1. Introduction

Perhaps the most influential proposition that has arisen from post–war theories of economic growth is that the accumulation of human capital, in the form of educational expansion or otherwise, is a major determinant of overall GNP growth (see, for example, Schultz 1963, Lucas 1988). Davies and Whalley (1989) estimate the stock of human capital in the U.S. to be about three times as large as the stock of physical capital. Perceived deficiencies in the supply of skilled manpower to LDCs in the 1950s and 1960s led to rapid expansion of their formal education systems. Public education expenditures tripled in Asia and more than doubled in Africa and Latin America during the 1960s and 1970s. Since the early 1980s educational budgets in many LDCs have been absorbing between 20 and 35% of recurrent government expenditures (Todaro 1985).

Education is also a major factor in determining the distribution of gains from economic growth (World Development Report 1991). However, the impact of the education system on the way in which the distribution of income evolves over time is unclear. Public investments in education, health and nutrition can, in principle, reduce income inequality, while promoting economic development in other ways. However, a number of studies (see Bhagwati 1973, Jallade 1976, Harbison 1978) find that, contrary to this premise, the educational systems of many developing nations have actually increased income inequality. There are several reasons for this. Even when primary school education is subsidized, the private costs of sending children to school may be disproportionately high for the relatively poor. Children of primary school age are often needed to work on family farms. The opportunity cost of sending a child to school, in terms of home production, tends to be of less significance to higher–income families. The inegalitarian nature of many LDC educational systems is often compounded by substantial tuitions charged at the secondary level[1], and by highly subsidized university education (Nafziger 1990).

The impact that an individual's human capital has on economic activity and the income that he receives, depends crucially on the type of occupation he engages in. While the human capital of wage laborers is important, the quality of those who manage the production process may often be decisive in the overall performance of firms. To use Radner's (1992) terminology, "figuring out what to do" is often more skill–intensive than "doing it". Bruch and Hiemenz (1984), find that

[1] Annual tuition is sometimes equivalent to per capita national income.

managerial quality emerges as a major determinant of the economic performance of small scale industry in the ASEAN countries. The education and ability of individuals is a key determinant of their managerial capital and hence their occupational choice.

In many cases, underdeveloped markets for skilled labor force entrepreneurs to perform production, financial and technological management tasks themselves (Kilby 1971). A number of studies (e.g. Gupta 1987, Khanka 1990, Bates 1990) indicate a higher than average level of education amongst entrepreneurs, and a positive relationship between education and entrepreneurial performance. As Nafziger (1990) points out, education results in sounder business decisions, improves verbal skills, increases the rate of acquisition of new ideas and methods, and facilitates computation and recordkeeping.

The policy objectives of the government and the instruments used to achieve them affect the 'natural' relationship that would exist between growth and inequality in the absence of intervention. Meerman (1979), for example, finds that the income share of the poorest Malaysians increased by as much as 29% after budgetary effects were taken into account. Conversely, for the reasons discussed above, public expenditures on education often end up being concentrated among those from richer households, even if this is not the intention. Policies of progressive taxation or expenditures on education targetted towards low income groups are often argued to be a burden to growth. However, if there are decreasing returns to human capital in production, then such policies may be growth promoting.

Other economic institutions and government policies can distort the equilibrium in the labor market. Development policies, such as that of Singapore to phase out labor–intensive industries by deliberately raising wages (Kalirajan 1992), affect occupational choices and, hence, production. The proposition that higher growth may result from increased entrepreneurship has led to the adoption of enterprise–promoting policies in the form of discriminatory taxation, industrial protection, and small business subsidies (World Development Report 1990). Such policies have been criticized for increasing inequality, by effectively subsidizing those most likely to become entrepreneurs, the relatively rich, at the expense of the poor. Proponents of "trickle–down" economic policies argue, in response, that the gains from growth resulting from increased enterprise accrue to all agents in the economy, so that even the poorest would be made better off.

These issues are addressed here in the context of a growth model in which agents live for a single period and optimally invest a portion of their lifetime income in their childrens' well–being. The human capital that a child receives is a function both of this private parental investment and public investments by the government, financed by direct taxation. Each agent must decide whether to use his human capital endowment in managing a productive activity, or to work as a wage laborer at the prevailing equilibrium unskilled wage. Managerial talent varies across agents. Managerial capital embodies both raw managerial talent and human capital. If an agent becomes an owner/manager his managerial capital is combined with as much labor as desired at the equilibrium wage rate. Owner/management is attractive if an agent's endowment of human capital and managerial talent are sufficiently high.

Thus, in contrast to previous analyses of the growth–inequality relationship (e.g. Tamura 1991, Glomm and Ravikumar 1992, Bénabou 1993), occupations which play asymmetric roles in the production process are distinguished[2]. This occupational asymmetry gives rise to an endogenous distribution of income whose characteristics are an equilibrium outcome. The distribution of income and its relationship to the rate of growth depend intimately on the production functions of firms and the technology for human capital accumulation. The transitional dynamics and the associated endogenous evolution of the distributions of income and human capital are characterized. Individual lineage dynamics depend non–linearly on aggregate variables, so that the associated Markov process is non–stationary. Even so, the detrended distribution of income is shown to converge to an endogenous, non–degenerate, stationary distribution, which is independent of initial conditions. The law of motion describing the economy's long–run growth path converges to a simple first–order difference relationship. The growth rate is a function of structural and fiscal parameters and the Lorenz curve describing long–run inequality.

The role of private vs. public human capital investments is examined. There is a unique public–private investment ratio at which the growth rate is maximized and which is neutral with respect to inequality. The growth–maximizing public expenditure exceeds the level which maximizes the welfare of the current generation of tax payers. These effects take as given the human capital

[2] Banerjee and Newman (1993) and Lloyd–Ellis and Bernhardt (1993) also emphasize the role of occupational choice, but focus on the role of imperfect capital markets and do not consider sustained growth. Galor and Zeira (1993) show how occupational choice in the presence of non–convexities in human capital accumulation can effect the limiting distribution.

accumulation technology. The impact of differences in this technology across economies is captured by varying the relative responsiveness of human capital to public and private investments. It is shown that a shift from a more public to a more private education system results in greater long–run growth and greater inequality.

Growth is also influenced by redistributional policies. A mean–preserving reduction in the dispersion of human capital increases per capita income in that period. Although lump–sum redistribution of a given stock of human capital is implausible, the government can redistribute via its taxation or expenditure policies. However, the effects of doing so are not identical. If a perfect non–distortionary taxation system were available, growth and welfare maximization require a tax policy that leaves everyone with the same after–tax income. In the absence of such a tax system, it maybe be desirable to redistribute via targeting of expenditure. However, the incidence of public expenditure affects the aggregate level of human capital as well as its distribution. There is a unique public expenditure policy which maximizes aggregate human capital. Due to decreasing returns in production, the short–run growth–maximizing expenditure regime is somewhat more egalitarian than this. On a long–run balanced growth path, however, the relationship between growth and inequality resulting from alternative stationary expenditure regimes is positive. Moreover, maximizing the aggregate welfare of the current generation of taxpayers yields a non–degenerate optimal distribution of human capital. Thus, both the growth–inequality relationship and the optimal distribution are sensitive to the method of redistribution.

Finally, we consider the impact of fiscal distortions in the form of different taxes on managers relative to wage laborers. With no within–period externalities, taxing managers at a different rate to wage laborers reduces output. However, depending on parameters, increased fiscal incentives to managers can have surprising distributional effects. For example, a "trickle–down" effect may reduce after–tax income dispersion. The resulting reduction in the dispersion of human capital endowments is a positive externality on the next generation. Thus, a seemingly regressive, growth–promoting policy, in the form of relative tax incentives to owner/managers, may reduce both inequality and growth in the short run. In the long run, the impact of distortionary fiscal policy on the growth–inequality relationship is non–monotonic: a trade–off exists only over low levels of inequality.

The outline of the paper is as follows. Section 2 sets out the structure of the economy, and the optimal behavior of individual agents is derived in Section 3. Section 4 provides a detailed characterization of the general equilibrium. Section 5 details the equilibrium distributions of income and human capital. Section 6 analyzes the dynamics of the distributions of income and human capital, and the key macroeconomic variables. A closed–form expression for the long–run growth rate is derived as a function of both structural and fiscal parameters and the long–run Lorenz curve. Section 7 addresses the role of private and public investments and section 8 characterizes the effects of redistribution. Finally, in Section 9, the impact of fiscal incentives to enterprise is examined. All proofs and derivations are contained in the appendix.

2. The Environment

There are countably many time periods, $t = 0, 1, 2, ...$, and a continuum of family lineages with unit measure. Each agent is active for one period, then reproduces one agent so that the economy's population is constant over time. An agent is endowed with human capital determined by the preceding generation, which may be used to generate income during adult life. The agent allocates lifetime income toward consumption and to expand the opportunities of his offspring.

Agents are assumed to have identical preferences[3]:

$$u(c_t, h_t) = c_t^{\alpha} h_{t+1}^{\beta},$$

where $0 < \alpha < 1$, $0 < \beta < 1$, c_t is his consumption and h_{t+1} is the human capital of his offspring. Thus, agents value their offspring's level of human development, independently of the impact that it has on income. For example, parents value a child's health directly, and not the indirect consequences on income. This assumption significantly reduces the complexity of the equilibrium analysis.

An agent born at time $t + 1$ whose parent invested e_t and in whom the government invests a fraction ζ_t of its total expenditure E_t, receives human capital given by

$$h_{t+1} = \theta(\zeta_t E_t)^{\rho} e_t^{\mu},$$

[3] The results extend directly to any homothetic utility function.

where $0 < \rho < 1$ and $0 < \mu < 1$. Public and private investments are complements in the production function for human capital. Thus, the greater the private investment in a child's well being, the more human capital the child will accumulate per unit of public expenditure. One interpretation might treat private investment, e_t, as a nutritional input and public investment, E_t, as expenditure on a formal school system. The government spends all of its tax revenue on education. Thus, were aggregate income in the economy Y_t and the average tax rate τ, aggregate public investment would be $E_t = \tau Y_t$.

When they come of age, agents are distinguished by two characteristics: their level of human capital, h, and their managerial quality z. Managerial quality is drawn from a time–invariant uniform distribution with support $[0, Z]$. The distribution of managerial quality conditional on human capital, h, is assumed to be identical to the unconditional distribution. The distribution of human capital is determined endogenously as described below.

The production side of the economy parallels that described by Lucas (1978) and Jovanovic (1993)[4]. There are a continuum of production units each of which requires a single manager together with wage labor in order to produce. We refer to these as firms, but they could be thought of as projects, plants or departments of a firm. Each firm produces a single consumption good according to the common Cobb–Douglas production technology,

$$f(z, h, l_t) = zh^\gamma l_t^\delta,$$

where l_t is the number of wage laborers hired, h is the manager's human capital and z is his innate managerial quality. Thus, zh^γ may be regarded as the managerial capital of the firm. There is perfect competition amongst firms in the market for managers, so that they receive all the rents from production. An alternative formulation might interpret the manager as an entrepreneur setting up a firm and hiring labor himself. The resulting equilibrium would be identical. It is assumed that there are decreasing returns to scale in production, so that $\gamma + \delta < 1$.

There is no individual or aggregate uncertainty. At the beginning of each period agents observe their own human capital and managerial talent, and then choose between two alternative lifetime occupations. If his managerial capital is sufficient, then an agent becomes an owner/manager. If not, he works as a wage laborer, leaving his human capital endowment unused. The occupational

[4] I am grateful to Aloysius Siow for bringing this paper to my attention.

choice of an agent with human capital h and managerial quality z, determines his lifetime income given the equilibrium obtaining during that period, $y(h, z, w_t)$.

3. Optimal behavior

An agent with income y_t, facing a tax rate τ and a share of public investment in his child's human development, ζ, solves the following utility maximization problem:

$$\max_{c_t, e_t} \; c_t^{\alpha} h_{t+1}^{\beta}$$

$$\text{s.t.} \quad c_t + e_t = (1-\tau)y$$

$$h_{t+1} = \theta(\zeta E_t)^{\rho} e_t^{\mu}.$$

The optimal consumption and educational expenditure policies are, respectively

$$c_t^* = (1-\sigma)(1-\tau)y_t \qquad e_t^* = \sigma(1-\tau)y_t,$$

where $\sigma = \frac{\beta\mu}{\alpha+\beta\mu}$. Thus, each agent expends a constant share of his after–tax income on his offspring's well–being. The greater the responsiveness of human capital production with respect to private investment, μ, the greater the share of income allocated to human capital accumulation.

A firm that hires a manager with human capital h and managerial quality z, chooses the quantity of wage labor it hires to solve

$$\max_{l_t} \; zh^{\gamma} l_t^{\delta} - w_t l_t - \pi,$$

where w_t is the period t equilibrium wage rate and π is the payment made to the manager. The firm's labor demand at the wage rate w_t, is therefore

$$l(h, z, w_t) = \Big(\frac{\delta z h^{\gamma}}{w_t}\Big)^{\frac{1}{1-\delta}}.$$

With perfect competition in the market for managers, the firm earns zero profits and the manager receives all of the rent associated with his managerial capital:

$$\pi(h, z, w_t) = (1-\delta)\Big(\frac{\delta}{w_t}\Big)^{\frac{\delta}{1-\delta}} (zh^{\gamma})^{\frac{1}{1-\delta}}.$$

If the manager is interpreted as an entrepreneur, $\pi(h, z, w_t)$ would represent his profits. The firm's output is given by

$$q(h, z, w_t) = \left(\frac{\delta}{w_t}\right)^{\frac{\delta}{1-\delta}} (zh^\gamma)^{\frac{1}{1-\delta}}.$$

The human capital of wage–laborers, while valued ex-ante, has no impact on output[5].

