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Abstract

This paper advances a model which can account for these five prominent facts of monetary economics. (i) Money is held in non trivial amounts even though it is a dominated asset. (ii) High rates of inflation lead to low rates of growth. (iii) Monetary injections momentarily depress the rate of interest. (iv) Hyper-inflations can be ended with a minimal loss of output. (v) Liquidity crises lead to recessions. The model advanced develops the second variation in Woodford (1990). The model is designed to capture three features of reality, which form the basis for modeling money as a dominated asset: Income and expenditures by households and firms are not synchronized. Money is used to transfer income from the time it is received to the time it is spent because real assets are costly to liquidate, and capital markets are imperfect. And money improves the allocation of resources by providing flexibility to the timing of consumption and investment. In this model, monetary shocks affect total factor productivity. Hence, the model offers a link between the real business cycle model and a monetary approach to the business cycle. (JEL E40, E31, O42)

I. Introduction

Five prominent facts of monetary economics can be summarized as follows: (i) Money is held in non trivial amounts even though it is a dominated asset. (ii) High average rates of inflation decrease economic growth (Andres and Hernando [1997]). (iii) Monetary injections momentarily depress the rate of interest (Christiano, Eichenbaum, and Evans [1997]). (iv) Hyper-inflations can be ended with minimal loss of output (Sargent [1983]). (v) Large monetary disturbances, associated with liquidity crises, lead to severe recessions (Friedman and Schwartz [1963]).[1] This paper advances a simple monetary model which can account for these facts.

The model advanced in this paper develops the second variation in Woodford (1990).[2] In the model, the income received by households and firms normally does not match their expenditure profiles. Money facilitates transferring income from the time it is received to the time it is spent because real assets are costly to liquidate, and capital markets are imperfect so financial assets do not fully circumvent these liquidation costs. Thus, money enhances the production capabilities of the economy because it provides a vehicle for households and firms to improve the timing not only of their consumption but also of their investments. The model in this

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[1] A sixth prominent fact could be the positive relationship at high frequencies between the growth rate of output and inflation (Phillips curve). However, recent research (e.g. Backus and Kehoe [1992]) has shown that, without some qualification, this relationship does not exist. In the present model, the co-movement of output and inflation at high frequencies induced by monetary shocks depends on several factors, among them the duration of the shocks. (See sections IV and VI).

[2] Woodford (1990) is linked to the literature following Bewley (1980). Another example in this literature with capital that is not perfectly liquid is Aiyagari and Gertler (1991). See this paper for other related work.

paper differs from Woodford (1990) in two ways, both important for explaining the five prominent facts stated above. First, the production technology yields constant returns to scale with respect to all types of capital, so the model allows for endogenous growth as in Barro (1990). Second, investment projects take time to be completed, so production activities overlap. As a result, investors simultaneously hold money and capital even though money is an asset dominated by the rate of return of capital when projects are carried up to their maturity.[3]

The model explored in this paper has close connections with other monetary models. Similar to the models with search (Diamond [1984] and Kiyotaki and Wright [1989]), with overlapping generations (Samuelson [1958]), with spatially separated agents (Townsend [1980]), and with incomplete insurance (Bewley [1980]), money in the present model facilitates trade across individuals with income and expenditure profiles that are not synchronized, and who live in economies with incomplete markets. Thus, similar to all these models, money is valued without cash-in-advance constraints. In another connection, similar to the liquidity models following the work of Lucas (1990) and Fuerst (1992), money in the present model plays a role financing production activities. However, the present model is tractable without "split family" devices and cash-in-advance constraints. By comparison with all these models, the most salient feature of the model presented in this paper is the role of money in the allocation of capital across alternative projects. This feature is crucial to most results reported.

In Section II, I describe the economy and the equilibrium concept in detail. In summary,

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[3] Also, Woodford (1990) refers to the only asset issued by the government as national debt, while in this paper it is referred as money. This distinction is only terminological.

the model develops the following parable. Production projects take two periods to mature, and the various production projects in the economy overlap. So in every period, some projects are at their initial stage (plowing), some are at their second stage (seeding), and some have just matured. Projects require investment of real resources for plowing and seeding, and yield a harvest of output once they mature. Investors can participate at most in one project at a time. At plowing time, investors have no problem in financing their consumption and investment expenditures because in the same period they have just harvested their previous project. However, at seeding time, investors face a liquidity problem because they have no earnings in this period. This problem cannot be trivially solved because the investments they made at plowing time are physically irreversible, and capital markets are imperfect, so it is costly to mortgage their capital. In the equilibrium studied, this financial problem is solved by holding money; that is, part of the harvest of one project is sold for money which is kept until the seeding time of the next project.

In Section III, I study the role of money in promoting efficiency and growth. The main point of this section is that money improves efficiency by allowing a better allocation of resources, in general, and capital, in particular. This role is enhanced as the return on money nears the social return on capital, and so as the rate of inflation falls. (This effect parallels the arguments in favor of a high return on liquid assets advocated by McKinnon (1973)). Unfortunately, in a no intervention equilibrium with a fixed supply of money, the rate of return on money approaches the growth rate of output, which is lower than the social return on capital. Consequently, this equilibrium is inefficient. In fact, this inefficiency satisfies the strong concept of dynamic inefficiency: consumption could be raised at all periods. Curiously, when this happens all the standard tests of dynamic inefficiency that have sprung out of the overlapping generations model are invalid. The economy remains dynamically inefficient even though there is a fixed supply of money, the growth rate of output is lower than the marginal product of capital, and capital income is larger than investment. In this economy, the source of dynamic inefficiency is not too much capital, but capital being invested too early in the life of projects.

In Section IV, I study the effect of monetary injections on the rate of interest. This section shows that this model offers similar liquidity effects to the Grossman and Weiss (1983) and Rotemberg (1984) model. Namely, if monetary injections were to go exclusively to present buyers (of goods), which at the beginning of the period hold all the money, they would be neutral in the sense that they would not affect real variables and interest rates. Prices would just increase proportionately to the quantity of money. However, if some of the money goes to sellers (investors having a harvest and so demanding money), monetary injections have real effects. In the latter case, as a result of a monetary injection, real purchasing power is transferred from buyers to sellers. These sellers will increase their real demand for money, so in equilibrium prices cannot increase proportionately to the quantity of money in the same period of the injection. Some of the price increases induced by the extra supply of money will come in the future, so temporarily expected inflation rises and real interest rates fall. Simultaneously, sellers increase their consumption and investment, while buyers are able to consume and invest less. Therefore, the harvest following the injection will be small, while the harvest following that one will be large. For a few periods, relatively small and large harvests will succeed one another. In periods with abundant harvests the intertemporal price of goods will be low (low interest rates), while with scarce harvests the intertemporal price of goods will be high (high interest rates).

