Meetings with costly participation

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Abstract

We study a collective decision-making process in which people who are interested in an issue are invited to attend a meeting, and the policy chosen is a compromise among the preferences of those who show up. We show that in an equilibrium the number of attendees is small and their positions are extreme, and that when the compromise is the median, the outcome is likely to be random. The model and its equilibria are consistent with evidence on the outcome of hearings on US *regulatory policy.* (*JEL* D7, H0, L5)

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We study a model of collective decision-making. Each member of a group of people independently decides whether to participate, at a cost, in a decision-making process whose outcome is a compromise among the participants' favorite positions. In deciding whether or not to participate, each person compares the cost of doing so with the impact of her presence on the compromise. Our analysis focuses on the determination of the set of participants and the resulting collective decision.

Much US federal regulation is made by such a process. Federal regulators are required to seek out and respond to public comment on proposed regulations (see Section III); participation in the regulatory process is costly because participants spend time preparing submissions and may have to travel to hearings. Our model of collective decision-making is a stylized description of the rules used to determine regulations. Existing analyses of the regulatory process typically ignore procedural details (as in Gary S. Becker, 1985), or characterize optimal procedures (as in Jean-Jacques Laffont and Jean Tirole, 1993, chs. 11, 15).

Our model also describes other forms of participatory democracy. Examples include the ancient Athenian *ekklesia* (the primordial democratic assembly), parent-teacher associations, faculty associations, neighborhood associations, and many societies and clubs.

Under a wide range of conditions equilibria have the following features.

Nonparticipation of moderates In any equilibrium a bloc of moderates does not participate.

Low participation The proportion of individuals in a large group who participate is small.

Randomness of the outcome If the compromise is the median and individuals are prevented from attending with arbitrarily small probability then the outcome varies randomly and is likely to be extreme.

I. The Model

We are interested in collective decision processes in which the group of people affected by the decision have heterogeneous preferences, participation in the decision-making process is costly, and the outcome is a compromise among the participants' preferences. We study the following model.

A group of n people must collectively choose a *policy*, a point in a compact convex subset of $\mathbb{R}^\ell$ (that for convenience contains 0). We refer to a policy also as a *position*. We denote person i's favorite policy by x_i.

Each person cares about the remoteness of the collectively chosen policy from her favorite policy. Specifically, person i's valuation of the policy x is $v(x_i - x)$, where $v : \mathbb{R}^\ell \to \mathbb{R}_-$ is a continuous *valuation function*, with $v(0) = 0$. (If x is random, each person's valuation is the expected value of $v(x_i - x)$.) We assume that v decreases in each direction: for each $d \in \mathbb{R}^\ell$, $v(\alpha d)$ decreases in α for $\alpha \geq 0$. This assumption allows v to be asymmetric: a person's dislike for a policy may depend on the direction in which it differs from her favorite policy. If each person's valuation of a policy x depends only on the (Euclidean) distance between x and her favorite policy, we say that the valuation function is *symmetric*, and, with a slight abuse of notation, denote person i's valuation of policy x by $v(\|x_i - x\|)$.

Each person chooses whether or not to attend a meeting, at which a policy is selected. Denote by a_i the action of person i—either *Attend* or *Do not attend*. Every person who attends a meeting bears the cost $c > 0$. We refer to a person who attends as an "attendee" or "participant".

Given an action profile a (a list of actions, one for each person), let $X(a)$ be the list of the participants' favorite positions. We use set notation for operations on lists of positions: $X \setminus \{x\}$ is the list that differs from X only in that one copy of the position x is excluded, and $X \cup \{x\}$ is the list consisting of all elements of X with the addition of the position x.

The *outcome* of the action profile a is a policy $m(X(a))$, where m is the *compromise function*. We assume that if no one attends the meeting, the compromise is an arbitrary "default policy"; if a single person attends, the compromise is her favorite policy; and any person's switching from attendance to nonattendance moves the compromise further from her favorite policy. In a one-dimensional policy space ($\ell = 1$), the median satisfies all these conditions[1]. For a policy space of any dimension any weighted mean satisfies all the conditions.

Some of our results apply to arbitrary compromise functions, while others are restricted to the median over a one-dimensional policy space. The median has special appeal as a model of compromise over a one-dimensional space. First, it is the only policy not defeated by any other policy in majority-rule

[1]If the number of attendees is odd, the median is the middle favorite policy of the attendees. If the number of attendees is even and positive, we take the median to be the mean of the two middle favorite policies of the attendees. (Taking it instead to be a random variable that assigns probability $\frac{1}{2}$ to each middle position in this case makes little difference to the character of the equilibria.)

two-way contests[2]. Second, it is the unique member of the core of the coalitional game in which each majority coalition can enforce any outcome while each minority coalition is powerless.[3] Third, it is a subgame perfect equilibrium outcome of any binary agenda game in which no voter's strategy in any subgame is weakly dominated, and is the only such outcome if the number of attendees is odd. This fact is particularly relevant in a study of meetings, because binary agenda games are intended to model committee procedure. Fourth, if everyone knows that the outcome will be the median of the policies they announce, no player has an incentive to announce a policy other than her favorite policy: the announcement of her favorite policy weakly dominates every other announcement. Finally, if the outcome is determined by competition for votes by two parties, the median voter theorem implies that both parties will propose the median favorite policy of the participants.

In summary, we study a strategic game in which the players are the n people, each player's set of actions is $\{Attend, Do\ not\ attend\}$, and each player's payoff to an action profile is equal to her valuation of the outcome of this profile, less c if she attends. A (Nash) equilibrium of the game is an action profile for which no player is better off changing her action, given all the other players' actions.

We interpret a Nash equilibrium as a steady state. Each player, through her experience in similar situations, knows the players who will participate and their positions (or, in the case of a mixed strategy equilibrium, the partic-

[2]When the number of attendees is odd, the median defeats every other policy in majority-rule two-way contests (it is the "Condorcet winner").

[3]Consequently the median is the outcome of any stationary subgame perfect equilibrium of the extensive game modeling committee procedure studied by Eyal Winter (1997).

ipation probabilities and positions of those whose participation probabilities are positive); each player makes her participation decision optimally, given this knowledge. Each player does not necessarily know the positions of players who do not participate; an external agent (e.g. regulator) that organizes a meeting may not know the favorite position of *any* player prior to the meeting.

II. Intuition

A. *Nonparticipation of Moderates (Section V)*

In order for a player's participation to be worthwhile, her withdrawal must sufficiently increase the distance of the compromise from her favorite policy. Suppose that the valuation function is concave and the sensitivity of the compromise to a player's withdrawal is nondecreasing in the distance of the player's favorite position from the compromise. Then the further a player's favorite position is from the compromise, the more her payoff changes if she withdraws. (Refer to Figure 1.) Thus in an equilibrium only players whose favorite positions are sufficiently far from the compromise attend; similarly, only players whose favorite positions are sufficiently close to the compromise do not attend.

The mean has the required nondecreasing sensitivity, as does any weighted mean for weights sufficiently close to uniform. The median has nondecreasing sensitivity when the number of attendees is even, and we show that the equilibrium number of attendees is indeed even when the compromise function is the median and the valuation function is concave and symmetric.

