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INVESTMENT IRREVERSIBILITY IN GENERAL EQUILIBRIUM
Capital Accumulation, Interest Rates, and the Risk Premium

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INVESTMENT IRREVERSIBILITY IN GENERAL EQUILIBRIUM
Capital Accumulation, Interest Rates, and the Risk Premium

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Abstract

This paper provides a tractable general equilibrium model that can accommodate shocks and irreversibility constraints both at the firm and at the aggregate level. Under realistic conditions, irreversibility not only prevents capital destruction, but it also promotes capital creation. Moreover, irreversibility depresses risk-free rates, especially in booms, and magnifies risk premia, especially in downturns. In addition to the theoretical analysis, two numerical examples illustrate the quantitative effects of investment irreversibility in the face of either large rare depressions or mild frequent recessions.

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1. Introduction

Many forms of investment are irreversible or they can be reversed only at a high cost. A growing body of literature (surveyed by Pindyck [1991], Dixit and Pindyck [1994] and Hubbard [1994]) has shown that irreversibility is important for firms' investment decisions under uncertainty. We still know very little about the consequences, however, of investment irreversibility for the economy as a whole, that is, in general equilibrium. In particular, what are the consequences of investment irreversibility for capital accumulation, growth, interest rates, and the risk premium? To answer this question, this paper analyzes a tractable general equilibrium model that can accommodate shocks and irreversibility constraints both at the firm and at the aggregate level. This theoretical analysis is complemented by a numerical evaluation of the effects of investment irreversibility.

Firm behaviour analyses (e.g. Bertola [1988], Pindyck [1988], and Dixit [1989]) have shown that when investment is irreversible the rental cost of capital must incorporate the option value lost as a result of constraining the future capital stock when investment takes place. This "rental cost" effect makes firms more reluctant to invest in irreversible projects than they would be to invest in reversible ones. In the long run, the "direct" effect of the irreversibility constraint preventing capital destruction counteracts the "rental cost" effect deterring capital creation. As Abel and Eberly (1995) have shown, in the long run, the average capital stock of a firm with irreversible investment may be larger or smaller than the average capital stock of a firm with flexible investment. All these important results, hinge on exogenous paths for demand conditions and for interest rates. Irreversibility constraints may have an effect on both the demand for goods and their intertemporal price (interest rates). So these firm behaviour analyses must be extended to address issues such as how investment irreversibility affects aggregate investment and growth.

As shown by Leahy (1993), an endogenous derivation of prices in a competitive equilibrium changes completely the economic rationale of how irreversibility affects investment decisions at the firm level. However, irreversibility has a similar effect on aggregate investment to the effect it has for a single firm. Leahy's analysis, though, is conducted assuming exogenous interest rates and assuming demand functions with no income effects. So it abstracts from the type of general equilibrium effects that are the focus of this paper.

A different strand of the literature has analyzed investment irreversibility in the framework of the neoclassical growth model. Sargent (1980), Olson (1989), and Christiano and Fisher (1994) impose the irreversibility constraint on stochastic versions of the neoclassical growth model with a representative agent and a single good. These papers show that standard dynamic programming techniques are useful to characterize the optimal consumption paths. They report, however, that the neoclassical growth model with a single type of capital has two discouraging properties: Except for some general results such as existence and monotonicity, the policy functions are not analytically tractable. Moreover, except for rare large shocks, the aggregate dynamics in modern economies are not variable enough for the irreversibility constraint to be binding. Therefore, it appears that if irreversibility is important in modern economies it must be at a disaggregated level.

In partial equilibrium, Bertola and Caballero (1994) show that uncertainty and irreversibilities at the firm level are relevant to aggregate investment. In general equilibrium, Dow and Olson (1992), Boldrin, Christiano, and Fisher (1995), Christiano and Fisher (1995), Coleman (1996), and Veracierto (1997) extend the neoclassical growth model by adding multiple sectors.[1] With multiple sectors, the neoclassical growth model is even less tractable than with a single sector, and these papers rely largely on numerical simulations. These simulations show that constraints on the movement of capital across sectors are more important than constraints on the aggregate level of capital. And interestingly, irreversibility may either increase or reduce aggregate investment, even when no constraints are binding. Moreover, several of these papers report important effects of irreversibility on interest rates.

In summary, the extant literature shows that investment irreversibility in general equilibrium does not necessarily depress capital accumulation, and it has important effects on interest rates. These two finding raise the question of what are the general equilibrium effects of investment irreversibility that are abstracted from partial equilibrium analyses.

This paper recasts the stochastic growth models referenced in the three previous paragraphs with the following assumptions: First, all production processes yield the same output which can be consumed or invested in many types of capital. Second, previous papers have assumed that the alternative to capital is a fixed (or exogenously growing) factor of production. Instead this paper assumes, as many other endogenous growth models have done, that all inputs are a type of capital; thus "human capital" replaces "labor". Third, the aggregate production function yields constant returns to scale. And fourth, preferences are homothetic and modelled along the lines of Epstein and Zin (1989, 1991) and Weil (1989, 1990). These preferences include as a special case the standard isoelastic time additive preferences, but they allow to distinguish the intertemporal elasticity of substitution from the inverse of the coefficient of relative risk aversion. These two parameters play important and distinct roles in the analysis.

The assumptions of the model allow for multiple stochastic shocks, multiple types of capital, and irreversibility constraints for each type of capital. Thus, the model can accommodate firms faced with investment irreversibility constraints and with (possibly) idiosyncratic shocks. At the same time, the model is analytically tractable, so it is useful to describe the general equilibrium effects of investment irreversibility.

The model reveals that in general equilibrium the "rental cost" effect is a substitution effect, which is counterbalanced by a "wealth" effect induced by the constraint that irreversibility imposes on the set of feasible paths. That is, irreversibility makes the representative consumer poorer. Assuming consumption is a normal good, this "wealth" effect leads to lower consumption, higher savings, and thus to higher investment. Moreover, if the intertemporal elasticity of substitution is less than one (which all empirical studies strongly support) the "wealth" effect outweighs the "rental cost" effect, so investment irreversibility not only prevents capital destruction, as stressed by Abel and Eberly (1995), but it also induces capital creation.[2]

One is tempted at this point to conclude that if irreversibility prevents capital destruction and promotes capital creation, it must foster growth. Unfortunately, this is not necessarily the case because irreversibility also changes the capital mix: Irreversibility increases the rental cost of inflexible types of capital. Thus, investment is diverted from irreversible into flexible types of capital at the cost of lowering the aggregate return of capital. Consequently, irreversibility has an ambiguous effect on growth.

