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Taxation incentives and deadweight loss in a system of intergovernmental transfers*

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Abstract

Intergovernmental transfer programs in many federal systems, including Canada, attempt to equalize differences in subnational jurisdictions' tax capacities on the basis of the so-called representative tax system (RTS). It is shown that RTS equalization grants effectively compensate local governments for a portion of the deadweight loss associated with taxes, and consequently the grants may tend to increase the distortionary tax rates chosen by local governments. This may be the case even when equalization is confined to tax bases which are themselves non-distortionary, such as the taxation of pure economic rents. These insights are then applied to a discussion of the design of an optimal revenue-sharing scheme for a federation, given the implementation constraints facing the central government.

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1 Introduction

In federal systems, intergovernmental transfer programs are frequently designed and implemented to serve regional equity objectives and promote efficient tax and fiscal policies for subnational governments. Such programs act analogously to systems of redistributive taxation of individual income, targetting transfers to low-revenue regions and, implicitly or explicitly, taxing high-revenue regions. As such, they may create incentives for subnational governments to adopt suboptimal tax policies in order to avoid taxation or to induce transfers through the fiscal federal system. In this paper, we analyze the effects of a system of intergovernmental transfers on the "tax effort" of welfare-maximizing subnational governments, and we explore the implications of the analysis for the optimal design of such transfer programs.

The potential for transfers to distort tax policy incentives of subnational governments is particularly clear when transfers are based on the so-called representative tax system. The RTS approach, which is the basis for equalization in Canada and many other federal systems,[1] is a canonical example of a transfer formula with an explicitly redistributive aspect. Under Canada's system of equalization, eligible provinces are compensated from federal revenues for the difference between a standard level of tax revenues and the revenue the province is deemed to be able to raise, if it were to apply national average tax rates to its tax bases. Thus the program aims to equalize differences in tax revenue, but implements transfers through an indirect formula, based on differences in observed tax bases. When all provinces choose identical tax rates, the formula does indeed result in revenue equalization. But to the extent that local tax bases are elastic with respect to distortionary tax rates, provinces can induce larger equalization transfers by increasing tax rates. Additional federal transfers then partially offset the deadweight losses resulting from higher tax rates and the consequent distortions. In effect, equalization reduces the notional elasticity of tax bases used by welfare-maximizing governments in calculating second-best distortionary tax rates, and tax rates can rise in consequence.

The pure transfer component of such a policy, however, allows a portion of local government spending to be financed through non-distortionary means and, if private consumption in normal in utility, this income effect tends to reduce distortionary tax rates, offsetting the substitution effect just identified. In Section 3 of the paper, we consider a partial equilibrium model of a single government that is a recipient of equalization transfers and chooses tax and spending policies to maximize utility of a representative agent, taking the parameters of the equalization formula as given. We explore cases in which the substitution effect of equalization is dominant, so that distortionary tax rates of the receiving jurisdiction must rise as a result of equalization. We conclude that equilibrium with equalization results in a lower lever of welfare of the representative consumer than would a lump-sum transfer to the jurisdiction of an equal amount. In Section 4, the analysis is extended to the case of multiple tax bases, including non-distortionary taxes on resource rents, a special case which has received much attention in the previous literature.

The positive analysis of incentives for local tax effort under equalization raises questions of the optimal design of a federal revenue-sharing formula. In Section 5, we provide a preliminary analysis of the issue. If federal authorities are concerned about regional equity, transfers should be sensitive to differences in local tax revenues and responsive to changes over time in revenues. Given government's limited information about underlying tax capacities, this requires transfers to

[1] Similar formulas are employed in, for instance, Australia, Denmark and Switzerland (Ahmad and Thomas, 1996), as well as a number of developing countries (Shah, 1994).

be calculated based on observables such as actual revenues, which are prone to manipulation by tax authorities in subnational governments. We therefore examine equalization transfers as a problem in second-best optimal policy design, given the informational limitations of the federal government and the political or constitutional constraints imposed on it by federal decentralization. We consider a simplified version of this problem and derive its implications for such practical policy matters as: (i) the appropriate rate of "tax back" of local revenues; (ii) the optimal degree of inclusion of highly variable tax bases (such as, in the Canadian context, resource revenues) in the formula; and (iii) the role of intergovernmental transfers in providing revenue stabilization for subnational governments, while preserving appropriate incentives for tax effort.

2 Previous literature and policy analysis

Economic analysis of equalization in Canada has adduced rationales for the program based on principles of both efficiency and equity. In a federal state, with decentralized tax and fiscal policies, it is argued, federal transfers may be required to correct resource misallocations that result from a variety of fiscal spillovers among jurisdictions. (Such transfers may serve federal objectives of inter-regional equity as well.) While the efficiency analysis of transfers is conducted in a second-best environment in which local government policies may be inefficient, it is typically assumed federal authorities have access to first-best taxes and subsidies to correct interjurisdictional spillovers. Issues of information and implementation, and of the potential response of local authorities to the existence of federal transfers, typically do not arise. In what follows, we review briefly the previous literature on equalization and explain why issues of implementation have not been the primary focus of the analysis.

In a classic contribution, Boadway and Flatters (1982), building on the work of Flatters et al. (1974), construct a model of fiscally-induced migration of labour in a federation.[2] They show that, if labour is freely mobile among jurisdictions and workers locate to maximize utility derived from private income and locally-provided public goods, then labour may be misallocated in decentralized equilibrium, to the extent that jurisdictions differ in their endowments of source-based tax revenues. Labour migrates to regions with high "net fiscal benefits," reducing the marginal product of labour there below the efficient level. In this view, productive efficiency requires transfers to high-wage regions (which have low net fiscal benefits) in order to equalize the marginal product of labour in all jurisdictions. The fiscal externality in the model is internalized by federally mandated transfers equal to the difference between a region's actual source-based revenue and the average level for the federation. In the model, such conditional transfers are implementable, because taxes are non-distortionary, so that jurisdictions have no incentive to alter tax rates in response to the equalization policy.[3] In the presence of deadweight costs of subnational taxation, however, such transfers calculated on the basis of actual revenues create obvious incentive problems. A system of pure federal revenue sharing induces a free-rider problem, as all jurisdictions have an incentive to reduce tax rates in order to shift the burden of taxation to citizens of other jurisdictions. In this context, issues of implementation in the design of transfer formulas assume primary importance.

[2] See also Stiglitz (1983) and Wildasin (1991) for analysis of related issues.

[3] Since tax policies do distort the migration decision, jurisdictions may be induced to manipulate transfers in order to influence taxpayers' location decisions. Although Boadway and Flatters (1982) do not consider this possibility, Boadway (1982) shows that, in some environments, subnational governments can do no better than to set tax policies without regard for such strategic interactions.

More recently, some authors have considered the setting of distortionary tax rates in models of fiscal federalism. Dahlby and Wilson (1994) examine the optimal design of equalizing transfers when subnational governments may impose distortionary taxes on many tax bases. They show that a federal planner that seeks to maximize the sum of utilities of representative citizens of each jurisdiction should design intergovernmental transfers that implement a Ramsey tax rule for the nation—viz. taxes that equalize the marginal excess burden of taxation for each commodity and each region. They examine how transfer formulas can be adjusted for elasticity differences in order to meet this objective. In the model, however, subnational governments do not behave strategically with respect to the transfer formula, and issues of implementation do not arise. (In contrast, this paper considers how the federal planner can elicit the necessary information from sophisticated subnational governments in order to implement tax policies that equalize the marginal cost of public funds in all regions.)

