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WORKING PAPER
NUMBER UT-ECIPA-YAMASHIG-95-03

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September 9, 1995

ISSN 0829-4909

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On-line Version: http://www.epas.utoronto.ca:5680/wpa/wpa.html

Fairness in Markets and Government Policies

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September 9, 1995

JEL Classification: D51, D63, H20, H41

Keywords: Fairness, Competitive Equilibrium, Government Policy

Abstract: An allocation is said to be "weakly envy-free" if there is no individual who envies others with the same or smaller endowments. The requirement is a necessary condition for an allocation to be envy-free [Foley (1967)]. We show that competitive equilibrium allocations in exchange economies are weakly envy-free, i.e., this weak notion of fairness is achieved in markets. Since governments are expected to improve fairness in economies, it seems natural to require government policies satisfy this minimum requirement, which we refer to as no-envy criterion. We show that well-known equity criteria for government policies need not satisfy the no-envy criterion. Since the benefit tax (e.g., Lindahl tax) in general does not satisfy the no-envy criterion either, we argue that it is not a fair tax to finance pure public goods and show a result which supports income tax and commodity tax as fair taxes. The result suggests that the no-envy criterion may be indeed adopted by governments as an equity criterion. It is also argued that when public goods are not pure the benefit tax must be introduced because the income tax and the commodity tax no longer satisfy the no-envy criterion.

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†I wish to thank Professor Koichi Tadenuma for his comments on the earlier version of this paper. I am also grateful to the participants of the seminar at University of Western Ontario for their comments.

1 Introduction

It is widely perceived that markets cannot attain "fairness" by themselves[1] and thus some form of government policies are necessary.[2] At the same time, it seems that markets achieve some notion of fairness because, for example, the Japanese government is sometimes considered to be unfair for not opening its economy suggesting that markets can attain some notion of equity. The purpose of this paper is to clarify the notion of fairness achieved in the exchange markets and study its implications for government policies.[3]

In 1967, Foley introduced a famous equity criterion in his discussion of optimal government policies in a general equilibrium model; an allocation is said to be envy-free, if no one envies others under the allocation.[4] Since the equity criterion allows individuals to judge the fairness of allocation based on their own preferences and does not require them to know the preferences of others, this concept is likely to be accepted as an equity criterion for allocations in large economies where it is practically impossible to know the preferences of others [cf. Pazner (1977), Yamashige (1994)].[5]

In this paper, we introduce a weaker version of this concept;[6] an allocation is said to be "weakly envy-free" if there is no individual who envies others with the same or smaller endowments. The condition is a necessary condition for an allocation to be envy-free and

[1] In this paper, by "markets" we always mean competitive markets (markets under perfect competition). We also use the term "fairness" and "equity" interchangeably.

[2] For example, Atkinson-Stiglitz (1980, p.6) states as follows: "Pareto efficiency does not ensure that the distribution that emerges from the competitive process is in accord with the prevailing concepts of equity (whatever these may be). One of the primary activities of the government is indeed redistribution."

[3] We only work with an exchange economy because of its simplicity. Consideration of production activities seems to be an interesting topic because the concept of fairness discussed in this paper is well known to have a non-existence problem in economies with production [Pazner-Schmeidler (1974)].

[4] Besides its intuitive appeal as an equity criterion, it is known to have some nice properties: it does not require interpersonal comparability of preferences nor cardinality of utility functions [cf. Varian (1974)].

[5] Yamashige (1994) argued that when each individual has some information on the welfare of some other individuals, for example, of those with handicaps, the appropriate notion of fairness is not envy-free principle but is similar to the one discussed by Rawls (1971). Hence, the equity criterion for government policies discussed in this paper (based on the envy-free principle) should be understood to be applied to the individuals with no special characteristics that others can identify.

[6] An equity criterion (concept of fairness) reduces the number of admissible allocations. We say that an equity criterion is stronger than another if the set of admissible allocations under the criterion is a subset of the one under the other criterion.

essentially requires that the envy-structure in the initial state should not be overturned in the final state. Since the endowment of each individual is his/her contribution to the society, the condition may be also seen as a requirement that each individual should not envy those who make the same or smaller contribution to a society.[7] We show that competitive equilibrium allocations in exchange economies are weakly envy-free, i.e., this weak notion of fairness is achieved in markets.[8] This fact indeed may be a reason for why the market system has been a socially stable allocation mechanism in many countries.

Since the competitive equilibria satisfy this weak notion of fairness, it seems natural to expect that government policies, which are supposed to improve the fairness of an economy, should not violate this minimum requirement for fairness. A government policy is said to be weakly envy-free, or said to satisfy no-envy criterion, if under the policy no individual envies others with the same or smaller endowments. Hence, the no-envy criterion for government policies requires that the envy-structure in the competitive equilibrium without public sectors should not be overturned in the presence of public sectors.[9] We take this equity criterion as the weakest criterion that every government policy must satisfy and study how this criterion restricts the set of admissible policies. The results indicate that our new equity criterion for government policies is indeed different from other equity criteria and that it sheds new light on the fairness of government policies.

Fairness of government policies has been an important issue in public economics [Pigou (1947), Atkinson-Stiglitz (1980), Musgrave (1959, 1990) and references therein]. We show that even though the no-envy equity criterion is consistent with the horizontal equity the stronger

[7] A sophist Hippias of Elist in the ancient Greek, according to de la Mora (1984, p.4), classified the "envy" into two types, which "Aristotle later on will establish as the proper definition of indignation and envy": The "just" one is "the envy towards undeserving people when they receive honors" and the "unjust" one is the envy "of those who envy good people". Hence, we may be able to say, roughly, that our "weakly envy-free" criterion is a principle to prohibit causing "just" envy and let the "unjust" one as it is.

[8] This observation shows that the concept of fairness attained in markets does depend on each individual's initial states and therefore differs from those proposed by Rawls (1971) and Harsanyi (1955). See, e.g., Sen (1970, Chapter 9) for more details.

[9] A similar requirement for the notion of fairness is considered in Feldstein (1976) and King (1983), where they require that "the tax system should not lead to changes in the ranking of utilities". We will discuss the difference between this notion and ours in Section 3.4.

equity criteria (e.g., principles of vertical equity) need not satisfy the no-envy criterion. We also show that the Lindahl tax (benefit tax) [e.g., Lindahl (1919), Wicksell (1896)] does not satisfy our no-envy equity criterion either, implying that the Lindahl tax system is not a fair tax system to finance pure public goods.

Then, we show that when we require government policies satisfy the criterion for any economy, the no-envy criterion is characterized by a condition on individual budget constraints, which are satisfied under the standard income tax system and commodity tax system. Since these tax systems are adopted by many governments, the result suggests that the universal no-envy equity criterion may be indeed adopted by many governments. We also discuss the existence of Pareto optimal government policies which satisfy the universal no-envy criterion and how the results will be changed when public goods are not pure. It is shown that the benefit tax must be introduced to finance impure public goods from the view point of fairness because the income tax and the commodity tax no longer satisfy the no-envy equity criterion.

The paper is organized as follows. In the next section, we discuss various equity criteria for general allocation mechanisms and show that the competitive equilibrium allocation mechanism is weakly envy-free. In Section 3, we introduce a government, define our no-envy criterion, and discuss its relationship with other equity criteria for government policies and the Lindahl tax. In Section 4, we require universality for government policies and show that this requirement provides an important condition for the no-envy equity criterion, which is preference-independent. Section 5 concludes the paper. In Appendix, we define the "income-based no-envy equity criterion" which strengthens the no-envy equity criterion and has some interesting policy implications on the income tax base.

2 Fairness in Markets

Markets usually do not treat individuals symmetrically. The more an individual is endowed the better off he/she becomes. This asymmetry seems to be a reason for why people consider that markets fail to attain equity. In order to analyze the asymmetry in markets, we define allocation mechanisms each of which assigns an economy an allocation of commodities and study various equity criteria for allocation mechanisms.[10] Then we define the competitive equilibrium allocation mechanism and study its property.

2.1 Preliminaries

Let $N = \{1, \ldots, n\}$ be a set of individuals. Let $X \subset \mathbf{R}^m$ be a commodity space. Let $u_i : X \to \mathbf{R}$ be a utility function and an endowment of an individual $i \in N$. Let $\omega_i \in X$ be an endowment of $i \in N$. Since an individual i is characterized by a pair (ω_i, u_i), an exchange economy is defined by $e = (N; \omega_1, \ldots, \omega_n; u_1, \ldots, u_n)$. Let $\mathbf{E}$ be a space of economies.

We consider assignment rules which are dependent on each individual's name and endowment. Formally, for each $i \in N$, an assignment function is defined by a mapping $\tilde{x}_i : X \times \mathbf{E} \to X$ such that $\tilde{x}_i(\omega_i; e) \in X$ represents the commodity bundle of individual $i \in N$ in an economy $e \in \mathbf{E}$ when his/her endowment is $\omega_i \in X$. Given a collection of assignment functions $\{\tilde{x}_i\}_{i \in N}$, a mapping $\tilde{x} : \mathbf{E} \to X^n$ defined by $\tilde{x}(e) = (\tilde{x}_1(\omega_1; e), \ldots, \tilde{x}_n(\omega_n; e))$ for each $e \in \mathbf{E}$ is called an allocation mechanism if it satisfies the resource constraint $\sum_{i \in N} \tilde{x}_i(\omega_i; e) \leq \sum_{i \in N} \omega_i$. Hence, an allocation mechanism is associated with a collection of assignment functions which satisfies the resource constraint in each economy.

We now define two envy-free principles. Roughly speaking, the first one says that no one envies other individuals' allocations[11] whereas the second one says that no one envies other individuals' endowments[12] under the given allocation mechanism $\tilde{x}$.