In Section V, I show that, despite the short-run dynamics described in Section IV, inflations can be ended without loss of output. In fact, if the new monetary policy is credible, a good way to end inflations is "cold turkey" (hold money constant form now on).[4] If an inflationary episode is ended "cold turkey," the immediate impact is a sharp increase in the demand for money which leads to a drop in the price level and a simultaneous increase in the real return on money. The real return on money actually overshoots its new long-run level. These monetary phenomena go hand in hand with improvements in the allocation of capital and increases in investment, so output starts growing faster immediately.

In Section VI, I subject the economy in the model to severe monetary disturbances. These disturbances have the form of liquidity crises: a portion of the money stock becomes illiquid for one period. Numerical simulations show that these liquidity crises lead to severe recessions which leave a permanent mark on the long-run path of output. During these recessions, some investors have projects with highly productive uses of capital, but they lack the liquidity to finance their investments. At the same time, other investors have output in their hands, but they lack a good use for it. This output ends up either being sold at a low (intertemporal) price or self-invested in a low productivity use of capital; so this output becomes underemployed.

Finally, Section VII summarizes the main results, and discusses some directions for

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[4] Empirical evidence gathered by Ball (1994) shows that ending inflations "cold turkey" induces lower sacrifice ratios than more gradual approaches. This empirical finding is a puzzle if the major source of interaction between money and output is price rigidity, but it is consistent with the model presented here.

further research. Among these directions, it points that the effect of money on total factor productivity, due to the role of money in the allocation of capital, might provide a useful link between the real business cycle model and a monetary approach to the business cycle.

II. A global economy

Consider a planet which axis of rotation is tilted 23.5 degrees toward the ecliptic. This tilt induces a seasonal pattern in both hemispheres, in such a way that winters in the South coexist with summers in the North. The only economic good in the planet is perishable and harvested in the summer. To obtain a harvest next summer, investors must sacrifice real resources to plow this summer and to seed in the intervening winter. Investors cannot be simultaneously plowing in one hemisphere while seeding in the other, so they can only invest in one productive project at a time. Moreover, capital markets are imperfect, so titles on the ownership of capital in one hemisphere cannot be sold or mortgaged in the other. Monitoring costs are too large for this possibility to become economically viable. Instead, to finance consumption and investment while seeding, investors sell a portion of the present harvest for money, and spend this money on goods while seeding. As a result, some of the North harvest goes to the South and vice versa.

In the summer, investors allocate their wealth into consumption, plowing capital, and money. The flow budget constraint for a representative investor at this stage is:

$$c_t + k_{pt} + m_t = x_t; \tag{1}$$

where c_t = consumption in period t;

k_{pt} = plowing capital invested in period t;

m_t = real money demand in period t; and

x_t = real wealth at the beginning of period t.

In the winter, investors own some plowing capital, but this capital is illiquid. They also own some money which they can spend buying consumption and seeding capital, or they can keep for the next period. The flow budget constraint for a representative investor at this stage is:

$$c_t + k_{st} + m_t = \frac{m_{t-1} p_{t-1}}{p_t}; \tag{2}$$

where k_{st} = seeding capital invested in period t; and

p_t = nominal price of the good in period t.

In both summer and winter, consumption, capital, and real money balances must be nonnegative:

$$c_t \geq 0; k_{pt} \geq 0; k_{st} \geq 0; \text{and } m_t \geq 0, \text{ for all } t. \tag{3}$$

The size of the harvest in period $t + 2$ depends on the plowing and seeding capitals respectively invested in periods t and $t + 1$, so the wealth at the beginning of period $t + 2$ is:

$$x_{t+2} = F(k_{pt}, k_{st+1}) + \frac{m_{t+1} p_{t+1}}{p_{t+2}}; \tag{4}$$

where F = gross production function. For ease of notation, the surviving portions of k_{pt} and k_{st+1} are incorporated in the function F. The function F is positive, twice continuously differentiable, increasing, concave, and homogeneous of degree one.

The investors' horizon is infinite; their preferences are time additive; and, their instantaneous utility function is isoelastic:

$$V_t = \sum_{i=0}^{\infty} \beta^i \frac{c_{t+i}^{1-\sigma}}{1-\sigma}; \tag{5}$$

where V_t = utility in period t;

β = discount factor (positive and less than 1); and

σ = inverse of the intertemporal elasticity of substitution (positive, and if $\sigma = 1$, the instantaneous utility should be understood as logarithmic).

The nominal quantity of money is constant. The only economic transactions in the economy consist of exchanges of money and the good performed in a market that takes place every period. When it is summer in the North, this market clears when the demand for the good by the South equals the net supply of the good by the North, that is:

$$c_t^S + k_{st}^S = x_t^N - c_t^N - k_{pt}^N ; \qquad \textbf{(6)}$$

where superscripts denote South and North. Using the budget equations (1) and (2), market clearing can be also stated in terms of money:

$$m_{t-1}^S \frac{p_{t-1}}{p_t} - m_t^S = m_t^N . \qquad \textbf{(7)}$$

When it is summer in the South, the two superscripts must be reversed.

Each hemisphere has a large number of investors. Thus, the economy has settled into a recursive monetary competitive equilibrium (henceforth equilibrium) where the sequence of prices $\{p_t, p_{t+1}, \ldots\}$, the allocation of harvests, and the real quantity of money satisfy:

(i) prices are a function of the fundamental state variables (to be specified below);

(ii) money is valued;

(iii) investors have perfect foresight about prices and take them as given;

(iv) investors maximize V_t subject to the constraints (1), (2), (3), and (4); and

(v) the market that exchanges money and the good clears every period.

Moreover, the equilibrium into which the economy has settled has the property that money is a dominated asset, that is the marginal products of plowing and seeding capital for the period they are invested exceed the rate of return on money. The fact that the marginal product of seeding capital exceeds the rate of return on money implies that investors spend all their money at seeding time. From now on, I will focus exclusively in equilibria where money is a dominated asset and all choices, except for the demand for money at seeding time, are interior. This greatly facilitates the exposition and, as it turns out, the equilibrium paths in all the thought experiments of this paper satisfy these properties.

The assumptions made on preferences and technology allow for the existence of a representative investor from the South and a representative investor from the North. So in a money dominated equilibrium the fundamental state variables at the beginning of a period are the size of the harvest in this period and the amount of plowing capital invested in the previous period. Furthermore, the homotheticity of preferences coupled with constant returns to scale in production imply that equilibrium relative prices depend only on the ratio of these two state variables.