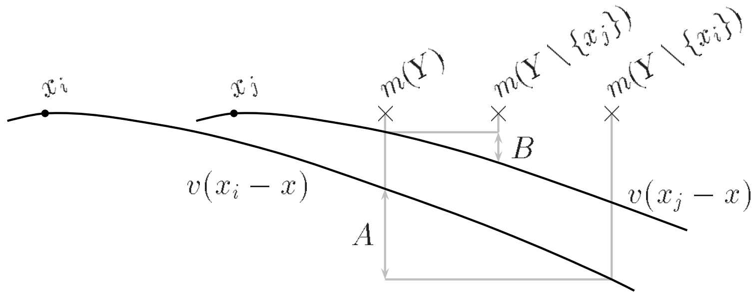

Figure 1. When the list of attendees' favorite positions is Y, the compromise is $m(Y)$. If player i withdraws, the compromise moves to $m(Y \setminus \{x_i\})$, while if player j withdraws, the compromise moves a smaller distance, to $m(Y \setminus \{x_j\})$. Because of the concavity of the valuation function, the change (A) in player i's valuation exceeds the change (B) in player j's valuation.

B. *Low Participation (Section VI)*

Suppose that the impact of an attendee's withdrawal on the compromise decreases to zero as the number of participants increases. Because the impact of an attendee's withdrawal on the compromise has to be large enough to offset the cost of attendance, we deduce that the equilibrium number of attendees is relatively small. We show, in fact, that in a large population the fraction of attendees is close to zero.

The mean satisfies this condition of decreasing impact. The median does not: a player's withdrawal may have a large impact on the median even in a large population. However, if randomness is added to the model, then the median *does* satisfy this condition—no matter how small the amount of randomness. Specifically, assume that with fixed probability $p > 0$ any player who intends to participate is (independently) prevented from doing so (e.g. she gets a flat tire on the way to a meeting). Then given the players' intentions, the set of attendees is random. In order for a player's attendance to be worthwhile, the expected impact of her attendance must be large enough. Hence the probability must be high enough that there is a large gap on at least one side of the median of the attendees' favorite positions. But this is possible only if the degree of randomness is small, which in turn is possible in a large population only if the fraction of attendees is small (given the fixed probability $p > 0$). We conclude that participation is low in a large population when the compromise function is the median and there is a positive probability that an intended participant will be prevented from attending.

C. *Randomness of the Outcome (Section VII)*

We find that when the compromise function is the median, the outcome is likely to be highly random. This finding has two bases.

First, in a wide range of circumstances the game has no pure strategy equilibrium: in all equilibria some individuals attend with positive probability less than one. In such a mixed strategy equilibrium there is, with high probability, an interval of moderates who do not participate. The reason is the same as before: a participant's withdrawal must significantly affect the outcome in order for her participation to be worthwhile. Thus even a small amount of randomness in the players' equilibrium actions generates significant randomness in the equilibrium outcome.

Second, we show that in the modified model in which chance events prevent intended participants from attending, the equilibrium probability that the outcome is extreme is significant. As in an equilibrium in the absence of chance events, two groups of extremists of equal or almost equal sizes intend to participate. Thus the outcome is extreme unless equal numbers of intended participants on each side are prevented from attending. Because each intended participant is independently prevented from attending with the same probability, the probability of this occurring is bounded away from one independent of the population size, and thus the probability of an extreme outcome does not approach zero in a large population.

III. Anecdotal Evidence

The properties of the process by which much US federal regulation is made correspond to those of our model.[4] The requirement that regulators seek out and respond to public comment about proposed regulation is contained in legislation and court cases. The Administrative Procedure Act requires that all US federal regulatory agencies "shall give interested persons an opportunity to participate in the rule making through submission of written data, views, or arguments with or without the opportunity for oral presentation"[5]. The Magnuson Fishery Management and Conservation Act specifies the way in which interested parties' views will be heard: it requires "public hearings ... to allow all interested persons the opportunity to be heard in the development of fishery management plans"[6]. Finally, *Corrosion Proof Fittings v. Environmental Protection Agency* establishes that public opinion must be adequately consulted. In this case, the court vacated proposed regulation because the Environmental Protection Agency prematurely ended public hearings and deprived the

[4] We exclude cases in which regulatory discretion is eliminated by statute.
[5] Title 5 *U.S. Code* §553(c), 1988 edition.
[6] Title 16 *U.S. Code* §1852(h)(3), 1988 edition.

public of sufficient opportunity to "comment [on], analyze, and influence the [regulatory] proceedings"[7].

Specifically, the regulation of New England federal fisheries and Rhode Island state fisheries corresponds closely to our model. Fishers often travel long distances to attend public hearings, and regulation depends in part on the positions taken by attendees at these hearings. Both regulatory bodies occasionally change policies dramatically from one meeting to the next, solely because of changes in the set of participants in the hearings. For example, George Allen (1991) describes a conflict between conservation-minded sport fishers and extraction-minded commercial fishers in Rhode Island. Both groups took fairly extreme positions and the attendance at two successive public hearings was lopsided in different directions, producing a policy that was first pro-conservation and then pro-extraction. Similarly, the record of the public hearings held by the New England Fishery Management Council (1985, p. 9.45) describes a conflict between two different groups of fishers (gillnetters and trawlers), who attended successive public hearings in lopsided proportions. As in Rhode Island, the result was a policy that first favored one group, then the other. In both cases the meetings were dominated by groups with extreme preferences about the policy.

Randomness of the outcome and the nonparticipation of moderates are also apparent in the history of timber harvesting in the Pacific Northwest. Kathie Durbin (1996) chronicles timber policy there from the early 1970's until the mid 1990's. This policy was formed with very little input from moderates: the

[7]In this case public participation is mandated by the Toxic Substances Control Act rather than the Administrative Procedure Act.

factions involved were primarily environmentalists and timber interests. Both groups appear to have preferences sharply at odds with the median preference in the region—one group is prepared to forego all other activities to save trees, while the other wants to cut all trees down. Further, the policy on timber extraction fluctuated dramatically from year to year.

Another situation to which our model applies is the Athenian assembly (*ekklesia*) of the fifth century BC. Any citizen could attend the assembly, and although items could be put on the agenda only by a council of 500 people, participants could make alternative proposals—compromise was possible (Mogens H. Hansen, 1991, pp. 138–139; Josiah Ober, 1989, p. 109). Attendance was costly, and the evidence suggests that attendance was not more than 6,000 of the approximately 30,000 eligible, although at least 12,000 citizens lived close to the assembly place (Hansen, 1976; R. K. Sinclair, 1988). In the fourth century a payment for attendance (of approximately a day's wage) was introduced to improve attendance; Aristotle (1959, p. 283) writes that "When the democracy was first restored, no payment was allowed for attendance at the Assembly, with the result that absenteeism was common. The [council of 500] tried all sorts of tricks to get the citizens to come and ratify the votes, but in vain."

IV. Related Literature

The focus of our model is the determination of the set of participants in a meeting and its implications for the action decided. The determination of the outcome *given* the set of participants is the focus of many strategic models of bargaining. Most of these models study the division of a pie, rather than the selection of a policy in some space. Winter's (1997) model is an exception. He studies a strategic model of committee procedure whose stationary subgame perfect equilibrium, under our assumptions, leads to the median of the participants' favorite positions.

Models of costly voting (for example, Thomas R. Palfrey and Howard Rosenthal 1983, 1985) bear a family resemblance to our model. However, the questions addressed are different from those we study, and voters' incentives differ fundamentally from the players' incentives in our model. A citizen's vote affects the outcome only if it is cast for one of the two leading alternatives; a citizen cannot introduce new alternatives. This assumption is appropriate when studying voting in elections, but not when studying the compromise reached in a meeting.

In the voting literature, Timothy J. Feddersen's (1992) model is closest to ours. Feddersen analyzes a game in which citizens simultaneously decide whether to vote and the policy to vote for; the policy with the most votes wins. When the policy space is one-dimensional, in every equilibrium exactly two policies receive votes, and these policies tie. The set of equilibrium pairs is large; in an example with quadratic preferences, every possible policy is part of some equilibrium.