Whether or not an agent with human capital h becomes an owner/manager depends upon whether his managerial quality realization is large enough to yield an income which exceeds the wage. The managerial quality of the agent who is just indifferent between management and wage labor is defined by

$$\pi(h, z^c, w_t) = w_t.$$

Solving for z^c yields the critical quality function

$$z^c(h, w_t) = \frac{w_t}{\Delta h^\gamma},$$

where $\Delta = \delta^\delta(1-\delta)^{1-\delta}$. Agents with $z < z^c(h, w_t)$ prefer laboring to management. Agents with $z \geq z^c(h, w_t)$ prefer to become owner/managers and earn economic rent over and above the equilibrium wage.

Assume, initially, that the tax rate, τ, and the share of public investment ζ are constant across all lineages. Then, lineage human capital evolves according to

$$h_{t+1} = \theta[\tau Y_t]^\rho[\sigma(1-\tau)y(h_t, z_t, w_t)]^\mu,$$

where z_t is the cost draw of the t–th member of the lineage. The support of the distribution of human capital endowments in period $t+1$ is given by $[\underline{h}_{t+1}, \bar{h}_{t+1}]$, where

$$\underline{h}_{t+1} = \theta[\tau Y_t]^\rho[\sigma(1-\tau)w_t]^\mu; \qquad \bar{h}_{t+1} = \theta[\tau Y_t]^\rho[\sigma(1-\tau)\bar{\pi}_t]^\mu,$$

and $\bar{\pi}_t$ is the upper support on the distribution of profits from the previous period.

At this stage it is helpful to 'normalize' both the distributions of income and human capital. Let ν represent the human capital of an agent as a fraction of the upper support on the distribution.

5 Allowing their human capital to enter, would not change the qualitative results of the paper so long as the fraction of agents becoming owner/managers continues to increase with human capital.

Thus, an agent's human capital can be expressed as $h = \nu \bar{h}_t$. Similarly, let $\eta(\nu, z, w_t)$ represent the income of an agent of type (ν, z) relative to the maximum income, $\bar{\pi}_t$. Thus, lineage relative human capital evolves according to

$$\nu_{t+1} = [\eta(\nu_t, z_t, w_t)]^{\mu}.$$

An agent's relative human capital endowment depends not only on the relative human capital and managerial quality of his parent, but also on the past distribution of relative human capital via its effect on the equilibrium wage, w_t. Therefore, it is not possible to analyze the behavior of the economy by studying a single lineage because the transition function varies from period to period as the equilibrium wage rate evolves. The distribution of relative human capital evolves through time according to a time–varying, endogenous probability transition function $P_t(\nu'|\nu)$ so that

$$G_{t+1}(\nu') = \int_{\nu} P_t(\nu'|\nu) G_t(d\nu)$$

where $G_0(\cdot)$ is given.

4. Equilibrium

The time t aggregate supply of wage labor, $L_t^S(w)$, is given by

$$L_t^S(w) = \int_{\nu} \int_0^{z_c(\nu \bar{h}_t, w)} \frac{1}{Z}\, dz G_t(d\nu) = \left(\frac{w}{Z\Delta}\right) \frac{\Psi_t}{\bar{h}_t^{\gamma}},$$

where

$$\frac{\Psi_t}{\bar{h}_t^{\gamma}} = \frac{1}{\bar{h}_t^{\gamma}} \int_{\nu} \frac{1}{\nu^{\gamma}} G_t(d\nu)$$

is an "occupational shift factor". A second–order stochastic increase in the distribution of relative human capital, causes the occupational shift factor, and hence the supply of labor, to fall. With decreasing returns to human capital in production, $\gamma < 1$, transferring human capital from rich to poor lineages increases the fraction of agents from poorer households becoming owner/managers, at a given wage, by more than it reduces the fraction of those from rich households doing so.

The aggregate demand for labor $L_t^D(w)$ is given by

$$L_t^D(w) = \left(\frac{\delta}{2-\delta}\right) \left(\frac{Z\Delta}{w}\right)^{\frac{1}{1-\delta}} \bar{h}_t^{\frac{\gamma}{1-\delta}} \Phi_t - \left(\frac{\delta}{2-\delta}\right) \frac{w\Psi_t}{Z\Delta \bar{h}_t^{\gamma}},$$

where

$$\bar{h}_t^{\frac{\gamma}{1-\delta}}\Phi_t = \bar{h}_t^{\frac{\gamma}{1-\delta}}\int_\nu \nu^{\frac{\gamma}{1-\delta}} G_t(d\nu)$$

is a "scale factor". A second–order stochastic increase in the distribution of human capital causes the scale factor, and hence the demand for wage labor, to rise. With decreasing returns to scale in production, $\gamma+\delta<1$, transferring human capital from rich to poor lineages increases the demand for labor from firms managed by agents from poorer backgrounds by more than it reduces that of firms managed by those from richer ones. The actual demand for labor is reduced by the fact that only a fraction of agents become owner/managers in equilibrium. Thus, the occupational factor has a negative impact on the aggregate demand for labor; the greater the supply of labor, the lower the supply of managers.

A **competitive equilibrium** for an economy with initial distribution of relative human capital $G_t(\cdot)$ and upper support on human capital $\bar{h}_t$ is a tuple $\{w_t, L_t\}$ such that:

1. Given the wage w_t, each agent selects his occupation to maximize utility.
2. Given the wage w_t, firms choose their labor demand to maximize profits.
3. Firms earn zero profits.
4. The market for wage labor clears : $L_t^S(w_t) = L_t^D(w_t) = L_t$.

The labor market clearing conditions yield the following equilibrium values :

The equilibrium wage :

$$w_t = \Delta Z\Big(\frac{\delta\Phi_t}{2\Psi_t}\Big)^{\frac{1-\delta}{2-\delta}}\bar{h}_t^{\gamma}.$$

The equilibrium labor force :

$$L_t = \Big(\frac{\delta}{2}\Big)^{\frac{1-\delta}{2-\delta}}\Phi_t^{\frac{1-\delta}{2-\delta}}\Psi_t^{\frac{1}{2-\delta}}.$$

The equilibrium fraction of owner/managers :

$$M_t = 1 - L_t.$$

Equilibrium aggregate output :

$$Y_t = \frac{\Delta Z}{\delta}\Big(\frac{\delta}{2}\Big)^{\frac{2(1-\delta)}{2-\delta}}\Phi_t^{\frac{2(1-\delta)}{2-\delta}}\Psi_t^{\frac{\delta}{2-\delta}}\bar{h}_t^{\gamma}.$$

Observe that, although the equilibrium wage and output depend on both the level and dispersion of human capital, the equilibrium wage–labor force, L_t, and fraction of managers, M_t, depend only on the dispersion via its impact on Φ_t and Ψ_t.

Figure 1 goes here

Labor market equilibrium is illustrated in Figure 1. The area OwBL represents the total wage bill in the economy; the rest is equal to managerial earnings. The area wAB represents the producer surplus in the labor market and the area BCDL represents the rent accruing to owner/managers. Thus,

Lemma 1 : *Aggregate current output, Y_t, is equal to the area below the upper envelope generated by the supply and demand curves for labor.*

It follows that any change in the distribution of human capital which causes the demand and supply schedules for labor to shift up will result in an increase in the wage, w_t, and aggregate output, Y_t. In particular,

Lemma 2 : *A second–order stochastic increase in the distribution of actual human capital increases aggregate income, Y_t.*

With decreasing returns to human capital in production, $\gamma + \delta < 1$, a transfer of human capital from the child of a rich agent to the child of a poor one, increases both the likelihood that the child of the poor agent becomes an owner/manager and his income if he becomes one, by more than it decreases that of the child of the rich agent. Thus, any second–order stochastic increase in the distribution of human capital causes both the supply of management and the demand for labor to rise.

5. Distributions and inequality

At time t, a proportion $1 - \frac{z_c(\nu \bar{h}_t, w_t)}{Z}$ of agents with relative human capital ν realize a level of managerial quality high enough to make management a desirable occupation. Let $\tilde{z}_t(\eta, \nu)$ be the value of z such that $\pi(\nu \underline{h}_t, z, w_t) = \eta \bar{\pi}_t$. Then

$$\tilde{z}_t(\eta, \nu) = \frac{w_t^\delta \bar{\pi}_t^{1-\delta} \eta^{1-\delta}}{\Delta \bar{h}_t^\gamma \nu^\gamma} = \frac{Z \eta^{1-\delta}}{\nu^\gamma}.$$

Thus, $\tilde{z}(\eta, \nu)$ is the fraction of agents with relative human capital ν, earning income less than a fraction η of the highest income in the economy. The remaining fraction $\frac{z_c(\nu \bar{h}_t, w_t)}{Z}$ of such agents become wage laborers and receive relative income $\underline{\eta}_t = \frac{w_t}{\bar{\pi}_t}$. Hence, the distribution of relative income conditional on ν at time t can be represented by the cumulative distribution function

$$F_t(\eta|\nu) = \begin{cases} 0 & \text{if } \eta < \underline{\eta}_t, \\ \frac{\eta^{1-\delta}}{\nu^\gamma} & \text{if } \underline{\eta}_t \leq \eta \leq \nu^{\frac{\gamma}{1-\delta}}, \\ 1 & \text{otherwise,} \end{cases} .$$

This distribution is a mixed distribution, consisting of a point mass of wage laborers at $\underline{\eta}_t$ and a continuous density of relative incomes $f_t(\cdot)$ on the interval $[\underline{\eta}_t, \nu^{\frac{\gamma}{1-\delta}}]$. The unconditional distribution of income at time t is

$$F_t(\eta) = \int_\nu F_t(\eta|\nu) G_t(d\nu) = \int_{\eta^{\frac{1-\delta}{\gamma}}}^1 \frac{\eta^{1-\delta}}{\nu^\gamma} G_t(d\nu) + G_t(\eta^{\frac{1-\delta}{\gamma}}).$$

Using the change of variable $\eta = \hat{\nu}^{\frac{1}{\mu}}$ we can derive directly the conditional distribution of relative human capital

$$P_t(\hat{\nu}|\nu) = \begin{cases} 0 & \text{if } \hat{\nu} < \underline{\nu}_{t+1}, \\ \frac{\hat{\nu}^{\left(\frac{1-\delta}{\mu}\right)}}{\nu^\gamma} & \text{if } \underline{\nu}_{t+1} \leq \hat{\nu} \leq \nu^{\frac{\mu\gamma}{1-\delta}}, \\ 1 & \text{otherwise,} \end{cases}$$

where $\underline{\nu}_{t+1} = \underline{\eta}_t^\mu$. But $P_t(\hat{\nu}|\nu)$ is just the probability transition function for relative human capital. Hence,

Lemma 3 : *The distribution of relative human capital evolves according to*

$$G_{t+1}(\hat{\nu}) = \int_{\hat{\nu}^{\left(\frac{1-\delta}{\mu\gamma}\right)}}^1 \frac{\hat{\nu}^{\left(\frac{1-\delta}{\mu}\right)}}{\nu^\gamma} G_t(d\nu) + G_t(\hat{\nu}^{\frac{1-\delta}{\mu\gamma}}).$$

Observe that the evolution of $F_t(\cdot)$ and $G_t(\cdot)$ depends on the associated conditional distributions $F_t(\cdot|\nu)$ and $P_t(\cdot|\nu)$, respectively. The conditional distributions, in turn, depend only on the evolution of the lower support of the distribution of relative income,

$$\underline{\eta}_t = \frac{w_t}{\bar{\pi}_t} = \left(\frac{\delta \Phi_t}{2\Psi_t}\right)^{\frac{1}{2-\delta}}.$$

But Φ_t and Ψ_t depend only on the distribution of relative human capital and so,

Lemma 4 : *The distributions of relative income $F_t(\cdot)$ and relative human capital $G_t(\cdot)$ evolve independently of the levels of aggregate variables.*

In particular, the distributions evolve independently of the magnitudes of the wage w_t, aggregate income Y_t and the upper support on the distribution of human capital levels, $\bar{h}_t$. An increase in the dispersion of endowed human capital causes the occupational factor, Ψ_t, to decrease and the scale factor Φ_t to increase. It follows that the lower support on relative incomes, $\underline{\eta}_t$, rises and the dispersion in the distribution of human capital, endowed to the succeeding generation, declines.

The inequality associated with income distributions can be compared in terms of their Lorenz curves. These show the share of total income received by the poorest fraction p of the population as p varies from 0 to 1:

$$\Lambda_t(p) = \frac{1}{Y_t}\int_0^{\hat{\eta}_t(p)} \eta \bar{\pi}_t F_t(d\eta) = \frac{1}{\eta_t^m}\int_0^{\hat{\eta}_t(p)} \eta F_t(d\eta),$$

where $\hat{\eta}_t(p)$ is defined implicitly by $F_t(\hat{\eta}_t) = p$ and $\eta_t^m = \frac{Y_t}{\bar{\pi}_t}$ is the mean relative income.

