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On the efficiency of uniform taxation in a many-consumer economy

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Abstract

We derive necessary and sufficient conditions on preferences for uniform taxation to be Pareto efficient in a many-consumer economy. We argue that uniformity is desirable only under conditions far more restrictive than suggested in previous literature, most of which is based on analysis of a single-consumer economy. When the only feasible direct tax is a poll tax, uniformity holds if and only if consumers have identical, linear Engel curves. When income can be redistributed optimally through direct taxes, uniformity also holds if wage elasticities of demand for all goods are constant and equal for all consumers.

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1 Introduction

Economic policy-makers often take uniformity of commodity tax rates to be a natural objective for the tax system. Proponents of broad-based value-added and sales taxation and, more recently, of flat-rate income tax systems typically base their arguments in part on the presumption that the tax system should, where possible, leave relative prices of commodities undistorted. This view is perhaps most succinctly advanced by McLure (1987), who writes, "That the economic effects of the VAT are rather bland ... is one of its chief advantages. After all, the primary purpose of taxation is (or at least should be) to raise revenue, not to change economic behavior."

This view has likewise received considerable support from optimal tax theorists. While the optimal tax literature has long made clear that optimality of uniform taxation obtains only in special cases, standard results in the literature are typically interpreted as indicating that uniform taxation is a reasonable policy objective. For example, Atkinson and Stiglitz (1980) argue that analysis of Pareto efficient taxation "provides some limited support for the ... broad view that direct taxes are superior on both efficiency and equity counts. Not just in the case of the linear expenditure system, but in a much wider class of demand systems, there is no need to employ differentiated indirect taxation to achieve an optimum." This conclusion, and similar statements by Deaton (1981) and others, is based on analysis of a single-consumer economy and the sufficient conditions on preferences for optimal uniformity derived in that context. While the single-consumer framework is acknowledged to be restrictive, it is typically regarded as a reasonable simplification, generating conclusions about the efficiency of uniform commodity taxation in a many-consumer economy. Moreover, these papers argue, when planners have access to a reasonable set of direct tax instruments, distributional objectives are also probably best achieved through direct taxation, rather than differentiated commodity taxation.

This note reconsiders this standard interpretation. We argue that analysis of a many-consumer economy indicates that uniform taxation is likely to be undesirable on grounds of both efficiency and equity. In general, results from the single-consumer case do not generalize to the many-consumer economy—even when distributional considerations do not arise—because aggregate demands do not in general resemble those of a representative agent.

To establish this, we study Pareto efficient linear commodity taxation in a many-household economy. We show in Section 2 that, when the only feasible direct tax is a poll tax, a sufficient condition for Pareto efficient tax rates to be uniform is that consumer preferences be separable in the subset of taxed commodities, and that preferences for the taxed goods, conditional on labour supply (the numeraire commodity), are identical with linear Engel curves. If *all* Pareto efficient allocations are to entail uniform taxation (that is, if the optimality of uniform taxes is to be robust to changes in welfare weights applied to consumers), moreover, this condition is also necessary.

These results apply when direct taxation is restricted to poll taxes. A richer set of direct tax instruments can make uniform taxes more attractive, since direct taxation can offset the income and distributional effects of commodity taxes. To explore this possibility, in Section 3 we consider the opposite polar case, in which unrestricted personalized lump-sum taxes are available, but the planner is still constrained to raise some revenue through the commodity tax system.[1] In this more

[1]The approach used in this section relies on characterization of optimal tax rates in terms of the Scitovsky expenditure function, which was introduced to the study of optimal tax problems by Blackorby et al. (1990). A related problem of uniform commodity taxation was considered by Blackorby et al. (1990), and the results presented in Section 3 are related to some of their results. Here, however, we restrict attention to the case in which leisure is

general problem, the conditions under which uniformity is optimal are expanded, although perhaps not greatly. The previous condition—identical conditional preferences for taxed commodities, with linear Engel curves—remains sufficient for uniform taxes to be Pareto efficient. In this case, however, a second condition on preferences is also sufficient for uniformity. Under this condition, conditional preferences for taxed goods may differ in arbitrary ways among consumers, but wage elasticities are constant and equal for all goods and all consumers. If there is sufficient diversity in individual preferences, moreover, this condition is also necessary for uniformity. In general, it is clear that both these restrictions on preferences are very special ones. Thus even pure efficiency considerations will typically dictate that tax rates depart from uniformity. The implications of these conditions for consumer behaviour are analyzed in Section 4 of the paper.