In a money dominated equilibrium, the budget constraint (2) holds with $m_t = 0$. In addition, investors' optimal plans must satisfy the following conditions. In the winter, one unit of the good must bring the same utility to an investor either if it is immediately consumed, or if it is invested and the gross return consumed in the next period, that is:

$$c_t^{-\sigma} = \beta F_{st} c_{t+1}^{-\sigma}; \tag{8}$$

where F_{st} = marginal product of seeding capital in period t. In the summer, the marginal return of investing in plowing capital is equalized with the marginal return of acquiring money (p_t units) which is to be invested in seeding capital next period (p_t / p_{t+1} units), that is:

$$F_{pt} = \frac{F_{st+1} p_t}{p_{t+1}}; \tag{9}$$

where F_{pt} = marginal product of plowing capital in period t. Finally, in the summer, a unit of output must bring the same utility to an investor either if it is consumed immediately, or if it is sold for money (p_t units) which is to be spent on consumption next period (p_t / p_{t+1} units), that is:

$$c_t^{-\sigma} = \frac{\beta p_t c_{t+1}^{-\sigma}}{p_{t+1}}. \tag{10}$$

Conditions (8) to (10), together with the equations describing the budget constraints characterize the investors' optimal plans. For later reference, it is useful to combine (8) to (10), adjusting for time subscripts, to get the Euler equation relating consumption in two consecutive summers:

$$c_t^{-\sigma} = \beta^2 F_{pt} c_{t+2}^{-\sigma}. \tag{11}$$

To conclude this section, remark that if money is dominated by seeding capital: $F_{st+1} > p_{t+1} / p_{t+2}$, it must also be dominated by plowing capital when held up to maturity $F_{pt} > p_t / p_{t+2}$ (combining the previous inequality with (8)). The fact that investors are marginally indifferent between money and plowing capital in the summer is due to the liquidity services money provides in the winter.

III. Efficiency and growth.

In this section, I explore the long-run implications of the model presented in Section II, by analyzing the balanced path of an economy in a money dominated equilibrium. By construction, I show that under certain conditions this path exists. Next, I focus on the efficiency properties of an equilibrium along the balanced path, and how efficiency is affected by interventions that alter the long-run rate of inflation.

Without loss of generality suppose period t is a summer in the North. With the proper ratio of the two fundamental state variables, $x^N_t \;/\; k^S_{pt\text{-}1}$, the economy follows the balanced path along which both the North and the South grow at the same constant rate g, and the harvest of period t in the North is $(1 + g)$ times the harvest in period t - 1 in the South: $x^N_t = (1 + g)\; x^S_{t\text{-}1}$. To prove this conjecture, I construct the balanced path implied by an equilibrium with money as a dominated asset. That is, I find the rates of inflation and growth, and a set of ratios that characterizes the initial condition $(x^N_t \;/\; k^S_{pt\text{-}1})$ and the allocation of harvests ($k^S_{st} / k^S_{pt\text{-}1}$, $k^N_{pt} / k^S_{pt\text{-}1}$, and c^S_t / c^N_t) along the conjectured balanced path. Then, I show that the balanced path so constructed satisfies, with some restrictions on preferences and technology, the conditions defining an equilibrium with money as a dominated asset.

If a balanced path exists, the summer consumption, plowing capital, and money demand in either hemisphere are proportional to the size of the harvest (because of constant relative prices and homothetic preferences). Thus, the market clearing condition (7) implies that the rate of deflation along the balanced path is the rate of growth g. Therefore, the capital allocation condition (8) implies:

$$F_p = F_s(1 + g). \qquad \textbf{(12)}$$

(From now on, time subscripts are dropped for variables that are constant along a balanced path.) Moreover, the Euler equation (11) implies:

$$(1+g)^2 = \left(\beta^2 F_p\right)^{\frac{1}{\sigma}}. \tag{13}$$

By substitution, equations (12) and (13) simplify to:

$$F_p^{2-\frac{1}{\sigma}} = \beta^{\frac{2}{\sigma}} F_s^2. \tag{14}$$

Since *F* is homogenous of degree one, the marginal products F_p and F_s are a function of the proportion at which the two types of capital are employed, so (14) determines the ratio $k^S_{st} / k^S_{pt\text{-}1}$. With this ratio the balanced path is easily described: Equation (12) determines g. By definition of a balanced path, the ratio $k^N_{pt} / k^S_{pt\text{-}1}$ equals $1 + g$. Equation (10) implies $c^S_t / c^N_t =$ $(1 + g)^{1/\sigma - 1}\,\beta^{1/\sigma}$. Finally, because along a balanced path both North and South employ the two types of capital in the same proportion, the initial state variables must satisfy: $x^N_t \ / \ k^S_{pt\text{-}1} =$ $F(1, k^S_{st} / k^S_{pt\text{-}1})$. This completes the description of the balanced path.

Growth, and so the return on money in the balanced path, is bounded by the rate of growth that would result from investing the entire harvests. Namely,

$$x_{t+2} = F(k_{pt}, k_{st+1}) = F_p\left[k_{pt} + k_{st+1}(1+g)^{-1}\right] < F_p x_t; \tag{15}$$

where the first equality is (4) in a money dominated equilibrium, the second equality follows from constant returns to scale and (12), and the inequality at the end follows from the budget constraints (1) and (2), and positive consumption. Expression (15) implies that along a balanced path $(1 + g)^2 < F_p$, which, with (12), implies:

$$1 + g < F_s. \tag{16}$$

Since along a balanced path the rate of growth is the return on money, (16) implies that money is a dominated asset.

For existence of the conjectured balanced path, we need that equation (14) has a real solution, and the objective of investors (5) converges. Both conditions require some restrictions on preferences and technology. For example, they are both satisfied if $\sigma \geq 1$, and F_p and F_s are positive functions of the ratio k_{st} / k_{pt-1} with an image spanning $(0, \infty)$.

The balanced path just described is not Pareto efficient. For efficiency, the allocation of capital must satisfy that the present marginal product of plowing capital, which remains invested for two periods, is equal to the marginal product of rolling an investment in seeding capital for the next two periods:

$$F_{pt} = F_{st} F_{st+1}. \tag{17}$$

Comparison with (9) implies that this equality would be satisfied in a monetary equilibrium if the deflation factor were F_{st}. But, as seen, along an equilibrium balanced path, the deflation factor is lower than F_{st} and equal to $1 + g$. Consequently, the equilibrium balanced path is not efficient. In fact, along a balanced path (13) and (15) imply:

$$F_p < (F_s)^2. \tag{18}$$

Thus, along an equilibrium path, capital is misallocated. Too much capital is invested while plowing when investors are fully liquid but the marginal product of capital is low, and too little capital is invested while seeding when investors have limited liquidity but the marginal product of capital is high.