Distribution $F_1(\cdot)$ **Lorenz dominates** distribution $F_2(\cdot)$ if

$$\Lambda_1(p) \geq \Lambda_2(p) \quad \forall\ p.$$

That is, the entire Lorenz curve of distribution $F_1(\cdot)$ lies above that of $F_2(\cdot)$. Lorenz dominance is a sufficient condition for a reduction in inequality as measured by most commonly used indices of inequality. The slope of the Lorenz curve at a given point is equal to the ratio of the relative income of the corresponding quantile of the distribution to the mean income of the distribution[6]. Figure 2 illustrates a typical Lorenz curve for this economy. The linear segment OA corresponds to that part of the population engaged in wage laboring and has slope

$$\underline{s}_t = \frac{w_t}{Y_t} = \frac{\underline{\eta}_t}{\eta_t^m}.$$

[6] The coordinates of the Lorenz curve are given parametrically by $p = F(\hat{\eta})$ and $\Lambda(p) = \frac{1}{\eta^m}\int_0^{\hat{\eta}_t} \eta\, F(d\eta)$ Differentiating each with respect to $\hat{\eta}$ yields $\frac{d\Lambda}{dp} = \frac{\hat{\eta}}{\eta^m}$.

Note that given the Cobb–Douglas technology, the vertical component of this segment is always equal to δ. The convex segment, AB, corresponds to those agents who become owner/managers. The slope of the Lorenz curve at B is given by

$$\bar{s}_t = \frac{\bar{\pi}_t}{Y_t} = \frac{1}{\eta_t^m}.$$

Since the the Lorenz function depends only on the distribution of relative income, it follows from Lemma 4 that

Corollary : *The Lorenz curve describing income inequality evolves independently of the level of aggregate income.*

Figure 2 goes here

Lorenz dominance is a rather demanding criterion, since it does not allow inequality comparisons when Lorenz curves intersect. A less demanding measure of inequality is the Gini coefficient, equal to twice the area between the Lorenz curve and the 45^o line:

$$I_t = 1 - 2\int_0^1 \Lambda_t(p)dp.$$

Lorenz dominance is a sufficient, but not necessary condition for declining inequality, as measured by the Gini coefficient, I_t. Even where Lorenz dominance is unambiguous, the Gini coefficient is a convenient summary measure of the extent of inequality.

6. Dynamics of inequality and growth

We focus on the transitional dynamics for an economy in which all lineages start out at $t = 0$ with the same human capital, h^0. A fraction $1 - z_c(h^0, w_0)$ of agents become owner/managers and earn variable profits according to their productivity draw. Wage laborers invest a fraction of the after–tax wage, in the human capital of their offspring. However, because period 0 managerial incomes differ, the period 1 distribution of human capital is non–degenerate. In particular, the distribution of relative human capital in period 0, $G_0(\cdot)$, dominates that in period 1, $G_1(\cdot)$ in the first–order stochastic sense. More generally,

Proposition 1 : *For all t, the distributions of relative human capital, $G_t(\cdot)$, and relative income, $F_t(\cdot)$, dominate those at time $t+1$ in the first–order stochastic sense.*

To see this, consider any period $t \geq 1$ such that the distribution of relative human capital, $G_t(\cdot)$, is stochastically dominated by $G_{t-1}(\cdot)$. To begin with, fix the upper support on the distribution of human capital, $\bar{h}_t$. Then, this would be equivalent to a first–order stochastic decrease in the distribution of actual human capital. The supply of management and the demand for wage labor would fall, so that the wage falls. With $\bar{h}_t$ constant, the maximum income, $\bar{\pi}_t$, would rise so that the lower support on relative incomes, $\frac{w_t}{\bar{\pi}_t}$, falls. It follows that the distribution of relative income, $F_t(\cdot)$, decreases in the first–order stochastic sense.

Now suppose that aggregate human capital grows, so that the upper support, $\bar{h}_t$, increases over time. This offsets the impacts of dispersion on the supply and demand for labor, perhaps causing the wage to rise. However, the positive impact of the increase in $\bar{h}_t$ on the maximum income, $\bar{\pi}_t$, exceeds the negative impact of the wage increase, so that $\bar{\pi}_t$ still rises. In fact, the lower support on relative incomes, $\frac{w_t}{\bar{\pi}_t}$, is independent of $\bar{h}_t$ (see Lemma 2). It follows that the time t distribution of relative incomes, $F_{t-1}(\cdot)$, stochastically dominates that at t, $F_t(\cdot)$, and, hence, $G_t(\cdot)$ stochastically dominates $G_{t+1}(\cdot)$. By induction from period 1, Proposition 1 follows. The evolution of the distribution of relative human capital is illustrated in Figure 3.

Figure 3 goes here

Since the distribution of relative income stochastically decreases, mean relative income, η_t^m, falls. It follows that the slope of the Lorenz curve at $p = 1$, $\bar{s}_t = \frac{1}{\eta_t^m}$, rises. Although the wage falls relative to the maximum income, the slope of the Lorenz curve segment OA, $\underline{s}_t = \frac{w_t}{Y_t}$, may rise or fall. Were it to rise, the Lorenz curves in adjacent periods would intersect. Hence, a necessary condition for Lorenz dominance is that the wage–output ratio falls monotonically (see Figure 2). With the Cobb–Douglas technology, $w_t L_t = \delta Y_t$, so that the fraction of wage–laborers, L_t, must rise monotonically. This, in turn, depends on the wage–elasticity of demand for labor and, hence, on the labor share of total output, δ:

Proposition 2 : *There exists a $\underline{\delta} > 0$ such that if the labor share of output is sufficiently high, $\delta > \underline{\delta}$, then inequality increases monotonically through time, in the Lorenz dominance sense.*

Recall from Section 4 that the size of the equilibrium labor force, L_t, is independent of the upper support on human capital, $\bar{h}_t$. Hence, we need only consider whether or not L_t rises in response to stochastic decrease in the distribution of relative human capital. This depends on the relative elasticities of supply and demand. The elasticity of supply is constant at unity. The elasticity of demand for labor, evaluated in equilibrium, is

$$\frac{w_t}{L_t}\frac{dL_t^D}{w_t} = -\Big(\frac{1+\delta}{1-\delta}\Big).$$

If δ is sufficiently high, the elasticity of demand for labor is large and the equilibrium labor force rises.

In fact, a monotonically increasing slope $\underline{s}_t$ is also sufficient to ensure that the entire Lorenz curve at time t lies outside that at time $t-1$. This is because the convexity of the Lorenz curve increases. Since the slope at $p=1$, $\bar{s}_t$, rises, it follows that the Lorenz curves can intersect at most once. If $\underline{s}_t$ declines, however, they never intersect.

Since there are decreasing returns to private investment in human capital, $\mu < 1$, and decreasing returns to human capital in production, $\gamma + \delta < 1$, the marginal impact of a rich agent's human capital on his child's human capital endowment decreases over time relative to that of a poor agent. In particular, the lower support on the distribution of relative human capital, ν_t, decreases and converges to a constant value $\underline{\nu}^*$. It follows that

Proposition 3 : *The process governing the evolution of the distribution of relative human capital converges to a stationary Markov process with a stationary transition function given by*

$$P^*(\hat{\nu}|\nu) = \begin{cases} 0 & \textit{if } \hat{\nu} < \underline{\nu}^*, \\ \frac{\hat{\nu}^{\left(\frac{1-\delta}{\mu}\right)}}{\nu^{\gamma}} & \textit{if } \underline{\nu}^* < \hat{\nu} \leq \nu^{\frac{\mu\gamma}{1-\delta}}, \\ 1 & \textit{otherwise.} \end{cases}$$

The process is monotone since the distribution of relative human capital amongst the offspring of an agent with high human capital dominates that amongst the offspring of an agent with lower human capital in the first–order stochastic sense. Moreover, the process satisfies the Monotone Mixing Condition (Hopenhayn and Prescott 1992). That is, for any $\tilde{\nu} \in (\underline{\nu}^*, 1)$, after a sufficient number of generations, there is a positive probability that the descendant of an agent with ν in any neighborhood of 1 receives relative human capital below $\tilde{\nu}$, and a positive probability that the eventual descendant of an agent with ν in any neighborhood of $\underline{\nu}^*$ receives relative human capital above $\tilde{\nu}$. As a result, after a sufficient number of generations, the distribution of relative human capital amongst the descendants of the richest and poorest agents living at time t converge to identical distributions. By monotonicity, the distribution of relative human capital amongst the descendants of all agents living at time t must converge to the same distribution, and so:

Proposition 4 : *The distributions of relative human capital and relative income converge to unique time–invariant limiting distributions $G^*(\cdot)$ and $F^*(\cdot)$, respectively. These distributions are independent of initial conditions.*

Since, the distribution of relative human capital, $G_t(\cdot)$, converges, so also must the distribution of relative income, $F_t(\cdot)$. It follows that, the extent of relative inequality as measured by the Lorenz curve converges. In particular, the slopes of the Lorenz curve at $p = 0$ and $p = 1$ converge to steady state values $\underline{s}$ and $\bar{s}$, respectively, and the Gini coefficient converges to its long–run value, I_{LR}. The distribution of actual incomes is not stationary in the long run. If mean income rises (falls), then the distribution of actual incomes grows (declines) in the first–order stochastic sense.

Since the distribution of relative human capital converges to a stationary distribution, the scale and occupational shift factors converge to constant values Φ and Ψ, respectively. Since Y_t is continuous in Φ_t and Ψ_t, it is in fact possible to solve for a closed–form solution for the long–run dynamics of aggregate output:

Proposition 5 : *As the distribution of relative human capital converges to $G^*(\cdot)$, the law of motion for output converges to*

$$Y_{t+1} = \Gamma Y_t^{(\rho+\mu)\gamma},$$

where

$$\Gamma = \frac{\theta^{\gamma}\tau^{\rho\gamma}[\sigma(1-\tau)\bar{s}]^{\mu\gamma}}{\underline{s}^{\delta}\bar{s}^{1-\delta}}.$$

Since the size of the labor force depends only on the distribution of relative human capital, it converges to a constant long–run value, L. Since, $w_t L = \delta Y_t$, it follows that

Corollary : *The wage rate, w_t, grows at the same long–run rate, Γ, as output.*

Assuming that $\Gamma > 1$, the economy follows one of three distinct long–run growth paths. If $[\rho+\mu]\gamma > 1$, then increasing returns to scale in education are great enough to more than offset the decreasing returns to human capital in production and the economy follows an explosive growth path. If $[\rho+\mu]\gamma < 1$, then the effects of decreasing returns to human capital in production dominate the returns to scale in education and aggregate income converges to the steady state level

$$Y^{*} = \Gamma^{\frac{1}{1-[\rho+\mu]\gamma}}.$$

Finally, if $[\rho+\mu]\gamma = 1$, so that the increasing returns in education just compensate the decreasing returns in production, the economy follows a balanced growth path with a constant growth rate, $\Gamma - 1$. In each of these three cases, inequality is constant along the growth path. Indeed, if the only difference between each scenario stems from differences in the returns to public investments in education, ρ, then the extent of long–run inequality is the same in each case.

The long–run relationship between growth and inequality is captured by the relative incomes of the poorest and the richest agents, $\underline{s}$ and $\bar{s}$ respectively. It is tempting to analyze this relationship by considering the effects of exogenous changes in the distribution of income on the rate of growth. However, both the long–run distribution of income and the growth rate are jointly determined by the structural and fiscal parameters of the economy. Thus, we must consider how they change simultaneously when these parameters are varied.

7. Public vs. private investment

In this section we consider two alternative aspects of the roles of public and private human capital investments in determining the long–run growth–inequality relationship. Firstly, we consider how the relationship changes as the share of public relative to private investment changes, taking the technology for human capital accumulation as given. Secondly, we show how the relationship varies with the relative sensitivity of human capital accumulation to each source of investment.

A change in the average tax rate, τ, does not affect income inequality in the current period or in the long run. In addition, because it does not influence occupational choice, such a change does not affect current output, Y_t. However, since it represents the share of investment in human development coming from public relative to private sources, a change in the tax rate does affect the growth rate and aggregate welfare.

Proposition 6 : *There exists a unique growth–maximizing average tax rate given by*

$$\tau^* = \frac{\rho}{\rho + \mu}.$$

This tax rate is greater than that which maximizes the welfare of the current generation.

This result is similar to Barro's (1990) proposition that the growth rate first rises with the ratio of government expenditures to GNP, but declines with excessive government spending. In our context, government expenditure goes to the production of human capital, not output. The growth–maximizing tax policy does not account for the fact that increasing the tax rate reduces the consumption of the current generation. However, this does not imply that a government wishing to maximize the welfare of the current generation would set the tax rate to zero, because agents care about their offspring. In particular, if the welfare function is utilitarian, then aggregate social welfare is

$$W_t = A_t(1-\tau)^{\alpha+\mu\beta}\tau^{\rho\beta}\int \eta^{\alpha+\mu\beta}F_t(d\eta).$$

where $A_t = (1-\sigma)^{\alpha}\sigma^{\mu\beta}\theta^{\beta}\bar{\pi}_t^{\alpha+\mu\beta}Y_t^{\rho\beta}$. Maximizing W_t with respect to τ yields

$$\tau^{**} = \frac{\rho}{\frac{\alpha}{\beta} + \rho + \mu}.$$

The welfare–maximizing tax rate, τ^{**}, is less than the growth–maximizing rate, τ^*, by an amount which depends upon the importance of consumption relative to a child's human development in the agents' utility function, $\frac{\alpha}{\beta}$.

Now consider the effect of increasing the responsiveness of human capital with respect to private investment, μ, relative to that with respect to public investment, ρ, holding $\rho+\mu$ constant. This change in the human capital accumulation technology affects both growth and inequality.

