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Miquel Faig* and Pauline M. Shum**

Abstract

This paper provides a discrete-time framework for analyzing a firm's investment and financial choices under uncertainty. The investment decision is incremental and subject to a parameterized degree of irreversibility. Corporate taxes are asymmetric, but we allow imperfect carry-forward or carry-back of losses. Personal taxes are also levied. The paper focusses on the impact of various tax rules and corporate borrowing constraints on the firm's choices particularly the desired capital stock. The paper also highlights the effects of depreciation, interest rates, and personal taxes. Numerical simulations complement the analytical results to illustrate when taxes and borrowing constraints matter the most.

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I. Introduction

Following the 1986 Tax Reform Act in the United States, the tax system in most developed countries is either being challenged or undergoing reform. In this revisionist movement, the taxation of corporate income has attracted disparate proposals. On the one hand, populist groups demand that profitable corporations pay a larger share. On the other hand, economists advocate a fully integrated corporate and personal income tax system. The argument for integration is grounded on the efficiency gains from taxing all capital income at the same rate.

In the public finance literature, the classic Samuelson (1964) result is that a corporate tax system with full loss offset and full deductibility of economic depreciation is neutral. Of course, it is now well-known that this neutrality result breaks down with an asymmetric corporate tax (i.e., when there is limited loss offset) and uncertainty. However, the size of the distortion on investment is not clear. Based on the assumption that investment is fully flexible, economists have long argued that the tax treatment of corporate income can have large effects on firms' investment behavior. However, there is very little empirical evidence to support this claim (see Chirinko (1993)).[1] This has led some researchers to abandon the traditional approach to modelling investment in favor of a framework that allows investment to be "irreversible".[2]

Dual to the controversy over the effect of corporate taxation on investment is the controversy over the interaction between the firm's financial and investment decisions. In a Modigliani-Miller world, the two are independent. This independence also breaks down with an asymmetric corporate tax and uncertainty (DeAngelo and Masulis (1980)), since both financial and investment decisions affect the firm's marginal tax rate. The extent of the interaction has not been widely examined. In a recent study, Mauer and Triantis (1994) conclude from their numerical stimulations that the effects of alternative financing rules on investment are negligible. However, the generality of their result has not been tested.

The purpose of this paper is to study the two related issues discussed above: 1) the effects of an asymmetric corporate tax system on investment, and 2) the interaction between financial and investment decisions. To make the investment decision more interesting and realistic, we employ a modern model of irreversible investment.

An asymmetric corporate tax system is one in which the government does not treat the firm's profits and losses equally. When taxable income is positive, taxes are payable immediately. When taxable income is negative however, the firm is not entitled to a tax refund at the corporate tax rate. Rather, the government allows the loss to be "carried back" to offset previously taxed income or "carried forward" to offset future income, both with some constraints.[3] While previous theoretical studies model only on the loss carry-forward provision (Mayer (1986), Fane (1987), Keen and Schiantarelli (1991), Shum (1994)), we model both the loss carry-forward and carry-back provisions.

Irreversibility is an important consideration in the modern theory of investment. For example, it may alter tax incentives, or it may make investment especially sensitive to different forms of risk, such as uncertainty over future product prices. Related to this paper, Mackie-Mason (1990) and McKenzie (1994) also investigate the implications of corporate taxes for irreversible investment, but our focus is quite different from theirs. In Mackie-Mason (1990), the investment decision is for a single "on-off" project. In our model, the investment decision is incremental. That is, the firm continues to decide whether or not to add to its capital stock period after period.

In McKenzie (1994), a full loss offset tax system is assumed. In our model, the corporate tax is asymmetric and losses can be carried forward or carried back, though imperfectly. Further, neither of these papers consider a joint financing decision as we do.

Most previous studies of corporate taxation treat investment and financing decisions separately. Three exceptions are Mayer (1986), Dammon and Senbet (1988) and Mauer and Triantis (1994). In these papers, the firm optimizes its financing and investment choices at the same time. Both Mayer (1986) and Dammon and Senbet (1988) assume that investment is completely and costlessly reversible. Using the U.K. imputation corporate tax framework with loss carry-forward only, Mayer demonstrates that the firm's optimal financial policy fluctuates to absorb any shocks to its loss carry-forward, thereby leaving investment in the following period independent of past losses. Dammon and Senbet extend DeAngelo and Masulis (1980) analysis of the impact of an asymmetric corporate tax on the firm's optimal debt policy to include an endogenous investment decision. Finally, Mauer and Triantis (1994) analyze the interaction of the firm's investment and financing decisions in a continuous-time contingent claims framework. In particular, they focus on the roles of production and financial flexibilities in these decisions. Results from their numerical simulations demonstrate that higher production flexibility, as a result of lower cost of shutting down and reopening the production facility, increases debt capacity. Higher financial flexibility, as a result of lower recapitalization costs, also has similar effects. Further, Mauer and Triantis find that production and financial flexibilities are essentially substitutes, and the impact of debt financing on investment is economically insignificant.[4]

This paper is organized as follows. In section II, we explain the general framework and derive the first-order financing and investment conditions. In section III, we discuss analytical results for the simple case in which there is no loss carry-forward or carry-back. In section IV, we extend some of our theoretical findings from section III to the general case with loss carry-forward and carry-back. We also complement these findings with numerical simulations aimed at identifying when corporate taxation matters and when the effects of financial constraints on investment are sizable. Moreover, since some of the comparative static results are ambiguous (due to opposing effects that arise from investment irreversibility), numerical simulations allow us to determine at what parameter values the sign changes. A summary section concludes the paper.

II. The model

Instead of using a continuous-time contingent claims model as in most of the studies mentioned in the introduction, we employ a discrete-time Markovian framework to model the firm's problem. The latter is a more intuitive framework for modelling the annual corporate tax, as well as the dynamic provisions of loss carry-back and carry-forward. It also enables us to use a new methodology to analyze irreversible investment and to obtain new analytical results.

We assume that capital takes one period to build. At the beginning of each period, the firm must make its investment and financing decisions before the price of its product is realized (at the end of the period). The objective of the firm is to maximize:[5]

$$V = \max_{\{k,b\}} E\{i - t - kr(1 - \tau_b) + \beta V'\} \tag{1}$$

where

$V =$ value of the firm (after distributing past profits and before knowing the price of the product).

$V^{/}$ = value of the firm in the next period.

k = capital stock, $k \in K = \mathbb{R}_{+}$.

b = debt (one-period bonds).

i = revenue from production net of capital consumption cost.

t = total corporate and personal tax liabilities.

r = risk-free rate of interest, $r > 0$.

τ_b = personal tax rate on interest income, $\tau_b \geq 0$.

β = discount factor, $\beta \in (0,1)$.

The term $kr(1 - \tau_b)$ in (1) represents the interest cost of capital net of personal taxes, where the unit price of capital is normalized to one. The interest rate on debt is the risk-free rate because there is no risk of bankruptcy.[6]

There are two constraints in the firm's optimization problem:

$$k \geq x_0, \quad \text{and} \tag{2}$$

$$b \leq k. \tag{3}$$

where

x_0 = the stock of capital that survives from the last period. Specially, x_0 is a positive fraction μ (the survival rate) of the last period's capital stock.[7]

The first constraint (2) captures the fact that once a capital investment is made, it is irreversible. The second constraint (3) restricts the firm's borrowing to no more than the value of its capital. We do not impose a lower bound for debt. A negative b simply means that the firm invests in bonds rather than issuing them.

The price of the firm's product, p, is a continuous random variable which follows a Markov process and takes on values within a bounded interval $P = [\underline{p}, \bar{p}] \subset \mathbb{R}_+$. If the price in the last period is p_0, the distribution of p is determined by the transition function $Q(p_0,\mathrm{d}p)$. Q has the Feller property (i.e., if v is bounded and continuous in P, then $\int v(p)Q(p_0,\mathrm{d}p)$ is bounded and continuous for all $p_0 \in P$) and is monotone (i.e., if v is non-decreasing in P, then $\int v(p)Q(p_0,\mathrm{d}p)$ is non-decreasing for all $p_0 \in P$). This monotonic property implies that the stochastic process of p is persistent: the probability of a high p increases with p_0.

The firm's revenue net of capital consumption costs is:

$$i = F(k,p) - \delta k \tag{4}$$

where

$F =$ revenue function.

$\delta =$ capital consumption rate.

The function $F{:}K\mathrm{x}P \rightarrow \mathbb{R}$ is continuously differentiable, bounded, and concave with respect to k. As functions of p, both $F(k,\cdot)$ and $\partial_k F(k,\cdot)$ are increasing. For all $p \in P$, $\lim_{k\rightarrow 0} \partial_k F(k,p) = \infty$, and $\exists\, \bar{k} \in K$ such that $\partial_k F(k,p) \leq \delta$ for all $k \geq \bar{k}$. In general, we do not assume that $F(0,p) = 0$ to allow for fixed production costs that are already committed and/or revenues from other investments that are independent of k.

In models with perfectly flexible investment, depreciation plays only one role - it affects the opportunity cost of capital. When investment is irreversible however, depreciation has an additional effect: a larger depreciation rate relaxes the irreversibility constraint (2), as the firm inherits less capital from the previous period. To distinguish between these two effects, we study the impact of depreciation on the opportunity cost of capital through the capital consumption rate

δ in (4), and the impact of depreciation on the irreversibility constraint through the survival rate μ in (2). In the comparative statics section, we assume that δ and μ are independent parameters in order to analyze the two effects separately. Also, having individual parameters allows for capital consumption costs, such as maintenance costs, which are not negatively related to the survival rate.[8]

To focus on the main issues, we adopt the usual assumption that at the end of each period, the firm distributes *all* of its earnings to shareholders as dividends. Miller (1977), DeAngelo and Masulis (1980) and Dammon and Senbet (1988) assume that the personal tax rate on equity is zero. In our model, we assume that τ_c is the composite tax rate on both corporate income and dividends.

The total corporate and personal tax liability of the firm, t, is:

$$t = \tau_b rb + \tau_c S(c) - \tau_c S(y_0) \tag{5}$$

where

$c =$ $i - rb + y_0$, taxable income.

$y_0 =$ cumulated profits/losses carried over from the previous period.

$\tau_c =$ composite tax rate on corporate income and dividends.

$S =$ truncation function for values lower than zero; that is, S maps a positive number onto itself and a non-positive number onto zero.

