinations, and it seems reasonable that this uncertainty leads to estimates of destination values that reflect the individuals’ expected values of these destinations, as required by Proposition 1. In other cases, it is less clear that revealed preference data reflect the appropriate information. For example, individuals make housing choices when they are uncertain of their values for nearby amenities, but it is unclear how such uncertainty translates into the prices that these individuals pay for their housing. In sum, it seems reasonable to expect our theory of stated preference cost benefit analysis to extend to revealed preference cost benefit analysis in private value environments, and possibly also in common value environments.
8. Conclusion

The clarity of the language with which legislators, regulators, and professional economists call for calculations of the economic costs and benefits of policy decisions conceals the difficulty of such calculations. In response to this difficulty, the US Congressional Budget Office has issued guidelines on how to perform a cost benefit analysis (United States Office of Management and Budget, 2003), as has at least one panel of distinguished economists (Arrow et al., 1996). Both sets of instructions implicitly adopt the standard, now widespread in the profession, that a cost benefit analysis is good if it produces an accurate estimate of the costs and benefits of the policy in question.

We propose a different standard: a cost benefit analysis is good or bad according to whether it leads to a better public decision than competing decision rules. With this standard in mind, we compare a stylized cost benefit decision rule with a referendum, another widely used institution for making public decisions.

Our conclusions depend sensitively on the extent to which agents behave strategically in their interactions with the analyst performing the cost benefit analysis.

Under the assumption of truthful reporting, a cost benefit analysis elicits cardinal information about preferences whereas a referendum elicits only ordinal information. If this cardinal information is important, then cost benefit analysis leads to a better decision than does a referendum. Conversely, a referendum can aggregate widely-dispersed information, whereas cost benefit analysis simply recovers a common prior. If information about the state of the world is important, then a referendum leads to a better decision than does a cost benefit analysis. This logic leads to the conclusion that a cost benefit analysis is superior to a referendum in private value environments and inferior in common value environments.

For a cost benefit analysis conducted with a stated preference methodology, it is of interest to examine the implications of strategic responses to survey questions. When agents participate and report strategically, a cost benefit analysis and a referendum elicit qualitatively identical behavior, and the two methods always result in the same policy choice. Thus, if we believe that agents behave strategically in cost benefit analyses, there is no reason to prefer one decision rule over the other.

Having said this, there are two reasons to believe that the widely maintained assumption of truthful reporting in stated preference cost benefit analyses is more reasonable than the assumption of strategic behavior. First, stated preference surveys are not generally administered with the same solemnity as are
referenda, and the link between survey responses and outcomes is more subject to doubt than in the case of a referendum. Thus, it is probably not reasonable to expect the same degree of strategic behavior from people participating in a cost benefit analysis as from those participating in a referendum. Second, if agents respond strategically to stated preference cost benefit analyses, from Proposition 2 we ought to observe only extreme reports in any stated preference survey conducted in a private value environment. That this pattern of responses is not widely observed suggests that either private value environments or strategic behavior is rare. Third, even if we suspect strategic behavior in stated preference cost benefit analyses, it is implausible to suspect such behavior in travel cost and hedonic cost benefit analyses. Therefore, if strategic responses to stated preference surveys are common, we ought to see systematic differences in the conclusions of analyses based on revealed and stated preferences. The survey of many revealed and stated preference cost benefit analyses in Carson et al. (1996) does not support this conclusion.

Without taking a stand on whether agents approach stated preference cost benefit analyses strategically, we can make a strong statement about the comparative advantages of the two decision rules: cost benefit analyses are always at least as good as referenda for private value problems, whereas referenda are always at least as good as cost benefit analyses for common value problems. If our skepticism about strategic behavior in cost benefit analyses is warranted, a stronger statement is possible: cost benefit analyses are strictly better than referenda for private value problems, whereas referenda are strictly better than cost benefit analyses for common value problems.