Proposition 7 : *There exists a $\underline{\delta} > 0$ and a $\bar{\tau} < 1$ such that if*

(a) the labor share of output is sufficiently high, $\delta > \underline{\delta}$, and

(b) the average tax rate is sufficiently low, $\tau < \bar{\tau}$,

then economies with greater returns to private investment in human capital, μ, holding total returns to scale, $\rho + \mu$, constant, exhibit both greater long–run growth and greater inequality, measured in the Lorenz dominance sense.

An increase in the returns to private investment in human capital, μ, causes a first–order stochastic decrease in the long–run distribution of relative human capital. Provided the labor share of output, δ, is sufficiently large, it follows that long–run inequality increases in the Lorenz dominance sense. Moreover, as μ increases, agents devote a larger share, σ, of their after–tax income to the human development of their offspring and the impact of it on human capital increases. On the other hand, as ρ falls, the impact of government expenditure declines. Provided that the tax rate, τ, is not too high, however, the long–run growth rate rises. This result is similar to Glomm and Ravikumar's (1992) comparison of public and private education regimes, except that here there are a continuum of intermediate regimes represented by alternative accumulation technologies. As $\mu \to 0$ the education system tends to a purely public one; as $\mu \to 1$ it tends to a more private one.

8. Redistribution policy

The distribution of human capital in the economy at the beginning of each period is not, in general, efficient. If it were possible to redistribute the endowed stock of aggregate human capital, H_t, then output could be increased (see Lemma 2). However, direct lump–sum redistribution of a given stock of human capital is implausible. Although the government can redistribute via taxation and public expenditure policies, the impacts of doing so are not identical. This section examines the sensitivity of the growth–inequality relationship to the method of redistribution, in both the short– and long–run. Moreover, the aggregate welfare–maximizing taxation and public expenditure policies are characterized.

Suppose first that tax rates can be set according to income level, $\tau(\eta)$. Such redistributive taxes are non–distortionary because they do not affect the occupational choices of agents. If such a tax system were available, then growth and welfare maximization mandates that taxes be set so that all agents have the same after–tax income:

Proposition 8: *The aggregate welfare of the current generation, W_t, the aggregate human capital, H_{t+1}, and output, Y_{t+1}, of their offspring and long–run growth, Γ, are maximized by the non–distortionary tax regime*

$$\tau^*(\eta) = 1 - \frac{Y_t(1-\tau)}{\eta\bar{\pi}_t}.$$

Since there are decreasing returns to private investment in human capital, $\mu < 1$, transferring a unit of after–tax income from a rich agent to a poor one raises the human capital of the poor agent's child by more than it reduces that of the richer agent's child. This represents a second–order stochastic increase in the distribution of actual human capital and, hence, an increase in the output produced by the next generation.

Since $\alpha + \mu\beta < 1$, the aggregate welfare of the current generation of taxpayers is the expected value of a concave function of after–tax income:

$$W_t = A_t\tau^{\rho\beta}\int[(1-\tau(\eta))\eta]^{\alpha+\mu\beta}F_t(d\eta).$$

Thus, since current per capita income remains unchanged, aggregate welfare rises. Moreover, the resulting second–order stochastic increase in the distribution of human capital reduces inequality and raises the aggregate welfare of the succeeding generation. Finally, if this progressive policy were fixed permanently, long–run after–tax inequality, I_{LR}, would be zero and long–run growth, Γ, would be maximized.

One suspects that such a strongly progressive tax policy is infeasible. In many LDCs official rates of taxation are often quite progressive, but effective tax rates are not. One problem lies in the collection of taxes when the full incomes of agents cannot be observed. A full analysis of the design of an incentive compatible collection mechanism is outside the scope of this paper. In addition, the incompetence or under payment of tax collectors can often result in undesirable outcomes (see Besley and McClaren 1993). Thus, while there is some scope for redistribution via the tax system, it is likely to be inefficient.

Whether it is because the effects are regarded as more certain or because it is more administratively–efficient, redistributive policy is often pursued via expenditure policies. Suppose public investments in education can be targetted according to parental income. Let the share of total public expenditure received by a child whose parent earned relative income η be denoted by $\zeta(\eta)$. Then the aggregate human capital endowed to the offspring of the current generation is

$$H_{t+1} = \theta(\tau Y_t)^{\rho}[\sigma(1-\tau)\bar{\pi}_t]^{\mu} \int \zeta(\eta)^{\rho}\eta^{\mu}F_t(d\eta).$$

Proposition 9: *Aggregate human capital is maximized by the public expenditure policy,*

$$\zeta^*(\eta) = \Big(\frac{\eta^{\frac{\mu}{1-\rho}}}{\int \hat{\eta}^{\frac{\mu}{1-\rho}} F_t(d\hat{\eta})}\Big).$$

The transfer of one unit of public expenditure from a poor lineage to a rich one has two opposing effects on aggregate human capital. First, since private and public expenditures are complementary inputs, the offspring of the rich lineage benefits more from a given amount of public expenditure. Second, with decreasing returns to public expenditure, $\rho < 1$, the marginal benefit decreases with each additional unit of expenditure. As a result, there exists an optimal expenditure policy such that for each income level these effects just offset each other.

We can now consider the impact of the expenditure regime on aggregate output, Y_{t+1}. We restrict the set of possible expenditure regimes to be of the form

$$\zeta(\eta, k) = \frac{\eta^k}{\int \hat{\eta}^k F(d\hat{\eta})}.$$

If $k = 0$, there is constant expenditure across all income levels. If $k < 0$, the expenditure policy is progressive, whereas if $k > 0$, the expenditure policy is regressive. Restricting expenditure regimes to take this form allows a direct comparison with the aggregate human capital maximizing policy. In that case, $k = \frac{\mu}{1-\rho}$. Moreover, such expenditure regimes result in income distributions which can be ordered by Lorenz dominance:

Lemma 5 : *There exists a $\underline{\delta} > 0$ such that if the labor share of output is sufficiently high, $\delta > \underline{\delta}$, then a more regressive expenditure regime, k, increases inequality in the Lorenz dominance sense.*

The expenditure policy which maximizes output is more egalitarian than that which maximizes aggregate human capital, but less egalitarian than a policy which allocates the same level of human capital to all agents in the economy. In fact, aggregate output, Y_{t+1}, is a single–peaked function of k.

Proposition 10 : *At each date t, there exists a public expenditure policy, $\zeta(\eta, k_t^*)$, where*

$$k_t^* \in \left[-\frac{\mu}{\rho}, \frac{\mu}{1-\rho}\right],$$

such that a more regressive policy raises short–run growth if $k < k_t^*$ and lowers it if $k > k_t^*$.

If $k > \frac{\mu}{1-\rho}$, then a reduction in the regressitivity of the expenditure regime, k, at time t causes a second–order stochastic increase in the distribution of human capital and, hence, increases aggregate output, Y_{t+1}. As k falls below $\frac{\mu}{1-\rho}$, aggregate human capital falls, offsetting the positive effects of reduced dispersion on aggregate output. Eventually, as k is decreased further, Y_{t+1} begins to fall. If $k = -\frac{\mu}{\rho}$, there is zero dispersion in the distribution of human capital, so that any further decrease in k represents a second–order decrease in the distribution of human capital and, hence, a reduction in Y_{t+1}. The short–run growth–maximizing regime, k_t^*, depends on the distribution of income in the previous period and, hence, varies over time.

Now suppose that the public expenditure regime, k, is held constant. Such a policy interferes with the convergence properties of the distribution of relative human capital. A sufficient condition for the distribution of relative human capital to converge to a stationary distribution, for an expenditure policy k, is

$$\frac{(\mu + \rho k)\gamma}{1-\delta} < 1.$$

If k is sufficiently large, the process governing evolution of $G_t(\cdot)$ may not become stationary and the law of motion for aggregate output may not converge. If k satisfies the above convergence condition, however, the law of motion for output under a given permanent expenditure regime, k, converges.

Proposition 11: *There exists a* $\underline{\delta} > 0$ *such that if*

(a) the labor share of output is sufficiently high, $\delta > \underline{\delta}$*, and*

(b) the distribution of relative human capital converges, $\frac{(\mu+\rho k)\gamma}{1-\delta} < 1$*,*

then economies with more regressive public expenditure regimes, k, exhibit both greater long–run growth and greater inequality, measured in the Lorenz dominance sense.

On a stationary long–run growth path,

$$\frac{Y_{t+1}}{Y_t} = \Big(\frac{H_{t+1}}{H_t}\Big)^{\gamma}.$$

Thus, a policy which increases the long–run rate of human capital accumulation also increases long–run growth. If the convergence condition holds and the economy is on a long–run balanced growth path, $(\rho + \mu)\gamma = 1$, then it must be the case that $k < \frac{\mu}{1-\rho}$. From Proposition 9, human capital growth increases with k for all $k < \frac{\mu}{1-\rho}$. Since, long–run inequality, I_{LR}, also increases with k, Proposition 11 follows. Thus, the implied relationship between long–run growth and inequality is in the opposite direction to that under alternative non–distortionary tax regimes. If $(\rho+\mu)\gamma < 1$, it is possible for convergence to hold when $k = \frac{\mu}{1-\rho}$. In this case, long–run output, Y^*, is maximized but of course, long–run growth is zero.

Finally, consider the optimal expenditure policy from the perspective of the current generation of tax payers. Their aggregate utilitarian welfare is

$$W_t = A_t(1-\tau)^{\alpha+\mu\beta}\tau^{\rho\beta}\int \zeta(\eta)^{\rho\beta}\eta^{\alpha+\mu\beta}F_t(d\eta).$$

Proposition 12: *The aggregate welfare of the time t generation of taxpayers is maximized by the public expenditure policy*

$$\hat{\zeta}_t(\eta) = \Bigg(\frac{\eta^{\left(\frac{\alpha+\beta\mu}{1-\beta\rho}\right)}}{\int \hat{\eta}^{\left(\frac{\alpha+\beta\mu}{1-\beta\rho}\right)}F_t(d\hat{\eta})}\Bigg).$$

This policy is less progressive than the welfare–maximizing tax policy.

The progessivity of the policy relative to that which maximizes short–run growth depends on the parameters of the individual utility function and on the human capital accumulation technology. If $\frac{\alpha}{1-\beta} < \frac{\mu}{1-\rho}$, then the welfare maximizing policy is more progressive. Moreover,

Corollary : *If* $\frac{\alpha}{1-\beta} < \frac{\mu}{1-\rho}$, *the welfare–maximizing expenditure regime yields an optimal, stationary long–run level of inequality.*

The optimal level of inequality is not zero, but is less than that imposed by an educational expenditure regime aimed at maximizing growth.

9. Fiscal distortions and "trickle down".

So far we have assumed that the fiscal system treats wage laborers and owner/managers symmetrically. In fact, the effective tax rates faced by individuals in different occupations may be very different due to investment credits and other subsidies or due to politically determined differences in statutory tax rates[7]. Let the effective tax rates on wages and managerial incomes be τ_w and τ_m, respectively. With Cobb–Douglas technology, the average tax rate is then given by

$$\tau = \delta\tau_w + (1-\delta)\tau_m.$$

The managerial quality of the marginal agent with relative human capital ν is now defined by

$$(1-\tau_m)\pi(\nu\underline{h}_t, z_c, w_t) = (1-\tau_w)w_t.$$

In the following analysis, the tax rates τ_w and τ_m are varied holding the average tax rate, τ, constant.

The government can encourage agents to engage in entrepreneurship or management activities by offering increased fiscal incentives in the form of reduced taxes relative to those on wage laborers. Whether this results in increased output depends on the distortion already imposed by the existing tax system.

[7] Although the focus here is on different tax rates, similar distortions could be created by factors other than the tax system. For example, unionization of the non–management work force, barriers–to–entry faced by managers or the risk of appropriation of business assets from entrepreneurs.

Proposition 13: *Current output, Y_t, is maximized when effective tax rates are equal across occupations, $\tau_m = \tau_w$.*

The relationship between current aggregate output, Y_t, and the relative tax rate on managers, $\frac{\tau_m}{\tau_w}$, is shown for a numerical example in Figure 4[8]. There are no within–period externalities and so, given the distribution of human capital, management is not under–supplied in equilibrium, unless $\tau_w \neq \tau_m$. Any difference in tax rates creates a distortion that reduces current output. Fiscal incentives, in the form of relative tax breaks to owner/managers increase output only if the existing tax system sets greater effective tax rates on management than on wage laborers. Otherwise, this positive effect on aggregate output is offset by the reduced output of each firm due to the increase in the equilibrium wage.

Figure 4 goes here

The fact that the equilibrium wage increases when the relative tax burden on managers is reduced, suggests the possibility of a "trickle down" effect over some ranges. This is measured here by considering the effect on the after–tax wage. Since utility is monotonically increasing in after–tax income, the after–tax wage corresponds to a Rawlsian measure of social welfare.

Proposition 14 : *The welfare of wage laborers is maximized by a unique, time–invariant pair of tax rates (τ_m^*, τ_w^*), such that*
(a) if the managerial share of output exceeds that of wage labor, $\delta < \frac{1}{2}$, then the optimal tax rate on owner/managers exceeds that on labor, $\tau_m^* > \tau_w^*$, and
(b) if the labor share of output exceeds that of management, $\delta > \frac{1}{2}$, then the optimal tax rate on labor exceeds that on owner/managers, $\tau_m^* < \tau_w^*$.

If the relative tax rate on management, $\frac{\tau_m}{\tau_w}$, is high, then reducing it causes the equilibrium wage rate to rise to the extent that the after–tax wage, $(1-\tau_w)w_t$, rises. However, if $\frac{\tau_m}{\tau_w}$ is sufficiently low, the rise in the wage, w_t, is small enough to be completely offset by the rise in the tax rate of wage laborers, τ_w. The optimal tax ratio varies systematically with the factor shares of output.

