Flat tax reform*

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Abstract

This paper studies the welfare effects of tax reforms moving in the direction of flat-rate taxation of income. The welfare impact of such reforms can be decomposed into two effects: (i) the increase in welfare associated with substitution among taxed commodities, and (ii) the loss in welfare associated with substitution between commodities and leisure. On balance, flat tax reforms are desirable when inter-commodity substitution effects are large relative to commodity–leisure substitution effects. The analysis allows us to reconcile the apparently conflicting results of the tax reform and optimal taxation literatures.

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1 Introduction

Recently, proponents of flat-rate taxation of labour income have enjoyed renewed attention in policy debates in the U.S. and elsewhere. One argument common among policy analysts is that a movement toward taxing all transactions at a common rate reduces distortions in economic behaviour and the associated deadweight costs.[1] Optimal tax theory has long made clear, however, that this view may be misguided. A system of flat-rate taxation creates a tax wedge in the relative prices of taxed goods and the numeraire commodity (leisure, say), and this "one big distortion" can in general be less advantageous than the many, smaller distortions of a differentiated commodity tax system.

This paper analyzes the relationship between these arguments and develops a new heuristic for analyzing flat-rate taxation. We provide a simple formula for calculating the impact of flat tax reforms on consumer welfare, which consists of the welfare gain from substitution among taxed

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[1] Another issue commonly addressed in flat tax proposals, but not addressed here, is tax rate progressivity. The analysis of this paper can however be applied to a number of related issues in the theory of taxation, including arguments for broad-based sales, value-added, and consumption taxes, and analysis of the effects of exemptions and credits within the income tax system.

commodities, and the welfare loss from substitution between commodities and leisure. Thus it is shown that flat tax reforms are more desirable the greater the degree of substitution possible among taxed commodities, and the less the degree of substitution between leisure and taxed commodities.[2] Informational requirements of our approach are parsimonious, moreover, requiring information only on initial tax rates and market demand elasticities.

As well as providing simple rules for evaluating flat tax reforms, the analysis establishes a link between the earlier literatures on optimal taxation and on piecemeal tax reform, and helps to reconcile their sometimes disparate conclusions. Most research on the optimality of flat-rate taxation has examined the role of substitution between commodities and leisure in the analysis, but has largely ignored substitution among taxed commodities. Thus Corlett and Hague (1953) established that, beginning from a uniform tax system, increasing a tax on a complement to leisure would increase consumer welfare. Sadka (1977) showed that uniform commodity taxation is optimal if and only if compensated wage elasticities of demand for all commodities are equal at the optimum. More recently, Besley and Jewitt (1995) and others have extended this approach, establishing separability restrictions on preferences that are consistent with the Sadka condition and which are sufficient for optimality of uniform taxes. In contrast, Hatta (1986) examined the welfare impact of a particular tax reform moving in the direction of uniform taxation, which is related to the flat tax reforms studied in this paper. Hatta argued such reforms improve welfare when demands for *taxed* commodities satisfy a net substitutability property, described below.

Unlike the Hatta approach, the welfare decomposition derived in this paper relies on inherent substitutability (i.e. Slutsky negativity) of expenditure-minimizing demands, rather than a restriction on pairwise substitution effects. This paper also makes clear that it is the magnitude of such substitution effects relative to commodity–leisure substitution effects that is relevant to the analysis. This emendation of Hatta's approach also permits us to reconcile results with those of the optimal tax literature, and to offer a new interpretation of earlier conditions sufficient for optimality of uniform taxes. It is shown that Sadka's preference restriction implies that a piecemeal flat tax reform always improves welfare, as gains from commodity substitution dominate the leisure distortion. Conversely, the welfare gains of flat tax reform are "robust," holding for any initial tax rates, only if consumer preferences satisfy Sadka's restriction.

Section 2 introduces the model of taxation in a single-consumer economy. Sections 3 and 4 show how the welfare impact of a flat tax reform depends on the interaction of revenue and distortionary effects of the initial tax system. Section 5 presents the main "substitutability" result, decomposing the welfare change into two effects, the increase in welfare associated with substitution among taxed commodities and the offsetting decrease in welfare associated with substitution between commodities and leisure. The result is then related to the optimal tax literature. Section 6 discusses how the results can be extended to an economy with many consumers. Section 7 concludes the paper.

