Anonymous Lindahl Pricing in a Tiebout Economy with Crowding Types

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Abstract

The "crowding types" model of a local public goods economy makes a distinction between crowding effects and tastes of agents. Decentralization of the core is possible both with anonymous admission prices that depend only on publicly observable information and, when technology is linear, with Lindahl (per unit) prices. Lindahl price systems are superior to admission prices since they can be specified with only a finite number of prices. Here, we show that the core is equivalent to the set of nonanonymous Lindahl equilibria, even with nonlinear technology. The core is generally larger, however, than the set of anonymous Lindahl equilibria.

1. Introduction

The classic notion of a Lindahl equilibrium requires that agents with different tastes pay different prices for public goods. Samuelson (1954) noted that agents may therefore find it in their best interests to conceal their true preferences. As a result, it may be impossible to achieve Lindahl equilibrium outcomes in a market context. Tiebout (1956) observed, however, that many types of public goods are "local" rather than "pure". Tiebout suggested that competition between local jurisdictions for members would lead to a near-optimal, market-like outcome since agents would find it in

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their best interests in this case to reveal their preferences through their choices of jurisdictions.

Bewley (1981) emphasized that Tiebout's hypothesis hinges on the possibility of decentralizing efficient allocations with prices that are anonymous in the sense that they do not depend on tastes or any other private information possessed by agents. It has now been shown in several forms that if small groups are effective and if crowding is nondifferentiated,[1] then such anonymous price systems exist and decentralize core states of the economy.[2] On the other hand, when small groups are effective and crowding is differentiated, decentralization of the core is still possible, but not, in general, by an anonymous price system. (See Wooders (1985,1997), references therein, and Scotchmer and Wooders (1986); all of these papers require prices that depend on the type of an agent, where "type" includes "taste type.").

In Conley and Wooders (1997) we introduce a new model of local public goods economy with differentiated crowding in which prices that decentralize the core need not depend on the preferences of agents. The key difference between our new model, called a *crowding types model,* and the standard differentiated crowding model is that we distinguish between two separate sets of characteristics for agents. The first set consists of tastes and endowments. These are unobservable and do not enter into the objectives or constraints of other agents. The second set consists of crowding characteristics that enter into the utility or production functions and therefore affect the welfare of others. We assume that crowding characteristics are observable, and allow their effects to be either positive or negative. Both preferences and production possibilities are defined to depend on the crowding profile of a jurisdiction, that is, the numbers of agents of each crowding type represented. For example, gender may be a crowding characteristic. We are typically able to distinguish males from females and we may have preferences about the proportion of males to females at a dance. On the other hand, it is difficult to tell which band another agent most prefers. This taste characteristic, however, does not directly affect the welfare of others.[3]

The central question studied in Conley and Wooders (1997) is the equivalence of the core and anonymous Tiebout admission equilibria.[4] A Tiebout admission price

[1]Small groups are effective if all or almost all gains from scale in population can be realized by groups bounded in size, and this bound is small relative to the total population. We provide a formal definition in Section 3. Note also that "nondifferentiated" crowding has been called "anonymous" crowding in some of the previous literature. Crowding is differentiated if different types of agents crowd each other differently. Our usage of the term "differentiated" is motivated by the differentiated commodities literature.

[2]This was result was demonstrated in Wooders (1978), using a two-part anonymous price system. See also Scotchmer and Wooders (1987), which shows that the same result holds with admission prices. Related discussions of the properties of price systems supporting optimal states of the economy appear in McGuire (1972), Hamilton (1975), Boadway (1980), Berglas and Pines (1980,1981) and Barham and Wooders (1998).

[3]A recent paper by Epple and Romano (1994) makes a similar distinction in a more applied context. See Conley and Wooders (1996) for a more detailed discussion of the relationship between the model of Epple and Romano and the crowding types model.

[4]A related result is contained in Cole and Prescott (1997) for valuation equilibria.

system specifies a single admission price that an agent of a given crowding type must pay to be allowed to join a jurisdiction with a given profile of agents and given level of public goods. Note that since the crowding types of agents are publicly observable and relevant to other agents, the price system for public goods is allowed to distinguish between different crowding types. Since tastes are not observable the price system cannot charge agents different prices on the basis of their taste type. The one price (or tax) assigned to a crowding profile and the public goods levels provided by a jurisdiction seems very much in the spirit of Tiebout's original notion of competitive jurisdictions. There is, however, one important criticism of this price system that may be made. Notice that for each crowding profile there is a continuum of prices, one for each possible level of public goods provision. This ensures that the price system is complete.[5] In fact, there is a continuum of prices required for *each* jurisdiction. This means that in general the informational requirements are significantly greater than those of a competitive equilibrium in a private goods exchange economy.

In this paper we study the relationship between the core and both the Lindahl equilibrium and admission equilibrium concepts for local public goods economies with crowding types. The difference between the Lindahl and admission price equilibrium concepts is that the former does not specify a separate price for every possible level of public good for a given jurisdiction. Instead, for each jurisdiction, a Lindahl price system specifies a participation price (a lump sum tax) and a per unit price of public good for each type of agent.[6] Thus, to specify a Lindahl price system we require exactly two numbers for each agent type for each possible profile of crowding types. Another way to look at this is that for each jurisdiction and each agent type represented in that jurisdiction, the admissions price system specifies an arbitrary nonlinear function from public good levels onto total tax liability, whereas the Lindahl price system species an affine function from public good level onto total tax liability. The most important advantage of the Lindahl price system is that it requires less information to specify.

We also study the relationship between the core and Lindahl equilibrium outcomes with anonymous, as well as nonanonymous, pricing. A price system is said to be *anonymous* if all agents of the same crowding type in the same jurisdiction pay the same level of taxes, and in addition, all jurisdictions with the same crowding profile offer the same prices.[7] A *nonanonymous* price system, on the other hand, may discriminate between agents on the basis of both their crowding and taste types. We show that when small groups are strictly effective – that is, when all gains to

[5] A discussion of complete price systems in economies with local public goods is provided in Conley and Wooders (1998).

[6] Note that these prices are not "personalized," but still not truly anonymous since they require private information – the type, including the taste type, of each agent. As emphasized by Barham and Wooders (1998), this is a two-part pricing system. For two-part pricing in economies with private goods, see, for example, Oi (1976) and Vohra (1990).

[7] Another definition of anonymity would be to require that the same prices be available to all agents of the same crowding type. The equilibrium of Wooders (1978), for example, would satisfy this definition.

collective activities can be realized by groups bounded in size – then the core and the set of *nonanonymous* Lindahl equilibria are equivalent.[8] We also show, however, that in general the *anonymous* Lindahl equilibria and the core are not equivalent.