The personal tax liability on interest income is $\tau_b rb$. The corporate tax liability is $\tau_c S(c) - \tau_c S(y_0)$. To see this, let us consider all of the possible values of y_0 and c. First, if y_0 is negative, it represents a cumulated loss which is a tax shield to the firm. In this case, corporate taxes are $\tau_c c$ if $c \geq 0$, and zero otherwise. Second, if y_0 is positive, it represents a cumulated profit, and the firm is able to carry back any losses that it may incur in the current period. In this case, the firm gets a refund equals to $\tau_c y_0$ if current losses exceed y_0 (so that $c < 0$), it gets a refund equals to -$\tau_c(i - rb)$ if current losses do not exceed y_0 (so that $c \geq 0$), and it pays taxes equal to $\tau_c(i - rb)$ if the firm earns a profit.

Since carry-forward and carry-back do not accrue any interest and have a time limit, cumulated profits and losses depreciate as time passes. For simplicity, we assume a constant discount factor ρ for both profits and losses. Hence, the cumulated profit from the current period to be carried over to the next period is $y = \rho c$.

The value of the firm, V, is a function of the three state variables x_0, y_0 and p_0. V includes the value of the firm's commitment to a minimum capital stock of x_0 in the following period due to the irreversibility constraint (which alone would be non-positive) and the value of cumulated profits or losses y_0 (which alone would be non-negative). If the firm owns the capital it uses, its market value at the beginning of the period is $x_0 + V$. The value function in the next period, V', is endogenous to the model. If the investment horizon is finite, V' can be easily obtained by solving the problem backward from the last period to the first. If the horizon is infinite however, V' must be obtained using limiting arguments (or numerical calculations), as is done in the next section. In any case, for a given V', the firm's financial and capital decisions can be characterized using standard calculus. (The differentiability and other properties of V' are established in section III).

The first-order condition for the optimal financing of a given k is:

$$\tau_b \leq \tau_c Pr(c \geq 0) - \beta\rho E\{\partial_y V'\} \qquad \text{with equality if } b < k. \qquad \textbf{(6)}$$

where

$\Pr(c \geq 0)$ = probability that $c \geq 0$, that is , probability that the firm can use all of its interest tax shield in the current period.

The L.H.S. of (6) is the marginal cost of an additional dollar of debt interest to bondholders who have to pay \$ τ_b for each extra dollar of interest income. The R.H.S. of (6) is the benefit of an additional dollar of debt interest to the firm from tax sheltering. This marginal benefit is the expected savings on corporate taxes minus (plus) the expected discounted value of an additional dollar of cumulated profits (losses) carried over to the next period. If the marginal tax cost and benefit of debt are equal, the choice of b is interior. If the marginal cost is lower than the marginal benefit, the firm finances all of its capital investment with debt.

The optimal condition for capital investment is:

$$E\{R\partial_k F\} \leq (r + \delta)E\{R\} - \beta\mu E\{\partial_x V'\}, \quad \text{with equality if } k > x_0. \tag{7}$$

where

R = $1 - \tau_c L(c \geq 0) + \beta\rho E\partial_x V'$ (marginal retention rate or one minus the marginal effective tax rate); and

L = indicator function for positive values; that is, it maps positive numbers on to one, and non-positive numbers on to zero.

The L.H.S. of (7) is the expected after-tax marginal revenue of capital. Note that because of capital irreversibility and the provisions of loss carry-forward and carry-back, the firm's investment decision is a dynamic problem. The R.H.S. of (7) is the expected marginal cost of capital, which includes the expected after-tax interest and capital consumption costs of capital, minus the expected marginal value of capital in the next period. The last term arises from the irreversibility constraint, and will be discussed in more detail in the next section. If the expected after-tax marginal revenue is lower than the expected marginal cost at x_0, the firm will not invest because k is constrained to be greater than or equal to x_0.

III. Optimal policies without loss carry-forward and carry-back

The first-order conditions (6) and (7) determine the firm's optimal debt, $\hat{b}$, and capital, $\hat{k}$, respectively. However, in order to describe $\hat{b}$ and $\hat{k}$ precisely, we must first characterize the value function in the next period, V'. The task of this section is to simultaneously characterize V' and the optimal policies $\hat{b}$ and $\hat{k}$ for an infinite horizon. As a first pass, we assume no loss carry-forward and carry-back provisions in this section. This implies that y_0 is no longer a state variable in the value function. We also assume in this section that the discount factor, β, is an independent parameter. We do so to distinguish the direct effects that τ_b and r have on $\hat{b}$ and $\hat{k}$ from the indirect effects that they have through β.

Let T be the operator that maps next period's value function, V', onto this period's value function, V, as implied by the maximization problem (1). Standard results from stochastic dynamic programming (see Stokey, Lucas, and Prescott (1989) sections 9.1 and 9.2, pp. 241-270) are:

PROPOSITION 1: *T* is a contraction mapping in the space of continuous and bounded functions with the sup norm, so this mapping has a unique fixed point V^* which is the value function of the firm with an infinite horizon. For all $p_0 \in P$, $V^*(\cdot, p_0)$ is continuously differentiable, bounded, non-increasing, and concave in *K*. And, for all $x_0 \in K$, $V^*(x_0, \cdot)$ is increasing in *p*.

Using the structure of the model, we can characterize the value function V^* more precisely, especially its first derivative with respect to x_0 . This derivative is particularly important for the firm's investment decision since it affects $\hat{k}$. Before we begin, let $\bar{k}$ be the capital stock which satisfies $\partial_k F(\bar{k},\bar{p}) = \delta$, where δ is the capital consumption rate introduced in (4). Also, let $\mathscr{F}$ be the set of real functions which in the domain $[0,\bar{k}]x[\underline{p},\bar{p}]$, satisfies the properties of V^* in proposition 1, and that their first derivatives with respect to k are bounded above by 0 and below by $J = -(r + \delta) / \mu(1 - \beta)$. The rest of this section is organized as follows. First, we characterize the firm's financing and investment decisions with the conjecture $V' \in \mathscr{F}$. Second, we prove that the firm's value function $V^* \in \mathscr{F}$. Third, we analyze how $\partial_x V^*$ is affected by p_0 and by various model parameters.

We begin with the firm's optimal financing decision for a given k. The first-order condition (6) without loss carry-forward and carry-back is:

$$\Pr(c \geq 0) \geq \frac{\tau_b}{\tau_c}, \qquad \text{with equality if } \hat{b} < k; \tag{8}$$

where $c = F(k,p) - \delta k - r\hat{b}$. To understand the implications of this condition, let p^* be the price level that satisfies:

$$\Pr(p \geq p^*) = \frac{\tau_b}{\tau_c}; \tag{9}$$

(p^* depends on p_0). Condition (8) implies that if $\hat{b}$ is interior (less than k), then $\Pr(c \geq 0) = \tau_b/\tau_c$. Hence, since c is an increasing function of the continuous random variable p, if $\hat{b}$ is interior, then $c = 0$ when $p = p^*$. In an interior debt solution, the firm's economic profit evaluated at p^*, $F(k,p^*)$

- δk - rk, is less than c (because $\hat{b} < k$), and so it is negative. Conversely, (8) and (9) imply that if the financing decision is at the corner where $\hat{b} = k$, then the set of prices that yields $c \geq 0$ contains the interval $[p^*,\bar{p}]$. Hence, when $\hat{b} = k$, $F(k,p^*) - \delta k - rk \geq 0$. Therefore, the optimal debt choice is interior if and only if $F(k,p^*) - \delta k - rk < 0$.

The above analysis is illustrated in graph 1 (p. 31). Note that the economic profit curve, $F(k,p^*) - \delta k - rk$, is concave with respect to k and has an interior maximum. Consequently, the subset of K for which the financing decision is constrained at $\hat{b} = k$ is either a bounded interval or empty (if the entire function lies below zero). Intuitively, $\hat{b}$ is at the corner where $\hat{b} = k$ in the interval of k where the firm is most profitable, and seeks to shelter its income with as much debt as possible. On the other hand, if the firm's income is low enough to exhaust the tax benefit of debt, it will choose an interior $\hat{b}$, which implies $\Pr(c \geq 0) = \tau_b/\tau_c$ from (8). In this case, $\hat{b} = [F(k,p^*) - \delta k] / r$, since c evaluated at $p = p^*$ equals to zero. As illustrated in graph 1, the profitable region of k that is associated with a corner debt solution is in the middle section.

The optimal financing policy, $\hat{b}$, determines the effective tax rates that the firm faces for each realization of p. Therefore, the portion of profit the firm retains after tax, R (see the definition below equation (7)), depends on k, p, and the other determinants of $\hat{b}$ such as p_0. Note that since $\hat{b}$ is independent of V', so is R.

To illustrate the optimal investment choice graphically, let $\phi(k,p_0)$ be:

$$\phi(k,p_0) = \mathrm{E}\Big\{\beta\mu\partial_x V'(\mu k,p) + \Big[\partial_k F(k,p) - \delta - r\Big]R(k,p,p_0)\Big\} \tag{10}$$

where the expectation is taken with respect to p conditional on p_0. Using (10) and the first-order condition (7), the optimal capital stock $\hat{k}$ is characterized by:

$$\phi(\hat{k},p_0) \leq 0, \text{ with equality if } \hat{k} > x_0 \qquad \textbf{(11)}$$

Condition (11) is illustrated in graph 2 (p. 32). As a function of k, ϕ is continuous,[9] decreasing,[10] positive at a neighborhood of 0, and negative at $\bar{k}$. Therefore, there is a k^* for which $\phi(k^*,p_0) = 0$, where k^* is the desired capital stock. The optimal capital is $\hat{k} = \max\{x_0, k^*\}$. Intuitively, the function ϕ represents the expected net marginal value of capital for a given k. As long as ϕ is positive, the firm gains by choosing a larger capital stock. If ϕ is negative however, the firm regrets having inherited such a large capital stock from the past. So the firm is either constrained in the current period by the inherited capital stock, or it chooses the capital stock k^* at the point where $\phi = 0$. In our comparative static analysis, we will focus on k^*, since once k^* is determined, $\hat{k}$ is determined.

The Envelope Theorem together with the optimal capital choice implies:

$$\partial_x V(x_0,p_0) = \min\{0,\phi(x_0,p_0)\} \qquad \textbf{(12)}$$

This equation is also illustrated in graph 2 (p. 32). If the inherited capital stock, x_0, is less than k^*, the irreversibility constraint $k \geq x_0$ is not binding, and the marginal value of past capital commitments is zero. Conversely, if x_0 exceeds k^*, $\hat{k} = x_0$. In this case, each additional unit of x_0 increases the inherited capital stock by μ, and so the marginal value of past capital commitment is $\partial_x V(x_0,p_0) = \phi(x_0,p_0)$.