Several authors have considered the potential for distortions in local tax policies resulting from the equalization formula, but previous analysis has been descriptive in its nature and incomplete in its treatment. Such issues were first raised in a theoretical model by Courchene and Beavis (1973), who considered the potential for receiving jurisdictions to manipulate the formula through its dependence on national average tax rates and tax bases. These considerations are also discussed by Bird and Slack (1990) and Usher (1995). Courchene (1994) extended the analysis to endogenize jurisdictions' own tax bases, arguing that, in some cases, equalization might deter receiving jurisdictions from developing new revenue sources, since additional revenues are implicitly "taxed back" through the entitlement formula. He argues that an optimal program would take into account such disincentive effects for local tax effort, and proposes partial equalization of revenue differences. In this paper, these arguments are extended, and it is shown that the potential for local governments to manipulate the formula and distort tax policies is far more general than has previously been recognized.

3 Equalization and local tax policies

For the sake of concreteness, the analysis which follows focusses on the Canadian equalization formula. Equalization entitlements are presently calculated for each of 37 separate revenue categories. A jurisdiction's per capita entitlement in a revenue category is equal to its per capita tax base "deficiency" in the category, relative to a per capita standard for the category, multiplied by the calculated national average tax rate for the category. (Under the current Representative Five Province Standard (RFPS) system, standard tax bases are calculated as the weighted average per capita tax base of the five "standard" provinces of Quebec, Ontario, Manitoba, Saskatchewan, and British Columbia.[4]) Equalization entitlements are summed over all revenue categories; jurisdictions with positive calculated entitlements receive a transfer from the federal government equal to the entitlement,[5] whereas jurisdictions with negative calculated entitlements receive a net transfer of zero. Thus equalization is an asymmetric revenue sharing scheme, raising revenues of deficient jurisdictions to the standard level, but not taxing jurisdictions with larger-than-average tax bases,

[4]The RFPS was instituted in 1982, replacing the earlier national average standard. The reasons for the RFPS are discussed briefly in Section 5.

[5]Since 1982, the cumulative growth rate of total transfers to jurisdictions under equalization cannot exceed the rate of GNP growth.

as would a symmetric scheme.[6]

3.1 Basic analytics of equalization

To discuss the analytics of the formula, define X_{pj} as the measured tax base of jurisdiction p in revenue category j. Let P represent an index set of all jurisdictions and $S \subset P$ represent an index set of standard jurisdictions. Let n_p represent the population of jurisdiction p and $n = \sum_{p \in S} n_p$. Then the per capita equalization entitlement of jurisdiction p for category j is

$$\begin{aligned} e_{pj} &= \bar{t}_j \left(\frac{\sum_{i \in S} X_{ij}}{n} - \frac{X_{pj}}{n_p} \right) \\ &= \bar{t}_j (\bar{x}_j - x_{pj}), \end{aligned} \tag{1}$$

where

$$\bar{t}_j = \frac{\sum_{i \in P} t_{ij} X_{ij}}{\sum_{i \in P} X_{ij}} \tag{2}$$

is the national average tax rate and $\bar{x}_j$ is the weighted-average per capita base of standard jurisdictions and x_{pj} is the per capita base of jurisdiction p for the category. Consider the case of a receiving jurisdiction; viz. one for which

$$\sum_j e_{pj} \geq 0.$$

Define the jurisdiction's own-source revenue for category j as $\tilde{R}_{pj} = t_{pj} x_{pj}$. The per capita total revenue of p in category j, net of equalization transfers is

$$\begin{aligned} R_{pj} &= \tilde{R}_{pj} + e_{pj} \\ &= \bar{t}_j \bar{x}_j + (t_{pj} - \bar{t}_j) x_j. \end{aligned} \tag{3}$$

Because receiving jurisdictions' tax rates and bases influence national average tax rates and, for standard jurisdictions $p \in S$, representative standard bases, local tax policies interact to influence equalization entitlements in complicated ways. In particular, when a receiving jurisdiction has a large share of national revenues in a category, tax rates have strong effects on equalization transfers and receiving jurisdictions have perverse incentives in choosing tax policies. Such effects of the particular accounting treatment of the equalization formula were first analyzed by Courchene and Beavis (1973) and have been stressed by Boadway and Hobson (1993) and Usher (1995), among others. This paper, in contrast, is primarily concerned with the economic effects of equalization, resulting from the elasticity of tax bases with respect to tax rates, and initially we analyze incentives when $\bar{t}_j$ and $\bar{x}_j$ are invariant to the receiving jurisdiction's tax rate t_{pj}. Unlike the earlier papers, this analysis applies equally to jurisdictions that are small relative to the federation (and which therefore have only negligible effect on national averages) and to large jurisdictions.

It is useful to summarize some of the simple comparative statics of equalization transfers with respect to key exogenous variables, in order to highlight the mechanics of the program and the

[6] Boadway and Hobson (1993) refers to asymmetric equalization as a "gross" scheme and symmetric equalization as a "net" scheme.

incentives it creates. In what follows, we consider for simplicity the case of a receiving jurisdiction not in the representative standard.

(i) Effect of exogenous increases in own-source revenue: Differentiating (3) with respect to x_{pj},

$$\frac{\partial R_{pj}}{\partial x_{pj}} = t_{pj} - \bar{t}_j \geq 0 \quad \text{if and only if} \quad t_{pj} \geq \bar{t}_j. \tag{4}$$

An exogenous positive shock to the receiving jurisdiction's tax base increases net revenue if and only if the local tax rate exceeds the national average for the category. If $\bar{t}_j > 0$, moreover, the rise in net revenue is less than the rise in own-source revenue, as equalization transfers are reduced: equalization induces positive "tax back" of local revenues. A number of commentators have argued that this feature of equalization attenuates the incentives of local government to develop new revenue sources (Courchene, 1994).

(ii) Effect of an increase in own tax rate: Suppose all cross-price elasticities are zero. Then

$$\frac{\partial R_{pj}}{\partial t_{pj}} = x_{pj} + (t_{pj} - \bar{t}_j)\frac{\partial x_{pj}}{\partial t_{pj}} + (\bar{x}_j - x_{pj})\frac{\partial \bar{t}_j}{\partial t_{pj}}. \tag{5}$$

In contrast, the derivative of own-source revenue is

$$\frac{\partial \tilde{R}_{pj}}{\partial t_{pj}} = x_{pj} + t_{pj}\frac{\partial x_{pj}}{\partial t_{pj}}. \tag{6}$$

Assume $\partial x_{pj}/\partial t_{pj} < 0$. Then

$$\frac{\partial R_{pj}}{\partial t_{pj}} \geq \frac{\partial \tilde{R}_{pj}}{\partial t_{pj}} \quad \text{if} \quad x_{pj} \leq \bar{x}_j.$$

(Note this is a weak sufficient condition if $\bar{t}_j$ is large relative to t_{pj}.) Equalization increases the slope, or *tax responsiveness*, of the revenue function of any category in which the receiving jurisdiction is deficient. It is this property that is crucial to the analysis that follows.

(iii) Effect of increase in national average tax rates:

$$\left.\frac{\partial R_{pj}}{\partial \bar{t}_j}\right|_{t_{pj}\ \text{constant}} = \bar{x}_j - x_{pj}. \tag{7}$$

Increases in tax rates of other jurisdictions, holding revenue shares fixed, result in increases in transfers to jurisdictions with tax base deficiencies in the category. Moreover, since

$$\left.\frac{\partial^2 R_{pj}}{\partial t_{pj}\partial \bar{t}_j}\right|_{t_{pj}\ \text{constant}} = -\frac{\partial x_{pj}}{\partial t_{pj}} > 0, \tag{8}$$

increases in other jurisdictions' tax rates raise the tax-responsiveness of net revenue.