Capital being invested too early in the life of projects means that the inefficiency of the equilibrium balanced path takes the strong form of dynamic inefficiency: consumption could be raised at all periods by shifting resources from plowing to seeding. Indeed, let 0 be the initial period. Consider the following reallocation from plowing to seeding capital: $\mathrm{d}k_{st} = -\mathrm{d}k_{pt} > 0$ for all $t \geq 0$, and $\mathrm{d}k_{st+1} = \mathrm{d}k_{st}(1+g)$. Equation (13) implies $\mathrm{d}x_t = 0$ for all $t \geq 2$. Moreover, $\mathrm{d}x_1 > 0$ because the projects maturing at 1 got more seeding capital with the same plowing capital. Thus, by comparison with the original balanced path, the amount of output left for consumption at $t = 1$ rises after the reallocation without reducing consumption at any other period. (Continuity allows this gain to be spread to all other periods.)

If a government were to introduce newly created money by paying interest on money, this policy would have no real effects because the real return on money would not be affected. Suppose, however, that the government decides to change the supply of money with a subsidy to investors paid with newly created money. If the subsidy is proportional to consumption with a constant rate over time, it has no direct effect on any margin of choice described in the first order conditions (8) to (10). The subsidy though creates an inflationary process. Thus, in a balanced path the rate of return on money is no longer the rate of growth, instead the market equilibrium condition (7) implies:

$$\frac{p_{t-1}}{p_t} = \frac{m_t^N}{m_{t-1}^S(1+\gamma)} = \frac{1+g}{1+\gamma}; \tag{19}$$

where γ is the rate of growth of the money supply. Likewise, the allocation of capital along a balanced path is transformed into:

$$F_p = F_s \frac{1+g}{1+\gamma}. \quad \textbf{(20)}$$

With a positive growth rate of the money supply γ, the rate of return on money falls, and the misallocation of capital worsens. Combining (13) and (20), one can easily check that as γ increases both $k^S_{st} / k^S_{pt\text{-}1}$ and g fall. If instead of a subsidy the government imposes a tax, the rate of growth of the money supply is negative. In this case, as long as the return on money remains lower than the return on capital, the allocation of capital improves and g increases. In conclusion, if no interest is paid on money, inflation unambiguously reduces efficiency and growth along a balanced path.

IV. Liquidity effects

In this section, I study the dynamics of the model out of the balanced path. In particular, I focus on the adjustments that take place after a monetary injection. As is common in growth models explicit solutions out of the balanced path exists only for a few special cases. For this reason, this section presents the most interesting of these special cases, and it checks its robustness with numerical simulations.

Out of the balanced path, the model can be solved explicitly with unit elasticity of substitution both in preferences and gross production; that is, the special case with logarithmic preferences and Cobb-Douglas gross production: $V_t = \sum_{i=0}^{\infty} \beta^i \ln c_{t+i}$; and $F(k_{pt}, k_{st+1}) = A k_{pt}^{\alpha} k_{st+1}^{1-\alpha}$. With this specialization of F, the two types of capital are implicitly assumed to fully depreciate. (The numerical simulations relax this assumption). In this special case, equations (6) to (11) describing an equilibrium path can be explicitly solved. Assuming period *t* is a summer in the North, and money remains a dominated asset, the optimal plan for Northern investors is:

$$c_t^N = (1-\beta)x_t^N\,;\ k_{pt}^N = \beta^2\alpha x_t^N\,;\ \text{and } m_t^N = (\beta-\alpha\beta^2)x_t^N\,. \tag{21}$$

Thus, in this special case, the demand for money is proportional to output, as in the classical quantity equation of money. The optimal plan for Southern investors is:

$$c_t^S = \frac{1-\beta}{1-\alpha\beta}\frac{M_t^S}{p_t};\ \text{and } k_{st}^S = \frac{\beta(1-\alpha)}{1-\alpha\beta}\frac{M_t^S}{p_t}. \tag{22}$$

where M^S_t = nominal quantity of money at the beginning of period *t.* Equilibrium in the money market implies :

$$m_t^N = \frac{M_t^S}{p_t}. \tag{23}$$

Combining (22) and (23) implies that consumption and seeding capital in the South is also proportional to output x^N_t. In the next period, the inherited plowing capital in the South $k^S_{pt\text{-}1}$ and the seeding capital invested in period *t* determines the size of the harvest x^S_{t+1}. Again, a similar analysis applies in the latter period with reversed superscripts. Alternating superscripts, equations (21) to (23) recursively describe the equilibrium path for given initial conditions x^N_t and $k^S_{pt\text{-}1}$.

With a constant money supply, $M_t^S = m_{t-1}^S p_{t-1}$, so equations (21) and (23) imply:

$$\frac{p_t}{p_{t-1}} = \frac{m_{t-1}^S}{m_t^N} = \frac{x_{t-1}^S}{x_t^N}. \tag{24}$$

That is, with abundant harvests, the good is cheap both in terms of money and in terms of future goods: the real interest rate is low. This provides an equilibration mechanism that drives the economy toward the balanced path. The hemisphere with a low harvest enjoys a high return on money, and so it experiences faster growth than the other hemisphere.

Let us consider now monetary injections modeled as one time unexpected handouts of money to either investors in the North (having a harvest) or investors in the South (having money). If the monetary injection goes to the South, the only effect of the injection is a rise in the price level proportional to the increase in the nominal quantity of money. With the injection, the nominal money supply includes not only the money Southerners acquired in the previous period, but also the money handed to them. Because the value of money in period t is determined by the demand for money by Northerners, all the equilibrium conditions continue to hold with a proportional increase in the price level. In contrast, if the monetary injection goes to the North, the injection has real effects in the economy. With the injection, the wealth of Northerners x^N_t includes both the harvest they collected and the real value of money handed to them. Their optimal plan is still described by (21), which implies that their demand for consumption, investment, and money demand is raised proportionately with their newly raised wealth. For an equilibrium to exist, the demand for the good by the South falls by the same amount that it rises in the North. So, the price level p_t must rise. However, the price level cannot rise proportionally with the quantity of money because that would contradict that the real demand for money (by Northerners) increases after the injection. Thus, some of the price increases are delayed to the future, which implies a rise in expected inflation and a fall on the real rate of interest.

The effects of the monetary injection linger over many periods. In the initial period, the monetary injection raises the plowing capital invested in the North, while depresses the seeding capital invested in the South. As a consequence, the next harvest after the injection will fall, while the harvest following that one will rise. For several periods, relatively small and large harvests will succeed one another. In periods with abundant harvests, real interest rates will be low, while in periods with scarce harvest real interest rates will be high.

Figures 1 and 2 portray the dynamics of economies that have been subjected to a 1 percent monetary injection handed to the owners of the initial harvest. The appendix describes the parameter values of these economies in detail. In summary, both economies are calibrated to roughly match the rate of growth and the rate of return on capital in the United States along a balanced path. Plowing capital is interpreted as an aggregate of durable forms of physical and human capital with partial depreciation every period, while seeding capital is interpreted as materials and other circulating capital which fully depreciate every period. Figure 1, portrays an economy with a Cobb-Douglas production function. While, Figure 2 portrays an economy with a CES production function where the two types of capital are fairly strong complements (the elasticity of substitution is 1/3).