These results contrast with existing results for both the nondifferentiated and differentiated crowding models. For the case of nondifferentiated crowding, the anonymous admission price equilibria and core are equivalent, and are both equivalent to Lindahl price equilibria when the technology is linear (Wooders 1978). This implies the same relationships hold for the nonanonymous equilibria. In the differentiated crowding model on the other hand, the nonanonymous Lindahl and admission price equilibria are equivalent to the core, but both the anonymous Lindahl equilibria and admission price equilibria are generally contained in the core. Thus, in the existing models, the possibility of decentralizing the core with anonymous or nonanonymous admission prices implies (at least under some conditions) that it is also possible to decentralize, respectively, with anonymous or nonanonymous Lindahl prices. We find this relationship does not hold up for the crowding types model. Even though the anonymous admission price equilibria and the core are equivalent, regardless of the technology the anonymous Lindahl price equilibria are generally contained in the core.

2. The Model

We consider an economy in which agents are ascribed two characteristics, tastes and crowding types. An agent has one of T different sorts of tastes or preference maps, denoted by $t \in 1, \ldots, T \equiv \mathcal{T}$ and one of C different sorts of crowding types, denoted $c \in 1, \ldots, C \equiv \mathcal{C}$. We make no assumption about correlation between c and t.[9]

The total population of agents is denoted $N = (N_{11}, \ldots, N_{ct}, \ldots, N_{CT})$, where N_{ct} is interpreted as the total number of agents with crowding type c and taste type t in the economy. A *jurisdiction* $m = (m_{11}, \ldots, m_{ct}, \ldots, m_{CT}) \leq N$ describes a group of agents, where m_{ct} denotes the number of agents with crowding type c and taste type t in the group. The *crowding profile* of a jurisdiction m is a vector $\psi(m) = (\psi_1(m), ..., \psi_C(m))$, where $\psi_c(m) = \sum_t m_{ct}$. The crowding profile simply lists the numbers of agents of each crowding type in the jurisdiction m. The set of all feasible jurisdictions is denoted by $\mathcal{N}$. We denote the set of jurisdictions that contain at least one agent of type ct by $\mathcal{N}_{ct}$, and similarly, the set of jurisdictions that

[8] Since Wooders (1978) a number of papers have demonstrated that the assumption of strict small group effectiveness permits equivalence of the core and equilibrium states of the economy for finite economies. This holds even with multiple public goods (cf., Wooders (1985,1997)). The weaker assumptions of small group effectiveness and/or per capita boundedness (finiteness of the supremum of equal treatment payoffs) implies asymptotic equivalence. Such results hold in a variety of contexts (cf., Wooders (1992,1993)).

[9] Note that in an interesting paper Helsley and Strange (1991) consider related questions in a model where exclusion is costly. However, they allow only one crowding type – that is, crowding is anonymous – and all agents have the same utility function. Thus, the problems of interest in the current paper do not appear in their framework.

contain at least one agent of crowding type c by $\mathcal{N}_c$. A *partition* n of the population is a set of jurisdictions $\{n^1, ..., n^K\}$ such that $\sum_k n^k = N$. We will write $n^k \in n$ when a jurisdiction n^k is in the partition n. It will sometimes be useful to refer to individual agents whom we denote by $i \in \{1, \ldots, I\} \equiv \mathcal{I}$, where $I = \sum_{c,t} N_{ct}$. Let $\theta : \mathcal{I} \to \mathcal{C} \times \mathcal{T}$ be a mapping describing the crowding and taste types of individual agents; thus, $|\{i \in \mathcal{I} : \theta(i) = (c,t)\}| = N_{ct}$. A coalition is a subset of $\mathcal{I}$. With a slight abuse of notation, if individual i is a member of a coalition of agents described by m, we shall write $i \in m$. When it will not cause any confusion, we shall also sometimes refer to a coalition described by m as the coalition m.

Consider an economy with one public good y that is provided by jurisdictions and one private good x. Each agent is a member of exactly one jurisdiction. An agent $i \in \mathcal{I}$ of taste type t is endowed with ω_t of the private good[10] and has a quasi-linear utility function $u_t(x, y, m) = x + h_t(y, m)$ where $i \in m$ and y is the quantity of public good produced in the jurisdiction m. For all $t \in \mathcal{T}$ and all $m \in \mathcal{N}$ with $m_{ct} > 0$ for some $c \in \mathcal{C}$, the following conditions are satisfied:

Monotonicity: For all $y, \hat{y} \in \Re$ with $y \geq \hat{y}$,

$$h_t(y, m) \geq h_t(\hat{y}, m). \tag{2.1}$$

Convexity: For all $y, \hat{y} \in \Re$ and for all $\alpha \in [0, 1]$,

$$\alpha h_t(y, m) + (1 - \alpha) h_t(\hat{y}, m) \geq h_t(\alpha y + (1 - \alpha)\hat{y}, m). \tag{2.2}$$

Differentiability: The function $h_t(y, m)$ is differentiable with respect to y.

TAC: If for $\widehat{m} \in \mathcal{N}$ it holds that $\psi(\widehat{m}) = \psi(m)$, then for all $y \in \Re_+$,

$$h_t(y, m) = h_t(y, \hat{m}). \tag{2.3}$$

Convexity and monotonicity are standard assumptions that allow optimal allocations to be supported by per unit prices. We assume differentiability to make the proofs more transparent. It is possible to prove equivalent theorems without differentiability. The last assumption, TAC, is called *taste anonymity in consumption,* and deserves some discussion. In words, TAC requires that if two jurisdictions produce the same level of public goods and have the same crowding profiles, then agents view them as equivalent. That is, agents care only about the crowding types of the agents with whom they share the public goods and not about their preferences. It is probably

[10] Formally, this implies that agents with the same tastes but different crowding characteristics have the same endowment. This does not cost any degree of generality since there is no requirement that agents of taste type t have different preferences from agents of type t'. Thus, we can consider agents of the same crowding type with the same preferences but different endowments to be different taste types.

more appropriate to view this as a definition of a crowding type than as a restriction on preferences.

The cost in terms of private good of producing y public good for a jurisdiction with membership m is given by the function $f(y,m)$. We make four assumptions on the production function for each $m \in \mathcal{N}$.

Monotonicity: For all $y, \hat{y} \in \Re_+$ with $y \geq \hat{y}$, it holds that $f(y,m) \geq f(\hat{y},m)$.

Convexity: For all $y, \hat{y} \in \Re_+$ and for all $\alpha \in [0,1]$ it holds that $\alpha f(y,m) + (1-\alpha) f(\hat{y},m) \geq f(\alpha y + (1-\alpha)\hat{y}, m)$.

Differentiability: $f(y,m)$ is differentiable with respect to y.

TAP: If for $\widehat{m} \in \mathcal{N}$ it holds that $\psi(\widehat{m}) = \psi(m)$, then for all $y \in \Re_+$, $f(y,m) = f(y,\hat{m})$.

