University of Toronto
Department of Economics

Working Paper 282

Another Look at the Identification of Dynamic Discrete Decision Processes: With an Application to Retirement

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March 27, 2007

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This version: March 2007
First version: May 2002

Abstract

This paper deals with the estimation of the behavioral and welfare effects of counterfactual policy interventions in dynamic structural models where all the primitive functions are nonparametrically specified (i.e., preferences, technology, transition rules, and distribution of unobserved variables). It proves the nonparametric identification of agents' decision rules, before and after the policy intervention, and of the change in agents' welfare. Based on these results we propose a nonparametric procedure to estimate the behavioral and welfare effects of a general class of counterfactual policy interventions. The nonparametric estimator can be used to construct a test of the validity of a particular parametric specification. We apply this method to evaluate hypothetical reforms in the rules of a public pension system using a model of retirement behavior and a sample of workers in Sweden.

Keywords: Dynamic discrete decision processes; Nonparametric identification; Counterfactual policy interventions; Retirement behavior.

JEL: C14, C25, C61, D91, J26.

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*I would like to thank Russell Davidson, Jim Heckman, Chun-Yu Ho, Rasmus Lentz, Thierry Magnac, Rosa Matzkin, Pedro Mira, Salvador Navarro, Whitney Newey, John Rust, Steven Stern, Tom Stoker, Chris Taber, and Gustavo Vicentini for comments and helpful discussions. I want to thank also comments from audiences at the following conferences and seminars: the Northamerican winter meeting of the Econometric Society (Boston, 2006), the Canadian Econometric Study Group (Vancouver, 2005), the Society of Economic Dynamics conference (Vancouver, 2006), University of Chicago, Duke, Georgia, Indiana (CAEPR), MIT, Pittsburgh, Washington University-St. Louis, and Western Ontario.

1 Introduction

Discrete choice dynamic structural models have proven useful tools for the assessment of public policy initiatives (Wolpin, 1996). These econometric models have been applied to the evaluation of economic policies, hypothetical or factual, such as welfare programs (Sanders and Miller, 1997, Keane and Moffit, 1998, and Keane and Wolpin, 2000), unemployment insurance (Ferrall, 1997), social security pensions (Berkovec and Stern, 1991, and Rust and Phelan, 1997), patents regulation (Pakes, 1986), educational policies (Eckstein and Wolpin, 1999, and Keane and Wolpin, 1997 and 2001), contraceptive choice (Hotz and Miller, 1993), regulation on labor contracts (Aguirregabiria and Alonso-Borrego, 1999, and Rota, 2004), programs on child poverty (Todd and Wolpin, 2003), scrapping subsidies (Adda and Cooper, 2000), or regulation of nuclear plants (Sturm, 1991, and Rust and Rothwell, 1995), among others.

A common feature of the econometric models in these applications is the parametric specification of the structural functions in the model, i.e., utility function, technology, transition probabilities of state variables, and the probability distribution of unobservable variables.[1] These parametric models contrast with the emphasis on robustness and nonparametric specification that we find in other econometric approaches to the evaluation of public policies. In particular, the literature on evaluation of treatment effects has emphasized the importance of a nonparametric specification of the distribution of unobservables to obtain robust results (see Heckman and Robb, 1985, Manski, 1990, and more recently Heckman and Smith, 1998, and Heckman and Vytlacil, 1999 and 2005). Though robustness is an important argument in favor of this evaluation approach, at its current stage it has limitations to evaluate counterfactual policies, to estimate welfare effects, to incorporate transitional dynamics, or to allow for general equilibrium effects. It is in this context where dynamic structural models can be particularly useful. These structural models incorporate assumptions on individual be-

[1] Two exceptions are the semiparametric models in Taber (2000) and Heckman and Navarro (2004), where utilities are parametrically specified but the distribution of unobservable variables is nonparametric.

havior and on the equilibrium concept, e.g., rational expectations, competitive equilibrium. In addition to these economic assumptions, does the identification of these models require a parametric specification of the primitives? This question is the main theme of this paper.[2]

In this paper we show that it is possible to use nonparametrically specified dynamic structural models to evaluate the effects of counterfactual policy interventions. The paper has three main contributions. First, for a broad class of models, data and policies, we show that agents' behavior before and after the policy intervention, and the change in agents' welfare are nonparametrically identified. Second, based on this identification result we propose a nonparametric method to estimate the behavioral and welfare effects of counterfactual policy interventions. When the effect of interest is conditional on agents' state variables, the estimator is subject to the standard "curse of dimensionality" in nonparametric estimation (i.e., the speed of convergence of the estimator to its true value decreases with the number of explanatory variables). However, the estimates of unconditional effects are root-n consistent. Therefore, it is possible to obtain precise estimates of policy effects even when the specification of the structural model contains a relatively large number of state variables. As a third contribution, we apply this method to evaluate hypothetical reforms in the rules of a public pension system using data of male blue-collar workers in Sweden. This application illustrates how the method can be used to obtain precise estimates of behavioral and welfare effects which do not rely on any parametric assumption on the primitives of the model.

As shown by Rust (1994) and Magnac and Thesmar (2002), the differences between the utilities of two choice alternatives cannot be identified in dynamic decision models even when the researcher "knows" the time discount factor, the probability distribution of the unobservables, and the transition probabilities of the state variables. This under-identification

[2] The economic content of a dynamic structural model does not rest on the choice of a particular family of parametric functions for the primitives but on general assumptions which are essentially nonparametric such as: the selection of the relevant decision and state variables; independence assumptions between unobservable variables and some observables; the stochastic structure of the transition probabilities of the state variables (e.g., which variables follow exogenous transitions, and which variables are endogenous and why); monotonicity and concavity assumptions of some primitive functions; specification of individual heterogeneity; or the equilibrium concept that is used. Of course, these assumptions remain (and become more transparent!) once we remove parametric assumptions from our structural models.

result contrasts with the identification of utility differences in static (i.e., not forward looking) decision models (see Matzkin, 1992). This paper takes a different look at the problem of nonparametric identification of dynamic decision models. Instead of looking at the nonparametric identification of the utility function we consider the identification of the behavioral and welfare effects of counterfactual policy changes. More specifically, we prove the identification of agents' choice probability functions and surplus functions associated with hypothetical policy interventions. We show that knowledge of the current utility function or of utility differences is not necessary to identify these counterfactual functions. These counterfactuals depend on the distribution of unobservables and on the difference between the present value of choosing always the same alternative and the value of deviating one period from that behavior. We show that these objects are identified under similar conditions as in static models.

The class of dynamic discrete structural models that has been most commonly used in empirical applications is the one in Rust (1994) that assumes that unobserved state variables are not correlated over time. This paper also considers Rust's framework. Understanding identification in this framework is a necessary first step before considering models with more general structure for the unobservables.[3] As we have explained in footnote 2, this class of models incorporate explicitly important economic assumptions such as rational expectations and behavior, an equilibrium concept, etc. An open question is which of these assumptions are really necessary for identification and which ones might be relaxed. This paper does not study this type of identification issue, but we think that the results in this paper are a necessary first step before studying the empirical content of *deeper* economic assumptions.

During the last years, there has been an increasing interest in the nonparametric identification, and in the validation, of dynamic structural models. Recent important contributions in this area Magnac and Thesmar (2002), Pesendorfer and Schmidt-Dengler (2003), Heckman and Navarro (2005), Aguirregabiria (2005), and Bajari and Hong (2006).

[3]Though the identification results in this paper might be extended to models with unobserved heterogeneity *ala Heckman-Singer*, we have not explored this possibility in this paper.

The rest of the paper is organized as follows. In section 2 we set up the model, the basic assumptions and the type of counterfactual policy experiments that we want to evaluate. Section 3 presents the identification results. In section 4 we describe the estimation procedure. The empirical application is presented in section 5. We summarize and conclude in section 6.

2 Model

2.1 Framework and basic assumptions

Time is discrete an indexed by t. Consider an agent who has preferences defined over a sequence of states of the world between periods 0 and T, where the time horizon T can be finite or an infinite. A state of the world has two components: a vector of state variables s_t that is predetermined before period t, and a discrete decision $a_t \in A = \{0, 1, ..., J\}$ that the agent chooses at period t. The decision at period t affects the evolution of future values of the state variables. The agent's preferences over possible sequences of states of the world can be represented by the time-separable utility function $\sum_{j=0}^{T} \beta^j \; U_t(a_{t+j}, s_{t+j})$, where $\beta \in [0, 1)$ is the discount factor and $U_t(a_t, s_t)$ is the current utility function at period t. The agent has uncertainty about future values of state variables. His beliefs about future states can be represented by a sequence of Markov transition probability functions $F_t(s_{t+1}|a_t, s_t)$. These beliefs are *rational* in the sense that they are the true transition probabilities of the state variables. Every period t the agent observes the vector of state variables s_t and chooses his action $a_t \in A$ to maximize the expected utility

$$E\left(\sum_{j=0}^{T} \beta^j \; U_t(a_{t+j}, s_{t+j}) \mid a_t, s_t\right). \tag{1}$$

Let $\alpha_t(s_t)$ and $V_t(s_t)$ be the optimal decision rule and the value function at period t, respectively. By Bellman principle of optimality the sequence of value functions can be obtained using the recursive expression:

$$V_t(s_t) = \max_{a \in A} \left\{U_t(a, s_t) + \beta \int V_{t+1}(s_{t+1}) \; dF_t(s_{t+1}|a, s_t)\right\} \tag{2}$$

And the optimal decision rule $\alpha_t(s_t)$ is equal to $\arg\max_{a \in A}\{U_t(a, s_t) + \beta \int V_{t+1}(s_{t+1}) dF_t(s_{t+1}|a, s_t)\}$.

Suppose that we observe a random sample of agents who behave according to this model. We index agents in the sample by $i \in \{1, 2, ..., n\}$. As it is typically the case in micro panels, we observe each individual over a short period of time: e.g., two, three periods. Without loss of generality for our identification results, we will consider that each individual is observed for two periods. It is important to note that in this paper the time index t does not represent calendar-time but the agent-specific period in the agent's decision problem. To emphasize this agent-specific time, we refer to t as the *agent's age*. In some parts of the paper we will also emphasize this point by using the variable t_i to represent the agent $i's$ age. For each agent in the sample, the econometrician observes his action, a_{it}, and a subvector x_{it} of the vector of state variables s_{it}, i.e., $x_{it} \subset s_{it}$. In many applications of dynamic decision models, the researcher also observes an outcome variable y_{it} that is a component of the current payoff function. For instance, in a model of firm behavior the researcher may observe a component of the profit function such as output, revenue or the wage bill. In a model of individual behavior the econometrician may observe individual earnings, that is a component of current utility. In summary, the data set can be described as:

$$Data = \{a_{it},\ x_{it},\ y_{it} : i = 1, 2, ..., n;\ t = t_i, t_i + 1\} \tag{3}$$

Note that the outcome variable y_{it} is neither a decision nor a state variable, but it is an outcome that depends on the agent's decision and state variables. We represent this relationship using the structural outcome equation:

$$y_{it} = h_t\left(a_{it}, x_{it}, \omega_{it}\right) \tag{4}$$

where $\omega_{it} \subset s_{it}$ is an unobserved state variable. For example, if y_{it} is firm's output, then $h_t(.)$ is a production function and ω_{it} is an unobserved productivity shock. Or in an application where y_{it} represents individual's earnings, we have that $h_t(.)$ is an earnings function and ω_{it} is a shock in earnings. Therefore, x_{it} and ω_{it} are components of the vector of state variables s_{it}. We represent the rest of the state variables using the vector ε_{it}, that contains all the unobserved state variables, for the researcher, which do not enter in the outcome function h_t. Thus, $s_{it} = (x_{it}, \omega_{it}, \varepsilon_{it})$.

Let $a \in A$ be an arbitrary choice alternative, not necessarily the optimal alternative. *Without loss of generality* we can write the one-period utility associated with a as the sum of two components:

$$
\begin{aligned}
U_t(a, s_{it}) &= u_t(a, x_{it}, \omega_{it}) + \varepsilon_t(a, x_{it}, \omega_{it}), \\
\text{where:} \quad & u_t(a, x_{it}, \omega_{it}) \equiv Median\left(U_t(a, s_{it}) \mid x_{it}, \omega_{it}\right), \\
\text{and:} \quad & \varepsilon_t(a, x_{it}, \omega_{it}) \equiv U_t(a, s_{it}) - u_t(a, x_{it}, \omega_{it})
\end{aligned}
\tag{5}
$$

For the sake of notational simplicity we use $\varepsilon_{it}(a)$ instead of $\varepsilon_t(a, x_{it}, \omega_{it})$, and the vector ε_{it} to represent $\{\varepsilon_{it}(0),\ \varepsilon_{it}(1),\ ...,\ \varepsilon_{it}(J)\}$. By construction, the random variables in ε_{it} have median equal to zero and are median independent of x_{it} and ω_{it}.[4]

We consider the following assumptions on the joint distribution of the state variables.

ASSUMPTION 1: *The cumulative transition probability of the state variables factors as:*

$$
F(s_{i,t+1}|a_{it}, s_{it}) = F_\omega(\omega_{i,t+1}|\omega_{it})\ F_{\varepsilon|x}(\varepsilon_{i,t+1}|x_{i,t+1})\ F_x(x_{i,t+1}|a_{it}, x_{it}) \tag{6}
$$

where $F_\omega(.|\omega)$, $F_\varepsilon(.|x)$ and $F_x(.|a,x)$ are distribution functions. That is: (A) ω_{it} follows an exogenous Markov process; (B) ε_{it} may depend on contemporaneous x_{it} (e.g., heterocedasticity) but not on previous values of a, x or ε; (C) conditional on x_t and a_t, the vector x_{t+1} does not depend on ω_t and ε_t; and (D) $F_{\varepsilon|x}$ is continuously differentiable and strictly increasing with support the Euclidean space.