We can see from (10) that the unconstrained capital stock k^* falls short of the capital stock which equates the expected marginal product of capital to the capital consumption rate plus the real interest rate (Jorgenson (1963)) for two reasons. First, $\beta\mu E\partial_x V' \leq 0$. Because capital is irreversible, investment carries the extra cost of increasing future capital commitments. Hence, a larger k reduces the firm's value in the next period. Second, an asymmetric corporate tax system taxes the firm more heavily during good times when both the marginal product of capital and marginal taxes are high than it subsidizes the firm during bad times when both the marginal product of capital and marginal taxes are low. In other words, $\partial_k F(k,p) - \delta - r$ and $R(k,p,p_0)$ are negatively correlated. A negative covariance and $R \leq 1$ imply that:

$$E[(\partial_k F(k,p) - \delta - r)R(k,p,p_0)] \leq E[\partial_k F(k,p) - \delta - r]. \tag{13}$$

As shown in graph 2, the irreversibility of capital and the asymmetry of the corporate tax system imply that $\phi(k,p_0) \leq \mathrm{E}[\partial_k F(k,p) - \delta - r]$, and so k^* falls short of the capital stock which equates $\mathrm{E}[\partial_k F(k,p) - \delta - r]$ to zero.

We turn now to establish that $V^* \in \mathcal{F}$:

PROPOSITION 2: If $V' \in \mathcal{F}$, then $V \in \mathcal{F}$. Moreover, the sequence $\partial_x(T^n V')$ converges uniformly to $\partial_x V^*$. As a result, the firm's value function $V^* \in \mathcal{F}$.

<u>Proof</u>

Let S the set of feasible states: $S = \{(k,p) \in K\mathrm{x}P \mid k \leq \bar{k}\}$; to abbreviate an element of S will be denoted s. The proof has two parts:

1. **If $V' \in \mathcal{F}$, then $V \in \mathcal{F}$:** Because of proposition 1, we must just prove the bounds on $\partial_x V(s)$ for all $s \in S$. The bound $\partial_x V(s) \leq 0$ follows directly from (12). Since $\partial_x V' \geq J$ by assumption, and $(\partial_k F - \delta - \mathrm{r})R \geq (1 - \beta)\mu J$ by the definition of J together with $0 \leq R \leq 1$ and $\partial_k F \geq 0$, (12) also implies the other bound: $\partial_x V(s) \geq \beta\mu J + (1\text{-}\beta)\mu J = \mu J \geq J$. Therefore, $V \in \mathcal{F}$.
2. **The sequence $\partial_x(T^n V')$ converges uniformly to $\partial_x V^*$:** This result follows from the

boundedness and convexity of the capital stocks in S, and the concavity and the differentiability of V^* and all the members in the sequence T^nV' as a function of k (Rockafellar 1970, Theorem 25.7 p. 248)■

The convergence result in proposition 2 is now used to characterize how $\partial_x V^*$ and the optimal capital choice depends on p_0 and the model parameters $(\beta,\mu,r,\delta,\tau_c,\tau_b)$:

PROPOSITION 3: In the absence of an asymmetric corporate tax, the function $\partial_x V^*(x_0,p_0;\beta,\mu,r,\delta)$ is non-decreasing with respect to p_0 and non-increasing with respect to β, μ, r, and δ. Moreover, the desired capital k^* is increasing with p_0 if Q is strictly monotone, it is non-increasing with respect to β and μ, and it is decreasing with respect to r and δ.

Proof

Let V' belong to the subset of $\mathscr{F}$ which satisfies the property that $\partial_x V'$ is non-decreasing with respect to p. The monotonicity of the transition function Q (see p.6) implies that $E(\beta\mu\partial_x V' + \partial_k F - \delta - r)$ is non-decreasing with respect to p_0. Thus, the Envelope Theorem (12) implies that $\partial_x V$ is non-decreasing with respect of p_0. Recursively, the whole sequence $\partial_x(T^nV')$ is non-decreasing with respect to p_0. Further, since $\partial_x(T^nV')$ converges uniformly to $\partial_x V^*$, $\partial_x V^*$ is non-decreasing with respect to p_0. Consequently, for a firm with an infinite horizon, the $\phi(k,p_0)$ line in graph 2 shifts (weakly) up as p_0 increases, and the optimal capital choice k^* either increases or remains constant.

Analogously, let $V' \in \mathscr{F}$ and $\partial_x V'$ be non-increasing with respect to β, μ, r and δ. The expression $E(\beta\partial_x V' + \partial_k F - \delta - r)$ is non-increasing with respect to β, r, μ, and δ. Thus, the Envelope Theorem (12) implies that $\partial_x V$ is non-increasing with respect of β, r, μ, and δ.

Recursively, the whole sequence $\partial_x(T^nV^{\prime})$ is non-increasing with respect to β, r, μ, and δ. Further, since $\partial_x(T^nV^{\prime})$ converges uniformly to ∂_xV^*, ∂_xV^* is non-increasing with respect to β, μ, r, and δ. As a result, for a firm with an infinite horizon, as either β, μ, r, or δ increases, the $\phi(k,p_0)$ line in graph 2 shifts (weakly) down, and the optimal capital choice k^* either falls or remains constant.

If Q is strictly monotone, $E\partial_kF$ is increasing with p_0. Now, the line $\phi(k,p_0)$ shifts strictly up as p_0 increases and k^* increases. Similarly, the line $\phi(k,p_0)$ shifts strictly down as either r or δ increase, so k^* falls ■

If the discount factor, β, and the real interest rate, r, were negatively related (for example, $\beta = [1 + r(1 - \tau_b)]^{-1}$) instead of being independent, proposition 3 implies that r has an uncertain effect on k^*. On the one hand, a rise in r raises the opportunity cost of capital, which tends to reduce the optimal capital stock. On the other hand, a rise in r reduces β, which tends to increase the optimal capital stock as the firm discounts more heavily the negative expected marginal value of capital in the next period ($E\partial_xV^{\prime}$). In other words, a smaller discount factor reduces the negative impact of having to commit to a larger capital stock in the future. This ambiguous effect of the real interest rate on the optimal capital stock for *marginal* investment decisions parallels the ambiguous effect it has for the option to invest discussed in Dixit and Pindyck (1994).

In a similar vein, if the capital survival rate, μ, and the capital consumption rate, δ, were negatively related (for example, $\mu = 1 - \delta$) instead of being independent, proposition 3 implies that δ has an ambiguous effect on k^*. On the one hand, a rise in δ increases the opportunity cost of capital, which tends to reduce k^*. On the other hand, a rise in δ reduces the survival rate of capital, which tends to increase k^* as the irreversibility constraint (2) is relaxed.

PROPOSITION 4: Suppose $F(0,p) \geq 0$ for all p. The function $\partial_x V^*(x_0,p_0;\beta,\mu,r,\delta,\tau_b,\tau_c)$ is non-decreasing with respect to p_0 and τ_b, and non-increasing with respect to β, μ, r, and δ. The function $\partial_x V^*(x_0,p_0;\beta,r,\mu,\delta,\tau_b,\tau_c)$ / $(1-\tau_c)$ is non-increasing with respect to τ_c. Moreover, the desired capital k^* is increasing with p_0 if Q is strictly monotone, it is non-decreasing with respect to τ_b, it is non-increasing with respect to β, μ, and τ_c, and it is decreasing with respect to r and δ.

Proof

If $F(0,p) \geq 0$ for all p, then the economic profit schedule $F(k,p) - \delta k - rk$ in graph 1 crosses the vertical axis at or above the origin: The firm has no losses at low levels of capital. Consequently, $R = 1$ only if k is so large that $\partial_k F - \delta - r < 0$.

Since p^* is defined to satisfy $\Pr(p \geq p^*) = \tau_b/\tau_c$ and the transition function Q is monotonic, p^* is non-decreasing with respect to p_0 and τ_c, and non-increasing with τ_b. Therefore, with the help of graph 1, we can easily check that R is a non-increasing function of p, p_0 and τ_b, a non-decreasing function of r and δ, and an independent function of β, μ, and $\partial_x V'$. Moreover, R / $(1 - \tau_c)$ is an non-decreasing function of τ_c. Define $\psi(k,p_0,\hat{p}_0)$ to be:

$$\psi(k,p_0,\hat{p}_0) \ = \ \mathrm{E}\Big\{\beta\mu\partial_x V'(\mu k,p) \ + \ \Big[\partial_k F(k,p) - \delta - r\Big] R(k,p,\hat{p}_0) \ \ |p_0\Big\} \tag{14}$$

Let V' belong to the subset of $\mathscr{F}$ which satisfies the property that $\partial_x V'$ is non-decreasing with respect to p. Since the step function R is non-increasing with respect to p and falls from 1 to $1-\tau_c$ only if $\partial_k F - \delta - r < 0$, the term inside the expectation in (14) is non-decreasing with respect to p. Thus, the monotonicity of the transition function Q implies that ψ is non-decreasing with respect to p_0. Likewise, since R is non-increasing with respect to $\hat{p}_0$ and falls from 1 to $1-\tau_c$ only if $\partial_k F - \delta - r < 0$, the function ψ is also non-decreasing with respect to $\hat{p}_0$. Consequently, since $\phi(k,p_0) = \psi(k,p_0,p_0)$ and ψ is non-decreasing with respect to the second and third arguments, ϕ is non-decreasing with respect to p_0. Thus, the Envelope Theorem (12) implies that $\partial_x V$ is non-decreasing with respect of p_0. Recursively, the whole sequence $\partial_x(T^n V')$ is non-decreasing with respect to p_0. And since $\partial_x(T^n V')$ converges uniformly to $\partial_x V^*$, $\partial_x V^*$ is non-decreasing with respect to p_0. As a consequence, for an infinite horizon firm as p_0 increases the $\phi(k,p_0)$ line in graph 2 will shift (weakly) up, and so k^* will either increase or remain constant.

Let $V' \in \mathscr{F}$ and $\partial_x V'$ be non-increasing with respect to r. Since the step function R is non-decreasing with respect to r and jumps from $1-\tau_c$ to 1 only if $\partial_k F - \delta - r < 0$, the expression $E[\beta\mu\partial_x V' + (\partial_k F - \delta - r)R]$ is decreasing with respect to r. Thus, the Envelope Theorem (12) implies that $\partial_x V$ is decreasing with respect of r. Recursively, the whole sequence $\partial_x(T^n V')$ is decreasing with respect to r. Further, since $\partial_x(T^n V')$ converges uniformly to $\partial_x V^*$, $\partial_x V^*$ is non-increasing with respect to r. As a consequence, for a firm with an infinite horizon, as r rises the $\phi(k,p_0)$ line in graph 2 will shift down, and so k^* falls. The effects of δ, τ_b, and τ_c as stated in the proposition follow from similar proofs. Finally, the proof for the effects of β and μ on $\partial_x V$ and k^* is analogous to proposition 3 since these two parameters do not affect R■

If the discount factor, β, and the personal income tax rate, τ_b, were positively related (for example, $\beta = [1 + r(1-\tau_b)]^{-1}$) instead of being independent, proposition 4 implies that τ_b has an uncertain effect on k^*. On the one hand, a rise in τ_b reduces the gap between the taxation of corporate and personal capital, which tends to increase the optimal capital stock invested in the corporate sector. On the other hand, a rise in τ_b increases β. This tends to reduce the optimal capital stock because the firm will discount less heavily the negative expected marginal value of capital in the next period ($E\partial_x V^{/}$), i.e., the firm will put more weight on the negative impact of having to commit to a larger capital stock in the future.