The "wealth" effect of irreversibility on investment is transmitted to firms' decisions that are analyzed in the partial equilibrium models not only through the demand conditions for their output, but also through real interest rates. Under some conditions, the dual effect of depressing the demand for goods when irreversibility constraints are not binding is a reduction in their intertemporal price: the real interest rate. Conversely, the dual effect of constraining the supply of goods when irreversibility constraints are binding is an increase in the real interest rate.

Interest rates are not the only price affected by the irreversibility constraints. The price of installed capital is, of course, more directly affected. With investment irreversibility the price of installed capital is not necessarily equal to the cost of producing capital, because of the possibility of capital losses. Changes in the price of capital is a key ingredient to generating endogenous market returns processes with a realistic degree of variability, which in turn is a key ingredient to having production models with realistically large risk premia.[3]

Two numerical examples illustrate the quantitative effects of irreversibility. In the first example, large rare shocks generate depressions with a depth and duration similar to that of the Great Depression. In the second example, mild frequent shocks generate booms and recessions with a depth and duration similar to that of an average business cycle. With large shocks, even if they are Hicks neutral, investment irreversibility has important positive effects on growth and risk premia, and negative effects on interest rates. With mild shocks, the irreversibility constraints may still have important effects on risk premia and interest rates, but only if the shocks are sufficiently non-neutral.

Section 2 presents the model. Section 3 analyses the effects of investment irreversibility.

Section 4 illustrates the quantitative effects of investment irreversibility. Finally, section 5 concludes with a discussion of the results.

2. **The Model**

The economy has a representative consumer. In each period, the consumer's problem is to consume or invest the output just obtained from a stochastic production process. All output is homogeneous, but capital is differentiated into multiple types. In the present, the vector of capital stocks inherited from the past is $k \in \mathbb{R}^N$, and the realized value of the stochastic shocks is $z \in \mathbb{R}^M$. The vector z follows a Markov process endowed with the Feller property. A gross production function F maps (k, z) onto gross output:

$$x = F(k,z). \tag{1}$$

The gross output x includes both production and the capital stock inherited from the past valued at cost. The function F is concave, linearly homogeneous, and continuously differentiable in k, and measurable in z.

In the allocation of x, the consumer faces a set of irreversibility constraints on each one of the capital stocks:

$$k_i' \geq \mu_i k_i, \quad \text{for } i = 1,\ldots,N; \tag{2}$$

where $\mu_i \geq 0$ is the survival rate of the capital stock i and a prime on a variable denotes the value of the variable next period. This specification accommodates flexible capital stocks with a zero μ_i and irreversible capital stocks with a positive μ_i.[4] In addition to the irreversibility constraints, the consumer faces a standard resource constraint:

$$c + \sum_{i=1}^{N} k_i^{\prime} \leq x; \tag{3}$$

where c is consumption.

The preferences of the consumer are recursive, homothetic, and independent across states for atemporal lotteries, but not necessarily time additive. Specifically, the consumer is endowed with the parametric version of Kreps and Porteus preferences introduced by Epstein and Zin (1989):[5]

$$u = \left[(1-\beta) c^{1-\sigma} + \beta \left(\mathrm{E} u^{\prime\, 1-\gamma} \right)^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}; \tag{4}$$

where u is present utility; γ is the coefficient of relative risk aversion for atemporal lotteries; and σ is the inverse of the intertemporal elasticity of substitution along a deterministic path. Both σ and γ are assumed positive. The expectation E is conditional on present information.

The representative consumer maximizes (4) subject to (2) and (3). A solution to this optimal growth problem exists under some restrictions on F (see Epstein and Zin (1989)) which are assumed to be satisfied throughout the paper. Similarly, a proof analogous to Epstein and Zin (1989) shows that there is a value function V which maps the vector (k, z) onto the maximized utility of the consumer. Standard recursive dynamic programming arguments imply that, with respect to k, V is continuously differentiable, concave, and linearly homogeneous.[6]

Using ν_i and λ as the Lagrange multipliers for constraints (2) and (3), the first order conditions to the optimal growth path are:

$$(1-\beta) V^{\sigma} c^{-\sigma} = \lambda; \tag{5}$$

and

$$\lambda - V^{\sigma}\beta\left(EV'^{1-\gamma}\right)^{\frac{\gamma-\sigma}{1-\gamma}}EV'^{-\gamma}\partial_i V' = \nu_i \geq 0, \quad \text{with equality if } k_i' > \mu_i k_i. \tag{6}$$

Despite the length of these expressions, the intuition behind them is simple. Condition (5) equates the marginal utility of consumption to the marginal value of wealth (λ). Condition (6) states that each constraint (2) penalizes the consumer for a marginal value ν_i which is equal to the difference between the marginal value of wealth and the marginal value of k_i'. Moreover, ν_i is always non-negative, and it is zero when the constraint is not binding.

Using the envelope theorem, we can now derive a useful consumption rule from these first order conditions. The envelope theorem states:

$$\partial_i V = \lambda\partial_i F - \mu_i\nu_i, \quad \text{for } i = 1...N. \tag{7}$$

Multiplying both sides of this equality by k_i and adding for all i, we obtain:

$$V = \lambda\tilde{x}; \quad \text{where } \tilde{x} = x - \sum_{i=1}^{N}(1 - q_i)\mu_i k_i, \quad \text{and } q_i = 1 - \frac{\nu_i}{\lambda}. \tag{8}$$

q_i is the real price (in units of the consumption good) of capital i ("Tobin's q"), and it is obtained by subtracting the capital losses to the cost of one unit of capital. Consequently $\tilde{x}$ is the gross output valued at cost minus the aggregate capital losses incurred, or equivalently $\tilde{x}$ is the market value of gross output. Finally, using (5) and (8), we obtain

$$c = (1 - \beta)^{\frac{1}{\sigma}}\lambda^{1-\frac{1}{\sigma}}\tilde{x}. \tag{9}$$

Because preferences are homothetic, consumption is proportional to the consumer's wealth, which here corresponds with $\tilde{x}$. Consumption also depends on the marginal value of wealth λ. An increase in λ induces a negative substitution effect and a positive wealth effect on consumption. If $\sigma > 1$, the wealth effect dominates. All this is familiar. The novelty here is that $\tilde{x}$ is not predetermined but is endogenous to the present decisions of the representative consumer.