Our analysis thus generates conditions considerably more restrictive than the previous literature. In particular, Deaton (1979) studied linear commodity taxation in a single-consumer economy where lump sum taxation is not feasible and determined that uniformity is optimal if preferences are implicitly homothetically separable.[2] In a recent contribution, Besley and Jewitt (1995) generalize the Deaton result for the special case in which the single consumer has no lump-sum income. In contrast, Atkinson and Stiglitz (1976) study an economy with a single consumer whose wage rate is unknown to the planner and allow the planner a full array of non-linear taxes in place of linear commodity taxes. They show that uniform commodity taxation is Pareto efficient when preferences are directly separable in taxed and untaxed commodities. Our analysis differs from these approaches in that we explicitly model an economy with many consumers and make different assumptions about feasible direct tax instruments. Deaton and Stern (1986) examine a related problem in a many-consumer economy, in which only linear commodity taxes are admissible and direct taxation can depend on observable demographic characteristics of consumers. They show that uniformity is optimal when, in addition to the Atkinson–Stiglitz condition, conditional preferences for taxed goods have linear Engel curves and are identical except for "minimum requirements" bundles that are uncorrelated with the social marginal utility of income. This paper extends of their analysis in seeking necessary and sufficient conditions on preferences for uniformity (Section 2) and in considering more general direct tax possibilities (Section 3).

2 Commodity taxation with a poll tax

Consider a competitive economy composed of H consumers with preferences represented by direct utility functions $U^h(x,y)$, $h = 1,\dots,H$, where x is a K-vector of consumption goods and y is leisure. Producer prices of the K consumption goods are taken to be fixed and normalized to unity. Let $H \geq K$. The wage rate available to individual h is w^h. Labour of different consumers is assumed to be taxed at a uniform rate (presumably because differences in wage rates are unobservable) which we normalize to zero by choosing the labour of one consumer as the numeraire commodity. Linear taxes t are applied to the consumption goods, so that consumer prices are $q = \mathbf{1} + t$. Define the expenditure function of consumer h by

$$e^h(q, w^h, u^h) = \min q \cdot x + w^h y \quad \text{s.t.} \quad U^h(x,y) \geq u^h. \tag{1}$$

We assume the standard properties of the expenditure functions, given as follows.

the only untaxed commodity, and consumers may face heterogeneous wage rates. Results in this setting are clearer and more closely related to the previous literature.

[2]The terminology, which is explained in Section 3, is due to Gorman (1976) and Blackorby et al. (1990).

Assumption 1 *Household expenditure functions have the following properties:*

1. *e^h is non-decreasing, concave, and degree-one homogeneous in (q, w^h);*
2. *e^h is continuously differentiable in (q, w^h).*

If each consumer h receives an identical lump-sum transfer or poll tax G, then individual expenditure is $e^h(q, w^h, u^h) = G + w^h T^h$, where T^h is the individual's labour endowment. (There are no endowments of the taxed commodities.) A Pareto efficient tax policy then solves

$$\max \sum_h \beta^h u^h \tag{2}$$

subject to the individual budget constraints

$$e^h(q, w^h, u^h) \leq G + w^h T^h \tag{3}$$

and the government budget constraint

$$t \cdot X(q, w, u) \geq \bar{R} - HG \tag{4}$$

where $\beta^h \geq 0$ is the welfare weight assigned by the planner to consumer h, and $X_k = \sum_h x_k^h$ is aggregate demand for commodity k.

Defining λ^h to be the Lagrange multiplier associated with each of the individual budget constraints (3) and μ to be the multiplier for (4), first-order necessary conditions[3] are, for q_k,

$$\sum_h \left(\frac{\lambda^h}{\mu} - 1 \right) x_k^h = \sum_i t_i X_{ki} \tag{5}$$

and, for G, $\sum_h \lambda^h = \mu H$. (In deriving (5), we have used the fact that $e_k^h = x_k^h$, by Shephard's lemma.) In the many-consumer case, optimal taxes depart from the Ramsey–Samuelson rule of equiproportionate reductions in compensated demands. Demand reductions should be smaller for commodities for which welfare-weighted average demands are higher. The λ^h terms, which represent the marginal social valuation of a lump-sum transfer to h, are determined from the welfare weights to satisfy

$$\frac{\lambda^h}{\mu} = \frac{1}{e_u^h} \left(\sum_i t_i \frac{\partial x_i^h}{\partial u^h} + \frac{\beta^h}{\mu} \right) \tag{6}$$

where $1/e_u^h$ is the marginal utility of income to h, and the first term in parentheses on the right-hand side of (6) represents the consequences for government revenue of a transfer to h.