IV. The general case

When losses can be carried back or forward, propositions 1 and 2 are generalized by:

PROPOSITION 5: Let T be the operator that maps next period's value function, $V^{/}$, onto this period's value function V as implied by the maximization problem (1). T is a contraction mapping in the space of continuous and bounded functions with the sup norm, so this mapping has a unique fixed point V^* which is the value function of the firm with an infinite horizon. The function V^* can be decomposed as:

$$V^*(x_0,y_0,p_0) = \tau_c S(y_0) - \tau_b y_0 + W^*(x_0,y_0,p_0); \tag{15}$$

where, in the range of admissible values for its arguments, the function W^* is continuously differentiable, bounded, non-increasing and concave with respect to x_0 and y_0, and increasing with respect to p_0. Moreover, the first derivatives of W^* are bounded: $-(r + \delta) / \mu(1 - \beta) \leq \partial_x W^* \leq 0$, and $-(\tau_c - \tau_b) \leq \partial_y W^* \leq 0$. Finally, for all W which satisfies the properties stated about W^*, the sequence $[\partial_x W^n, \partial_y W^n]$ where $W^n(x_0,y_0,p_0) = T^n[\tau_c S(y_0) - \tau_b y_0 + W(x_0,y_0,p_0)]$ converges uniformly to $[\partial_x W^*, \partial_y W^*]$.

The complexity of the general case in which losses can be carried back or forward makes the derivation of monotonicity results analogous to propositions 3 and 4 cumbersome, if not impossible. To complement the theoretical findings, we employ numerical simulations to examine the impact of several important factors on the firm's optimal decisions. Also, since the effects of depreciation and the interest rate are ambiguous (due to opposing effects that arise from investment irreversibility), numerical simulations help us determine at what parameter values the sign changes. In the following subsections, we focus on the impact of 1) investment irreversibility, 2) the rate of depreciation, 3) the interest rate, 4) cumulated profits or losses, 5) the carry-forward and carry-back provisions, 6) the personal tax rate, 7) constraints on debt financing, and 8) the asymmetric corporate tax rate.

In the simulations, the revenue function is: $i = pAk^{1-\alpha} - \delta k - g$; where A and α are parameters of the production function, and g is a pre-committed fixed cost. The stochastic process of p is the following martingale: given last period's price p_0, the price in the current period, p, increases to θp_0 with probability π_1, remains constant with probability π_2, or falls to $\theta^{-1}p_0$ with probability π_3, where $\theta > 1$.[11] Further, the probabilities are parameterized so that the expected value of p is p_0.

In Table 1 (p. 31), we show the parameter values that are used in the simulations. Only a few of the values merit explanations. We pick $A = 2.8$ so that $k^* = 100$ if investment is completely flexible and the tax system is neutral (corporate income is taxed symmetrically and at the same rate as other capital income). Similarly, we pick $g = 13$, so that $V(0,0,1) = 0$ given the values of the other parameters in the base scenario. Finally, to emphasize on the consequences of capital irreversibility, we pick the survival rate μ to be 1 (so the 0.1 capital consumption rate must be understood as maintenance costs).

The desired capital stock k^* in the base scenario is 59. That is, the imposition of the irreversibility constraint and asymmetric taxation leads to a drop of 41 percent of the capital stock. This drop is mainly due to irreversibility, which, even with neutral taxation, reduces the desired capital stock to 63.4. Still, asymmetric taxation has a non-trivial effect on investment. When neutral taxation is replaced by the asymmetric system described in Table 1, the desired capital stock drops by 4 percent (from 100 to 96) if investment is flexible, and it drops by 7 percent (from 63.4 to 59) if investment is irreversible.

Effects of investment irreversibility

[Graph 3, p. 33]

The effect of irreversibility on the desired capital stock declines very fast as the survival rate departs from 1. Once the survival rate is below 0.9, most of the effects of the irreversibility constraint disappear. A survival rate of 0.9 gives to the firm enough flexibility to adjust most of its capital stock as product prices vary over time.

Effects of the depreciation rate

[Graph 4, p.34]

The rate of depreciation affects the firm's desired capital stock through the irreversibility constraint and the opportunity cost of capital. As discussed in the section III, the impact of depreciation through these two channels provide opposite investment incentives for the firm, so that the overall effect of depreciation on the desired capital stock is ambiguous. Graph 4 illustrates this ambiguity. In the simulations reported in graph 4, the survival rate is 1 minus the depreciation rate, while the capital consumption rate is a fixed 10 percent (due for example to maintenance costs) plus the depreciation rate. For high depreciation rates, the negative effect of depreciation through a higher cost of capital is dominant, so that a rise in the depreciation rate reduces the desired capital stock. In contrast, for low depreciation rates, the positive effect of depreciation through a lower survival of capital is dominant, so that a rise in the depreciation rate actually raises the desired capital stock!

The firm's optimal debt decreases with a higher rate of depreciation. This is a result of the tax shield substitution effect (DeAngelo and Masulis (1980)) between debt interest and depreciation allowances.

Effects of the interest rate

[Graph 5, p.35]

The effect of the interest rate on the desired capital stock is also ambiguous. As discussed in the section III, interest rates affect investment in opposite directions through the discount factor and the opportunity cost of capital. Graph 5 illustrates this ambiguity. For high interest rates, the negative effect of a higher interest rate through a higher cost of capital is dominant, so that a rise in the interest rate reduces the desired capital stock. In contrast, for low interest rates, the positive effect of a higher interest rate through heavier discounting of potential future losses due to an excessive inherited stock of capital is dominant. So a rise in the interest rate actually increases the desired capital stock!

The firm's optimal debt declines with a higher rate of interest because a smaller debt level can generate the same amount of interest expense when the interest rate goes up.

Effects of cumulated profits or losses

[Graph 6, p.36]

When the firm has cumulated losses, it has an incentive to generate more income in order to write off these losses. In this sense, the provision of loss carry-forward acts as an "automatic stabilizer". It stimulates investment in capital when the firm has cumulated losses, presumably during recessionary times. However, the size of these effects is rather small: 2 percent (from 59.6 to 58.4) when cumulated profits go up from -4 to 5.[12]

The effect of cumulated profits on the financial structure of the firm is much stronger. Debt and cumulated losses are tax shield substitutes, so the optimal debt increases with cumulated profits until the constraint on debt is binding.

Effects of the loss carry-forward and carry-back provisions

[Graph 7, p.37]

The ability of the firm to carry losses forward and back reduces the asymmetry of corporate taxation. As losses carried forward or back are discounted less heavily, that is, as the discount factor ρ increases, the desired capital stock rises. Graph 7 shows that this effect, while perceptible, is not very strong. As ρ increases from 0 to 1, k^* rises from 58.1 to 60.4, amounting to only a 4 percent increment.[13]

As for the firm's financial decision, ρ has a positive impact on debt as long as this choice is interior. This positive impact is stronger when losses have been incurred in the past (not shown). The reason is that in this case the firm has little incentive to use debt unless the corporate tax system is relatively symmetric, so that its interest tax shield is less likely to be wasted.

Effects of the personal income tax rate

[Graph 8, 38]

As discussed in the previous section, the tax rate on personal income τ_b has two contradictory effects on the desired capital stock, k^*. An increase in τ_b reduces the relative tax rate on corporate income, τ_c - τ_b, encouraging investment in the corporate sector. At the same time, an increase in τ_b lowers the firm's discount rate, thus making the future more important. If capital is irreversible, the firm is more cautious in its investment as it discounts less heavily the negative impact of committing to a larger capital stock in the future. Our simulation results show that for the parameters considered, the overall impact of τ_b on k^* is negative. If τ_b rises from 0 to 60 percent, k^* falls from 64.8 to 53.7. Hence, the influence of the irreversibility constraint outweighs the tax effects.

The tax on personal income is the only disadvantage of using debt to finance the firm's investment. Hence, an increase in the personal income tax rate, τ_b, is expected to have a negative impact on $\hat{b}$. However, this impact might be offset by the fact that a higher τ_b will reduce the discount rate, thereby increasing the present value of the interest tax shield. Graph 8 illustrates that for our parameter values debt and the personal tax rate are negatively related. The discontinuous drop in debt at τ_b = 0.48 is due to the discretization of the stochastic process of the output price.

Effects of constraints on debt financing

In this subsection, we investigate the robustness of the Mauer and Triantis (1994) result that the ability to issue debt has a negligible impact on investment in a multiperiod framework.[14] In our model, this ability is parameterized by *M*, the maximum amount of debt that a firm can borrow as a fraction of its capital stock. (This type of constraints are popular covenants in bond contracts.)

[Graph 9, 39]

The willingness to borrow depends primarily on the profitability of the firm. Graph 9 illustrates how the effect of financial constraints varies for different levels of the fixed cost, g. The significance of the borrowing constraints decreases as fixed costs rise. With large fixed costs, the desired capital stock, k^*, becomes less responsive to changes in the debt constraint. In fact, when $g = 25$, k^* is *invariant* with respect to the portion of investment which can be financed with debt. The reason is that when fixed production costs are large, the firm will always opt for an interior debt solution by holding bonds. Accordingly, the borrowing constraint becomes irrelevant. In contrast, with small fixed costs the firm has more taxable income, and hence it places a greater value on the interest tax shield. In this case, the borrowing constraint does matter for the desired capital and the size of the effect is quite strong: For $g = 0$, as the maximum debt/capital ratio rises from 0 to 1, k^* increases from 44.4 to 66.4. In conclusion, financial constraints do exert a significant impact on the desired capital stock in some instances, in our model this happens when firms are highly profitable.