Multiplying the first order conditions (6) by k_i and adding the resulting equations for all i, we obtain:

$$\lambda(\tilde{x} - c) = V^{\sigma}\beta\mathrm{E}\left(V^{\prime\,1-\gamma}\right)^{\frac{1-\sigma}{1-\gamma}}. \tag{10}$$

Using this expression and (7), the first order condition (6) can be transformed into a pricing formula for the capital i:

$$q_i = \mathrm{E}\,\frac{V^{\prime\,-\gamma}\lambda^{\prime}}{\mathrm{E}\,V^{\prime\,-\gamma}\lambda^{\prime}\tilde{R}}\,(\partial_i F^{\prime} - \mu_i + \mu_i q_i^{\prime}). \tag{11}$$

The price of one unit of installed capital is equal to the expected present value of the gross return next period. The gross return next period is the expression in parenthesis. This expression is multiplied by a set of contingent prices denoting the relative value of output next period in terms of output this period. This contingent prices can be transformed using (8) and (9) in the numerator and (10) in the denominator to obtain the standard Euler equation:

$$q_i = E\left[\beta\left(\frac{c^{\prime}}{c}\right)^{-\sigma}\tilde{R}^{\frac{\sigma-\gamma}{1-\gamma}}\right]^{\frac{1-\gamma}{1-\sigma}}(\partial_i F^{\prime} - \mu_i + \mu_i q_i^{\prime}); \quad \text{where } \tilde{R} = \frac{\tilde{x}^{\prime}}{qk^{\prime}}. \tag{12}$$

That is, the contingent prices depend on consumption growth (c'/c) and the market return ($\tilde{R}$) in a fairly simple fashion. As a pricing formula, this Euler equation is just an application of Epstein and Zin (1989) and Weil (1989). The novelty here is how consumption and the market return are related to the irreversibility constraints. This novelty, as the next section shows, is crucial to understand the behavior of interest rates and risk premia.

3. **The Effects of Irreversibility**

This section provides a comparison between an economy with the irreversibility constraints (2) and an economy where these constraints are relaxed. The aim is to pin down the effects of irreversibility on capital accumulation, growth, interest rates, and risk premia.

Capital Accumulation

Suppose two identical economies except that in one all μ_i's are zero (flexible economy) while in the other some μ_i's are positive (irreversible economy). Using (8) and (9), the consumption rule for both economies can be described as:

$$c = \left[(1 - \beta) V^{\sigma - 1} \tilde{x}\right]^{\frac{1}{\sigma}}; \tag{13}$$

except, of course, that V and $\tilde{x}$ will differ in each economy. Let superscript F denote the flexible economy and the superscript I denote the irreversible economy. Since the constraints in (2) restrict the set of feasible paths of the irreversible economy versus those of the flexible economy, $V^F(k,z) \geq V^I(k,z)$. Moreover, since $q_i = 1$ for all i in the flexible economy, while $q_i \leq 1$ in the irreversible economy, $\tilde{x}^F(k,z) = x(k,z) \geq \tilde{x}^I(k,z)$. Therefore, if $\sigma \geq 1$, the consumption rule (13) implies $c^F(k,z) \geq c^I(k,z)$. Then, the resource constraint (3) implies:

> **Proposition 1:** If the intertemporal elasticity of substitution is between zero and one ($\sigma \geq 1$), investment in the flexible economy does not exceed investment in the irreversible economy. In addition, this inequality is strict if some irreversibility constraints are presently binding ($\tilde{x}^F > \tilde{x}^I$), or if $\sigma > 1$ and some constraints are expected to be binding sometime in the future ($V^F > V^I$).

If no constraints are presently binding, the elimination of the irreversibility constraints increases the marginal value of the now more productive wealth. This induces a negative substitution effect and a positive wealth effect on consumption. If $\sigma \geq 1$, the wealth effect at least balances the substitution effect. Moreover, if some irreversibility constraints are presently binding, the direct effect of the elimination of these constraints revalues the capital stock and so rises wealth and consumption.

Fixed Factors

As in other second best analyses, the partial elimination of some constraints does not necessarily move the economy in the same direction as the total elimination of all constraints. The following example helps to understand the lack of strong results in previous general equilibrium models with irreversible investment. In these studies, output was a strictly concave function of capital, so they were assuming the existence of some factors which either were fixed or growing at an exogenous rate. Without loss of generality, let the capital stock 1 be fixed, in the sense that the constraint (2) must now hold with equality. Using analogous derivations to those in section 2, we obtain the consumption rule (13), except that now q_1 may either be above or below unity. Let us compare this economy with a flexible economy with $\mu_i = 0$ for $i = 2,..., N$. The removal of the constraints on all capital stocks except 1, unambiguously raises the utility of the representative consumer V with the same type of substitution and wealth effects as in Proposition 1. However, this removal may induce capital losses on the fixed factor, so it may actually depress wealth. An analogous result to Proposition 1 requires now that these potential capital losses are sufficiently low:

Proposition 2: When some factors are fixed, we can guarantee that investment in the flexible economy does not exceed investment in the irreversible economy if, in addition to $\sigma \geq 1$, $\tilde{x}^F(k,z) \geq \tilde{x}^I(k,z)$.

Relocation Versus Irreversibility

A common motivation for irreversibility in partial equilibrium analyses is the difficulty of relocating installed capital from a particular use to another. This strong form of irreversibility is captured in this model when the set of constraints (2) applies to each particular use of capital. A weaker constraint is to allow relocation. This is achieved by replacing the set of constraints (2) with this unique constraint on the total capital stock:

$$\sum_{i=1}^{N} k_i' \geq \mu \sum_{i=1}^{N} k_i \tag{14}$$

We can now evaluate the constraint on relocation by comparing an economy with complete irreversibility (I superscript) with another with relocation (R superscript). If $\mu_i \geq \mu$ for all $i = 1,..., N$, the set of feasible paths with constraint (14) includes the feasible paths with the set of constraints (2). For states in which the constraint (14) is not presently binding, Proposition 1 is valid for exactly the same reason. For states in which the constraint (14) is binding, we have:

$$\sum_{i=1}^{N} k_i'^{R} = \mu \sum_{i=1}^{N} k_i \leq \sum_{i=1}^{N} \mu_i k_i \leq \sum_{i=1}^{N} k_i'^{I}. \tag{15}$$

Consequently:

Proposition 3: If $\sigma \geq 1$ and $\mu_i \geq \mu$, investment in the economy with relocation does not exceed investment in the economy with complete capital irreversibility.[7]

If one is willing to let μ vary with k, the assumption $\mu_i \geq \mu$ is stronger than needed for Proposition 3. All that is required about μ for this proposition, is the first weak inequality in (15); that is, μ can be as high as the convex average of the μ_i's with weights equal to the relative abundance of each type of capital. So, the average survival rate for the aggregate capital stock understates the severity of the irreversibility constraints in the economy. Conversely, when aggregation is feasible, the survival rate for the aggregate model must be not lower, and in some instances much higher, than the average survival rate of the aggregate capital stock. The following examples illustrate this last point.