Feddersen's model leaves open the question of how agents coordinate on a particular pair of policies. In a model of elections, it may be reasonable to leave this coordination problem open—arguably the role of parties is to help solve it. But in a model of meetings, in which the participants are well-informed about each others' preferences, it is reasonable to assume that the participants will coordinate only on policies that reflect some kind of compromise among their favorite positions. Our compromise function may thus be viewed as a solution of the coordination problem in Feddersen's model.

In the literature on economic regulation, our approach is most closely related to that of Becker (1983, 1985). Becker proposes that regulators respond to "pressure" from various interest groups, and that regulation favors groups better able to apply pressure. Exogenous "influence functions" describe how regulators respond to political pressure. Unlike Becker, we explicitly model the process by which regulation is selected. In consequence, Becker's approach is more general, while our approach fits a particular class of regulatory problems better. Our model makes predictions very different from those of Becker's. In particular, our approach endogenizes the formation of factions and predicts that outcomes vary from meeting to meeting. Becker's factions are exogenous and the outcome of his model is unique and deterministic.

Several models share significant features with ours. Feddersen and Wolfgang Pesendorfer (1996) analyze a voter's decision to participate in an election in a setting with imperfect information; they find that some types of voters will choose strategic abstention. This result is driven by the way that elections aggregate information. The aggregation of information is also at the heart of Susanne Lohmann's (1993) model. Individuals may take costly political action to signal their information to a leader who aims to please the median voter. In an equilibrium extremists are the activists. Jeffrey Zwiebel (1995) analyzes the adoption of a new technology by a heterogeneous group of managers. Like us he finds that the set of players separates into extremists and moderates: good and bad managers adopt a new technology, average managers do not. These results are driven by managers' desires to influence their own wages, not by any effort to manipulate a collective decision. Michael R. Baye et al. (1993) find that outcomes are random in a model of lobbying based on an "all pay" auction. They are primarily interested in the decision by a policy maker to exclude some interested parties; their analysis does not apply when regulators are under a statutory obligation to allow public participation. Avinash Dixit and Mancur Olson (1998), in a model closely related to that of Palfrey and Rosenthal (1984), explore the decisions of the members of a homogeneous group to participate in the private provision of a public good. They study symmetric mixed strategy equilibria, focusing on the probability with which the good is provided.

V. Nonparticipation of Moderates

We first give conditions under which there is an equilibrium in which no "moderate" attends, and all "extremists" attend: that is, a person attends if and only if her favorite position is sufficiently far from the central position.

Say that a list of positions is *symmetric* if the number of occurrences of $-x$ in the list is the same as the number of occurrences of x, and a compromise function m is *symmetric* if for any symmetric list X of positions and any position x we have $m(X) = 0$ and $m(X \cup \{x\}) = -m(X \cup \{-x\})$. Say that a symmetric compromise function m has **nondecreasing sensitivity on symmetric lists** if for any symmetric list X of positions, the effect on the compromise of a player's withdrawal is a nondecreasing function of the distance of her favorite position from $m(X) = 0$; that is, for any $x \in X$ and $x' \in X$ with $\|x\| > \|x'\|$ we have

$$\|x - m(X \setminus \{x\})\| - \|x\| \geq \|x' - m(X \setminus \{x'\})\| - \|x'\|.$$

For a one-dimensional policy space the median is symmetric and has nondecreasing sensitivity; for a space of any dimension, any weighted mean in which the weights are symmetric about 0 is symmetric, and any such weighted mean in which the weights are sufficiently close to uniform has nondecreasing sensitivity.[8]

PROPOSITION 1: *Suppose that the valuation function is concave and symmetric, the set of all the players' positions is symmetric, the default policy is* 0*, and the compromise function is symmetric and has nondecreasing sensitivity on symmetric lists. Then for some real number $z^* \geq 0$ there is an equilibrium in which every player i for whom $\|x_i\| > z^*$ attends and every player i for whom $\|x_i\| < z^*$ does not attend.*

(A proof is in the appendix, together with all other proofs not given in the text.) An example of a symmetric equilibrium in a two-dimensional policy space is shown in Figure 2.

We now give a lower bound on the distance of any attendee's favorite position from the compromise and an upper bound on the distance of any nonattendee's favorite position from the compromise. This result depends on

[8]Precisely, a sufficient condition for the weighted mean $\sum w(x)x / \sum w(x)$ to have nondecreasing sensitivity is that whenever $\|x\| > \|x'\|$ we have $(w(x') - w(x))/w(x') \leq \frac{3}{5}(\|x\| - \|x'\|)/\|x\|$.

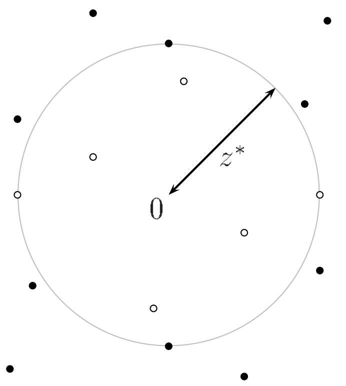

Figure 2. An example of a symmetric equilibrium in two dimensions, which exists under the conditions in Proposition 1. Each small disk is the favorite position of a player who attends, while every small circle is the favorite position of a player who does not attend. Every player whose favorite position is outside the gray circle attends, while no player whose favorite position is inside the circle does so. Players whose favorite positions are on the circle may or may not attend.

the concavity of the valuation function, but not on any characteristics of the compromise function.

When m is the mean, for any finite list X of positions with at least two members and any $x \in X$ we have $\|x - m(X \setminus \{x\})\| = (k/(k-1))\|x - m(X)\|$, where k is the number of members of X. For other compromise functions the ratio $\|x - m(X \setminus \{x\})\|/\|x - m(X)\|$ may depend on X (not only on its cardinality), and on x. For any $k \geq 2$ define the *withdrawal sensitivity* $\overline{\beta}(k)$ to be the highest value of this ratio for all k-member lists X and all $x \in X$:

$$\overline{\beta}(k) = \sup_{\{(X,x):|X|=k,\, x\in X,\text{ and } x\neq m(X)\}} \frac{\|x - m(X \setminus \{x\})\|}{\|x - m(X)\|}.$$

Because we assume that any person's withdrawal moves the compromise away from her favorite position, we have $\overline{\beta}(k) \geq 1$. Similarly, for any $k \geq 1$ define the *attendance sensitivity* $\underline{\beta}(k)$ to be the highest value of the ratio $\|x - m(X \cup$

$\{x\})\|/\|x - m(X)\|$:

$$\underline{\beta}(k) = \sup_{\{(X,x):|X|=k \text{ and } x \neq m(X)\}} \frac{\|x - m(X \cup \{x\})\|}{\|x - m(X)\|};$$

we have $\underline{\beta}(k) \leq 1$.

For any integer $k \geq 2$, define $\underline{z}(k)$ to be ∞ if $\overline{\beta}(k) = 1$ and 0 if $\overline{\beta}(k)$ is ∞, and define $\overline{z}(k)$ to be ∞ if $\underline{\beta}(k) = 1$; otherwise define $\underline{z}(k)$ and $\overline{z}(k)$ to be the unique solutions of

$$v(\underline{z}(k)) - v(\overline{\beta}(k)\underline{z}(k)) = c$$

$$v(\underline{\beta}(k)\overline{z}(k)) - v(\overline{z}(k)) = c.$$

The condition defining $\underline{z}(k)$ is illustrated in Figure 3.

