LIQUIDITY EFFECTS WITH LONG LIVED PRODUCTION PROJECTS

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Abstract:

This paper explores the effects of monetary shocks on the allocation of factors of production. We analyze these effects when money plays a role in improving the timing of the transactions undertaken by entrepreneurs. Such improvement is facilitated by money's important role in providing liquidity to entrepreneurs. Using a model in which production processes take time to mature and where credit contracts are not enforceable, we show the consequences of monetary shocks for the allocation of resources and the real business cycle. Our analysis reveals that such shocks disrupt the allocation of resources with important effects on total factor productivity.

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1. INTRODUCTION

In 1998, United States firms held on average 36 percent of M1[1]. This fact suggests that money balances are an important element of firms' production process. The present paper explores the effects of monetary shocks on the allocation of factors of production. In particular, we analyze these effects when money plays a role in improving the timing of the transactions undertaken by entrepreneurs. Such improvement is facilitated by money's important role in providing liquidity to the entrepreneurs. In the absence of liquidity, firms have to either time their expenditures to coincide with their income or invest in assets associated with transactions costs that are significantly high for short holding periods. Monetary injections tend to disrupt this liquidity function of money since the extra liquidity that they provide is not proportional to previous holdings of money and they are not distributed to entrepreneurs with the best investment opportunities. This disruption is likely to occur, in particular, in countries where access to credit is linked to political affiliation and friendship connections. To a lesser extent, such phenomena are also witnessed in the United States where monetary contractions dry the liquidity of small firms in a much larger proportion than large firms. The heterogeneous effects of monetary tightness on firms of varying sizes has been documented and analyzed by Gertler and Gilchrist (1994). The aim of this paper is to introduce a simple model to analyze the

[1]This estimate is calculated from the Federal Reserves' Flow of Funds, September 1999, as the percentage of total checkable deposits and currency held by non-financial businesses (which includes non-farm non-financial commercial businesses, non-farm non-corporate businesses and farm businesses). The average reported here corresponds to the average of the four quarters of 1998. The average percentage for the first two quarters of 1999 is 38%.

dynamic effects of monetary injections when money provides a liquidity service to entrepreneurs while abstracting from credit markets and heterogeneously sized firms.

Our model builds on Woodford (1990) along similar lines to Faig (1998). Production projects take time to mature and different economically productive activities overlap in their timing. In other words, only a portion of projects yields output in each period. Furthermore, entrepreneurs can participate in, at most, one project at a time. As in Lucas (1980), these agents have no recourse to credit because credit contracts cannot be enforced. In such an environment, earnings from projects currently yielding output easily contribute to financing those entrepreneurs' consumption and investment expenditures. Meanwhile, entrepreneurs who cannot rely on earnings from projects not yet matured require another source of liquidity to finance their expenditures. Such asymmetric opportunities for self-financing motivates the need for money in our model. That is, once entrepreneurs receive output from their projects, they sell a portion of it for money, which is consequently used to consume and invest in periods when projects are not yet matured.

Our model departs from Woodford (1990) in that production projects take time to mature and thus production activities overlap. The overlapping structure is adopted in order to easily endogenize the presence of entrepreneurs with different marginal rates of return on their investments, and hence money's important role in the allocation of the factors of production. The major difference from our model and Faig (1998) is the introduction of workers supplying labor to the entrepreneurs. This characterization allows us to extend the analysis of monetary shocks to their effects on real wages and employment. Moreover, the inclusion of a labor force in the model provides a realistic tone in which it is no longer necessary to assume that monetary balances in the economy are uniquely held by entrepreneurs.

Our model contributes to the limited participation literature that stresses the liquidity role of money and the real effects of the relocations of liquidity that follow monetary injections. Seminal contributions to this literature are those of Grossman and Weiss (1983) and Rotemberg (1984). However, in these models money is only held by households and thus has no direct role in financing purchases of productive inputs. Lucas(1990), Fuerst(1992) and the long literature following them augmented the framework by incorporating this financing role of money. Nonetheless, these models still rely on the existence of a representative firm and are consequently mute with respect to the effects of monetary injections on the allocation of productive resources across alternative projects. Additionally, for the purpose of tractability, these models adopt a family structure in which wealth is occasionally shared by all its members, while at other times, monetary holdings cannot be transferred from one household member to another. With this family structure, tractability is gained at the expense of eliminating long lasting dynamic effects, at least without ad-hoc costs of adjusting cash balances as in Christiano and Eichembaum (1992). In our model, this split family device is not necessary.