[8] Parameter values are Z=2, θ=10, α=β=τ=0.5, δ=0.3, γ=0.625, μ=ρ=0.8.

Because of the distortionary impact on aggregate output, it is not possible to completely order the income distributions resulting from different fiscal incentive regimes, according to Lorenz dominance. However, Figure 5 illustrates the general result that the relationship between the Gini coefficient, I_t, and the tax ratio, $\frac{\tau_m}{\tau_w}$, is U-shaped.

Figure 5 goes here

The fiscal incentive regime in period t has repercussions for the level and dispersion of human capital endowed to the next generation:

Proposition 15 : *The output produced by the next generation, Y_{t+1}, is maximized by a time t pair of tax rates $(\hat{\tau}_m^t, \hat{\tau}_w^t)$ such that*
(a) if the managerial share of output exceeds that of wage labor, $\delta < \frac{1}{2}$, then $1 < \frac{\hat{\tau}_m^t}{\hat{\tau}_w^t} < \frac{\tau_m^*}{\tau_w^*}$, and
(b) if the labor share of output exceeds that of management, $\delta > \frac{1}{2}$, then $1 > \frac{\hat{\tau}_m^t}{\hat{\tau}_w^t} > \frac{\tau_m^*}{\tau_w^*}$.

Suppose, as in part (a), the labor share of output is less than that of owner/managers, $\delta < \frac{1}{2}$. As long as $\tau_m < \tau_w$ an increase in the relative tax on owner/managers raises current output, Y_t, and the after–tax wage, $(1 - \tau_w)w_t$. This results in a second–order stochastic increase in the distribution of after–tax income and hence, in the distribution of human capital endowed to the next generation, $G_{t+1}(\cdot)$. Conversely, as long as $\frac{\tau_m}{\tau_w} > \frac{\tau_m^*}{\tau_w^*} > 1$ an increase in the relative tax rate on managers decreases current output and the after–tax wage. This leads to a second–order stochastic decrease in the human capital of the succeeding generation. It follows, from Lemma 2, that the relative tax rates at which Y_{t+1} is maximized must lie in the interval, $[1, \frac{\tau_m^*}{\tau_w^*}]$. An analagous argument establishes part (b) of the proposition.

Note, in particular, that even if a reduction in the relative tax on managers reduces current output, the resulting fall in the dispersion of human capital can have an offsetting positive externality on the output of future generations.

Finally, suppose that changes in relative tax rates are permanent. Figure 6 illustrates the resulting non–monotonic relationship between long–run growth and the Gini coefficient for a simulated example, when $\delta < \frac{1}{2}$. Suppose the relative tax on managers, $\frac{\tau_m}{\tau_w}$, is initially high. As it is decreased, after–tax inequality tends to fall and growth increases. However, once the tax ratio becomes sufficiently low, further reductions in inequality come only at the expense of a lower long–run growth rate[9]

Figures 6 goes here

10. Conclusion

This paper has explored the endogenous relationship between growth and inequality that arises when occupational asymmetries due to differences in education and ability are present. The non–linear dynamic evolution of the distributions of income and human capital were detailed and a closed–form expression for the relationship between long–run growth and inequality derived. This relationship is determined by the occupational choices of individuals, by the technology for human capital accumulation and by the intervention of the government. The impact of government policies depend on the relative importance of private and public sources of growth and on the instruments that the government has at its disposal. Attempting to reduce inequality through increasingly progressive expenditure policy lowers long–run growth, while doing so through progressive non–distortionary taxation policy raises it. However, if only distortionary tax instruments are available, then a more egalitarian fiscal regime, in the form of increased relative taxes to management, may increase or decrease growth depending on the extent of existing distortions.

The results of this paper imply, in particular, that one must be very careful when interpreting observed cross–sectional relationships between growth and inequality when they are jointly endogenously determined, especially in the presence of government intervention. The window provided by this and related frameworks can help to disentangle the determinants of this relationship.

[9] Under alternative parameterizations of the model where $\delta > \frac{1}{2}$ the relationship is also non–monotonic. However, reducing the tax ratio sufficiently can eventually cause the Gini coefficient to rise again.

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Technical Appendix

Derivation of the equilibrium wage rate:
The aggregate supply of labor is given by

$$L_t^S(w) = \int_\nu \int_0^{z_c(\nu,w)} \frac{1}{Z}\, dz G_t(d\nu) = \int_\nu \frac{z_c(\nu,w)}{Z} G_t(d\nu) = \big(\frac{w}{Z\Delta}\big)\frac{\Psi_t}{\bar{h}_t^\gamma}$$

where $\Delta = \delta^\delta(1-\delta)^{1-\delta}$ and $\Psi_t = \int_\nu \frac{1}{\nu^\gamma} G_t(d\nu)$. The aggregate demand for labor $L_t^D(w)$ is given by

$$L_t^D(w) = \int_\nu \int_{z_c(\nu,w)}^Z \frac{l(\nu\bar{h}_t, z, w)}{Z}\, dz G_t(d\nu) = \Big(\frac{\delta}{w}\Big)^{\frac{1}{1-\delta}} \bar{h}_t^{\frac{\gamma}{1-\delta}} \int_\nu \nu^{\frac{\gamma}{1-\delta}} \int_{z_c(\nu,w)}^Z \frac{z^{\frac{1}{1-\delta}}}{Z}\, dz G_t(d\nu).$$

Integrating over $z's$ gives,

$$L_t^D(w) = \Big(\frac{1-\delta}{2-\delta}\Big)\Big(\frac{\delta}{w}\Big)^{\frac{1}{1-\delta}} \bar{h}_t^{\frac{\gamma}{1-\delta}} \int_\nu \nu^{\frac{\gamma}{1-\delta}} \frac{[Z^{\frac{2-\delta}{1-\delta}} - z_c(\nu,w)^{\frac{2-\delta}{1-\delta}}]}{Z} G_t(d\nu)$$

Substituting the expression for $z_c(\nu,w)$ and re-arranging yields

$$L^D(w) = \Big(\frac{\delta}{2-\delta}\Big)\Big(\frac{Z\Delta}{w}\Big)^{\frac{1}{1-\delta}} \Phi_t \bar{h}_t^{\frac{\gamma}{1-\delta}} - \Big(\frac{\delta}{2-\delta}\Big)\Big(\frac{w}{Z\Delta}\Big)\frac{\Psi_t}{\bar{h}_t^\gamma}.$$

where $\Phi_t = \int_\nu \nu^{\frac{\gamma}{1-\delta}} G_t(d\nu)$. The equilibrium condition is given by $L_t^S(w_t) = L_t^D(w_t)$, so that

$$\big(\frac{w_t}{Z\Delta}\big)\frac{\Psi_t}{\bar{h}_t^\gamma} = \Big(\frac{\delta}{2-\delta}\Big)\Big(\frac{Z\Delta}{w}\Big)^{\frac{1}{1-\delta}} \Phi_t \bar{h}_t^{\frac{\gamma}{1-\delta}} - \Big(\frac{\delta}{2-\delta}\Big)\Big(\frac{w}{Z\Delta}\Big)\frac{\Psi_t}{\bar{h}_t^\gamma}.$$

Rearranging and solving for the time t equilibrium wage yields

$$w_t = Z\Delta\Big(\frac{\delta\Phi_t}{2\Psi_t}\Big)^{\frac{1-\delta}{2-\delta}} \bar{h}_t^\gamma.$$

Proof of Lemma 1: Aggregate income is the sum of total profits and total wages,

$$Y_t = \int_\nu \int_{z_c(\nu,w_t)}^Z \frac{\pi(\nu\bar{h}_t, z, w_t)}{Z} dz G_t(d\nu) + w_t L_t.$$

Since both profits and $z^c(\cdot)$ are continuous and differentiable with respect to the wage,

$$Y_t = \int_{w_t}^\infty \int_\nu \Big[-\int_{z^c(\nu,w)}^Z \frac{\pi_w(\nu\bar{h}_t, z, w)}{Z} dz - \frac{w z_w^c(\nu,w)}{Z}\Big] G_t(d\nu) dw.$$

Since $\pi_w(\nu\bar{h}_t, z, w) = -l(\nu\bar{h}_t, z, w)$ this can be written

$$Y_t = \int_{w_t}^\infty \Big[L_t^D(w) - \int_\nu \frac{w z_w^c(\nu,w)}{Z} G_t(d\nu)\Big] dw + w_t L_t.$$

But

$$\frac{dwz^c(\nu,w)}{dw} = wz^c_w(\nu,w) + z(\nu,w),$$

so that

$$\begin{aligned} Y_t &= \int_{w_t}^{\infty} \Big[L_t^D(w) - \frac{1}{Z}\int_{\nu}\Big(\frac{dwz^c(\nu,w)}{dw} - z^c(\nu,w)\Big)G_t(d\nu)\Big]dw + w_t L_t \\ &= \int_{w_t}^{\infty} [L_t^D(w) + E_t(w)]dw + w_t L_t + w_t E_t, \end{aligned}$$

which is the area under the upper envelope created by the supply and demand curves. ∎

Proof of Proposition 1: Since $G_{t-1}(\cdot)$ FSD (first–order stochastically dominates) $G_t(\cdot)$, it follows that $\Phi_t < \Phi_{t-1}$ and $\Psi_t > \Psi_{t-1}$. The lower bound on the distribution of relative income is given by

$$\frac{w_t}{\bar{\pi}_t} = \Big(\frac{\delta\Phi_t}{2\Psi_t}\Big)^{\frac{1}{2-\delta}}.$$

Thus, it must be the case that

$$\frac{w_t}{\bar{\pi}_t} < \frac{w_{t-1}}{\bar{\pi}_{t-1}}.$$

Trivially, therefore, $F_t(\eta) > F_{t-1}(\eta)\ \forall\ \eta < \frac{w_t}{\bar{\pi}_t}$. Now consider the change in the distribution of income for $\eta > \frac{w_t}{\bar{\pi}_t}$:

$$F_t(\eta) - F_{t-1}(\eta) = \int_{\eta^{\frac{1-\delta}{\gamma}}}^{1} \frac{\eta^{1-\delta}}{\nu^{\gamma}}[G_t(d\nu) - G_{t-1}(d\nu)] + G_t(\eta^{\frac{1-\delta}{\gamma}}) - G_{t-1}(\eta^{\frac{1-\delta}{\gamma}}).$$

Partial integration yields

$$F_t(\eta) - F_{t-1}(\eta) = \int_{\eta^{\frac{1-\delta}{\gamma}}}^{1} \Big(\frac{-\gamma\eta^{1-\delta}}{\nu^{\gamma+1}}\Big)[G_{t-1}(\nu) - G_t(\nu)]d\nu.$$

Since $G_{t-1}(\cdot)$ FSD $G_t(\cdot)$ this expression is negative. It follows that $F_{t-1}(\cdot)$ FSD $F_t(\cdot)$. An analagous argument establishes that if $G_{t-1}(\cdot)$ FSD $G_t(\cdot)$ then $G_t(\cdot)$ FSD $G_{t+1}(\cdot)$. It is trivially true that $G_0(\cdot)$ FSD $G_1(\cdot)$ and so, by induction, Proposition 1 must hold. ∎

Derivation of the equilibrium elasticity of demand for labor, $\theta^D(w_t)$:
Differentiating the aggregate demand for labor with respect to w and multiplying by w yields

$$w\frac{dL_t^D}{dw} = -\Big(\frac{1}{1-\delta}\Big)\Big(\frac{\delta}{2-\delta}\Big)\Big(\frac{Z\Delta}{w}\Big)^{\frac{1}{1-\delta}}\Phi_t\bar{h}_t^{\frac{\gamma}{1-\delta}} - \Big(\frac{\delta}{2-\delta}\Big)\frac{w\Psi_t}{Z\Delta\bar{h}_t^{\gamma}}.$$

But, in equilibrium,

$$\frac{w_t\Psi_t}{z\Delta\bar{h}_t^{\gamma}} = L_t \quad \text{and} \quad \Big(\frac{Z\Delta}{w_t}\Big)^{\frac{1}{1-\delta}}\Phi_t\bar{h}_t^{\frac{\gamma}{1-\delta}} = \Big(\frac{2\Psi_t}{\delta\Phi_t}\Big)^{\frac{1}{2-\delta}}\Phi_t = \Big(\frac{2}{\delta}\Big)^{\frac{1}{2-\delta}}\Phi_t^{\frac{1-\delta}{2-\delta}}\Psi_t^{\frac{1}{2-\delta}} = \frac{2L_t}{\delta}.$$

Substitution yields

$$w_t\frac{dL_t^D}{dw} = -\Big(\frac{1}{1-\delta}\Big)\Big(\frac{\delta}{2-\delta}\Big)\Big(\frac{2L_t}{\delta}\Big) - \Big(\frac{\delta}{2-\delta}\Big)L_t.$$

Dividing by L_t and re-arranging yields the elasticity of demand evaluated in equilibrium:

$$\theta^D(w_t) = -\Big(\frac{2+\delta-\delta^2}{(2-\delta)(1-\delta)}\Big) = -\Big(\frac{1+\delta}{1-\delta}\Big).$$

Proof of Proposition 2: An intuitive proof that $L_t > L_{t-1}$ for sufficiently large δ, was given in the text. Here, a somewhat more direct, though less intuitive, proof is given. The ratio of the fraction of agents in the labor force at time t to that at time $t-1$ is given by

$$\frac{L_t}{L_{t-1}} = \Big(\frac{\Phi_t}{\Phi_{t-1}}\Big)^{1-\frac{1}{2-\delta}}\Big(\frac{\Psi_t}{\Psi_{t-1}}\Big)^{\frac{1}{2-\delta}}.$$

If $G_{t-1}(\cdot)$ FSD $G_t(\cdot)$, then $\Phi_t < \Phi_{t-1}$ and $\Psi_t > \Psi_{t-1}$. If δ is sufficiently large, then the effect of the increase in the occupational factor is weighted by more than the decrease in the shift factor so that $L_t > L_{t-1}$. To see this, observe that as $\delta \to 1$,

$$\frac{L_t}{L_{t-1}} \to \frac{\Psi_t}{\Psi_{t-1}} > 1.$$

Thus, by continuity, there exists a δ sufficiently large that L_t rises and, hence, $\underline{s}_t$ falls.