In both economies, the monetary injection leads to an immediate fall on real interest rates. In Figure 1, with a high elasticity of substitution in production, the fall is small, but the dynamic effects linger for a long time. In Figure 2, with a low elasticity of substitution in production, the immediate fall is large, but the dynamic effects vanish in a few periods. In both economies, the monetary injection worsens the allocation of capital, which leads to a drop in output for the next harvest with minor effects on the other harvests to come.[5]

V. Stopping Inflations

This section shows the dynamic adjustments that take place after an inflation is ended "cold turkey," that is when inflation is ended by holding the money supply constant at the inherited level in all future periods. I assume that investors did not anticipate that inflation would end, so previous to the change of policy the economy had settled in a high inflation balanced path. Also, I assume that once the policy has changed investors have perfect foresight about the future path of inflation. With this perfect foresight assumption, the fact that the economy was in an inflationary process in the past is irrelevant for present and future plans. Inflation only leaves its mark on the present state variables, for concreteness, the harvest in the North x^N_t and the inherited plowing capital in the South k^S_{pt-1}. The economy will not jump to the new balanced path because with inflation seeding capital is scarce relative to plowing capital, so when the inflation ends the ratio x^N_t / k^S_{pt-1} is lower than its new long run value.

Figures 3 and 4 compare the equilibrium paths with and without an end to the growth of the money supply. The calibration of the economies in these two figures is respectively the same as that of Figures 1 and 2. In both figures, the initial balanced path was one with an 8 percent annual rate of inflation. This rate of inflation may appear benign, but in the model all monetary assets earn no interest, so the annual rate of return on monetary assets is 10 percent below the

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[5] In the figures, output at $t + 2$is measured by $F(k_{pt}, k_{st+1}) - (1 - \delta)k_{pt}$. Thus, it is a measure of final output. In contrast, GDP in the national accounts try to allocate this output to the various periods the production projects last.

rate of return on these assets for an economy with a constant money supply. This drop is extremely large and economically important: The semiannual rates of growth of the two economies jump respectively from 0.3 and 0.28 percent to 1 percent.

Both figures show that immediately after inflation is ended, the price level falls and the real quantity of money jumps up. With lower inflation the real demand for money increases, since its nominal supply is fixed, prices must fall to achieve equilibrium. The size of this price fall depends positively on the elasticity of money: high with high elasticity of substitution in production (Figure 3), and low with low elasticity of substitution in production (Figure 4). Simultaneously, the real interest rate increases well above its new long run value because the present harvest is scarce relative to the harvests to come.

The jumps on the real quantity of money and the real interest rate bring an immediate improvement in the allocation of capital from plowing to seeding. This improvement is behind the sharp increase in output in the first two harvests after inflation has ended. Moreover, the higher expected return on capital combined with the end to consumption subsidies, leads to a reallocation of resources from consumption to investment. The combination of more and better allocated capital leads to a faster rate of growth of output than that of the high inflation economy.

VI. Liquidity crises

In Section IV, I showed that monetary injections tend to momentarily depress the real rate of interest. However, the impact of these monetary disturbances on output does not have the expected sign: injections induce a recession as the allocation of capital worsens, while (not shown) contractions induce a boom as the allocation of capital improves. This section presents a different type of monetary disturbance which induces a positive correlation between money and output.

The monetary disturbance studied in this section aims to capture the type of event that Friedman and Schwartz (1963) identify as leading to sharp recessions, for example the banking crisis during the Great Depression which culminated with the Banking Holiday of March 1933. During these crises, the money supply contracts, not because monetary authorities conduct open market operations, but because of a combination of bank closures and a shift from deposits to currency. Moreover, all these crises have been transitory, so investors must expect a subsequent increase in the money supply. Consequently, I assume that liquidity crises are a temporary drop in the money supply without explicit transfers across individuals. Specifically, I assume that a portion of the money supply turns illiquid for one period. Money holders correctly realize that next period they will own the portion of the money supply which they cannot spend this period because it has turned illiquid.

Figures 5 and 6 illustrate the impact of a liquidity crisis in which the money supply temporarily falls by 30 percent. (This is approximately the drop in the money supply from 1929 to 1933). Both figures show that the liquidity crises are the onset of a severe recession: output falls respectively by -16 and -13 percent. This is most remarkable because the model, in its present form, rules out unemployed resources and leisure, so these drops in output roughly correspond to drops in total factor productivity. During the downward phase of these recessions, seeding capital drops dramatically, and so their marginal product is very high. At seeding time, investors lack the liquidity to finance their highly productive investments. Simultaneously, at the other hemisphere with a harvest, investors find themselves with output that sells at a low intertemporal price and if self-invested earns a low marginal product. At the end of the recession, output converges to new balanced paths at a level of output significantly lower than the original paths. The gap between the two paths is 0.35 and 0.3 percent respectively.

Figures 5 and 6 differ importantly from one another on the initial drop of the price level, and the associated impact on the real quantity of money. With a Cobb-Douglas production function, investors with a harvest are quite willing to substitute plowing capital for seeding capital, so their real demand for money drops. In this case, the 30 percent drop in the nominal quantity of money is split between a 16 percent drop in the price level and a 12 percent drop in the real quantity of money (and a 2 percent combined effect). With a CES production function with high complementarity between the two types of capital, investors with a harvest increase their real demand for money to ensure that they can seed their fields the following period. In this case the 30 percent drop in the nominal quantity of money is more than fully absorbed by a 31 percent drop in the price level. As a result, the real quantity of money increases by 1 percent. This is extremely important. Drops in the nominal quantity of money associated with liquidity crises can cause severe recessions even though, as it happened from 1929 to 1931, the real quantity of money increases in the initial phase of the recession.

VIII. Conclusion

The model of this paper provides an interesting avenue to think about money and its role in the economy. Apart from the details, the defining characteristics of the model are a combination of three facts that form the basis for the demand for money as a dominated asset: Income and expenditures by households and firms are not synchronized. Money is used to transfer income from the time it is received to the time it is spent because of a combination of the illiquidity of real assets and the imperfection of capital markets. And money improves the allocation of resources by providing flexibility to the timing of consumption and investment. The resulting model implies the following facts. Money is held in non trivial amounts even though it is an asset dominated by the rate of return on capital. Along balanced paths, high rates of inflation lead to low rates of growth. Monetary injections momentarily depress the rate of interest. Inflation, in general, and hyper-inflations in particular, can be ended without a temporary loss in output. And liquidity crises lead to severe recessions, even though the real quantity of money does not necessarily fall at the onset of these recessions.

In this paper, I have intentionally stripped the model from all unnecessary complications. Subsequent work, some of it in progress, must address important extensions. Most notably, it must be interesting to allow for unemployed resources and leisure, to differentiate outside liabilities between money and national debt, to incorporate banking, and to add uncertainty. These extensions will refine the modeling of employment, monetary injections, liquidity crises, and expectations. However, I anticipate that these extensions will not overturn the basic logic and results of the present paper.