When uniform taxes are optimal, there exists a scalar τ such that $t_i = \tau q_i$. Since $e^h(q, w^h, u^h)$ is degree-one homogeneous in (q, w^h),

$$\sum_i q_i e_{ki}^h(q, w^h, u^h) = -w^h e_{k0}^h(q, w^h, u^h), \tag{7}$$

[3] For a discussion of necessity, see Besley and Jewitt (1995).

where e_{k0}^h denotes $\partial e_k^h/\partial w^h$. Hence optimal taxes are uniform—using (5) and (7)—if and only if there exist scalars $(\alpha^1, \ldots, \alpha^H)$, where $\alpha^h = \tau^{-1}(1 - \lambda^h/\mu)$, such that

$$\sum_h w^h e_{k0}^h(q, w^h, u^h) = \sum_h \alpha^h e_k^h(q, w^h, u^h) \quad \text{(all } k\text{)}. \tag{8}$$

Notice that $\sum_h \alpha^h = 0$ since $\sum_h \lambda^h/H = \mu$.

Thus conditions on preferences for optimal uniform taxes depend on the welfare weights assigned to consumers, through their influence on the α^h terms. If economists' prescriptions for uniformity are to be robust, however, in the sense that they are valid regardless of which Pareto efficient allocation the planner chooses to implement through the tax system, then we require that (8) hold for all vectors of weights α^h summing to zero.[4] This imposes strong restrictions on the class of consumer preferences for which uniformity is optimal, given in the following proposition. (The proofs of this and other results are in an appendix.)

Proposition 1 *Uniform commodity taxation is optimal given identical lump-sum taxation if there exist degree-one homogeneous functions $\phi(q)$ and $\psi(q)$ and functions $f^h(\phi, w^h, u^h)$ such that*

$$e^h(q, w^h, u^h) = f^h(\phi(q), w^h, u^h) + \psi(q). \tag{9}$$

Conversely, every constrained Pareto efficient allocation with identical lump-sum taxation entails uniform commodity taxation only if (9) *holds.*

The implications of efficient uniform taxation are therefore far more restrictive in a many-consumer economy than in the single-consumer case studied by Deaton (1979). The restrictions on preferences given by Proposition 1 are examined more fully in Section 4. Some immediate discussion of the implications of these conditions may however be useful to the reader. Given (9), preferences for taxed commodities must be weakly separable and, conditional on labour supply, Engel curves must be linear and identical for all consumers. Equivalently, when this condition holds, the direct utility function has the representation

$$U^h(x, y) = V^h(\Gamma(x), y), \tag{10}$$

where Γ is quasi-homothetic and identical for all consumers.

3 Efficient commodity taxation: The general case

The previous section derived necessary and sufficient conditions on preferences for all Pareto efficient tax systems to entail uniform taxation of commodities. Uniformity was shown to be far more restrictive in a many-consumer economy than suggested by analysis of the single-consumer case in Deaton (1979). These results were obtained, however, under the assumption that the only lump-sum tax available to the planner was a poll tax, implying that a uniform commodity tax system is equivalent to a linear income tax, which is perhaps unlikely to be efficient in general. A more taxes and might therefore eliminate the need to apply differentiated rates to commodity demands, as argued by Atkinson and Stiglitz (1976) and others. To evaluate this argument, we now examine a far broader system of direct taxation. In the section, it is assumed that the planner can use personalized lump-sum taxes and transfers to achieve the desired degree of (first-best efficient) redistribution among consumers, but is constrained to raise some revenue through distortionary commodity taxation. While this environment is somewhat artificial, it allows us to focus on the efficiency effects of commodity taxation, without regard to its redistributive impacts. Moreover, it represents the most appropriate generalization of the single-consumer optimal tax problem to a many-consumer economy.