[2]To avoid a misunderstanding common in the literature, we stress there is nothing special about leisure in the analysis, unless it is chosen arbitrarily to be the numeraire. The planner could choose to tax all commodities including labour at a uniform rate relative to, say, the tax on shoelaces. In this case, it is elasticities of shoelace demand that are relevant to the analysis.

2 The model

The model is standard. A single consumer supplies labour services x^0 at wage rate q^0 and uses the proceeds to purchase produced commodities $x = (x^1, \ldots, x^n)$ at prices $q = (q^1, \ldots, q^n)$. For convenience, we will refer to the extended price and quantity vectors as $Q = (q^0, q)$ and $X = (x^0, x)$, respectively. By convention, we measure labour time supplied as a negative number, so that $x^0 < 0$; in contrast, there are positive net demands for produced commodities, so that $x > 0$. The consumer's budget constraint is then $Q'X \leq 0$. Consumer preferences are represented by the utility function $U(X)$. The associated indirect utility and expenditure functions are

$$v(Q) = \max_X \{U(X) : Q'X \leq 0\} \tag{1}$$

$$e(Q, u) = \min_X \{Q'X : U(X) \geq u\}. \tag{2}$$

With the assumption that U is strictly concave, increasing and C^2, the compensated demands attaining the minimum in (2) are single-valued, continuously differentiable in Q, and satisfy $X(Q, u) = e_Q(Q, u)$ (Shephard's lemma).[3] Since compensated demands are degree-zero homogeneous in prices and have symmetric derivatives,

$$Q'x^i_Q = Q'X_i = 0 \qquad \text{(all } i\text{)}.$$

This property is used repeatedly in what follows.

Production in the economy is linear, with p^i units of labour required to produce one unit of good i. Competitive profit maximization results in producer prices equal to p, when the pre-tax wage p^0 is normalized to one. Government revenue is derived from linear taxes on transactions at rate $t^i = q^i - p^i$. There is no lump-sum taxation. Since consumer demands are degree-zero homogeneous in consumer prices, it is possible to normalize taxes so that $t^0 = 0$: labour is initially untaxed in the model. Define the extended producer price and tax vectors $P = (1, p)$ and $T = Q - P$, and let $R(Q, u) = (Q - P)'X(Q, u)$ denote government revenue. Government consumes an exogenously given G units of labour, so that the aggregate production possibility constraint is $P'X + G = 0$. A tax policy (Q, u) is then feasible given revenue requirements if and only if

$$e(Q, u) = 0 \tag{3}$$

$$R(Q, u) = G. \tag{4}$$

Note that (3) and (4) imply that the production constraint holds identically.

3 Revenue and welfare effects of reform

A *tax reform* from initial prices Q is defined as a differentiable path $Q(\theta)$ where θ is a scalar parameter and $Q(0) = Q$. We are interested in local tax reforms which move smoothly in the direction of flat-rate taxes on labour income in the following sense: beginning from any initial tax system, consumer prices are "shrunk" proportionately toward producer prices, while the tax on labour

[3] Throughout the paper, superscripts represent elements of a vector, whereas subscripts represent partial derivatives. Where the subscript is a vector, the result is a column vector of partial derivatives; hence $e_Q = (\partial e/\partial q^0, \ldots, \partial e/\partial q^n)'$. Similarly, X_i denotes $(\partial x^0/\partial q^i, \ldots, \partial x^n/\partial q^i)'$ and X_Q is the Slutsky matrix.

income is increased to maintain revenue neutrality. This is a natural notion of a comprehensive reform towards flat taxation, inasmuch as all ad valorem tax rates are reduced proportionately and replaced by a tax on income at a common rate. For such a reform,

$$dq = -t\, d\theta. \tag{5}$$

Our preliminary results, however, apply to any feasible tax reform dq.