What are the implications of these effects for the tax rates chosen by local governments? In the remainder of this section, a simple model of tax policy decisions is introduced and the impact of equalization is examined.

3.2 A model of optimal taxation

3.2.1 Economic and fiscal environment

Consider a local economy with two private goods, consumption x and labour l, and a single public good g. The local government levies a distortionary tax on consumption at specific rate t in order to finance public sector expenditures, and chooses tax and fiscal policy to maximize welfare of a representative consumer.[7] Suppose that utility of the representative agent is additively separable in g,

$$U(x, l, g) = u(x, l) + b(g), \tag{9}$$

where u is increasing in x, decreasing in l, and concave in both its arguments, and b is increasing and concave in g. The assumption of separability in (9), while probably excessively restrictive, permits an analysis of governmental transfers that is not complicated by the existence of direct substitution effects between private and public consumption.

Normalize producer prices to unity and define the agent's indirect utility from private consumption conditional on tax policy as

$$v(t) = \max u(x, l) \quad \text{s.t.} \quad (1+t)x = l. \tag{10}$$

Let $(x(t), l(t))$ solve (10). Note that v is decreasing and concave in t by standard arguments.

3.2.2 Tax optima

As a benchmark for the analysis of equalization, consider initially second-best tax policies for a jurisdiction receiving a lump-sum, unconditional grant $\bar{e} > 0$. Define $R(t) = tx(t)$ as local own-source revenue. A tax optimum solves

$$\max v(t) + b(g) \quad \text{s.t.} \quad g = R(t) + \bar{e}. \tag{11}$$

If t^0 solves (11) then

$$-\frac{v_t}{b_g} = x(t^0)\left(1 - \frac{t^0}{1+t^0}\epsilon(t^0)\right), \tag{12}$$

where $\epsilon = -\partial \log x / \partial \log(1+t)$ is the elasticity of demand for x. By Roy's identity, $v_t = -\lambda x$, where λ is the marginal utility of private income. Hence

$$\frac{\lambda}{b_g} = 1 - \frac{t^0}{1+t^0}\epsilon. \tag{13}$$

At the optimum, in the absence of transfers, the marginal rate of substitution between public and private spending is set equal to the marginal cost of public funds.

[7] The case of a government maximizing a concave welfare function of the utilities of many heterogeneous consumers is a straightforward but unedifying extension. Alternatively, under some additional conditions, the representative consumer can be regarded as the median voter in the jurisdiction, who is decisive in choosing tax policies under majority rule.

Consider the introduction of a program of revenue equalization that pays the local government a transfer

$$e(t, \bar{t}) = \bar{t}(\bar{x} - x(t)). \tag{14}$$

By assumption, the national tax rate $\bar{t}$ and standard base $\bar{x}$ are positive. The transfer is financed externally to the jurisdiction, and the local tax authority is assumed to regard $\bar{t}$ and $\bar{x}$ as exogenous parameters, invariant to t. In the presence of equalization, the jurisdiction's optimal tax policy maximizes welfare of the representative agent, subject to the modified budget constraint that public spending equal own-source revenue plus the equalization transfer. Thus a tax optimum under equalization solves

$$\max v(t) + b(g) \quad \text{s.t.} \quad g = R(t) + e(t, \bar{t}). \tag{15}$$

If a tax rate $t^* > 0$ solves (15) then

$$\frac{\lambda}{b_g} = 1 - \frac{t^* - \bar{t}}{1 + t^*}\epsilon. \tag{16}$$

Comparing (16) to (13), it is evident that equalization induces a substitution effect which lowers the effective marginal cost of public funds, if $\epsilon > 0$ and $\bar{t} > 0$, and leads to *ceteris paribus* increases in local tax rates. Intuitively, the decline in tax bases associated with distortionary taxation is partially offset through transfers, lowering the burden to local taxpayers of taxation. The effect of equalization on the effective marginal cost of public funds is particularly clear when $t = \bar{t}$: in this event, the right-hand side of (16) reduces to unity, so that the effective marginal excess burden of local taxation is zero when the local tax rate is set equal to the standard level $\bar{t}$. Hence equalization tends to raise tax rates above the chosen standard level.[8]

We formalize this intuition as follows. Unlike more traditional approaches to such comparative static results, the proofs below do not rely on convexity of the optimization problem or differentiability of the optimizer, properties which cannot be guaranteed in second-best optimal tax problems. Instead, a technique similar to the ordinal approach to monotone comparative statics of Milgrom and Shannon (1994) is adopted.

Proposition 1 *Let t^0 solve* (11) *for some $\bar{e}$, and suppose that $e(t^0, \bar{t}) = \bar{e}$. Then $t^* \geq t^0$ if $x(t)$ is non-increasing in t.*

Proof. Since t^0 is optimal for (11),

$$v(t^0) + b(t^0 x(t^0) + \bar{e}) \geq v(t) + b(tx(t) + \bar{e})$$

for all $t \leq t^0$. Since x is non-increasing in t by assumption, e is non-decreasing in t by (14). Hence $\bar{e} = e(t^0, \bar{t}) \geq e(t, \bar{t})$ for all $t \leq t^0$, so that

$$v(t^0) + b(t^0 x(t^0) + e(t^0, \bar{t})) \geq v(t) + b(tx(t) + e(t, \bar{t}))$$

for all $t \leq t^0$. It follows that if t^* solves (15) then $t^* \geq t^0$. $\square$

[8] I thank Dan Usher for pointing this out.

Proposition 1 states that equalization induces substitution toward higher rates of distortionary taxation by welfare-maximizing local governments. A natural inference is that equalization creates welfare losses relative to a benchmark program paying equal unconditional grants. This inference is indeed correct, although welfare analysis of the program is deferred until Section 4.1 below. Proposition 1 does not however generate empirical predictions about the effect of equalization on observed local tax rates, as the transfers also induce income effects which can lead to reductions in distortionary taxation. In some restrictive cases, however, it is possible to state unambiguously that income effects do not offset the substitution effect identified in Proposition 1. The most immediate such case occurs when local social welfare is quasi-linear in public spending, so that by assumption there are no income effects. The following proposition states that, in the quasi-linear case, equalization results in a higher local tax rate than would be chosen for any level of the lump-sum grant $\bar{e}$.

Proposition 2 *Suppose that $b(g) = \theta g$, for some $\theta > 0$. Then $t^* \geq t^0$ for all $\bar{e}$ if x is non-increasing in t.*

Proof. By the definitions of the tax optima (11) and (15) for the case of quasi-linear preferences for public spending,

$$v(t^0) + \theta(t^0 x(t^0) + \bar{e}) \geq v(t^*) + \theta(t^* x(t^*) + \bar{e})$$
$$v(t^*) + \theta(t^* x(t^*) + e(t^*, \bar{t})) \geq v(t^0) + \theta(t^0 x(t^0) + e(t^0, \bar{t})),$$

which implies, given (14),

$$\theta\bar{t}(x(t^0) - x(t^*)) \geq 0.$$

Hence if x is non-increasing in t then $t^* \geq t^0$. □

Our analysis in the introductory subsection indicated that a jurisdiction that was deficient in a tax base had an unambiguous incentive to increase tax rates, if its objective were to maximize net revenue. This observation can be extended to the case of welfare-maximizing governments as well. Suppose that, in the absence of equalization, the tax rate t^0 is chosen such that $x(t^0) < \bar{x}$, so that the jurisdiction is deficient in the tax base relative to the standard level. Then the following proposition demonstrates that introduction of the equalization grant results in an increase in the local tax rate.