Let each consumer h receive a lump-sum transfer G^h, so that after-tax income is $w^h T^h + G^h$.. The transfers are again financed with linear commodity taxes, so that the government budget constraint is

$$t \cdot X(q, w, u) \geq \sum_h G^h, \tag{11}$$

where $X(q, w, u)$ is the vector of aggregate compensated demands for taxed commodities at wages $w = (w^1, \ldots, w^H)$ and utilities $u = (u^1, \ldots, u^H)$. The institutional constraint on commodity taxation is

$$t \cdot X(q, w, u) \geq \bar{R}. \tag{12}$$

Define the *Scitovsky expenditure function*[5] as

$$E(q, w, u) = \min q \cdot \sum_h x^h + \sum_h w^h y^h \quad \text{s.t.} \quad U^h(x^h, y^h) \geq u^h, \quad (\text{all } h), \tag{13}$$

the minimum cost of attaining utility profile u at prices (q, w). Then a tax policy is feasible if

$$E(q, w, u) \leq \bar{R} + \sum_h w^h T^h. \tag{14}$$

If a tax policy is Pareto efficient, then there exist non-negative welfare weights $(\beta^1, \ldots, \beta^H)$ for which the policy solves

$$\max \sum_h \beta^h u^h \tag{15}$$

subject to the feasibility constraint (14) and commodity tax revenue constraint (12). Let the Lagrange multipliers associated with these constraints be μ and θ respectively. First-order necessary conditions for the problem are the familiar Ramsey requirement of (approximate) proportional reduction in aggregate compensated demands, or

$$\sum_i t_i E_{ki} = \frac{\theta - \mu}{\mu} E_k \quad (\text{all } k). \tag{16}$$

When taxes are uniform, there exists a scalar τ such that $t_i = \tau q_i$, so that (16) reduces to

$$\tau \sum_i q_i \frac{E_{ki}(q, w, u)}{E_k(q, w, u)} = \frac{\theta - \mu}{\mu} \quad (\text{all } k). \tag{17}$$

[5]The Scitovsky expenditure function is discussed by Blackorby et al. (1990, 1993).

Since $E(q,w,u)$ is degree-one homogeneous in (q,w),

$$\sum_i q_i E_{ki}(q,w,u) = -\sum_h w^h E_{kh}(q,w,u), \tag{18}$$

where E_{kh} denotes $\partial E_k/\partial w^h$. Hence uniform taxation satisfies first order conditions if and only if, for all goods k,l,

$$\sum_h w^h \left(\frac{E_{kh}}{E_k} - \frac{E_{lh}}{E_l} \right) = 0 \tag{19}$$

or equivalently if and only if

$$\sum_h w^h \frac{\partial(E_k/E_l)}{\partial w^h} = 0 \quad (\text{all } k,l). \tag{20}$$

The condition states that, at the optimum point, relative aggregate demands for the taxed commodities are invariant to local changes in wages, holding their distribution fixed. If the uniformity result is to be robust to arbitrary changes in the environment $(T^1, \ldots, T^H, \bar{R})$, moreover, this condition must hold identically in prices, implying relative demands are degree-zero homogeneous in the wage profile. Equivalently, aggregate demands for commodities differ only by a multiplicative term that is degree-zero homogeneous in w, and they therefore can be written

$$E_k(q,w,u) = E_1(q,w,u)\Gamma^k(q,w,u) \quad (k = 2, \ldots, K) \tag{21}$$

where Γ^k is some function degree-zero homogeneous in w. A characterization of expenditure functions implied by (21) is given in the following proposition.

Proposition 2 *Uniform commodity taxation is Pareto efficient if there exist functions $\Phi(q,w,u)$ and $F(\Phi,w,u)$ such that*

$$E(q,w,u) = F(\Phi(q,w,u),w,u) \tag{22}$$

where Φ can be chosen to be degree-one homogeneous in q and degree-zero homogeneous in w. Conversely, uniform taxation is Pareto efficient for all consumer endowments and revenue requirements only if (22) holds.

The separable form (22) is the natural generalization to a many-consumer economy of Deaton (1979), where it is established that uniform commodity taxation is optimal in a single-consumer economy with no lump-sum taxation if and only if preferences are implicitly homothetically separable in taxed commodities.[6] Indeed, Proposition 2 includes the Deaton result as a special case for $H = 1$. Here, the possibilities are expanded, because conditional demands for the taxed commodities can depend on a degree-zero-homogeneous index of wage inequality, as well as on commodity prices.

[6] Besley and Jewitt (1995) study the single-consumer case and show that (22) is not necessary when the consumer's lump-sum income is zero. In this case, the budget constraint restricts prices to a manifold of smaller dimension than K so that (20) cannot be integrated in a neighbourhood of the optimum to yield (22). In our case, the issue does not arise, since endowments may be varied. The Deaton results are discussed at greater length in Section 4 below.