Definition 1 *An allocation mechanism $\tilde{x}$ is said to be posterior-envy-free in an economy $e \in \mathbf{E}$ if $u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_j(\omega_j; e))$ for all $i \in N$ and $j \neq i$.*

[10] This approach is similar to the one taken by Thomson (1983) who studies various notions of fairness by analyzing allocation mechanisms (which he calls choice correspondences) with permutations of endowment vectors. Our goal in this section is to study allocation mechanisms to figure out the fairness embodied in the competitive equilibrium allocation mechanism.

[11] This definition coincides with the traditional definition of envy-free allocation by Foley (1967) except that our definition is for allocation mechanisms rather than allocations.

[12] This notion is considered by Varian (1976) in the context of the competitive equilibrium allocations and is called "opportunity fairness". Thomson (1994) extended this idea by considering collections of choice sets to study the notions of "equal opportunity" and examine the relationship with other equity criteria.

Definition 2 *An allocation mechanism $\tilde{x}$ is said to be prior-envy-free in an economy $e \in \mathbf{E}$ if $u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_i(\omega_j; e))$ for all $i \in N$ and $j \neq i$.*

We say that an allocation mechanism $\tilde{x}$ is posterior-envy-free if it is posterior-envy-free in every economy $e \in \mathbf{E}$. We also say that an allocation $\tilde{x}(e)$ is posterior-envy-free if the allocation mechanism $\tilde{x}$ is posterior-envy-free in an economy $e \in \mathbf{E}$. These two conventions are used for other definitions of envy-free criteria.

Example 1. An allocation mechanism $\tilde{x}$ is said to be *trivial* if for each $e \in \mathbf{E}$ and $i \in N$, $\tilde{x}_i(\omega_i; e) = \omega_i$. In this case, two definitions are the same because $\tilde{x}_j(\omega_j; e) = \omega_j = \tilde{x}_i(\omega_j; e)$. $\square$

Example 2. Consider an allocation mechanism $\tilde{x}$ associated with the assignment functions such that $\tilde{x}_i(\omega_i; e) = \tilde{x}_j(\omega_i; e)$ for all $i \in N$, $j \neq i$ and $e \in \mathbf{E}$. This allocation mechanism is an example of the anonymous allocation mechanism. In this case, the two definitions are also the same since $\tilde{x}_j(\omega_j; e) = \tilde{x}_i(\omega_j; e)$. $\square$

Example 3 (Posterior-Envy-Free $\not\Rightarrow$ Prior-Envy-Free). Let $X = \{(x_1, x_2) \in \mathbf{R}^2 : 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1\}$ and suppose there are only two individuals, i and j. Consider a set of economies $\mathbf{E}_1 = \{(\omega_i, \omega_j; u_i, u_j) \in \mathbf{E} : u_i(x) = 2x_1x_2, u_j(x) = 2x_1 + x_2, \omega_i + \omega_j = (1,1)\}$. For each $e \in \mathbf{E}_1$, define assignment functions by

$$\tilde{x}_i(\omega_i; e) = \begin{cases} (\frac{1}{2}, \frac{1}{2}) & \text{if } \omega_{i1} + \omega_{i2} < 1 \\ (0,1) & \text{if } \omega_{i1} + \omega_{i2} \geq 1 \end{cases} \text{ and } \tilde{x}_j(\omega_j; e) = \begin{cases} (\frac{1}{2}, \frac{1}{2}) & \text{if } \omega_{j1} + \omega_{j2} \geq 1 \\ (1,0) & \text{if } \omega_{j1} + \omega_{j2} < 1 \end{cases}.$$

It is easy to check that this pair of assignment functions constitutes an allocation mechanism (on $\mathbf{E}_1$). Roughly speaking, the commodity bundles are allocated so that the lower income individual is given his/her most preferable bundle under the constraint $\tilde{x}_{i1}(\omega_i; e) + \tilde{x}_{i2}(\omega_i; e) = \tilde{x}_{j1}(\omega_j; e) + \tilde{x}_{j2}(\omega_j; e) = 1$. Suppose that $\omega_i = (0,0)$ and $\omega_j = (1,1)$. Then, according to the allocation mechanism, $\tilde{x}_i(\omega_i; e) = (\frac{1}{2}, \frac{1}{2})$ and $\tilde{x}_j(\omega_j; e) = (\frac{1}{2}, \frac{1}{2})$ under which $u_i(\tilde{x}_i(\omega_i; e)) = \frac{1}{2}$ and $u_j(\tilde{x}_j(\omega_j; e)) = \frac{3}{2}$. Notice that this allocation is posterior-envy-free because the assignments are the same. The allocation, however, is not prior-envy-free because $u_j(\tilde{x}_j(\omega_j; e)) = \frac{3}{2} < 2 = u_j(\tilde{x}_j(\omega_i; e))$, i.e., j envies i's endowment. $\square$.

Example 4 (Prior-Envy-Free $\not\Rightarrow$ Posterior-Envy-Free). Consider the same example above except for the allocation mechanism. For each $e \in \mathbf{E}_1$, redefine the assignment functions by

$$\tilde{x}_i(\omega_i; e) = (1,0) \;\; \forall \omega_i \in X \text{ and } \tilde{x}_j(\omega_j; e) = (0,1) \;\; \forall \omega_j \in X.$$

Suppose $\omega_i = (0,0)$ and $\omega_j = (1,1)$. The allocation is prior-envy-free because $u_i(\tilde{x}_i(\omega_i; e)) = 0 = u_i(\tilde{x}_i(\omega_j; e))$ and $u_j(\tilde{x}_j(\omega_j; e)) = 1 = u_j(\tilde{x}_j(\omega_i; e))$. It is not, however, posterior-envy-free because $u_j(\tilde{x}_j(\omega_j; e)) = 1 < 2 = u_j(\tilde{x}_i(\omega_i; e))$, i.e., j envies i's assignment. $\square$

These examples motivate the following stronger notion of envy-free.[13]

Definition 3 *An allocation mechanism $\tilde{x}$ is said to be strongly envy-free in an economy $e \in \mathbf{E}$ if and only if it is both posterior-envy-free and prior-envy-free in an economy $e \in \mathbf{E}$.*

Although these are important notions of fairness, they seem too strong in the sense that there are many allocations that we feel fair even if we envy others. The less ambitious but still important notion of fairness may be the following; an allocation mechanism should guarantee that each individual does not envy others with the same or smaller endowments. The equity criterion is very weak in the sense that it is only violated if an individual feels that other individuals with the same or smaller endowments get better off under the allocation mechanism. We believe that the criterion is important because it essentially requires that an allocation mechanism should not overturn the envy-structure in an economy.[14] Since the endowment of each individual is his/her contribution to the society, we can also see this condition as a requirement that individual should not envy those who made the same or smaller contribution to a society, which we believe captures some sense of fairness.

Definition 4 *An allocation mechanism $\tilde{x}$ is said to be weakly posterior-envy-free in an economy $e \in \mathbf{E}$ if and only if $\omega_i \geq \omega_j \Rightarrow u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_j(\omega_j; e))$.*

[13] This is indeed stronger than the traditional definition of envy-free allocation by Foley (1967) because we also require allocations to be prior-envy-free.

[14] We are implicitly assuming that the utility functions are increasing.

Definition 5 *An allocation mechanism $\tilde{x}$ is said to be weakly prior-envy-free in an economy $e \in \mathbf{E}$ if and only if $\omega_i \geq \omega_j \Rightarrow u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_i(\omega_j; e))$.*

Definition 6 *An allocation mechanism $\tilde{x}$ is said to be weakly envy-free in an economy $e \in \mathbf{E}$ if and only if it is both weakly posterior-envy-free and weakly prior-envy-free in $e \in \mathbf{E}$.*

These are indeed weaker requirements because the condition $u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_j(\omega_j; e))$ (or $u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_i(\omega_j; e))$) has to hold only when $\omega_i \geq \omega_j$. It is easy to check that each weaker condition is a necessary condition for the corresponding stronger one. Hence, it should be guaranteed more easily and also would be accepted more easily by many people as a minimum requirement for fairness. Example 3 and 4 above also show that there is no inclusion-relation between the weaker versions of posterior-envy-free and prior-envy-free.

2.2 Fairness of Competitive Equilibrium Allocation Mechanism

We are ready to study an important allocation mechanism, which actually motivated the weaker definitions of envy-free criterion. Let $\Delta = \{(q_1, \ldots, q_m) \in \mathbf{R}^m : \sum_{k=1}^m q_k = 1 \text{ and } q_k > 0 \ \forall k = 1, \ldots, m\ \}$ be a space of price vectors. Define a budget correspondence $B : \Delta \times X \longrightarrow\!\!\!\!\to X$ by $B(q, \omega_i) = \{x \in X : q \cdot x \leq q \cdot \omega_i\}$.

Definition 7 *An allocation mechanism $\tilde{x}$ is said to be a competitive equilibrium allocation mechanism if for any economy $e \in \mathbf{E}$, there is $q_e \in \Delta$ such that $\tilde{x}_i(\omega_i; e) \in \arg\max_{x \in B(q_e, \omega_i)} u_i(x)$ for each $i \in N$.*

It is well known that under certain conditions this mapping is well defined [see, e.g., Debreu (1959)]. An important property of the competitive equilibrium allocation mechanism $\tilde{x}$ is that if $\tilde{x}$ is weakly prior-envy-free then it is weakly posterior-envy-free.