Reductions in commodity taxes require an increase in labour taxation to maintain revenue neutrality. To see how much labour taxation must adjust, define $q^0 = \phi(q,u)$ as the wage which allows the consumer to attain utility u at prices q, so that

$$e(\phi(q,u),q,u) \equiv 0 \tag{6}$$

and define *wage-compensated government revenue* as

$$R^c(q,u) = R(\phi(q,u),q,u). \tag{7}$$

A tax policy q is then feasible if and only if

$$R^c(q,u) = G. \tag{8}$$

Under weak conditions, R^c may be interpreted as a compensation function, measuring the consumer's willingness to pay for a tax reform. This simplifies the welfare evaluation of reforms considerably. To see this, note that when tax rates are changed income effects in commodity demands also lead to changes in government revenue. The total marginal effect of t^i on revenue is

$$\frac{d}{dq^i}R(Q,v(Q)) = R_i(Q,u) - \frac{R_u(Q,u)}{e_u(Q,u)}x^i(Q,u) \tag{9}$$

since (3) implies $v_i = -x^i/e_u$. When $dR/dq^i > 0$ for $i > 0$, the tax on commodity i is said to be *revenue increasing.* In contrast, increases in t^0 represent increases in *subsidization* of labour, since $q^0 = 1 + t^0$ is the consumer's after-tax wage rate. Hence the labour tax is revenue increasing when $dR/dq^0 < 0$. With this definition, Proposition 1 establishes a condition under which the welfare effects of any feasible tax reform dq may be identified with its effects on wage-compensated revenue.

Proposition 1 *Suppose the labour tax is revenue increasing. Then for any feasible tax reform dq, $du \geq 0$ if and only if*

$$dR^c = R_q^{c\prime} dq \geq 0. \tag{10}$$

Proof. A tax reform is feasible when it satisfies (8). Thus

$$du = -\frac{1}{R_u^c}\left(R_q^{c\prime} dq\right).$$

Differentiating (7) with respect to u yields

$$\begin{aligned} R_u^c &= R_u - \frac{e_u}{x^0}R_0 \\ &= -\frac{e_u}{x^0}\frac{d}{dq^0}R(Q,v(Q)). \end{aligned}$$

Since $x^0 < 0$ and $dR/dq^0 < 0$, $R_u^c < 0$. Hence $du \geq 0$ if and only if $dR^c \geq 0$. □

In seeking to characterize welfare-improving tax reforms, it is possible to ignore income effects and search for cases in which wage-compensated revenue rises.

4 Tax-dependent reform evaluations

Section 3 showed that welfare effects of tax reforms depend solely on their effects on an appropriately defined compensated tax revenue function. Revenue effects of a reform depend both on the tax rates initially levied and on the responsiveness of tax bases to rate changes. This section provides a joint condition on initial tax rates and elasticities that is necessary and sufficient for a tax reform to be welfare increasing, and relates it to a common measure of tax distortions. The next section then relates the welfare impact to underlying properties of consumer preferences.

To analyze welfare effects of reforms, it is convenient to have a measure of the compensated revenue effects of a tax base. Suppose that a marginal increase in t^i were compensated by a lump-sum transfer to keep utility constant. The change in revenue, net of the compensating transfer, per dollar transferred, is

$$\varepsilon^i = \frac{\partial[R(Q,u) - e(Q,u)]/\partial t^i}{\partial e(Q,u)/\partial t^i} = \frac{t' x_i}{x^i} \tag{11}$$

(Since Slutsky derivatives are symmetric, $\varepsilon^i = t' x^i_q / x^i$, a first-order approximation to the percentage change in x^i resulting from the imposition of taxes t, which Mirrlees (1976) labels the *index of discouragement* for commodity i. The relevance of this measure is discussed further below.) The marginal effect of q^i on wage-compensated revenue is, differentiating (6) and (7),

$$\begin{aligned} R^c_i &= R_i - \frac{x^i}{x^0} R_0 \\ &= x^i(\varepsilon^i - \varepsilon^0). \end{aligned} \tag{12}$$

The total differential of wage-compensated revenue is

$$\begin{aligned} dR^c &= \sum_{i=1}^{n} x^i(\varepsilon^i - \varepsilon^0) dq^i \\ &= L \sum_{i=1}^{n} w^i(\varepsilon^i - \varepsilon^0) d\log q^i, \end{aligned} \tag{13}$$

where $L = -q^0 x^0$ is labour income, and $w^i = q^i x^i / L$ is the expenditure share of commodity i.