But separability has additional implications in the many-consumer economy, since by definition of the Scitovsky expenditure function

$$E(q, w, u) = \sum_h e^h(q, w^h, u^h) \tag{23}$$

so that E is additively separable in the (w^h, u^h) pairs. The joint implications of the separability condition (22) and additive separability in (w^h, u^h) pairs impose strong restrictions on admissible forms of $e^h(q, w^h, u^h)$. The following preliminary results establishes a structure for the expenditure functions that is necessary for (22)–(23).

Proposition 3 *The expenditure function has the form* (22) *only if there exist functions* $g^h(q, u^h)$, $\psi^h(q)$, *and* $f^h(g, w^h, u^h)$, *each degree-one homogeneous in its price arguments, such that*

$$e^h(q, w^h, u^h) = f^h(g^h(q, u^h), w^h, u^h) + \psi^h(q) \tag{24}$$

The proposition establishes that, when optimal taxes are uniform, preferences of each consumer are quasi-homothetically implicitly separable in the taxed commodities.[7] This does not exhaust the implications of optimal uniformity in the many-consumer case, however, since homogeneity of relative demands in wages (implied by (20)) is not satisfied for all preferences with the structure (24). To establish a fuller characterization of preferences consistent with uniformity, an additional monotonicity assumption is required to eliminate uninteresting special cases.

Assumption 2 *For all consumers h, compensated labour income $w^h(T^h - y^h(q, w^h, u^h))$ is increasing in w^h.*

Differentiating (24) and applying Euler's theorem,

$$w^h(T^h - y^h) = f_1^h(g^h(q, u^h), w^h, u^h)g^h(q, u^h).$$

Hence Assumption 2 holds, given preferences of the form (24), if and only if f_1^h is increasing in w^h. This monotonicity property is used in the proof of the following propositions. The first result shows that the characterization of preferences for the case of the poll tax, analyzed in Section 2, remains sufficient for uniformity in the general case, with the additional restriction that consumers' "minimum requirements" bundles are zero on average in the population.

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[7] Preferences are *implicitly separable* in a partition of commodities (x, y) if there exists a representation of preferences such that

$$U^h(x, y) = u \iff T^h(x, u) = S^h(y, u).$$

Preferences are *quasi-homothetically implicitly separable* in the commodities x if they are implicitly separable and the aggregator T^h is quasi-homothetic in x. When this is so, the conditional expenditure function for x has the form

$$\begin{aligned} C^h(q, \gamma, u) &= \min\{q \cdot x : T^h(x, u) \geq \gamma\} \\ &= g^h(q, u)\gamma + \psi^h(q, u) \end{aligned}$$

where γ is sub-group utility for x. It is then easily verified that the expenditure function has a form satisfied by (24). See Gorman (1976) for details.

Proposition 4 *Suppose that expenditure functions have the form* (24) *with*

$$g^h(q, u^h) = \phi(q) \tag{25}$$

for all h, and Assumption 2 holds. Then the Scitovsky expenditure function has the form (22) *if and only if*

$$\sum_h \psi^h(q) = 0 \quad \textit{for all } q \tag{26}$$

in (24).

The condition in Proposition 6 that expenditure on minimum requirements sum to zero (i.e., $\sum_h \psi^h(q) = 0$) is clearly unreasonable (cf. Blackorby et al., 1993). In particular, this condition is invariant to population size if and only if $\psi^h(q) \equiv 0$ for all h. This additional restriction implies conditional preferences for taxed goods are identical and homothetic for all consumers, as

$$e^h(q, w^h, u^h) = f^h(\phi(q), w^h, u^h). \tag{27}$$

With unrestricted personalized lump-sum taxation, uniform commodity taxation is efficient if consumer preferences are strongly similar, as in the restrictive case of poll taxes studied in Section 2. But can uniformity be efficient when preferences for taxed commodities vary more widely among consumers than permitted by Proposition 4? The answer is affirmative, although the conditions remain highly restrictive. To consider this possibility, suppose that there is sufficient diversity of preferences for taxed commodities, in the sense that the vectors of marginal demands $(g_1^h, \ldots, g_K^h)$ in (24) of all consumers are linearly independent for all prices and utility profiles. (In other words, no consumer's demands for commodities can be written as a linear combination of the others'.) This eliminates the possibility that preferences satisfy the condition of Proposition 4, and leads to the following characterization of preferences necessary and sufficient for uniformity.