Effects of the asymmetric corporate tax rate

[Graph 10, p.40]

Asymmetric corporate taxation affects negatively the desired capital stock, k^*. Graph 10 illustrates how the desired capital stock is affected by variations in the corporate tax rate τ_c and the constraints on debt financing. Increases in the corporate tax rate have a weak effect on k^* if investment can be fully debt financed (if the maximum debt-to-capital ratio is one): as τ_c rises from 0.3 to 0.8, k^* falls by 9 percent. In contrast, if restriction is placed on debt financing, the effect of τ_c on k^* becomes stronger overall. In the extreme, if the firm is not allowed to issue any debt, k^* falls by 31 percent as τ_c rises from 0.3 to 0.8. Curiously though, there are local non-monotonic regions which can only be ruled out in the absence of the fixed cost.

V) Conclusion

This paper provides a discrete-time framework for analysing irreversible investment under uncertainty. The framework is flexible enough to incorporate a very general stochastic process for output prices, time to build, asymmetric corporate taxes, imperfect carry-back and carry-forward of tax losses, and corporate financial constraints. The firm's investment decision also differs from previous research on irreversible investment in that it is incremental.

In the paper, we highlight the counter-intuitive effects that arise when investment is irreversible. Assuming fairly "realistic" parameter values, we demonstrate in one example that the desired capital stock may actually increase with a higher real interest rate. In another, which we believe has never been provided before, we show that a larger depreciation rate may actually increase the desired capital stock.

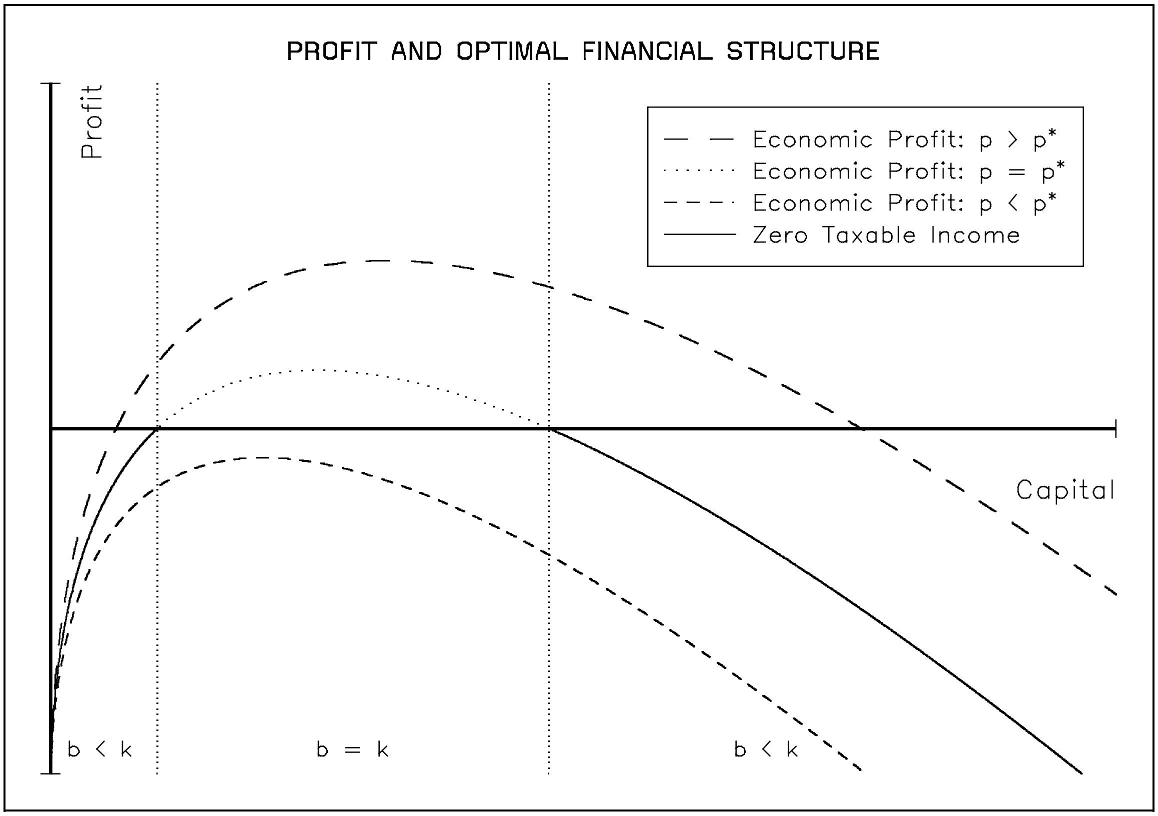
Finally, the paper inquires when asymmetric corporate taxation and corporate financial constraints matter. As it turns out, the two issues are closely related. When financial constraints on corporate debt are likely to be binding in the near future, the corporate tax rate has a strong impact on the desired capital stock. Vice-versa, when financial constraints on corporate debt are not likely to be binding, i.e. when the debt/equity choice is interior, the corporate tax rate has a weak impact on the expected costs and benefits of capital, and so on the desired capital stock. Finally, the tightening of the corporate financial constraints, for example by reducing the maximum amount of debt that can be issued as a portion of the capital stock, has, by itself, a strong impact on the desired capital stock when firms are profitable and/or corporate tax rates are high.

Table 1

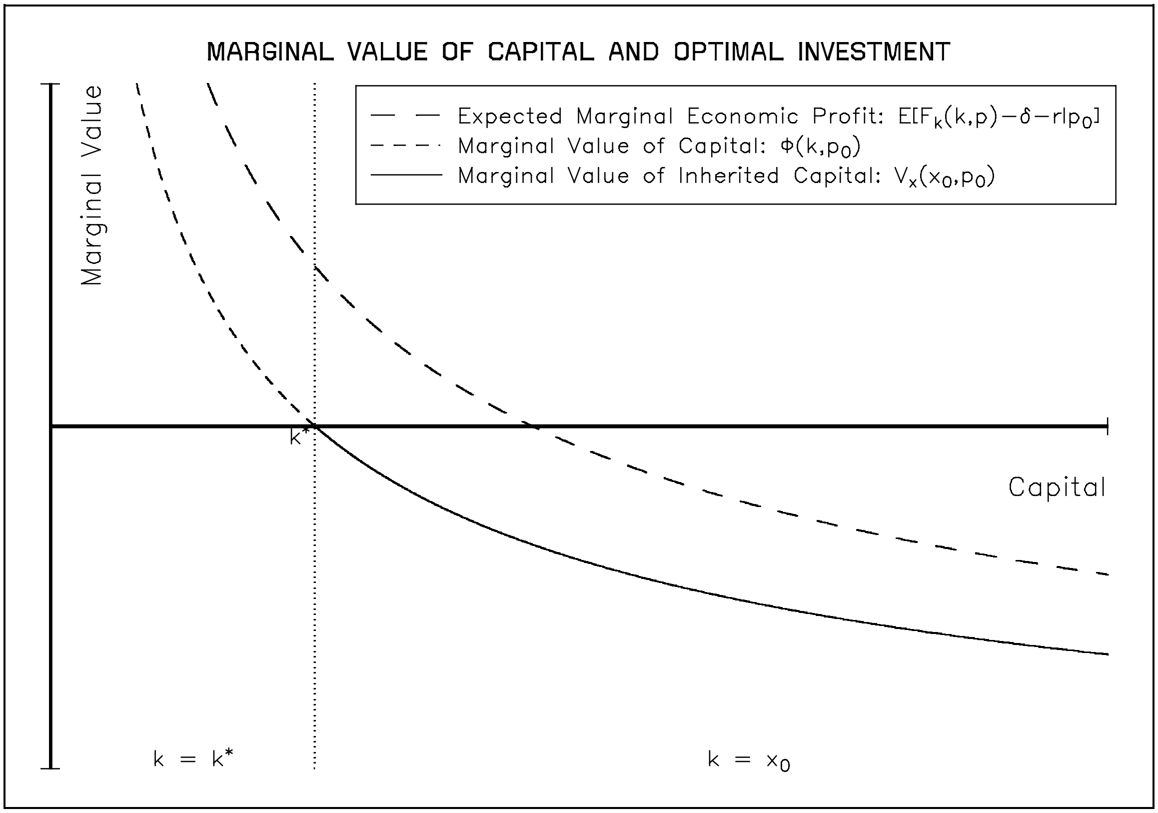
Parameter values for the numerical simulations

Parameter	**Base Value**	**Values in experiments**
Real interest rate, r	0.04	0.01 to 0.11
Survival rate of capital, μ	1.00	0.8 to 1
Capital consumption rate, δ	0.10	0.1 to 0.3
Parameters in revenue function		
α (capital elasticity)	0.50	-
A (normalization)	2.80	-
g (fixed cost/profitability of firm's other activities)	13	0 to 25
Shock to output price, θ	1.10	1.00 to 1.20
Probability of no shock, π_2	0.5	-
Initial output price, p_0	1	-
Personal income tax rate, τ_b	0.3	0 to 0.60
Combined corporate and personal capital gains tax rate, τ_c	0.6	0.30 to 0.80
Percentage of loss carried back or forward, ρ	0.6	0 to 1
Maximum debt (as a fraction of capital)	1	0 to 1
The discount factor, β	$1/[1 + r(1 - \tau_b)]$	-
Initial cumulated profits/losses, y_0	0	-4 to 5