It has thus been established that if $G_{t-1}(\cdot)$ FSD $G_t(\cdot)$, $\bar{s}_t > \bar{s}_{t-1}$ and, if δ is sufficiently large, $\underline{s}_t < \underline{s}_{t-1}$. These are necessary, but not sufficient, conditions for Lorenz dominance. If the Lorenz curve at time t is more convex than that at time $t-1$ on the interval $p \in [L_t, 1]$, then these two conditions are also sufficient for the Lorenz curve at time t to lie everywhere outside that at time $t-1$.

Let $\hat{\eta}_t(p)$ be defined by $F_t(\hat{\eta}_t) = p$. Then the change in the convexity of the Lorenz curve is given by

$$\begin{aligned}\Lambda_t''(p) - \Lambda_{t-1}''(p) &= \frac{\hat{\eta}_t'(p)}{\eta_t^m} - \frac{\hat{\eta}_{t-1}'(p)}{\eta_{t-1}^m} \\ &= \frac{1}{f_t(\hat{\eta}_t(p))\eta_t^m} - \frac{1}{f_{t-1}(\hat{\eta}_{t-1}(p))\eta_{t-1}^m}.\end{aligned}$$

Clearly, if $G_{t-1}(\cdot)$ FSD $G_t(\cdot)$, then $\eta_t^m < \eta_{t-1}^m$. The distribution of relative income for $p > L_t$ has slope given by the density function

$$f_t(\eta) = (1-\delta)\int_{\eta^{\frac{1-\delta}{\gamma}}}^1 \frac{\eta^{-\delta}}{\nu^\gamma} G_t(d\nu) \quad \forall\ \eta \in (\underline{\eta}_t, 1).$$

Observe that $f_t(1) = 0$ and $f_t'(\eta) < 0$. The change in density over time is given by

$$f_t(\eta) - f_{t-1}(\eta) = (1-\delta)\int_{\eta^{\frac{1-\delta}{\gamma}}}^1 \frac{\eta^{-\delta}}{\nu^\gamma}[g_t(\nu) - g_{t-1}(\nu)]d\nu.$$

Thus, $f_t(\eta) < f_{t-1}(\eta)$ iff $g_t(\nu) < g_{t-1}(\nu)$ for all $\nu \geq \underline{\nu}_{t-1}$. But, a similar argument implies that $g_t(\nu) < g_{t-1}(\nu)$ iff $g_{t-1}(\nu) < g_{t-2}(\nu)$ for all $\nu \geq \underline{\nu}_{t-2}$. Since, $g_1(\bar{\nu}) < g_0(\bar{\nu})$ it follows, by induction, that the density decreases for any relative income level, η, between time $t-1$ and t. Since, the slope is decreasing with η and is equal to zero at $\eta = 1$, it also follows that the density decreases for any quantile p. In other words it must be the case that $f_t(\hat{\eta}_t(p)) < f_{t-1}(\hat{\eta}_{t-1}(p))$ for all $p \in [L_t, 1]$. Hence, it follows that

$$\Lambda_t''(p) > \Lambda_{t-1}''(p) \quad \forall\ p \in [L_t, 1].$$

■

Proof of Proposition 3: We first show that the process governing the evolution of the distribution of relative human capital converges to a stationary Markov process. A necessary and sufficient condition for this to be true is that the lower bound on the distribution $\underline{\nu}_t$ converges to some value $\underline{\nu}^*$. Then the probability transition function is given by

$$P^*(\hat{\nu}|\nu) = \begin{cases} 0 & \text{if } \hat{\nu} < \underline{\nu}^*, \\ \frac{\hat{\nu}^{\left(\frac{1-\delta}{\mu}\right)}}{\nu^{\gamma}} & \text{if } \underline{\nu}^* < \hat{\nu} \leq \nu^{\frac{\mu\gamma}{1-\delta}}, \\ 1 & \text{otherwise.} \end{cases}$$

The lower bound on the distribution of relative human capital is given by

$$\underline{\nu}_{t+1} = \left(\frac{\delta\Phi_t}{2\Psi_t}\right)^{\frac{\mu}{2-\delta}}.$$

$G_t(\cdot)$ decreases through time in the first–order stochastic sense, so that $\underline{\nu}_t$ decreases. Whatever the distribution, it must be the case that $\Phi_t > \underline{\nu}_t^{\frac{\gamma}{1-\delta}}$ and $\Psi_t < \frac{1}{\underline{\nu}_t^{\gamma}}$. Thus, it follows that

$$\underline{\nu}_{t+1} > \left(\frac{\delta}{2}\right)^{\frac{\mu}{2-\delta}} \underline{\nu}_t^{\frac{\mu\gamma}{1-\delta}}.$$

If $\frac{\mu\gamma}{1-\delta} < 1$, the sequence $\underline{\nu}_t$ must therefore be bounded below by

$$\underline{\nu} = \left(\frac{\delta}{2}\right)^{\frac{\mu}{2-\delta}\left(\frac{1-\delta}{1-\delta-\mu\gamma}\right)}.$$

Since, it is decreasing and bounded below the sequence $\underline{\nu}_t$ must therefore converge. Hence it follows that there exists $\underline{\nu}^*$ such that for all $\epsilon > 0$, there exists a $T > 0$ such that

$$P_t(\cdot|\nu) - P^*(\cdot|\nu) < \epsilon \;\forall\; \nu,\; t > T.$$

Thus, the process converges to a stationary Markov process. Moreover, this process is monotone since the distribution of relative human capital amongst the offspring of an agents with high human capital dominates that amongst the offspring of agents with lower human capital in the first–order stochastic sense.

A sufficient condition for the distribution to converge is that $P^*(\cdot|\nu)$ satisfies the Monotone Mixing Condition (see Hopenhayn and Prescott, 1992). The MMC requires that there exists a relative human capital level $\hat{\nu} \in (\underline{\nu}^*, 1)$ such that for all $\epsilon > 0$ there exists an N such that

$$P^N([\underline{\nu}, \hat{\nu}]|1) > \epsilon \quad \text{and} \quad P^N([\hat{\nu}, 1]|\underline{\nu}) > \epsilon.$$

To verify the first condition, consider an agent with the highest possible level of relative human capital, $\nu = 1$. The probability that such an agent becomes a worker is

$$\frac{z_c(\bar{h}_t, w_t)}{Z} = \left(\frac{\delta\Phi_t}{2\Psi_t}\right)^{\frac{1-\delta}{2-\delta}} > \left(\frac{\delta}{2}\right)^{\frac{1-\delta}{2-\delta}\left(\frac{1-\delta}{1-\delta-\mu\gamma}\right)} > 0.$$

Since there is always a positive probability that the offspring of the richest entrepreneur becomes a worker, it follows immediately that the first condition holds. To verify that the second condition holds, consider the offspring of a worker, offspring who inherit the lowest possible relative human capital,

$$\underline{\nu}_{t+1} = \left(\frac{\delta\Phi_t}{2\Psi_t}\right)^{\frac{\mu}{2-\delta}}.$$

The probability that such an agent becomes an entrepreneur is

$$\frac{z_c(\underline{\nu}_t, w_t)}{Z} = \frac{w_t}{\Delta \underline{\nu}_t^\gamma \bar{h}_t^\gamma} = \frac{1}{\underline{\nu}_t^\gamma}\left(\frac{\delta \Phi_t}{2\Psi_t}\right)^{\frac{1-\delta}{2-\delta}} = \left(\frac{\delta}{2}\right)^{\frac{1-\delta-\mu\gamma}{2-\delta}} < 1.$$

Thus, there is always a positive probability of an agent becoming an entrepreneur, even at the lowest possible level of human capital. For any $\varpi > 0$, there is positive probability the child of a worker receives a quality draw $z \in [Z - \varpi, Z]$. Similarly, with positive probability his grandchild receives a quality draw $z \in [Z - \varpi, Z]$, and so on. For a sufficiently small ϖ and after a sufficiently large number of generation, say N, there is positive probability that the N^{th} descendant of the worker receives a relative human capital level in any neighbourhood of 1. ■

Proof of Proposition 4: As $G_t(\cdot)$ converges to $G^*(\cdot)$, both Φ_t and Ψ_t converge to their long–run values Φ and Ψ, respectively. Since it is continuous in Φ_t and Ψ_t, L_t also converges to its long–run value:

$$L = \left(\frac{\delta}{2}\right)^{\frac{1-\delta}{2-\delta}} \Phi^{\frac{1-\delta}{2-\delta}} \Psi^{\frac{\delta}{2-\delta}}.$$

Recall that the upper support on the distribution of human capital is given by

$$\bar{h}_{t+1} = \theta \tau^\rho [\sigma(1-\tau)]^\mu Y_t^\rho \bar{\pi}_t^\mu.$$

But $Y_t = w_t \frac{L}{\delta}$ and $\bar{\pi}_t = \left(\frac{2\Psi}{\delta\Phi}\right)^{\frac{1}{2-\delta}} w_t$. Substitution yields

$$\bar{h}_{t+1} = \theta \tau^\rho [\sigma(1-\tau)]^\mu \left(\frac{L}{\delta}\right)^\rho \left(\frac{2\Psi}{\delta\Phi}\right)^{\frac{\mu}{2-\delta}} w_t^{\rho+\mu}.$$

But

$$w_{t+1} = Z\Delta \left(\frac{\delta\Phi}{2\Psi}\right)^{\frac{1-\delta}{2-\delta}} \bar{h}_{t+1}^\gamma.$$

Substituting for $\bar{h}_{t+1}$ using this identity yields the equation of motion for the wage

$$w_{t+1} = Z\Delta\theta^\gamma \tau^{\rho\gamma} [\sigma(1-\tau)]^{\mu\gamma} \left(\frac{L}{\delta}\right)^{\rho\gamma} \left(\frac{\delta\Phi}{2\Psi}\right)^{\frac{1-\delta-\mu\gamma}{2-\delta}} w_t^{(\rho+\mu)\gamma}.$$

But $w_t = \frac{\delta}{L} Y_t$ and so substituting for the wage and re-arranging yields the equation of motion for output

$$Y_{t+1} = Z\Delta\theta^\gamma \tau^{\rho\gamma} [\sigma(1-\tau)]^{\mu\gamma} \left(\frac{L}{\delta}\right)^{1-\mu\gamma} \left(\frac{\delta\Phi}{2\Psi}\right)^{\frac{1-\delta-\mu\gamma}{2-\delta}} Y_t^{(\rho+\mu)\gamma}.$$

But re-arranging once more yields

$$Y_{t+1} = Z\Delta\theta^\gamma \tau^{\rho\gamma} [\sigma(1-\tau)]^{\mu\gamma} \left(\frac{Y_t}{w_t}\right)^{\delta} \left(\frac{Y_t}{\bar{\pi}_t}\right)^{1-\delta-\mu\gamma} Y_t^{(\rho+\mu)\gamma}.$$

Since $\frac{w_t}{Y_t} = \underline{s}$ and $\frac{\bar{\pi}_t}{Y_t} = \bar{s}$, the result follows. ■

Proof of Proposition 5 : The time $t+1$ distribution of relative human capital is given by

$$G_{t+1}(\hat{\nu}) = \int_{\hat{\nu}^{\left(\frac{1-\delta}{\mu\gamma}\right)}}^{1} \frac{\hat{\nu}^{\left(\frac{1-\delta}{\mu}\right)}}{\nu^\gamma} G_t(d\nu) + G_t(\hat{\nu}^{\left(\frac{1-\delta}{\mu\gamma}\right)}).$$

Differentiating w.r.t. μ yields for all $\hat{\nu} > \underline{\nu}_{t+1}$,

$$\frac{dG_{t+1}}{d\mu} = -\Big(\frac{1-\delta}{\mu^2}\Big)\int_{\hat{\nu}^{\left(\frac{1-\delta}{\mu\gamma}\right)}}^{1} \frac{\hat{\nu}^{\left(\frac{1-\delta}{\mu}\right)}\ln(\hat{\nu})}{\nu^{\gamma}} G_t(d\nu) > 0.$$

Now consider the lower bound on the distribution of relative human capital, $\underline{\nu}_{t+1}$. Differentiating w.r.t. μ yields

$$\frac{d\underline{\nu}}{d\mu} = \Big(\frac{1}{2-\delta}\Big)\Big(\frac{\delta\Phi_t}{2\Psi_t}\Big)^{\frac{\mu}{2-\delta}} \ln\Big(\frac{\delta\Phi_t}{2\Psi_t}\Big) < 0,$$

since $\Phi_t < 1$ and $\Psi_t > 1$. Thus, an increase in μ causes a first–order stochastic decrease in $G_{t+1}(\cdot)$ in every time period relative to what it would otherwise have been. By induction this implies that the long–run stationary distribution decreases in the first–order stochastic sense. As in Proposition 2, this is a necessary condition for increasing inequality in the Lorenz dominance sense. However, sufficiency requires that the long–run Lorenz curve becomes more convex.