Note that zero-degree homogeneity of demands implies $q^0 x^j_0 + q' x^j_q = 0$, so that

$$\sum_{i=1}^{n} w^i(\varepsilon^i - \varepsilon^0) = -\sum_{j=1}^{n} t^j \left(\sum_{i=1}^{n} q^i x^j_i + q^0 x^j_0 \right) = 0.$$

Consequently, the summation in (13) equals the covariance of ε^i and $d\log q^i$, weighted by initial expenditure shares, say $\rho(\varepsilon, d\log q)$. Proposition 1 implies

$$du \geq 0 \quad \Longleftrightarrow \quad \rho(\varepsilon, d\log q) \geq 0. \tag{14}$$

It is interesting to note that the analysis gives rise to a standard optimal tax rule, since (14) implies q is optimal only if $\rho(\varepsilon, d\log q) = 0$ for all dq. This in turn requires ε is a vector constant, or

$$\varepsilon^i = \varepsilon^0 \quad (\text{all } i > 0).$$

This condition reproduces the Ramsey–Samuelson rule that an optimal tax system leads to (approximately) equiproportionate reduction in all commodity demands.

When flat tax reforms are considered, $d\log q = -\tau\ d\theta$, where $\tau^i = t^i/q^i$ is the ad valorem tax rate. Then (14) states $du \geq 0$ if and only if $\rho(\varepsilon, \tau) \leq 0$. This establishes the following characterization of desirable flat tax reforms.

Proposition 2 *Suppose that the labour tax is revenue increasing. Then a flat tax reform increases welfare if and only if the expenditure-weighted covariance of τ^i and ε^i is negative.*

To understand the result, observe that ε^i is related to the *marginal excess burden* (MEB) of tax base i, defined as the marginal compensation required for an additional dollar of compensated revenue derived from the tax base, when the revenue is returned to the consumer as a lump sum.[4] The marginal compensating variation for an increase in t^i is $e_i(Q,u)$ and, since revenues are rebated, the tax rate change required to generate one dollar in revenue is $1/R_i(Q,u)$. The MEB is therefore

$$\frac{e_i(Q,u)}{R_i(Q,u)} - 1 = \frac{x^i}{x^i(1+\varepsilon^i)} - 1 = -\frac{\varepsilon^i}{1+\varepsilon^i}.$$

Since the MEB is decreasing in ε^i, Proposition 2 indicates that a flat tax reform increases welfare when, on average, high tax rates are levied on bases with high MEBs. This is a natural conclusion, given that flat tax reforms move all tax rates toward the average.

An alternative interpretation of the condition relies on the equivalence between our inefficiency measure and *compensated radial elasticities* (CREs) of tax bases, noted by Fane (1991). The CRE of tax base i is defined as the percentage change in compensated demand for i given a one per cent increase in all taxes, or

$$\left.\frac{d\log x^i(P+\alpha T, u)}{d\alpha}\right|_{\alpha=1} = \frac{t' x^i_q}{x^i} = \varepsilon^i.$$

In a flat tax reform, percentage demand changes are proportional to $-\varepsilon^i$. If the change is smaller on average for high-tax commodities, so that $\rho(\tau,\varepsilon) \leq 0$, then compensated revenue rises with the reform, and Proposition 1 indicates that welfare increases in consequence.

4.1 The Hatta analysis

The objective of this paper—to relate welfare effects of flat tax reforms to substitution effects in consumption—is not new to the literature. Hatta (1986) examines reforms in which, beginning from any initial commodity tax vector, the highest tax rate (t^n, say) is decreased, while the lowest tax rate (t^1) is increased to maintain revenue neutrality. For such a reform, (10) simplifies to

$$dR^c = (\varepsilon^1 - \varepsilon^n)\ d\theta$$

and welfare rises if and only if $\varepsilon^1 > \varepsilon^n$. In this section, Hatta's result is summarized, in order to facilitate comparison with the results which follow.

[4] A number of definitions of marginal excess burden have been proposed in the literature. Ahmad and Stern (1984), for example, define the *marginal social cost of finance* in a similar fashion, but using uncompensated demands. For a discussion of alternative approaches, see Ballard and Fullerton (1992).