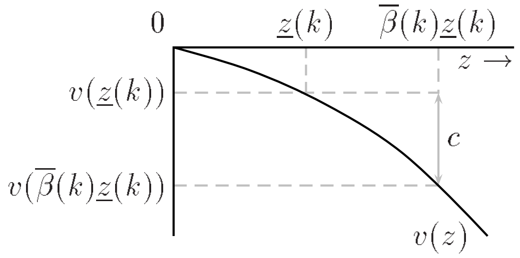

Figure 3. An illustration of the condition defining $\underline{z}(k)$.

The next result says that if the valuation function is concave then in any equilibrium in which there are k attendees, the distance of every attendee's favorite position from the compromise is at least $\underline{z}(k)$, and the distance of every nonattendee's favorite position from the compromise is at most $\overline{z}(k)$. Further, if the compromise is the mean then any configuration that satisfies these conditions is an equilibrium.

PROPOSITION 2: *Suppose that the valuation function is concave and symmetric. In any equilibrium in which $k \geq 2$ players attend we have*

$$\|x_j - m(Y)\| \geq \underline{z}(k) \qquad \textit{if } j \textit{ attends} \tag{1}$$

$$\|x_j - m(Y)\| \leq \overline{z}(k) \qquad \textit{if } j \textit{ does not attend} \tag{2}$$

where Y is the list of the attendees' favorite positions (so that, in particular, $|Y| = k$). If m is the mean and a is an action profile for which $k \geq 2$ players attend and the list of attendees' positions satisfies (1) and (2), then a is an equilibrium.

(For an arbitrary value of k, we can find lists Y of attendees' favorite positions that satisfy (1) and (2). In order for Y to correspond to an equilibrium, it must, in addition, have exactly k members.)

This proposition does not restrict the behavior of any player j for whom the distance between x_j and the compromise is between $\underline{z}(k)$ and $\overline{z}(k)$. Thus its bite is greater the larger is $\underline{z}(k)$ and the smaller is $\overline{z}(k)$. It implies that every equilibrium has the general character illustrated in Figure 4.

When the compromise function is the median, we can fully characterize all equilibria. We show that the number of attendees in any equilibrium is even. From Proposition 2 every attendee's favorite position is at least $\underline{z}(k)$ (which for the median is independent of k) from the compromise, so there is a gap of length at least $2\underline{z}(k)$ of nonattendees around the median. Thus any nonattendee outside the gap who switches to attendance moves the median by at least $\underline{z}(k)$. Given the strict concavity of the valuation function, we conclude that no nonattendee's favorite position can be too far from the median.

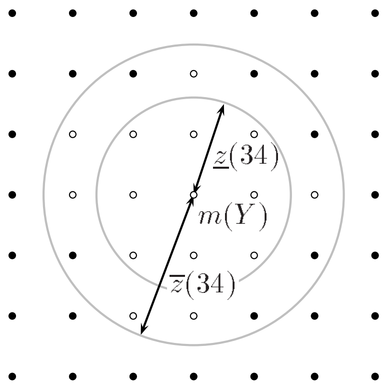

Figure 4. The character of an equilibrium (with 34 attendees) in two dimensions when the compromise function is the mean, as given by Proposition 2. Each small disk is the favorite position of a player who attends; each small circle is the favorite position of a player who does not attend. Every player whose favorite position is outside the outer gray circle attends, while no player whose favorite position is inside the inner circle does so. Players whose favorite positions lie between the two circles may or may not attend.

Precisely, define $\underline{y} = 2\underline{z}(k)$ and let $\overline{y}$ be the unique solution of $-v(\frac{1}{2}\overline{y}) = c$; for any $z \geq 0$, define $\Delta(z)$ to be the unique solution of $v(\Delta(z)) - v(z + \Delta(z)) = c$. Then we have the following result.

PROPOSITION 3: *Suppose that the policy space is one-dimensional, the compromise function is the median, the valuation function is strictly concave and symmetric, the list of all the players' favorite positions is symmetric, and the default policy is* 0. *An action profile with at least one attendee is an equilibrium if and only if the number of attendees is even, the distance between the favorite positions x_h and $x_i > x_h$ of the two central attendees h and i is at least $\underline{y}$, the distance between x_{h+1} and x_{i-1} is at most $\overline{y}$ if $i \geq h + 2$, and no player whose position is less than $x_h - \Delta(\frac{1}{2}(x_i - x_h))$ or more than $x_i + \Delta(\frac{1}{2}(x_i - x_h))$ does not attend.*

(If $c \geq -v(K)$, where $K = \max_j |x_j|$, there is an equilibrium in which no one attends, and if $c > v(K) - v(2K)$, there is no other equilibrium.) The result is illustrated in Figure 5.

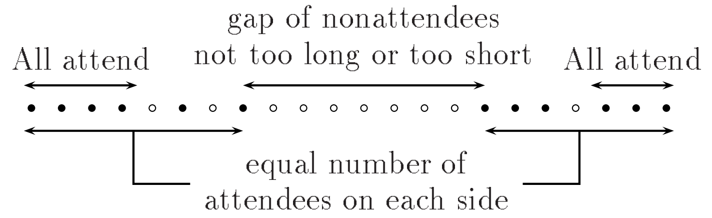

Figure 5. The structure of an equilibrium when the compromise function is the median and the valuation function is concave and symmetric, as given by Proposition 3. Each disk represents the favorite policy of an attendee and each circle represents the favorite policy of a nonattendee.

All the results so far assume that the valuation function is concave and symmetric. We now present two results that show that even in the absence of these assumptions, in every equilibrium in a one-dimensional policy space players whose favorite positions are close to the compromise do not attend.

The first result says that under weak conditions (primarily continuity) on the compromise function, any equilibrium has a gap of nonattendees on at least one side of the compromise. The idea is that there cannot be attendees close to the compromise on both sides because the withdrawal of any such player would have little effect on the compromise.

A compromise function m is **regular** if for any list X of positions and any $x \in X$, $m(X)$ is a convex combination of x and $m(X \setminus \{x\})$. (In a one-dimensional policy space, this condition holds if and only if when $x \leq m(X)$ we have $m(X \setminus \{x\}) \geq m(X)$, and when $x \geq m(X)$ we have $m(X \setminus \{x\}) \leq m(X)$.) A compromise function m is **continuous** if for any sequences $\{x^n\}_{n=1}^{\infty}$ and $\{y^n\}_{n=1}^{\infty}$ with $\lim_{n\to\infty} \|x^n - y^n\| = 0$ we have $\lim_{n\to\infty} \sup_X \|m(X \cup \{x^n\}) - m(X \cup \{y^n\})\| = 0$. (These conditions are satisfied by the median and by any weighted mean with a continuous weighting function.)

PROPOSITION 4: *Suppose that the policy space is one-dimensional and the compromise function is regular and continuous. Then there exists $\delta > 0$, independent of the number of players and their favorite positions, such that in every equilibrium at least one of the open intervals $(M-\delta, M)$ and $(M, M+\delta)$ contains no attendee's favorite position, where M is the equilibrium compromise.*

We argue later (Section VII) that when the compromise is the median in a one-dimensional policy space and the valuation function is not symmetric or not concave, pure strategy equilibria typically do not exist. Thus mixed strategy equilibria (which exist because the game is finite) are of particular interest. The next result says that in a large population, all mixed strategy equilibria share the main features of the equilibria characterized in Proposition 3 for symmetric concave valuation functions: there is an interval of positions such that most players whose favorite positions are in the interval participate with at most small probability; the sum of the participation probabilities of the players whose favorite positions are to the left of the interval is close to the sum of the participation probabilities of the players whose favorite positions are to the right of the interval; and within these two extreme groups most of the participation probabilities are close to either 0 or 1.