Proposition 3 *Let t^0 solve (11) for $\bar{e} = 0$, and suppose that $x(t^0) \leq \bar{x}$. Then $t^* \geq t^0$ if x is non-increasing in t.*

Proof. Note $x(t^0) \leq \bar{x}$ implies $e(t^0, \bar{t}) \leq 0$. Apply the proof of Proposition 1 with $\bar{e} = 0$. □

4 Extensions of the model

This section briefly explores tax policy incentives for subnational governments under equalization in a number of contexts that are related to, but more general than, the simple model of Section 3. Section 4.1 examines the effects of an equalization program when governments have access to several, inter-related final demand tax bases. Section 4.2 considers the special case of non-distortionary taxation of rents associated with fixed factors of production, such as resource rents. The issue of equalization of such source-based, non-distortionary tax bases has been the subject of much attention in the previous literature (e.g. Boadway and Hobson, 1993), and so the analysis of Section 3 is extended to deal with this case. It is shown that equalization of only such source-based taxes, which is occasionally recommended in the literature, may lead to distortions related to those described in Section 3, even when the tax bases are themselves non-distortionary. Section 4.3 briefly discusses tax competition and fiscal externalities, and shows that the welfare implications of the partial equilibrium model may change when the potential for tax competition exists.

4.1 Multiple tax bases

In practice, in Canada and other federal systems, equalization is applied not to a single tax base, as assumed in Section 3, but to many distinct tax bases. This section extends the analysis of optimal subnational tax policies under equalization to the case of multiple final demand tax bases and shows that the principal conclusions of Section 3—that equalization results in higher rates of distortionary taxation and lower levels of consumer welfare than equivalent lump-sum transfers—also obtain in this more general context.

The model is largely the same as that of Section 3. There is a single representative citizen resident in the local jurisdiction with preferences $u(x, l) + b(g)$, where $x \in \mathbf{R}^K$ is a K-dimensional vector of commodity demands. Producer prices of the commodities are fixed and normalized to unity, and $t \in \mathbf{R}^K$ is a vector of specific tax rates levied by the local government on commodity demands.

Local government tax revenues are subject to a symmetric equalization scheme, given a representative tax system with standard bases $\bar{x} \in \mathbf{R}^K$ and standard tax rates $\bar{t} \in \mathbf{R}^K$. The equalization entitlement of a local government adopting tax rates t is $e(t, \bar{t})$, given by (14) above, where tax and quantity variables are now interpreted as vectors. Define

$$W(\bar{t}) = \max_{(t,g)} v(t) + b(g) \quad \text{s.t.} \quad t \cdot x(t) + e(t, \bar{t}) \geq g \tag{17}$$

as the optimal level of social welfare in the local jurisdiction given equalization at the standard level $\bar{t}$, and let $(t^*(\bar{t}), g^*(\bar{t}))$ solve (17). As before, comparative static analysis of the effects of equalization on optimal tax rates t^* and social welfare W are complicated by the income effects associated with intergovernmental transfers. For this reason, consider the corresponding compensated optimization problem

$$T(w, \bar{t}) = \min_{(t,g)} g - \bar{t} \cdot \bar{x} - (t - \bar{t}) \cdot x(t) \quad \text{s.t.} \quad v(t) + b(g) \geq w. \tag{18}$$

Thus $T(w, \bar{t})$ defines the net lump-sum transfer to the local government required to achieve social welfare w, given that an equalization standard $\bar{t}$ is in place and that local tax rates are chosen optimally. Define $(t^c(w, \bar{t}), g^c(w, \bar{t}))$ as the optimizers in (18). The function T plays the same role in the analysis as does the consumer expenditure function in the standard analysis of optimal tax policies of a single government. Several properties of T, analogous to properties of the expenditure function, are useful for the results that follow. First, by definition, W and T are algebraic inverses given $\bar{t}$, viz.

$$T(W(\bar{t}), \bar{t}) \equiv 0. \tag{19}$$

Second, if T is differentiable, by the envelope theorem,

$$\frac{\partial T(w, \bar{t})}{\partial \bar{t}_k} = x_k(t^c(w, \bar{t})) - \bar{x}_k \quad k = 1, \ldots, K. \tag{20}$$

Hence T is decreasing in $\bar{t}_k$ if and only if the local jurisdiction is deficient in tax base k, given its optimal policy. Third, T is concave in $\bar{t}$.[9]

When $\bar{t} = 0$, the transfer formula (14) reduces to the case of no equalization transfer, which is taken to be the benchmark case against which outcomes under equalization are compared. Let $w^0 = W(0)$ be social welfare in the absence of equalization and $t^0 = t^c(w^0, 0)$ be the associated vector of tax rates. When government has access to many tax bases, it is not possible to establish conditions under which individual tax rates are greater under equalization than under an equivalent lump-sum transfer, as it was in the case of a single tax base, because substitution effects among commodities can offset the direct tax-increasing effect of equalization. It can be established, however, that equalization results in greater aggregate deadweight loss for the citizen of the receiving jurisdiction than would an equivalent lump-sum grant. Thus, while individual tax rates may rise or fall as a result of a compensated change to an equalization system, the level of taxation must rise in the aggregate. The proof of this proposition is simple. By definition, $T(w^0, \bar{t})$ is the net transfer to the region required to attain benchmark welfare level w^0. Equivalently, $T(w^0, \bar{t})$ is the consumer's compensating variation for the introduction of equalization at standard tax rates $\bar{t}$. The actual transfer to the local government under equalization is

$$\bar{t} \cdot (\bar{x} - x(t^c(w^0, \bar{t})) = \bar{t} \cdot DT(w^0, \bar{t}),$$

where $DT(w, \bar{t}) = (\partial T(w, \bar{t})/\partial t_k)$ is the vector of partial derivatives of T. Since T is concave in $\bar{t}$ and $T(w^0, 0) = 0$ by construction,

$$\bar{t} \cdot DT(w^0, \bar{t}) \leq T(w^0, \bar{t}) \tag{21}$$

for all $\bar{t} \geq 0$. Conversely, suppose that the distortionary effects of taxation are not higher under equalization, so that (21) holds as an equality for all $\bar{t}$. Then DT is independent of $\bar{t}$, so that (20) implies x is independent of t. Thus equalization does not result in higher deadweight loss than the lump-sum transfer only if market demands for commodities are perfectly inelastic with respect to prices. The result is stated formally as follows.

Proposition 4 *The representative citizen's compensating variation for the introduction of equalization is never less than the associated transfer. Moreover, the compensating variation equals the actual transfer if and only if $x(t)$ is independent of t.*

It is in this sense that equalization unambiguously increases the distortionary effect of local government taxes and reduces welfare of the representative citizen.

[9] The proof is standard. Choose any $\bar{t}_1$ and $\bar{t}_2$ and let $\bar{t}_0 = \lambda\bar{t}_1 + (1-\lambda)\bar{t}_2$ for $\lambda \in [0,1]$. Let (t_i, g_i) solve (18) for $\bar{t}_i$, $i = 0, 1, 2$. By definition of T,

$$g_i - \bar{t}_i \cdot \bar{x} - (t_i - \bar{t}_i) \cdot x(t_i) \leq g_0 - \bar{t}_i \cdot \bar{x} - (t_0 - \bar{t}_i) \cdot x(t_0)$$

for $i = 1, 2$. Multiplying and summing the two inequalities,

$$\lambda T(w, \bar{t}_1) + (1-\lambda) T(w, \bar{t}_2) \leq g_0 - \bar{t}_0 \cdot \bar{x} - (t_0 - \bar{t}_0) \cdot x(t_0) = T(w, \bar{t}_0),$$

so that T is concave. Q.E.D.