Assumption 1 is based on Rust's conditional independence assumption (Rust, 1994), but it is more general than Rust's because it allows for the unobservable ω_{it}. Under this assumption the optimal decision rule $\alpha_t(s_{it})$ can be described as:

$$
\alpha_t(s_{it}) = \arg\max_{a \in A} \{\ v_t(a, x_{it}, \omega_{it}) + \varepsilon_{it}(a)\ \} \tag{7}
$$

[4] A more common condition in the discrete choice literature is that ε_i has zero mean and is mean independent of the observable state variables. Instead, we consider median independence because it simplifies the nonparametric estimation of the distribution of ε.

where the functions $v_t(0, x, \omega)$, $v_t(1, x, \omega)$, ..., $v_t(J, x, \omega)$ are *alternative-specific value functions* and they are defined as:

$$v_t(a, x, \omega) \equiv u_t(a, x, \omega) + \beta \int \max_{a' \in A} \{v_{t+1}(a', x', \omega) + \varepsilon'(a')\} \; F_\omega(d\omega'|\omega) F_\varepsilon(d\varepsilon'|x') F_x(dx'|a, x) \tag{8}$$

The optimal decision rule represents individuals' behavior. Individuals' welfare is given by the value function $V_t(s_t) = \max_{a \in A} \{v_t(a, x_t, \omega_t) + \varepsilon_t(a)\}$. For the econometric analysis it is convenient to define versions of these functions which are integrated over the unobservables in ε. The *choice probability function* is defined as:

$$P_t(a|x_t, \omega_t) \equiv \int I\{\alpha(x_t, \omega_t, \varepsilon_t) = a\} \; F_{\varepsilon|x}(d\varepsilon_t) \tag{9}$$

And the *integrated valued function* is:

$$V_{\sigma,t}(x_t, \omega_t) \equiv \int V_t(s_t) \; F_{\varepsilon|x}(d\varepsilon_t) = \int \max_{a \in A} \{ \; v_t(a, x_t, \omega_t) + \varepsilon_t(a) \; \} \; F_{\varepsilon|x}(d\varepsilon_t) \tag{10}$$

To complete the description of the model we have to establish how the outcome variable y_{it} enters into the utility function. Assumption 2 establishes that the utility function $u_t(a, x, \omega)$ is linear in the outcome variable.

ASSUMPTION 2: *The utility function $u_t(a, x_t, \omega_t)$ has the following form:*

$$u_t(a, x_t, \omega_t) = \psi_t(x_t) \; y_t + c_t(a, x_t) \tag{11}$$

where the function $\psi_t(.)$ is positive valued, and $c_t(., .)$ is a real valued function.

The structural functions of the model are $\{\psi_t, \; h_t, \; c_t, \; \beta, \; F_\omega, \; F_\varepsilon, \; F_x\}$. This is the so called *model structure.*

2.2 Policy interventions

We want to evaluate the behavioral and welfare effects of an hypothetical policy intervention that modifies the current utility function. Let U_t be the utility function in the *data generating process*, and let U_t^* be the utility function under the counterfactual policy. Assumption 3 describes the class of policy interventions that we consider in this paper.

ASSUMPTION 3: *The counterfactual policy is such that: (A) It can be represented as a change in the payoff or utility function, from function U_t to function U_t^*. (B) Though the econometrician does not know neither function U_t nor U_t^*, he knows the difference between these two functions in units of the outcome variable y_t, i.e., he knows the function $\tau_t(a,s)$ where:*

$$\tau_t(a,s) = \frac{U_t^*(a,s) - U_t(a,s)}{\psi_t(x)} \tag{12}$$

And (C) the function $\tau_t(a,s)$ does not depend on ε, i.e., $\tau_t(a,s) = \tau_t(a,x,\omega)$.

The function τ_t represents the policy intervention and it is known to the researcher, though the functions U_t and U_t^* are unknown. Assumption 3 establishes that the policy intervention should be such that it can be measured in units of one of the outcome variable y. This assumption includes a broad class of counterfactual experiments. For instance, any change in the vector of outcome functions $h_t(a,x,\omega)$ is a particular case of this class of experiments. Note that the function τ_t may depend on (a,x,ω) in a completely unrestricted way, but it cannot depend on the unobservable ε. We provide several examples to illustrate how general is this class of policy interventions.

EXAMPLE 1: Consider a model of retirement from the labor force as in Rust and Phelan (1997). The outcome variable in this model is individual earnings. The type of policies that we can evaluate includes: policies that modify retirement benefits such as changes in the minimum and normal retirement age, or changes in the discount (premium) for early (late) retirement; policies that affect labor earnings such as a wage tax; or an hypothetical change in the risk aversion parameter.

EXAMPLE 2: Consider a model of occupational choice as in Keane and Wolpin (1997) where individual earnings are observable. Some examples of policies that we can evaluate in this model are a change in returns to schooling in a certain occupation, or a change in the costs of schooling.

EXAMPLE 3: Consider a dynamic model of firm investment in physical capital where the researcher observes firm output. In this model we can evaluate hypothetical changes in the production function parameters; or a sales tax.

Let $\{P_t\}$ and $\{V_{\sigma,t}\}$ be the choice probability functions and the integrated value functions before the policy intervention. And let $\{P_t^*\}$ and $\{V_{\sigma,t}^*\}$ be these functions after the policy intervention. We represent the behavioral effects of the policy by comparing the functions P_t^* and P_t. Similarly, the difference between the functions $V_{\sigma,t}^*$ and $V_{\sigma,t}$ represents the welfare effects of the policy. We are interested in the nonparametric estimation of the functions P_t, P_t^* and $V_{\sigma,t}^* - V_{\sigma,t}$. Given probability functions $\{P_t\}$ and $\{P_t^*\}$, we can simulate the future evolution of the variables $\{x_t, y_t, a_t\}$ starting from the initial period in which the hypothetical policy is implemented. That is, we can obtain the *transition dynamics* associated with the policy change and so derive the cross-sectional distribution of the state variables one period, two periods, etc, after the policy change.

2.3 An example: A model of capital replacement

Dynamic structural models of machine replacement have been considered before by, among others, Rust (1987), Das (1992), Kennet (1993), Rust and Rothwell (1995), Cooper, Haltiwanger and Power (1999), Adda and Cooper (2000) and Kasahara (2004). Most of these studies use the estimated structural model to evaluate the behavioral effects of a policy change. Kennet (1993) studies how deregulation of the US airline industry affected the number of aircraft engine hours between major overhauls. Rust and Rothwell (1995) analyze the impact on the operation of US nuclear power plants of an increase in the intensity of safety regulation by the US Nuclear Regulatory Commission after an accident on March 1979. Adda and Cooper (2000) evaluate the effects of a policy in France in which the government subsidized the replacement of old cars with new ones. Kasahara (2004) examines the impact on firms' investment in equipment of a temporary increase in import tariffs in Chile.

Consider a firm that produces a good using capital and some perfectly flexible inputs. The firm has multiple plants and each plant consists of only one machine. Production at different plants is independent and therefore we can concentrate in the decision problem for an individual plant or machine. Let x_t represent *time since last machine replacement,* that is the age of the existing machine at the beginning of month t. And let $a_t \in \{0,1\}$ be the indicator of the decision of replacing the old machine by a new machine at the beginning of month t. Therefore, the age of the machine that is used during month t is $(1-a_t)x_t$, and the transition of the variable x is:

$$x_{t+1} = (1-a_t)\ \ (x_t+1) + a_t \tag{13}$$

The output that a machine produces during month t depends on the age of the machine, and on a productivity shock ω_t that follows a Markov process.[5] The profit function has the following form:

$$U_t = Y((1-a_t)x_t, \omega_t) - MC((1-a_t)x_t, \varepsilon_t^{MC}) - a_t\ RC(x_t, \varepsilon_t^{RC}) \tag{14}$$

where $Y(.,.)$ is the production function; $MC(.,.)$ is the maintenance cost, that depends on the age of the machine and on a random shock ε_t^{MC} that is unobservable to the researcher; and $RC(.,.)$ is the replacement cost net of the scrapping value of the retired capital. The replacement cost is realized only if the machine is replaced and it depends on the age of the replaced machine (not the used one) and on a random shock ε_t^{RC} that is also unobservable to the researcher.

To show that this model has the form that we have postulated in Assumptions 1 and 2, consider the following definitions: (1) the function $c(0,x_t)$ is the median of $MC(x_t, \varepsilon_t^{MC})$ conditional on x_t; (2) the function $c(1,x_t)$ is the median of $MC(0,\varepsilon_t^{MC})+RC(x_t,\varepsilon_t^{RC})$ conditional on x_t; (3) the variable $\varepsilon_t(0)$ is the difference between $MC(x_t,\varepsilon_t^{MC})$ and its conditional median $c(0,x_t)$; and (4) the variable $\varepsilon_t(1)$ is the difference between $MC(0,\varepsilon_t^{MC})+RC(x_t,\varepsilon_t^{RC})$ and its conditional median $c(1,x_t)$. Given these definitions, we have that:

$$U_t = Y((1-a_t)x_t, \omega_t) - c(a_t, x_t) - \varepsilon_t(a_t) \tag{15}$$

[5] The amount of variable inputs is a function the age of the machine and the productivity shock and therefore we can omit it from our analysis. Note that the "productivity shock" may include shocks in the prices of variable inputs.

where $\varepsilon_t(0)$ and $\varepsilon_t(1)$ are zero conditional median random variables.

Suppose that we are interested in evaluating the effects of a counterfactual policy that modifies firms' replacement costs. This policy tries to promote the retirement of old capital by providing a subsidy that depends on the age of the retired capital. The amount of the subsidy, that coincides with the change in the current profit function associated with the policy, is:

$$\tau(a_t, x_t) = \begin{cases} 0 & if \quad x_t < x^*_{\texttt{low}} \\ a_t \ (\tau_0 - \tau_1 \ [x_t - x^*_{\texttt{low}}]) & if \quad x^*_{\texttt{high}} \leq x_t \leq x^*_{\texttt{low}} \\ 0 & if \quad x_t > x^*_{\texttt{high}} \end{cases} \tag{16}$$

where $\tau_0 > 0$, $\tau_1 > 0$, $x^*_{\texttt{low}}$ and $x^*_{\texttt{high}}$ are parameters that characterize the policy. The subsidy is zero if replacement takes place either too early (i.e., before age $x^*_{\texttt{low}}$) or too late (i.e., after age $x^*_{\texttt{high}}$). For replacement ages within the range $[x^*_{\texttt{low}}, x^*_{\texttt{high}}]$ the subsidy is strictly positive and it decreases linearly with the age of capital. This type of policy has been common in many countries and it has been motivated as part of an environmental policy to reduce emissions of carbon dioxide.

3 Identification

Suppose that we have a random sample of individuals with information on the variables $\{a_{it_i}, a_{i,t_i+1}, x_{it_i}, x_{i,t_i+1}, y_{it_i}, y_{i,t_i+1}\}$. As usual, we study identification with a very large (i.e., infinite) sample of individuals.[6] For the sake of simplicity, we concentrate in binary choice models. However, the identification results and the estimation method can be generalized to the multinomial case. Consider the binary choice case where $a \in \{0, 1\}$. For notational simplicity we use $P_t(x, \omega)$ to denote $P_t(1|x, \omega)$. Also, we assume that the outcome function h is identified without having to estimate the rest of the structural model. There are different conditions under which one can consistently estimate wage equations or production functions using instrumental variables or control function approaches which do not require the estimation of the complete structural model (see Olley and Pakes, 1996, and Imbens and

[6] Furthermore, we assume that the sample has variability over the whole support of the observable variables: $A \times [1, T]^2 \times X^2 \times Y^2$. This assumption of full-support variation is needed to identify the reduced form of the model.

Newey, 2002). We provide an examples for the identification of h in the empirical application of retirement behavior in section 4.

ASSUMPTION 4: *The outcome functions $h_t(0, x, \omega)$ and $h_t(1, x, \omega)$ are real valued functions such that: (A) they are identified from the data $\{a_{it_i}, a_{i,t_i+1}, x_{it_i}, x_{i,t_i+1}, y_{it_i}, y_{i,t_i+1}\}$; (B) they are strictly monotonic in ω, such that we can invert these functions to obtain $\omega_{it} = h^{-1}(a_{it}, x_{it}, y_{it})$; (C) ω_{it} is a continuous random variable with support Ω; and (D) the transition probability $F_\omega(\omega_{t+1}|\omega_t)$ is such that if $q(\omega)$ is a strictly increasing function, then $E(q(\omega_{t+1})|\omega_t) = \int q(\omega_{t+1})F_\omega(d\omega_{t+1}|\omega_t)$ is also a strictly increasing function of ω_t.*

Assumption 4 implies that the transition probability functions F_x and F_ω are nonparametrically identified. We can identify F_x on $A \times X^2$ from the transition probabilities $\Pr(x_{i,t_i+1}|a_{it_i}, x_{it_i})$ in the data. Furthermore, under Assumption 4 the values of ω_{it_i} and ω_{it_i+1} can be consistently estimated and we can treat these variables as observables. Therefore, F_ω is also identified on $\Omega \times \Omega$ from the probabilities $\Pr(\omega_{it_i+1}|\omega_{it_i})$ in the data. It is also clear that we can identify the choice probability functions $P_t(x, \omega)$ on $X \times \Omega$ from the probabilities $\Pr(a_{it_i} = 1|x_{it_i}, \omega_{it_i}, t_i = t)$ in the data. However, without further restrictions, we cannot identify the structural functions $\{\psi_t,\ c_t,\ F_{\varepsilon|x,t}\}$. This is the case both in decision models where agents are forward looking (i.e., $\beta > 0$) and in models where agents are myopic (i.e., $\beta = 0$). In this paper we are not interested in the identification of $\{\psi_t,\ c_t,\ F_{\varepsilon|x,t}\}$ but in the functions P_t^* and $V_{\sigma,t}^* - V_{\sigma,t}$ associated with a counterfactual policy intervention. We show below that the following Assumption 5, together with Assumptions 1 to 4, is sufficient to identify the functions P_t^* and $V_{\sigma,t}^* - V_{\sigma,t}$.

ASSUMPTION 5: *Define the functions $\tilde{h}_t(x, \omega) \equiv h_t(1, x, \omega) - h_t(0, x, \omega)$. This function $\tilde{h}_t(x, \omega)$ is: (A) strictly increasing in ω; and (B) for any $x \in X$, it is unbounded from above, such that for any constant $k \geq \tilde{h}_t(x, -\infty)$ there exits a value $\omega \in \Omega$ such that $\tilde{h}_t(x, \omega) = k$.*

For the sake of presentation, we start showing identification in a myopic version of the model.