Our results suggest that a determination of whether or not to rely on referenda or cost benefit analyses for any given public decision depends on whether valuations are private or common. This question appears to be difficult to answer ex ante for any given public decision. However, the fundamental difference between private value and common value environments is that preferences in common value environments change systematically after a policy decision is taken, whereas preferences in private value decisions do not. This suggests the possibility of distinguishing common value from private value decisions ex post. In principal, a collection of studies performed ex ante and ex post on different public decisions could allow regulators to determine classes of public decisions where common values are important and where they are not. This information would, in turn, allow a determination of when referenda should be expected to outperform cost benefit analyses and when they should not.
9. Appendix

In the proofs, an equivalent version of our definition of an equilibrium (Definition 1) is useful. Consider a player choosing between two strategies that generate the same payoff given the other players’ strategies. When $\epsilon$ is small, for any positive integer $k$, the effect on the player’s expected payoff of the involuntary nonparticipation of $k + 1$ or more players is negligible compared with the effect of the involuntary nonparticipation of $k$ players. Therefore, the requirement that a player’s strategy be a best response to the other players’ strategies for all small $\epsilon$ means that the player’s choice between the strategies is based on the case of the involuntary nonparticipation by the smallest number of players for which the expected payoffs of the strategies differ. If, for example, the player’s strategies $r_j$ and $r_j'$ yield her the same expected payoff when all players participate and also over all the cases in which one of the other players involuntarily does not participate, but the expected payoff of $r_j$ over all cases in which two of the other players involuntarily do not participate is higher than the expected payoff of $r_j'$ in this case, then she chooses $r_j$.

Precisely, a useful implication of our definition of equilibrium is that

a strategy profile is an equilibrium if and only if for each agent and each change in her strategy, one of the following conditions is satisfied:

- For every integer $k$ with $0 \leq k \leq n - 1$, the change does not affect her expected payoff when each set of $k$ nonparticipants is equally-likely.
- For the smallest integer $k$ for which the change does affect her expected payoff when each set of $k$ nonparticipants is equally-likely, it decreases this expected payoff.

**Proposition 2** Consider cost benefit analysis with strategic participation and reporting.

(a) (Pure common values) Suppose that $p_0 = p_1 = 0$ and that at least one agent is informed.

(i) Every strategy profile in which each informed agent reports $-B$ in state 0 and $B$ in state 1 and the report of each uninformed agent lies in $(0, B/|U|)$ is an equilibrium, where $U$ is the set of uninformed agents.

(ii) In every equilibrium the correct (welfare-maximizing) policy is chosen in each state.
(b) (Pure private values) Suppose that $p_i = p_u = 0$ and that there is at least one partisan of each type.

(i) The game has a unique equilibrium, in which every 0-partisan reports $-B$ and every 1-partisan reports $B$, and the policy chosen is the one favored by the majority of agents.

(ii) The policy selected in the unique equilibrium differs from the correct (welfare-maximizing) policy if $\frac{1}{2} < p_1 < \frac{\pi_0}{(\pi_0 + \pi_1)}$ or if $\frac{\pi_0}{(\pi_0 + \pi_1)} < p_1 < \frac{1}{2}$.

Proof. (ai) The outcome of any such strategy profile is policy 1 in state 1 and policy 0 in state 0. This outcome is the best possible outcome for every agent, so no deviation can improve any agent’s payoff when all agents participate.

Now consider a perturbation of the game in which some agents do not participate. If at least one informed agent participates, the outcome is policy 1 in state 1 and policy 0 in state 0. If only uninformed agents participate, the outcome is policy 1. In both cases, the outcome is the best possible outcome for every participant (given that $\alpha$, the prior probability of state 0, is less than $\frac{1}{2}$), so that no deviation by any agent increases her payoff. Thus any such strategy profile is an equilibrium.

(ii) We first argue that in every equilibrium, every informed agent’s report is positive in state 1 and negative in state 0.

Suppose to the contrary that the report of some informed agent $j$ in state 0 is nonnegative. Then if no other agent participates, the outcome is policy 1 if $j$’s report is positive and each policy with probability $\frac{1}{2}$ if $j$’s report is 0. Thus a deviation by $j$ to $-B$ improves the outcome (to policy 0) in state 0 if no other agent participates and either improves the outcome or does not affect the outcome for any other set of reports. Thus every informed agent’s report in state 0 is negative in any equilibrium. Similarly every informed agent’s report in state 1 is positive in any equilibrium.

Now consider the behavior of uninformed agents. Let $r$ be a strategy profile and for each state $z$ let

$$R(z) = \sum_{j \in I} r_j(z) + \sum_{j \in U} r_j$$

be the sum of the reports in state $z$, where $I$ is the set of informed agents and $U$ is the set of uninformed agents. Assume that the outcome in state 1 is not correct, so that $R(1) \leq 0$. By the previous argument, $r_j(1) > 0$ and $r_j(0) < 0$ for every $j \in I$. Thus $R(0) < R(1)$, $R(0) < 0$, and $r_j < 0$ for some $j \in U$. Denote by $m$ the uninformed agent for whom $r_m$ is smallest (or any such agent if there is more than one such agent). (That is, $r_m < 0$ and $r_m \leq r_j$ for all $j \in U$.)
We claim that there is an alternative report \( r'_m \) for agent \( m \) and a nonnegative integer \( k \) such that a deviation by \( m \) to \( r'_m \) does not affect the outcome if fewer than \( k \) other agents do not participate and increases \( m \)'s expected payoff over all cases in which \( k \) other agents do not participate.