4.2 Resource taxes

Theoretical literature studying federal fiscal equalization has frequently given special attention to resource tax bases or, more generally, taxation of rents accruing to fixed factors of production within a jurisdiction. From the perspective of equalization policy, such taxes have two special features. First, taxation of economic rents is in principle non-distortionary. Thus it might be expected that the tendency for equalization transfers to enhance the distortionary effects of local taxation would not apply to such taxes. Second, resource revenues are source-based taxes. To the extent that the size of source-based tax bases differ and workers are mobile among jurisdictions, labour may be misallocated in decentralized equilibrium, as migrants move in response to differences in net fiscal benefits, rather than differences in the gross marginal product of labour. This observation has led some analysts to adduce efficiency arguments in favour of equalization of source-based taxes in particular (Boadway and Hobson, 1993).

When the tax policy decisions of local governments are regarded as endogenous to the equalization policy, this conclusion may be reversed. Equalization has the potential to correct fiscal externalities associated with interjurisdictional differences in net fiscal benefits, but the greater excess burden of taxation associated with the policy may more than offset the welfare gains resulting from improved interjurisdictional allocation. This observation remains true even when equalization is confined to source-based resource revenues, and we adopt the (probably extreme) assumption that such taxes are non-distortionary.

To establish this, consider again the three-good model of Section 3, extended to incorporate equilibrium producer surplus, which may be interpreted as resource rents. The representative citizen in the local jurisdiction chooses consumption x and labour supply l to solve

$$v(\tau, w) = \max_{(x,l)} u(x, l) \quad \text{s.t.} \quad x = (1-\tau)wl \tag{22}$$

given the gross wage rate w and income tax rate τ. Let $l((1-\tau)w)$ solve (22). The competitive production sector hires labour to produce the consumption good with the decreasing-returns-to-scale technology $y = f(l)$, so that $f' > 0$ and $f'' < 0$ by assumption. Profit-maximizing labour demand $l^d(w)$ solves

$$F(w) = \max_{l} f(l) - wl. \tag{23}$$

Producer surplus $F(w)$ accrues in the first instance as a rent to owners of a fixed factor of production, but all the rent is taxed by the local government to finance a portion of public spending g. Notice that this fixed-factor levy is *per se* non-distortionary, and that it is always optimal for a welfare-maximizing government to tax away 100 per cent of the rent.

A competitive private market equilibrium given tax policy τ is therefore described by a labour supply function $l((1-\tau)w)$ and demand function $l^d(w)$ solving (22) and (23) respectively, and a gross wage rate $w^*(\tau)$ such that

$$l((1-\tau)w^*(\tau)) = l^d(w^*(\tau)). \tag{24}$$

Let $l^*(\tau) = l((1-\tau)w^*(\tau))$ be equilibrium labour supply and $F^*(\tau) = F(w^*(\tau))$ be the equilibrium fixed-factor tax base. Observe that

$$\frac{dF^*(\tau)}{d\tau} = -l^*(\tau)\frac{dw^*(\tau)}{d\tau} \leq 0 \quad \text{if and only if} \quad \frac{dw^*(\tau)}{d\tau} \geq 0. \tag{25}$$

so that local increases in income tax revenue "crowd out" fixed-factor tax revenue.

An optimal local tax policy equates the marginal rate of substitution between public and private consumption to the effective marginal cost of public funds, given the transfer formula, so that

$$\frac{v_\tau}{b_g} = \frac{dg}{d\tau}. \tag{26}$$

Consider first the case of an unconditional lump-sum transfer $\bar{e}$ to the local jurisdiction. The government budget constraint is

$$g(\tau) = \tau w^*(\tau) l((1-\tau) w^*(\tau)) + F(w^*(\tau)) + \bar{e}. \tag{27}$$

Total differentiation of (24) yields

$$\frac{dw^*(\tau)}{d\tau} = \frac{w}{1-\tau} \frac{\epsilon^s}{\epsilon^s + \epsilon^d} \tag{28}$$

where ϵ^s and ϵ^d are the wage elasticities of supply of and demand for labour, respectively.

Differentiating (27) and substituting (28) leads to the following expression for marginal tax revenue under the lump-sum federal grant:

$$\frac{dg}{d\tau} = wl \frac{\epsilon^d}{\epsilon^s + \epsilon^d} \left(1 - \frac{\tau}{1-\tau} \epsilon^s\right). \tag{29}$$

In this case, optimal tax policy takes account of the effect of the wage tax on fixed-factor rents accruing to the local government, and the marginal cost of public funds is increased proportionally to $(\epsilon^s + \epsilon^d)/\epsilon^d$.

When fixed-factor rents are equalized among jurisdictions in the federation, the feedback from distortionary local taxes to rents is eliminated, the effective marginal cost of public funds is reduced, and distortionary tax rates may rise in consequence. Consider a transfer formula $e(\tau) = \bar{F} - F(w^*(\tau))$, which equalizes source-based taxes at some exogenous level $\bar{F}$. The government budget constraint is

$$g(\tau) = \tau w^*(\tau) l((1-\tau) w^*(\tau)) + \bar{F}. \tag{30}$$

Differentiating (30) and substituting (28),

$$\frac{dg}{d\tau} = wl \left(\frac{\epsilon^d}{\epsilon^s + \epsilon^d} \left(1 - \frac{\tau}{1-\tau} \epsilon^s\right) + \frac{\epsilon^s}{\epsilon^s + \epsilon^d} (1-\tau) \right). \tag{31}$$

Comparing (29) and (31), it can be seen that equalization increases the tax-responsiveness of net local revenues, relative to the lump-sum federal grant, and decreases the effective marginal cost of public funds. Hence, compensating for income effects, equalization results in higher optimal tax rates than an equivalent lump-sum grant. The techniques applied in the proof of Proposition 1 also establish the following formal statement of the result. Let τ^0 maximize welfare subject to (27) and τ^* maximize welfare subject to (30).

Proposition 5 *Suppose that* $e(\tau^0) = \bar{e}$. *Then* $\tau^* \geq \tau^0$ *if* $l((1-\tau)w)$ *is non-increasing in* τ *and* $l^d(w)$ *is non-increasing in* w.

When labour is elastically supplied and demanded, the income-compensated effect of equalization is to increase distortionary taxes. This is so even if equalization is confined to a revenue category, such as rents to fixed factors of production, which is itself non-distortionary.

4.3 Fiscal externalities and tax competition

The analysis of the paper has been conducted in a partial equilibrium context of a single local jurisdiction and a federal authority. In this environment, undistorted local government tax policy is second-best optimal, so that an equalization grant results in welfare losses, relative to an equivalent lump-sum grant. More generally, however, it is well understood that strategic interactions among governments may lead to equilibrium tax policies that are inefficient in decentralized equilibrium. In such circumstances, the welfare implications of the analysis may be undermined and possibly reversed.

When tax bases are mobile among jurisdictions in a federation, local government tax policies have external effects on residents of other jurisdictions, as each jurisdiction's choices of tax rates influence the level and tax responsiveness of revenues in other jurisdictions. If local governments choose tax policies independently, then such fiscal externalities are not accounted for in decision-making, and Nash equilibrium tax rates can depart from second-best optimal levels. In some circumstances, it can be established that Nash equilibrium tax rates of the decentralized game are below second-best levels.[10] When this is the case, RTS equalization can reduce the effective tax responsiveness of local government revenues, raise equilibrium tax rates, and result in welfare gains, relative to the decentralized Nash equilibrium. In effect, equalization serves partially to "cartelize" local governments, inducing them to internalize a portion of the fiscal externalities resulting from local tax policy. This observation echoes the casual argument of many authors that fiscal federal systems tend to centralize authority in a federation and may result in higher levels of taxation (citations). It is interesting to note, however, that this phenomenon can be attributed to the direct substitution effects of an equalization grant, which may not exist for other forms of federal transfer arrangements.