Subsequent work must also address the following question raised by the effects of monetary disturbances on total factor productivity: Is the role of money in the allocation of capital, as stressed in this paper, the missing link between the real business cycle model and a monetary approach to the business cycle? The real business cycle model in almost 20 years of intensive development has not been able to document what events precipitate the technological regressions that according to the model trigger recessions. In contrast, long ago, Friedman and Schwartz (1963) gave a great thrust to a monetary approach to business cycles by documenting that monetary disturbances were behind most recessions the United States had experienced. Nevertheless, monetary theories have been confronted with the difficulty of explaining why both real wages and total factor productivity are pro-cyclical. The role of money in the allocation of capital may successfully link both approaches and help to overcome their difficulties.

Appendix

CALIBRATION OF NUMERICAL SIMULATIONS

In all the simulations depicted the period is thought to be half year. The parameters have been chosen so along a balanced path with a constant money supply, the growth rate is 1 percent (semiannual) and the net marginal product of plowing capital, which is invested for 2 periods, is 7.2 percent (annual). With logarithmic preferences this implies $\beta = 1.025^{-1}$. The depreciation rate of plowing capital is 10 percent (annual), while seeding capital fully depreciates in one period. The technology differs across simulations. In Figures 1, 3, and 5, the technology is:

$$F(k_{pt}, k_{st+1}) = A k_{pt}^{\alpha} k_{st+1}^{1-\alpha} + (1-\delta) k_{pt}.$$

In Figures 2, 4, and 6, the technology is:

$$F(k_{pt}, k_{st+1}) = A\left[\alpha k_{pt}^{-\rho} + (1-\alpha) k_{st+1}^{-\rho}\right]^{-\frac{1}{\rho}} + (1-\delta) k_{pt};$$

with $\rho = 2$. In both sets of simulations, the values of A and α have been assigned so, with $\delta = 0.1$ and the net marginal product of plowing capital = 7.2 percent (annual), the ratio k_{st+1} / k_{pt} is 0.4 along a balanced path. As a result, in Figures 1, 3, and 5, $A = 1.14$ and $\alpha = 0.29$, and in Figures 2, 4, and 6, $A = 0.94$, and $\alpha = 0.71$. Previous literature give little guidance to the relative importance of the two types of capital. With $k_{st+1} / k_{pt} = 0.4$ along a balanced path, the total capital invested in a project increases by 35 percent from the first period to the second period. This is consistent with the strong seasonality of most industries, toward the season where they yield their final output. Although qualitative results are not sensitive to this ratio, as it increases money becomes more important in the allocation of capital, and so monetary shocks have stronger real effects.

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Output
Deviation in percent
0.1
-0.1
-0.3
-0.5
-0.7
0
2
4
6
8
10
12
Period