Lemma 1 *Every competitive equilibrium allocation mechanism is weakly posterior-envy-free if it is weakly prior-envy-free.*

Proof. Let $e \in \mathbf{E}$ be arbitrary and $(\tilde{x}(e), q_e)$ be a competitive equilibrium. Let $\omega_i \geq \omega_j$ and assume $u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_i(\omega_j; e))$ for each $i \in N$. Then, $u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_i(\omega_j; e)) = \max_{x \in B(q_e, \omega_j)} u_i(x) \geq u_i(\tilde{x}_j(\omega_j; e))$, where the last inequality is due to $\tilde{x}_j(\omega_j; e) \in B(q_e, \omega_j)$. ∎

Namely, if $\tilde{x}$ is any competitive equilibrium allocation mechanism, then the prior criterion is stronger. Now, we have the following important result, which can be seen as a claim that the markets are fair to some degree.[15]

Proposition 1 *Every competitive equilibrium mechanism is weakly envy-free.*

Proof. By the previous lemma, we have only to show that a competitive equilibrium allocation is weakly prior-envy-free for any economy. Let $e \in \mathbf{E}$ be arbitrary and $(\tilde{x}(e), q_e)$ be a competitive equilibrium. Let $\omega_i \geq \omega_j$. Then, $B(q_e, \omega_i) \supseteq B(q_e, \omega_j)$ because if $x \in B(q_e, \omega_j)$ then $q_e \cdot x \leq q_e \cdot \omega_j \leq q_e \cdot \omega_i$, i.e., $x \in B(q_e, \omega_i)$. Therefore, $u_i(\tilde{x}_i(\omega_i; e)) = \max_{x \in B(q_e, \omega_i)} u_i(x) \geq \max_{x \in B(q_e, \omega_j)} u_i(x) = u_i(\tilde{x}_i(\omega_j; e))$, i.e., the allocation is weakly prior-envy-free. ∎

The following proposition shows that when utility function of each individual is increasing, the strongly envy-free allocation is attained if and only if the income of each individual is the same.[16] The result is not true for the Foley's definition of envy-free allocation, i.e., posterior-envy-free allocation. The equal income is a sufficient condition but not the necessary condition.

Proposition 2 *Let u_i be increasing for each $i \in N$ and let $(\tilde{x}(e), q_e)$ be a competitive equilibrium. $\tilde{x}(e)$ is strongly envy-free if and only if $q_e \cdot \omega_i = q_e \cdot \omega_j$ for all $i, j \in N$.*

Proof. Let $\tilde{x}$ be a competitive equilibrium allocation mechanism. Suppose $q_e \cdot \omega_i = q_e \cdot \omega_j$ for all $i, j \in N$. Since every competitive equilibrium allocation is weakly envy-free, $\omega_i \geq \omega_j$ implies $u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_i(\omega_j; e))$. Hence, by assumption, we have $u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_i(\omega_j; e))$ for all

[15] Schmeidler-Vind (1972) has reported a similar result: In the competitive equilibrium each individual's net trades is at least as good as the net trade of any other individual.

[16] This result is already suggested in Varian (1974) in his discussion of "opportunity fairness" which corresponds to our prior-envy-free principle.

$i \in N$ and $j \neq i$. Namely, $\tilde{x}(e)$ is strongly envy-free. Conversely, suppose $\tilde{x}(e)$ is strongly envy-free. Suppose that $q_e \cdot \omega_i \neq q_e \cdot \omega_j$ for some $i \neq j$. Then, either $q_e \cdot \omega_j > q_e \cdot \omega_i$ or $q_e \cdot \omega_i > q_e \cdot \omega_j$. Without loss of generality, we can assume $q_e \cdot \omega_j > q_e \cdot \omega_i$. Then, $B(q_e, \omega_i) \subset B(q_e, \omega_j)$ (strict subset). Then, under the assumption that u_i is increasing, it is easy to show

$$u_i(\tilde{x}_i(\omega_i; e)) = \max_{x \in B(q_e, \omega_i)} u_i(x) < \max_{x \in B(q_e, \omega_j)} u_i(x) = u_i(\tilde{x}_i(\omega_j; e)),$$

which contradicts that $\tilde{x}(e)$ is strongly envy-free. This completes the proof. ∎

As far as the competitive equilibrium is concerned, the following result is also useful. (Since the proof is similar to the one for Proposition 1 above, we omit it.)

Proposition 3 *Let $(\tilde{x}(e), q_e)$ be a competitive equilibrium in an economy $e \in \mathbf{E}$. Then,*

$$q_e \cdot \omega_i \geq q_e \cdot \omega_j \Rightarrow u_i(\tilde{x}_i(\omega_i; e)) \geq u_i(\tilde{x}_i(\omega_j; e)).$$

The condition in the definition above is a stronger requirement than being weakly prior-envy-free (and thus weakly envy-free) because $\omega_i \geq \omega_j$ implies $q_e \cdot \omega_i \geq q_e \cdot \omega_j$ but $q_e \cdot \omega_i \geq q_e \cdot \omega_j$ does not necessarily imply $\omega_i \geq \omega_j$. This idea is used to strengthen the definition of no-envy criterion for government policies in Appendix.

Despite these facts about the competitive equilibrium allocation mechanism, we sometimes experience bad feelings that others with the same or smaller endowment got better off. This may be because an allocation is determined not only by markets but also by governments. It seems that if such a situation indeed occurred due to a government policy we would feel unfair. The reason that the U.S. government blames the Japanese government may be that this situation is indeed happening due to the Japanese government policies.

In other words, this weak notion of fairness seems to be a natural requirement that the government policies must satisfy because every competitive equilibrium satisfies it. In the following, we adopt this concept as an equity criterion for government policies and discuss its relation with various other equity criteria well known in the theory of public economics.

Remark 1. In this paper, we consider that the fairness of markets is captured by the fact that the competitive equilibrium allocations are weakly envy-free. In relation to the Lindahl tax discussed later, we may also be able to argue that markets are fair because each individual is paying the price which is equal to the benefit from the (last unit of) commodity. Probably, our feeling that markets are fair relies on these two aspects of the competitive equilibrium allocation and it is a kind of miracle that the markets in an economy with pure private goods possess these two properties simultaneously. As we will see later, this miracle does not happen when there are public goods (externalities in general).[17] We take the no-envy principle as the primary criterion for fairness in this paper because it is practically impossible to know the preferences (and thus benefits from commodities) of other individuals. See Yamashige (1994) for more discussion on this point of view. □

3 No-Envy Criterion for Government Policies

In this section, we introduce a government in the exchange economy above and define the concept of weakly envy-free government policies. Then we compare this equity criterion with other well known equity criteria and the benefit principle for government policies.

3.1 Preliminaries

As before, there are m commodities. Let $\{1, 2, \ldots, \ell, \ell+1, \ldots, m\}$ be a set of commodities. We assume that the first ℓ commodities are publicly provided but also can be individually provided and purchased by each agent. (For example, 'defense' is not only publicly provided but also privately provided.) Each publicly provided good is assumed to be pure, i.e., equally consumed by each individual. (We see some implications of relaxing this assumption later.)

We assume that a government policy can be divided into two parts: supply of public goods (policies which directly affect utility functions) and the taxation (policies which affect budget

[17] Sato (1985, 1987) elaborated on the notions of fairness based on the benefit principle.

constraints).[18] Let $Y \subset \mathbf{R}^\ell$ ($\ell \leq m$) be a space of public goods and let T be an abstract space of tax policies for each individual. By letting $Z \subset \mathbf{R}^{m-\ell}$ be a space of purely private goods, we have the commodity space $X = Y \times Z$. Since the utility from publicly provided goods and publicly purchased goods may be different for each commodity $\{1, \ldots, \ell\}$, it is useful to define utility function for each vector of public goods. Let $u_i : X \times Y \to \mathbf{R}$ be a utility function of an individual $i \in N$ such that $u_i(x; g)$ represents the utility of an individual $i \in N$ when the commodity bundle is $x \in X$ and the level of public goods is $g \in Y$. Let $\omega_i \in X$ be an endowment of $i \in N$.

Let $\Gamma = Y \times T^n$ be a space of possible policies. Let $G = (g, t_1, \ldots, t_n) \in \Gamma$ be a generic notation for a government policy. For each $i \in N$, by $t_i = 0$ we mean that there is no tax policy for i. A budget correspondence of each individual is given by $B : \Delta \times X \times T \longrightarrow\!\!\!\!\to X$ such that $B(q, \omega_i, t_i) \subset X$ represents the set of affordable bundles for individual $i \in N$ when the market price is $q \in \Delta$, endowment is $\omega_i \in X$ and the tax policy to this individual is $t_i \in T$.

In the following, we are only interested in the competitive equilibrium allocation mechanism under government policies. We consider an assignment rule which is dependent on an individual's name, endowment and government policy. Namely, the assignment function of $i \in N$ under a government policy $G \in \Gamma$ is a mapping $\tilde{x}_i^G : X \times T \times \mathbf{E} \to X$ such that $\tilde{x}_i^G(\omega_i, t_i; e)$ represents the commodity bundle of individual $i \in N$ in an economy $e \in \mathbf{E}$ under a government policy $G \in \Gamma$ when his/her endowment is ω_i and the tax policy on him/her is t_i. Given a collection of assignment functions $\{\tilde{x}_i^G\}_{i \in N}$, a mapping $\tilde{x}^G : \mathbf{E} \to X^n$ defined by $\tilde{x}^G(e) = (\tilde{x}_1^G(\omega_1, t_1; e), \ldots, \tilde{x}_n^G(\omega_n, t_n; e))$ for each $e \in \mathbf{E}$ is called an allocation mechanism under a government policy G if it satisfies the resource constraints: $\sum_{i \in N} \tilde{x}_{ik}^G(\omega_i, t_i; e) + g_k \leq \sum_{i \in N} \omega_{ik}$ for each $k \in \{1, \ldots, \ell\}$ and $\sum_{i \in N} \tilde{x}_{ik}^G(\omega_i, t_i; e) \leq \sum_{i \in N} \omega_{ik}$ for each $k \in \{\ell + 1, \ldots, m\}$.