3.1 Myopic model

Suppose that agents are not forward looking, i.e., $\beta = 0$. Then, the counterfactual choice probability function is:

$$
\begin{aligned}
P_t^*(x,\omega) \quad &= \quad \Pr\left(\ U_t^*(1,s) \geq U_t^*(0,s) \mid x,\omega\right) \\
&= \quad \Pr\left(\frac{U_t^*(1,s) - U_t(1,s)}{\psi_t(x)} + \frac{U_t(1,s)}{\psi_t(x)} \geq \frac{U_t^*(0,s) - U_t(0,s)}{\psi_t(x)} + \frac{U_t(0,s)}{\psi_t(x)} \mid x,\omega\right)
\end{aligned}
\tag{17}
$$

Define the functions $\tilde{c}_t(x) \equiv (c_t(1,x) - c_0(1,x))/\psi_t(x)$ and $\tilde{\tau}_t(x,\omega) \equiv \tau_t(1,x,\omega) - \tau_t(0,x,\omega)$, and the random variable $\tilde{\varepsilon}_t \equiv (\varepsilon_t(0) - \varepsilon_t(1))/\psi_t(x)$. Let $F_{\tilde{\varepsilon}|x,t}$ be the CDF of the random variable $\tilde{\varepsilon}_t$ conditional to x. Then, using these definitions, the previous expression of $P_t^*(x,\omega)$ can be written as:

$$
P_t^*(x,\omega) \qquad F_{\tilde{\varepsilon}|x,t}\left(\tilde{\tau}_t(x,\omega) + \tilde{h}_t(x,\omega) + \tilde{c}_t(x)\right) \tag{18}
$$

By the conditions in Assumptions 3 and 4, the functions $\tilde{\tau}_t$ and $\tilde{h}_t$ are known to the researcher. Equation (18) shows that the identification of P_t^* requires one to identify the functions $F_{\tilde{\varepsilon}|x,t}$ and $\tilde{c}_t$. The relationship between these functions and the *factual* reduced form probability function P is:

$$
P_t(x,\omega) \quad = \quad F_{\tilde{\varepsilon}|x,t}\left(\tilde{h}_t(x,\omega) + \tilde{c}_t(x)\right) \tag{19}
$$

Proposition 1 establishes the nonparametric identification of the counterfactual probability function P_t^*.

PROPOSITION 1: Suppose that Assumptions 1 to 5 hold and $\beta = 0$. For any period t, define the set $[X \times \Omega]_t^* \equiv \{(x,\omega) \in X \times \Omega : \tilde{\tau}_t(x,\omega) + \tilde{h}_t(x,\omega) \geq \tilde{h}_t(x,-\infty)\}$. The counterfactual choice probability function P_t^* is identified over the set $[X \times \Omega]_t^*$. For any $(x_0,\omega_0) \in [X \times \Omega]_t^*$ this probability can be obtained as:

$$
P_t^*(x_0,\omega_0) \quad = \quad P_t(x_0,\omega_t^{(\tau)}(x_0,\omega_0)) \tag{20}
$$

where $\omega_t^{(\tau)}(x_0,\omega_0)$ is a function from $[X \times \Omega]_t^*$ into Ω that is implicitly defined as the value $\omega \in \Omega$ that solves the equation $\tilde{h}_t(x_0,\omega) = \tilde{\tau}_t(x_0,\omega_0) + \tilde{h}_t(x_0,\omega_0)$. Furthermore, the function

$\tilde{c}_t$ is identified on X and $F_{\tilde{\varepsilon}|x,t}$ is identified on the set $\tilde{u}_t(X \times \Omega) \times X$, where $\tilde{u}_t(X \times \Omega) \subseteq \mathbb{R}$ is the space of real values that the function $\tilde{h}_t(x, \omega) + \tilde{c}_t(x)$ can take.

Proof. By Assumption 5, the function $\omega_t^{(\tau)}(x_0, \omega_0)$ exists and is unique at any point $(x_0, \omega_0) \in [X \times \Omega]_t^*$. Using this function, it is clear that:

$$\begin{aligned} P_t^*(x_0, \omega_0) &= F_{\tilde{\varepsilon}|x_0,t}\left(\tilde{\tau}_t(x_0, \omega_0) + \tilde{h}_t(x_0, \omega_0) + \tilde{c}_t(x_0)\right) \\ &= F_{\tilde{\varepsilon}|x_0,t}\left(\tilde{h}_t(x_0, \omega_t^{(\tau)}(x_0, \omega_0)) + \tilde{c}_t(x_0)\right) \\ &= P_t(x_0, \omega_t^{(\tau)}(x_0, \omega_0)) \end{aligned}$$

Given that the factual choice probability function P_t is identified on $X \times \Omega$ and the function $\omega_t^{(\tau)}$ is identified on $[X \times \Omega]_t^*$, it is clear that P_t^* is identified $[X \times \Omega]_t^*$. We now prove the identification of functions $\tilde{c}_t$ and $F_{\tilde{\varepsilon}|x,t}$. Define $P_t^{-1}(x, 0.5)$ as the value of ω that solves the equation $P_t(x, \omega) = 0.5$. By Assumptions 1(D) and 5(A), the probability function P_t is strictly monotonic in ω and therefore there is a unique value ω that solves $P_t(x, \omega) = 0.5$, i.e., $P_t^{-1}(x, 0.5)$ is identified for any $x \in X$. Since $Median(\tilde{\varepsilon}_t|x) = 0$, we have that $P_t(x, \omega) = 0.5$ implies that $\tilde{h}_t(x, \omega) + \tilde{c}_t(x) = 0$. Therefore, given $P_t^{-1}(x, 0.5)$ we can identify $\tilde{c}_t(x)$ as $\tilde{c}_t(x) = -\tilde{h}_t(x, P_t^{-1}(x, 0.5))$. The function $\tilde{c}_t$ is identified on X. Now, define the function $\omega_t^*(x, u)$ from $X \times \tilde{u}_t(X \times \Omega)$ into Ω such that this function provides the value of ω that solves the equation $\tilde{h}_t(x, \omega) + \tilde{c}_t(x) = u$. By definition of $\omega_t^*(x, u)$ we can obtain this function for $u \neq 0$ by solving in ω the equation $\tilde{h}_t(x, \omega) = u + \tilde{h}_t(x, P_t^{-1}(x, 0.5))$. Then, by construction we have that, for any $(x, u) \in X \times \tilde{u}_t(X \times \Omega)$, $F_{\tilde{\varepsilon}|x,t}(u) = P_t(x, \omega_t^*(x, u))$. Thus, $F_{\tilde{\varepsilon}|x,t}$ is identified. Q.E.D. ■

Remark 1. The (differential) outcome function $\tilde{h}_t$ plays a key role in the identification result in Proposition 1 and on the rest of identification results in this paper. The identification of P_t^* is possible because $\tilde{h}_t$ is identified and it depends monotonically on a continuous variable ω that does not enter in $\tilde{c}_t$ and $F_{\tilde{\varepsilon}|x,t}$.

Remark 2. The counterfactual probability function is identified on the set $[X \times \Omega]_t^*$ that is a subset of $X \times \Omega$ where the factual P_t is identified. However, there are different cases in which $[X \times \Omega]_t^* = X \times \Omega$. For instance, that is the case when the range of variation of the utility differences is the whole real line, or when $\tilde{\tau}_t$ is a positive valued function. The latter occurs in applications where the outcome variable y has a lower bound at zero (e.g., output, earnings, revenue), and we consider a counterfactual policy that increases the differential outcome for any possible value of (x, ω). For instance, an increase in the returns of college degree in a model for the decision of going to college.

Proposition 2 establishes the identification of the welfare effect function $(V_{\sigma,t}^*(x, \omega) - V_{\sigma,t}(x, \omega))/\psi_t(x)$, that is measured in units of the outcome variable y. We use the following Lemma in the proof of Proposition 2.

LEMMA 1: Consider the utility maximization problem $\max\{u(0) + \varepsilon(0), u(1) + \varepsilon(1)\}$. *McFadden's social surplus is defined as the expected surplus of behaving optimally instead of choosing always alternative zero: that is,* $E_{\tilde{\varepsilon}}(\max\{\tilde{u} - \tilde{\varepsilon}; 0\})$, *where* $\tilde{u} \equiv u(1) - u(0)$ *and* $\tilde{\varepsilon} \equiv \varepsilon(0) - \varepsilon(1)$. *Let* $F_{\tilde{\varepsilon}}$ *be the CDF of* $\tilde{\varepsilon}$, *and let* P *be the probability* $\Pr(\tilde{\varepsilon} \leq \tilde{u}|\tilde{u})$, *i.e.,* $P = F_{\tilde{\varepsilon}}(\tilde{u})$. *If* $F_{\tilde{\varepsilon}}$ *is a continuous and strictly increasing function, then the social surplus is a function of* P *and* $F_{\tilde{\varepsilon}}$ *only. That is,*

$$E_{\tilde{\varepsilon}}(\max\{\tilde{u} - \tilde{\varepsilon}; 0\}) = G(P, F_{\tilde{\varepsilon}}) \equiv \int \max\{F_{\tilde{\varepsilon}}^{-1}(P) - \tilde{\varepsilon}; 0\} dF_{\tilde{\varepsilon}}(\tilde{\varepsilon}). \tag{21}$$

Proof. Since $F_{\tilde{\varepsilon}}$ is continuous and strictly increasing, we have that $\tilde{u} = F_{\tilde{\varepsilon}}^{-1}(P)$, where $F_{\tilde{\varepsilon}}^{-1}$ is the inverse function of $F_{\tilde{\varepsilon}}$. Then, $E_{\tilde{\varepsilon}}(\max\{\tilde{u} - \tilde{\varepsilon}; 0\}) = \int \max\{\tilde{u} - \tilde{\varepsilon}; 0\} dF_{\tilde{\varepsilon}}(\tilde{\varepsilon}) = \int \max\{F_{\tilde{\varepsilon}}^{-1}(P) - \tilde{\varepsilon}; 0\} dF_{\tilde{\varepsilon}}(\tilde{\varepsilon})$. Q.E.D. ■

PROPOSITION 2: Under Assumptions 1 to 5 and $\beta = 0$ *the functions* $(V_{\sigma,t}^*(x, \omega) - V_{\sigma,t}(x, \omega))/\psi_t(x)$ *are nonparametrically identified. We can obtain these functions as:*

$$(V_{\sigma,t}^*(x, \omega) - V_{\sigma,t}(x, \omega))/\psi_t(x) = \tau_t(0, x, \omega) + G\left(P_t^*(x, \omega), F_{\tilde{\varepsilon}|x,t}\right) - G\left(P_t(x, \omega), F_{\tilde{\varepsilon}|x,t}\right) \tag{22}$$

where $G(P, F)$ is *McFadden's surplus function as defined in Lemma 1.*

Proof. We omit here the time subindex. When $\beta = 0$, the integrated value function is

$V_\sigma(x,\omega) = \int \max_{a \in A}\{u(a,x,\omega) + \varepsilon(a)\} dF_{\varepsilon|x}(\varepsilon)$. Then,

$$
\begin{aligned}
V_\sigma(x,\omega)/\psi(x) &= \frac{1}{\psi(x)} \int \max_{a \in A} \{ \ u(a,x,\omega) + \varepsilon(a) \ \} \, dF_{\varepsilon|x}(\varepsilon) \\
&= \frac{u(0,x,\omega)}{\psi(x)} + \int \max \left\{ \ \tilde{h}(x,\omega) + \tilde{c}(x) - \tilde{\varepsilon} \ ; \ 0 \right\} dF_{\tilde{\varepsilon}|x}(\tilde{\varepsilon}) \\
&= \frac{u(0,x,\omega)}{\psi(x)} + G\left(P(x,\omega), F_{\tilde{\varepsilon}|x}\right)
\end{aligned}
$$

where we have used Lemma 1 and the property $\tilde{h}(x,\omega) + \tilde{c}(x) = F^{-1}_{\tilde{\varepsilon}|x}(P(x,\omega))$. Given that $(u^*(0,x,\omega) - u(0,x,\omega))/\psi(x) = \tau(0,x,\omega)$, we have that $(V^*_\sigma(x,\omega) - V_\sigma(x,\omega))/\psi(x)$ is equal to $\tau(0,x,\omega) + G\left(P^*(x,\omega), F_{\tilde{\varepsilon}|x}\right) - G\left(P(x,\omega), F_{\tilde{\varepsilon}|x}\right)$. Since P, P^* and $F_{\tilde{\varepsilon}|x}$ are identified (Proposition 1), the surplus function, and therefore $(V^*_\sigma(x,\omega) - V_\sigma(x,\omega))/\psi(x)$, is identified. Q.E.D. ■

3.2 Dynamic model

We now study the identification of counterfactual choice probabilities when agents are forward looking, i.e., when $\beta > 0$. For the sake of simplicity, we present the proof in the context of a finite horizon model, i.e., $T < \infty$. However, the proof can be extended to infinite horizon models.[7] The factual choice probability function is:

$$
P_t(x,\omega) \;\; = \;\; F_{\tilde{\varepsilon}|x,t}\left(\tilde{v}_t(x,\omega)\right) \tag{23}
$$

where $\tilde{v}_t(x,\omega) \equiv (v_t(1,x,\omega) - v_t(0,x,\omega))/\psi_t(x)$ is the *differential value function.* The counterfactual choice probability function is $P^*_t(x,\omega) = F_{\tilde{\varepsilon}|x,t}(\tilde{v}^*_t(x,\omega))$, where $\tilde{v}^*_t$ is the differential value function after the policy change. We show in this section that the functions $F_{\tilde{\varepsilon}|x,t}$, $\tilde{v}^*_t$ and P^*_t are identified under the similar conditions as in Proposition 1. However, there are some differences in the identification results of the myopic and the dynamic or forward-looking models. In the dynamic model we cannot identify current utility differences or any other function that depends only on preferences and not on agents' beliefs. That is, we cannot

[7] The proof of identification in a model with infinite horizon appears in an earlier version of this paper which is available online at `ideas.repec.org` and at `papers.ssrn.com`.

separately identify agents' preferences and agents' beliefs. Despite this under-identification of preferences, we can identify the counterfactual choice probabilities associated to the class of experiments defined in Assumption 3. We also need an additional assumption on the function $\psi_t(x)$.