Aggregation

As is well known, perfect commodity aggregation attains only with either perfect substitutability or perfect complementarity. Perfect substitutability yields a degenerate result: If two types of capital are perfect substitutes and one is more irreversible than the other, the representative consumer will only invest in the more flexible type, and the more flexible type will be the only type of capital in the long run. Perfect complementarity yields interesting results when survival rates are diverse or shocks are non-neutral.

Suppose an economy has two indispensable types of capital: one with a low depreciation rate so μ_1 is close to 1, and the other with a unit depreciation rate so μ_2 is zero. Further suppose that both capital stocks are perfect complements. For example, each car requires four wheels, cars last long, and tires last for one period. Faced with catastrophic shocks, the representative consumer will choose to let some cars idle and perfect aggregation will no longer apply. However, as long as shocks are not catastrophic, the representative consumer will equip each car with four wheels. Consequently, the economy will work as if the survival rate for the aggregate capital stock is μ_1: The irreversibility of cars forces an apparent irreversibility on the stock of tires!

Consider another example. Suppose an economy with two types of capital with a common survival rate μ. The two types of capital are perfect complements in production, but their productivity varies over time to make the ratio at which they are both useful to alternate from one period to the next. For example, cows and refrigerators are perfect complements in the production of milk. In the summer cows give large amounts of milk, while in the winter fewer refrigerators are required to keep the milk cool. So the desired ratio of cows over refrigerators alternates from a low in the summer to a high in the winter. Since investment takes one period to be productive, investment, if unconstrained, is cow-intensive in the summer and refrigerator-intensive in the winter. Suppose, in addition, that the production of milk is subject to neutral total factor productivity random shocks. With adverse shocks, the representative consumer smoothes consumption by investing less. If a winter adverse shock is sufficiently severe, the irreversibility constraint on cows will be binding. However, as long as the shock is not catastrophic, the representative consumer will continue to invest in refrigerators, so that all the next summer's milk can be kept cool. Consequently, the non-neutral shocks on the productivity of cows and refrigerators, lead the economy to work as if the survival rate for the aggregate capital stock is higher than common survival rate μ!

<u>Capital Mix</u>

In the aggregate, the intuition from partial equilibrium analyses that irreversibility is bad for investment is misleading. With some qualifications, this intuition is nonetheless correct when applied to the composition of capital; that is, the "rental cost" effect drives investment away from irreversible types of capital and towards flexible types of capital. Suppose capital

F is flexible while capital I is irreversible. Subtracting equations (7) applied to next period for $i = F$ and I,

$$\partial_F V' - \partial_I V' = \lambda' (\partial_F F' - \partial_I F') + \mu_I \nu_I' \qquad \textbf{(16)}$$

Multiplying both sides of (16) by $V'^{-\gamma}$ and taking the conditional expectation of the resulting expressions, we obtain:

$$\mathrm{E}V'^{-\gamma}(\partial_F V' - \partial_I V') = \mathrm{E}V'^{-\gamma}\lambda'(\partial_F F' - \partial_I F') + \mathrm{E}V'^{-\gamma}\mu_I \nu_I'. \qquad \textbf{(17)}$$

If the irreversibility on capital I is not binding, the first order condition (6) implies that the left hand side of (17) is zero. Also, (6) applied to the next period implies that $\mathrm{E}V'^{-\gamma}\mu_I \nu'_I$ is non-negative. Consequently,

$$E\left[V'^{-\gamma}\lambda'(\partial_F F' - \partial_I F')\right] \leq 0. \qquad \textbf{(18)}$$

The expression $V'^{-\gamma}\lambda'$ stands for the contingent value to the representative consumer of one good next period, so:

> **Proposition 4:** When the representative consumer is investing in capital I, so the irreversibility constraint on capital I is not binding, the expected value of the marginal product of capital I is at least as large as the expected value of the marginal product of flexible capital F. Moreover, if there is a positive probability that the irreversibility constraint on I will be strictly binding next period, this inequality is strict.

The qualification brought from the general to the partial equilibrium analyses is that the value of goods is endogenous. Controlling for the expected marginal product of capital, investment is discouraged not only by irreversibility, but also by a low co-movement with the contingent value of goods. This is the central idea of portfolio allocation, and it is applied next to the choice between capital and bonds.

Interest Rates and Risk Premia

Flexible economies with technological shocks face multiple puzzles about the average and the cyclical variation of risk-free rates and risk premia (see Rouwenhorst [1995]). First, the average predicted risk-free rate is unrealistically high. Second, the average predicted risk premium is unrealistically low. Third, the predicted risk-free rate is too pro-cyclical to be realistic. And fourth, the predicted risk premium is not sufficiently counter-cyclical to be realistic. Under some conditions, investment irreversibility contributes to the resolution of all these puzzles by lowering the risk-free rate in good times, without necessarily lowering the risk-free rate in bad times. Moreover, it delivers counter-cyclical risk premia.