Proposition 5 *Suppose that the matrix $G = [g_k^h]$ has full row rank K for all (q, w, u), and that Assumption 2 holds. Then the Scitovsky expenditure function has the form* (22) *if and only if* (26) *holds and there exists a scalar $\sigma > 0$ such that*

$$e^h(q, w^h, u^h) = \frac{1}{1-\sigma} g^h(q, u^h)^{1-\sigma}(w^h)^\sigma + \psi^h(q) \tag{28}$$

for all h.

When this condition holds, preferences are implicitly but not directly separable in taxed commodities. Unlike Proposition 4, conditional preferences for taxed commodities are unrestricted in this case, but preferences for labour supply are restricted to be strongly similar among consumers. In particular, (28) implies that compensated labour supply functions are affine transforms of a common iso-elastic function for all consumers, since

$$L^h(q, w, u) = T^h - e_0^h(q, w, u) = \alpha^h + \beta^h(q, u) w^{\sigma - 1}.$$

Note that we again require minimum requirements to be zero on average in the population. If $\psi^h(q) = 0$ for all h, then the expenditure function is a Cobb-Douglas function of the wage and the (heterogeneous) price indices for taxed commodities, with a common share parameter for all consumers.[8]

The various results of this section can be summarized as follows.

Proposition 6 *A necessary condition for Pareto efficiency of uniform taxation is that preferences have the form given by Proposition 3. Moreover, sufficient conditions for uniformity are that preferences additionally satisfy the restrictions of Propositions 4 or 5. Conversely, if there is sufficient diversity of preferences, uniform taxation is Pareto efficient if and only if the conditions of Proposition 5 hold.*

4 Implications for consumer behaviour

This paper has sought conditions on preferences guaranteeing optimality of linear or proportional income taxes alone, without differential indirect taxation of commodities, in an economy with many consumers. Thus we have considered a more general economy than the single-consumer case of Deaton (1979), and we have considered more restrictive tax policies than the optimal non-linear taxes admitted in Atkinson and Stiglitz (1976). Thus it unsurprising that we find optimality of direct taxation alone requires more restrictive conditions than in the earlier papers. In this section, the results are reviewed and their implications for commodity demand structures are compared.

In the single-consumer case with $H = 1$, Deaton (1979) showed that the implicitly homothetically separable form

$$e(q, w, u) = F(\Phi(q, u), w, u). \tag{29}$$

was necessary and sufficient for optimal uniformity of indirect taxes. Homothetic implicit separability places restrictions on wage elasticity of relative compensated commodity demands, as we have noted. In particular, (29) implies

$$\frac{\partial x_k / x_l}{\partial w} = 0 \quad \text{for all } k, l. \tag{30}$$

Atkinson and Stiglitz (1976) consider an economy with a single consumer of unknown wage rate and show that an optimal non-linear tax on labour incomes, without differential taxation of commodity demands, is Pareto efficient if preferences are directly weakly separable:

$$U(x, y) = V(\Gamma(x), y). \tag{31}$$

Weak separability of commodities and leisure restricts the compensated wage elasticities of commodity demands. In particular, (31) implies compensated demands can be written $x_k(q, w, u) = \hat{f}_k(q, \gamma(q, w, u))$, where γ is sub-group utility for commodities. Hence relative compensated wage effects on commodity demands differ only through income effects:

$$\frac{\partial x_k / \partial w^h}{\partial x_l / \partial w^h} = \frac{\partial \hat{f}_k / \partial \gamma}{\partial \hat{f}_l / \partial \gamma} \quad \text{for all } k, l. \tag{32}$$

[8]This condition is reminiscent of Sadka (1977), who showed that a necessary and sufficient condition for uniformity in the single-consumer case is that wage elasticities of demand for all taxed commodities be equal. The condition here is far stronger, however, inasmuch as the elasticity is constant for all prices and equal for all consumers as well.

Preferences are weakly directly separable and homothetically implicitly separable if and only if preferences are homothetically directly separable. Hence Proposition 4 holds if and only if the Deaton and Atkinson–Stiglitz conditions both hold. (Here we exclude the case in (25) of non-trivial terms $\psi^h(q)$ summing to zero.) This imposes considerably more structure on commodity demands. In particular, for any consumer h,

$$\frac{x_k^h(q,w,u)}{x_l^h(q,w,u)} = \frac{\phi_k(q)}{\phi_l(q)} \quad \text{for all } k, l, \tag{33}$$

independent of w and u. Since ϕ is identical for all consumers, moreover, distribution of commodity demands is restricted as well.

$$\frac{x_k^h}{X_k} = \frac{f_1^h(\phi(q), w^h, u^h)}{\sum_h f_1^h(\phi(q), w^h, u^h)} \quad \text{for all } k. \tag{34}$$

Conditional on prices and the distribution of utilities, a consumer's share in aggregate demand is the same for all commodities.