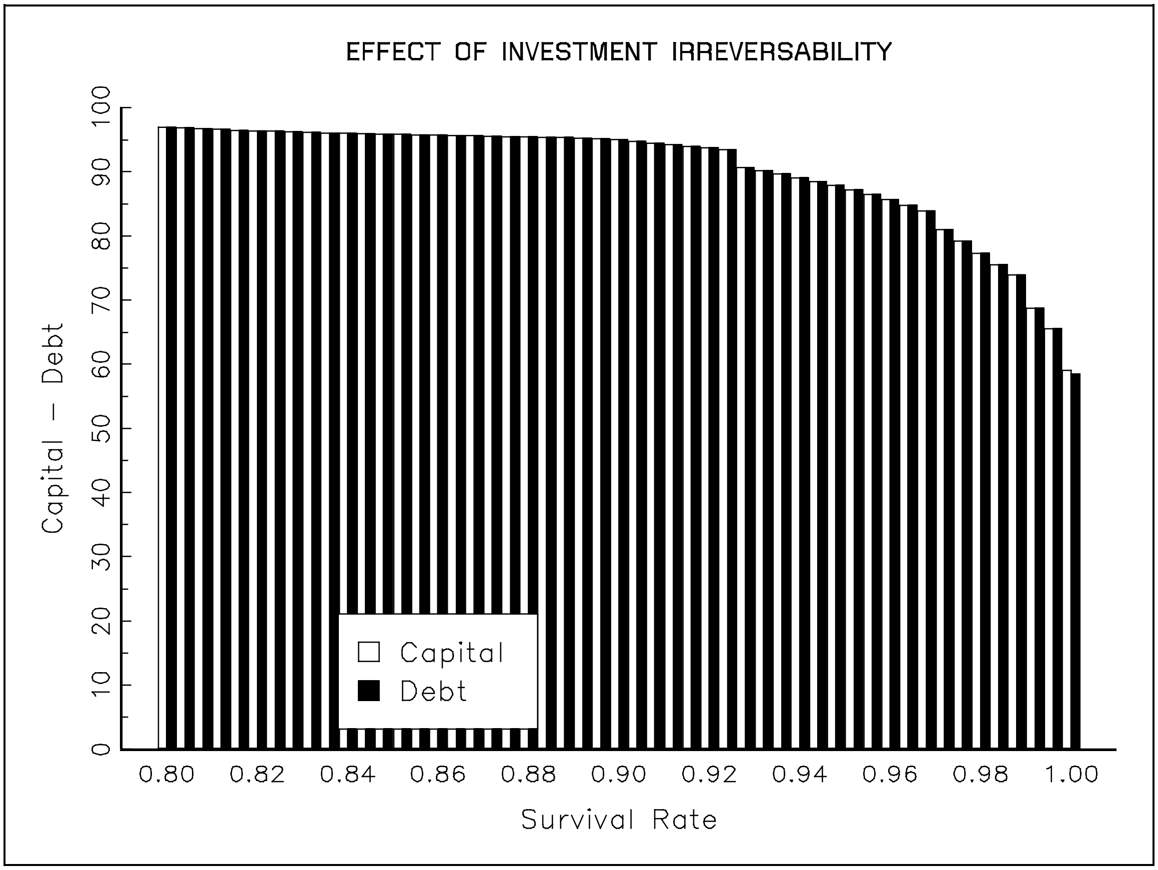
Graph 1


PROFIT AND OPTIMAL FINANCIAL STRUCTURE
Profit
Economic Profit: p > p*
Economic Profit: p = p*
Economic Profit: p < p*
Zero Taxable Income
Capital
b < k
b = k
b < k

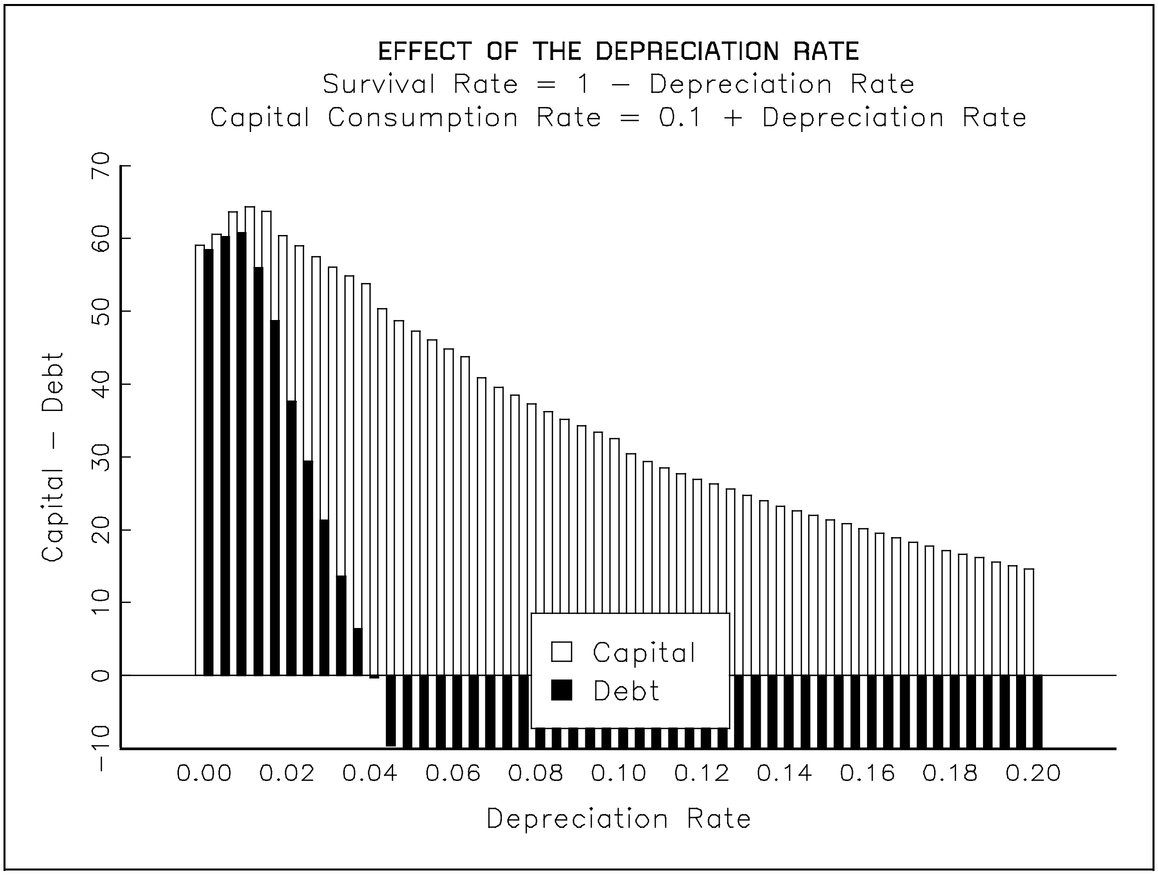
Graph 2


MARGINAL VALUE OF CAPITAL AND OPTIMAL INVESTMENT
Expected Marginal Economic Profit: $E[F_k(k,p)-\delta-r|p_0]$
Marginal Value of Capital: $\Phi(k,p_0)$
Marginal Value of Inherited Capital: $V_x(x_0,p_0)$
Marginal Value
Capital
k^*
$k = k^*$
$k = x_0$

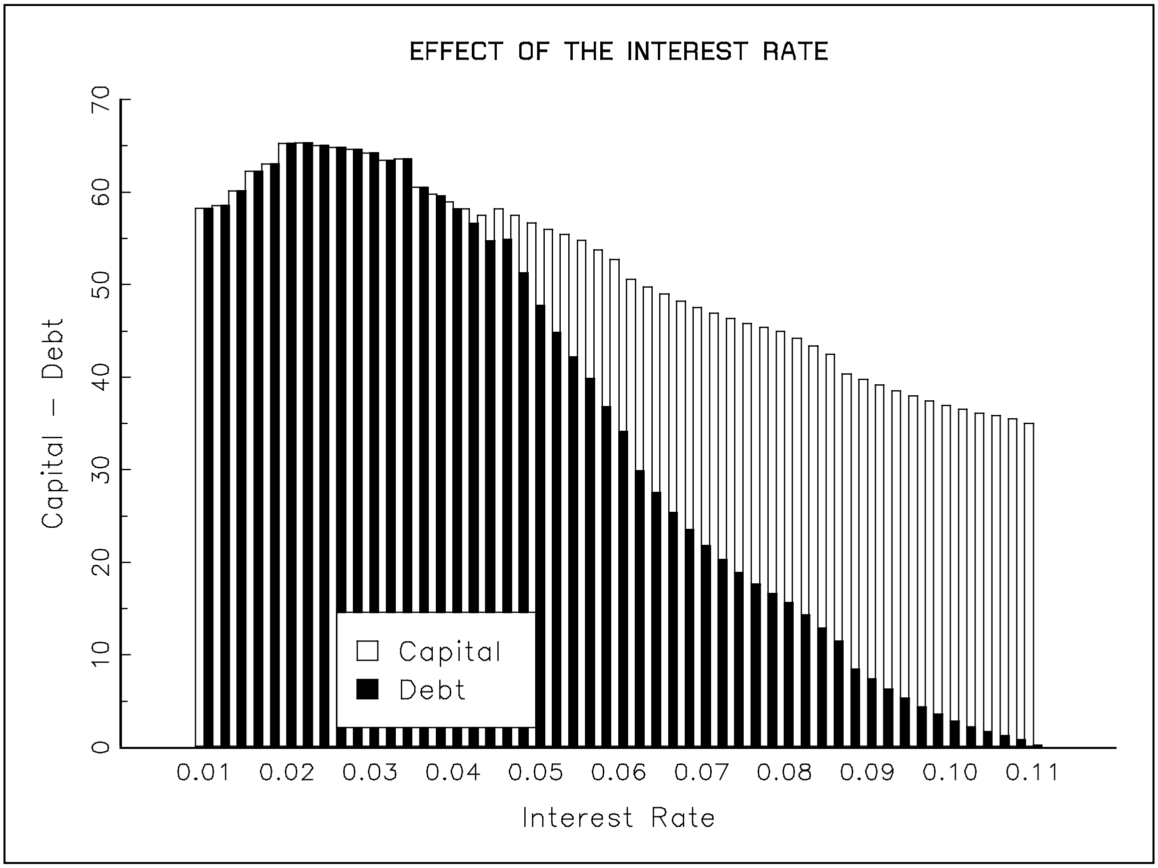
Graph 3


EFFECT OF INVESTMENT IRREVERSABILITY
Capital - Debt
0
10
20
30
40
50
60
70
80
90
100
Capital
Debt
0.80
0.82
0.84
0.86
0.88
0.90
0.92
0.94
0.96
0.98
1.00
Survival Rate