PROPOSITION 5: *Suppose that the policy space is one-dimensional and the compromise function is the median. Then there are positive numbers λ, γ, γ', and γ'', independent of the number of players and their favorite positions,* *such that for any mixed strategy equilibrium there is a position z such that*

$$\sum_{\{i:z\leq x_i\leq z+\lambda\}} p_i \leq \gamma, \quad \sum_{\{i:x_i<z\}} p_i(1-p_i) \leq \gamma', \quad \sum_{\{i:x_i>z+\lambda\}} p_i(1-p_i) \leq \gamma'$$

and

$$\left|\sum_{\{i:x_i<z\}} p_i - \sum_{\{i:x_i>z+\lambda\}} p_i\right| \leq \gamma'',$$

where p_i is the equilibrium participation probability of player i. In particular, $\sum_i p_i(1-p_i) \leq \gamma + 2\gamma'$.

The idea behind this result is that a player's participation is worthwhile only if it is sufficiently likely to significantly improve the median from her point of view. This requires a sufficiently high probability that the participants' positions have a significant gap (the interval $[z, z+\lambda]$) and that the numbers of participants on each side of this gap differ by at most 1. Now, the more random is the action profile, the less likely is such a gap to exist. Hence the amount of randomness ($\sum_i p_i(1-p_i)$) in an equilibrium is limited, and in a large population most of the participation probabilities must be close to 0 or 1.

VI. Low Participation

As the number of attendees increases without bound, the impact of any one player's attendance on the mean favorite position decreases to zero. The same is true of any weighted mean for which all relative weights go to zero as the number of attendees increases without bound. We show that for any compromise function with this property the equilibrium fraction of the population that attends converges to zero as the population size increases without bound.

The logic is that a player's attendance is worthwhile only if her withdrawal significantly changes the compromise, so that when the impact of a player's attendance decreases as the number of players increases, attendance is worthwhile only when the number of attendees is small.

Precisely, for a fixed compact policy space, say that a compromise function m reflects **small influence in a large meeting** if

$$\lim_{k \to \infty} \sup_{\{Y : |Y| \geq k\}} \sup_{y_i \notin Y} \|m(Y \cup \{y_i\}) - m(Y)\| = 0.$$

PROPOSITION 6: *Suppose the compromise function reflects small influence in a large meeting. Then for every valuation function and cost of attendance there exists an integer k, independent of the number of players in the population, such that in any (pure) equilibrium the number of attendees is no more than k.*

The median does not reflect small influence in a large meeting. If, for example, the policy space is the one-dimensional interval $[-1, 1]$, half of the attendees' favorite positions lie in $[-1, -\frac{1}{2}]$, and half of these positions lie in $[\frac{1}{2}, 1]$, a switch to attendance of a player whose favorite position is in $[\frac{1}{2}, 1]$ changes the median from 0 to at least $\frac{1}{2}$, regardless of the number of attendees. Indeed, Proposition 3 shows that when the compromise function is the median and the valuation function is concave and symmetric, the equilibrium fraction of the population that attends stays essentially constant as the size of the population increases.

However, if we perturb the model by adding a small amount of noise then the resulting (stochastic) compromise function reflects small influence in a large meeting. Specifically, suppose that with (small) probability p an exogenous event prevents any given player from attending, and these events are independent across players; suppose also that a player incurs the cost c only if she decides to attend and (with probability $1-p$) is not prevented from doing so. As we argued at the end of Section V, the greater the randomness in the set of players who attend, the smaller the impact of any player's switching between attendance and nonattendance. Thus in an equilibrium the amount of randomness is small. Given the exogenous probability p with which players are prevented from attending, the randomness is small only when the number of intended participants is small—if this number increases without bound as the size of the population increases, then the probability of any given player's attendance having a large influence on the median approaches zero. Thus given $p > 0$, in any equilibrium the number of attendees is small. (In particular, there is a discontinuity at $p = 0$ in the limiting fraction of the population that attends as the population size increases without bound.)

PROPOSITION 7: *Consider a sequence* $\{P^n\}_{n=2}^{\infty}$ *of populations such that (i) in each population* P^n*, the policy space is the one-dimensional interval* $[-1, 1]$ *and the default policy is 0, (ii) in* P^n *the players' favorite positions* $x_1^n, \ldots, x_n^n$ *are symmetric about 0 and satisfy* $x_1^n = -1$*,* $x_n^n = 1$*,* $x_i^n \leq x_{i+1}^n$ *for* $1 \leq i \leq n-1$*, and (iii)* $\lim_{n\to\infty} \max_i |x_i^n - x_{i-1}^n| = 0$*. Suppose that the compromise function is the median, the valuation function v is concave and symmetric, and* $c < -v(1)$*. Suppose that each player who decides to attend is independently prevented from doing so with probability* $p > 0$*. Then for each value of n the game for population* P^n *has a (pure) equilibrium in which the set of players* *who decide to attend is* $\{1, \ldots, j_n(p)\} \cup \{n - j_n(p) + 1, \ldots, n\}$*, where* $j_n(p) \geq 1$*. The function* j_n *remains bounded as* $n \to \infty$*.*

VII. Randomness of the Outcome

When the policy space is one-dimensional and the compromise function is the median, the outcome of an equilibrium in which there are two sets of participants separated by a large gap (like those in Proposition 3 when $\underline{y}$ is large) is very sensitive to a change in any player's action, which changes the compromise from the middle to one end of the gap. This suggests that in the presence of even a small amount of randomness, the equilibrium outcome may vary dramatically.

One source of such randomness is the players' behavior in a mixed strategy equilibrium. We argue that under a wide range of circumstances the game has no pure strategy equilibrium, so that randomness is inevitable: *all* equilibria are mixed. Specifically, if c is small enough that there is no equilibrium in which no one attends or one person attends, the game has no pure strategy equilibrium when the number of players is large if the valuation function is either (i) concave and sufficiently asymmetric, or (ii) strictly convex on each side of 0. We give the ideas behind the nonexistence of a pure equilibrium with an even number of participants in each case; the ideas for an odd number of participants is similar. For case (i), denote the two central attendees by j and $k > j$. Suppose that players are more sensitive to changes in positions to the left of their favorite positions than they are to changes in positions to the right. Now, j's desire to attend depends upon the gap of nonattendees being sufficiently long, while $k-1$'s desire not to attend depends upon this gap being sufficiently short. When the number of players is large these two requirements are incompatible. For case (ii), a player whose favorite position is close to the outcome is more sensitive to a change in the outcome than a player whose favorite position is far away, so that j's desire to attend conflicts with $j+1$'s desire not to attend. (We omit the straightforward details of the arguments.)

We now show that the randomness in any mixed strategy equilibrium causes the variance of the compromise to be high. By Proposition 5, in any equilibrium there is, with high probability, a gap of nonattendees, and any single player's change in attendance could significantly affect the median. This suggests that if any player i has an attendance probability $p_i \in (0,1)$ then the variance of the compromise is high.

PROPOSITION 8: *Suppose that the policy space is one-dimensional and the compromise function is the median. Then there exists $\eta > 0$, independent of the number of players, such that the variance of the compromise in any mixed strategy equilibrium is at least $\eta \max_i p_i(1-p_i)$, where p_i is the equilibrium probability of player i's attendance.*

Another source of randomness is exogenous, as in the model of Proposition 7 in which each player is prevented from attending with positive probability p. In this case we conclude not only that equilibrium outcome is random, but that, no matter how small p, in a large population the probability of an extreme outcome is bounded away from zero. From Proposition 7, for any population size the equilibrium set of intended participants is the union of two sets, one containing people with favorite positions close to -1 and one containing people with favorite positions close to 1. Even though these two sets have the same number of intended participants, the probability that they yield the same number of actual attendees is bounded above by a number less than 1 for all n. Thus the probability that the outcome is close to -1 or 1 (rather than being close to 0) is bounded away from zero independent of n.