The time t density function for human capital is

$$g_{t+1}(\hat{\nu}) = -\Big(\frac{1-\delta}{\mu}\Big)\hat{\nu}^{\left(\frac{1-\delta}{\mu}-1\right)}\int_{\hat{\nu}^{\frac{1-\delta}{\mu\gamma}}}^{1} \frac{1}{\nu^{\gamma}} g_t(\nu)d\nu.$$

Differentiating w.r.t. μ yields

$$\begin{aligned}\frac{dg_{t+1}(\hat{\nu})}{d\mu} = &- \Big(\frac{1-\delta}{\mu^2}\Big)\hat{\nu}^{\left(\frac{1-\delta}{\mu}-1\right)}\int_{\hat{\nu}^{\frac{1-\delta}{\mu\gamma}}}^{1} \frac{1}{\nu^{\gamma}} g_t(\nu)d\nu \\ &- \Big(\frac{1-\delta}{\mu}\Big)\Big(\frac{1-\delta}{\mu^2}\Big)\hat{\nu}^{\left(\frac{1-\delta}{\mu}-1\right)}\ln\hat{\nu}\int_{\hat{\nu}^{\frac{1-\delta}{\mu\gamma}}}^{1} \frac{1}{\nu^{\gamma}} g_t(\nu)d\nu \\ &+ \Big(\frac{1-\delta}{\mu}\Big)\Big(\frac{1-\delta}{\mu^2}\Big)\hat{\nu}^{\left(\frac{1-\delta}{\mu\gamma}-1\right)}\ln\hat{\nu}\, g_t(\hat{\nu}^{\left(\frac{1-\delta}{\mu\gamma}\right)}).\end{aligned}$$

Re–arranging yields

$$\begin{aligned}\frac{dg_{t+1}(\hat{\nu})}{d\mu} = &- \Big[1 + \Big(\frac{1-\delta}{\mu}\Big)\ln\hat{\nu}\Big]\Big(\frac{1-\delta}{\mu^2}\Big)\hat{\nu}^{\left(\frac{1-\delta}{\mu}-1\right)}\int_{\hat{\nu}^{\frac{1-\delta}{\mu\gamma}}}^{1} \frac{1}{\nu^{\gamma}} g_t(\nu)d\nu \\ &+ \Big(\frac{1-\delta}{\mu}\Big)\Big(\frac{1-\delta}{\mu^2}\Big)\hat{\nu}^{\left(\frac{1-\delta}{\mu\gamma}-1\right)}\ln\hat{\nu}\, g_t(\hat{\nu}^{\left(\frac{1-\delta}{\mu\gamma}\right)}).\end{aligned}$$

The second term is negative. The first term is negative if

$$1 + \Big(\frac{1-\delta}{\mu}\Big)\ln\hat{\nu} > 0.$$

The lower bound $\underline{\nu}$ on relative human capital was derived in Proposition 3 and hence a sufficient condition is

$$\frac{\mu}{2-\delta}\Big(\frac{1-\delta}{1-\delta-\mu\gamma}\Big)\ln\Big(\frac{\delta}{2}\Big) \geq -\frac{\mu}{2-\delta}.$$

Re–arranging and observing that $\left(\frac{1-\delta}{1-\delta-\mu\gamma}\right) > 1$, yields the sufficient condition

$$\ln\Big(\frac{\delta}{2}\Big) \geq -\frac{2-\delta}{1-\delta}.$$

It is easily verified that this inequality holds for all sufficiently large δ (equality occurs for δ approximately equal to 0.2). Thus, for sufficiently large δ, an increase in μ causes $g_{t+1}(\hat{\nu})$ to decrease relative to what it otherwise would have been given $G_t(\cdot)$. By induction, it follows that the long–run density of relative human capital and, hence, the that of relative income, decreases. By the same reasoning as that in Proposition 2, the long–run level of inequality increases with μ, in the Lorenz dominance sense.

The impact of an increase in μ relative to ρ on Y_{t+1} given the equilibrium obtaining in period t comes from the impact on $G_{t+1}(\cdot)$ and on $\bar{h}_{t+1}$. To begin with, fix $\bar{h}_{t+1}$. A first–order stochastic decrease in $G_{t+1}(\cdot)$ results in an increase in the demand for labor and a decrease in the supply. From Lemma 1 this implies that Y_{t+1} increases. Now consider the impact on $\bar{h}_{t+1}$. In logarithms we have

$$\ln \bar{h}_{t+1} = \ln \theta + \mu \ln\Big(\frac{\beta\mu}{\alpha+\beta\mu}\Big) + \Big(\frac{1}{\gamma} - \mu\Big)\ln(\tau Y_t) + \mu \ln((1-\tau)\bar{\pi}_t).$$

Differentiating yields

$$\frac{d\ln \bar{h}_{t+1}}{d\mu} = \frac{\alpha}{\alpha+\beta\mu} + \ln\Big[\frac{\sigma(1-\tau)\bar{\pi}_t}{\tau Y_t}\Big].$$

This expression is positive for τ sufficently small. One strongly sufficient condition is $\tau < \frac{\eta^m}{\eta^m+\sigma}$. By continuity, we can always find a number $\bar{\tau}$ such that if $\tau < \bar{\tau}$, the overall impact of the increase in μ on Y_{t+1} is positive relative to the value that would have otherwise obtained. By induction it then follows that the long–run growth rate increases with μ. ∎

Proof of Proposition 6 : The indirect utility function of an agent with relative income η is given by

$$v_t(\eta) = A_t(1-\tau)^{\alpha+\mu\beta}\tau^{\rho\beta}\eta^{\alpha+\mu\beta}$$

where $A_t = (1-\sigma)^\alpha\sigma^{\mu\beta}\theta^\beta Y_t^{\alpha+(\mu+\rho)\beta}$. Integrating over all relative income levels yields the aggregate social welfare

$$W_t = A_t(1-\tau)^{\alpha+\mu\beta}\tau^{\rho\beta}\int \eta^{\alpha+\mu\beta}F_t(d\eta).$$

Maximizing with respect to τ yields the result. ∎

Proof of Lemma 3 : Let $\hat{G}_t(\cdot)$ denote the distribution of human capital levels, h. Then the scale and occuptional shift factors can be expressed as

$$\Phi_t\bar{h}_t^{\frac{\gamma}{1-\delta}} = \int_h h^{\frac{\gamma}{1-\delta}}\hat{G}_t(dh) \quad \text{and} \quad \frac{\Psi_t}{\bar{h}_t^\gamma} = \int_h \frac{1}{h^\gamma}\hat{G}_t(dh).$$

Since, $h^{\frac{\gamma}{1-\delta}}$ is an increasing, concave function of h and $h^{-\gamma}$ is a decreasing, convex function of h, a second–order stochastic increase in the distribution of human capital causes the scale factor to increase and the occuptional shift factor to decrease. As a result, the demand schedule for labor shifts to the right and the supply schedule shifts to the left. From Lemma 1, this implies that aggregate output, Y_t, must increase. ∎.

Proof of Proposition 7: The Lagrangian for the social planning problem is

$$\ell = \theta[\tau Y_t]^\rho[\sigma\bar{\pi}_t]^\mu\int_\eta[1-\tau(\eta)]^\mu\eta^\mu f_t(\eta)d\eta + \lambda[Y_t(1-\tau) - \bar{\pi}_t\int_\eta[1-\tau(\eta)]\eta f_t(\eta)d\eta].$$

The first–order necessary conditions are

$$\theta[\tau Y_t]^\rho[\sigma\bar{\pi}_t]^\mu\mu[1-\tau^*(\eta)]^{\mu-1}\eta^\mu f_t(\eta) = \lambda\eta f_t(\eta) \quad \forall\ \eta,$$

$$\bar{\pi}_t \int_\eta [1-\tau^*(\eta)]\eta f_t(\eta)d\eta = Y_t(1-\tau).$$

Solving for τ^*, yields the optimal policy function,

$$\tau^*(\eta) = 1 - \frac{Y(1-\tau)}{\eta\bar{\pi}_t}.$$

Since this represents a second–order stochastic increase in the distribution of human capital to the point where all agents have the same level of human capital, aggregate output growth must also be maximized. Finally, solving for the welfare–maximizing, non–distortionary tax policy yields the same policy function. ∎

Proof of Proposition 8: The Lagrangian for the social planner's problem is

$$\ell = \theta[\tau Y_t]^\rho[\sigma(1-\tau)\bar{\pi}_t]^\mu \int_\eta \zeta(\eta)^\rho \eta^\mu f_t(\eta)d\eta + \lambda[1 - \int_\eta \zeta(\eta)f_t(\eta)d\eta],$$

where $\lambda \geq 0$ is the Lagrange multiplier. The first order necessary conditions for this problem are

$$\theta[\tau Y_t]^\rho[\sigma(1-\tau)\bar{\pi}_t]\rho\zeta(\eta)^{\rho-1}\eta^\mu f_t(\eta) = \lambda f_t(\eta) \quad \forall\ \eta$$

$$\int_\eta \zeta(\eta)f_t(\eta)d\eta = 1.$$

Solving for ζ yields the optimal policy function

$$\zeta^*(\eta) = \frac{\eta^{\frac{\mu}{1-\rho}}}{\int \eta^{\frac{\mu}{1-\rho}} f_t(\eta)d\eta}.$$

Proof of Lemma 4 : This is analagous to the proof in Proposition 5 that an increase in μ results in an increase in inequality in the Lorenz dominance sense for δ sufficiently large. Just replace μ with $\mu + \rho k$ everywhere in the argument. ∎

Proof of Proposition 9 : As argued in the text, if there exists a unique, aggregate output maximizing value of k_t^*, it must occur between the two bounds stated in the Proposition. Suppose we start at $k = -\frac{\mu}{\rho}$ and let k rise. The addition to output resulting from the rising aggregate level of human capital is offset by the reduction resulting from the increasing dispersion. A sufficient condition for Y_{t+1} to be a single–peaked function of k is that the additions are decreasing in magnitude, and the size of the reductions are increasing monotonically as k rises.

The maximum level of human capital at time $t+1$ under the redistribution policy is given by

$$\bar{h}_{t+1} = \theta[\tau Y_t]^\rho[\sigma(1-\tau)\bar{\pi}_t]^\mu \Big(\frac{1}{\int \eta^k F_t(d\eta)}\Big).$$

Differentiating w.r.t. k yields

$$\frac{d\bar{h}_{t+1}}{dk} = -\bar{h}_{t+1}\Big(\frac{\int \eta^k \ln(\eta)F_t(d\eta)}{\int \eta^k F_t(d\eta)}\Big) > 0.$$

Differentiating a second time yields

$$\frac{d^2\bar{h}_{t+1}}{dk^2} = \bar{h}_{t+1}\Big(\frac{[\int \eta^k F_t(d\eta)]\int[\eta^k \ln(\eta)]^2 F_t(d\eta) - 2\big[\int \eta^k \ln(\eta)F_t(d\eta)\big]^2}{\big[\int \eta^k F_t(d\eta)\big]^2}\Big) < 0.$$

where the sign follows from Jensen's inequality and the fact that

$$\int \eta^k F_t(d\eta) < 1.$$

Thus, as k rises the maximum level of human capital increases at a decreasing rate.

The scale factor at time $t+1$ can be expressed as a function of the distribution of income at time t,

$$\Phi_{t+1} = \int \eta^{\frac{(\mu+\rho k)\gamma}{1-\delta}} F_t(d\eta).$$

Differentiating w.r.t. k yields

$$\frac{d\Phi_{t+1}}{dk} = \int \Big(\frac{\rho\gamma\eta^{\frac{(\mu+\rho k)\gamma}{1-\delta}} \ln(\eta)}{1-\delta} \Big) F_t(d\eta) < 0.$$

Differentiating a second time yields

$$\frac{d^2\Phi_{t+1}}{dk^2} = \int \Big(\frac{\rho\gamma\eta^{\frac{(\mu+\rho k)\gamma}{1-\delta}} \ln(\eta)}{1-\delta} \Big)^2 F_t(d\eta) > 0.$$

Thus, as k rises, the scale factor decreases at an increasing rate.

Similarly, the occupational shift factor can be expressed as

$$\Psi_{t+1} = \int \Big(\frac{1}{\eta}\Big)^{(\mu+\rho k)\gamma} F_t(\eta).$$

Differentiating w.r.t. k yields

$$\frac{d\Psi_{t+1}}{dk} = \int \rho\gamma \Big(\frac{1}{\eta}\Big)^{(\mu+\rho k)\gamma} \ln\Big(\frac{1}{\eta}\Big) F_t(d\eta) > 0.$$

Differentiating a second time yields

$$\frac{d^2\Phi_{t+1}}{dk^2} = \int \Big[\rho\gamma \Big(\frac{1}{\eta}\Big)^{(\mu+\rho k)\gamma} \ln\Big(\frac{1}{\eta}\Big)\Big]^2 F_t(d\eta) > 0.$$

Thus, as k rises, the occupational shift factor increases at a increasing rate.