First suppose that \( r_m - R(1) < B \). Choose \( \delta > 0 \) such that \( r_m - R(1) + \delta \leq B \) and \( \delta < R(1) - R(0) \). Then \( m \) can deviate to the report \( r'_m = r_m - R(1) + \delta \) (> \( r_m \)), and if she does so the sum of the reports in state 1 becomes positive (it becomes \( \delta \)) while the sum of the reports in state 0 remains negative (it becomes \( R(0) - R(1) + \delta \)). Thus \( m \)'s deviation induces a better outcome, and increases her payoff. We conclude that if \( r_m - R(1) < B \) then \( r \) is not an equilibrium.

Now suppose that \( r_m - R(1) = B \). Then \( R(1) < 0 \), because \( r_m < 0 \). If \( m \) deviates to \( r'_m = B \), the sum of the reports becomes \( R(1) - r_m + B = 0 \) in state 1 and \( R(0) - r_m + B < R(1) - r_m + B = 0 \). Thus the outcome improves because the policy chosen in state 1 changes from 0 to a 50–50 mixture of 0 and 1 while the policy chosen in state 0 remains 0. Hence \( r \) is not an equilibrium.

Finally suppose that \( r_m - R(1) > B \). In this case, if all agents participate, no deviation by \( m \)—even a deviation to \( B \)—affects the outcome in either state. Thus to determine whether \( r \) is an equilibrium we need to consider the consequences of agents not participating. Suppose that one agent does not participate. Denote by \( j \) the agent for whom \( r_j \) is smallest among the agents other than \( m \). The sum of the reports when \( j \) does not participate is \( R(1) - r_j \). Given that \( r_j \geq r_m \), we have \( R(1) - r_j \leq R(1) - r_m < -B < 0 \) (given \( r_m - R(1) > B \)). That is, when \( j \) does not participate, the sum of the reports in state 1 (and therefore also the sum in state 0) remains negative.

There are two cases to consider. If \( r_m - R(1) + r_j \leq B \), then by the arguments in the previous two paragraphs, if agent \( j \) does not participate, agent \( m \) has a deviation that changes the outcome to either policy 1 or a 50–50 mixture of the two policies in state 1 and retains policy 0 in state 0, thus increasing her payoff. Further, the report of every other agent is at least \( r_j \), so when any other single agent does not participate this deviation either increases \( m \)'s payoff or has no effect on the outcome. Thus if \( r_m - R(1) + r_j \leq B \) then \( m \) has a deviation that increases her expected payoff over all cases in which one agent does not participate, so that the strategy profile is not an equilibrium.

If \( r_m - R(1) + r_j > B \), then no deviation by agent \( m \) affects the outcome in either state when at most one agent does not participate. In this case, we can take the agent \( j' \) whose report is smallest among the agents other than \( m \) and \( j \), and repeat the argument of the previous paragraph for the case in which \( j \) and \( j' \) do not participate. We can continue in the same way for larger sets of nonparticipants, at each step adding the agent whose report is smallest among the reports of the remaining agents. For each number \( \ell \) of nonparticipants, one
of the following two cases occurs.

(i) Agent \( m \) has a deviation that changes the policy from 0 to 1 (or a 50–50 mixture of 0 and 1) in state 1 and retains policy 0 in state 0 for some set (or sets) of \( \ell \) nonparticipants and does not affect the policy in either state for other sets of \( \ell \) nonparticipants. In this case \( r \) is not an equilibrium.

(ii) No deviation by agent \( m \) affects the outcome in either state for any set of \( \ell \) nonparticipants.

In case (ii) we increment \( \ell \) and proceed to the next step. If we reach \( \ell = |U| - 1 \), the process terminates with case (i) because when the set of nonparticipants consists of all the uninformed agents except \( m \), every participant is informed other than \( m \) and hence (by the earlier argument) makes a positive report in state 1.

We conclude that in any equilibrium the outcome in state 1 is correct. A similar argument shows that in any equilibrium the outcome in state 0 is correct.