Again, monotonicity is used to guarantee supporting prices exist, and differentiability simplifies the proofs. We show below that convexity is all that is required for nonanonymous Lindahl decentralization of core states in the crowding types model, as well as in the differentiated and nondifferentiated models. We also show that strengthening convexity to constant returns to scale (CRS)[11] is not sufficient to guarantee anonymous Lindahl decentralization of the core in the crowding types model even though it is sufficient in the nondifferentiated crowding model. The last assumption on production, called *taste anonymity in production,* is motivated in the same way as TAC above. TAP dictates that only the crowding type of an agent affects the production opportunities of a jurisdiction. For example, it is easier to build houses if you have a carpenter in your jurisdiction but the fact that he might be a Cubs fan should not make any difference in production.

Because of differentiability, monotonicity and convexity or constant returns to scale (CRS), the marginal willingness to pay and the marginal cost functions are well defined and single valued.

$$\text{Marginal willingness to pay:} \quad mwp_t\,(y,m) = \frac{\partial h}{\partial y}. \tag{2.4}$$

$$\text{Marginal cost:} \quad mc\,(y,m) = \frac{\partial f}{\partial y}. \tag{2.5}$$

A *feasible state* of the economy $(X,Y,n) \equiv ((x_1,\ldots,x_I),(y^1,\ldots y^K),(n^1,\ldots n^K))$ consists of:

(a) an allocation of private goods to agents $X = (x_1,\ldots,x_I)$;

[11] By CRS we mean that the production technology available to a jurisdiction m satisfies constant returns to scale, that is: For all $y, \hat{y} \in \Re$ and for all $\alpha > 0$ it holds that $\alpha f(y,m) = f(\alpha y, m)$.

(b) a public goods production plan for each jurisdiction, $Y = (y^1, \ldots y^K)$; and

(c) a partition $n = (n^1, \ldots n^K)$ such that

$$\sum_k \sum_{ct} n^k_{ct}\omega_t - \sum_i x_i - \sum_k f(y^k, n^k) \geq 0. \tag{2.6}$$

A coalition is a subset of agents who may act collectively to improve upon a state of the economy for the members of the coalition. Since there is only one private good and thus no gains from trade between jurisdictions, to define the core of the economy there is no loss of generality in restricting coalitions to be jurisdictions. Following our convention, we'll refer to individual agents i as members of coalitions $m \in \mathcal{N}$.

A coalition $m \in \mathcal{N}$ is said to *improve upon a state* (X, Y, n) if there exists a distribution of private good $\widehat{x} \equiv \{\widehat{x}_i\}_{i \in m}$ and a level of public good $y \in \Re_+$ such that:

1. $$\sum_{i \in m}(\omega_i - \widehat{x}_i) - f(y, m) \geq 0, \tag{2.7}$$

   and

2. for all $i \in m$, where agent $\theta\,(i) = (c, t)\,,\ i \in n^k$ and $n^k \in n$

$$u_t(\widehat{x}_i, y, m) > u_t(x_i, y^k, n^k).^{12} \tag{2.8}$$

The coalition m is said to *improve with the allocation* $(\widehat{x}, y, m)$. A feasible state of the economy $(X, Y, n) \in F$ is a *core state of the economy* or simply a *core state* if it cannot be improved upon by any coalition m. This definition simply says that a feasible state is in the core if it is not possible for a coalition of agents to break away and, using only its own resources, provide all its members with preferred consumption bundles.

A key assumption that drives core equivalence results is small group effectiveness, the assumption that all or almost all gains to collective activities can be realized by groups bounded in absolute size. There are many ways in which to formalize this notion which has its origin in the sixth assumption made by Tiebout in his original paper. We choose to adopt a relatively strong version because of the ease with which it allows us to obtain our results. It is possible to prove essentially the same propositions with weaker assumptions at the cost of complicating the proofs. In any event, we say an economy satisfies *strict small group effectiveness*, (SSGE), if there exists a positive integer B such that:

[12] We require that all agents be strictly better off in the improving coalition. Because agents are assumed to have quasi linear preferences, this entails no loss of generality from the usual definition. If it is possible to make one agent strictly better off while not harming the other agents, it is necessarily possible to make all agents in the coalition strictly better off as well.

1. For all core states (X, Y, n) and all $n^k \in n$, it holds that $\sum_{ct} n^k_{ct} \leq B$.
2. For all $c \in \mathcal{C}$ and all $t \in \mathcal{T}$ it holds that $N_{ct} > B$.

The first condition any state in which includes at least one jurisdiction with more than B agents can be improved upon. In other words coalitions larger than B do strictly worse that coalitions with B agents or fewer. The second condition says that there are at least B agents of each type in the economy.

Remark. Other forms of SSGE that would suffice include the assumption that there is a "minimum efficient scale" for coalitions, that is, there is a bound B such that all possibilities for "improvement" can be exhausted by coalitions containing fewer than B members (Wooders 1983) or that admissible coalition sizes are bounded (Kaneko and Wooders 1996)). The term SSGE is usually used for assumptions that ensure the equal treatment property of the core in finite economies whereas SGE, ensuring only that *almost* all gains to collective activities can be realized by groups bounded in absolute size implies that, as economies grow large, asymptotically cores and approximate cores are *nearly* equal treatment (Wooders (1980,1994)[13]). SGE is a mild yet powerful assumption and, if there are sufficiently many players of each type, equivalent to the apparently mild condition of boundedness of the supremum of equal treatment payoff (cf., Wooders (1983,1994)).

One immediate consequence of SSGE is that the core has the equal treatment property (Wooders 1983, Theorem 3). We restate this result here.

Theorem 1. *The equal-treatment property of the core.* Let (X, Y, n) be a core state of an economy satisfying SGE. For any two individuals $i, \hat{i} \in \mathcal{I}$ such that $\theta(i) = \theta(\hat{i}) = (c, t)$, if $i \in n^k$ and $\hat{i} \in n^{\hat{k}}$ then $u_t(x_i, y, n^k) = u_t(\hat{x}_{\hat{i}}, \hat{y}, n^{\hat{k}})$.

Proof. Since the proof is available in Conley and Wooders (1997) we do not provide a formal proof here. The argument is essentially the same as that of Wooders (1983,Theorem 3). Suppose there are two agents of the same type in different jurisdictions who are treated unequally by some core allocation. Then the coalition consisting of those agents in the jurisdiction containing the "better-off" of the two agents replace the best off agent with the worse-off agent and all agents in the coalition could be better off. If the two agents treated unequally are in the same jurisdiction, from 1. of the definition of SSGE there is another jurisdiction containing additional agents of the same type. Then the above argument can be applied to the agents in these two jurisdictions. ■

[13] The result also appears in Shubik and Wooders (1982) and, with an assumption of SSGE, in SUNY-Stony Brook Working paper #184 (1977).