ASSUMPTION 6: *The function $\psi_t(x)$ is time invariant and it only depends on time-invariant individual characteristics: i.e., $\psi_t(x) = \psi$ where ψ may vary over individuals.*

Proposition 3 provides a characterization of the choice probability function that will be useful to identify and to estimate the counterfactuals.

PROPOSITION 3: For any age t, the optimal choice probability function P_t has the following form:

$$P_t(x,\omega) \equiv F_{\tilde{\varepsilon}|x,t}\left(\tilde{V}_t^{(y)}(x,\omega) + \tilde{V}_t^{(c)}(x) + \tilde{V}_t^{(opt)}(x,\omega)\right) \tag{24}$$

where: (1) $\tilde{V}_t^{(y)}(x,\omega) \equiv V_t^{(y)}(1,x,\omega) - V_t^{(y)}(0,x,\omega)$, and $V_t^{(y)}(a,x,\omega)$ is the expected, discounted value of the sum of current and future realizations of the outcome variable y if the current choice is a and then alternative 0 is chosen forever in the future; (2) $\tilde{V}_t^{(c)}(x) \equiv V_t^{(c)}(1,x) - V_t^{(c)}(0,x)$, and $V_t^{(c)}(a,x)$ is the expected, discounted value of the sum of current and future realizations of the component c of the utility function if the current choice is a and then alternative 0 is chosen forever in the future; and (3) $\tilde{V}_t^{(opt)}(x,\omega) \equiv V_t^{(opt)}(1,x,\omega) - V_t^{(opt)}(0,x,\omega)$, where $V_t^{(opt)}(a,x,\omega)$ is the value of behaving optimally in the future minus the value of choosing always alternative 0, given that the current choice is a. These functions are formally defined as:

$$\begin{aligned}
V_t^{(y)}(a,x,\omega) &= h_t(a,x,\omega) + \sum_{j=1}^{T-t} \beta^j \left[\int h_{t+j}(0,x^{(j)},\omega^{(j)})\ dF_0^{(j)}(x^{(j)}|a,x) dF_\omega^{(j)}(\omega^{(j)}|\omega)\right] \\
V_t^{(c)}(a,x) &= \frac{c_t(a,x)}{\psi} + \sum_{j=1}^{T-t} \beta^j \left[\int \frac{c_{t+j}(0,x^{(j)})}{\psi}\ dF_0^{(j)}(x^{(j)}|a,x)\right] \\
V_t^{(opt)}(1,x,\omega) &= \sum_{j=1}^{T-t} \beta^j \left[\int G(P_{t+j}(x^{(j)},\omega^{(j)}), F_{\tilde{\varepsilon}|t+j,x^{(j)}})\ dF_0^{(j)}(x^{(j)}|a,x) dF_\omega^{(j)}(\omega^{(j)}|\omega)\right]
\end{aligned} \tag{25}$$

where $G(P,F)$ is the surplus function that we have defined in Lemma 1; $F_\omega^{(j)}(\omega^{(j)}|\omega)$ is the j-periods-forward transition probability function of ω; and $F_0^{(j)}(x^{(j)}|a,x)$ is the j-periods-forward transition probability of x given that the agent chooses alternative 0 at every period in the future.[8]

Proof. For notational simplicity we omit ω as an argument in the different functions. Given the definition of the surplus function $G(P,F)$ in Lemma 1, we have that:

$$\int \max\left\{\frac{v_t(0,x)+\varepsilon_t(0)}{\psi}\ ;\ \frac{v_t(1,x)+\varepsilon_t(1)}{\psi}\right\} dF_{\varepsilon|x,t}(\varepsilon) \quad = \frac{v_t(0,x)}{\psi} + G(P_t(x), F_{\tilde{\varepsilon}|x,t})$$

Solving this expression in equation (8) that defines the conditional choice value function $v_t(a,x)$, we have that:

$$\begin{aligned}\frac{v_t(a,x)}{\psi} &= h_t(a,x) + \frac{c_t(a,x)}{\psi} + \beta \int \frac{v_{t+1}(0,x^{(1)})}{\psi} dF(x^{(1)}|a,x) \\ &+ \beta \int G(P_{t+1}(x^{(1)}), F_{\tilde{\varepsilon}|x^{(1)},t+1})\ dF(x^{(1)}|a,x)\end{aligned}$$

We can apply the same decomposition to the value $v_{t+1}(0,x^{(1)})$ that appears in this expression. If we do this, we get:

$$\begin{aligned}\frac{v_t(a,x)}{\psi} &= h_t(a,x) + \frac{c_t(a,x)}{\psi} + \beta \int h_{t+1}(0,x^{(1)}) dF(x^{(1)}|a,x) + \beta \int \frac{c_{t+1}(0,x^{(1)})}{\psi}\ dF(x^{(1)}|a,x) \\ &+ \beta^2 \int \frac{v_{t+1}(0,x^{(2)})}{\psi}\ dF_0^{(2)}(x^{(2)}|a,x) \\ &+ \beta \int G(P_{t+1}(x^{(1)}), F_{\tilde{\varepsilon}|x^{(1)},t+1})\ dF(x^{(1)}|a,x) + \beta^2 \int G(P_{t+2}(x^{(2)}), F_{\tilde{\varepsilon}|x^{(2)},t+2})\ dF_0^{(2)}(x^{(2)}|a,x)\end{aligned}$$

If we continue applying the decomposition to $v_{t+2}(0,x^{(2)})$, $v_{t+3}(0,x^{(3)})$, and so on, we get:

$$\begin{aligned}\frac{v_t(a,x)}{\psi} &= h_t(a,x) + \textstyle\sum_{j=1}^{T-t} \beta^j \left[\int h_{t+j}(0,x^{(j)})\ dF_0^{(j)}(x^{(j)}|a,x)\right] \\ &+ \frac{c_t(a,x)}{\psi} + \textstyle\sum_{j=1}^{T-t} \beta^j \left[\int \frac{c_{t+j}(0,x^{(j)})}{\psi}\ dF_0^{(j)}(x^{(j)}|a,x)\right] \\ &+ \textstyle\sum_{j=1}^{T-t} \beta^j \left[\int G(P_{t+j}(x^{(j)}), F_{\tilde{\varepsilon}|x^{(j)},t+j})\ dF_0^{(j)}(x^{(j)}|a,x)\right]\end{aligned}$$

---

[8]That is, the one-period forward transition is $F_0^{(1)}(x^{(1)}|a,x) = F_x(x^{(1)}|a,x)$; two-periods forward we have $F_0^{(2)}(x^{(2)}|a,x) = \int F_x(x^{(2)}|0,x^{(1)})F_x(x^{(1)}|a,x)dx^{(1)}$; three periods forward, $F_0^{(3)}(x^{(3)}|a,x) = \int\int F_x(x^{(3)}|0,x^{(2)})F_x(x^{(2)}|0,x^{(1)})F_x(x^{(1)}|a,x)dx^{(1)}dx^{(2)}$, and so on.

Given the definitions in (25), this shows that $v_t(a,x)/\psi = V_t^{(y)}(a,x)+V_t^{(c)}(a,x)+V_t^{(opt)}(a,x)$. This implies that $P_t(x) = \Pr(\tilde{\varepsilon}_t \leq (v_t(1,x) - v_t(0,x))/\psi) = F_{\tilde{\varepsilon}|t,x}(\tilde{V}_t^{(y)}(x) + \tilde{V}_t^{(c)}(x) + \tilde{V}_t^{(opt)}(x))$. Q.E.D. ■

Proposition 3 establishes that we can decompose additively the differential value function $\tilde{v}_t$ into the functions $\tilde{V}_t^{(y)}$, $\tilde{V}_t^{(c)}$, and $\tilde{V}_t^{(opt)}$. This decomposition is not arbitrary. We show below that we can identify these three components of the differential value function, and that these components, together with $F_{\tilde{\varepsilon}|x,t}$, is all what we need in order to construct the counterfactual choice probability P_t^*. First, note that the functions $\tilde{V}_t^{(y)}$ and $\tilde{V}_t^{(c)}$ do not depend on the optimal behavior of the individual. Therefore, it is straightforward to show that these functions after the policy intervention are:

$$\tilde{V}_t^{(y)*}(x,\omega) + \tilde{V}_t^{(c)*}(x) = \tilde{V}_t^{(y)}(x,\omega) + \tilde{V}_t^{(c)}(x) + \tilde{\Upsilon}_t(x,\omega) \tag{26}$$

where $\tilde{\Upsilon}_t(x,\omega) \equiv \Upsilon_t(1,x,\omega) - \Upsilon_t(0,x,\omega)$ and

$$\Upsilon_t(a,x,\omega) \quad = \quad \tau_t(a,x,\omega) + \sum_{j=1}^{T-t} \beta^j \left[\int \tau_{t+j}(0,x^{(j)},\omega^{(j)}) \; dF_0^{(j)}(x^{(j)}|a,x) dF_\omega^{(j)}(\omega^{(j)}|\omega) \right] \tag{27}$$

Given the functions $\{\tau_t\}$, the discount factor and the transition probabilities, it is straightforward to construct $\tilde{\Upsilon}_t(x,\omega)$, which is identified.

The value function $V_t^{(opt)}$ depends on the agent's future optimal behavior and therefore on the optimal choice probability functions from period $t+1$ until period T. To emphasize this dependence, we incorporate future choice probabilities as an explicit argument in this function: $\tilde{V}_t^{(opt)}(x,\omega;\{P_{t+j} : j > 0\})$. Note that all the primitives that enter explicitly in this value function (i.e., transition probabilities, distribution of $\tilde{\varepsilon}$, and discount factor) are policy invariant: they are the same before and after the policy change. Therefore, the only way in which the value function $\tilde{V}_t^{(opt)}$ is affected by the policy change is through the change in the optimal behavior between periods $t+1$ and T, i.e., through the change in the optimal choice probabilities between $t+1$ and T. Taking into account these considerations, we can write the counterfactual choice probability functions as follows:

$$P_t^*(x,\omega) = F_{\tilde{\varepsilon}|x,t}\left(\tilde{\Upsilon}_t(x,\omega) + \tilde{V}_t^{(y)}(x,\omega) + \tilde{V}_t^{(c)}(x) + \tilde{V}_t^{(opt)}(x,\omega;\{P_{t+j}^* : j>0\})\right) \tag{28}$$

Proposition 4 establishes the identification of the counterfactual choice probabilities.

PROPOSITION 4: Suppose that Assumptions 1 to 6 hold and that the discount factor β is known. For any period t, define the set $[X \times \Omega]_t^* \equiv \{(x,\omega) \in X \times \Omega : \tilde{v}_t^*(x,\omega) \geq \tilde{v}_t(x,-\infty)\}$. The counterfactual choice probability functions $\{P_t^*\}$ are identified over the sets $\{[X \times \Omega]_t^*\}$. Starting at the last period, $t=T$, the probability $P_t^*(x_0,\omega_0)$ can be obtained recursively as:

$$P_t^*(x_0,\omega_0) \quad = \quad P_t(x_0, \omega_t^{(\tau)}(x_0,\omega_0)) \tag{29}$$

where $\omega_t^{(\tau)}(x_0,\omega_0)$ is a function from $[X \times \Omega]_t^*$ into Ω that is implicitly defined as the value $\omega \in \Omega$ that solves the equation:

$$\tilde{V}_t^{(y)}(x_0,\omega)+\tilde{V}_t^{(opt)}(x_0,\omega;\{P_{t+j} : j>0\}) = \tilde{\Upsilon}_t(x_0,\omega_0)+\tilde{V}_t^{(y)}(x_0,\omega_0)+\tilde{V}_t^{(opt)}(x_0,\omega_0;\{P_{t+j}^* : j>0\}) \tag{30}$$

Furthermore, the differential value functions $\{\tilde{V}_t^{(c)}\}$ are identified on X, and the probability distributions $\{F_{\tilde{\varepsilon}|x,t}\}$ are identified on $\tilde{\nu}_t(X \times \Omega) \times X$, where $\tilde{\nu}_t(X \times \Omega)$ is the space $\{\tilde{\nu}_t(x,\omega) : (x,\omega) \in X \times \Omega\}$.

Proof: We start at last period T and then proceed backwards. At period T, there is no future and therefore the decision problem is static. We can apply Proposition 1 to show the identification of functions P_T^*, $\tilde{c}_T$ and $F_{\tilde{\varepsilon}|x,T}$. Let (x_0,ω_0) be a point in the set $[X \times \Omega]_T^*$ and let $\omega_T^{(\tau)}(x_0,\omega_0)$ be the function that is defined in the enunciate of the Proposition. For period T, $\omega_T^{(\tau)}(x_0,\omega_0)$ is implicitly defined as the value $\omega \in \Omega$ that solves the equation $\tilde{h}_T(x_0,\omega) = \tilde{\tau}_T(x_0,\omega_0) + \tilde{h}_T(x_0,\omega_0)$. By Assumption 5, the function $\omega_T^{(\tau)}(x_0,\omega_0)$ exists and is unique at any point $(x_0,\omega_0) \in [X \times \Omega]_T^*$. Using this function, it is clear that:

$$\begin{aligned} P_T^*(x_0,\omega_0) &= F_{\tilde{\varepsilon}|x_0,T}\left(\tilde{\tau}_T(x_0,\omega_0) + \tilde{h}_T(x_0,\omega_0) + \tilde{c}_T(x_0)\right) \\ &= F_{\tilde{\varepsilon}|x_0,T}\left(\tilde{h}_T(x_0,\omega_T^{(\tau)}(x_0,\omega_0)) + \tilde{c}_T(x_0)\right) \\ &= P_T(x_0,\omega_T^{(\tau)}(x_0,\omega_0)) \end{aligned}$$