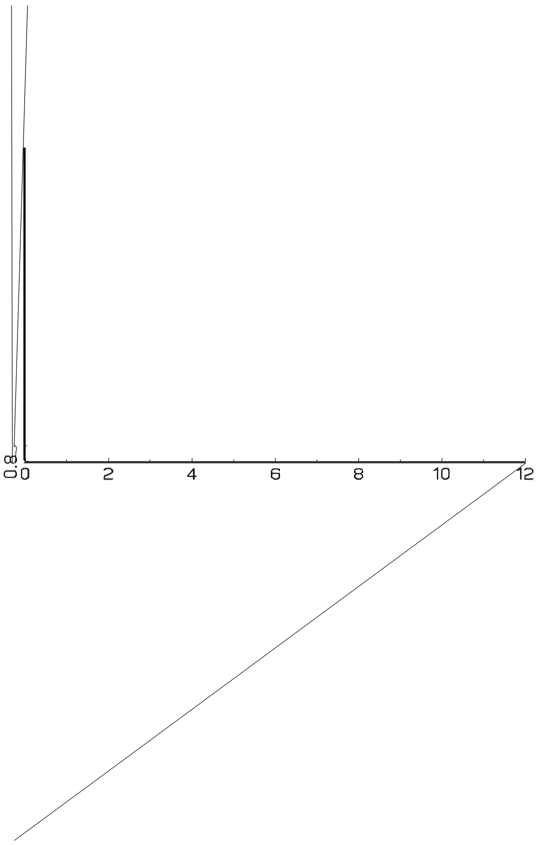
0.8
0
2
4
6
8
10
12

Figure 2: MONETARY INJECTION

CES Production Function: Rho = 2

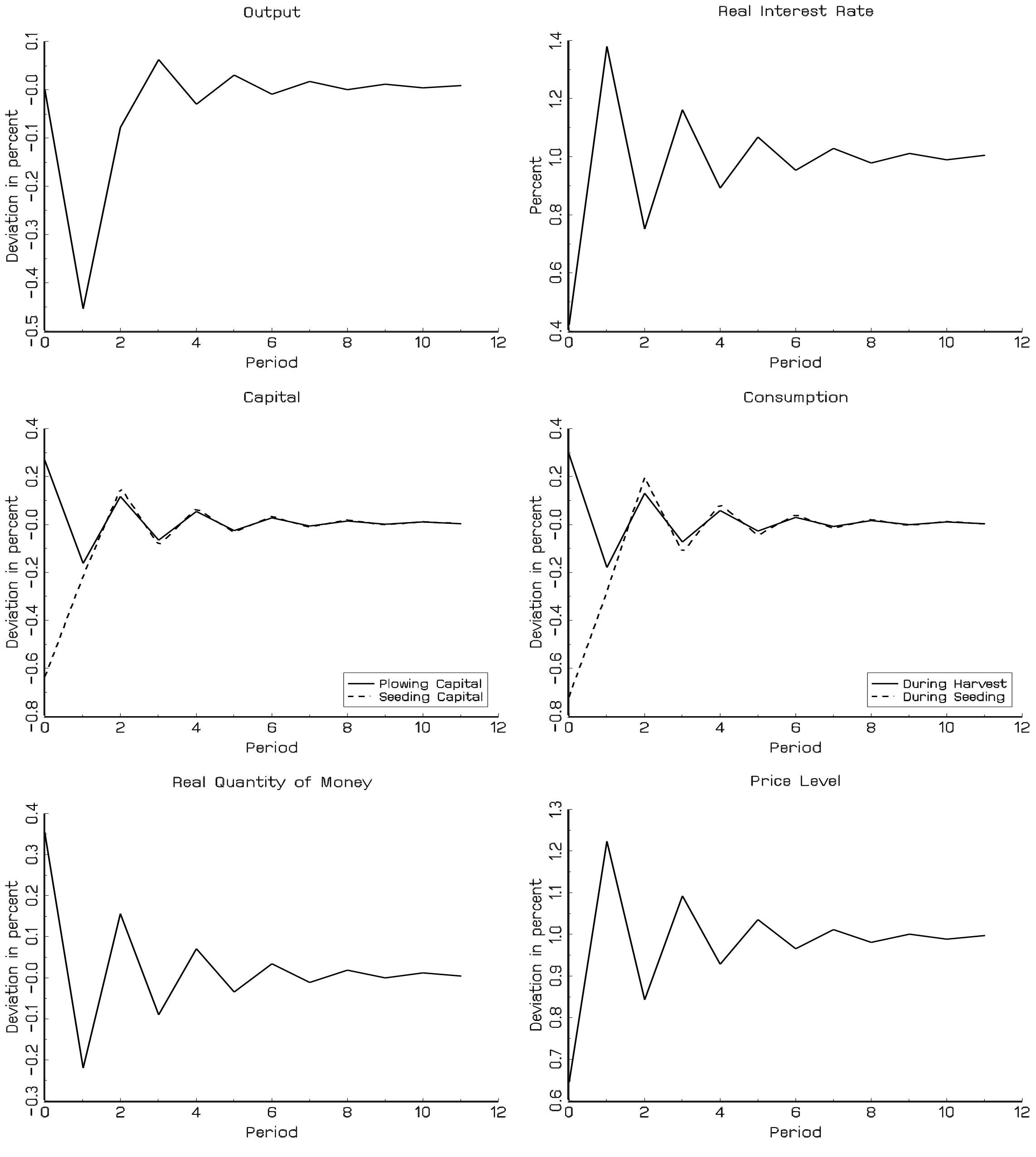

Figure 3: STOPPING INFLATION

Cobb-Douglas Production Function

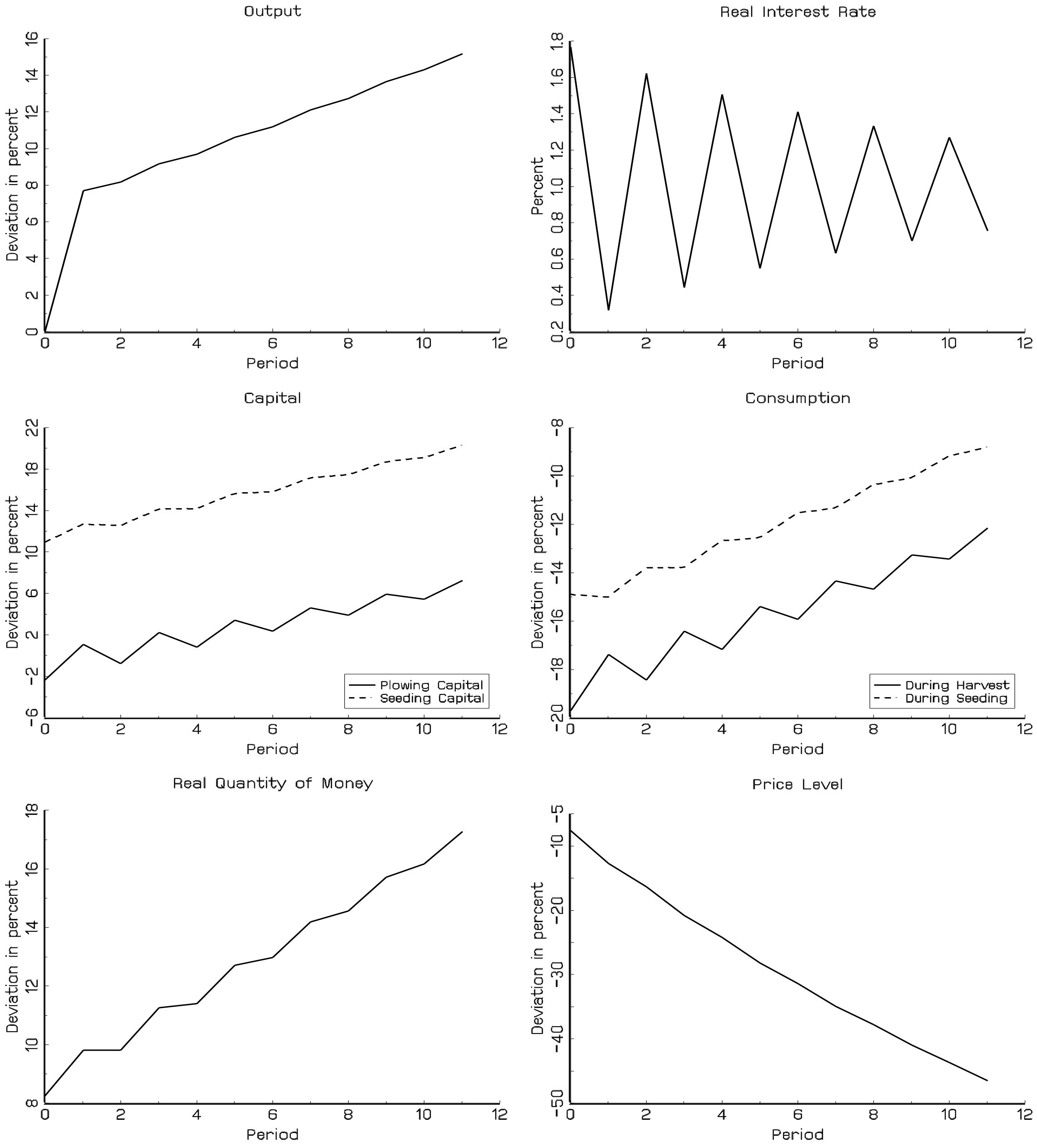