2.1. Nonemptiness of cores and approximate cores

For our results on the core and on equivalence of the core and equilibrium outcomes to have significance, it must be shown that they can be applied to an interesting class of economies. It is well-known, however, that, unless agents can be partitioned into "optimal"[14] jurisdictions, the core will be empty (cf., Wooders (1978,1980,1983) or Shubik and Wooders (1983)). Unless we directly impose "balancedness" assumptions ensuring nonemptiness of the core for all replications of the economy, or assumptions ensuring that agents can be partitioned into "optimal" jurisdictions, we have no result showing nonemptiness of the core. However, under our assumption of strict $SSGE$ it holds that all sufficiently large economies have non-empty approximate cores where the approximate core concept requires that, when (at most) a small percentage of agents is ignored, then the core of the economy with agent set consisting of the remaining agents is nonempty. See Wooders (1980,1983), Kaneko and Wooders (1982) or Shubik and Wooders (1983), for example, or, for the most recent of such results, Kovalenkov and Wooders (1997). Our claims about the core are therefore broadly applicable since they immediately apply to cores of "large" subeconomies (containing "most" of the total population).

For ease in illustrating the nature of nonemptiness of approximate core results, let us suppose that essential coalition sizes are bounded. That is,

(*) There is a bound B' such that for any type (c,t), any x, y, and any jurisdiction m with $\sum m_{c't'} > B'$ and $m_{ct} \neq 0$, it holds that $u_t(x,y,m) < u_t(\omega_t, 0, 1_{ct})$ where 1_{ct} denotes the jurisdiction containing only one agent of type (c,t).[15]

The following result then holds.[16]

Proposition 1. Assume (*) holds. Let $\varepsilon > 0$ be given. Then there is a constant η such that if, $\sum_{ct} N_{ct} > \eta$, for some subpopulation $N^0 \leq N$ the economy with total population N^0 has a nonempty core and $\frac{\sum_{ct} N_{ct} - \sum_{ct} N^0_{ct}}{\sum_{ct} N_{ct}} \leq \varepsilon$.

[14] While this is clear in the case of one-private good, in fact, for multiple private goods the definition of an "optimal" jurisdictions may be unclear. For equilibrium, an optimal jurisdiction can be defined relative to equilibrium prices; an optimal jurisdiction is an equilibrium jurisdiction. For the core, a jurisdiction is optimal if it appears in a core state of the economy. More generally, given a Pareto-optimal state, a jurisdiction is optimal if it appears in that state of the economy.

Since economy-wide relative scarcities of commodities are relevant, the "optimal" autarchic (no-trade) jurisdiction is not necessarily consistent with equilibrium, the core, or Pareto-optimal states of the economy. As pointed out to us by David Pines, other authors have considered autarchic optimal jurisdictions in the many-private-good case as analogous to autarchic jurisdictions in the one-private-good case. In the special case of one private good, however, gains to trade of private commodities need not be taken into account in defining truly optimal jurisdictions, while with many private goods the possibility of such gains cannot be ignored.

[15] Such assumptions are common in the literature on economies with local public goods, cf., Wooders (1978, Theorem 5) . The strong form of the assumption is made here for convenience.

[16] Note that this result is independent of the utility functions and extends a result of Kaneko and Wooders (1982) for games with a fixed distribution of player types.

Proof. See Kovalenkov and Wooders (1997, Theorem 1). ■

In application to our model, Proposition 1 immediately implies that when a small percentage (ε) of players is ignored, for an economy containing a large subset of agents (at least $1 - \varepsilon$ percent of the agents), the (exact) core is nonempty. For conditions under which the core coincides with the equilibrium states of the economy, Proposition 1 therefore implies existence of an approximate equilibrium, where a small percentage of players are not optimizing.

Typically, approximate core concepts have one of two forms. Besides the form discussed above, introduced in Shubik and Wooders (1983), an approximate core concept may require that no group of agents can significantly improve upon their payoff; see, for example, Wooders (1980,1983) or numerous subsequent papers. Application of the general game theoretic results to economies including those herein as special cases is carried out in Wooders (1987, 1997). In particular, Wooders (1987) contains several results showing that with multiple private and public goods, approximate cores of sufficiently large economies are nonempty even under the mild condition of boundedness of per capita payoffs. These results can be applied to show existence of equilibrium; see Wooders (1985,1997). Similar results can be easily obtained for the model of the current paper. Thus, for economies satisfying SSGE, our results are not vacuous and in fact, analogous hold under the apparently mild condition of SGE.

3. Lindahl Equilibrium and the Core

In this section, we first define the notion of a nonanonymous Lindahl equilibrium for a Tiebout economy with crowding types. We prove that nonanonymous Lindahl equilibrium outcomes are equivalent to the core and therefore (by Conley and Wooders (1997)) to the set of Tiebout admission equilibrium outcomes. Next, we define an anonymous Lindahl equilibrium for this economy. We provide an example which shows that in general, the core is larger than the set of anonymous Lindahl equilibrium outcomes. We also discuss why this result is contrary to the intuition we get from the existing literature.

In the situations of interest in this paper, from Theorem 1 all states of the economy in the core have the equal treatment property. Thus, it is without loss of generality that we require Lindahl prices to be the same for all participants of the same type in the same jurisdiction; with more notation, this can be shown to be a consequence of strict small group effectiveness.

3.1. Nonanonymous Lindahl equilibrium

A nonanonymous Lindahl price system specifies a participation price and a per unit price of public good for each type of agent. The price system is complete in the sense that it provides prices for all possible jurisdictions, including those that do not appear in the equilibrium partition. Formally, a price system states, for each $c \in \mathcal{C}$ and $t \in \mathcal{T}$, a function $\lambda_{ct} : \mathcal{N}_{ct} \rightarrow \Re^2$ assigning a pair of prices to each possible jurisdiction. It is convenient to decompose this function into two parts, $p_{ct} : \mathcal{N}_{ct} \rightarrow \Re$, and $q_{ct} : \mathcal{N}_{ct} \rightarrow \Re$; thus, $\lambda_{ct}(m) \equiv (p_{ct}(m), q_{ct}(m))$. The first component $p_{ct}(m)$ of this pair is interpreted as a participation price, a lump-sum amount, that an agent of type ct must pay to be admitted to a jurisdiction with composition m. The second component $q_{ct}(m)$ is the price an agent of type ct must pay per unit of public good in the jurisdiction m.

A *nonanonymous Lindahl price system* is just a list $\lambda = \{\lambda_{11}, \ldots, \lambda_{CT}\}$ of price systems, one for each type of agent.

A *nonanonymous Lindahl equilibrium* consists of a feasible state of the economy $(X, y, n) \in F$ and a price system λ such that

1. for all $n^k \in n$, all individuals $i \in n^k$ with $\theta(i) = (c,t)$, all alternative coalitions $m \in \mathcal{N}_{ct}$, and levels of public good $y \in \Re_+$,

$$\omega_t - p_{ct}(n^k) - q_{ct}(n^k)y^k + h_t(y^k, n^k) \geq \omega_t - p_{ct}(m) - q_{ct}(m)y + h_t(y,m); \quad (3.1)$$

2. for all possible coalitions $m \in \mathcal{N}$ and all $y \in \Re_+$,

$$\sum_{ct} q_{ct}(m)y + \sum_{ct} m_{ct}p_{ct}(m) - f(y,m) \leq 0; \quad (3.2)$$

3. for all $n^k \in n$,

$$\sum_{ct} q_{ct}(n^k)y^k + \sum_{ct} n^k_{ct}p_{ct}(n^k) - f(y^k, n^k) = 0. \quad (3.3)$$

Some existence results for a Lindahl equilibrium with nonanonymous crowding are provided by Wooders (1997, Theorem 3 and Corollary 2).[17] These results apply to the situation we are considering.

