A simple proof of the efficiency of the poll tax

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1 Introduction

In a recent article in this Journal, Richard M. Peck argues that imposition of an excise tax may constitute a potential Pareto improvement over an equal-yield uniform lump-sum tax (or poll tax). Professor Peck argues that, when the sum of taxpayers' compensating variations for a movement to excise taxation is positive, "those who prefer the excise tax can compensate those who prefer the uniform lump-sum tax. Therefore imposing an equal yield lump-sum tax in place of an excise tax on an entire economy may actually reduce aggregate welfare." (Peck, 1998, p. 243)

Since an allocation with a lump-sum tax is Pareto efficient, whereas an allocation with an excise tax generally is not, it would be surprising indeed if winners could compensate losers for a move to the latter. In this note, I show that the result is merely an instance of the long-established difficulties in using the sum of compensating variations for policy evaluation, which says nothing about the inefficiency of poll taxes.

It is shown that, if preferences are convex, imposition of an equal-yield excise tax *never* constitutes a true potential Pareto improvement (PPI) over lump-sum taxation. That is, compensation for a move to excise taxation is never possible, and the Hicks–Kaldor compensation test fails. If Scitovsky indifference curves cross and compensation is restricted to transfers of produced commodities rather than endowments, however, then the converse may also hold. Lump-sum taxation need not constitute a PPI over excise taxation, and winners could not compensate losers for a move in the opposite direction. This possibility is, however, independent of the sign of the sum of compensating variations for the policy change. The results I present are not new—indeed most are simple corollaries of results found in Blackorby and Donaldson (1990), for example—but are often overlooked in policy analysis. The problem was first pointed out by Boadway (1974), who established that the sum of compensating variations for a move between two Pareto-efficient allocations is non-negative, and is generally positive. It follows that a movement to an inefficient allocation, as in the excise tax case, can also lead to a positive aggregate compensating variation, as long as the distortion in demands is not too great.

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2 Results

Consider an economy consisting of two commodities, x and z, and H consumers. Commodities are produced under constant returns to scale at exogenous producer prices $(p, 1)$. Consumer i has exogenous lump-sum income y^i, measured in units of the numeraire, and preferences for consumption represented by a continuous, monotone, quasi-concave utility function $U^i(x^i, z^i)$. If the consumer prices of (x, z) are $(q, 1)$, the associated indirect utility and expenditure functions are denoted $V^i(q, y^i)$ and $E^i(q, u^i)$.

Let $Y = \sum_i y^i$ be aggregate income and $(X, Z) = (\sum_i x^i, \sum_i z^i)$ be aggregate demands. The government revenue requirement is R units of the numeraire, to be raised through imposition of an excise tax on x at rate t, or a uniform lump-sum tax $T = R/H$ on all consumers. Let (X_E, Z_E) denote aggregate demands when the excise tax is imposed, and (X_L, Z_L) denote aggregate demands under the lump-sum tax. If the allocations are feasible given revenue requirements, then

$$pX_i + Z_i = Y - R \qquad (i = E, L). \tag{1}$$

Consider a move from lump-sum taxation to excise taxation. An consumer's *compensating variation* c^i for the change is the amount of income the consumer would be willing to pay to have the change implemented; thus[1]

$$V^i(p + t, y^i - c^i) = V^i(p, y^i - T) \equiv u^i_L,$$

or, letting $u^i_E = V^i(p + t, y^i)$,

$$\begin{aligned} c^i &= y^i - E^i(p + t, u^i_L) \\ &= E^i(p + t, u^i_E) - E^i(p + t, u^i_L). \end{aligned} \tag{2}$$

When $c_A = \sum_i c^i > 0$, the net amount consumers would pay to move to excise taxation is positive, suggesting that aggregate consumer surplus has been generated by the switch. Professor Peck demonstrates that $c_A > 0$ is indeed possible and concludes that, in such cases, excise taxation constitutes a PPI over lump-sum taxation.

But the existence of aggregate consumer surplus for the change does not imply that winners could actually provide lump-sum compensation to losers and generate a Pareto improvement. To see this, for any vector of utilities $u = (u^1, \dots, u^H)$ define the *Scitovsky set* $B(u)$ as the set of aggregate consumption vectors which can be distributed to taxpayers in a way which yields each one a utility level at least as great as his or her utility level in u. Formally,

$$B(u) = \left\{ (\sum_i x^i, \sum_i z^i) : U^i(x^i, z^i) \geq u^i, i = 1, \dots, H \right\}. \tag{3}$$

Note that $B(u)$ is merely the set summation of individual no-worse-than sets and so is convex. Let

$$E(q, u) = \min\{qX + Z : (X, Z) \in B(u)\}$$

be the support function for $B(u)$. Note that $E(q, u) = \sum_i E^i(q, u^i)$.