For a given $\bar{h}_{t+1}$, increasing k, therefore, causes the labor demand curve to shift to the left and the labor supply curve to shift to the right at an increasing rate. Thus, for a given $\bar{h}_{t+1}$, increasing k causes Y_{t+1} to decrease at a increasing rate. Conversely, for given Φ_{t+1} and Ψ_{t+1}, increasing k causes Y_{t+1} to increase at a decreasing rate. Taken together, these effects imply that Y_{t+1} is a single-peaked function of k, with a maximum in the interval $k \in [-\frac{\mu}{\rho}, \frac{\mu}{1-\rho}]$. ∎

Proof of Proposition 10: First observe that under the permanent expenditure regime k, the lower bound on the distribution of relative human capital is given by

$$\underline{\nu}_{t+1} = \Big(\frac{w_t}{\bar{\pi}_t}\Big)^{\mu+\rho k} = \Big(\frac{\delta\Phi_t}{2\Psi_t}\Big)^{\frac{\mu+\rho k}{2-\delta}}.$$

As in Proposition 4, if $\frac{(\mu+\rho k)\gamma}{1-\delta} < 1$, the sequence $\underline{v}_t$ converges and, hence, the distribution of relative human capital converges to a stationary distribution. As in Proposition 5, the law of motion for aggregate output also converges.

On the stationary growth path, aggregate human capital at time t is given by

$$H_t = \int_\nu \nu G(d\nu)\bar{h}_t.$$

Aggregate output is given by

$$Y_t = \frac{\Delta Z}{\delta}\Big(\frac{\delta}{2}\Big)^{\frac{2(1-\delta)}{2-\delta}} \Phi^{\frac{2(1-\delta)}{2-\delta}} \Psi^{\frac{\delta}{2-\delta}} \bar{h}_t^{\gamma}.$$

This implies that

$$\frac{Y_{t+1}}{Y_t} = \Big(\frac{\bar{h}_{t+1}}{\bar{h}_t}\Big)^{\gamma} = \Big(\frac{H_{t+1}}{H_t}\Big)^{\gamma}.$$

Thus, if the long–run rate of aggregate human capital accumulation increases, so also must the long–run growth rate. In particular, if $k < \frac{\mu}{1-\rho}$, increasing k raises long-run growth.

However, it is easily verified that if $(\rho+\mu)\gamma = 1$ and if $\gamma + \delta < 1$, then the sufficient condition for convergence $\frac{(\mu+\rho k)\gamma}{1-\delta} < 1$ cannot be satisfied for any $k \geq \frac{\mu}{1-\rho}$. It follows that, on a stationary growth path, k must be less that $\frac{\mu}{1-\rho}$ and so, for δ sufficiently large, the relationship between long–run growth and inequality measured in the Lorenz dominance sense must be positive. ∎

Proof of Propostion 11: The Lagrangian for the social planner's problem is

$$\ell = \theta[\tau Y_t]^{\rho\beta}[\sigma(1-\tau)\bar{\pi}_t]^{\alpha+\mu\beta}\int \zeta(\eta)^{\rho\beta}\eta^{\alpha+\mu\beta}f_t(\eta)d\eta + \lambda[1 - \int \zeta(\eta)f_t(\eta)d\eta].$$

This problem is analogous to that in Proposition 9 and hence yields a similar solution. ∎

Proof of Proposition 12: With differential tax rates the aggregate supply of labor is now

$$L_t^S(w) = \frac{wT}{Z\Delta}\frac{\Psi_t}{\bar{h}_t^{\gamma}}$$

and the aggregate demand for labor $L_t^D(w)$ is

$$L_t^D(w) = \Big(\frac{1-\delta}{2-\delta}\Big)\Big(\frac{\delta}{w}\Big)^{\frac{1}{1-\delta}}\Big[Z^{\frac{1}{1-\delta}}\Phi_t\bar{h}_t^{\frac{\gamma}{1-\delta}} - \frac{1}{Z}\Big(\frac{wT}{\Delta}\Big)^{\frac{2-\delta}{1-\delta}}\frac{\Psi_t}{\bar{h}_t^{\gamma}}\Big].$$

The equilibrium condition is $L_t^S(w_t) = L_t^D(w_t)$, and solving for the equilibrium wage, as before, yields

$$w_t = Z\Delta\Big[\Big(\frac{\delta}{(2-\delta)T + \delta T^{\frac{2-\delta}{1-\delta}}}\Big)\frac{\Phi_t}{\Psi_t}\Big]^{\frac{1-\delta}{2-\delta}}\bar{h}_t^{\gamma}.$$

Clearly w_t is declining with T. Substituting for w_t in the labor supply function yields the equilibrium proportion of the population in the labor force,

$$L_t = T\Big(\frac{\delta}{(2-\delta)T + \delta T^{\frac{2-\delta}{1-\delta}}}\Big)^{\frac{1-\delta}{2-\delta}}\Phi_t^{\frac{1-\delta}{2-\delta}}\Psi_t^{\frac{1}{2-\delta}}.$$

Differentiating with respect to T yields

$$\frac{dL_t}{dT} = \frac{L_t}{T}\Big(\frac{(2-\delta)T + \delta T^{\frac{2-\delta}{1-\delta}} + T}{(2-\delta)T + \delta T^{\frac{2-\delta}{1-\delta}}}\Big) > 0.$$

The level of aggregate output and, hence, the pre-tax aggregate income is

$$Y_t = \frac{Z\Delta}{\delta} T \Big(\frac{\delta}{(2-\delta)T + \delta T^{\frac{2-\delta}{1-\delta}}} \Big)^{\frac{2(1-\delta)}{2-\delta}} \Phi_t^{\frac{2(1-\delta)}{2-\delta}} \Psi_t^{\frac{\delta}{2-\delta}} \bar{h}_t^{\gamma}.$$

Differentiating with respect to T yields

$$\frac{dY_t}{dT} = \frac{Y_t}{T}\Big(1 - \frac{2(1-\delta) + 2\delta T^{\frac{1}{1-\delta}}}{2-\delta+\delta T^{\frac{1}{1-\delta}}}\Big) = \frac{Y_t}{T}\delta\Big(\frac{1-T^{\frac{1}{1-\delta}}}{2-\delta+\delta T^{\frac{1}{1-\delta}}}\Big).$$

Thus, if $T < 1$, $\frac{dY_t}{dT} > 0$ and if $T < 1$, $\frac{dY_t}{dT} < 0$. Hence, current output Y_t is maximized when $T = 1$. ■

Proof of Propostion 13 : Note first that fiscal incentive regimes are neutral with respect to the relative profits of entrepreneurs. Thus, to show that distribution 1 Lorenz dominates distribution 2, it is sufficient to show that the slope of Lorenz curve 1 at $p = 0$ is greater and the slope at $p = 1$ is lower, than those of Lorenz curve 2.

Consider first the ratio of after-tax wages to the mean after-tax income. This is the slope of the segment of the after-tax Lorenz curve corresponding to those agents in the work-force,

$$\underline{s}_t' = \frac{(1-\tau_w)w_t}{(1-\tau)Y_t} = \Big(\frac{\delta}{\delta T + 1 - \delta}\Big)\Big(\frac{(2-\delta)T + \delta T^{\frac{2-\delta}{1-\delta}}}{\delta}\Big)^{\frac{1-\delta}{2-\delta}}\Big(\frac{1}{\Phi_t^{\frac{1-\delta}{2-\delta}}\Psi_t^{\frac{1}{2-\delta}}}\Big).$$

Differentiating w.r.t. T yields after some manipulation

$$\frac{d\underline{s}_t'}{dT} = \underline{s}_t'\Big(\frac{(1-\delta)^2 - (1-\delta)\delta T^{\frac{1}{1-\delta}} - \delta T}{[(2-\delta)T + \delta T^{\frac{2-\delta}{1-\delta}}][\delta T + 1 - \delta]}\Big).$$

This imples in particular that

$$\text{sign}\Big[\frac{d\underline{s}_t'}{dT}\Big] = \text{sign}\Big[(1-\delta)^2 - (1-\delta)\delta T^{\frac{1}{1-\delta}} - \delta T\Big].$$

The term in brackets on the right hand side of this equation decreases with T for all T. Now if $T = 1$, then

$$\frac{d\underline{s}_t'}{dT} = \underline{s}_t'\Big(\frac{1}{2} - \delta\Big).$$

Next, consider the slope of the after–tax Lorenz curve at $p = 1$,

$$\bar{s}_t' = \frac{(1-\tau_e)\bar{\pi}_t}{(1-\tau)Y_t} = \frac{1}{T}\Big(\frac{1}{\delta T + 1 - \delta}\Big)\Big(\frac{(2-\delta)T + \delta T^{\frac{2-\delta}{1-\delta}}}{\Phi_t}\Big).$$

Differentiating yields

$$\frac{d\bar{s}_t'}{dT} = \bar{s}_t'\Big(\frac{\delta}{1-\delta}\Big)\Big[\frac{T^{\frac{\delta}{1-\delta}}}{2-\delta+\delta T^{\frac{1}{1-\delta}}} - \frac{1-\delta}{\delta T + 1 - \delta}\Big].$$

This implies, in particular, that

$$\text{sign}\Big[\frac{d\bar{s}_t'}{dT}\Big] = \text{sign}\Big[\delta^2 T^{\frac{1}{1-\delta}} + (1-\delta)T^{\frac{\delta}{1-\delta}} - (2-\delta)(1-\delta)\Big].$$

The term in brackets on the right hand side of this identity increases with T for all T. Now, if $T=1$ then

$$\frac{d\bar{s}_t'}{dT} = \bar{s}_t'\Big(\frac{\delta}{1-\delta}\Big)\Big(\delta - \frac{1}{2}\Big).$$

We can now prove each of the statements in Proposition 13.

(a) If $\delta = \frac{1}{2}$, then if $T=1$, $\frac{d\underline{s}_t'}{dT} = \frac{d\bar{s}_t'}{dT} = 0$. Since, $\frac{d\underline{s}_t'}{dT}$ decreases with T, it follows that $\frac{d\underline{s}_t'}{dT} < 0$ for all $T > 1$ and $\frac{d\underline{s}_t'}{dT} < 0$ for all $T < 1$. Similarly, since $\frac{d\bar{s}_t'}{dT}$ increases with T, it follows that $\frac{d\bar{s}_t'}{dT} > 0$ for all $T > 1$ and $\frac{d\bar{s}_t'}{dT} < 0$ for all $T < 1$. Thus inequality falls with T for all $T < 1$ and rises with T for all $T > 1$, in the Lorenz dominance sense.

(b) If $\delta > \frac{1}{2}$, then if $T=1$, $\frac{d\underline{s}_t'}{dT} < 0$ and $\frac{d\bar{s}_t'}{dT} > 0$. Since, $\frac{d\underline{s}_t'}{dT}$ decreases with T, it follows that $\frac{d\underline{s}_t'}{dT} < 0$ for all $T > 1$. Similarly, since $\frac{d\bar{s}_t'}{dT}$ increases with T, it follows that $\frac{d\bar{s}_t'}{dT} > 0$ for all $T > 1$. Thus inequality rises with T for all $T > 1$, in the Lorenz dominance sense. Moreover, inequality rises with T for $T > T^*(\delta)$ for some $T^*(\delta) < 1$. Note that the value of T at which inequality can be said to rise with T, in the Lorenz dominance sense, is the maximum of the values of T at which $\underline{s}_t'$ and $\bar{s}_t'$ change direction. This turning point depends only on δ and hence is time–invariant.

(c) If $\delta < \frac{1}{2}$, then if $T=1$, $\frac{d\underline{s}_t'}{dT} > 0$ and $\frac{d\bar{s}_t'}{dT} < 0$. Since, $\frac{d\underline{s}_t'}{dT}$ decreases with T, it follows that $\frac{d\underline{s}_t'}{dT} > 0$ for all $T < 1$. Similarly, since $\frac{d\bar{s}_t'}{dT}$ increases with T, it follows that $\frac{d\bar{s}_t'}{dT} < 0$ for all $T < 1$. Thus inequality falls with T for all $T < 1$, in the Lorenz dominance sense. Moreover, inequality falls with T for $T < T^*(\delta)$ for some $T^*(\delta) > 1$. Note that the value of T at which inequality can be said to rise with T in the Lorenz dominance sense if the minimum of the values of T at which $\underline{s}_t'$ and $\bar{s}_t'$ change direction. This turning point depends only on δ and hence is time–invariant.

Proof of Proposition 14: First note that, as shown by Shorrocks (1983), Lorenz dominance plus a rising mean, implies second-order stochastic dominance.

(a) If $T > 1$ a reduction in T reduces Y_t. If $\delta = \frac{1}{2}$, then, from Proposition 15, a reduction in T results in a reduction in after–tax inequality in the Lorenz dominance sense. Together, these effects imply a second–order stochastic increase in actual after–tax income, which results in a second–order stochastic increase in the distribution of actual human capital. With decreasing returns to human capital in production this results in an increase Y_{t+1}. A reverse argument establishes that H_{t+1} and Y_{t+1} both decrease as a result of fiscal incentives, when $T < 1$.

(b) If $\delta > \frac{1}{2}$, a reduction in T when $T > T^*(\delta)$ results in a reduction in after–tax inequality in the Lorenz dominance sense. If $T > 1$ this also result in an increase in Y_t so that the impact would be the same as in (a). However, if $T < 1$, then the reduction in T results in a decrease in Y_t. It follows that there is some $T_t^{**} \in [T^*(\delta), 1]$ such that the effect of the reduction in inequality (in the Lorenz dominance sense) on aggregate human capital outweighs the fall in Y_t, for $T > T_t^{**}$. In general, T_t^{**} is time varying.

(c) An analogous argument to that in (b) establishes this result. ∎