A feasible state $(X, y, n) \in F$ is a *nonanonymous Lindahl state of the economy* if there is a price system λ such that (X, y, n) and λ constitute a nonanonymous Lindahl equilibrium.

Our first theorem is that a nonanonymous Lindahl state of the economy is also a state of the economy in the core. This implies equivalent results for both the differentiated and nondifferentiated crowding models.[18] All proofs are contained in the Appendix.

Theorem 2. *Equilibrium states are in the core.* Let (X, y, n) and λ constitute a nonanonymous Lindahl equilibrium. Then (X, y, n) is a core state of the economy.

[17] This holds for both admission prices and Lindahl prices when crowding is differentiated; see also Scotchmer and Wooders (1986) and Wooders (1985).

[18] Note that the nondifferentiated crowding model is identical to the crowding types model with only one crowding type. The differentiated crowding model is identical to the crowding types model with $C = T$, and $N_{ct} = 0$, for $c \neq t$.

Note that we do not need to assume any type of convexity. We only use monotonicity, implied by the quasi-linearity of utility functions. A First Welfare Theorem is an immediate corollary of Theorem 1.

Our next theorem is somewhat more difficult to prove. It says that any core state can be decentralized as a nonanonymous Lindahl state provided that small groups are strictly effective and the economy satisfies monotonicity, convexity and differentiability assumptions. Taken together with Theorem 1, this means that under monotonicity, convexity and differentiability, the core and the set of nonanonymous Lindahl states are equivalent. Theorem 2, like Theorem 1, also complements results in the literature on differentiated crowding models.[19]

Theorem 3. Assume that utility and cost functions satisfy monotonicity, convexity, and differentiability and in addition, the economy satisfies (SGE). Let (X, y, n) be a state of the economy in the core. Then there exists a price system λ such that (X, y, n), and λ constitute a nonanonymous Lindahl equilibrium.

Note that Theorem 2 holds independently of TAC and TAP. Since prices are nonanonymous, there is no need to require that only the crowding type of an agent affects the welfare of the other agents.

3.2. Anonymous Lindahl equilibrium

An anonymous Lindahl equilibrium is the same as a nonanonymous Lindahl equilibrium with one important distinction – in an anonymous Lindahl equilibrium, prices for public goods and participation prices are defined to depend only on the crowding types of individuals. We will define the equilibrium using the same notation as in the definition of the nonanonymous Lindahl equilibrium but without the subscript t on prices. Formally, for each $c \in \mathcal{C}$, there is a price system given by a mapping $\lambda_c : \mathcal{N}_c \rightarrow \Re^2$. It is convenient to decompose λ_c into two mappings: $p_c : \mathcal{N} \rightarrow \Re$, and $q_c : \mathcal{N}_c \rightarrow \Re$. Thus $\lambda_c(m) \equiv (p_c(m), q_c(m))$, where $p_c(m)$ is interpreted as the lump sum participation price that an agent of crowding type c must pay to be admitted to a jurisdiction with composition m and $q_c(m)$ is the price an agent of this type pays per unit of public good provided in m. An *anonymous Lindahl price system* $\lambda = \{\lambda_1, \ldots, \lambda_C\}$ is a list of price systems, one for each crowding type.

A *anonymous Lindahl equilibrium* consists of a feasible state of the economy, $(X, y, n) \in F$, and price system λ such that:

1. for all $n^k \in n$, all individuals $i \in n^k$ with $\theta(i) = (c, t)$, all alternative coalitions $m \in \mathcal{N}_c$, and level of public goods $y \in \Re_+$,

$$\omega_t - p_c(n^k) - q_c(n^k)y^k + h_t(y^k, n^k) \geq \omega_t - p_c(m) - q_c(m)y + h_t(m, y); \quad (3.4)$$

[19] We include the result here for completeness. Essentially the same result appears in Scotchmer and Wooders (1986). General equilibrium versions of this result and other closely related results appear in Wooders (1985,1997).

2. for all possible coalitions $m \in \mathcal{N}$ and all $y \in \Re_+$,

$$\sum_{i \in m} q_c(m)y + \sum_{ct} m_{ct} p_c(m) - f(y, m) \leq 0; \tag{3.5}$$

3. for all $n^k \in n$,

$$\sum_{i \in n^k} q_c(n^k)y^k + \sum_{ct} n^k_{ct} p_c(n^k) - f(y^k, n^k) = 0. \tag{3.6}$$

4. for all $m, \hat{m} \in \mathcal{N}$ with $\psi(m) = \psi(\hat{m})$ it holds that $\lambda(m) = \lambda(\hat{m})$.

A feasible state $(X, y, n) \in F$ is an *anonymous Lindahl state of the economy* if there is a price system λ such that (X, y, n) and λ constitute an anonymous Lindahl equilibrium.

Our first step is to show that any anonymous Lindahl state is in the core. Since an anonymous equilibrium is also a nonanonymous equilibrium, the result is a corollary of Theorem 1.

Theorem 4. Let (X, y, n) and λ constitute an anonymous Lindahl equilibrium. Then (X, y, n) is a core state.

In the nondifferentiated crowding model, under constant returns to scale of the production technology available to a jurisdiction, the core states of the economy can be decentralized through anonymous Lindahl prices. This is proven in Wooders (1978). (See also Barham and Wooders (1998)), who obtain the same result with nonanonymous Lindahl equilibria.) Constant returns to scale may be a very restrictive condition on technology, but it appears indispensable. We provide the following example to demonstrate this point.

Example 1: Nonequivalence of the core and the set of anonymous Lindahl equilibria with nondifferentiated crowding and decreasing returns to scale.

Consider a world with one crowding type – crowding is nondifferentiated – and two taste types. Suppose that crowding occurs in consumption and that only members of two-person jurisdictions get utility from public good – a matching problem. The utility functions of the two types, denoted by ℓ and h, are as follows:

$$U_\ell(x, y, m) = \begin{cases} x + 2y \text{ if } m_\ell + m_h = 2, \text{and} \\ x \text{ otherwise} \end{cases} \tag{3.7}$$

and

$$U_h(x,y,m) = \begin{cases} x + 4y \text{ if } m_\ell + m_h = 2, \text{and} \\ x \text{ otherwise.} \end{cases} \tag{3.8}$$

We may think of these two types as low demanders (ℓ) and high demanders (h) of public good relative to private good. Let the endowments of each type be ten: $\omega_\ell = \omega_h = 10$. The total cost of producing public good is given by the function:

$$TC(y,m) = y^2. \tag{3.9}$$

This implies that the marginal cost is given by:

$$MC(y,m) = 2y. \tag{3.10}$$

Finally, suppose that there are 100 agents of each of these two types in the economy. We claim that there is a state of the economy in the core with 50 jurisdictions each consisting of two low demanders only and 50 jurisdictions each consisting of two high demanders only. The jurisdictions containing only low demanders each produce 2 units of public good, and the jurisdictions containing only high demanders will each produce 4 units of public good. In this state of the economy $U_\ell = 12$ and $U_h = 18$.