Figure 4: STOPPING INFLATION

CES Production Function: Rho = 2

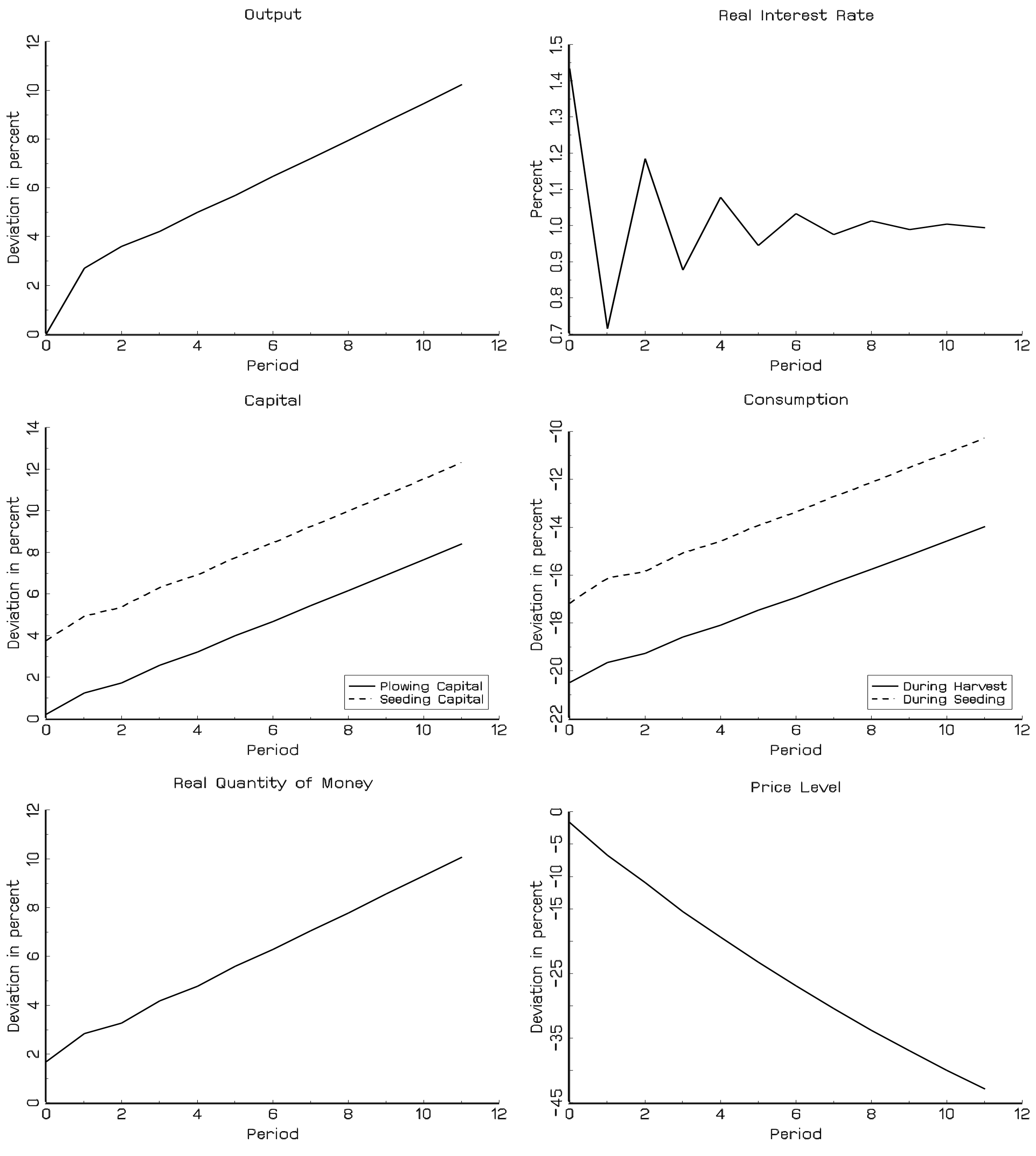

Figure 5: LIQUIDITY CRISIS

Cobb-Douglas Production Function

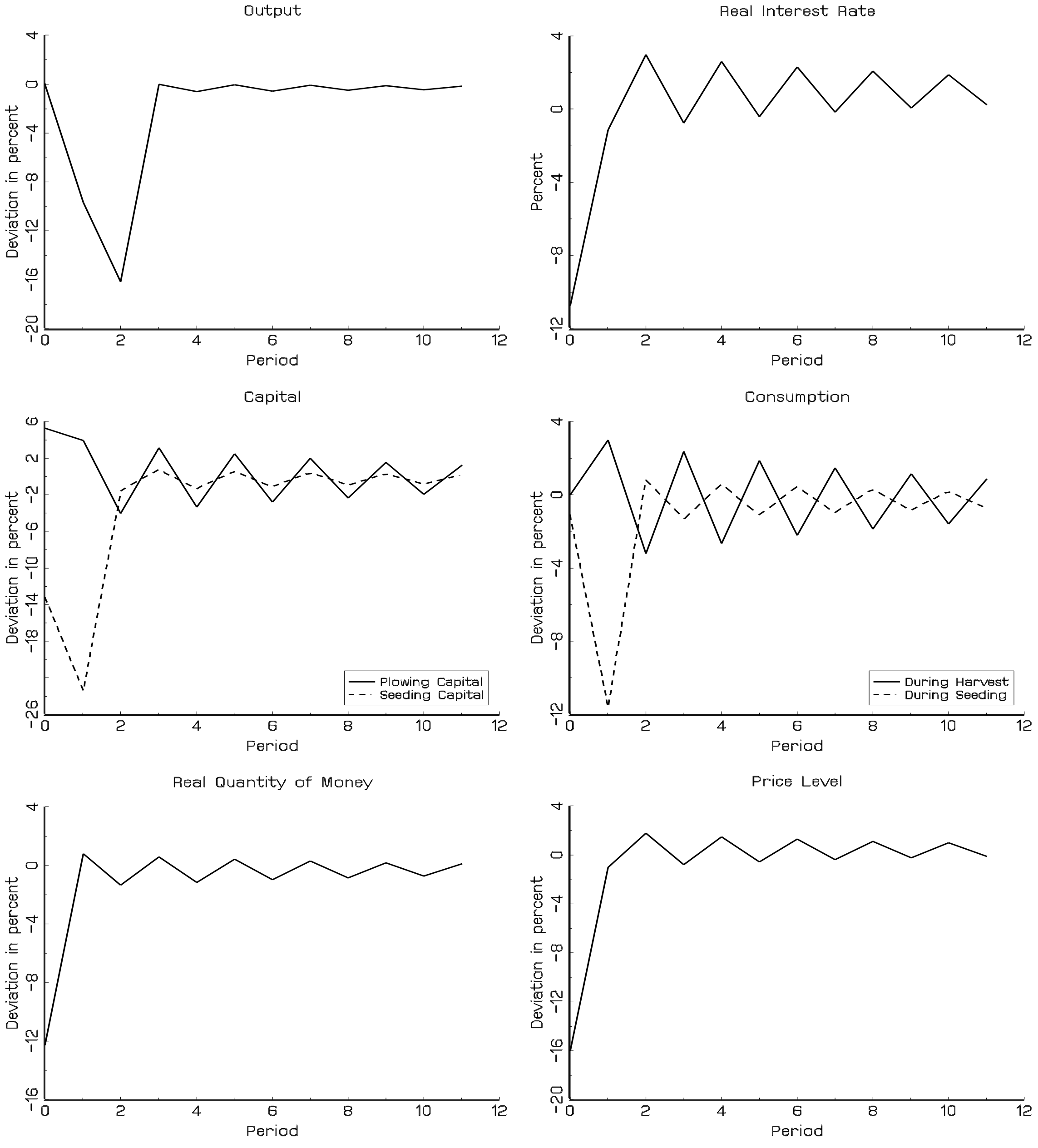

Figure 6: LIQUIDITY CRISIS

CES Production Function: Rho = 2