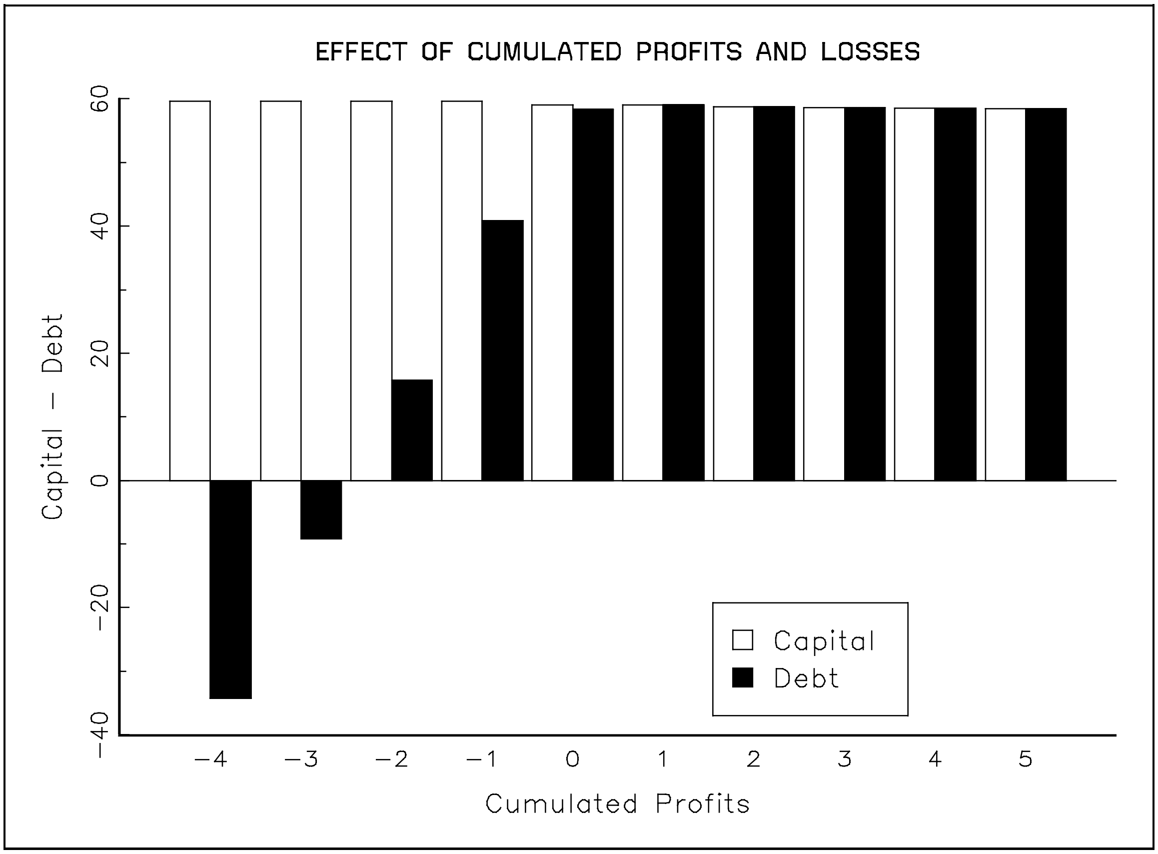
Graph 4


EFFECT OF THE DEPRECIATION RATE
Survival Rate = 1 − Depreciation Rate
Capital Consumption Rate = 0.1 + Depreciation Rate
Capital − Debt
70
60
50
40
30
20
10
0
−10
0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16 0.18 0.20
Depreciation Rate
Capital
Debt

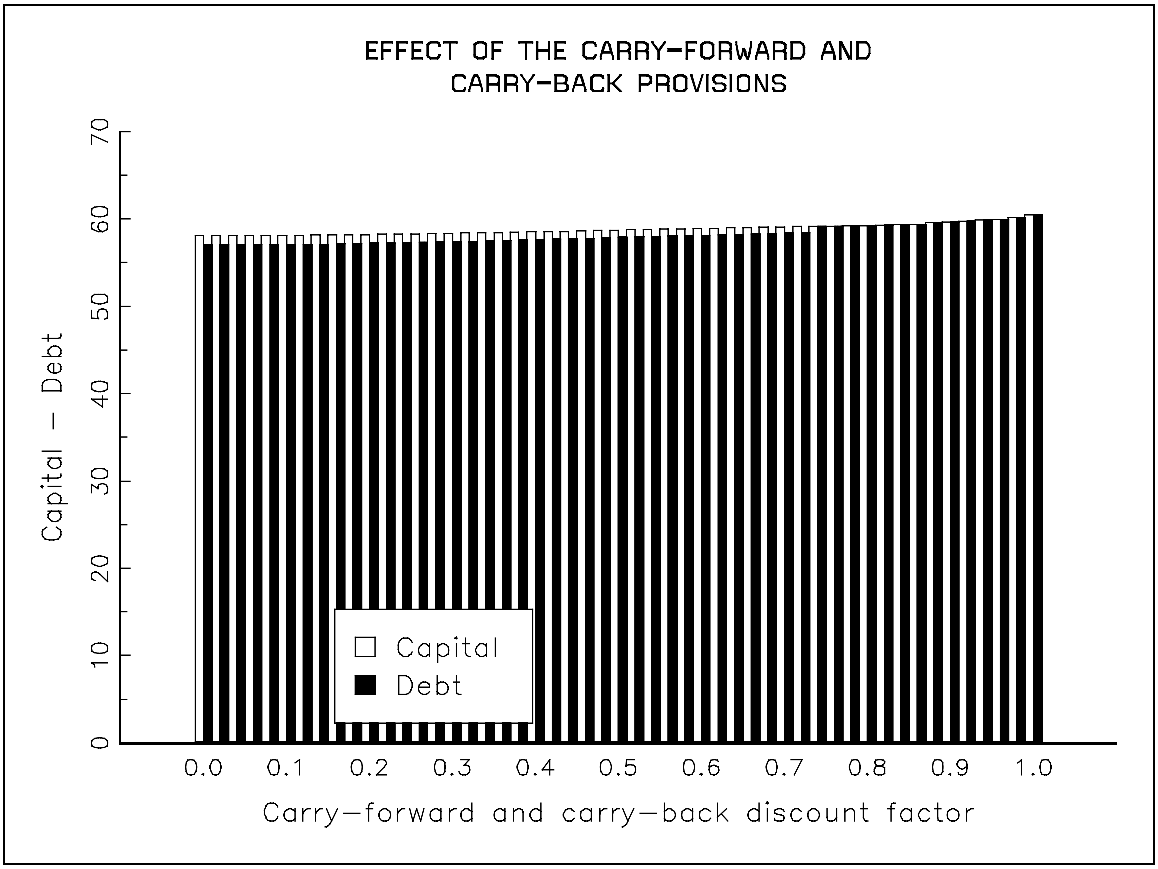
Graph 5


EFFECT OF THE INTEREST RATE
Capital - Debt
0
10
20
30
40
50
60
70
Capital
Debt
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
Interest Rate

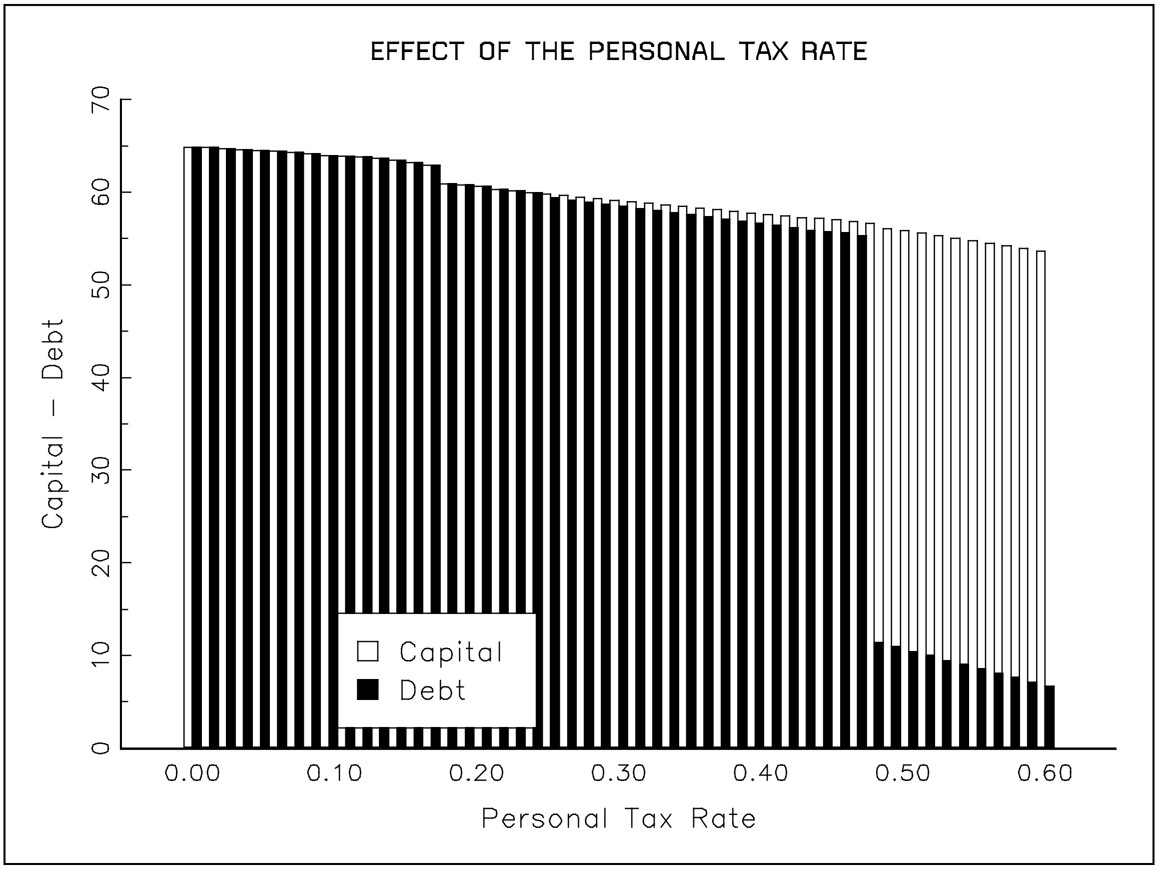
Graph 6


EFFECT OF CUMULATED PROFITS AND LOSSES
Capital – Debt
60
40
20
0
–20
–40
–4
–3
–2
–1
0
1
2
3
4
5
Cumulated Profits
Capital
Debt