Call the jurisdictions with type ℓ's type L jurisdictions and those with type h's, type H jurisdictions. Obviously the state of the economy described above cannot be improved upon by any jurisdiction containing any number other than two agents since members of such jurisdictions get zero utility from public good. Consider then a jurisdiction consisting of one agent of type ℓ and one of type h. The most total utility that such a jurisdiction can generate can be found by solving the following equation:

$$\max_y 2y + 4y - y^2. \tag{3.11}$$

Thus, the mixed jurisdiction should produce three units of public good, yielding an aggregate utility of 29, which is less than the total of 30 units of utility these two agents are assigned by the core state. Thus, the state of the economy described above is in the core.

We now try to decentralize the given core state through anonymous Lindahl prices. Note that since there is only one crowding type, the same prices are faced by all agents. Consider type L jurisdictions (producing only two units of public good.). Observe that if two units of public good are produced by the firm, it must be that

$2q^L = MC(2, m) = 2y = 4$. Otherwise, since the firm is a profit maximizer, it would choose to produce a different level of public good. We have an analogous expression for the second type of jurisdiction (producing four units of public good). We conclude that $q^L = 2$,and $q^H = 4$. Since there must be zero profits in equilibrium or else we will have entry by other firms, the participation prices must refund the surplus to the agents. It is easy to verify the profits are: $\pi^L = 2q^L y - y^2 = 4$ and $\pi^H = 2q^H y - y^2 = 16$. This means the participation prices must be: $p^L = -2$ and $p^H = -8$. To summarize:

$$p^L = -2,\ q^L = 2; \qquad p^H = -8,\ p^H = 4 \tag{3.12}$$

The prices above induce the firms to produce the specified levels of public goods while making zero profit. Unfortunately, these prices do not decentralize the core from the consumer side. At these prices, agents of type h can make infinite utility by joining a jurisdiction of type L. To see this, observe that the utility that an agent of type h gets by joining such a jurisdiction is:

$$4y - p^L - q^L y + 10 = 12 + 2y. \tag{3.13}$$

Thus, such agents face a marginal cost of 2 for producing public good under these prices, but receive a marginal benefit of 4. It is therefore optimal for these agents to demand infinite levels of public good.

We conclude that the only prices which decentralize the core from the producer side must lead to adverse selection on the consumer side so the core cannot be decentralized with anonymous Lindahl prices. Note that this example holds for both the crowding types model and the anonymous crowding model, which is just one special case.•

Example one does not tell us whether or not the core of a crowding types economy can be decentralized by anonymous Lindahl when the technology is linear. There is a strong intuition from the existing literature that such a decentralization should be possible. The argument is the following. In the nondifferentiated crowding model, the core is equivalent to the set of anonymous admission price equilibria. This model is essentially a crowding types model with only one crowding type. Thus, one admission price function per jurisdiction suffices to decentralize the core. When we generalize the model to allow for more than one crowding type we find that it is still possible to decentralize the core with an anonymous admission price system. Now we need one admission price function for each crowding type per jurisdiction (see Conley and Wooders (1997,1998). We also know that the core and anonymous Lindahl equilibria are equivalent in a nondifferentiated crowding model under CRS production. Again, we need exactly one pair prices to decentralize core since there is essentially only one crowding type in this model. We would expect, therefore, that the core of a crowding types economy with linear technology should also be decentralizable with a similar set anonymous Lindahl prices, one pair for each crowding type per jurisdiction.

A Lindahl price system, however, has more burdens placed on it than an admission prices system. Lindahl prices must solve both the adverse selection problem and the public goods allocation problem. It turns out that in general, equating the marginal costs and benefits of public goods within a jurisdiction to solve the public goods allocation problem uses up too many degrees of freedom and makes it infeasible to solve the adverse selection problem of agent allocation at the same time. The following example demonstrates this point.

Example 2: Nonequivalence of the core and the anonymous Lindahl equilibrium with more than one crowding type and constant returns to scale.

Consider the classic marriage problem. This is a special case of the more general class of local public goods problems in which agents get utility from being in two-person coalitions with the opposite crowding type, and nothing otherwise. We have two crowding types, boys and girls, indexed by b and g, respectively, and two patterns of tastes, indexed by 1 and 2. Denote the set of coalitions that get positive utility as follows:

$$\mathcal{N}^{+} \equiv \{m \in \mathcal{N} \mid m_{b1} + m_{b2} = 1 = m_{g1} + m_{g2}\}. \tag{3.14}$$

This describes the set of jurisdictions that have exactly one boy and one girl. (Note that $m_{bi} = 1$ if and only if $m_{bj} = 0$ and for $j \neq i$ and similarly for m_{gi} and m_{gj}.) The two taste types have utility functions given by:

$$u_1(x, y, m) = x + \sqrt{y} - 10 \text{ for all } m \in \mathcal{N}^{\,+}, \text{ and zero otherwise;} \tag{3.15}$$

$$u_2(x, y, m) = x + 2\sqrt{y} - 10 \text{ for all } m \in \mathcal{N}^{\,+}, \text{ and zero otherwise.} \tag{3.16}$$

Assume that endowments of private good are ten for all agents. Also suppose public goods y can be produced according to the following total cost function:

$$TC(y) \;=\; y. \tag{3.17}$$

Suppose that the population is as follows: $N = (N_{g1}, N_{g2}, N_{b1}, N_{b2}) = (6, 6, 3, 9)$. There will be three types of marriages: those containing two agents of taste type 1, those containing two agents of taste type 2, and marriages which are mixed by taste type. We index the two segregated types of marriages by $s1$, and $s2$, respectively, and the integrated marriage by i

1. Consider the two possible mixed marriages first. The profile of agents, m, is either $(1, 0, 0, 1)$, or $(0, 1, 1, 0)$ in this case. When a mixed marriage maximizes its total utility it equates the sum of the marginal benefits with the marginal cost of public goods. The sum of marginal benefits is the following:

$$MB_1 + MB_2 = \frac{1}{2}y^{-.5} + y^{-.5} = \frac{3}{2}y^{-.5} \tag{3.18}$$

Since marginal cost equals one, the efficient level of public good is found by solving:

$$1 = \frac{3}{2} y^{-.5}. \tag{3.19}$$

Thus,

$$y^i = \frac{9}{4}. \tag{3.20}$$

This implies that the total transferable utility available to this coalition is:

$$U^i = \sqrt{\frac{9}{4}} + 2\sqrt{\frac{9}{4}} - \frac{9}{4} = \frac{9}{4}. \tag{3.21}$$