Given that the factual choice probability function P_T is identified on $X \times \Omega$ and the function $\omega_T^{(\tau)}$ is identified on $[X \times \Omega]_T^*$, it is clear that P_T^* is identified on $[X \times \Omega]_T^*$. Following the same arguments as in Proposition 1, we can prove the identification of functions $\tilde{c}_T$ (which is equal to $\tilde{V}_T^{(c)}$) and $F_{\tilde{\varepsilon}|x,T}$. Now, consider the decision problem at some period $t < T$. Suppose that the sequence of future probability functions $\{P_{t+j}^* : j > 0\}$ and the sequence of future distribution functions $\{F_{\tilde{\varepsilon}|x,t+j} : j > 0\}$ are identified. Then, it is clear that the function $\tilde{V}_t^{(opt)}(.;\{P_{t+j}^* : j > 0\})$ is also identified because it only depends on $\{P_{t+j}^* : j > 0\}$ and $\{F_{\tilde{\varepsilon}|x,t+j} : j > 0\}$. The functions $\tilde{\Upsilon}_t$, $\tilde{V}_t^{(y)}$ and $\tilde{V}_t^{(opt)}(.;\{P_{t+j} : j > 0\})$ are also identified. By Assumptions 4 and 5 the function $\omega_t^{(\tau)}$ exists and is unique at any point in the set $[X \times \Omega]_t^*$. And given the identification of $\tilde{\Upsilon}_t$, $\tilde{V}_t^{(y)}$, $\tilde{V}_t^{(opt)}(.;\{P_{t+j}^* : j > 0\})$, and $\tilde{V}_t^{(opt)}(.;\{P_{t+j} : j > 0\})$, it is clear that $\omega_t^{(\tau)}$ is identified on $[X \times \Omega]_t^*$. Using this function, we have that:

$$\begin{aligned} P_t^*(x_0, \omega_0) &= F_{\tilde{\varepsilon}|x_0,t}\left(\tilde{\Upsilon}_t(x_0,\omega_0) + \tilde{V}_t^{(y)}(x_0,\omega_0) + \tilde{V}_t^{(c)}(x_0) + \tilde{V}_t^{(opt)}(x_0,\omega_0;\{P_{t+j}^* : j > 0\})\right) \\ &= F_{\tilde{\varepsilon}|x_0,t}\left(\tilde{V}_t^{(y)}(x_0,\omega_t^{(\tau)}(x_0,\omega_0)) + \tilde{V}_t^{(c)}(x_0) + \tilde{V}_t^{(opt)}(x_0,\omega_t^{(\tau)}(x_0,\omega_0);\{P_{t+j} : j > 0\})\right) \\ &= P_t(x_0, \omega_t^{(\tau)}(x_0,\omega_0)) \end{aligned}$$

Therefore, P_t^* is identified. Following the same arguments as in Proposition 1, we can prove the identification of functions $\tilde{V}_t^{(c)}$ and $F_{\tilde{\varepsilon}|x,t}$. Thus, by a backwards induction argument, the sequence of functions $\{P_t^*, \tilde{V}_t^{(c)}, F_{\tilde{\varepsilon}|x,t} : t = 1, 2, ..., T\}$ is identified. Q.E.D. ■

The functions $\{\tilde{V}_t^{(c)}\}$ depend on preferences and on transition probabilities of the state variables. Without further restrictions we cannot separately identify preferences. A restriction that identifies the utility function $c(a,x)/\psi$ is the "normalization" $c(0,x) = 0$ for any $x \in X$. By the definition of $\tilde{V}_t^{(c)}$ in equation (25) it is simple to verify that under this restriction we have that $V_t^{(c)}(0,x) = 0$ and $\tilde{V}_t^{(c)}(x) = V_t^{(c)}(1,x) = c(1,x)/\psi$. Furthermore, this normalization is innocuous for the type of counterfactual experiments that we consider in this paper (i.e., Assumption 3). That is, if the true $c(0,x)$ is not zero, then it is no longer true that our estimator of $\tilde{V}_t^{(c)}$ is a consistent estimator of $c(1,x)/\psi$, but this misspecification does not affect the consistency of our estimation of P_t^* because P_t^* depends on $c(1,x)/\psi$

and $c(0,x)/\psi$ only though $\tilde{V}_t^{(c)}$. Nevertheless, this normalization is not innocuous for the evaluation of other type of policy interventions which we do not consider in this paper, such as those that modify the transition probabilities of the state variables or the discount factor.

Proposition 5 establishes the identification of the welfare effect function $\left(V^*_{\sigma,t} - V_{\sigma,t}\right)/\psi$.

PROPOSITION 5: Under the conditions in Proposition 4 the welfare effect functions $\left(V^*_{\sigma,t} - V_{\sigma,t}\right)/\psi$ *are identified. We can obtain these functions as:*

$$\begin{aligned}(V^*_{\sigma,t}(x,\omega) - V_{\sigma,t}(x,\omega))/\psi &= \Upsilon_t(0,x,\omega) + \left[G\left(P^*_t(x,\omega), F_{\tilde{\varepsilon}|x,t}\right) - G\left(P_t(x,\omega), F_{\tilde{\varepsilon}|x,t}\right)\right] \\ &+ \left[\tilde{V}_t^{(opt)}(0,x,\omega;\{P^*_{t+j} : j > 0\}) - \tilde{V}_t^{(opt)}(0,x,\omega;\{P_{t+j} : j > 0\})\right]\end{aligned} \tag{31}$$

where $G(P,F)$ is McFadden's surplus function as defined in Lemma 1.

Proof. By the definition of the value functions $V^*_{\sigma,t}$ and $V_{\sigma,t}$, we have that:

$$\begin{aligned}&\left(V^*_{\sigma,t}(x,\omega) - V_{\sigma,t}(x,\omega)\right)/\psi \\ &= \left[\int \max_{a\in A}\{v^*_t(a,x,\omega) + \varepsilon(a)\}\, dF_{\varepsilon|x,t}(\varepsilon) - \int \max_{a\in A}\{v_t(a,x,\omega) + \varepsilon(a)\}\, dF_{\varepsilon|x,t}(\varepsilon)\right]/\psi \\ &= \left[v^*_t(0,x,\omega) - v_t(0,x,\omega)\right]/\psi + G\left(P^*_t(x,\omega), F_{\tilde{\varepsilon}|x,t}\right) - G\left(P_t(x,\omega), F_{\tilde{\varepsilon}|x,t}\right)\end{aligned}$$

We know from Proposition 3 that $v(0,x,\omega)/\psi = V_t^{(y)}(0,x,\omega) + V_t^{(c)}(0,x,\omega) + V_t^{(opt)}(0,x,\omega;P)$ and $v^*(0,x,\omega)/\psi = \Upsilon_t(0,x,\omega) + V_t^{(y)}(0,x,\omega) + V_t^{(c)}(0,x,\omega) + V_t^{(opt)}(0,x,\omega;P^*)$. Solving these expressions into the equation for $\left(V^*_{\sigma,t}(x,\omega) - V_{\sigma,t}(x,\omega)\right)/\psi$, we can get equation (31). Proposition 5 establishes the identification of all the functions in the right hand side of this equation. Q.E.D. ■

4 Estimation method

Suppose that we have a random sample of N individuals, indexed by i, for which we observe actions, states and outcomes at two consecutive periods: $\{a_{it_i}, a_{i,t_i+1}, x_{it_i}, x_{i,t_i+1}, y_{it_i}, y_{i,t_i+1}\}$. The method proceeds in two steps.

Step 1: Estimation of the outcome function h, the transition probabilities F_ω and F_x, and the (factual) choice probability functions $\{P_t\}$. First, we estimate the outcome functions h_t.

Given these functions we can get the residuals $\hat{\omega}_{it_i} = \hat{h}_{t_i}^{-1}(y_{it_i}, a_{it_i}, x_{it_i})$ which are consistent estimates of the true $\omega's$. Then, we use $\{\hat{\omega}_{it_i}\}$ to obtain a kernel estimator of the transition density function of ω. For instance,

$$\hat{f}_\omega(\omega'|\omega) = \left[\sum_{i=1}^{N} K\left(\frac{\omega - \hat{\omega}_{it_i}}{b_n^\omega}\right)\right]^{-1} \left[\sum_{i=1}^{N} K\left(\frac{\omega - \hat{\omega}_{it_i}}{b_n^\omega}\right) K\left(\frac{\omega' - \hat{\omega}_{i,t_i+1}}{b_n^\omega}\right)\right] \tag{32}$$

where b_n^ω is the bandwidth and $K(.)$ is the kernel function. Similarly, we estimate the transition density function f_x and the choice probability functions P_t using a kernel method. For instance,

$$\hat{P}_t(x,\omega) \quad = \quad \frac{\sum_{i=1}^{N} a_{it_i}\ I\{t_i = t\}\ K\left(\frac{x - x_{it_i}}{b_n^x}\right) K\left(\frac{\omega - \hat{\omega}_{it_i}}{b_n^\omega}\right)}{\sum_{i=1}^{N} I\{t_i = t\}\ K\left(\frac{x - x_{it_i}}{b_n^x}\right) K\left(\frac{\omega - \hat{\omega}_{it_i}}{b_n^\omega}\right)} \tag{33}$$

In all these kernel estimations we use cross-validation to select the optimal bandwidth.[9]

Step 2: Recursive Backwards estimation of the functions $\{\hat{P}_t^*\}$, $\{\omega_t^{(\tau)}\}$ *and* $\{F_{\tilde{\varepsilon}|x,t}\}$.

> **[At period T]** Starting at the last period T, we can estimate $\omega_T^{(\tau)}(x_0, \omega_0)$ using the estimator $\hat{\omega}_T^{(\tau)}(x_0, \omega_0)$ that solves in ω the equation $\hat{h}_T(x_0, \omega) = \tilde{\tau}_T(x_0, \omega_0) + \hat{h}_T(x_0, \omega_0)$. For instance, suppose that $\tilde{h}_t(x, \omega) = g_t(x)\exp(\sigma_t\omega)$, where $g_t(.)$ is nonparametrically specified. This is the class of outcome function that we use in

[9] In some applications, the kernel estimate of the function P may not be strictly monotonic in ω. Monotonicity of $\hat{P}$ is necessary for the subsequent estimation of the threshold functions ω_t^τ and therefore for the estimation of the effects of the counterfactual policy. If that is the case, we can impose monotonicity using the isotonic-smooth (IS) kernel estimator proposed by Mukerjee (1988) and Mammen (1991). The estimator can be defined in two steps. Suppose that the observations have been sorted with respect to the variable $\hat{\omega}$, such that $\hat{\omega}_1 \leq \hat{\omega}_2 \leq ... \leq \hat{\omega}_N$. The first step is an *isotonic regression* for $\{a_i\}$ on $\{\hat{\omega}_i\}$:

$$\hat{P}_I(x_i, \hat{\omega}_i) = \max_{s\leq i} \min_{t\geq s} \frac{\sum_{j=s}^{t} a_j}{t - s + 1}$$

The second step introduces smoothing by using a Nadaraya-Watson kernel estimator where the dependent variable is the isotonic regression $\{\hat{P}_I((x_i, \hat{\omega}_i)\}$ and the explanatory variable is $\hat{\omega}$.

$$\hat{P}_{IS}(x,\omega) = \frac{\sum_{i=1}^{N} \hat{P}_I(x_i, \hat{\omega}_i)\ I\{t_i = t\}\ K\left(\frac{x - x_i}{b_n^x}\right) K\left(\frac{\omega - \hat{\omega}_i}{b_n^\omega}\right)}{\sum_{i=1}^{N} I\{t_i = t\}\, K\left(\frac{x - x_i}{b_n^x}\right) K\left(\frac{\omega - \hat{\omega}_i}{b_n^\omega}\right)}$$

the empirical application in section 4. Then, there is the following closed form expression for $\hat{\omega}_T^{(\tau)}(x_0, \omega_0)$:

$$\hat{\omega}_T^{(\tau)}(x_0, \omega_0) = \omega_0 + \frac{1}{\hat{\sigma}_T} \log \left(1 + \frac{\tilde{\tau}_T(x_0, \omega_0)}{\hat{g}_T(x_0) \exp\{\hat{\sigma}_T \omega_0\}} \right) \tag{34}$$

We calculate $\hat{\omega}_T^{(\tau)}(x_0, \omega_0)$ for every $x_0 \in X$ and over a finite grid of points $\hat{\Omega}$ that is included in Ω. Then, we estimate $P_T^*(x_0, \omega_0)$ as:

$$\begin{aligned} \hat{P}_T^*(x_0, \omega_0) &= \hat{P}_T(x_0, \hat{\omega}_T^{(\tau)}(x_0, \omega_0)) \\ &= \frac{\sum_{i=1}^N a_{it_i} \ I\{t_i = T\} \ K\left(\frac{x - x_{it_i}}{b_n^x}\right) K\left(\frac{\hat{\omega}_T^{(\tau)}(x_0, \omega_0) - \hat{\omega}_{it_i}}{b_n^\omega}\right)}{\sum_{i=1}^N I\{t_i = T\} \ K\left(\frac{x - x_{it_i}}{b_n^x}\right) K\left(\frac{\hat{\omega}_T^{(\tau)}(x_0, \omega_0) - \hat{\omega}_{it_i}}{b_n^\omega}\right)} \end{aligned} \tag{35}$$

We can also estimate the distribution function $F_{\tilde{\varepsilon}|x,T}$ as:

$$\begin{aligned} \hat{F}_{\tilde{\varepsilon}|x_0,T}(u_0) &= \hat{P}_T(x_0, \hat{\omega}_T^*(x_0, u_0)) \\ &= \frac{\sum_{i=1}^N a_{it_i} \ I\{t_i = T\} \ K\left(\frac{x - x_{it_i}}{b_n^x}\right) K\left(\frac{\hat{\omega}_T^*(x_0, u_0) - \hat{\omega}_{it_i}}{b_n^\omega}\right)}{\sum_{i=1}^N I\{t_i = T\} \ K\left(\frac{x - x_{it_i}}{b_n^x}\right) K\left(\frac{\hat{\omega}_T^*(x_0, u_0) - \hat{\omega}_{it_i}}{b_n^\omega}\right)} \end{aligned} \tag{36}$$

where $\hat{\omega}_T^*(x_0, u_0)$ is the value of ω that solves the equation $\hat{h}_T(x_0, \omega) = u_0 + \hat{h}_T(x_0, \hat{P}_T^{-1}(x_0, 0.5))$. Following with the example with $\tilde{h}_t(x, \omega) = g_t(x) \exp(\sigma_t \omega)$, we have also a closed form expression for $\hat{\omega}_T^*(x_0, u_0)$:

$$\hat{\omega}_T^*(x_0, u_0) = \hat{P}_T^{-1}(x_0, 0.5) + \frac{1}{\hat{\sigma}_T} \log \left(1 + \frac{u_0}{\hat{g}_T(x_0) \exp\{\hat{\sigma}_T \ \hat{P}_T^{-1}(x_0, 0.5)\}} \right) \tag{37}$$

As with function $\hat{\omega}_T^{(\tau)}$, we calculate $\hat{\omega}_T^*(x_0, \omega_0)$ for every $x_0 \in X$ and over a finite grid of points $\hat{\Omega} \subset \Omega$.