Definition 8 *An allocation mechanism $\tilde{x}^G$ is said to be a competitive equilibrium allocation mechanism under a government policy $G \in \Gamma$ if and only if for any economy $e \in \mathbf{E}$ there exists a price system $q_e^G \in \Delta$ such that $\tilde{x}_i^G(\omega_i, t_i; e) \in \arg\max_{x \in B(q_e^G, \omega_i, t_i)} u_i(x; g)$ for each $i \in N$.*

[18] Even though the government's regulations are not explicitly considered, some form of regulations may be analyzed as long as they have the similar effects as the policies considered here.

3.2 No-Envy Criterion

Fairness of a government policy G can be judged by the properties of the allocation mechanism $\tilde{x}^G : \mathbf{E} \to X^n$. In this subsection, we propose our equity criterion (no-envy criterion) for government policies, which is based on the observations on the competitive equilibrium allocation mechanism studied in the previous section.

Definition 9 *A government policy G in e is said to be weakly posterior-envy-free if and only if $\tilde{x}^G(e)$ is weakly posterior-envy-free, i.e., $\omega_i \geq \omega_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_j^G(\omega_j, t_j; e); g)$.*

Definition 10 *A government policy G in e is said to be weakly prior-envy-free if and only if $\tilde{x}^G(e)$ is weakly prior-envy-free, i.e., $\omega_i \geq \omega_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$.*

Definition 11 *A government policy G in an economy e is said to be weakly envy-free if and only if it is both weakly prior-envy-free and weakly posterior-envy-free.*

We first show that the posterior notion is weaker than the prior notion. The proof is essentially the same as the one for Lemma 1.

Lemma 2 *Every weakly prior-envy-free government policy is weakly posterior-envy-free.*

Proof. Let $\tilde{x}^G$ be weakly envy-free. Let $\omega_i \geq \omega_j$ and assume that $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$. Then, $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g) = \max_{x \in B(q_e^G, \omega_j, t_j)} u_i(x; g) \geq u_i(\tilde{x}_j^G(\omega_j, t_j; e); g)$, where the last inequality is due to the fact that $\tilde{x}_j^G(\omega_j, t_j; e) \in B(q_e^G, \omega_j, t_j)$. Namely, the no-envy criterion in the posterior sense is indeed weaker. ∎

The following example shows that the weaker version is too weak to give us a satisfactory sense of equity in the presence of a government policy.

Example 5. Let $X = \mathbf{R}^2$ and $Y = \mathbf{R}$. Suppose $\omega_i = \omega_j = (1, 1)$ and that a government deprives half of the endowment of individual i as a tax and imposes no tax on j.[19] Suppose

[19] In the following examples, we often assume for simplicity that a change in tax policies for a small number of individuals does not change the equilibrium price, which will be justified in large economies implicitly assumed in this paper.

$g = 0$. Then a competitive equilibrium allocation under G may be determined as in the left diagram of Figure 1. The figure shows that this is a weakly posterior-envy-free government policy because both are satisfied with their own choice and feel no envy to the choice of the other under G. This policy, however, does not seem to be "fair". The policy is not weakly prior-envy-free because if i is given in a position of j (i.e., if there is no tax) then she could have enjoyed higher utility than she does now. In this example, it seems that the policy is not fair because the tax policy is not "symmetric" and thus we can still use the no-envy criterion in the posterior sense by adding a requirement that tax must be symmetric (or anonymous). The following example shows that this is not yet satisfactory. Assume that tax system is to triple the endowment if it is smaller than $(1, 1)$. Suppose $\omega_i = (1, 1)$ and $\omega_j = (\frac{1}{2}, \frac{1}{2})$. Under the tax policy, the final allocation may be the one in the right diagram of Figure 1 which is weakly posterior-envy-free. But again this does not look like a fair policy. Especially, i would strongly feel that the policy is unfair. Needless to say, this is not weakly prior-envy-free because i whose endowment is larger than that of j wishes that his endowment were ω_j. $\square$.

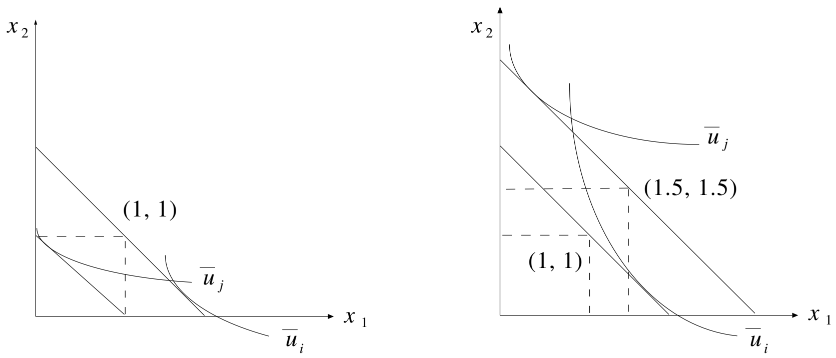

Figure 1: Diagrams for Example 5

Based on these observations and Proposition 1, we propose our no-envy criterion as follows.[20]

[20] Needless to say, the stronger requirement can be given by the following criterion. *A government policy G is said to be strongly envy-free if and only if $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$, for all $i \in N$ and $j \neq i$.* One may want to impose this criterion on government policies but we think it is too strong because by Proposition 2 we know that it essentially requires the equal endowment for all individuals.

Definition 12 *A government policy G in an economy e is said to satisfy no-envy criterion, if and only if* $\omega_i \geq \omega_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$.

Namely, a government policy G is said to satisfy the no-envy equity criterion if and only if it is weakly envy-free. Notice that the criterion does not require interpersonal comparability nor the cardinality of preferences. We take this criterion as the minimum one that any government policy must satisfy. It seems worth pointing out that it is *not* necessary that $\omega_i = \omega_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$ nor $\omega_i > \omega_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) > u_i(x_i^G(\omega_j, t_j; e); g)$. What we can say is that under a government policy any individual should not envy other individuals whose endowment is the same or smaller.

A natural question is about its relationship with other equity criteria for government policies. Although the no-envy criterion does not require the interpersonal comparison of welfare, to study the relation we assume that the utility functions below are defined to be interpersonally comparable. The next table which is based on the following proposition will be later used to discuss the relation between the no-envy criterion and other equity criteria.

Table 1: No-Envy Criterion

Initial Condition		Required Condition
Endowment	Utility Function	
$\omega_i = \omega_j$	$u_i = u_j$	$u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = u_j(\tilde{x}_j^G(\omega_j, t_j; e); g)$
$\omega_i = \omega_j$	$u_i \neq u_j$	$u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$ & $u_j(\tilde{x}_j^G(\omega_j, t_j; e); g) \geq u_j(\tilde{x}_j^G(\omega_i, t_i; e); g)$
$\omega_i > \omega_j$	$u_i = u_j$	$u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_j(\tilde{x}_j^G(\omega_j, t_j; e); g)$
$\omega_i > \omega_j$	$u_i \neq u_j$	$u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$

Proposition 4 *Under weakly envy-free policies,*

(1) $\omega_i = \omega_j$ & $u_i = u_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = u_j(\tilde{x}_j^G(\omega_j, t_j; e); g)$.

(2) $\omega_i = \omega_j$ & $u_i \neq u_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$ &

$$u_j(\tilde{x}_j^G(\omega_j, t_j; e); g) \geq u_j(\tilde{x}_j^G(\omega_i, t_i; e); g).$$

(3) $\omega_i > \omega_j$ & $u_i = u_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_j(\tilde{x}_j^G(\omega_j, t_j; e); g)$.

(4) $\omega_i > \omega_j$ & $u_i \neq u_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$.

Proof. Let G be a weakly envy-free government policy.

(1) Let $\omega_i = \omega_j$ and $u_i = u_j$. By assumption, we have $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_j^G(\omega_j, t_j; e); g)$ and $u_j(\tilde{x}_j^G(\omega_j, t_j; e); g) \geq u_j(\tilde{x}_j^G(\omega_i, t_i; e); g)$. Hence, $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = u_j(\tilde{x}_j^G(\omega_j, t_j; e); g)$.

(2) This follows from the definition.

(3) By definition, $\omega_i > \omega_j$ implies $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_j^G(\omega_j, t_j; e); g) = u_j(\tilde{x}_j^G(\omega_j, t_j; e); g)$.

(4) In this case $\omega_i > \omega_j$ implies $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$ and there is no condition for the utility of j. ∎

Careful readers may have already recognized that the no-envy criterion does not impose any restrictions on the policies for individuals whose endowments are not comparable. This weakness can be strengthened when we define "income-based no-envy equity criterion". The criterion, however, entails some conceptual difficulty on the choice of the price system to calculate income. Hence, even though the difficulty itself also leads us to an interesting policy implication, we differ the discussion to the appendix.

3.3 Relationship with Traditional Equity Criterion (Horizontal Equity)

In the field of public economics, two equity criteria for government policies are well known.[21] The first principle is that the government should treat "equals" equally, which is known as horizontal equity. Although there is an argument about how "equals" should be defined, there will be little controversy for regarding the two individuals with the same endowment and the same preference as "equals". Hence, we adopt the following definition of horizontal equity.

[21] The following two equity criteria are usually discussed in the context of "ability to pay" approach where individuals are usually assumed to have the same utility function. We do not adopt this assumption in the next two subsections. See Remark 2 at the end of subsection 3.4 for more discussion on this issue.