[1] In Peck (1998), this defines $-c^i$. I have followed most other authors in reversing the sign of the measure.

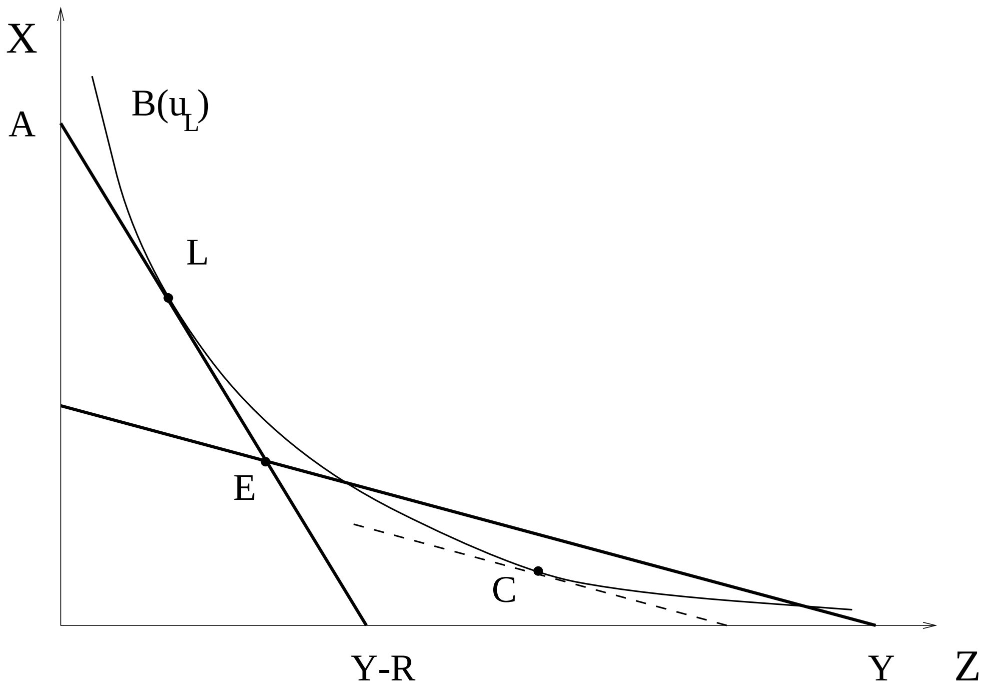

Figure 1: No potential Pareto improvements from the lump-sum tax.

The allocation (x_E, z_E) is a *potential Pareto improvement* over the allocation (x_L, z_L) if it possible to construct lump-sum transfers of commodities among taxpayers such that all are at least as well off after the transfers as at (x_L, z_L), and at least one taxpayer is strictly better off. Equivalently, excise taxation constitutes a PPI over lump-sum taxation if $(X_E, Z_E) \in B(u_L)$. It is easy to see, however, that this can never be the case.[2] Since $E(p, u_L) = pX_L + Z_L = Y - R$, and $B(u_L)$ is a convex set, the line $\{(\tilde{X}, \tilde{Z}) : p\tilde{X} + \tilde{Z} = Y - R\}$ is a supporting hyperplane for $B(u_L)$ at (X_L, Z_L). Thus

$$(X_E, Z_E) \in \text{int } B(u_L) \implies pX_E + Z_E > Y - R$$

which contradicts the feasibility condition (1) for (X_E, Z_E).

A simple, graphical version of the argument can be seen by inspecting Fig. 1. The boundary of $B(u_L)$ is tangent to the production possibility frontier, which is labelled $AY{-}R$, at the aggregate demand vector (X_L, Z_L), which is labelled L. If (X_E, Z_E) is feasible given the revenue requirement R, it must also lie on this line. Hence (X_E, Z_E) cannot lie on the interior of $B(u_L)$. Nevertheless, the sum of compensating variations for the change may be positive. Summing compensating variations in (2) gives

$$\begin{aligned} c_A &= Y - E(p + t, u_L) \\ &= E(p + t, u_E) - E(p + t, u_L) \end{aligned} \tag{4}$$

[2]The following argument can be applied without change to analyze excise taxation of multiple commodities.

In Fig. 1, c_A is therefore the horizontal distance from the actual budget line for the excise tax to the line with slope $-1/(p+t)$ which is tangent to the boundary of $B(u_L)$ at point C. In the figure, the aggregate demand vector (X_E, Z_E), which is labelled E, has been drawn such that $c_A > 0$, despite the fact that no PPI exists for a move to excise taxation.