[At period t $<$ T] Using the definitions of functions $\tilde{V}_t^{(y)}$ and $\tilde{V}_t^{(opt)}$ we construct the estimators $\hat{V}_t^{(y)}(x_0, \omega_0)$, $\hat{V}_t^{(opt)}(x_0, \omega_0; \{\hat{P}_{t+j} : j > 0\})$ and $\hat{V}_t^{(opt)}(x_0, \omega_0; \{\hat{P}_{t+j}^* : j > 0\})$. Note that there is a recursive formula for $V_t^{(y)}$ and $V_t^{(opt)}$ in terms of only transition probabilities and $V_{t+1}^{(y)}$ and $V_{t+1}^{(opt)}$. Given these estimators, we can now obtain the estimators $\hat{\omega}_t^{(\tau)}$ and $\hat{\omega}_T^*$. Then, we get:

$$\hat{P}_t^*(x_0, \omega_0) = \frac{\sum_{i=1}^N a_{it_i} \; I\{t_i = t\} \;\; K\left(\dfrac{x - x_{it_i}}{b_n^x}\right) K\left(\dfrac{\hat{\omega}_t^{(\tau)}(x_0, \omega_0) - \hat{\omega}_{it_i}}{b_n^\omega}\right)}{\sum_{i=1}^N I\{t_i = t\} \;\; K\left(\dfrac{x - x_{it_i}}{b_n^x}\right) K\left(\dfrac{\hat{\omega}_t^{(\tau)}(x_0, \omega_0) - \hat{\omega}_{it_i}}{b_n^\omega}\right)} \tag{38}$$

and

$$\hat{F}_{\tilde{\varepsilon}|x_0,t}(u_0) = \frac{\sum_{i=1}^N a_{it_i} \; I\{t_i = t\} \;\; K\left(\dfrac{x - x_{it_i}}{b_n^x}\right) K\left(\dfrac{\hat{\omega}_t^*(x_0, u_0) - \hat{\omega}_{it_i}}{b_n^\omega}\right)}{\sum_{i=1}^N I\{t_i = t\} \;\; K\left(\dfrac{x - x_{it_i}}{b_n^x}\right) K\left(\dfrac{\hat{\omega}_t^*(x_0, u_0) - \hat{\omega}_{it_i}}{b_n^\omega}\right)} \tag{39}$$

This estimator of $\{\hat{P}_t^*\}$ is consistent and asymptotically normal under standard regularity conditions. The Nadaraya-Watson estimators in step 1 are consistent, and the estimators in step 2 are continuous and differentiable functions of the estimators in step 1. However, we do not derive in this paper the rate of convergence and the asymptotic distribution of our estimator of $\hat{P}_t^*$. In the empirical application that we present in section 5, we use the bootstrap method to approximate the standard errors of the estimated policy effects.

The computational cost of getting a point estimate using this procedure is equivalent to solving the dynamic programming problem once. Though the computational cost increases exponentially with the number of cells in the state space, it is clear that we can use this method for any dynamic programming model that we be solved once in a reasonable CPU time. Of course, obtaining bootstrap standard errors is computationally more intensive.

In some applications with not few observations or not enough sample variability, the nonparametric estimator of $\{\hat{P}_t^*\}$ can be quite imprecise. However, we may be interested in some aggregate average effects of the policy, such as the *Average Treatment Effect*, $ATE_t = E_{x,\omega}(P_t^*(x_{t_i}, \omega_{t_i}) - P_t(x_{t_i}, \omega_{t_i}))$. This parameter can be consistently estimated by using $(1/N)\sum_{i=1}^n I\{t_i = t\}(\hat{P}_t^*(x_{t_i}, \omega_{t_i}) - \hat{P}_t(x_{t_i}, \omega_{t_i}))$ which is typically a root-N consistent estimator and that can provide precise estimates even when the estimator of the functions have a large variance.

5 An application

This section presents an application of this methodology to evaluate the effects of an hypothetical reform in the social security pension system in Sweden. The main purpose of this application is to illustrate the implementation of the method and to show that it can provide meaningful results in relevant contexts. Our model of retirement behavior follows Rust and Phelan (1997) and Karlstrom, Palme and Svensson (2004). The reform that we consider consists in a delay of three years in eligibility ages of the public pension system. The minimum age to claim a public pension goes from 60 to 63 years, and the *normal* retirement age goes from 65 to 68. This type of reform has been and it is still considered in many OECD countries.

5.1 A model of retirement behavior

At the beginning of each year, individuals decide whether to continue working ($a_t = 1$) or to retire from the labor force ($a_t = 0$). This decision is irreversible. Individuals have a utility function that is additively separable in consumption (C_t) and leisure (L_t). More specifically,

$$\tilde{U}_t = \psi_C \; U_C(C_t) + \psi_L(t, m_t, \varepsilon_t) \; U_L(L_t) \tag{40}$$

where the functions ψ_C and $\psi_L(t, m_t, \varepsilon_t)$ capture individual heterogeneity in the marginal utilities of consumption and leisure, respectively[10]; t represents age; m_t is marital status; and ε_t is an individual idiosyncratic shock in the marginal utility of leisure that is unobservable to the econometrician (e.g., unobserved health status).

If the individual works, his hours of leisure are equal to $L(1, t, m_t, \varepsilon_t)$ and his annual earnings are equal to the labor earnings W_t. If the individual decides to retire, then his hours of leisure are $L(0, t, m_t, \varepsilon_t)$ and earnings are equal to retirement benefits B_t. Thus,

[10] The marginal utility of consumption ψ_C may depend on time-invariant individual characteristics.

we can write leisure as $L_t = L(a_t, t, m_t, \varepsilon_t)$ and annual earnings as $Y_t = a_t W_t + (1 - a_t) B_t$. Labor earnings depend on age and on a wage shock:

$$\log(W_t) = h_W(t) + \omega_{t+1} \tag{41}$$

$h_W(.)$ is a function and ω_{t+1} is an individual wage shock that follows a Markov process $\omega_{t+1} = \rho(\omega_t) + \xi_{t+1}$ where $\rho(.)$ is a function and ξ_{t+1} is the innovation of the process. The individual knows ω_t when he decides whether to retire at age t, but he does not know the innovation ξ_{t+1}. Retirement benefits depend on retirement age (ra_t) and on *pension points* (pp_t): $B_t = B(ra_t, pp_t)$. We describe this function in section 5.2 below.

The state variables of the model are ε_t, ω_t and $x_t = (t, m_t, ra_t, pp_t)$. Note that the retirement status (i.e., a_{t-1}) is implicitly given by the retirement age. Since the individual has uncertainty about current labor earnings, the relevant current utility U_t is the expected utility $E_t(\tilde{U}_t)$ where the information set at period t is $(a_t, x_t, \omega_t, \varepsilon_t)$.

$$U_t = E_t(\tilde{U}_t) \;\; = \;\; \psi_C \; E_t \left(U_C \left(C_t \right) \right) + \psi_L(t, m_t, \varepsilon_t) \; U_L \left(L(a_t, t, m_t, \varepsilon_t) \right) \tag{42}$$

Define the functions $c(0, t, m_t)$ and $c(1, t, m_t)$ and the variables $\varepsilon_t(0)$ and $\varepsilon_t(1)$ such that, for $a = 0, 1$:

$$\begin{aligned} c(a, t, m_t) &\equiv Median \left(\psi_L(t, m_t, \varepsilon_t) \; U_L \left(L(a, t, m_t, \varepsilon_t) \right) \; | \; t, m_t \right) \\ \varepsilon_t(a) &\equiv \psi_L(t, m_t, \varepsilon_t) \; U_L \left(L(a, t, m_t, \varepsilon_t) \right) - c(a, t, m_t) \end{aligned} \tag{43}$$

Using these definitions we can rewrite the utility function as:

$$U_t \;\; = \;\; \psi_C \; E_t \left(U_C \left(C_t \right) \right) + c(a_t, t, m_t) + \varepsilon_t(a_t) \tag{44}$$

We now show that the utility function U_t can be represented in the form that we postulate in Assumptions 1 and 2. That is, we have to show that the term $\psi_C \; E_t(U_C(C_t))$ can be written as ψy_t, where y_t is an observable variable for the researcher. If we could observe individuals' consumption C_t, we would need very weak assumptions to have the type of structure in Assumptions 1 and 2. However, as in many other previous econometric models of retirement, we do not observe individual consumption. Instead, we observe labor earnings

W_t for those individuals who are working, and potential retirement benefits B_t for every individual, working or not. Following Rust and Phelan (1997), we assume that: (1) consumption is proportional to earnings, and the proportionality may depend on age and marital status, $C_t = \lambda Y_t$; and (2) the utility of consumption is a CRRA function, $U_C(C_t) = C_t^\alpha$ where α is the parameter of relative risk aversion. Furthermore, we also assume that this parameter α is known to the researcher. We can evaluate a policy under different scenarios for the degree of risk aversion. Under these conditions we can write $\psi_C \; E_t(U_C(C_t))$ as ψy_t, where $\psi \equiv \psi_C \lambda^\alpha$, and $y_t \equiv E(Y_t^\alpha | a_t, t, m_t, ra_t, pp_t, \omega_t)$.

Now we show that Assumption 4 holds in this model: i.e., that the outcome function $y_t = h(a_t, x_t, \omega_t)$ is identified. When $a_t = 0$, we have that $h(0, x_t, \omega_t) = E_t(B_t^\alpha) = B(ra_t, pp_t)^\alpha$, which is a known function for the econometrician. If $a_t = 0$, then $h(1, x_t, \omega_t) = E(W_t^\alpha | x_t, \omega_t)$. Given the previous assumptions about labor earnings, we have that $W_t^\alpha = \exp\{\alpha h_W(t) + \alpha\rho(\omega_t) + \alpha\xi_{t+1}\}$, and $E_t(W_t^\alpha) = \exp\{\alpha h_W(t) + \alpha\rho(\omega_t)\}$. We now describe how $h_W(.)$ and $\rho(.)$ can be nonparametrically identified from data on W_t and t from the subsample of working individuals. Given that $\ln W_t = h_W(t) + \rho(\omega_t) + \xi_{t+1}$, and $\omega_t = \ln W_{t-1} - h_W(t-1)$, we can write:

$$\ln W_t = h_W(t) + \rho\left(\ln W_{t-1} - h_W(t-1)\right) + \xi_{t+1} \qquad \text{if } a_t = 1 \tag{45}$$

The innovation ξ_{t+1} is i.i.d. over time and unknown to the individual when he makes his decision at period t. Therefore, ξ_{t+1} is independent of a_t and it is also independent of t and W_{t-1}. The orthogonality condition $E(\xi_{t+1} | a_t = 1, t, W_{t-1}) = 0$ provides moment conditions that allow us to estimate nonparametrically the functions $h_W(.)$ and $\rho\,(.)$.

5.2 Social security pensions and the counterfactual reform

The form of the benefits function $B(.,.)$ depends on the rules of the pension system. For the case of social security pensions in Sweden, we have that:

$$B_t = B(ra_t, pp_t) = \begin{cases} 0 & if \quad ra_t < RA_{\min} \\ pp_t \;\; (1 + \kappa_1 (ra_t - RA_{\mathtt{norm}})) & if \quad RA_{\min} \leq ra_t < RA_{\mathtt{norm}} \\ pp_t \;\; (1 + \kappa_2 (ra_t - RA_{\mathtt{norm}})) & if \quad RA_{\mathtt{norm}} \leq ra_t < RA_{\max} \\ pp_t \;\; (1 + \kappa_2 (RA_{\max} - RA_{\mathtt{norm}})) & if \quad ra_t \geq RA_{\max} \end{cases} \tag{46}$$

where $RA_{\min}$, $RA_{\mathtt{norm}}$, $RA_{\max}$, κ_1 and κ_2 are policy parameters that characterize the function $B(.,.)$. More specifically: $RA_{\min}$ is the minimum retirement age; $RA_{\mathtt{norm}}$ is the "normal" retirement age; κ_1 is a permanent actuarial reduction in benefits per year of early retirement; and κ_2 is a permanent actuarial increase in benefits per month of delayed retirement. For our sample period, 1983-1997, the values of these parameters in Sweden were $RA_{\min} = 60$ years, $RA_{\mathtt{norm}} = 65$ years, $RA_{\max} = 70$ years, $\kappa_1 = 6.0\%$ and $\kappa_2 = 8.4\%$.

An individual's pension points pp_t are a deterministic function of his whole history of earnings. However, it turns out that for many public pension systems the transition rule of pension points can be very closely approximated by a Markov process. For instance, that is the case for social security pensions in US (see Rust and Phelan, 1997), for Germany (see Knaus, 2002), and for Sweden (see Karlstrom, Palme and Svensson, 2004). Following these previous studies, we assume that pension points follow the process:

$$\log(pp_{t+1}) = h_B(t, pp_t) + \eta_{t+1} \tag{47}$$

where $h_B(.,.)$ is a function and η_{t+1} is the innovation of the process.

As mentioned above, the counterfactual policy that we evaluate is a three years delay in the eligibility ages. The new eligibility ages are $RA^*_{\min} = 63$, $RA^*_{\mathtt{norm}} = 68$, $RA^*_{\max} = 73$ years. Figure 1 presents the age profiles of benefits function before and after the reform. This reform does not affect the current utility when working, and therefore $\tau(1, x_t) = 0$. The change in the current utility when retired $\tau(0, x_t)$, measured in units of the outcome variable (i.e., earnings), is just the vertical difference between the two benefits functions in figure 1. We represent the age profile of $\tau(0, x_t)/pp_t$ in figure 2.

5.3 Data

The data come from the Swedish Longitudinal Individual Panel (LINDA). This dataset has been used before by Karlstrom, Palme and Svensson (2004, KPS hereinafter) to estimate a (parametric) dynamic structural model of retirement. The sample in KPS includes blue-collar workers who were born between 1927 and 1940. The observation period is 1983 to 1997. We select the subsample of men from the sample used by KPS. Our sample contains 3,129 individuals and 34,593 observations during the period 1983-1997.