Definition 13 *A government policy is said to satisfy horizontal equity if and only if*

$$\omega_i = \omega_j \ \& \ u_i = u_j \ \Rightarrow \ u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = u_j(\tilde{x}_j^G(\omega_j, t_j; e); g).$$

Although it is an intuitive way to state the notion of horizontal equity, this definition is too weak in terms of its restriction on government policies because it only compares two individuals who are exactly the same. Hence, the principle of vertical equity which gives us conditions that government policies must satisfy for the cases in which individuals are different is expected to be strong enough to guarantee government policies to be fair.[22] Unfortunately, there has been little consensus about the definition of vertical equity. The only principle accepted seems to be the one implied by the definition of horizontal equity, which we now call the weakest definition of vertical equity.[23] We discuss stronger principles of vertical equity in the next subsection.

Definition 14 *A government policy is said to satisfy the weakest definition of vertical equity if and only if* $\omega_i \neq \omega_j$ *and/or* $u_i \neq u_j$ *need not imply* $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = u_j(\tilde{x}_j^G(\omega_j, t_j; e); g)$.

Needless to say, this weakest version does not impose any further restrictions on policies than the horizontal equity does. We call the these two equity criteria the traditional equity criterion. The conditions imposed by the traditional criterion are summarized in Table 2.

Table 2: Traditional Criterion

Initial Condition		Required Condition
Endowment	Utility Function	
$\omega_i = \omega_j$	$u_i = u_j$	Horizontal Equity: $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = u_j(\tilde{x}_j^G(\omega_j, t_j; e); g)$
$\omega_i = \omega_j$	$u_i \neq u_j$	Vertical Equity (Weakest Definition): no restriction
$\omega_i > \omega_j$	$u_i = u_j$	
$\omega_i > \omega_j$	$u_i \neq u_j$	

By comparing Table 2 with Table 1, we have the following observations about the relationship between the no-envy equity criterion and the traditional criterion.

[22] See Musgrave (1990) for a discussion on the relationship between horizontal equity and vertical equity.
[23] For example, Musgrave (1990) expresses it as "appropriate differentiation among unequals".

(i) No-envy criterion is consistent with the horizontal equity in the sense that the no-envy criterion satisfies and is satisfied under the horizontal equity because the criterion imposes the same condition as the one under the horizontal equity.

(ii) Traditional equity criterion does not guarantee that government policies satisfy the no-envy equity criterion. This is because we have stronger conditions (restrictions) under the no-envy equity criterion for the cases when $\omega_i \neq \omega_j$ and/or $u_i \neq u_j$.

The first result (i) is an important result for us because by this result we may regard our no-envy principle as the generalization of the principle of horizontal equity. It seems that the reason for the popularity of the principle of horizontal equity is indeed that it prevents the occurrence of envy among equals, i.e., the spirit of the principle of horizontal equity is that government policy should not cause envies among similar individuals. It is in this sense that we may regard our no-envy principle as the generalization of the principle of horizontal equity. As for (ii), it is enough to point out that two policies in Example 5 also indicate that the policies given there violate the no-envy criterion but not the traditional equity criterion.

3.4 Relationship with Other Equity Criteria (Vertical Equity)

Motivated by the observation that the traditional definition of horizontal equity is too weak, Feldstein (1976) introduced a new definition for horizontal equity in his discussion of tax-reform. As we will discuss below, the definition is very close to the definition for a well known principle of vertical equity, "equal (absolute) sacrifice". In this section, we show that Feldstein's criterion and the principles of "equal sacrifice" need not satisfy the no-envy criterion.

We start with Feldstein's equity criterion. Even though Feldstein was interested in the equity criterion for taxation, his definition can be naturally generalized for government policies.[24] Let $G = 0$ mean $g = 0$ and $t_i = 0$ (no tax policy) for all $i \in N$.

[24] In his words, the definition is given as follows. "*If two individuals would be equally well off (have the same utility level) in the absence of taxation, they should also be equally well off if there is a tax.* More generally, the introduction of a tax should not alter the ordering of individuals by utility level [Feldstein (1976, p.83)]."

Definition 15 *A government policy satisfies Feldstein's equity criterion if and only if*

$$u_i(\tilde{x}_i^0(\omega_i,0;e);0) = u_j(\tilde{x}_j^0(\omega_j,0;e);0) \quad \Rightarrow \quad u_i(\tilde{x}_i^G(\omega_i,t_i;e);g) = u_j(\tilde{x}_j^G(\omega_j,t_j;e);g) \ \&$$
$$u_i(\tilde{x}_i^0(\omega_i,0;e);0) > u_j(\tilde{x}_j^0(\omega_j,0;e);0) \quad \Rightarrow \quad u_i(\tilde{x}_i^G(\omega_i,t_i;e);g) > u_j(\tilde{x}_j^G(\omega_j,t_j;e);g).$$

A nice property of this definition is that it is indeed a stronger notion of the horizontal equity in the sense that it requires more restrictions on government policies than the traditional definition of horizontal equity. This is easily verified by observing that if $(\omega_i, u_i) = (\omega_j, u_j)$ then $u_i(\tilde{x}_i^0(\omega_i,0;e);0) = u_j(\tilde{x}_j^0(\omega_j,0;e);0)$ and thus if G satisfies Feldstein's equity criterion then $u_i(\tilde{x}_i^G(\omega_i,t_i;e);g) = u_j(\tilde{x}_j^G(\omega_j,t_j;e);g)$ must be satisfied, i.e., it satisfies the traditional equity criterion. On the other hand, there are many cases under which $(\omega_i, u_i) \neq (\omega_j, u_j)$ but $u_i(\tilde{x}_i^0(\omega_i,0;e);0) = u_j(\tilde{x}_j^0(\omega_i,0;e);0)$, i.e., Feldstein's criterion puts more restrictions on policies. Namely, it is stronger than the traditional one. We want to show, however, that this equity criterion need not satisfy the no-envy conditions. The problem is basically due to its requirement of interpersonal compatibility of welfare in the definition of equity.

Example 6. Let $X = \mathbf{R}^2$ and $Y = \mathbf{R}$. Consider two individuals with utility function $u_i(x;g) = g + 2(x_1)^{\frac{1}{4}}(x_2)^{\frac{1}{4}}$ and $u_j(x;g) = g + (x_1)^{\frac{1}{4}}(x_2)^{\frac{1}{4}} + 2$. Let $\omega_i = \omega_j = (4,4)$ and $q_{e1} = q_{e2} = \frac{1}{2}$. Then, it is easy to calculate $\tilde{x}_i^0(\omega_i,0;e) = \tilde{x}_j^0(\omega_j,0;e) = (4,4)$. Hence, $u_i(\tilde{x}_i^0(\omega_i,0;e);0) = 4 = u_j(\tilde{x}_j^0(\omega_j,0;e);0)$. Consider a tax policy which makes the endowments as $\omega_i = (1,1)$ and $\omega_j = (0,0)$. Let g be arbitrary. When there is no change in prices, $\tilde{x}_i^G(\omega_i,t_i;e) = (1,1)$ and $\tilde{x}_j^G(\omega_j,t_j;e) = (0,0)$. Then, $u_i(\tilde{x}_i^G(\omega_i,t_j;e);g) = 2+g = u_j(\tilde{x}_j^G(\omega_j,t_j;e);g)$. Namely, this government policy satisfies Feldstein's equity criterion. The policy, however, does not satisfy the no-envy criterion since $u_j(\tilde{x}_j^G(\omega_j,t_j;e);g) = 2+g < 3+g = u_j(\tilde{x}_i^G(\omega_i,t_i;e);g)$. The example when $g = 0$ may be illustrated in the following Figure 2. In the initial state (x^0) both had the utility level 4. Under a policy which satisfies the Feldstein's equity criterion, both have the utility level 2. The policy, however, is not fair from i's point of view since it makes i envy j in the sense that j's bundle after the policy $(\bar{x})$ is better than i's bundle (0). □

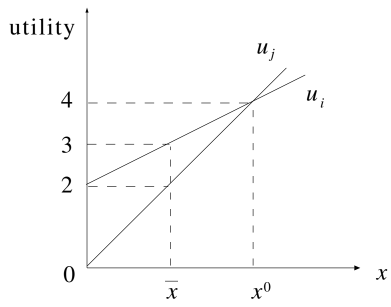

Figure 2: Diagram for Example 6

The policy in the example above is not fair because one individual (i in our case) feels that other individuals (j in our case) are treated favorably. Some people may still feel that the policy is fair. A possible reason is that it requires "equal sacrifice" to each individual. In this sense, Feldstein's criterion may be seen as the principle of vertical equity rather than the horizontal equity. In other words, the principle of "equal sacrifice" known as a criterion for vertical equity (cf. Musgrave [1990] and Mill [1848]) need not satisfy the no-envy criterion. Hence, the example above shows that the principle of "equal sacrifice" does not have to guarantee the no-envy conditions.

It turns out that the converse is not true either. Namely, there is no relation between the two definitions. To see it, let $\omega_i = \omega_j$. Suppose $\tilde{x}_i^G(\omega_i, t_i; e) = \tilde{x}_j^G(\omega_j, t_j; e)$ under a government policy G. Then it satisfies the no-envy criterion because $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$. But if u_i is not equal to u_j, $\tilde{x}_i^G(\omega_i, t_i; e) = \tilde{x}_j^G(\omega_j, t_j; e)$ does not guarantee $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = u_j(\tilde{x}_j^G(\omega_j, t_j; e); g)$ in general. Namely, a weakly envy-free policy need not satisfy Feldstein's equity criterion.