Hatta defines good i to be a substitute for the *compound* of other goods if

$$\sum_{j \neq i} p^j |t^j - t^i| x_j^i > 0,$$

i.e. if increasing prices of other goods in proportion to the distance between tax rates (t^i, t^j) results in an increase in the compensated demand for good i. Furthermore, define $\eta^i = q^0 x_0^i / x^i$ as the compensated wage elasticity of demand for i. Then Hatta shows (Theorem 2, p. 106) that decreasing t^n and increasing t^1 to maintain revenue neutrality is welfare improving if: (i) good 1 is a substitute for the compound of other goods and so is good n; and (ii) $\tau^n \eta^n \geq \tau^1 \eta^1$.

Hatta's result leaves questions unanswered, however.[5] While Hatta stresses the role of substitution among commodities, the result depends also on wage elasticities, and the relative importance of the two conditions is unexplored. Similarly, little attempt has been made to relate the conditions to those suggested by the optimal tax literature. Furthermore, compound elasticities are neither easily measured nor easily interpreted, so that verifying condition (i) appears to be difficult. (Note that goods are compound substitutes for all tax rates only if all goods are pairwise net substitutes, which is an unreasonably strong condition.)

The analysis in this paper differs from the Hatta approach in a number of ways: (i) in the reforms we consider, all taxes are moved toward uniformity (rather than merely the "outliers") which is perhaps a more plausible notion of flat tax reform; (ii) the welfare effects of reforms are related to the fundamental compensated price elasticities of demand, rather than to Hatta's compound elasticity construct; and (iii) the role of wage elasticities is explicitly considered, and results are reconciled with the literature on the optimality of uniform taxation, which appears prima facie to be inconsistent with the Hatta analysis.

5 Substitution effects and the desirability of reform

The preceding results give a clear prescription for flat tax reform. When (and only when) high tax rates are levied on bases that are on average more highly distorted, welfare is improved by a flat tax reform. In this section, we seek to elucidate what properties of consumer preferences are conducive to welfare gains from flat tax reforms.

Define $z(Q,u) = -x(Q,u)/x^0(Q,u)$ as the vector of compensated demands per unit labour supplied, and note that its Jacobian matrix is

$$z_q = -\frac{1}{x^0} \left(x_q - \frac{1}{x^0} x x_0' \right). \tag{15}$$

Expanding (13) with (11) yields

$$\begin{aligned} dR^c &= -\sum_{i=1}^{n} \sum_{j=1}^{n} t^i t^j \left(x_j^i - \frac{x^i}{x^0} x_j^0 \right) \, d\theta \\ &= x^0 \; t' z_q t \; d\theta. \end{aligned} \tag{16}$$

Recall that $x^0 < 0$, so that Proposition 1 implies the following.

[5] Gordon (1991) provides a more extensive discussion of the Hatta result along these lines.

Proposition 3 *Suppose that the labour tax is revenue increasing. Then a flat tax reform improves welfare if and only if*

$$t' z_q t \leq 0$$

where $z_q(Q, u)$ *is evaluated at* $u = v(Q)$.

The result indicates that flat tax reform improves welfare as long as substitution possibilities among taxed commodities are large relative to substitution possibilities between leisure and commodities. When this is so, reducing distortions among taxed commodities results in efficiency gains which more than offset the effects of the increased distortion in the relative price of consumption to leisure. This is seen most clearly by expanding (16) to obtain

$$\begin{aligned} dR^c &= -\left(t' x_q t - \frac{1}{x^0}(t'x)(x_0' t) \right) d\theta \\ &= -(t' x_q t - G\varepsilon^0)\, d\theta. \end{aligned} \tag{17}$$

The first term in parentheses represents the inter-commodity substitution effects of the reform. Since x_q, a minor of the Slutsky matrix, is negative semi-definite, this effect on welfare is unambiguously positive, as flat tax reform leads the consumer to substitute toward goods with high initial tax rates, raising compensated revenue. The second term in parentheses represents the commodity–leisure substitution effects of the reform. When $\varepsilon^0 < 0$, so that the marginal excess burden of labour taxation is positive, this effect offsets the gains from reducing commodity tax distortions. When substitution possibilities among commodities are large relative to commodity–leisure substitution possibilities, the net effect is positive, and welfare rises as a result of a flat tax reform. This conclusion parallels Hatta (1986), but makes explicit the role of both margins of substitution in the analysis.