The equilibrium risk-free rate consistent with the representative consumer choosing neither to borrow nor to lend is determined by a first order condition analogous (6):

$$\lambda = V^{\sigma}\beta\left(\mathrm{E}V'^{\,1-\gamma}\right)^{\frac{\gamma-\sigma}{1-\gamma}}\mathrm{E}V'^{\,-\gamma}\lambda'\,r; \tag{19}$$

where r is the gross real interest rate and $\lambda' r$ replaces $\partial_i V'$ as an application of the envelope theorem. Using (8), (10), and the definition of $\tilde{R}$, this equation is transformed into:

$$r = \mathrm{E}\psi\tilde{R}; \quad \text{where } \psi = \frac{V'^{\,-\gamma}\lambda'}{\mathrm{E}V'^{\,-\gamma}\lambda'}. \tag{20}$$

The gross interest rate is equal to the expected market return multiplied by contingent prices ψ denoting the relative value of future output at different states. The contingent prices ψ depend on the relative scarcity of goods. Using (8) and (9), this scarcity is measured by a geometric weighted average of consumption and the market value of gross output:

$$\psi = \frac{\left(c^{/a}\tilde{x}^{/1-a}\right)^{-\gamma}}{\mathrm{E}\left(c^{/a}\tilde{x}^{/1-a}\right)^{-\gamma}}, \quad \text{where} \quad a = \frac{1-\gamma^{-1}}{1-\sigma^{-1}}. \tag{21}$$

Similarly, using (20) the risk premium is:

$$\mathrm{E}\tilde{R} - r = \mathrm{E}\left(1 - \psi\right)\tilde{R}. \tag{22}$$

Irreversibility affects interest rates and risk premia both through the effect it has on the market return ($\tilde{R}$), because it opens the possibility to capital gains and losses, and through the contingent prices of future goods (ψ), because it affects the demand for and the supply of output at the various states of nature.

<u>A Simple Special Case</u>

To illustrate how irreversibility tends to lower the risk-free rate in good times without necessarily lowering neither the risk-free rate nor the risk premium in bad times, let us specialize the model to make these financial effects analytically tractable. Suppose a unique type of capital and a multiplicative independent and identically distributed (i.i.d.) stochastic shock, that is $x = zk$. Then, in a flexible economy, $V^F = v_0 zk$ for a positive number v_0, and in an irreversible economy, $V^I = v(z)zk$ for a positive function v. The function v is weakly increasing because for a given amount of goods zk presently available, the representative consumer is less constrained on future choices the lower is k and the higher is z. When the irreversibility constraint is binding, the value of installed capital in (12) simplifies to:

$$q = \left(\frac{z - \mu}{\mu}\right)^{\sigma} \frac{\beta}{1 - \beta} \left(\mathrm{E}\, v(z')^{1-\gamma}\right)^{\frac{1-\sigma}{1-\gamma}}. \tag{23}$$

Therefore, q increases with z as long as the irreversibility constraint is binding. It ceases to bind when $q = 1$.

In the flexible economy, the market return is z', so equation (20) determining the risk-free rate simplifies to:

$$r^F = \mathrm{E}\,\frac{z'^{-\gamma}}{\mathrm{E}z'^{-\gamma}}\,z'. \tag{24}$$

The contingent prices of future goods decrease with z' reflecting that in states of nature that z' is low goods are more scarce and more valuable (see Weil (1989) for further discussion of this equation). Since z' is i.i.d., r^F is constant. In the irreversible economy, the market return is

$$\tilde{R}^I = \frac{\tilde{x}'}{qk'} = \frac{\tilde{z}'}{q}, \quad \text{for } \tilde{z}' = z' + (q' - 1)\mu. \tag{25}$$

Using (8) and (20), the risk-free rate in the irreversible economy is:

$$r^I = \mathrm{E}\,\frac{V'^{1-\gamma}\tilde{z}'^{-1}}{\mathrm{E}V'^{1-\gamma}\tilde{z}'^{-1}}\,\frac{\tilde{z}'}{q}. \tag{26}$$

Proposition 5: In this special case, if $\gamma \geq 1$, the contingent prices of future goods for the irreversible economy are a mean preserving spread of the analogous prices for the flexible economy. (See the appendix for the proof).

This proposition implies that irreversibility raises the value of goods when z' is low and depresses them when z' is high, or equivalently, it penalizes the variability of returns (risk) more heavily. This reflects the extra scarcity induced by binding irreversibility constraints associated with low values of z'. The assumption $\gamma \geq 1$ is necessary because with irreversibility, even if the shock z is i.i.d., the market return is not: Capital losses today are expected to be reversed with capital gains some time in the future. For this reason, $\gamma \geq 1$ is necessary to ensure that the representative consumer values capital highly when output is scarce (wealth effect) as opposed to when the expected market return is high (substitution effect).

Proposition 5 has strong implications for the cyclical pattern of interest rates and risk premia. In good times, when z is high enough for the irreversibility constraint not to bind, $q = 1$, and thus the market return in the irreversible economy is $\tilde{z}'$. Then, if $\gamma \geq 1$, $r^I \leq r^F$ for two reasons. First, the possibility of capital losses implies that $\tilde{z}' \leq z'$ for all states of nature. Second, as seen, the contingent prices of future goods penalize risk more heavily in the irreversible economy than in the flexible economy. In contrast, in bad times, when z is low enough so $q < 1$, the market return is $\tilde{z}'/q > \tilde{z}'$. Then, the possible capital gains from an increase in q raises the expected market return. With i.i.d. shocks, the contingent prices of future goods do not depend on z and $\tilde{z}'$ is i.i.d., so r^I is counter-cyclical. The constraint irreversibility imposes on the supply of goods raises their intertemporal price: the risk-free rate. This same argument applies to the risk premium as long as this is positive (see (22)). Consequently, as z falls a portion of the higher expected market return goes to raise the risk-free rate while another portion goes to raise the risk premium.

This simple special case has been chosen for analytical tractability. However, realism demands multiple types of capital and persistent shocks. With persistence, a low value of z is not only an indicator of future expected capital gains, but it is also an indicator of low productivity of capital. Then, analytical tractability is largely lost. So the effect of persistent shocks will be illustrated with two numerical examples in the next section. These two numerical examples also incorporate multiple types of capital.

4. **Numerical Evaluation**

The previous section derives strong qualitative effects of investment irreversibility. Are these effects quantitatively important? This section addresses this question with two simple numerical examples.[8]

In the first example (Table 1), the model of section 2 is specialized to a CES technology which combines two types of capital (k_1,k_2) to obtain an homogeneous output y:

$$y = zA\left[\alpha k_1^{-\rho} + (1-\alpha) k_2^{-\rho}\right]^{-\frac{1}{\rho}}; \tag{27}$$

where z is the shock to total factor productivity, A is the average total factor productivity, $(\rho+1)^{-1}$ is the production elasticity of substitution, and α weighs the relative importance of k_1 and k_2. The capital stocks depreciate at the rates δ_1 and δ_2, so the gross production function is:

$$x = y + k_1(1-\delta_1) + k_2(1-\delta_2). \tag{28}$$

Type 1 capital is irreversible with a survival rate equal to $1-\delta_1$. Type 2 capital is flexible.