The example and the discussion above also indicate that another well known criterion for vertical equity, "equal marginal sacrifice", need not be consistent with no-envy criterion.[25] To show that a government policy which satisfies "equal marginal sacrifice" criterion need not satisfy the no-envy criterion, it is enough to modify Example 6 as follows.

[25]It is easy to see from the discussion just above that the no-envy criterion does not have to require equal marginal sacrifice.

Example 7. Consider an economy described in Example 6. The indirect utility function under $g = 0$ is calculated to be $v_i(q_1, q_2, y_i) = (4q_1q_2)^{-\frac{1}{4}}(2y_i^{\frac{1}{2}})$ and $v_j(q_1, q_2, y_j) = (4q_1q_2)^{-\frac{1}{4}}(y_j^{\frac{1}{2}})$, where $y_i = q_1\omega_{i1} + q_2\omega_{i2}$ and $y_j = q_1\omega_{j1} + q_2\omega_{j2}$. Consider a tax policy which makes the endowments as $\omega_i = (1, 1)$ and $\omega_j = (\frac{1}{2}, \frac{1}{2})$. When there is no change in prices, $\tilde{x}_i^G(\omega_i, t_i; e) = (1, 1)$ and $\tilde{x}_j^G(\omega_j, t_j; e) = (\frac{1}{2}, \frac{1}{2})$, and

$$\frac{\partial v_i(q_1, q_2, y_i)}{\partial y_i} = 1 = \frac{\partial v_j(q_1, q_2, y_j)}{\partial y_j}$$

Namely, under this tax policy marginal sacrifice is the same. The policy, however, does not satisfy the no-envy criterion because $u_j(\tilde{x}_j^G(\omega_j, t_j; e); 0) = 2 + \frac{1}{\sqrt{2}} < 3 = u_j(\tilde{x}_j^G(\omega_i, t_i; e); 0)$. □

The propositions and the discussion above show that the no-envy equity criterion is not necessarily guaranteed under well known equity criteria for vertical equity. Hence, the no-envy equity criterion can be seen as an independent equity criterion which is consistent with the traditional equity criterion and also provides the minimum conditions that any stronger notion of vertical equity must satisfy.

Remark 2. Traditionally, the two principles "horizontal equity" and "vertical equity" are discussed in the context of "ability to pay" approach which requires individuals' tax contribution depend on the their ability to pay. Since the ability is usually measured by income, the two principles are usually discussed under the implicit assumption that each taxpayer has the same utility function. Since this assumption does not generally hold in reality, we discussed the two equity criteria without such an assumption. It may be worth pointing out, however, that if we adopt the assumption then it is easy to check form Proposition 4 that both horizontal equity and vertical equity principles satisfy our no-envy criterion. Namely, the horizontal equity criterion and the vertical equity criteria in the traditional interpretation satisfies the no-envy criterion, which may be seen as an evidence that the no-envy criterion is a natural equity criterion. Our point in this subsection is that without the assumption the vertical equity criteria need not satisfy the no-envy equity criterion. □

3.5 Relationship with Benefit Principle (Lindahl Tax)

It seems worth discussing the implication of adopting no-envy equity criterion as a concept of fairness in relation to the benefit tax (Lindahl tax) [See, e.g., Wicksell (1896), Lindahl (1919) and Atkinson-Stiglitz (1980)]. As we have already pointed out in Section 1, no-envy criterion can be seen as an equity criterion which allows each individual, who do not necessarily know the preferences of others, to judge the fairness of allocations from their point of view (using his/her own utility function). Put differently, the no-envy criterion may not be a suitable concept of fairness if everyone knows the preferences of others.[26] To see this, let us consider the following example. (A more concrete example is rigorously discussed in Example 8 below.)

Suppose that you and your close friend have the same endowment but have different tastes over pure public goods. You do not appreciate the public goods at all but you know that your friend appreciates them very much. If you pay the same amount of tax, the final allocation is envy-free (notice that the public goods are pure, i.e., equally consumed). If you pay the Lindahl taxes, which requires you to pay the marginal utility from the public goods for each unit of public goods, then the final allocation is not likely to be envy-free because the benefits from the public goods are different. Which tax policy would be called fair? Since you know that your friend appreciates public goods much more than you do, you may feel that he should pay more for the public goods than you do, i.e., Lindahl tax (or benefit tax) seems to be fair to you. Hence, if each individual knows the preferences of others, the envy-free equity criterion may not be an appropriate notion of equity.

Our point of view in this paper is that, in economies where it is practically impossible for individuals and governments to know the preferences of all individuals, a concept of fairness which allows each individual to make judgment based on their preferences will be more likely to be accepted by people [cf. Yamashige (1994)]. Hence, even if you feel that your friend should pay more taxes, you would probably agree to pay the same tax as your friend as an

[26] For example, the arguments by Wicksell (1896) and Lindahl (1919) seem to be based on the implicit assumption that there are certain ways to extract information about individuals' preferences over the public goods.

individual who knows that it is impossible for governments to know the preferences of all the individuals.[27] The next example indeed shows that the Lindahl tax system need not satisfy the no-envy equity criterion.

Example 8. Consider an economy with two individuals and one commodity whose price is equal to 1. The utility functions are given by $u_i(x;g) = x^{\frac{1}{4}}g^{\frac{3}{4}}$ and $u_j(x;g) = x^{\frac{3}{4}}g^{\frac{1}{4}}$ and each has one unit of endowment. Namely, i appreciates the public good more than j. First, a Lindahl tax equilibrium is a vector $(x_i, x_j; g, p_i, p_j)$ which is a solution to the problems

$$\max_{(x_i;g)} u_i(x_i;g) \quad s.t. \;\; x_i + p_i g = 1 \;\text{ and }\; \max_{(x_j;g)} u_j(x_j;g) \quad s.t. \;\; x_j + p_j g = 1$$

with the constraints $p_i g + p_j g = g$ and $x_i + x_j + g = 2$. A simple calculation shows that

$$(x_i, x_j; g, p_i, p_j) = (\frac{1}{4}, \frac{3}{4}, 1, \frac{3}{4}, \frac{1}{4}),$$

i.e., individual i who appreciates the public good more pays more $p_i = \frac{3}{4} > \frac{1}{4} = p_j$. The Lindahl tax system does not satisfy the no-envy equity criterion because $u_i(\frac{1}{4};1) = (\frac{1}{4})^{\frac{1}{4}} < (\frac{3}{4})^{\frac{1}{4}} = u_i(\frac{3}{4};1)$, i.e., i envies j. $\square$

In the theory of public economics, it seems that the Lindahl tax has been perceived as the ideal tax but the difficult one to implement because governments do not have information about the preferences of individuals. The discussion above indicates that governments need not have the pessimistic sentiment for not being able to use the Lindahl tax because it is not a fair tax system especially in economies where individuals judge fairness using their own utility functions (due to the lack of their knowledge about the preferences of others). To understand this point more vividly, suppose that you know that you and your neighbor have a similar income and preference over private goods but you do not know his/her preference over public goods. Suppose that a government knows individual preferences and collects the benefit tax for a pure public good, say “defense'. If you observe that the neighbor paid only half of the

[27]This conjecture would be more likely to be accepted especially if you and your friend could have the same influence on the decision for public goods.

tax you paid you would probably feel, unless you have complete trust on the government, that the tax policy is unfair because your neighbor can now have more private goods that you also wanted. Hence, in an economy where it is practically impossible to know the preferences of others, Lindahl tax is no more a fair tax.

4 Universal No-Envy Criterion

Although the results in the previous section allow us to know the relationship between the no-envy criterion and other equity criteria, it does not give us much insights on the properties of the no-envy criterion. In the following, we show that the no-envy criterion can be characterized by a much simpler set of conditions (restrictions) on government policies.

In designing a government policy, the requirement that the government must know the preferences of all individuals is strong. Furthermore, since the society is changing over time, this requirement is indeed costly. The task may be actually impossible considering the fact that individuals can lie about their preferences. Hence, in this section we require that government policies should satisfy the no-envy equity criterion in any possible economy (profile of preferences and endowments). The requirement will be called a universal no-envy equity criterion and the conditions that we obtain will be referred to as preference-independent conditions. This consideration leads us to a necessary and sufficient condition for government policies, which supports income tax system and commodity tax as fair tax systems.

4.1 Preference-Independent Conditions and Universal No-Envy Criterion

We first show a sufficient condition for the no-envy criterion in terms of the effects of government policies on budget sets. Since identifying budget sets is much easier than identifying preferences, it will serve as a good guideline for policy planners. Notice that even though the government seems to be forced to calculate the equilibrium price q_e^G before implementing a policy in the following criterion, this is not necessary because if the condition is true under

$q_e^G \in \Delta$ then it must be true for all $q \in \Delta$, i.e., q_e^G is irrelevant (can be arbitrary). The result says that if the tax system is set up to guarantee larger budget sets for individuals with larger endowments, any government policy satisfies the no envy criterion.