5 Toward a theory of optimal equalization

In the previous section, we studied the response of welfare maximizing governments to (a stylized version of) the formula currently used to calculate equalization transfers to provincial governments in Canada. It was shown that, for a small jurisdiction which regards national average tax rates as fixed, equalization induces a substitution effect which increases distortionary tax rates. In particular, it was demonstrated that a small jurisdiction receiving equalization would choose a tax rate higher than that chosen if equalization were replaced by a lump-sum transfer in the same amount. Moreover, it was demonstrated that increased tax rates have adverse consequences for national welfare—in the sense that representative citizens are worse off under equalization than they would be with unconditional interjurisdictional transfers of an equal amount.

But the case of lump-sum federal grants may represent an inappropriate benchmark for evaluating equalization programs. Such transfers could in principle be designed to meet the federal government's objectives for inter-jurisdictional equity, without the distortions associated with equalization identified in this paper. But a lump-sum grant system would be deficient, inasmuch as it would not provide local governments with the insurance for own-source revenues that is implicitly provided through equalization and revenue-sharing programs. Governments operate in stochastic environments, with local variability in revenues and incomes, and so, to the extent regional equity

[10] Tax competition can also result in higher tax rates, however. See Mintz and Tullkens (1986).

is desirable, revenue-sharing systems may be preferred to lump-sum grants, despite their distortionary effects. Given the limited information of federal authorities about underlying tax capacities, transfers are calculated on the basis of observables such as actual revenues, which are prone to manipulation by tax authorities in subnational governments. Revenue sharing consequently gives rise to free-riding problems, as subnational governments seek to shift the burden of taxation to citizens of other jurisdictions.

Federal transfers must therefore seek to redistribute revenue and insure subnational treasuries against revenue shocks, to the extent possible, while preserving incentives for local authorities to set tax rates correctly. From this perspective, the representative tax system approach to calculating equalization transfers appears *prima facie* to be anomalous. Under the RTS system, transfers to a local government depend on revenue realizations in other jurisdictions in the federation in addition to its own (a property we call "revenue interdependence" in what follows). As such, the RTS exposes subnational treasuries to additional risks, which undermine the insurance effects of the transfer system. It will be argued, however, that while revenue interdependence increases risk and reduces welfare relative to a full-insurance system in a first-best environment, it may improve incentives for local governments in choosing tax effort.

In this section, we examine a simple model of second-best federal transfer policies in such an environment. In the model, a utilitarian federal planner seeks to design a revenue-sharing formula for subnational governments to enhance inter-local equity. It is assumed that transfers can be based solely on the observed revenue raised by the subnational governments, which is stochastic. The assumption that extraneous information cannot be used is a strong one, but may be justified as a simple approximation to the institutional and informational constraints facing federal planners.[11]

We consider a simplified version of this problem and derive its implications for such practical policy matters as: (i) the appropriate rate of "tax back" of local revenues; (ii) the optimal degree of inclusion of highly variable tax bases (such as, in the Canadian context, resource revenues) in the formula; and (iii) the role of intergovernmental transfers in providing revenue stabilization for subnational governments, while preserving appropriate incentives for tax effort.

The model of the economic and fiscal environment facing local governments is substantially the same as in previous sections, except that we allow for arbitrary differences in representative citizens' preferences for public and private consumption. The representative citizen in jurisdiction i maximizes utility from private consumption, given the distortionary tax on consumption and given the wage rate w_i, so that

$$v^i(t_i, w_i) = \max u^i(x_i, l_i) \quad \text{s.t.} \quad (1 + t_i)x_i = w_i l_i. \tag{32}$$

Let $x^i(t_i, w_i)$ denote the maximizer in (32) and $R^i(t_i, w_i) = t_i x^i(t_i, w_i)$ denote local own-source tax revenue.

Local "tax capacities" $w = (w_1, \dots, w_P)$ are stochastic, with joint density $\gamma(w)$, and unknown to local tax authorities at the time tax rates are chosen. Let $\gamma^i(w_i)$ denote the marginal density of

[11] Certainly, if transfers could be made conditional on tax rates, and rates were accurately observed, then a federal planner could implement any desired allocation by means of forcing contracts, which specified arbitrarily large penalties for deviations from the stipulated tax policies. But policies of this type are politically and constitutionally feasible only in *de facto* unitary states, and it is more appropriate to assume that fiscal decentralization imposes real constraints on federal policies. Moreover, it is unclear whether utilitarian federal planners could credibly commit to imposing such penalties *ex post*.

w_i and

$$c^i(t_i) = -\int v^i(t_i, w_i)\gamma^i(w_i)dw_i$$

denote jurisdiction i's *ex ante* expected cost of taxation. It is assumed that c^i is increasing and convex in t_i. Local tax rates are then chosen to maximize $Eb^i(g_i) - c^i(t_i)$, where $b^i(g_i)$ is the perceived benefit of public spending g_i in jurisdiction i, subject to the government budget constraint and the given formula for intergovernmental equalization.

It is sometimes convenient to suppress w from the problem and to imagine local tax policies as choosing a distribution function for own-source revenues, which is parameterized by the tax rates t. To this end, let $F(R,t) = \text{Prob}(R^1(t_1, w_1) \leq R_1, \ldots, R^P(t_P, w_P) \leq R_P)$ be the joint c.d.f. for $R_1, \ldots, R_P$. In what follows, we will require the technical assumption that F be convex in t. (This is satisfied if R^i is concave and $\gamma(w)$ is uniform, for example.) Define $f(R,t)$ as the corresponding joint density and $f^i(R_i, t_i)$ as the marginal density for R_i. Finally, let $f_i(R,t) = \partial f(R,t)/\partial t_i$.

5.1 Centralized optimum

Before proceeding to the characterization of tax policies and equalization formulas in a federal state, consider the case of a centralized, unitary state, in which all tax policy and spending decisions are undertaken by a single national planner on behalf of citizens of the regions. Suppose that the national planner is a classical utilitarian and seeks to maximize an unweighted sum of utilities of representative citizens in each region, less the cost of net public funds, if any, provided from the federal budget to finance local public expenditures in excess of tax collections. If the "fiscal gap" between desired public spending and tax revenues in the regions is small, it is appropriate to regard the marginal cost of federal funds as some constant $(1+\lambda)$, where λ is the marginal excess burden of federal taxation. This assumption is maintained throughout the analysis that follows.

Define the federal objective, therefore, as

$$V^0(g,t,R) = \sum_{i=1}^{P} \Big[b^i(g_i) - c^i(t_i) - (1+\lambda)(g_i - R_i) \Big]. \tag{33}$$

In the unitary case, tax and spending decisions are then chosen to maximize

$$\int V^0(g,t,R) f(R,t) dR.$$

Assuming positive tax rates are optimal, first-order conditions, which in view of preceding assumptions are necessary for an optimum are

$$b^i_g(g^*_i) = 1 + \lambda \quad (\text{all } i) \tag{34}$$

$$\begin{aligned} c^i_t(t^*_i) &= \int V^0 f_i dR \\ &= (1+\lambda) \int R_i \frac{\partial f^i(R_i, t_i)}{\partial t_i} dR_i \end{aligned} \tag{35}$$

where the second equality in (35) follows from the fact that g^*_i is non-stochastic (and $\int f_i dR_i = 0$). In the centralized optimum, then, public spending is allocated among the regions in order to equalize the perceived marginal benefit of public spending in each region at the marginal cost of federal funds. This allocation may involve a net transfer to or from the federal treasury, depending on the expected marginal cost of the regions' own-source revenues. Equation (35) indicates that local tax rates are set such that the ratio of the expected marginal welfare cost of taxation to the expected marginal change in revenue equals the marginal cost of federal funds. Thus the effective marginal cost of local funds in this stochastic environment is

$$c_t^i(t_i) \Big/ \int R_i \frac{\partial f^i(R_i, t_i)}{\partial t_i} dR_i$$

which equals $1 + \lambda$ at the optimum. The centralized optimum is therefore consistent with the stated objective of Canada's equalization program to "ensure that provinces have sufficient revenues to provide reasonably comparable levels of public services at reasonably comparable levels of taxation," in the language of the Constitution Act. In this case, however, it is the expected marginal cost of public funds that is equalized among regions.