PROPOSITION 9: *Under the conditions of Proposition 7, the equilibrium outcome is a random variable that assigns equal probability to* $[-1, x_{j_n(p)}]$ *and* $[x_{n-j_n(p)}, 1]$*; the infimum over* n *of the probability it assigns to each interval is positive.*

VIII. Concluding Comments

Our model fits a participatory democracy in which people disagree about the best policy, each person's participation in the procedure used to choose a policy is costly, and the outcome of this procedure is a compromise among the participants' favorite policies. We show that the outcome will be based on the participation of a small number of extremists, and, when the compromise is the median of the participants' favorite positions in a one-dimensional policy space, is likely to be significantly random, swinging from one extreme to the other over time. Some examples of US regulatory decisions are consistent with this evidence.

We have assumed that all players' participation costs are the same. At least some of our results generalize to the case in which individuals' costs are heterogeneous but uncorrelated with position, and all costs exceed some minimum. In this case the analogue of the equilibria in Propositions 1–3 is an equilibrium in which players sufficiently close to the compromise do not participate, and the remaining participants are players whose costs are low relative to the distance of their favorite positions from the compromise.

The properties we find for the equilibrium compromise may, at least in some contexts, be undesirable. In particular, the fact that the outcome is likely to be random and extreme when the compromise is the median in a one-dimensional policy space suggests that mechanisms based on voting may be undesirable. A question that arises is the character of an "optimal" mechanism.

Appendix: Proofs of Results

PROOF OF PROPOSITION 1:

We construct a symmetric equilibrium. If the action profile in which no one attends is an equilibrium, we take z^* to be larger than the distance of every player's favorite position from 0. Otherwise, we successively add symmetric pairs of players to the set of attendees, starting with players whose favorite positions are furthest from 0; we continue as long as, after the addition of each pair, each member of the pair is not better off withdrawing. We argue that this procedure creates an equilibrium (possibly one in which all players attend); we take z^* to be the distance from 0 of the favorite position of the attendee whose position is closest to 0.

Let A be a set of attendees constructed by the procedure, and let Y be the list of their positions. We have $m(Y) = 0$ by the symmetry of Y. To show that A is an equilibrium, we need to show that no player in A is better off withdrawing, and no player outside A is better off attending.

By construction, no player in A whose favorite position is closest to 0 among the positions in A is better off withdrawing. By the nondecreasing sensitivity of m and the concavity of v, no other player in A is better off withdrawing.

Now consider a player, say i, furthest from 0 outside A. We need to show that i is not better off attending. By construction, if *both* x_i and $-x_i$ are added to Y then i is better off withdrawing: $v(\|x_i - m(Y \cup \{x_i, x_j\})\|) - v(\|x_i - m(Y \cup \{x_j\})\|) < c$, where $x_j = -x_i$. By the symmetry of m we have $m(Y) = m(Y \cup \{x_i, x_j\}) = 0$ and $m(Y \cup \{x_i\}) = -m(Y \cup \{x_j\})$. Thus $v(\|x_i\|) - v(\|x_i + m(Y \cup \{x_i\})\|) < c$. But $\|x_i\| - \|x_i - m(Y \cup \{x_i\})\| \leq \|x_i + m(Y \cup \{x_i\})\| - \|x_i\|$ by the triangle inequality, so by the concavity of v we have $v(\|x_i - m(Y \cup \{x_i\})\|) - v(\|x_i\|) < c$. Thus i is worse off if she attends.

Finally, no other player outside A is better off attending because of the concavity of v and the nondecreasing sensitivity of m.

PROOF OF PROPOSITION 2:

An action profile a in which k players attend is a Nash equilibrium if and only if

$$v(\|x_j - m(Y)\|) - v(\|x_j - m(Y \setminus \{y_j\})\|) \geq c \qquad \text{if } j \text{ attends}$$
$$v(\|x_j - m(Y \cup \{x_j\})\|) - v(\|x_j - m(Y)\|) \leq c \qquad \text{if } j \text{ does not attend,}$$

where Y denotes the list of the attendees' positions.

Now, $\|x_j - m(Y \setminus \{x_j\})\| \leq \underline{\beta}(k)\|x_j - m(Y)\|$ and $\|x_j - m(Y \cup \{x_j\})\| \leq \overline{\beta}(k)\|x_j - m(Y)\|$. Thus in any equilibrium

(A1) $v(\|x_j - m(Y)\|) - v(\underline{\beta}(k)\|x_j - m(Y)\|) \geq c \qquad$ if j attends

(A2) $v(\overline{\beta}(k)\|x_j - m(Y)\|) - v(\|x_j - m(Y)\|) \leq c \qquad$ if j does not attend.

By the concavity of v, these conditions are equivalent to (1) and (2).

If m is the mean, we have $\|x_j - m(Y \setminus \{x_j\})\| = \underline{\beta}(k)\|x_j - m(Y)\|$ and $\|x_j - m(Y \cup \{x_j\})\| = \overline{\beta}(k)\|x_j - m(Y)\|$, so that a is an equilibrium if and only if it satisfies (A1) and (A2).

PROOF OF PROPOSITION 3:

Let a be an equilibrium with at least one attendee. First we argue that the number of attendees in a is even. If the number of attendees is odd, the median is the central attendee's favorite position, say x_i. (Refer to Figure 6.) Let the neighboring attendees' positions be x_h and x_j, and denote $y = \frac{1}{2}(x_h + x_j)$ (or $y = 0$ if i is the only attendee). If $x_i = y$, the withdrawal of player i has no effect on the outcome. Thus in an equilibrium $x_i \neq y$. Now consider a player, say ℓ, whose favorite position is symmetric with x_i about y. If this player attends, the distance the median moves is the same as the distance it moves if player i withdraws, so that, using the strict concavity of the valuation function, if player i is not better off withdrawing then player ℓ is better off attending. Thus the configuration is not an equilibrium.

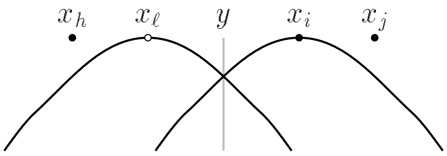

Figure 6. An illustration of the argument that when the compromise function is the median there is no equilibrium in which the number of attendees is odd. If either player i withdraws or player ℓ attends, the median changes from x_i to y.

Now let h and i be the two central attendees, so that $m(Y) = \frac{1}{2}(x_h + x_i)$. By Proposition 2, we have $x_i - x_h \geq 2\underline{z}(k) = \underline{y}$. If $i \geq h+2$ then players $h+1$ and $i-1$ do not attend. In order for their nonattendance to be optimal, we need $v(\frac{1}{2}(x_h+x_i)-x_{h+1}) \geq -c$, or, given the symmetry of the list of positions, $v(\frac{1}{2}(x_{i-1}-x_{h+1})) \geq -c$, or $x_{i-1}-x_{h+1} \leq \overline{y}$. Finally, suppose that $x_\ell < x_h - \Delta(x_i - x_h)$. If ℓ does not attend, her gain from switching to attendance is $v(x_h - x_\ell) - v(\frac{1}{2}(x_h + x_i) - x_\ell) - c$, or $v(x_h - x_\ell) - v(x_h - x_\ell + \frac{1}{2}(x_i - x_h)) - c$. Because $x_h - x_\ell > \Delta(x_i - x_h)$ and v is concave, we deduce that ℓ's gain to switching to attendance is positive. A similar argument shows that a player ℓ for whom $x_\ell > x_i + \Delta(\frac{1}{2}(x_i - x_h))$ must attend.