5.1 Optimal flat taxation: A reconciliation

When z_q is negative semi-definite at initial prices q, Proposition 3 implies that flat tax reform is desirable for any initial tax rates and producer prices. More generally, when substitution possibilities among taxed commodities are large relative to commodity–leisure substitution possibilities, a flat tax reform is a “robust” policy recommendation, in the sense that it improves welfare for a large set of initial tax systems.

This logic may be extended to derive a sufficient condition for the optimality of flat-rate taxation. Suppose that z_q is globally negative semi-definite and t^0 is globally revenue increasing. Then Proposition 3 indicates that flat tax reform is (weakly) welfare improving beginning from any initial tax system. It follows that the set of tax optima includes a flat-rate tax system when our sufficient conditions for local welfare improvements hold globally.[6]

Sadka (1977) showed that flat-rate taxation is optimal when wage elasticities of compensated demand for all commodities are globally equal, i.e. when there exists a scalar-valued function $\alpha(Q)$ such that, for all Q and $u = v(Q)$,

$$x_0(Q, u) = \alpha(Q) x(Q, u). \tag{18}$$

[6]Proof: Suppose any q^* is an optimal tax system. If the conditions of Proposition 3 hold globally, then there exists a path from q^* in $\mathbf{R}^n_+$ over which utility is non-decreasing, and which converges to a uniform tax system $\bar{q} = \bar{\alpha}\mathbf{1}$. Hence $\bar{q}$ is optimal.

Our analysis offers a further interpretation of the equal elasticities condition: it can be shown to be necessary and sufficient for z_q to be negative semi-definite, and hence for the global dominance of commodity substitution effects over commodity–leisure substitution effects.

Proposition 4 *$z_q(Q,u)$ is negative semi-definite at $u = v(Q)$ if and only if the Sadka equal-elasticities condition* (18) *holds at Q.*

Proof. (i) Sufficiency: For any square matrix A, let $A^{(m)}$ denote the $m \times m$ leading principal minor of A. Suppose that (18) holds at Q. Homogeneity of demands then implies $x_0^0 = \alpha x^0$, and (15) reduces to

$$z_q = -\frac{1}{x^0}\left(x_q - \frac{1}{x_0^0}x_0 x_0'\right)$$

so that its minor determinants are

$$\det z_q^{(m)} = \left(-\frac{1}{x^0}\right)^m \det\left(x_q^{(m)} - \frac{1}{x_0^0}\left(x_0 x_0'\right)^{(m)}\right)$$

The Slutsky matrix is

$$X_Q = \begin{bmatrix} x_0^0 & x_0' \\ x_0 & x_q \end{bmatrix}.$$

Applying a standard result on determinants of partitioned matrices (e.g., Murata, 1977),

$$\det X_Q^{(m+1)} = x_0^0 \det\left(x_q^{(m)} - \frac{1}{x_0^0}\left(x_0 x_0'\right)^{(m)}\right) = x_0^0(-x^0)^m \det z_q^{(m)}.$$

Since $x_0^0 < 0$,

$$(-1)^{m+1}\det X_Q^{(m+1)} \geq 0 \implies (-1)^m \det z_q^{(m)} \geq 0.$$

Hence z_q is negative semi-definite because X_Q is.

(ii) Necessity: Recall that $\eta^i = \partial \log x^i / \partial \log q^0$ denotes the wage elasticity of demand for i. Suppose (18) does not hold, so that there exists a commodity i with $\eta^0 > \eta^i$. It suffices to find a single vector $\hat{t} \in \mathbf{R}^n$ such that $\hat{t}' z_q \hat{t} > 0$. Consider

$$\hat{t} = \beta q + \delta e_i$$

where e_i is the unit vector with one as its ith element, and $\beta > 0$ and δ are arbitrary scalars. Observe that the homogeneity condition $q^0 x_0^i + q' x_q^i = 0$ implies

$$\begin{aligned} q' z_q &= q' x_q - \frac{1}{x^0}(q'x)x_0' \\ &= -q^0 x_0' + q^0 x_0' = 0_n' \end{aligned}$$

when the budget constraint $Q'X = 0$ is satisfied. Thus

$$\hat{t}' z_q \hat{t} = \beta\delta(e_i' z_q q) + \delta^2 z_i^i. \tag{19}$$

Applying the homogeneity condition once again,

$$\begin{aligned} e_i' z_q q &= -\frac{1}{x^0}\left(x_q^{i\,\prime} q - \frac{x^i}{x^0} x_q^{0\prime} q\right) \\ &= -\frac{1}{x^0}\left(-q^0 x_0^i + \frac{x^i}{x^0} q^0 x_0^0\right) \\ &= z^i(\eta^0 - \eta^i). \end{aligned}$$