The total productivity shock is assumed to take two possible values: a high value in normal times and a low value during a depression. The expected length of normal times is 100 years, while the expected length of a depression is 3.75 years. The latter is the length of the downturn phase of the Great Depression in the United States, which lasted from 1929(2) to 1933(1). The relative value of the shock between normal times and a depression is set so the average rate of decline of output during a depression matches the average rate of decline of output per capita during the downturn phase of the Great Depression.[9] The average total factor productivity is set so average net marginal product of capital is the conventional 0.065.

The parameters of the utility function are the following: The subjective discount factor β is 0.999 (a number close to but below unity). The intertemporal elasiticity of substitution is 0.1873, which allows output in the example to grow at the same rate that output per capita grew in the United States from 1869 to 1985.[10] The coefficient of relative risk aversion γ is 10. This is an intermediate value between those typically used in real business cycles and those required to generate realistic equity premia with the preferences used in this paper.[11]

Realistically, the depreciation rates for the two types of capital are assumed to differ substantially. The depreciation rate for the irreversible from of capital is assumed to be 0.01625. This is the mean of the conventionally used depreciation rates for structures 0.0125 (80 years duration) and basic human capital 0.02 (50 years duration). The depreciation rate for the flexible from of capital is assumed to be 0.15. This is a conventional rate for equipment, and it is also a reasonable rate for skills. The relative importance of the two types of capital α is set so the average rate of depreciation is 0.048.[12]

The remaining parameter is the production elasticity of substitution. This parameter controls how closely together the two types of capital move in equilibrium. Thus, it controls how the irreversibility constraint on type 1 capital affects total investment and so consumption. In this example, this parameter has been chosen so the average rate of decline of consumption during a depression matches the average rate of decline of consumption of non-durables and services per capita during the downturn phase of the Great Depression.[13]

With large rare depressions, investment irreversibility has noticeable effects both on capital accumulation and interest rates. Table 1 shows that the effect on the average propensity to consume is about 1 percent of output (0.4214 for the flexible economy versus 0.4129 for the irreversible economy). Therefore, the irreversible economy invests 1 percent of output more than the flexible economy. This extra capital accumulation leads the irreversible economy to grow at a rate 8 basis points faster than the flexible economy (0.0183 versus 0.0175). The effect on interest rates and risk premia is still stronger. The average interest rate in the flexible economy is 0.0591, and the average risk premium is a mere 39 basis points. In contrast, the average interest rate in the irreversible economy, is 0.0261, and the average risk premium is 392 basis points.

The cyclical pattern of rates of return on the flexible and the irreversible economies is in sharp contrast. In the flexible economy, the expected market rate of return is pro-cyclical due to the persistence of shocks and the pro-cyclical productivity of capital. In contrast, the expected market rate of return in the irreversibile economy is counter-cyclical due to the mean reversion of capital prices. The cyclical pattern of the risk-free rate is pro-cyclical in both economies. However, the cyclical pattern for the irreversible economy is very mild in contrast to the strong cyclical pattern for the flexible economy.

The standard deviations of the rates of growth of output and consumption are large, by comparisson with the historical experience of the United States from 1869 to 1985. For the irreversible economy, the standard deviation of the growth rate of output surpasses the standard deviation of the historical growth rate of per capita GDP (0.0758 versus 0.0547); while the standard deviation of the growth rate of consumption is virtually the same as the standard deviation of the historical growth rate of per capita consumption expenditures on non-durables and services (0.0377 versus 0.0379). This is paradoxical because this example abstracts from all fluctuations except for the Great Depression. The explanation to this paradox is twofold. First, the depth of the Great Depression was measured using quarterlized series as opposed to annual data. Second, the shape of a depression in the model is approximately that of a "U", while the shape of the Great Depression with annual data ressembles more that of a "V". In particular, in the model, at the end of a depression there is a tremendous increases in output and consumption (larger still that the decline at the outset). In contrast, output and consumption in the United States recovered slowly after the Great Depression barely reaching pre-depression levels by 1937. These counter-factual increases at the end of a depression are main contributors to the large standard deviations of output and consumption.

The numerical example 2 aims at evaluating the effects of investment irreversibility with fluctuations in output resembling those of an average business cycle. To this end, the numerical example 2 makes three modifications to the numerical example 1. First, shocks are frequent and mild: the average duration of the normal state is 4 years, and the example replicates the historical standard deviations of consumption and output. Second, a recovery state is added to the model to correct for the counter-factual fast up-turns:[14] during a recovery the shock takes an intermediate value between those it takes in the normal and the low states. And third, shocks are not Hicks neutral.

With the mild declines of output in an average recession, the investment irreversibility constraint would never be binding if the source of recessions were Hicks neutral technological shocks to the aggregate production function. Conterfactually, this would imply that recessions would not be associated with capital losses. For the investment irreversibility constraints to be binding during a typical recession non-neutral shocks are required. The source of this non-neutrality may be sectoral diversity, or it may also be a different nature of the shocks. This numerical example explores this second avenue with shocks to the depreciation rate of the flexible type of capital (skills and equipment). Formally, this is equivalent to a multiplicative shock to the efficiency of skills and equipment in the gross production function. That is, recessions are periods during which the efficiency of skills and equipment is partly lost.

In this second example, the effect of investment irreversibility is quite small on capital accumulation and growth. As Table 2 shows, the irreversible economy grows at a rate 2 basis points faster than the flexible economy. With frequent and mild shocks, the representative consumer responds to the irreversibiliby constraint mainly by diverting investment towards the flexible type of capital, and only secondarily by increasing the overall rate of investment.[15]

The effects of investment irreversibility on the risk-free rate and the risk premium remain important, although they are not as strong as in example 1.[16] In this example, the risk-free rate drops from 0.0537 in the flexible economy to 0.0406 in the irreversible economy. Conversely, the risk premium increases from 77 basis points to 234 basis points. Also, in this example the risk-free rate is counter-cyclical.

Overall, the two numerical examples illustrate that investment irreversibility helps to generate realistic features for asset returns. In addition to high variability of market returns and incongruent cyclicality of interest rates, investment irreversibility generates large risk premia (Mehra and Prescott [1985]), low risk-free rates (Weil [1989]), pro-cyclical risk premia (Ferson and Harvey [1991]), and mean reversion of capital prices (Fama and French [1988]). In both examples, the risk premium in the irreversible economy, even if large by comparison with the risk premium in the flexible economy, is still small by comparison with the 0.06 historical wedge between the real average returns on equity and Treasury bills. Likewise, the risk-free rate in the irreversible economy, even is low by comparison with the risk-free rate in the flexible economy, is still high by comparison to the 0.01 historical average real return on Treasury bills. Higher coefficients of risk aversion could generate these historical averages.[17] However, note that neither the return on equity is the market return, nor the real return on Treasury bills is risk-free. It is beyond the scope of these numerical examples to address these and related issues. The purpose of the examples is to illustrate the type of effects investment irreversibility induces.