(bi) First observe that if a 0-partisan whose report is greater than \(-B\) deviates to \(-B\), either the outcome does not change or it improves from the agent’s perspective. A deviation by a 1-partisan from a report of less than \(B\) to \(B\) has a similar effect.

To prove the result, we need to argue only that for any strategy profile \( r \) in which some 0-partisan reports more than \(-B\) or some 1-partisan reports less than \(B\), there is an agent \( m \) and a subset \( S \) of the other agents such that the deviation by \( m \) from \( r_m \) to \(-B\) (if \( m \) is a 0-partisan) or \( B\) (if \( m \) is a 1-partisan) strictly improves the outcome from \( m \)’s perspective when the set of nonparticipants is \( S \). Denote by \( m_0 \) the 0-partisan whose report is largest (or any such agent in the event of a tie) and by \( m_1 \) the 1-partisan whose report is smallest. Of these two agents, choose the one whose report deviates most from extreme report appropriate for her type. (That is, choose the 0-partisan if \( r_{m_0} > -r_{m_1} \), the 1-partisan if \( r_{m_0} < -r_{m_1} \), and either in the case of equality.) Denote this agent by \( m \), and assume, without loss of generality, that she is a 0-partisan. Let \( S \) be the set of all the other agents with the exception of a single 1-partisan, say \( m' \). When the set of nonparticipants is \( S \), there are exactly two participants, \( m \) and \( m' \). By the choice of \( m \), we have \( r_m + r_{m'} \geq 0 \) and \(-B + r_{m'} \leq 0 \), with at least one of these inequalities strict (otherwise \( r_m = -B \) and \( r_{m'} = B \)). Thus if \( m \) deviates to \(-B\) when the set of nonparticipants is \( S \), the outcome changes either from policy 1 to policy 0, from policy 1 to a 50–50 mixture of policies 0 and 1, or from a 50–50 mixture of policies 0 and 1 to policy 0. In all cases, the deviation increases \( m \)’s payoff, so \( r \) is not an equilibrium.

(bii) In the unique equilibrium, if \( p_1 > \frac{1}{2} \) then policy 1 is selected, if \( p_1 = \frac{1}{2} \) then each policy is selected with probability \( \frac{1}{2} \), and if \( p_1 < \frac{1}{2} \) then policy 0 is
selected. The welfare-maximizing policy is policy 1 if \(-(1 - p_1)\pi_0 + p_1\pi_1 > 0\), or 
\(p_1 > \frac{\pi_0}{(\pi_0 + \pi_1)}\), policy 0 if 
\(p_1 < \frac{\pi_0}{(\pi_0 + \pi_1)}\), and either policy if 
\(p_1 = \frac{\pi_0}{(\pi_0 + \pi_1)}\).
The result follows immediately. \(\square\)

**Proposition 3**  Consider a referendum with strategic participation and voting.

(a) (Pure common values) Suppose that \(p_0 = p_1 = 0\) and that at least one agent
is informed.

(i) The strategy profile in which every informed agent reports \(-1\) in
state 0 and 1 in state 1 and no uninformed agent votes is an equi-
librium.

(ii) In every equilibrium the correct (welfare-maximizing) policy is chosen
in each state.

(b) (Pure private values) Suppose that \(p_i = p_u = 0\) and that there is at least one
partisan of each type.

(i) The game has a unique equilibrium, in which every 0-partisan reports
\(-1\) and every 1-partisan reports 1, and the policy chosen is the one
favored by the majority of agents.

(ii) The policy selected differs from the correct (welfare-maximizing) pol-
icy if \(\frac{1}{2} < p_1 < \frac{\pi_0}{(\pi_0 + \pi_1)}\) or if \(\frac{\pi_0}{(\pi_0 + \pi_1)} < p_1 < \frac{1}{2}\).

**Proof.** (ai) We claim that the strategy profile \(r\) for which \(r_j(0) = -1\) and \(r_j(1) = 1\)
if \(j\) is informed and \(r_j = 0\) if she is uninformed is an equilibrium.

The outcome of this strategy profile is policy 1 in state 1 and policy 0 in state 0.
This outcome is the best possible outcome for every agent, so no deviation can
improve any agent’s payoff when all agents participate.

To show that the strategy profile is an equilibrium, we need to argue that
for each agent and each change in her strategy, either for every integer \(k\) with
\(0 \leq k \leq n - 1\), the change does not affect her expected payoff over all the
(equally-likely) sets of \(k\) nonparticipants, or for the smallest integer \(k\) for which
the change does affect her expected payoff over all the sets of \(k\) nonparticipants,
the change decreases this expected payoff.\(^{13}\)

\(^{13}\)The argument we use to show the corresponding result for cost-benefit analysis with strategic
reporting (Proposition 2) does not apply, because the outcome of the strategy profile \(r\) in the
perturbed game in which no informed agents participate is not the best outcome.