Under Proposition 5, relative consumer demands are again independent of wages but may differ across consumers in arbitrary ways, since

$$\frac{x_k^h(q,w,u)}{x_l^h(q,w,u)} = \frac{g_k^h(q,u^h)}{g_l^h(q,u^h)} \quad \text{for all } k, l. \tag{35}$$

In contrast, wage effects are restricted to be strongly similar across commodities and across consumers, since

$$\frac{\partial \log x_k^h}{\partial \log w^h} = 1 + \frac{\partial \log y^h}{\partial \log w^h} = \sigma \quad \text{for all } k, \text{ for all } h. \tag{36}$$

When the planner can impose a poll tax on all consumers, necessary and sufficient conditions for uniformity, presented in Proposition 1, are a weakened version of the preferences characterized in Proposition 4, inasmuch as quasi-homotheticity of conditional preferences must hold in place of homotheticity. Implied restrictions on commodity demands are weaker than (33)–(34), but not greatly so. Specifically, (33)–(34) hold with the deviation of individual demands from the average $x_k^h - \bar{x}_k$ in place of x_k^h.

Appendix

Proof of Proposition 1

Define

$$F_k^h(q,w,u) = \frac{e_k^h(q,w,u) - \bar{e}_k(q,w,u)}{\bar{e}_{k0}(q,w,u)}$$

where $\bar{e}_k = \sum_h w^h e_k^h$ and $\bar{e}_{k0} = \sum_h w^h e_{k0}^h$ are the wage-weighted averages of commodity demands and their wage derivatives, respectively. Then (8) is equivalent to

$$\sum_h \alpha^h \left(F_k^h(q,w,u) - F_l^h(q,w,u)\right) = 0 \quad (\text{all } k, l). \tag{37}$$

If (37) holds for all non-negative welfare weights and hence for all α such that $\sum_h \alpha^h = 0$, then, for all k, $F_k^h(q,w,u) = G^h(q,w,u)$, say, independent of k. From the definition of F_k^h,

$$e_k^h(q,w^h,u^h) = G^h(q,w,u)\bar{e}_{k0}(q,w,u) + \bar{e}_k(q,w,u).$$

But e_k^h is independent of (w^{-h},u^{-h}), so that there exist functions $\phi_k(q)$ and $\psi_k(q)$ such that

$$e_k^h(q,w^h,u^h) = G^h(q,w^h,u^h)\phi_k(q) + \psi_k(q).$$

Integration with respect to each q_k then yields (9). □

Proof of Proposition 2

The sufficiency of (22) for (20) follows immediately from differentiation. To show necessity, define

$$\Phi(q,w,u) = E(q,w/\|w\|,u) \tag{38}$$

where $\|\cdot\|$ is any norm; viz. in particular, $\|aw\| = a\|w\|$ for all scalars a. Notice that Φ is degree-zero homogeneous in w and degree-one homogeneous in q by construction and that, for all (q,w,u),

$$\frac{E_k(q,w,u)}{E_l(q,w,u)} - \frac{\Phi_k(q,w,u)}{\Phi_l(q,w,u)} = 0 \tag{39}$$

by virtue of degree-zero homogeneity of E_k/E_l in w. Hence the gradients of E and Φ with respect to q are collinear and the two functions are functionally dependent given (w,u). Thus there exists a (w,u)-dependent transform, say $F(\cdot,w,u)$, such that

$$E(q,w,u) = F(\Phi(q,w,u),w,u)$$

as required. □

Proof of Proposition 3

Fix $w^i = \bar{w}^i$ and $u^i = \bar{u}^i$ for all $i \neq h$. Then (22) implies

$$e^h(q,w^h,u^h) = \hat{f}^h(\phi^h(q,w^h,u^h),w^h,u^h) + \psi^h(q)$$

where

$$\hat{f}^h(b,w^h,u^h) = F(b,w^h,\bar{w}^{-h},u^h,\bar{u}^{-h})$$
$$\phi^h(q,w^h,u^h) = \Phi(q,w^h,\bar{w}^{-h},u^h,\bar{u}^{-h})$$