Now let a be an action profile that satisfies the conditions in the proposition. By arguments like those in the previous paragraph, no attendee is better off switching to nonattendance, and no nonattendee is better off switching to attendance.

PROOF OF PROPOSITION 4:

Suppose to the contrary that there is a sequence $\{a^\ell\}_{\ell=1}^\infty$ of equilibria (perhaps involving different numbers of attendees), for which $s^\ell \equiv \max\{|\underline{y}^\ell - m(Y^\ell)|, |\overline{y}^\ell - m(Y^\ell)|\} \to 0$, where Y^ℓ is the list of favorite positions of the attendees in a^ℓ, $\underline{y}^\ell$ is the largest member of Y^ℓ (strictly) less than $m(Y^\ell)$, and $\overline{y}^\ell$ is the smallest member of Y^ℓ (strictly) greater than $m(Y^\ell)$.

By regularity, $m(Y^\ell \setminus \{\underline{y}^\ell\}) \geq m(Y^\ell)$ and $m(Y^\ell \setminus \{\overline{y}^\ell\}) \leq m(Y^\ell)$. Now, $|\underline{y}^\ell - \overline{y}^\ell| \leq 2s^\ell$, so that by the continuity of m we have $|m(Y^\ell \setminus \{\underline{y}^\ell\}) - m(Y^\ell \setminus \{\overline{y}^\ell\})| \to 0$. We conclude that $|m(Y^\ell \setminus \{\underline{y}^\ell\}) - m(Y^\ell)| \to 0$ (and similarly $|m(Y^\ell \setminus \{\overline{y}^\ell\}) - m(Y^\ell)| \to 0$).

Now, because $v(0) = 0$ and v is continuous at 0, we conclude that for large enough n, the attendee in a^ℓ whose favorite position is $\underline{y}^\ell$ (and similarly the attendee whose favorite position is $\overline{y}^\ell$) is better off withdrawing, contradicting the assumption a^ℓ is an equilibrium.

PROOF OF PROPOSITION 5:

Assume, without loss of generality, that the policy space is the interval $[-1,1]$. For any $S \subseteq \{1,2,\ldots,n\}$, denote by $\mu(S)$ the equilibrium probability that the set of attendees is S:

$$\mu(S) = \left(\prod_{i\in S} p_i\right)\left(\prod_{i\notin S}(1-p_i)\right).$$

Let m_S be the median of $\{x_i \colon i \in S\}$. Given a player i, let $d_i(S) = |m_S - m_{S^i}|$, where S^i denotes the subset in which i's membership in S is changed: $S^i = S \cup \{i\}$ if $i \notin S$, and $S^i = S \setminus \{i\}$ if $i \in S$.

We show that in an equilibrium, every player i who attends with positive probability can, with some positive probability independent of n, change the equilibrium outcome by a positive amount (independent of n) by switching from attendance to nonattendance.

LEMMA A1: *There are positive real numbers α and β that do not depend on n, such that for all i with $p_i > 0$,*

$$\mu\{d_i \geq \alpha\} \geq \beta,$$

where $\mu\{d_i \geq \alpha\} = \sum_{\{S\colon d_i(S)\geq\alpha\}} \mu(S)$.

PROOF:

Because the valuation function v is continuous and the cost c of attending does not depend on n, a player i is willing to attend only if she has a good chance of having a large effect on m_S. Specifically, the expected change in m_S

from her attending must be at least a certain positive constant that does not depend on n. That is,

$$\mu(d_i) \equiv \sum_S \mu(S) d_i(S) \geq \delta > 0 \qquad \text{whenever } p_i > 0,$$

where $\delta > 0$ depends on neither n nor i. Hence $\mu(1-d_i) \leq 1-\delta$. Since $1-d_i$ is a nonnegative function, it follows from Markov's inequality that for any $t > 0$ we have $\mu\{1 - d_i \geq t(1-\delta)\} \leq 1/t$, or

$$\mu\{d_i > 1 - t(1-\delta)\} \geq 1 - 1/t.$$

Choosing t with $1 < t < 1/(1-\delta)$ and setting $\alpha = 1 - t(1-\delta) > 0$ and $\beta = 1 - 1/t > 0$, Lemma A1 follows.

For $-1 \leq a < b \leq 1$ and a subset S of players, say that the interval (a, b) is *balancing for S* if

$$\{i \in S : a \leq x_i \leq b\} = \varnothing \quad \text{and} \quad \Big| |\{i \in S \colon x_i < a\}| - |\{i \in S \colon x_i > b\}| \Big| \leq 1.$$

We write $\mu\{(a, b) \text{ is balancing}\} = \sum_{\{S:(a,b) \text{ is balancing for } S\}} \mu(S)$.

We now show that there is an interval $(z, z + \alpha)$ that is balancing with probability bounded away from zero.

LEMMA A2: *There are positive real numbers λ and ν that do not depend on n such that for each mixed strategy equilibrium there is a position z for which*

$$\mu\{(z, z + \lambda) \textit{ is balancing}\} \geq \nu.$$

PROOF:

If $p_i = 0$ for all i the statement is trivially satisfied, so assume $p_i > 0$ for some i. From Lemma A1 we have $\mu\{d_i \geq \alpha\} \geq \beta$. Thus from the definitions of d_i and the median, we conclude that

$$\mu\{\text{there exists } z \text{ such that } (z, z+2\alpha) \text{ is balancing}\} \geq \beta.$$

Now consider the $N = [2/\alpha]+1$ intervals of the form $(-1+\ell L, -1+(\ell+1)L)$ for $\ell = 0, \ldots, N-1$, each of length $L = 2/N < \alpha$. If there exists z such that $(z, z+2\alpha)$ is balancing, then one of these N intervals must itself be balancing. We thus have

$$\mu\{\text{there exists } \ell \text{ with } 0 \leq \ell \leq N-1 \text{ such that}$$
$$(-1+\ell L, -1+(\ell+1)L) \text{ is balancing}\} \geq \beta.$$

Hence, by sub-additivity of probabilities, there exists ℓ with

$$\mu\{(-1+\ell L, -1+(\ell+1)L) \text{ is balancing}\} \geq \beta/N.$$

Setting $z = -1+\ell L$, $\lambda = L$, and $\nu = \beta/N$, Lemma A2 follows.

We now prove the proposition. If it were false, there would be a sequence $\{(p_1^n, \ldots, p_n^n)\}_{n=1}^{\infty}$ of mixed strategy equilibria, where $(p_1^n, \ldots, p_n^n)$ is an equilibrium in a game with n players, such that one of the four expressions in the proposition would converge to infinity. We now argue that this is not the case.

Fix n, and let z, λ, and ν be as in Lemma A2. Let X_n, Y_n, and Z_n be the random variables equal to the number of attendees whose favorite positions are less than z, from z to $z+\lambda$, and greater than $z+\lambda$, respectively. From Lemma A2 we know that $\Pr(Y_n = 0) \geq \nu$ and $\Pr(|X_n - Z_n| \leq 1) \geq \nu$. We have $\Pr(Y_n = 0) = \prod_{\{i: z \leq x_i \leq z+\lambda\}} (1 - p_i)$, so that

$$\prod_{\{i: z \leq x_i \leq z+\lambda\}} (1 - p_i) \geq \nu.$$

But $1 - p_i \leq \exp(-p_i)$, so $\exp\left(-\sum_{\{i: z \leq x_i \leq z+\lambda\}} p_i\right) \geq \nu$, whence $\sum_{\{i: z \leq x_i \leq z+\lambda\}} p_i \leq \ln(1/\nu)$, establishing the first statement with $\gamma = \ln(1/\nu)$.