5. **Concluding Remarks**

For ease of exposition and analysis, the model presented in this paper has been set as an optimal growth problem. However, as is well known, the solution paths for these type of problems can be decentralized as competitive equilibria with the appropriate transversality conditions. Likewise, the irreversibility constraints are expressed as a simple inequality. However, costly reversion of investment should yield similar results (see Abel and Eberly [1996]). Finally, the irreversibility constraints have been modelled as physical constraints. However, to a certain degree, these constraints can also handle an extreme form of the "lemons problem" which would lead to the disappearance of some markets. Then, the interpretation of the irreversibility constraints would be that buying a car constitutes an irreversible commitment to a particular use for the car, for example being the means of transportation of the buyer. The buyer can still fully diversify the risk involved in this purchase, for example by selling shares on the ownership of the car. But at the end of the day, the only way the shareholders can get any dividend from the car is if the original buyer uses it. If this buyer does not need the car anymore, this is a social loss no matter how much diversification there was. If in addition to investment irreversibility capital markets were imperfect, the "lemons problem" would lead to stronger wealth effects than the ones modelled in this paper. Then, aggregation does not hold in general, and I conjecture that the effects of irreversibility on capital accumulation and interest rates would be even stronger.

APPENDIX

Proof of Proposition 5

As explained, the function v is positive and weakly increasing in z, so the expression $(V'z'_{-1})^{1-\gamma}$ is positive and weakly decreasing in z' if $\gamma \geq 1$. Likewise, q is positive and a weakly increasing function of z, so the expression $\tilde{z}'^{-1}z'$ (see (25)) is positive and weakly decreasing in z'. Therefore, the product $V'^{1-\gamma}\tilde{z}'^{-1}z'^{\gamma}$ is positive and weakly decreasing in z'.

Let $[\underline{z},\overline{z}]$ be the domain of the stochastic variable z, and $G(z',z)$ the distribution function of z' for a given z. Also, let $\psi^F = z'^{-\gamma}/Ez'^{-\gamma}$ and $\psi^I = V'^{1-\gamma}\tilde{z}'^{-1}/EV'^{1-\gamma}\tilde{z}'^{-1}$. Both continget prices ψ^F and ψ^I are functions of z'. From the definition of these prices, E $\psi^F(z')$ = E $\psi^I(z')$ = 1. Therefore, to prove that ψ^I is a mean preserving spread of ψ^F, we must show that

$$\int_{\underline{z}}^{\hat{z}} \psi^I(z')dG(z',z) \; - \; \psi^F(z')dG(z',z) \geq 0, \quad \text{for all } \hat{z} \in [\underline{z},\overline{z}]. \tag{A1}$$

Inequality (A1) holds trivially for $\hat{z} = \underline{z}$ and $\hat{z} = \overline{z}$. Suppose there is a $\hat{z}$ ε $(\underline{z},\overline{z})$ for which inequality (A1) does not hold. This implies the existence of a pair z_0 and z_1 that satisfies: $z_0 \leq \hat{z} < z_1$, $\psi^I(z_0) < \psi^F(z_0)$, and $\psi^I(z_1) > \psi^F(z_1)$. Using the definitions of ψ^F and ψ^I these inequalities imply that the value of $V'^{1-\gamma}\tilde{z}'^{-1}z'^{\gamma}$ for $z' = z_0$ is smaller than its value for $z' = z_1$. This contradicts that $V'^{1-\gamma}\tilde{z}'^{-1}z'^{\gamma}$ is a weakly decreasing function of z'.

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Table 1
NUMERICAL EXAMPLE 1

Gross production function: $x = zA[\alpha k_1^{-\rho} + (1-\alpha)k_2^{-\rho}]^{-1/\rho} + (1-\delta_1)k_1 + (1-\delta_2)k_2$.
Production elasticity of substitution $(1+\rho)^{-1} = 0.0292^{a}$.
Depreciation rate k_1 $(\delta_1) = 0.01625$.
Depreciation rate k_2 $(\delta_2) = 0.15$.
Average depreciation rate = 0.048.
Average net marginal product of capital = 0.065.
Markov process of z has 2 states: Normal (1) and Low (2).
Average value of shocks $z = 1$; low shock $z_2 = 0.5695^{a}\ z_1$.
Expected length of the normal state = 100 years.
Expected length of the low state = 3.75 years.
Transition probabilities matrix =

	0.99	0.2667
	0.01	0.7333

Subjective discount factor $(\beta) = 0.999$.
Intertemporal elasticity of substitution $(\sigma^{-1}) = 0.1873.^{a}$
Coefficient of relative risk aversion $(\gamma) = 10.0$.

	Normal	Low	Mean	Std.
IRREVERSIBLE INVESTMENT MODEL				
c/y	0.4083	0.5359	0.4129	0.0238
g_k	0.0193	-0.0077	0.0183	0.0070
g_y	0.0250	-0.1590^{b}	0.0183^{b}	0.0758
g_c	0.0222	-0.0860^{b}	0.0183	0.0377
q	1	0.6485	0.9874	0.0664
$\ln E\tilde{R}$	0.0617	0.1612	0.0653	0.0571
$\ln r$	0.0266	0.0117	0.0261	0.0055
FLEXIBLE INVESTMENT MODEL				
c/y	0.4149	0.5935	0.4214	0.0333
g_k	0.0186	-0.0107	0.0175	0.0075
g_y	0.0242	-0.1617	0.0175	0.0756
g_c	0.0207	-0.0663	0.0175	0.0271
q	1	1	1	0
$\ln E\tilde{R}$	0.0642	0.0301	0.0630	0.0088
$\ln r$	0.0601	0.0191	0.0591	0.0077

Note: Superscript ^{a} denotes parameters that have been adjusted to match the moments marked with superscript ^{b}. The parameters A and α have been calibrated so the net marginal product of capital and the average depreciation rate are 0.065 and 0.048 respectively along a balanced growth path. The ex-post growth rates of capital, output, and consumption (g_k, g_y, and g_c) are annual and continuously compounded. The remaning variables are: c/y = averaga propensity to consume, q = average price of capital in units of consumption, $E\tilde{R}$ = expected market return conditional on the present period information, and r = risk-free interest rate.