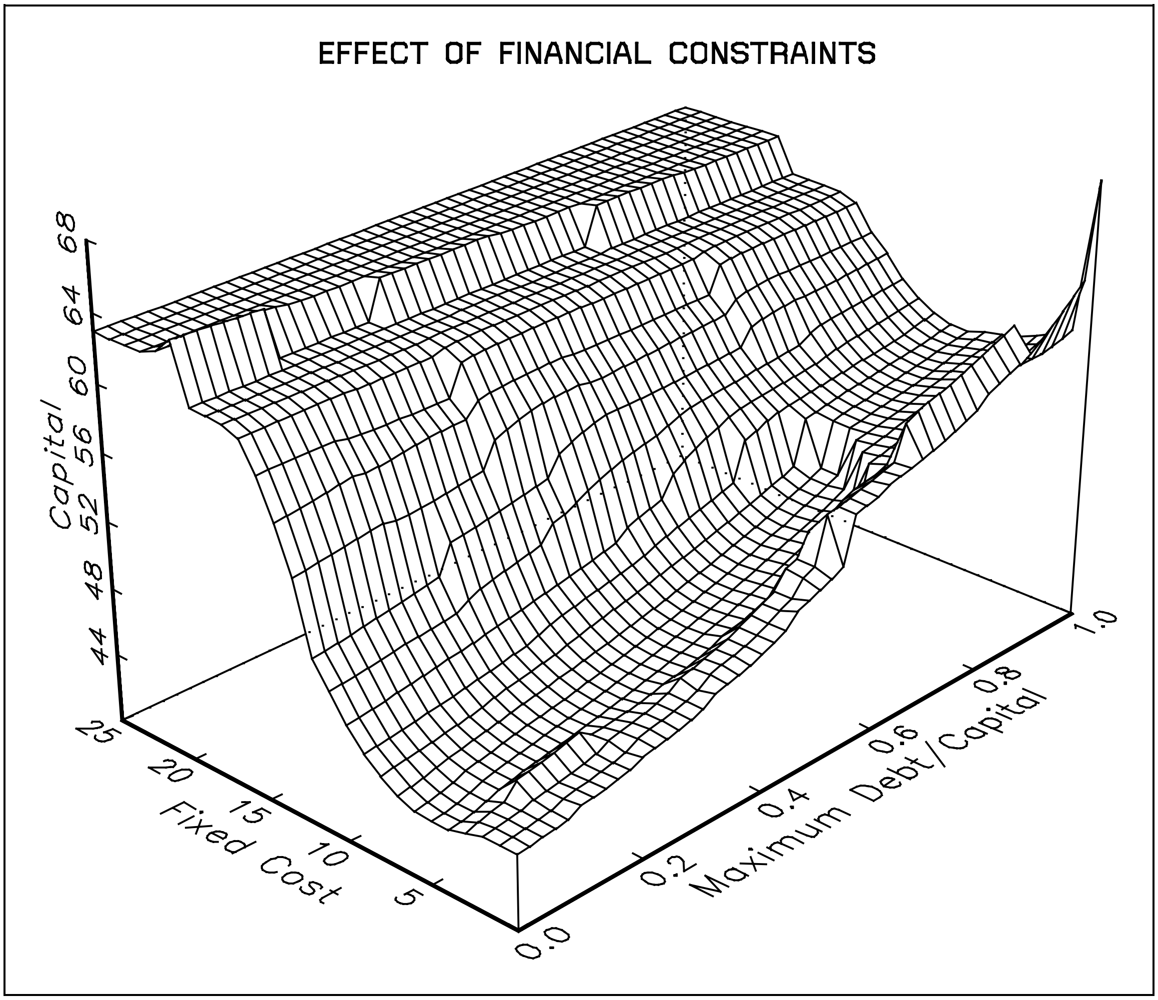
Graph 7


EFFECT OF THE CARRY-FORWARD AND
CARRY-BACK PROVISIONS
Capital - Debt
0
10
20
30
40
50
60
70
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Carry-forward and carry-back discount factor
Capital
Debt

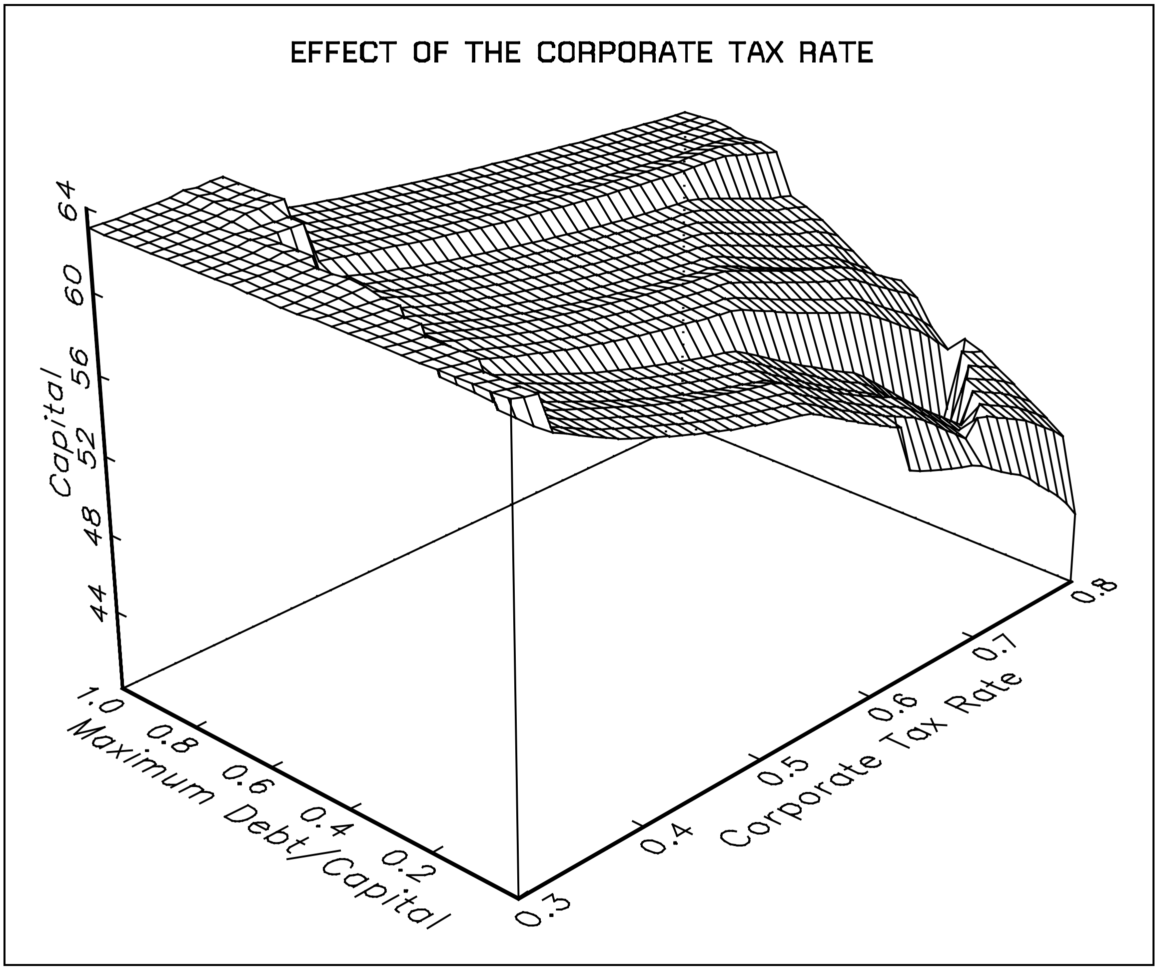
Graph 8


EFFECT OF THE PERSONAL TAX RATE
Capital – Debt
0
10
20
30
40
50
60
70
Capital
Debt
0.00
0.10
0.20
0.30
0.40
0.50
0.60
Personal Tax Rate

Graph 9


EFFECT OF FINANCIAL CONSTRAINTS
Capital
44
48
52
56
60
64
68
Fixed Cost
25
20
15
10
5
Maximum Debt/Capital
0.0
0.2
0.4
0.6
0.8
1.0

Graph 10


EFFECT OF THE CORPORATE TAX RATE
Capital
64
60
56
52
48
44
1.0
0.8
0.6
0.4
0.2
Maximum Debt/Capital
0.3
0.4
0.5
0.6
0.7
0.8
Corporate Tax Rate

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NOTES

1.One exception is a recent study by Cummins, Hasset and Hubbard (1995), who employ major tax reforms in the U.S. and several European countries as natural experiments.

2.There are several reasons why an investment may be irreversible. For example, if the capital is firm or industry specific, it will have little productive value in other firms or other industries. Even if the capital is not firm or industry specific, it is often partly irreversible because of the "lemons" problem, or because of government regulations. See Dixit and Pindyck (1994). For a unified framework that takes into account both irreversibility and adjusment costs, see Abel and Eberly (1994).

3.Losses can be carried back, without interest, for three years in the U.S. and in Canada. Losses can be carried forward, again without interest, for fifteen years in the U.S. and for seven years in Canada. Most European countries have less generous provisions.

4.There are several important differences between Mauer and Triantis' model and ours. First, investment in their model involves a single project with fixed output, while investment in our model is incremental. Second, they abstract from carry-back or carry-forward of losses, personal taxes, and explicit financial constraints - provisions which we include. Third, they assume sufficiently large bankruptcy costs such that their optimal debt solution is always interior. We allow for the possibility of both interior and corner debt choices. Finally, their model is in continuous time, while ours is in discrete time.

5.We assume that investors are risk-neutral. However, since the asymmetric tax system introduces a concavity into the firm's value function, investors behave as if they are risk-averse (Green and Talmor (1985)).

6.To abstract from issues arising from bankruptcy and reorganization in a dynamic setting, we assume that shareholders have unlimited liability. Shareholders in this case will also hold bonds in their portfolios as an insurance.

7.In our model, we treat the degree of irreversibility, μ, as an exogenous parameter, and analyze the effects of different μ's on the firm's optimal decisions. An interesting extension for future research would be to endogenize μ.

8.A natural extension is to let firms choose the degree of maintenance assuming that in this way they affect the survival rate. As expected, in an interior maintenance choice, firms reduce their maintenance when they inherit too much capital.

9.R is a discontinuous function. However, since p is a continuous random variable and R is continuous almost everywhere, ϕ is a continuous function of k.

10.Since the value function, V, is concave, we only need to show that $[\partial_k F(k,p) - \delta - r]R(k,p,p_0)$, is non-increasing with respect to k. The latter follows from: (i) $\partial_k F(k,p) - \delta - r$ is decreasing with respect to k because F is concave; (ii) $R(k,p,p_0)$ is a step function with values 1 and $1 - \tau_C$; and (iii) for a given p, as k increases, R jumps from 1 to $1-\tau_C$ only when $\partial_k F(k,p) - \delta - r$ is positive, and jumps from $1-\tau_C$ to 1 only when $\partial_k F(k,p) - \delta - r$ is non-positive. Graph 1 illustrates (iii) for p^*. The same argument of p^* applies to all $p < p^*$. If $p > p^*$, R is always equal to $1-\tau_C$ since $c > 0$ for all k.

11.Actually, we assume that p is a stochastic value within a small interval around θp_0, p_0, or $\theta^{-1}p_0$ respectively. Otherwise, the Feller property of Q would fail and the firms objective function will not be differentiable with respect to k and b. Also, p is bounded by two absorbing barriers, so the overall number of p states if finite (20).

12.To get a grasp of the scale of this gap, consider that the vector of profits for the three immediate states in the base scenario is (-1.7, 0.25, 2.4).

13.Recent empirical evidence supports this result. Devereux, Keen and Schiantarelli (1994) use panel data from 597 firms in the U.K. to analyze whether or not tax asymmetries are important for understanding observed investment behavior. They conclude that they are not, in the sense that specifications that capture the full complexities of the corporate tax system perform virtually no better than models that ignore the asymmetries altogether.

14.Mauer and Triantis (1994) argue that in the traditional static model where the firm's investment decision is "now or never", the value of the interest tax shield is exaggerated. In a multiperiod model, debt financing plays a smaller role because the firm gives up interest tax shields only in the period(s) it does not operate or invest.