Proposition 5 *A government policy G in an economy $e \in \mathbf{E}$ satisfies no-envy criterion if*

$$\begin{aligned} \omega_i = \omega_j \quad &\Rightarrow \quad B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_j, t_j) \text{ and} \\ \omega_i > \omega_j \quad &\Rightarrow \quad B(q_e^G, \omega_i, t_i) \supseteq B(q_e^G, \omega_j, t_j). \end{aligned}$$

Proof. Let $\omega_i = \omega_j$ and $B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_j, t_j)$. Let g be arbitrary. If $\tilde{x}_i^G(\omega_i, t_i; e)$ is the optimal commodity bundle in the competitive equilibrium under the policy G, then $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(x; g)$ for all $x \in B(q_e^G, \omega_i, t_i)$. Since $\tilde{x}_i^G(\omega_j, t_j; e) \in B(q_e^G, \omega_j, t_j) = B(q_e^G, \omega_i, t_i)$, we must have $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$, i.e., G is weakly envy-free. Let $\omega_i > \omega_j$ and $B(q_e^G, \omega_i, t_i) \supseteq B(q_e^G, \omega_j, t_j)$. If $B(q_e^G, \omega_i, t_i) \supset B(q_e^G, \omega_j, t_j)$, $u_i(\tilde{x}_i^G(\omega_i, t_j; e); g) \geq u_i(x; g)$ for all $x \in B(q_e^G, \omega_j, t_j)$. Since $\tilde{x}_i^G(\omega_j, t_j; e) \in B(q_e^G, \omega_j, t_j)$, we have $u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$, i.e., G is weakly envy-free. ∎

The condition is not generally a necessary condition in a given economy. In Figure 3, the allocation satisfies the envy-free criterion but the budget sets for i and j are different. It is,

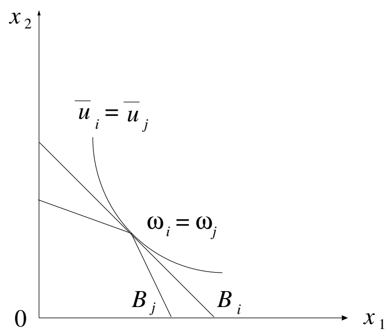

Figure 3: Preference-Independent Condition

however, indeed the necessary condition if we require a government policy to be universal, which is defined as follows.

Definition 16 *A government policy $G \in \Gamma$ is said to satisfy universal no-envy criterion if and only if the competitive equilibrium allocation under G satisfies the no-envy criterion for any economy e, i.e., if and only if $\tilde{x}^G(e)$ satisfies the no-envy criterion for any $e \in \mathbf{E}$.*

This definition reflects the idea that government policies must be designed so that they are robust for any change in tastes and endowments in the economy. We may argue that this requirement is necessary because it is practically impossible for governments to know the tastes of individuals. Since the competitive equilibrium allocation mechanisms without a government satisfy this property (Proposition 1), it would be natural to study such a class of government policies. The following proposition is an important and useful characterization of the universal no-envy equity criterion.

Proposition 6 *A government policy G satisfies the universal no-envy criterion if and only if*

$$\begin{aligned} \omega_i = \omega_j \quad &\Rightarrow \quad B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_j, t_j) \text{ and} \\ \omega_i > \omega_j \quad &\Rightarrow \quad B(q_e^G, \omega_i, t_i) \supseteq B(q_e^G, \omega_j, t_j) \text{ for all } e \in \mathbf{E}. \end{aligned}$$

Proof. By Proposition 5, we have only to show the necessity. Suppose G satisfies the universal no-envy criterion, i.e., $\omega_i \geq \omega_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g)$ for any economy. Let $\omega_i \geq \omega_j$ and $B(q_e^G, \omega_i, t_i) \not\supseteq B(q_e^G, \omega_j, t_j)$. Then there exists $\bar{x} \in B(q_e^G, \omega_j, t_j)$ such that $\bar{x} \notin B(q_e^G, \omega_i, t_i)$. Define the utility functions by $u_i(x; g) = u_j(x; g) = -\sum_{k=1}^m (x_k - \bar{x}_k)^2$ for all $g \in Y$. Then,

$$u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) = \max_{x \in B(q_e^G, \omega_i, t_i)} u_i(x; g) < 0 = \max_{x \in B(q_e^G, \omega_j, t_j)} u_i(x; g) = u_i(\tilde{x}_i^G(\omega_j, t_j; e); g),$$

which contradicts that G satisfies the universal no-envy criterion, i.e., $\omega_i \geq \omega_j \Rightarrow B(q_e^G, \omega_i, t_i) \supseteq B(q_e^G, \omega_j, t_j)$. Now, if $\omega_i = \omega_j$ then $B(q_e^G, \omega_i, t_i) \supseteq B(q_e^G, \omega_j, t_j)$ and $B(q_e^G, \omega_j, t_j) \subseteq B(q_e^G, \omega_i, t_i)$ and thus we have $B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_j, t_j)$. ∎

The result has an important implication for tax policies. We call a tax policy *anonymous* if t_i is the same for all $i \in N$.

Corollary 1 *A government policy satisfies the universal no-envy criterion only if the tax policy is anonymous, i.e., t_i is the same for all $i \in N$.*

Proof. Suppose $t_i \neq t_j$. Since G satisfies the universal no-envy criterion, we can assume $\omega_i = \omega_j$ under which $B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_j, t_j)$ must hold (Proposition 6). Since $\omega_i = \omega_j$, we have $B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_j, t_j)$ if and only if $t_i = t_j$, which is a contradiction. ∎

Table 3 summarizes the conditions imposed by the universal no-envy equity criterion. Notice especially that the conditions are satisfied under the standard income tax system and

Table 3: Universal No-Envy Criterion

Initial Condition	Required Condition
$\omega_i = \omega_j$	$B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_j, t_j)$
$\omega_i > \omega_j$	$B(q_e^G, \omega_i, t_i) \supseteq B(q_e^G, \omega_j, t_j)$

commodity tax system.[28] Since the conditions are observed by many governments, we think that the no-envy criterion is indeed adopted by many governments.[29] Even though the result seems to suggest that governments do not have to know the preferences of individuals, there are two caveats. First, from the viewpoint of equity, governments need not know individual preferences but from the viewpoint of efficiency, this is not true any more. Secondly, the preference-independent condition will not be obtained (and thus information on individual preferences will be required even from the viewpoint of equity) when the public goods are not pure. We discuss each issue in the following subsections.

[28] As we will discuss in Appendix, the stronger equity criterion, income-based equity criterion, may not be satisfied under the actual income tax system and commodity tax system.

[29] Traditionally, income and consumption have been considered as tax bases in the "ability to pay" approach because they can be seen as the indices of individual ability. In the ability-to-pay approach it is required that the individuals with more ability should pay more taxes. It is not clear, however, from which justice theory we can derive such a principle. It is our hope that we can answer this question in the near future based on the envy-free principle for fairness although the no-envy criterion proposed in this paper is so weak that there is no requirement that more able persons should pay more taxes.

4.2 Universal No-Envy Equity Criterion and Pareto Optimality

It was shown in the last section that the Lindahl tax, even though it is well known to attain Pareto optimality, does not necessarily satisfy the no-envy equity criterion. Furthermore, the result in the last subsection shows that conditions for the universal no-envy equity criterion must be preference-independent implying that the Lindahl tax does not satisfy the no-envy criterion in general, unless the "benefit" from public goods increases as endowment increases. One may then ask if there exists a government policy which satisfies the no-envy equity criterion, especially the universal no-envy equity criterion, and the Pareto optimality. For this question, it is useful to point out that Foley (1967) showed that there exists a Pareto optimal supply of public goods and a taxation scheme which satisfy our no-envy equity criterion. We think that this result is important because it provides us a foundation for calculating the optimal supply of public goods and the optimal taxes which satisfy the no-envy equity criterion. Unfortunately, to find such a government policy, the government should also have the information about the individual preferences. In our notations, the theorem can be stated as follows.[30]

Theorem [Foley (1967, Theorem 3.29)]

Under conditions in Foley (1967, Theorem 3.29), there exist Pareto optimal policy $G = (g, t_1, \ldots, t_n)$ *and* $s \in \mathbf{R}$ *such that* $t_i = s\, q_e^G \cdot \omega_i$ *for all* $i \in N$ *and* $\sum_{i \in N} t_i = \sum_{k=1}^{\ell} q_{ek}^G g_k$.

Namely, it is shown that a Pareto optimal supply of public goods is possible under the proportional tax on each individual's income. Now it is easy to check that under the proportional income tax, $\omega_i = \omega_j$ implies $B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_j, t_j)$ and $\omega_i > \omega_j$ implies $B(q_e^G, \omega_i, t_i) \supseteq B(q_e^G, \omega_j, t_j)$ and thus, by Proposition 5, the government policy above satisfies the universal no-envy criterion.[31]

[30] Diamantaras (1992) recently showed the existence of a Pareto optimal envy-free allocation in an economy with public goods under milder assumptions.

[31] Foley (1967, p.76) showed that the competitive equilibrium allocation under such G is envy-free when the initial endowment is the same, which seems to be his original intention for introducing now famous envy-free equity criterion.

Example 9. Consider an economy in Example 8. We find a Foley's equilibrium $(x_i, x_j; g, s)$. The equilibrium can be obtained by solving the following maximization problem

$$\max_{(x_i,x_j;g)} u_i(x_i; g) + u_j(x_j; g) \quad s.t. \;\; x_i + x_j + g = 2$$

with the constraint $x_i = (1-s) \cdot 1$ and $x_j = (1-s) \cdot 1$ (proportional income tax). Another easy calculation shows that the Pareto optimality condition (Samuelson condition) is given by $3x_i + \frac{1}{3}x_j = g$ and the Foley's equilibrium is $(x_i, x_j; g, s) = (\frac{3}{8}, \frac{3}{8}, \frac{5}{4}, \frac{5}{8})$. (It is not a surprise that the Lindahl tax equilibrium satisfies the Samuelson condition). In Foley's equilibrium, each individual pays the same tax $(\frac{5}{8})$ because both have the same endowment. Notice that we need information about the preference of each individual and we cannot separate the decisions on taxation and public goods to find the optimal government policy. □

As Foley's theorem and the calculation above shows, under certain conditions as long as the public goods are pure ("uniform" in Foley's term), there is a Pareto optimal level of public goods and a tax system which also satisfy our no-envy equity criterion. As Foley himself suggests, in that class of government policies, there may be a progressive income tax system which is consistent with some stronger notion of (vertical) equity and attains Pareto optimality. The inquiry for this question seems to be an important direction that we should pursue to extend our analysis.