Table 1 presents summary statistics for the variables that we use in this paper. Figure 3 shows the histogram of retirement ages for the subsample of 834 individuals who retire during the sample period. The average retirement age is 63.5 years and almost 46% of these individuals retired at age 65. Despite the peak at age 65, there is significant variability in retirement age. The range of retirement ages goes from 51to 69 years.

5.4 Step 1: Wage function and transition and choice probabilities

(a) Wage equation and stochastic process of the wage shock. Remember from section 5.1 that solving the stochastic process of the wage shock into the log-wage equation we get:

$$\log(W_{it}) = h_W(t) + \rho\left(\ln W_{i,t-1} - h_W(t-1)\right) + \xi_{i,t+1} \qquad \text{if } a_{it} = 1 \tag{48}$$

We observe W_{it} and $W_{i,t-1}$ only for the subsample of individuals who are still working at age t. However, under our assumption that $\xi_{i,t+1}$ is unknown to the individual when he makes his decision at age t, there is not selection bias in a least squares estimation of $h_W(.)$ and $\rho(.)$ using equation (48). In contrast, note that a least squares estimation of $h_W(.)$ based on equation $\log(W_{it}) = h_W(t) + \omega_{i,t+1}$ suffers of selection bias because $\omega_{i,t+1}$ is not mean independent of the individual's choice at period t. The Markov structure of the wage shock and our assumption on the arrival of information are both needed to control for potential selection bias. We consider a polynomial series approximation to the functions h_W and ρ, and we use Akaike's information criterion to choose the orders of these polynomials. Table 2 presents our estimates and figures 4 and 5 represent graphically these estimates. The function h_W is quadratic and ρ is cubic. Labor earnings reach their life-cycle maximum at age 60. There is strong persistence in the labor earnings shock ω. Given the residuals of this regression, $\{\hat{\xi}_{i,t+1}\}$, we estimate the density function of ξ using a kernel method with a Gaussian kernel and cross-validation for the choice of bandwidth. The estimated density, in figure 6, has very strong kurtosis at zero together with a long left tail. That is, innovations in labor earnings are typically very close to zero, but it is possible to observe a very negative shock (e.g., a period of unemployment during the year). Figure 7 shows that these properties also appear in the density of ω.

(b) Stochastic process of pension points. From section 5.2, the specification of this process is: $\log(pp_{i,t+1}) = h_B(t, pp_{it}) + \eta_{i,t+1}$, where $\eta_{i,t+1}$ is i.i.d. over individuals and over time. Again we use polynomial series for the function $h_B(.,.)$ and Akaike's information criterion to choose the order of the polynomial. Table 3 presents the estimated function $\hat{h}_B$. For any age, there is a monotonic relationship between current and next year pension points.

(c) Transition of marital status. The transition probability function of the *married dummy variable* m_t depends on age. Figure 8 presents our kernel estimates of the probabilities $\Pr(m_{t+1} = 1|m_t = 1, t)$ and $\Pr(m_{t+1} = 1|m_t = 0, t)$. Not surprisingly (especially for male, given that their mortality rate is higher than for women), there is very high persistence in marital status at these ages, and age has very small incidence in these transition probabilities. Thus, the estimated probabilities $\Pr(m_{t+1} = 1|m_t = 1, t)$ and $\Pr(m_{t+1} = 0|m_t = 0, t)$ are very close to one for every age t.

(d) Choice probability functions $P_t(m, pp, \omega)$. We have considered two estimation methods of the factual (and counterfactual) choice probabilities: (1) Nadaraya-Watson with cross-validation; and (2) a logit model with a very flexible specification in terms of the variables (t, m, pp, ω). The estimation results from the two methods are qualitatively identical. Table

4 presents our estimation of the logit model. The explanatory variables are dummies for each age between 55 and 68, a second order polynomial in age, the wage shock, the logarithm of pension points, and interactions of these variables with the married dummy. Figure 9 presents the age profile of the estimated probability of working evaluated at the mean values of the other explanatory variables. The most striking feature is the very low probability of working at the "normal" retirement age of 65. Though the probability decreases significantly between ages 60 and 64, the big jump occurs from age 64 to 65, i.e., from a value 0.8258 to 0.3280. The estimates are not very precise for ages older than 65 because there is a small number of individuals who are still working at these ages. Figure 10 represents the estimated probability of working as a function of the wage shock ω and evaluated at age 65 and the mean values of the other state variables. The estimated function is monotonic in ω. The estimation is quite precise for values of ω between values -1.0 and $+0.5$ which correspond to percentiles 3.5 and 98.5, respectively. Most importantly, the probability of working is very responsive to the wage shock. This probability goes from 0.17 for $\omega = -2.0$ to 0.97 for $\omega = 2.0$. This strong dependence of the working decision with respect to ω is important to identify the functions $\omega_t^{(\tau)}$ and ω_t^* (and therefore the counterfactuals P_t^*) over a wide range of variation.

5.5 Step 2: Counterfactual choice probabilities

We start at age $T = 75$, and use the backwards recursive method that we have described in section 4 to obtain the sequence of estimators $\{\hat{P}_t^*\}$, $\{\hat{\omega}_t^{(\tau)}\}$ and $\{\hat{F}_{\tilde{\varepsilon}|x,t}\}$ between $t = 50$ and $t = 75$. Figure 11 presents the estimates $\hat{P}_t$ and $\hat{P}_t^*$ for ages between 50 and 75 and evaluated at the mean values of the other state variables (m_t, pp_t, ω_t). The average retirement age goes from 63.47 to 64.95 years. The policy has several effects on the age profile of the probability of working. It increases between ages 60 and 63. There is a significant reduction at age 63. The probability of working increases between ages 64 and 67. There is also a new downward peak at age 68, the new 'normal' retirement age. Interestingly, the down peak at age 65 does not completely disappear after the reform. This could be due to misspecification of the model or to small-sample-bias in the nonparametric estimates. The peak at age 65 disappears only when we (artificially) increase the magnitude of the partial derivative of $\hat{P}_t$ with respect to ω_t. Therefore, it seems that the effect might be due to a downward bias in our estimate of this partial derivative. Alternatively, one might think that there are reasons, other than the social security system, why people retire at 65..Or at least, that there exit other reasons which are not captured by our simple model of retirement.

6 Summary and Conclusions

This paper presents a nonparametric approach to evaluate the behavioral and welfare effects of counterfactual policies using a dynamic structural model. The nonparametric structural model retains all the economic assumptions of a structural model (e.g., exogeneity/endogeneity assumptions, equilibrium concept, rational expectations, transition rules, independence assumptions, agents' information) but it is more robust than a parametric model because it relax the parametric assumptions. There are many situations in which a parametric model can be preferred either because its parsimony or to obtain more precise estimates. In these cases, our nonparametric approach can be used to test for the validity of a particular parametric specification and to search for a valid parametric model. The nonparametric approach has another interesting feature: it makes transparent the role that the substantial economic assumptions play in the model and in the evaluation of a policy. It is also a necessary first step to test the validity of these economic assumptions.

References

[1] Adda, J. and R. Cooper (2000): "Balladurette and Juppette: A Discrete Analysis of Scrapping Subsidies," *Journal of Political Economy*, 108, 778-806.

[2] Aguirregabiria, V. (2005): "Nonparametric Identification of Behavioral Responses to Counterfactual Policy Interventions in Dynamic Discrete Decision Processes," *Economics Letters*, 87, 393-398.

[3] Aguirregabiria, V. and C. Alonso-Borrego (1999): "Labor Contracts and Flexibility: Evidence from a Labor Market Reform in Spain," manuscript.

[4] Bajari, P. and H. Hong (2006): "Semiparametric Estimation of a Dynamic Game of Incomplete Information," NBER Technical Working Paper 320.

[5] Berkovec, J. and S. Stern (1991): "Job Exit Behavior of Older Men," *Econometrica*, 59, 189-210.

[6] Cooper, R., J. Haltiwanger and L. Power (1999): "Machine Replacement and the Business Cycle: Lumps and Bumps," *The American Economic Review*, 89, 921-946.

[7] Das, M. (1992): "A Micro-econometric Model of Capital Utilization and Retirement: The Case of the Cement Industry," *Review of Economic Studies*, 59, 277-297.

[8] Eckstein, Z. and K. Wolpin (1999): "Why Youth Drop out of High School: The Impact of Preferences, Opportunities and Abilities," *Econometrica,* 67, 1295-1339.

[9] Ferrall, C. (1997): "Unemployment Insurance Eligibility and the Transition from School to Work in Canada and the United States," *Journal of Business and Economic Statistics,* 15, 115-129.

[10] Heckman, J. and R. Robb (1985): "Alternative Methods for Evaluating the Impact of Interventions," in J. Heckman and B. Singer (eds.) *Longitudinal Analysis of Labor Market Data.* Cambridge University Press. Cambridge.

[11] Heckman, J. and Smith, J. (1998): "Evaluating the Welfare State", Frisch Centenary, Econometric Monograph Series, Cambridge: Cambridge University Press.

[12] Heckman, J., and E. Vytlacil (1999): "Local Instrumental Variables and Latent Variable Models for Identifying and Bounding Treatment Effects," Proceedings of the National Academy of Sciences, 96, 4730-4734.

[13] Heckman, J., and E. Vytlacil (2005): "Structural Equations, Treatment Effects, and Econometric Policy Evaluation," *Econometrica,* 73, 669-738

[14] Heckman, J. and S. Navarro (2005): "Dynamic Discrete Choice and Dynamic Treatment Effects," manuscript. The University of Chicago. Department of Economics.

[15] Hotz, J., and R.A. Miller (1993): "Conditional choice probabilities and the estimation of dynamic models," *Review of Economic Studies,* 60, 497-529.

[16] Imbens, G. and W. Newey (2002): "Identification and Estimation of Triangular Simultaneous Equations Models Without Additivity," NBER Technical Working Paper, #285.

[17] Karlstrom, A., M. Palme and I. Svensson (2004): "A dynamic programming approach to model the retirement behaviour of blue-collar workers in Sweden," *Journal of Applied Econometrics*, 19, 795-807.

[18] Kasahara, H. (2004): "Temporary Increases in Tariffs and Machine Replacement: The Chilean Experience,' manuscript, Queen's University, Kingston, Canada.

[19] Keane, M. and R. Moffitt (1998): "A Structural Model of Multiple Welfare Program Participation and Labor Supply," *International Economic Review*, 39, 553-89.

[20] Keane, M. and K. Wolpin (1997): "The Career Decisions of Young Men," *Journal of Political Economy*, 105, 473-522.

[21] Keane, M. and K. Wolpin (2000): "Estimating Welfare Effects Consistent with Forward-Looking Behavior. Part II: Empirical Results," PIER Working Paper 01-020. University of Pennsylvania.

[22] Keane, M. and K. Wolpin (2001): "The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment," *International Economic Review*, 42, 1051-1103.

[23] Kennet, M. (1993) "Did Deregulation Affect Aircraft Engine Maintenance? An Empirical Policy Analysis," *The RAND Journal of Economics,* 24, 542-558.

[24] Knaus, T. (2002): "The retirement decision in Germany: A dynamic programming model," manuscript. Freie Universität Berlin.

[25] Magnac, T. and Thesmar, D. (2002) "Identifying dynamic discrete decision processes". *Econometrica*, 70, 801-816.

[26] Mammen E. (1991): "Estimating a smooth monotone regression function,".*Annals. of Statistics*, 19, 724-740.

[27] Manski, C. (1990): "Nonparametric Bounds on Treatment Effects," *American Economic Review, Papers and Proceedings*, 80, 319-323.

[28] Mukerjee, H. (1988): "Monotone nonparametric regression," *Annals. of Statistics*, 16, 741-750.

[29] Olley, S. and Pakes, A. (1996). "The Dynamics Of Productivity In The Telecommunications Equipment Industry," *Econometrica*, 64, 1263-1297.

[30] Pakes, A. (1986): "Patents as Options: Some Estimates of the Value of Holding European Patent Stocks," *Econometrica,* 54, 755-84.

[31] Pesendorfer, M. and P. Schmidt-Dengler (2003): "Identification and Estimation of Dynamic Games," Working Paper. London School of Economics.

[32] Rota, P. (2004): "Estimating Labor Demand with Fixed Costs," *International Economic Review*, 45, 25-48.

[33] Rust, J. (1987): "Optimal replacement of GMC bus engines: An empirical model of Harold Zurcher," *Econometrica,* 55, 999-1033.

[34] Rust, J. (1994): "Structural estimation of Markov decision processes," in R. E. Engle and McFadden (eds.) *Handbook of Econometrics Volume 4*, North-Holland.

[35] Rust, J. and G. Rothwell (1995): "Optimal Response to a Shift in Regulatory Regime: The Case of the US Nuclear Power Industry," *Journal of Applied Econometrics*, 10, S75-S118.

[36] Rust, J. and C. Phelan (1997): "How Social Security and Medicare Affect Retirement Behavior in a World of Incomplete Markets," *Econometrica*, 65, 781-832.

[37] Sanders, S. and R. Miller (1997): "Human Capital Development and Welfare Participation" Carnegie-Rochester Conference Series on Public Policy, 46.

[38] Taber, C. (2000): "Semiparametric Identification and Heterogeneity in Discrete Choice Dynamic Programming Models," *Journal of Econometrics*, 96, 201-229.

[39] Todd, P. and K. Wolpin (2003): "Using a Social Experiment to Validate a Dynamic Behavioral Model of Child Schooling and Fertility: Assessing the Impact of a School Subsidy Program in Mexico," University of Pennsylvania - Department of Economics, PIER Working Paper No. 03-022.

[40] Wolpin, K. (1996): "Public-Policy Uses of Discrete-Choice Dynamic Programming Models," *American Economic Review*, 86, 427-32.

Table 1
Summary Statistics
3,129 blue-collar male workers. Cohorts 1927-1940
Years 1983 to 1997

Variable	Mean	Std. Dev.	Min	Max	# Obs.
Married	0.711	0.453	0	1	34,593
Annual Labor Earnings(1)	195.8	52.2	1.0	1,591.0	34,593
Pension Points	4.46	0.88	0.13	6.50	34,593
Retirement Age(2)	63.47	2.39	51	69	834

(1) In thousands of Swedish Kronas. In 1997, US$ 1 $\simeq$ 8 Swedish Kronas.
(2) Subsample of complete (uncensored) histories.