Table 2

NUMERICAL EXAMPLE 2

Gross production function: $x = A[\alpha k_1^{-\rho} + (1-\alpha)(zk_2)^{-\rho}]^{-1/\rho} + (1-\delta_1)k_1 + (1-\delta_2)(zk_2)$.
Production elasticity of substitution $(1+\rho)^{-1} = 0.0851$.[a]
Depreciation rate k_1 $(\delta_1) = 0.01625$.
Average depreciation rate k_2 $(\delta_2) = 0.15$.
Average depreciation rate = 0.048.
Average net marginal product of capital = 0.065.
Markov process of z has 3 states: Normal (1), Low (2), and Recovery (3).
Average value of shocks $z = 1$.
Low shock $z_2 = 0.7736$[a] z_1.
Recovery shock $z_3 = 0.5\ (z_1 + z_2)$.
Expected length of the normal state = 4 years.
Expected length of the low state = 1.4 years.
Expected length of the recovery state = 1 year.
Transition probabilities matrix =

$$\begin{matrix} 0.75 & 0 & 0.9 \\ 0.25 & 0.286 & 0.1 \\ 0 & 0.714 & 0 \end{matrix}$$

Subjective discount factor $(\beta) = 0.999$.
Intertemporal elasticity of substitution $(\sigma^{-1}) = 0.248$.[a]
Coefficient of relative risk aversion $(\gamma) = 10$.

	Normal	Low	Recovery	Mean	Std.
IRREVERSIBLE INVESTMENT MODEL					
c/y	0.3917	0.4090	0.4056	0.3980	0.0079
g_k	0.0300	0.0179	-0.0232	0.0183	0.0251
g_y	0.0433	-0.0766	0.0609	0.0183[b]	0.0547[b]
g_c	0.0346	-0.0482	0.0527	0.0183	0.0379[b]
q	1	0.9060	1	0.9780	0.0427
$\ln E\tilde{R}$	0.0479	0.1050	0.0640	0.0640	0.0683
$\ln r$	0.0249	0.0773	0.0459	0.0406	0.0234
FLEXIBLE INVESTMENT MODEL					
c/y	0.3936	0.4241	0.4027	0.4022	0.0126
g_k	0.0301	0.0176	-0.0245	0.0181	0.0257
g_y	0.0402	-0.0832	0.0804	0.0181	0.0623
g_c	0.0348	-0.0316	0.0275	0.0181	0.0278
q	1	1	1	1	0
$\ln E\tilde{R}$	0.0651	0.0425	0.0743	0.0614	0.0258
$\ln r$	0.0554	0.0384	0.0690	0.0537	0.0098

Note: See the explanation below Table 1.

NOTES

1.See also Caplin and Leahy (1993), Bertola (1994), Ricketts and McCurdy (1995), and Huffman and Wynne (1995), for models with sectoral shocks and irreversible investment.

2.This result is related but distinct from the demand for precautionary savings. As the example at the end of section 3 shows, investment irreversibility not only increases uncertainty, but it affects the expected return of capital as well.

3.Rouwenhorst (1995) and Tallarini (1997) show that production economies with flexible technologies cannot generate realistic asset rates of return. Boldrin, Christiano, and Fisher (1995) discuss the importance of capital gains and losses to produce a realistic variability for the return on the market portfolio. Beaudry and Guay (1996) show that adjustment costs are important to generate a more realistic cyclical pattern of interest rates.

4.One would expect μ_i to be equal to one minus the depreciation rate of capital i. The model allows this possibility, and it also allows μ_i to be determined in some other way due to maintenance costs, partial flexibility, or some exogenous growth. Finally, reversing the inequalities and crossing the subscripts (not done in this paper) handles time to build constraints.

5.This formula is valid for $\sigma \neq 1$ and $\gamma \neq 1$. For $\sigma = 1$, use a Cobb-Douglas aggregator. For $\gamma = 1$, use exp(E log(u')) instead of (E $u'^{1-\gamma})^{1/1-\gamma}$.

6.V is also concave with respect to z if the gross production function is concave with respect to z, and z follows a concave Markov process: $z' = T(z) + \varepsilon'$; where T is a concave function and ε' is a vector of white noise stochastic variables.

7.The conditions for strict inequality between investment in these two economies are those at the end of Proposition 1.

8.The computational algorithm used is similar to the one used by Coleman (1996), and it is described in Christiano and Fisher (1995) as the colocation method.

9.Before 1946, the only national accounts available are annual. To measure the average rate of decline of GDP from 1929(2) to 1933(1), I used seasonally adjusted industrial production to quarterlize the annual GDP figures for the years 1929 and 1933. The sources of these data are: U.S. Department of Commerce Survey of Current Business, Sep. 1994 and Aug. 1995 (annual national accounts). Economic Report of the President, 1996 (population). Annual Report of the Federal Reserve Board, 1994 (industrial production).

10.For a description of the historical United States series of consumption and output see Cecchetti, Lam, and Mark (1990).

11.See Kocherlakota (1996).

12.This is the rate found by Eisner (1988) for the a comprehensive measure of capital in the United States that includes human capital.

13.Before 1946, the only national accounts available are annual. To measure the average rate of decline of consumption of non-durables and services from 1929(2) to 1933(1), I used seasonally adjusted department store sales deflated by the consumer price index to quarterlize the consumption of non-durables and services for 1929 and 1933. The source of the department store sales is Annual Report of the Federal Reserve Bulletin, 1934.

14.The downturns at the begining of a recession are more realistic since historical data shows negative skewness of the rates of growth of output and consumption. Moreover, in this example the downturns are not all that sharp.

15.In fact, if one modifies δ_2 to be 0.25, then the flexible economy grows sligthly faster than the irreversible economy because of the "capital mix" effect.

16.The fact that large rare shocks have a stronger effect on the equity premium is known since Rietz (1988).

17.The coefficients of relative risk aversion required are: 11.3 for the numerical example 1, and 18.7 for the numerical example 2. The main effect of raising the risk aversion parameter is a drop of the risk-free rate. A secondary effect is an increase of the growth rate. The effect on other moments reported in tables 1 and 2 is fairly small if the elasticity of substituion is lowered to maintain the same growth rate.