2. For segregated marriages with agents of taste type 1, the population is $(1, 0, 1, 0)$. Proceeding as above, we have the following:

$$MB_1 + MB_2 = \frac{1}{2} y^{-.5} + \frac{1}{2} y^{-.5} = y^{-.5}. \tag{3.22}$$

Setting this equal to marginal cost gives us:

$$1 = y^{-.5}. \tag{3.23}$$

Thus,

$$y^{s1} = 1. \tag{3.24}$$

This implies that the total transferable utility available to this coalition is:

$$U^{s1} = \sqrt{1} + \sqrt{1} - 1 = 1. \tag{3.25}$$

3. For segregated marriages with agents of taste type 2, the population is $(0, 1, 0, 1)$. Proceeding as above, we have the following:

$$MB_1 + MB_2 = y^{-.5} + y^{-.5} = 2y^{-.5}. \tag{3.26}$$

Setting this equal to marginal cost gives us:

$$1 = 2y^{-.5}. \tag{3.27}$$

Thus,

$$y^{s2} = 4. \tag{3.28}$$

This implies that the total transferable utility available to this coalition is:

$$U^{s2} = 2\sqrt{4} + 2\sqrt{4} - 4 = 4. \tag{3.29}$$

One element of the core consists of three jurisdictions of type $s1$, six of type $s2$, and three of type i with a taste type 1 girl and a taste type 2 boy. The core allocation must satisfy the following equations in this case:

$$U_{g1} + U_{b1} = 1 \tag{3.30}$$

$$U_{g2} + U_{b2} = 4 \tag{3.31}$$

$$U_{g1} + U_{b2} = \frac{9}{4} \tag{3.32}$$

$$U_{g2} + U_{b1} \geq \frac{9}{4} \tag{3.33}$$

Where U_{ct} is the utility received by an agent of crowding type c and taste type t in the core. It is easy to verify that the following utility allocation satisfies these equations.

$$U_{g1} = \frac{1}{2},\ U_{g2} = \frac{9}{4}, U_{b1} = \frac{1}{2}, \text{and } U_{b2} = \frac{7}{4}. \tag{3.34}$$

Also observe that small groups are effective. The price system below is the only possible candidate to support this allocation. It is calculated by setting each agent's per unit price of public good equal to his marginal benefit and then setting his participation prices in a way that gives the utility levels above.

$$p_g^{s1} = 0, \quad q_g^{s1} = \frac{1}{2}, \tag{3.35}$$

$$p_b^{s1} = 0, \quad q_b^{s1} = \frac{1}{2}, \tag{3.36}$$

$$p_g^{s2} = -\frac{1}{4}, \quad q_g^{s2} = \frac{1}{2}, \tag{3.37}$$

$$p_b^{s2} = \frac{1}{4}, \quad q_b^{s2} = \frac{1}{2}, \tag{3.38}$$

$$p_g^{i} = \frac{1}{4}, \quad q_g^{i} = \frac{1}{3}, \tag{3.39}$$

$$p_b^{i} = -\frac{1}{4}, \quad q_b^{i} = \frac{2}{3}. \tag{3.40}$$

But observe that at these prices girls of type 2 would prefer to join the integrated marriages. If they do so, they set their marginal benefit equal to the per unit price:

$$y^{-.5} \quad = \quad \frac{1}{3} = q_g^i. \tag{3.41}$$

Solving this gives them a demand of $y = 9$. The agent pays a participation price of $\frac{1}{4}$, to join this coalition, and $\frac{1}{3}9 = 3$ in per unit contributions to the public good's cost.

This gives a total utility of:,

$$2\sqrt{9} - \frac{1}{4} - 3 = \frac{11}{4} > U_{g2} = \frac{9}{4}. \tag{3.42}$$

Thus, the prices above do not support the core allocation. We conclude that the core and anonymous Lindahl equilibrium are not equivalent in general even when the technology is linear.•

4. Conclusion

In this paper we have investigated the possibility of anonymous decentralizations of core allocations. From the existing literature, we know that anonymous decentralization through admission prices is possible in both the nondifferentiated crowding and crowding types model. This result does not depend on a convexity assumption on either preferences or technology. An undesirable feature of admission prices is that they are really price functions which give a separate price of admission for each crowding type to every conceivable jurisdiction type, producing every conceivable level of public goods. This means that such price systems are infinite dimensional. This in turn leads to an interest in Lindahl decentralization. Lindahl price functions are affine, and thus can be completely specified by a finite set of numbers. We know from existing literature that for the *nondifferentiated* crowding model, the core is equivalent to the anonymous Lindahl equilibria when the technology is linear and to the nonanonymous Lindahl equilibria when the technology is convex. We extend the latter result to the *crowding types* model to show that the core and *nonanonymous* Lindahl equilibrium are equivalent in the crowding types model when technology is convex. We are able to produce a counterexample, however, which demonstrates that the core is generally larger than the set of anonymous Lindahl equilibria.

5. Appendix

Theorem 2. *Equilibrium states are in the core.* Let (X, y, n) and λ constitute a nonanonymous Lindahl equilibrium. Then (X, y, n) is a core state of the economy.

Proof of Theorem 2. Suppose the Theorem is false. Then there is a Lindahl equilibrium, (X, y, n) and λ, such that (X, y, n) is not a core state. This means for some coalition m there is an improving allocation $(\widehat{x}, y, m)$. Since this allocation is feasible for m it holds that

1. $\sum_{ct} m_{ct}\omega_t - \sum \widehat{x}_i - f(y, m) \geq 0$ (the allocation is feasible) and

for each $i \in m$ with $\theta(i) = (c, t)$ where $i \in n^k$ in the equilibrium state,

$$u_t(\widehat{x}_i, y, m) > u_t(x_i, y^k, n^k). \tag{5.1}$$

From (1.) it follows that

$$\sum_{ct} m_{ct}\omega_t - \sum_{i\in m} \widehat{x}_i \geq f(y,m). \tag{5.2}$$

By profit maximization we know that $\lambda = (p,q)$ satisfies

$$\sum_{i\in m} q_{ct}(m)y + \sum_{ct} m_{ct}p_{ct}(m) - f(y,m) \leq 0, \tag{5.3}$$

and so

$$\sum_{i\in m} q_{ct}(m)y + \sum_{ct} m_{ct}p_{ct}(m) \leq \sum_{ct} m_{ct}\omega_t - \sum_{i\in m} \widehat{x}_i. \tag{5.4}$$

Thus, for at least one $i \in m$ with $\theta(i) = (c,t)$ it holds that

$$\omega_t - p_{ct}(m) - q_{ct}(m)y \geq \widehat{x}_i. \tag{5.5}$$

But then

$$u_t(\omega_t - p_{ct}(m) - q_{ct}(m)y, y, m) \geq u_t(\widehat{x}_i, y, m) > u_t(x_i, y^k, n^k), \tag{5.6}$$

which contradicts the supposition that jurisdiction n^k is an optimal choice for agent i under prices λ. ■

Theorem 3. Assume that utility and cost functions satisfy monotonicity, convexity, and differentiability and in addition, the economy satisfies SSGE. Let (X,y,n) be a state of the economy in the core. Then there exists a price system λ such that (X,y,n), and λ constitute a nonanonymous Lindahl equilibrium.