Table 2
Estimation of Wage Equation
and Stochastic Process of Wage Shock

Parameter(1)	Estimate	(Std. Error)
$h_{W,0}$ (constant)	4.7465	(0.2041)
$h_{W,1}$ (t)	0.0174	(0.0072)
$h_{W,2}$ (t^2)	-0.00014	(0.00006)
ρ_1	0.9161	(0.0037)
ρ_2	0.0019	(0.0044)
ρ_3	-0.0299	(0.0012)
Std.Dev. ξ	0.1607	
R-square	0.7160	
# Observations	30,630	

(1) The function $h_W(t)$ is $h_{W,0} + h_{W,1}\ t + h_{W,2}\ t^2$
The function $\rho(\omega)$ is $\rho_1\ \omega + \rho_2\ \omega^2 + \rho_3\ \omega^3$

Table 3
Estimation of Stochastic Process of Pension Points

Parameter	Estimate	(Std. Error)
constant	0.0287	(0.0155)
log(pp)	0.9630	(0.0049)
log(pp)^2	0.0161	(0.0004)
age	0.0009	(0.0005)
age^2	-6.64×10^{-6}	(4.53×10^{-6})
log(pp) $\times$ *age*	-0.0003	(0.0001)
Std.Dev. η	0.0116	
R-square	0.9971	
# Observations	30,630	

Table 4
Estimation of Choice Probability Function
(Probability of Working)

Variable	Estimate	(Std. Error)	Variable	Estimate	(Std. Error)
constant	32.5230	(8.8893)	*age*	-0.4328	(0.1450)
dummy age=55	-0.1483	(0.8785)	*log(pp)*	-5.496093	(5.1121)
dummy age=56	-1.1586	(0.6228)	*log(pp)*^{2}	-0.3162	(0.2612)
dummy age=57	-1.2248	(0.5914)	ω_t	8.7521	(3.2608)
dummy age=58	-1.1069	(0.6028)	ω_t^2	0.2580	(0.1364)
dummy age=59	-1.4228	(0.5969)	*age* $\times$ *log(pp)*	0.0929	(0.0820)
dummy age=60	-2.6664	(0.5925)	*age* $\times$ ω_t	-0.1104	(0.0486)
dummy age=61	-2.0099	(0.6365)	*age* $\times$ *married*	-0.0404	(0.0369)
dummy age=62	-2.0048	(0.6763)	*log(pp)* $\times$ ω_t	-0.4187	(0.3581)
dummy age=63	-1.9853	(0.7224)	*dummy married*	3.0896	(2.4262)
dummy age=64	-3.2234	(0.7647)	*married* $\times$ *log(pp)*	-0.3552	(0.4223)
dummy age=65	-5.1725	(0.8199)	*married* $\times$ ω_t	0.0413	(0.2940)
dummy age=66	-2.9375	(0.9134)			
dummy age=67	-2.2061	(1.011)	R-square	0.4644	
dummy age=68	-2.2779	(1.0870)	# Observations	31,464	

Figure 1. Age Profile of Pension Benefits (per pension point)

Before and After the Reform

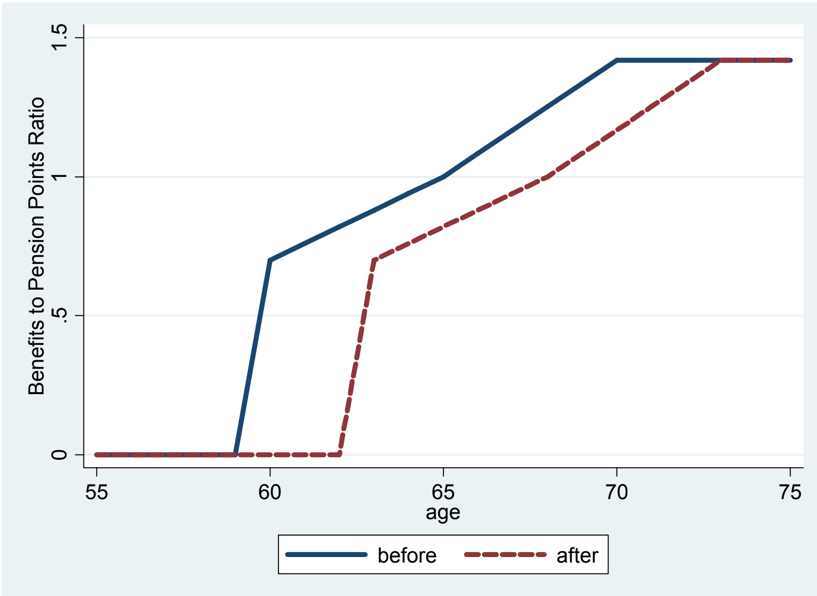

Figure 2. Function $\frac{\tau(\mathbf{0},\mathbf{t},\mathbf{ra},\mathbf{pp})}{\mathbf{pp}}$ associated with the Reform

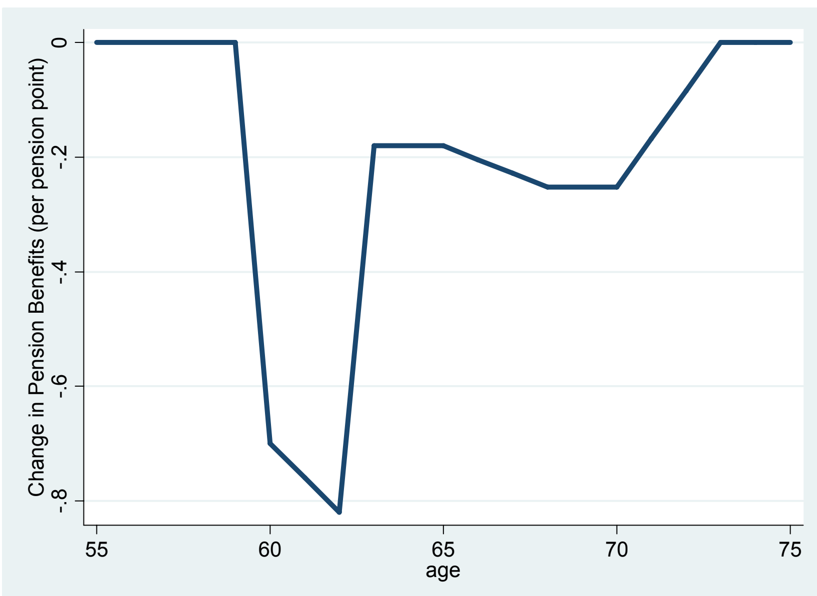

Figure 3. Histogram of Retirement Ages for the Subsample of 834 Men who Retire During the Sample Period

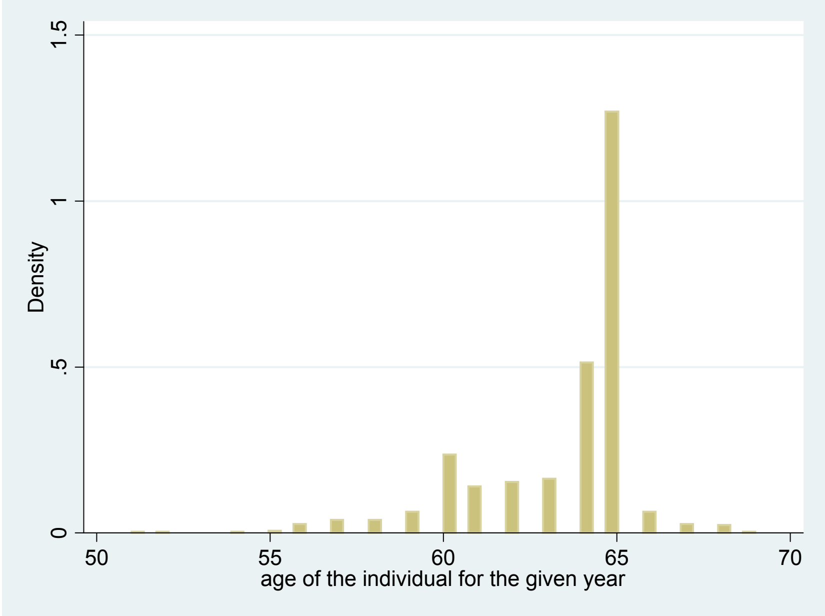

Figure 4. Estimated Function $h_W(t)$

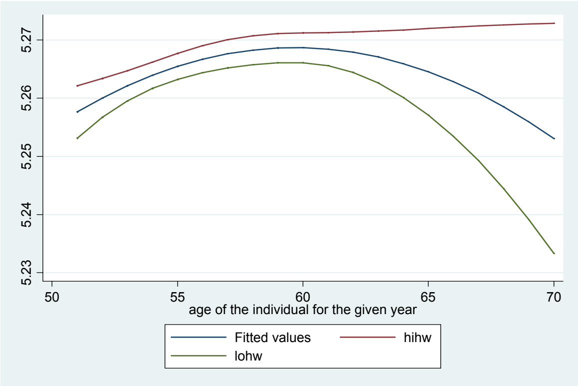

Figure 5. Estimated Function $\rho(\omega)$

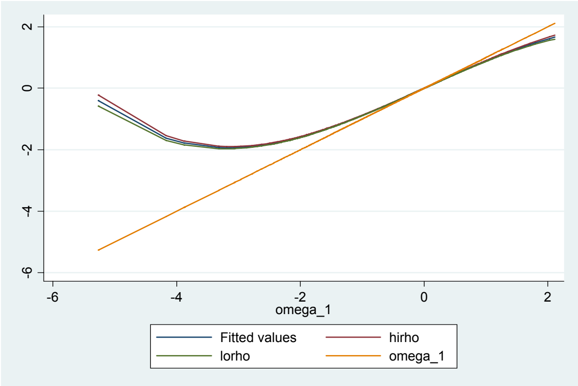

Figure 6. Estimated Density of ξ

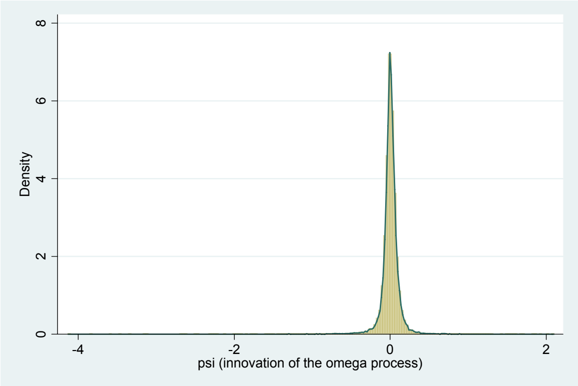

Figure 7. Estimated Density of ω

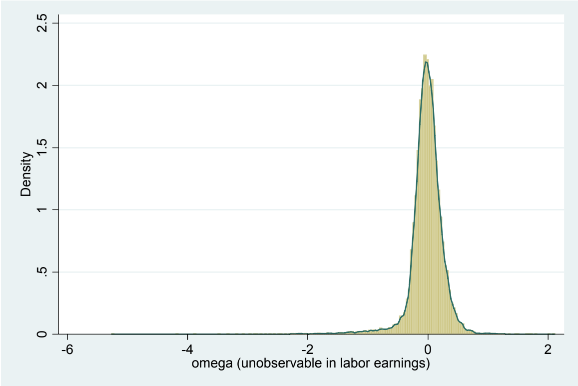

Figure 8. Transition Probabilities of Marital Status (Kernel Estimates)

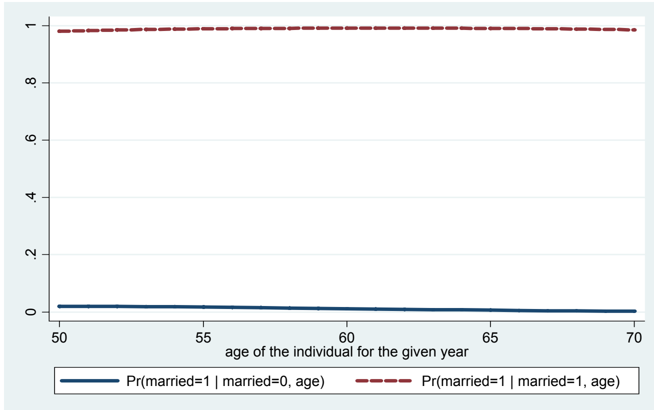

Figure 9. Prob. Working vs. age

(evaluated at mean values of m_t, pp_t and ω_t)

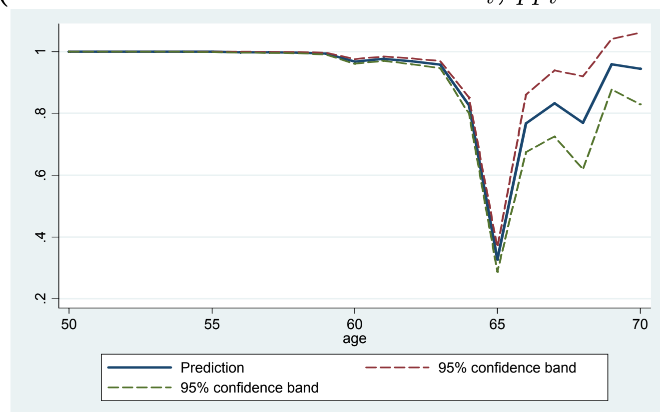

Figure 10. Prob. Working vs. ω_t

(evaluated at Age=65 and at mean values of m_t, pp_t)

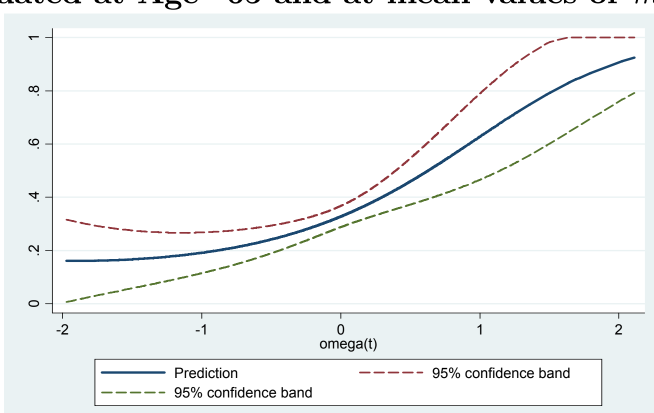

Figure 11. Factual an Counterfactual Probabilities

of Working at different ages

(Evaluated at mean values of m_t, pp_t, ω_t)