5.2 Transfer policies in a federal state

When local taxation and spending decisions are decentralized to local governments, but tax rates can be stipulated by federal authorities, then the centralized optimum can be implemented through stochastic lump-sum transfers to each jurisdiction equal to $g_i^* - R_i(t_i^*, w_i)$. When political or constitutional constraints preclude centrally directed tax policies, however, this allocation is not implementable, as local governments have no incentive to choose tax rates appropriately. (Indeed, since public spending is independent of local taxation with such a mechanism, equilibrium tax rates are zero for all jurisdictions.) Naturally, the precise nature of constrained optimal transfer mechanisms and the equilibrium tax decisions which result depend on assumptions about what information is available to federal authorities and admissible for transfer formulas under fiscal federalism, and what penalty schemes federal authorities can credibly commit to *ex ante*. In this section, we assume that the only information admissible for calculating federal-local transfers is the actual realization of revenues in each jurisdiction. This might be regarded as an extreme case, in which federal authorities are highly constrained. However, this approach generates exceptionally simple results, which yield a number of insights for more general questions of mechanism design in a federation.

Given these constraints, an optimal transfer mechanism is a vector of functions

$$g(R) = (g^1(R_1, \dots, R_P), \dots, g^P(R_1, \dots, R_P))$$

which solve

$$\max_{g(\cdot)} \int V^0(g(R), t, R) f(R, t) dR \tag{36}$$

subject to the condition that tax rates represent a Nash equilibrium for jurisdictions, viz.

$$t_i \in \arg\max \int \Big[b^i(g_i(R)) - c^i(t_i) \Big] f(R, t) dR. \tag{37}$$

In some situations, it is more convenient to replace (37) with its first-order equivalent,

$$c_t^i(t_i) = \int b^i(g(R)) \frac{f_i(R, t)}{f(R, t)} f(R, t) dR. \tag{38}$$

It is a standard result that (38) is necessary for (37) when the likelihood ratios f_i/f are monotone in R_i. In this case, pointwise maximization of (36) subject to (38) implies $g^*(\cdot)$ is optimal if there exists a vector $\mu = (\mu_1, \ldots, \mu_P)$ such that

$$b_g^i(g^*(R)) = \frac{1+\lambda}{1+\mu_i f_i/f} \quad (\text{all } i), \tag{39}$$

and (38) defines the optimal choice of t_i. In this case, the optimal mechanism departs from full equalization of marginal benefits of public spending and hence from full insurance against local revenue risk. While other characterizations of second-best actions are available, the analysis that follows will focus on the structure of the optimal equalization formula $g^*(\cdot)$. In particular, we seek characterizations of the optimal degree of revenue interdependence in the formula—viz. the manner in which the formula for a given jurisdiction i depends on revenue realizations in other jurisdictions R_{-i}.

Inspection of the formulas given above indicates that, because the federal authority is averse to regional inequality, it behaves in a risk-averse manner, seeking to offset local shocks to tax revenue through transfers, to the extent possible. Consequently, local treasuries are exposed to risk only in order to provide appropriate incentives for taxation, and calculated transfers should depend on realizations of local revenue only to the extent that realizations provide informative signals about the tax efforts of the jurisdictions. This notion of the role of risk in incentive problems has been formalized by Holmstrom (1979, 1982) as follows.

Given the parameterized density for revenues $f(R,t)$, a function $S^i(R)$ is said to be a *sufficient statistic* for t_i if there exist functions h^i, K^i such that

$$f(R,t) = h^i(R, t_{-i}) K^i(S^i(R), t). \tag{40}$$

Equivalently, S^i is sufficient for t_i if the distribution of R conditional on S^i is independent of t_i. That is, when S^i is observed, no other observable information provides an informative signal about t_i. It follows that optimal transfer mechanisms depend on R through a vector of sufficient statistics $S(R) = (S^1(R), \ldots, S^P(R))$, and only through $S(R)$. This is merely an application of the *Informativeness Principle* of Holmstrom (1979), which is stated formally in the following proposition.

Proposition 6 *Let $S(R)$ be a sufficient statistic for t. Then for any transfer mechanisms $g(R)$ there exists a vector of mechanisms $g^*(S) = (g^{1*}(S^1), \ldots, g^{P*}(S^P))$ that yields weakly greater national social welfare.*

Proof. Consider any $g(R)$. Let $\bar{f}(R|S,t)$ denote the distribution of R conditional on S and t. Define $g^{i*}(S^i)$ by

$$\begin{aligned} b^i(g^{i*}(S)) &= \int_{S^i(R)=S} b^i(g^i(R)) \bar{f}(R|S,t)\,dR \\ &= \int_{S^i(R)=S} b^i(g^i(R)) h^i(R, t_{-i})\,dR, \end{aligned} \tag{41}$$

where the second equality follows from sufficiency of S. By construction, if t_i^* solves (37) then

$$t_i^* \in \arg\max \int b^i(g^{i*}(S)) f(R,t)\,dR - c^i(t_i).$$

By concavity of b^i and (41), $Eb^i(g^{i*}(S)) \geq E(b^i(g^i(R))$. Hence

$$EV^0(g^*(S(R)), t^*, R) \geq EV^0(g(R), t^*, R).$$

$\square$

The proposition establishes that federal-local transfers should be based on relative tax performance of the jurisdictions to the extent that relative performance provides a useful signal about tax efforts. Revenue interdependence should be avoided because of its associated risks for local governments, except insofar as it improves incentives for local tax effort. This result perhaps contradicts conventional wisdom regarding the design of equalization programs. In the typical view, revenue interdependence is a natural feature of the system, since revenue shocks to a single jurisdiction should be shared among all members of the federation. In practice as well as in the model, however, equalization transfers are financed from federal revenue sources rather than from a pool of local government revenues. In consequence, full-insurance transfers to a jurisdiction depend not on the revenue realizations of other local governments, but on the marginal cost of federal funds, which is apt to be far less volatile than local revenues. (Indeed, the model adopts the perhaps extreme assumption that the marginal cost of federal funds is independent of local revenue shocks.)

With this result, a number of substantive characterizations of the structure of the optimal formula are possible. In particular, we address the ways in which transfers to a region should depend on revenues realized in other jurisdictions. It has been shown that, with the first-best centralized optimum, transfers are independent of conditions prevailing in other jurisdictions, since any such dependence would lower the degree of redistribution under the formula and expose subnational governments to unnecessary risks. In the second-best context, however, the experience of other jurisdictions may be informative about the tax effort of the receiving jurisdiction. Indeed, it can be shown that transfers should be independent if and only if regions are not macroeconomically integrated. More formally, a simple corollary to Proposition 6 establishes that transfers to jurisdiction i are independent of revenues R_{-i} if and only if w_i and w_{-i} are stochastically independent.