Proof of Theorem 3. Since the economy satisfies SSGE, regardless of their jurisdiction membership by Theorem 1 a core state assigns all agents of the same type the same utility level. Denote the utility received by agents of crowding type c and taste type t in the core state (X,Y,n) by U_{ct}. For all $c \in \mathcal{C}$ and $t \in \mathcal{T}$, denote the willingness of an agent of type (c,t) to pay to join a jurisdiction with profile $m \in \mathcal{N}_{ct}$ producing public good level y as

$$w_{ct}(y,m) \equiv \omega_t + h_t(y,m) - U_{ct}. \tag{5.7}$$

Now define the optimal level of public goods for each type of jurisdiction as follows:

$$y^*(m) = \left\{ y \in \Re_+ : \sum_{ct} m_{ct} mwp_{ct}(y,m) = mc(y,m) \right\}. \tag{5.8}$$

We next define a price system (p,q) by

$$q_{ct}(m) = mwp_{ct}(y^*(m), m) \text{ and} \tag{5.9}$$

$$p_{ct}(m) = w_{ct}(y^*(m), m) - y^* (m)\, mwp_{ct}(y^*(m), m). \tag{5.10}$$

1. We begin by showing that when agents maximize utility under these prices, they can do no better than in the jurisdictions to which they are assigned by the core state. Note first that, by construction, for any jurisdiction $m \in \mathcal{N}$ under the prices defined above a utility maximizing agent in m will always demand $y^*(m)$ public good as this equates his marginal benefit to his marginal cost, $q_{ct}(m)$. Also by construction, for all $m \in \mathcal{N}$, and all $c \in \mathcal{C}$ and $t \in \mathcal{T}$ it holds that

$$U_{ct} = \omega_t + h_t(y^*(m), m) - w_{ct}(y^*(m), m) = \tag{5.11}$$

$$\omega_t + h_t(y^*(m), m) - y^* (m)\, mwp_{ct}(y^*(m), m) - p_{ct}(m) =$$

$$\omega_t + h_t(y^*(m), m) - y^* (m)\, q_{ct}(y^*(m), m) - p_{ct}(m).$$

Thus, all agents are indifferent over all jurisdictions when choosing an optimal level of public good under the prices constructed above. In particular, choosing the bundle assigned by the core state is optimal. Therefore, these prices satisfy condition (1) of the definition of the nonanonymous Lindahl equilibrium.

2. Now we show that no jurisdiction can make positive profits under these prices. First observe that for any jurisdiction $m \in \mathcal{N}$, given this profile of agents, a profit maximizing firm will choose to produce $y^*(m)$ since by construction that choice equates marginal revenue $\sum_{ct} m_{ct} q_{ct}(m)$ and marginal cost. By construction,

$$U_{ct} = \omega_t + h_t(y^*(m), m) - y^* (m)\, q_{ct}(y^*(m), m) - p_{ct}(m). \tag{5.12}$$

Thus,

$$\sum_{ct} m_{ct} U_{ct} = \sum_{ct} m_{ct} \left(\omega_t + h_t(y^*(m), m) - y^* (m)\, q_{ct}(y^*(m), m) - p_{ct}(m)\right). \tag{5.13}$$

Suppose that the total revenue collected by a firm for producing the profit maximizing bundle of public good yielded positive profits:

$$\sum_{ct} m_{ct} \left(y^* (m)\, q_{ct}(y^*(m), m) + p_{ct}(m)\right) > f\left(y^*(m), m\right) \tag{5.14}$$

But then

$$\sum_{ct} m_{ct} U_{ct} < \sum_{ct} m_{ct} \left(\omega_t + h_t(y^*(m), m)\right) - f\left(y^*(m), m\right). \tag{5.15}$$

However, this implies that the agents in m can do better than they do in the core by producing $y^*(m)$ and distributing the surplus private good. This contradicts the hypothesis that (X, Y, n) is a core state. Thus, condition (2) of the definition of nonanonymous Lindahl equilibrium is satisfied.

3. Finally, we show that the jurisdictions in a core partition generate enough revenue at the constructed prices to pay for the public good level they provide. Consider any jurisdiction $n^k \in n$ that appears in the core state. From part (2) above we know that

$$\sum_{ct} n_{ct}^k \left(y^* \left(n^k\right) q_{ct}(y^*(n^k), n^k) + p_{ct}(n^k)\right) \leq f\left(y^*(n^k), n^k\right). \tag{5.16}$$

Suppose that

$$\sum_{ct} n_{ct}^k \left(y^* \left(n^k\right) q_{ct}(y^*(n^k), n^k) + p_{ct}(n^k)\right) < f\left(y^*(n^k), n^k\right). \tag{5.17}$$

By construction,

$$U_{ct} = \omega_t + h_t(y^*(n^k), n^k) - y^* \left(n^k\right) q_{ct}(y^*(n^k), n^k) - p_{ct}(n^k), \tag{5.18}$$

and so

$$\sum_{ct} n_{ct}^k U_{ct} = \sum_{ct} n_{ct}^k \left(\omega_t + h_t(y^*(n^k), n^k) - y^* \left(n^k\right) q_{ct}(y^*(n^k), n^k) - p_{ct}(n^k).\right) \tag{5.19}$$

But then

$$\sum_{ct} n_{ct}^k U_{ct} > \sum_{ct} n_{ct}^k \left(\omega_t + h_t(y^*(n^k), n^k)\right) - f\left(y^*(n^k), n^k\right). \tag{5.20}$$

However, by hypothesis that (X, Y, n) is a core state, and we so we know it is feasible for the jurisdiction n^k to produce the optimal level of public good $y^*(n^k)$ while providing its members with utility levels U_{ct}. This implies

$$\sum_{ct} n_{ct}^k U_{ct} = \sum_{ct} n_{ct}^k \left(\omega_t + h_t(y^*(n^k), n^k)\right) - f\left(y^*(n^k), n^k\right), \tag{5.21}$$

a contradiction, and so condition (3) of the definition of nonanonymous Lindahl equilibrium is satisfied by these prices.

■

Theorem 4. Let (X, y, n) and λ constitute an anonymous Lindahl equilibrium. Then (X, y, n) is a core state.

Proof of Theorem 4. Since the set of anonymous Lindahl equilibria are a subset of the set of nonanonymous Lindahl equilibria, this follows immediately from Theorem 1. ■

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