Choose δ small so that the second term in (19) is negligible, and

$$\hat{t}' z_q \hat{t} \approx \beta\delta z^i(\eta^0 - \eta^i) > 0 \qquad \text{for } \delta > 0,$$

a contradiction. Since the choice of good i was arbitrary, it follows that z_q is negative semi-definite only if (18) holds. □

When the equal-elasticities condition holds, an increase in labour taxation cannot distort relative commodity demands, so that the benefits of reducing distortions among taxed commodities must dominate. Conversely, for tax systems sufficiently close to uniformity, the positive commodity substitution effects of the reform are negligible, and distortions in leisure demand must dominate. This argument parallels Corlett and Hague (1953).[7]

6 Reform with many consumers

One of the most hotly debated aspects of flat tax reform is its distributional consequences, a consideration which has been absent from this analysis of a single-consumer economy.

The approach adopted here can be extended easily to analyze a many-consumer economy, to determine when *potential* Pareto improvements result from flat tax reforms, using the Scitovsky expenditure function

$$E(Q, u^1, \ldots, u^H) = \sum_{h=1}^{H} e^h(Q, u^h)$$

in place of the individual expenditure function, and interpreting results in terms of elasticities of aggregate demand.[8]

[7]Note that Proposition 4 may also be interpreted as demonstrating that the Sadka condition holds if and only if the Jacobian of wage-compensated demand functions $x(\phi(q,u), q, u)$ is negative semi-definite. Similarly, Besley and Jewitt (1995) have shown the Sadka condition holds if and only if wage-compensated labour supply $x^0(\phi(q,u), q, u)$ is independent of prices. The equivalence of these two conditions can be shown directly.

[8]One caveat is required, however. Guaranteeing that aggregate demand functions have a particular set of consistency properties may be difficult, because aggregate demands do not in general resemble those of a representative consumer. See Blackorby et al. (1990) for an analysis of this problem in the context of uniform optimal taxation.

When lump-sum compensation is possible for those who lose from the reform, these potential Pareto improvements can be made actual Pareto improvements. The amount of compensation required can in fact be computed directly from this analysis, since a taxpayer's marginal compensating variation for the reform can be shown to satisfy

$$dC^h = \frac{L^h}{H\bar{L}} \frac{dR^c}{1+\varepsilon^0} + L^{h0} \left(\frac{t'x^h}{L^h} - \frac{t'\bar{x}}{\bar{L}} \right) d\theta \tag{20}$$

where x^h is the vector of commodity demands and L^h is labour income for consumer h, and bars over variables indicate population averages. The first term in (20) is the share of consumer h in the aggregate efficiency change of the reform, where shares equal consumers' shares in aggregate labour income. The second term is the "transfer effect" of the reform. Since $t'x^h/L^h$ is a consumer' average propensity to pay tax out of labour income, this term is proportional to the deviation of the average effective tax rate from the mean and to labour income. Flat tax reform results in transfers away from consumers with average effective tax rates below the mean in the economy. Thus lump-sum compensation, if feasible, takes a simple form for flat tax reforms.

7 Conclusion

This paper has employed a new approach to examining the case for tax reforms moving in the direction of flat-rate taxation of labour income. It was shown the welfare impact of such reforms can be decomposed into two effects, the increase in welfare associated with substitution among taxed commodities and the offsetting decrease in welfare associated with substitution between commodities and leisure. On balance, flat tax reforms are therefore welfare improving when substitution effects in the demand for taxed commodities are sufficiently large, and substitution effects between leisure and taxed commodities sufficiently small. Our sufficient conditions for flat tax reform to be desirable are consistent with, but more intuitive than, those derived from the optimal tax literature.

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