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First consider an informed agent \( j \). If she follows \( r_j \) then the outcome is the correct policy in each state regardless of the set of nonparticipants, so any change in her strategy either has no effect on the outcome or makes it worse.

Now consider an uninformed agent \( j \). If the number of nonparticipants is \(|I| - 2\) or fewer, at least two informed agents participate, so that the difference between the sums of reports in the two states is at least 4. Hence no change in \( j \)’s action affects the outcome. Now suppose that the number of nonparticipants is \(|I| - 1\). Then if \( j \) follows her strategy \( r_j \), the outcome is the right policy in each state regardless of which \(|I| - 1\) agents do not participate. If \( j \) deviates to \(-1\), either the outcome does not change in either state or, if all \(|I| - 1\) nonparticipants are informed agents, the outcome remains correct in state 0 but changes to a \( \frac{1}{2} \)–\( \frac{1}{2} \) mixture of policies 0 and 1 in state 1, which is worse for her. If \( j \) deviates to 1, either the outcome does not change in either state or, if all \(|I| - 1\) nonparticipants are informed agents, the outcome remains correct in state 1 but changes to a \( \frac{1}{2} \)–\( \frac{1}{2} \) mixture of policies 0 and 1 in state 0, which also is worse for her.

We conclude that \( r \) is an equilibrium strategy profile.

(aii) The argument is closely related, though not identical, to the argument for the case of cost benefit analysis.

We first argue that in every equilibrium, every informed agent reports 1 in state 1 and \(-1\) in state 0.

Suppose to the contrary that \( r_j(0) \neq -1 \) for some \( j \in I \). Then a deviation by \( j \) to \(-1\) improves the outcome in state 0 if no other agent participates and either improves the outcome or does not affect the outcome for any other set of reports. Thus \( r_j(0) = -1 \) in any equilibrium. Similarly \( r_j(1) = 1 \) in any equilibrium.

Now let \( r \) be a strategy profile and for each state \( z \) let

\[
R(z) = \sum_{j \in I} r_j(z) + \sum_{j \in U} r_j
\]

be the sum of the reports in state \( z \). Assume that the outcome in state 1 is not correct, so that \( R(1) \leq 0 \). Because at least one agent is informed and \( r_j(1) = 1 \) and \( r_j(0) = -1 \) for every informed agent \( j \), we have \( R(1) - R(0) = 2|I| \geq 2 \) and \( \sum_{j \in U} r_j = R(1) - |I| \), so that in particular at least \( |I| - R(1) \) uninformed agents report \(-1\). Denote by \( m \) such an uninformed agent.

If \( R(1) \geq -2 \) then in the case that all agents participate, agent \( m \) has a deviation that improves the outcome in state 1 and does not change the outcome in state 0, so that the strategy profile is not an equilibrium. (If \( R(1) = -2 \), for example, \( m \) can deviate to a report of 1 and change the outcome in state 1 from policy 0 to a \( \frac{1}{2} \)–\( \frac{1}{2} \) mixture of policies 0 and 1, while keeping the outcome in state 0 equal to policy 0.)
Now suppose that $R(1) \leq -3$ and consider a deviation by $m$ to report 1 (instead of $-1$). If all agents participate or at most $-R(1) - 3$ do not participate, this deviation does not affect the outcome in either state. Now suppose that $k = -R(1) - 2$ agents do not participate. Unless all of these nonparticipants are uninformed agents for whom $r_j = -1$, the deviation by $m$ does not affect the outcome in either state, because it causes the sum of the reports in state 1 to increase to at most $-2$. If all nonparticipants are uninformed agents for whom $r_j(1) = -1$, the sum of the reports in state 1 when all participants follow their strategies is $R(1) - R(1) = -2$, so that the deviation by $m$ changes the sum of the reports to 0 and thus improves the outcome in state 1 to a $\frac{1}{2} - \frac{1}{2}$ mixture of the two policies. Thus the strategy profile is not an equilibrium.

A similar argument shows that a strategy profile that generates the wrong outcome in state 0 is not an equilibrium.

(bi) The argument is the same as in the proof of part bi of Proposition 2.
(bii) The argument is the same as in the proof of part bii of Proposition 2. □

References