The difficulty with the measure is that, if the income transfers envisaged were actually paid, then income effects would lead to a change in government revenue that would render the allocation infeasible. Since $E(p, u_L) = Y - R$, (4) can be expressed as

$$\begin{aligned} c_A &= E(p, u_L) - E(p+t, u_L) + R \\ &= [E(p, u_L) - E(p+t, u_L) + tX(p+t, u_L)] + t\,[X(p+t, u_E) - X(p+t, u_L)]\,. \end{aligned} \tag{5}$$

Since E is concave in price and $E_q(p+t, u_L) = X(p+t, u_L)$, the first term in brackets in (5) is non-positive, a measure of the distortionary effect of excise taxation. The second term in brackets may be positive, leading to $c_A > 0$, if income effects from compensation induce a sufficiently large fall in aggregate demand for the taxed commodity.

Alternatively, one may ask whether, beginning from the excise tax allocation, winners may compensate losers for a move to the poll tax.[3] This will be the case when $(X_L, Z_L) \in \text{int } B(u_E)$, which may or may not be satisfied.[4] Does the sign of c_A therefore indicate whether lump-sum taxation is a PPI over excise taxation? The answer again is no. Observe that c_A is minus the sum of equivalent variations for a change from excise to lump-sum taxation. Blackorby and Donaldson (1990) have demonstrated that a negative aggregate equivalent variation is neither necessary nor sufficient for a true potential Pareto improvement.[5]

An alternative explanation for the failure of the sum of compensating variations to accord with the compensation principle is that, given (4), c_A is a sum of money metrics for individual utility changes, measured at the distorted price $p + t$. It is then perhaps not surprising that c_A "overweights" utilities at the distorted allocation u_E, and indicates a potential efficiency gain where none exists. One might therefore expect that the sum of *equivalent* variations, which weights utility changes by undistorted prices, is an exact index for PPIs in both directions. But this is also not the case. The equivalent variation e^i of taxpayer i for a move to excise taxation solves $V^i(p+t, y^i) = u_E^i = V^i(p, y^i - T + e^i)$, so that

$$\begin{aligned} e_A = \sum_i e^i &= \sum_i E^i(p, u_E^i) - (Y - R) \\ &= E(p, u_E) - E(p, u_L). \end{aligned} \tag{6}$$

Analogous to (5), this can also be expressed as

$$e_A = E(p, u_E) - E(p+t, u_E) + tX_E. \tag{7}$$

[3]This is the version of the compensation test proposed by Hicks, in contrast to the usual version, associated with Kaldor. See Boadway and Bruce (1984) for a discussion.

[4]If $(X_L, Z_L) \notin B(u_E)$ then the two Scitovsky indifference curves must intersect, which may occur for almost all profiles of consumer preferences. That is, the strict social preference relation implied by PPI is neither complete nor asymmetric in general.

[5]In making this argument, I have restricted attention to compensation effected by lump-sum transfers of the aggregate production vector (X_L, Z_L). Lump-sum taxation is more likely to constitute a PPI if endowments are redistributed, and the aggregate production vector moves elsewhere on the production possibility frontier. But such compensation would be equivalent to imposing a *personalized* lump-sum tax system rather than a poll tax, which is another issue entirely.

Since $E(q,u)$ is concave in q and $E_q(p+t,u_E)=X_E$, $e_A \le 0$,with strict inequality if E is strictly concave at $(p+t,u_E)$, i.e. if there are substitution effects in aggregate demand for x. Thus the aggregate equivalent variation is non-positive, regardless of whether lump-sum taxation represents a PPI over excise taxation.

It is clear from Fig. 1 that if the two Scitovsky indifference curves do not cross then $c_A \le 0$ and these anomalies do not arise. In the same vein, Professor Peck notes that, when taxpayers' preferences are identical and homothetic, $c_A \le 0$. We may generalize and explain this observation as follows. Suppose that preferences satisfy the Gorman aggregation conditions, i.e.

$$E^i(q,u^i) = f(q)\phi^i(u^i) + g^i(q) \tag{8}$$

so that all consumers have parallel, linear Engel curves. Gorman (1955) showed that (8) is necessary and sufficient for Scitovsky indifference curves never to cross and hence, given the foregoing discussion, is sufficient for $c_A \le 0$. This may be verified directly by noting that under (8), using (4) and (6),

$$\begin{aligned} c_A &= f(p+t)\sum_i \left(\phi^i(u_E^i) - \phi^i(u_L^i)\right) \\ &= \frac{f(p+t)}{f(p)} e_A \le 0. \end{aligned}$$

When (8) holds, the allocation with lump-sum taxation is unambiguously a PPI over the allocation with excise taxation (viz. both the Hicks and Kaldor criteria are satisfied). In this case, use of c_A to measure the efficiency gain gives sensible results.

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