4.3 Universal No-Envy Criterion and Impure Public Goods

It is important to recognize that the preference-independent condition for the no-envy criterion was obtained because public goods are pure. We show that the condition is not sufficient nor necessary when public goods are not pure. The result implies that income tax system is in general not a fair tax system to finance impure public goods and suggests that we need to introduce the benefit tax for fairness. This case is important because the assumption that everyone uses the same amount of public goods is sometimes restrictive.[32]

[32]When we consider fairness among generations, even the pure public goods do not have a nature of equal consumption among individuals in different generations. Hence the consideration for the impure public goods

To analyze the problem, we assume that there exists a constraint function $\nu_i : Y \to Y$ such that $\nu_i(g) \in Y$ represents the amount of public goods an individual i uses and that this constraint is a result of some government policy, i.e., ν_i is part of a policy variable. For example, some government policy may be age-specific, location-specific, etc.[33] The utility function is now modified as $u_i(x; \nu_i(g))$. We do not have to change other structures (e.g., resource constraints) at all. An important consequence of this reformulation is that in this case we cannot characterize the universal no-envy criterion by the budget sets but we have to use the following definition. Notice that ν_i is replaced by ν_j in i's utility function to be compared because the constraint is individual-specific.

Definition 17 *A government policy G satisfies universal no-envy criterion, if and only if*

$$\omega_i \geq \omega_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); \nu_i(g)) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); \nu_j(g)) \; \forall e \in \mathbf{E}.$$

The following example shows that the preference-independent characterization of the universal no-envy criterion was possible because the public goods are assumed to be equally consumed by everyone.

Example 10. Consider two individuals who are exactly the same except for their constraint functions ν_i and ν_j. Suppose $X = \mathbf{R}^2$ and $Y = \mathbf{R}$, i.e., there are two commodities one of which is pure-private goods and the other is publicly (and also privately) provided. Suppose that the utility function is given by $u(x; \nu(g)) = x_1 + x_2 + \nu(g)$ for each i and j and the constraint functions are given by $\nu_i(g) = \frac{2}{3}g$ and $\nu_j(g) = \frac{1}{3}g$. Let $\omega_i = \omega_j = (1, 1)$. Assume $g = 3$ and that the tax on i and j is \$1. Let $q_{e1}^G = q_{e2}^G = 1$. Then the budget constraint is given by $x_1 + x_2 = 1$ for each individual. According to Proposition 6, this policy would satisfy the universal no-envy criterion if the constraint functions were the same. But it does not in this case because in any competitive equilibrium allocation under G, $u(x; \nu_i(g)) = 3 > 2 = u(x; \nu_j(g))$, i.e., j envies i

is useful to study intertemporally fair tax systems. As the result in this section suggests, the intertemporally fair tax system comes to have a flavor of benefit tax [Yamashige (1994)].

[33] It is very important that ν_i is determined by the government because otherwise, e.g., if it reflects individual decision over the use of public goods, the constraint functions should not be used.

despite the fact that they both had the same endowment and preference. Hence, Proposition 6 does not hold when public goods are not pure. To see a nature of the government policy which satisfies the no-envy criterion, suppose that the tax imposed on i is \$2. Then it satisfies the no-envy criterion because $u(x;\nu_i(g)) = 2 = u(x;\nu_j(g))$. Namely, by requiring higher tax on the individual who uses public goods more, the no-envy equity criterion was satisfied. □

The example above shows that the preference-independent condition given in Proposition 6 is no more sufficient nor necessary condition for the universal no-envy criterion. The example also suggests that the benefit tax must be introduced for each individual not to envy others who use public goods more. In general, however, if the usable levels of public goods are different among individuals then the government has to know the preferences of individuals to attain the no-envy criterion. Since knowing the preferences of individuals is difficult, if an impure public good can be supplied in the competitive market without any market failure, then it may be fair for governments to let it be traded in the markets because we know by Proposition 1 that the competitive equilibrium allocations are weakly envy-free.

5 Concluding Remarks

Everyone would agree that the issue of fairness is an important one but also a difficult one. Therefore, it is natural that we do not have yet a satisfactory equity criterion. In this paper, we proposed a weak equity criterion which is naturally guaranteed even in the competitive markets and requires that envy-structure in the initial states should not be overturned in the final states. We argued that the equity criterion has nice properties (no need for interpersonal comparison and cardinality of preferences) and is appealing especially in large economies where individuals do not know the preferences of others and thus should be allowed to judge equity based on their own preferences.

We have shown that the criterion is not necessarily guaranteed under various well known equity criteria for government policies proposed by Feldstein (1976), Mill (1848), Pigou (1947), and others. Since our criterion is consistent with the horizontal equity but is not guaranteed under other equity criteria, it may be seen as a generalization of the principle of horizontal equity which also provides a set of minimum conditions that any other equity criterion must satisfy. This view seems to be a natural one in the sense that the horizontal equity is accepted by people because if it is violated then someone will envy others and feel unfair, i.e., the spirit of the horizontal equity is to prevent the occurrence of envy among people. The results obtained for the universal no-envy equity criterion suggest that the criterion is indeed adopted by many governments, which may be natural because politicians are trying to reduce the constituents' complaints and design socially stable tax systems.

Although our results provide explanations for the present tax systems in many countries, such as the income tax to finance pure public goods and the benefit tax (user fees) for impure public goods, our criterion is very weak. It is so weak that the criterion may not be worth to be called a concept of fairness. For example, it does not impose any restriction on government policies which treat rich people favorably. We hope that our no-envy principle is accepted as a necessary condition for fairness and the stronger notions of equity, which may be truly called a concept of fairness, will be fruitfully developed based on this principle.

Appendix (Income-Based No-Envy Criterion)

It seems useful to strengthen the no-envy criterion by virtue of Proposition 3 in Section 2. Recall that $G = 0$ means that $g = 0$ and there is no tax policy.[34]

Definition 18 *A government policy G is said to satisfy income-based no-envy criterion if and only if*

$$q_e^0 \cdot \omega_i \geq q_e^0 \cdot \omega_j \Rightarrow u_i(\tilde{x}_i^G(\omega_i, t_i; e); g) \geq u_i(\tilde{x}_i^G(\omega_j, t_j; e); g).$$

[34] Notice that the following criterion is indeed a stronger than the endowment-based no-envy criterion. First, notice that given a price system q_e^0, $\omega_i \geq \omega_j$ implies $q_0^G \cdot \omega_i \geq q_e^0 \cdot \omega_j$ and thus if G satisfies the endowment-based no-envy criterion then it must satisfy income-based no-envy criterion. Next, notice that the income-based no-envy criterion compares every individual, whereas the endowment-based no-envy criterion compares the individuals whose endowments are comparable. Hence, the set of admissible government policies is much smaller than the set of weakly envy-free policies.

Since the competitive equilibrium satisfies the corresponding criterion (Proposition 3), it would be also justifiable to impose this condition for government policies in the sense the government should not change the envy-structure which was attained without the government policies.

A subtle issue is the choice of the price system in the definition of "income". Even though we adopted q_e^0, it seems that q_e^G or $q_e^{G_0}$ is also possible, where $G_0 = (g, 0, \ldots, 0)$. This problem is fairly important especially when we discuss the universal no-envy equity criterion. To explain why we chose q_e^0, consider a government policy which changes the income distribution via the change in the price system; e.g., suppose that, before any government policy, j had more income than i but, given a government policy, i now has more income than j. Since the principle of no-envy criterion is that the envy-structure in the competitive equilibrium without public sectors should not be overturned in the competitive equilibrium with public sectors, the best choice of the price to evaluate the income is q_e^0, the price before any policy is implemented.

Given such a definition, we can derive the following characterization of the income-based universal no-envy criterion (Table 4), which can be easily proved by modifying the proof for Proposition 6.

Table 4: Income-Based Universal No-Envy Criterion

Initial Condition	Required Condition
$q_e^0 \cdot \omega_i = q_e^0 \cdot \omega_j$	$B(q_e^G, \omega_i, t_i) = B(q_e^G, \omega_j, t_j)$
$q_e^0 \cdot \omega_i > q_e^0 \cdot \omega_j$	$B(q_e^G, \omega_i, t_i) \supseteq B(q_e^G, \omega_j, t_j)$

Even though the requirement that we should use q_e^0 has an advantage that when the governments *design* policies they can use income (which allows total ordering of individuals) rather than the endowment (which allows only partial ordering), the requirement also causes problems. First of all, in order to *check* the fairness of the current policy the government must know the price system when there is no government policy, which is usually difficult. Secondly, this requirement complicates the tax system to satisfy the no-envy equity criterion. Namely, if q_e^G were the price to evaluate the income then the standard income tax and the consumption tax system could have been the one that satisfies the no-envy equity criterion, which is not true any more when q_e^0 must be used.

Although using the income $q_e^G \cdot \omega_i$ as the tax base, for example, may be justifiable in practice, the argument just above actually has some interesting policy implications. For example, an implication may be that the government should tax all the gains that individuals obtain from public projects, or compensate the losses that individuals incur from the public projects. This is a fair policy in the sense that if this is not done individuals who did not get any gain form the policy or who did suffer from the policy will feel that the policy is unfair.[35] Since the potential economic gains from the government policies are also often the sources of political corruption, we believe that this type of policy must be strictly implemented for fairness not to overturn the envy-structure.

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