and

$$\psi^h(q) = -\sum_{i\neq h} e^i(q,\bar{w}^i,\bar{u}^i). \tag{40}$$

Define

$$\hat{e}^h(q,w^h,u^h) = \hat{f}^h(\phi^h(q,w^h,u^h),w^h,u^h).$$

The functions ϕ^h are degree-one homogeneous in q by the corresponding property of Φ; hence $\hat{e}^h$ is homothetic in q. Moreover, $\hat{e}^h(q,w^h,u^h)$ is degree-one homogeneous in (q,w^h) by the corresponding property of $e^h(q,w^h,u^h)$. Hence there exists a function $g^h(q,u^h)$, degree-one homogeneous in q, such that

$$\begin{aligned}\hat{e}^h(q,w^h,u^h) &= w^h\hat{f}^h(\phi^h(q/w^h,1,u^h),1,u^h)\\ &= w^h\tilde{f}^h(g^h(q/w^h,u^h),u^h)\\ &= w^h\tilde{f}^h(g^h(q,u^h)/w^h,u^h)\\ &=: f^h(g^h(q,u^h),w^h,u^h),\end{aligned}$$

The first equality follows from degree-one homogeneity of $\hat{e}^h$ in (q, w^h), the second from homotheticity of $\hat{e}^h$ in q, and the third from degree-one homogeneity of g^h in q. Finally, since g^h is degree-one homogeneous in q by construction and $\hat{e}^h$ is degree-one homogeneous in (q, w^h), it follows that $f^h(g^h, w^h, u^h)$ is degree-one homogeneous in (g^h, w^h), which completes the proof. □

Proof of Proposition 4

Sufficiency of the conditions for (22) follows from differentiation. To show necessity, differentiation of (24) implies

$$x_k^h(q, w^h, u^h) = f_1^h(g^h(q, u^h), w^h, u^h) g_k^h(q, u^h) + \psi_k^h(q). \tag{41}$$

Proposition 2 establishes that uniform taxes are optimal if and only if relative aggregate demands are independent of scaling of the wage profile, or equivalently if and only if, for all prices and utilities (q, w, u) and all scalars $\lambda > 0$,

$$\frac{E_k(q, \lambda w, u)}{E_k(q, w, u)} = \frac{E_1(q, \lambda w, u)}{E_1(q, w, u)}. \tag{42}$$

Summing (41) over h and substituting for terms involving λ, (42) becomes

$$\sum_h f_1^h(g^h, \lambda w^h, u^h) \left[\frac{g_k^h(q, u^h)}{E_k(q, w, u)} - \frac{g_1^h(q, u^h)}{E_1(q, w, u)} \right] + \sum_h \left[\frac{\psi_k^h(q)}{E_k(q, w, u)} - \frac{\psi_1^h(q)}{E_1(q, w, u)} \right] = 0. \tag{43}$$

Evaluating (43) for $\lambda = 1$ and using the resulting expression to eliminate the second summation yields

$$\sum_h \left[f_1^h(g^h, \lambda w^h, u^h) - f_1^h(g^h, w^h, u^h) \right] \left(\frac{g_k^h}{E_k} - \frac{g_l^h}{E_l} \right) = 0. \tag{44}$$

With (q, w, u) regarded as fixed, (44) defines a functional equation in λ. To simplify notation, define

$$v^h(\lambda) = f_1^h(g^h(q, u^h), \lambda w^h, u^h) - f_1^h(g^h(q, u^h), w^h, u^h) \tag{45}$$

$$d_k^h = \frac{g_k^h(q, u^h)}{E_k(q, w, u)} - \frac{g_1^h(q, u^h)}{E_1(q, w, u)} \tag{46}$$

so that (44) becomes

$$\sum_h v^h(\lambda) d_k^h = 0 \tag{47}$$

identically in λ.

Since $g^h(q, u^h) = \phi(q)$ for all h by hypothesis, (47) reduces to

$$\left(\frac{\phi_k}{E_k} - \frac{\phi_1}{E_1} \right) \sum_h v^h(\lambda) = 0.$$

Since $v^h(\lambda) > 0$ for all $\lambda > 1$ by Assumption 2, this implies $\phi_k / E_k = \phi_1 / E_1$ for all k and q. Because $E_k = \phi_k \sum_h f_1^h + \sum_h \psi_k^h$,

$$\frac{\phi_k(q)}{\phi_1(q)} = \frac{\sum_h \psi_k^h(q)}{\sum_h \psi_1^h(q)}.$$

Hence ϕ and $\sum_h \psi^h$ are functionally dependent, and an obvious change of variables then yields $\sum_h \psi^h(q) = 0$, which concludes the proof of Proposition 4. □

Proof of Proposition 5

Sufficiency again follo

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