Proposition 7 *The optimal transfer mechanisms $g^{i*}(R)$ are independent of R_{-i} if and only if (w_i, w_j) are stochastically independent for all $j \neq i$.*

Proof. If $(w_1, \ldots, w_P)$ are independent, then

$$f(R, t) = \prod_{i=1}^{P} f^i(R_i, t_i).$$

Hence R_i is sufficient for t_i by (40).

To establish the converse, suppose $g^{i*}(R) = \phi^i(R_i)$ for all i. If the first-order approach is valid then

$$b^i(\phi^i(R_i)) = \frac{1+\lambda}{1+\mu_i f_i/f}$$

for all R, implying the likelihood ratios f_i/f are independent of R_{-i} for all i. Integration yields $\log f = \sum_i \psi^i(R_i, t_i)$, using weak separability of f, which implies independence. $\square$

An example

When representative citizens' preferences for private goods have a simple log-linear form, a more explicit characterization of the optimal transfer formula is available. We will appeal to the following example to illustrate the qualitative results which follow.

Let private-goods preferences have the quasi-linear form

$$u^i(x_i, l_i) = A_i \log x_i - l_i. \tag{42}$$

Solution of the citizen's utility maximization problem (32) yields $x^i(t_i, w_i) = A_i w_i/(1+t_i)$. Define $\tau_i = t_i/(1+t_i)$ as the *ad valorem* tax rate for jurisdiction i. Then local revenue is

$$R_i = \tau_i A_i w_i.$$

With this change of variables, indirect utility from private goods is

$$v^i(\tau_i, w_i) = \log w_i + \log(1-\tau_i).$$

Suppose that each jurisdiction's tax capacity w_i can be decomposed multiplicatively into a common, national macroeconomic shock η and a jurisdiction-specific component ϵ_i, so that

$$w_i = \eta\, \epsilon_i \tag{43}$$

where η and each ϵ_i is independently log-normally distributed with known mean and variance. In this case, Holmstrom (1982) has demonstrated that a sufficient statistic for t_i depends only on own-source revenue R_i and a weighted geometric average of all jurisdictions' revenues,

$$\tilde{R} = \sum_{i=1}^{P} \alpha_i \log R_i, \tag{44}$$

where α_i is the precision (inverse of variance) of $\log \epsilon_i$.

Proposition 8 (Holmstrom (1982), Theorem 8) *Suppose that preferences take the form* (42) *and tax capacities take the form* (43). *Then there exists functions $S^i(R_i, \tilde{R})$ such that S^i is a sufficient statistic for t_i.*

The proposition states that, in simple environments, optimal transfers need depend only a jurisdiction's own revenue performance and a simple, weighted-average measure of national aggregate performance. In the aggregate measure, weights assigned to inferences about the common shock from each jurisdiction's performance are inversely proportional to the idiosyncratic noise in the jurisdiction's tax capacity. This is intuitive, as when variation in a jurisdiction's tax base is large and independent of economic conditions facing other jurisdictions, information derived from observation of that jurisdiction's experience is relatively valueless.

If the proposition is interpreted as a positive prediction about the nature of equalization formulas rather than a normative prescription, it is broadly consistent with the experience of Canada's equalization program. Under the current formula, the program calculates transfers based on revenue performance relative to a five-province standard, which excludes revenues of the oil-rich province of Alberta. This formula was instituted in 1982, after decades of experimentation with formulas which excluded all or part of resource revenues from equalization, and which appears to have been motivated at least in part by large and unpredicted swings in equalization entitlements following the oil price rises of the 1970s. Policy analysts have often criticized the five-province standard and the partial-inclusion formulas as being *ad hoc* and inconsistent with the efficiency and equity rationales for equalization. But our analysis suggests that exclusion of tax bases that are subject to large, idiosyncratic variations is appropriate, inasmuch as such tax bases provide little information about the tax effort of receiving jurisdictions. Linking equalization transfers to such revenue sources therefore increases revenue risk for receiving jurisdictions, while doing little to improve tax-effort incentives.

6 Conclusion

In a federal system of government, subnational governments are ceded authority to collect taxes and determine levels of public spending. When this is so, attempts by federal authorities to promote regional equity through transfers may be hampered by the distortions in incentives for tax authorities at the subnational levels that such transfers create. Fiscal federalism therefore creates conflicts between efficiency and equity analogous to those existing in personal tax systems. An equalization policy based on the representative tax system attempts to address these incentive problems by calculating transfer entitlements indirectly, using differences in revenues calculated at deemed rather than actual tax rates. But this formula may induce higher levels of distortionary taxation in receiving jurisdictions than is second-best optimal. In general, an optimal revenue sharing policy must be designed to provide appropriate incentives for taxation to subnational governments. Optimal policies entail imperfect redistribution of revenues and imperfect insurance against shocks to revenues. When local economic shocks are correlated, the use of relative standards of subnational tax effort improve the incentives for tax effort and enhance the degree of local redistribution that is implementable. The degree to which relative standards should be employed depends on the degree of integration of local economies.

References

AHMAD, E. AND R. THOMAS (1996): "Types of transfers: A general formulation," in *Financing decentralized expenditures: An international comparison of grants*, ed. E. Ahmad, Aldershot UK: Edward Elgar.

BIRD, R. AND E. SLACK (1990): "Equalization: The Representative Tax System Revisited," *Canadian Tax Journal*, 38, 913–927.

BOADWAY, R. AND F. FLATTERS (1982): "Efficiency and Equalization Payments in a Federal System of Government: A Synthesis and Extension of Recent Results," *Canadian Journal of Economics*, 15, 613–633.

BOADWAY, R. W. (1982): "On the Method of Taxation and the Provision of Local Public Goods: Comment," *American Economic Review*, 72, 846–851.

BOADWAY, R. W. AND P. A. HOBSON (1993): *Intergovernmental Fiscal Relations in Canada*, Toronto: Canadian Tax Foundation.

COURCHENE, T. J. (1994): *Social Canada in the Millenium: Reform Imperatives and Restructuring Principles*, Toronto: C.D. Howe Institute.

COURCHENE, T. J. AND D. A. BEAVIS (1973): "Federal-Provincial Tax Equalization: An Evaluation," *Canadian Journal of Economics*, 6, 483–502.

DAHLBY, B. AND L. WILSON (1994): "Fiscal Capacity, Tax Effort, and Optimal Equalization Grants," *Canadian Journal of Economics*, 27, 657–672.

FLATTERS, F., J. HENDERSON, AND P. MIESZKOWSKI (1974): "Public Goods, Efficiency and Regional Fiscal Equalization," *Journal of Public Economics*, 3, 99–112.

HOLMSTROM, B. (1979): "Moral Hazard and Observability," *Bell Journal of Economics*, 10.

HOLMSTROM, B. (1982): "Moral Hazard in Teams," *Bell Journal of Economics*, 13, 324–340.

MILGROM, P. AND C. SHANNON (1994): "Monotone comparative statics," *Econometrica*, 62, 157–180.

MINTZ, J. AND H. TULLKENS (1986): "Commodity Tax Competition Between Member States of a Federation: Equilibrium and Efficiency," *Journal of Public Economics*, 29, 133–172.

SHAH, A. (1994): "The Reform of Intergovernmental Fiscal Relations in Developing and Emerging Market Economies," Discussion Paper 23, The World Bank Policy and Research Series.

STIGLITZ, J. E. (1983): "The Theory of Local Public Goods Twenty-Five Years After Tiebout: A Perspective," in *Local Provision of Public Services: The Tiebout Model After Twenty-Five Years*, Academic Press, chap. 2, pp. 17–53.

USHER, D. (1995): *The Uneasy Case for Equalization Payments*, Vancouver: The Fraser Institute.

WILDASIN, D. E. (1991): "Income Redistribution in a Common Labour Market," *American Economic Review*, 81, 757–774.