Next, note that since X_n and Z_n are independent random variables, we have $\Pr(|X_n - Z_n| \leq 1) = \sum_z \Pr(Z_n = z) \Pr(|X_n - z| \leq 1)$, so since $\Pr(|X_n - Z_n| \leq 1) \geq \nu$, there exists z with $\Pr(|X_n - z| \leq 1) \geq \nu$. Now assume for contradiction that $\mathrm{Var}(X_n) \equiv \sum_{\{i: x_i < z\}} p_i(1 - p_i)$ is unbounded as a function of n. Then by the Lindeberg Central Limit Theorem (see e.g. Patrick Billingsley 1995, Theorem 27.2), since X_n is the sum of independent Bernoulli (bounded) random variables, the distribution of X_n is approximately normal for certain sufficiently large n. Specifically, given any $\epsilon > 0$, we can find n such that $\Pr(x < X_n \leq y) \leq \int_x^y N(\mathrm{E}(X_n), \mathrm{Var}(X_n); t) dt + \epsilon$, where $N(m, v; t)$ is the density function of the normal distribution with mean m and variance v. But as $v \to \infty$, we have $\sup_{m,t} N(m, v; t) \to 0$. Hence we have $\Pr(x < X_n \leq y) \to 0$ as $n \to \infty$ for any fixed $x < y$, which contradicts the assumption that $\Pr(|X_n - z| \leq 1) \geq \nu$.

We conclude that $\sum_{\{i: x_i < z\}} p_i(1 - p_i)$ is bounded, say by γ', as a function of n. This establishes the second statement of the result; the third statement follows similarly.

Finally, since $\Pr(|X_n - Z_n| \leq 1) \geq \nu$ and $\mathrm{Var}|X_n - Z_n| \leq \mathrm{Var}(X_n - Z_n) \leq 2\gamma'$, we have by Chebyshev's inequality (see e.g. Billingsley, 1995, p. 80) that if $\mathrm{E}|X_n - Z_n| > 1$, then

$$\begin{aligned}\nu \leq \Pr(|X_n - Z_n| \leq 1) &= \Pr\left(|X_n - Z_n| - \mathrm{E}|X_n - Z_n| \leq 1 - \mathrm{E}|X_n - Z_n|\right) \\ &\leq \Pr\left(\Big||X_n - Z_n| - \mathrm{E}|X_n - Z_n|\Big| \geq \mathrm{E}|X_n - Z_n| - 1\right) \\ &\leq \frac{\mathrm{Var}|X_n - Z_n|}{(\mathrm{E}|X_n - Z_n| - 1)^2} \leq \frac{2\gamma'}{(\mathrm{E}|X_n - Z_n| - 1)^2},\end{aligned}$$

so that

$$|\mathrm{E}(X_n) - \mathrm{E}(Z_n)| \leq \mathrm{E}|X_n - Z_n| \leq 1 + \sqrt{2\gamma'/\nu}\,.$$

Thus the result follows with $\gamma'' = \max(1,\ 1 + \sqrt{2\gamma'/\nu}) = 1 + \sqrt{2\gamma'/\nu}$.

PROOF OF PROPOSITION 6:

Because v is continuous and the set of policies is compact, v is *uniformly continuous* on $\{y - M : y \text{ and } M \text{ are policies}\}$. Thus there exists $\delta > 0$ such that $|v(y - M_1) - v(y - M_2)| < c$ for all y whenever $\|M_1 - M_2\| < \delta$.

Now, because m reflects small influence in a large meeting, there is a positive integer k such that

$$\sup_{\{Y : |Y| \geq k\}} \sup_{y_i \notin Y} \|m(Y \cup \{y_i\}) - m(Y)\| < \delta.$$

Thus for any action profile in which k or more players attend, an additional player who enters changes the compromise by less than δ, and hence changes her valuation of the compromise by less than c. Thus in any equilibrium at most k players attend.

PROOF OF PROPOSITION 7:

An argument that an equilibrium of the form given in the proposition exists follows the lines of the proof of Proposition 1. We have $j_n(p) \geq 1$ because $c < -v(1)$.

The number $j_n(p)$ remains bounded as a function of n by the following argument. As in Lemma A1, we know that for a player to want to attend, her presence must have probability at least β of moving the median by at least α, where $\alpha > 0$ and $\beta > 0$ are independent of n. But for $0 < p < 1$, the proof of Proposition 5 shows that if $j_n(p) \to \infty$ then the probability that the number of attendees in $\{1, \ldots, j_n(p)\}$ is within one of the number of attendees in $\{n - j_n(p) + 1, \ldots, n\}$ goes to 0. Further, as $n \to \infty$ the probability that there is some other gap of size α in the list of positions of actual attendees also goes to 0. We conclude that if $j_n(p) \to \infty$ as $n \to \infty$ then the probability that a change in the action of any given player moves the median by at least α goes to 0, contradicting the fact that the action profile in which the set of attendees is $\{1, \ldots, j_n(p)\} \cup \{n - j_n(p) + 1, \ldots, n\}$ is an equilibrium.

PROOF OF PROPOSITION 8:

Fix a player i. Let S be the random variable that represents the realized set of attendees in the equilibrium, and let $R = S \setminus \{i\}$ (so that $S = R$ if i does not attend, and $S = R \cup \{i\}$ if i does attend). We have

$$\begin{aligned}
\mathrm{Var}\,(m(S) \mid R) &\equiv \mathrm{E}\left([m(S) - \mathrm{E}\,(m(S) \mid R)]^2 \mid R\right) \\
&= p_i \left[m(R \cup \{i\}) - (p_i m(R \cup \{i\}) + (1 - p_i) m(R))\right]^2 \\
&\quad + (1 - p_i) \left[m(R) - (p_i m(R \cup \{i\}) + (1 - p_i) m(R))\right]^2 \\
&= p_i (1 - p_i) \left[m(R \cup \{i\}) - m(R)\right]^2 .
\end{aligned}$$

Now, from Lemma A1 in the proof of Proposition 5, there is probability at least β that R is such that $d_i(R) \equiv |m(R \cup \{i\}) - m(R)| \geq \alpha$. Hence,

$$\begin{aligned} \mathrm{Var}\,(m(S)) &\geq \mathrm{E}\,[\mathrm{Var}\,(m(S) \mid R)] \\ &= \mathrm{E}\left[p_i(1-p_i)\,(m(R \cup \{i\}) - m(R))^2\right] \\ &\geq p_i(1-p_i)\beta\alpha^2\,. \end{aligned}$$

Hence, the result follows with $\eta = \beta\alpha^2$.

PROOF OF PROPOSITION 9:

The equality of probabilities for the outcome to be in the two intervals $[-1, x_{j_n(p)}]$ and $[x_{n-j_n(p)}, 1]$ follows from the symmetry of v and the symmetry of the list of the players' favorite positions.

Denote by $q_n(p)$ the probability that the equilibrium outcome is in $[-1, x_{j_n(p)}] \cup [x_{n-j_n(p)}, 1]$. We have $q_n(p) \geq 1 - E$, where E is the probability that the realized numbers of attendees in $\{1, \ldots, j_n(p)\}$ and in $\{n - j_n(p) + 1, \ldots, n\}$ are equal. But each of these two numbers of attendees independently follows the Binomial$(j_n(p), p)$ distribution. Hence the probability E that the numbers are equal is bounded above by the probability of the mode of this distribution. Given $j_n(p) \geq 1$ (see Proposition 7), this probability is no larger than the probability of the mode of Binomial$(1, p)$, namely $\max(p, 1-p)$. Hence $\inf_n q_n(p) \geq 1 - E \geq 1 - \max(p, 1-p) > 0$, as claimed.

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