The role of money in providing liquidity to entrepreneurs in order to improve the timing of their transactions is easily captured in the overlapping production structure adopted in this paper. In the present setup, we find that monetary injections introduced through entrepreneurs at the stage of acquiring money momentarily depress the real rate of interest by increasing expected inflation. These injections lead with some delays to increases of output, employment, and real wages. The increases, however, tend to worsen the allocation of resources in the market production sector because they shift resources from entrepreneurs with the best investment opportunities to the entrepreneurs that receive the monetary injection. Also, monetary injections generate long lasting dynamic effects not only because of the wealth effects present in Grossman and Weiss (1983) and Rotemberg (1984), but also because of the relocations of resources across alternative long lasting projects.

The remainder of the paper is organized as follows. Section 2 describes the model and the overlapping production structure of the model economy. This section also defines the recursive monetary competitive equilibrium. Section 3 characterizes optimal individual behavior and the Euler equations relating to the individuals' decision process. Section 4 constructs the steady state that the economy would reach in the absence of shocks. Section 5 analyzes the stochastic dynamics around the steady state when the economy is subjected to monetary shocks. Section 6 provides a numerical evaluation of the stochastic dynamics following a one-time shock to the money supply. Finally, section 7 concludes and reiterates the main findings of the paper.

2. DESCRIPTION OF THE MODEL

The present model describes a closed economy with two types of individuals: workers and entrepreneurs. Workers are not endowed with technology to produce for the market and they supply labor, which is assumed to be perfectly mobile across sectors. Each entrepreneur owns the technology for a market production process and manages the firm using this technology. Because credit contracts cannot be enforced, entrepreneurs also own the capital invested in their firms. Financing a portion of the capital stock with credit, such as fixed capital, does not drastically change the model. However, the assumption that some portion of the resources invested in their firms must be self-financed is crucial and replaces the customary cash-in-advance constraint.

The model economy is characterized by two sectors with overlapping production processes. Production in each sector requires two periods in which labor and materials are employed to later yield output in the third period. The two sectors are identical except in the timing of their production process. In each period, one of the two sectors collects the output from a completed production process and starts the first stage of the next. Simultaneously, the other sector is in the second stage of a production process that will yield output one period later. We distinguish between two types of capital in either sector: circulating and fixed capital. While the former is used to finance labor and materials, the latter is exogenously determined. This simplification, made for analytical convenience, is supported by the fact that fixed capital depreciates slowly and thus fluctuates little over the business cycle.

For ease of exposition, we further assume that each sector is comprised of a representative firm managed by a representative entrepreneur. This would be the case if, for example, all entrepreneurs in a particular sector were identical. Each representative entity is endowed with the following technology:

(1) $$y_t = F(\bar{k}, x^1_{t-2}, n^1_{t-2}, x^2_{t-1}, n^2_{t-1});$$

where y_t = output in period t;

F = production function;

$\bar{k}$ = fixed capital;

x^i_t = materials employed in period t at stage of production i (i = 1 and 2); and

n^i_t = labor employed in period t at stage of production i (i = 1 and 2).

The function F is increasing in all arguments, concave, continuously differentiable, and linearly homogeneous, satisfying the Inada conditions for interior solutions.

During the first stage of production, an entrepreneur starts with a certain amount of liquid wealth comprised of the output just obtained and the real money balances carried from the previous period or distributed in a lump-sum fashion by the government:

(2) $$a_t^1 = y_t + \frac{M_{t-1}^2 + \Delta_t^1}{p_t};$$

where a_t^i = liquid wealth in period t at stage i of production (i = 1 and 2);

M_t^i = money acquired in period t at stage of production i (i = 1 and 2);

Δ_t^i = money received in period t at stage of production i (i = 1 and 2); and

p_t = price of output in period t.

In the first stage of production, liquid wealth is allocated to consumption, real money balances, and circulating capital. Since the entrepreneur purchases materials and hires labor with the circulating capital, the budget constraint faced by the entrepreneur in the first stage of production takes the following form:

(3) $$c_t^1 + x_t^1 + n_t^1 \frac{w_t}{p_t} + \frac{M_t^1}{p_t} = a_t^1;$$

where c_t^i = consumption in period t at stage i of production (i = 1 and 2);

w_t = wage rate in period t.

During the second stage of production, the liquid wealth of an entrepreneur is comprised of the monetary balances acquired in the first stage of production and any lump-sum monetary transfers received:

(4) $$a_t^2 = \frac{M_{t-1}^1 + \Delta_t^2}{p_t}.$$

This liquid wealth is used to purchase consumption, materials, labor, and possibly to carry forward some money balances to the next period. Therefore, the budget constraint faced by the entrepreneur in the second stage of production is represented as follows:

(5) $$c_t^2 + x_t^2 + n_t^2 \frac{w_t}{p_t} + \frac{M_t^2}{p_t} = a_t^2.$$

All the demand components in (3) and (5) must be non-negative.

Workers do not own any technology to produce for the market. Instead, they are endowed with time and a home production technology. Workers' time can be sold in the market as labor or can be directly employed in home production. For ease of exposition, we also assume the existence of a representative worker. The home production technology for this worker is specified as follows:

(6) $$c_t^h = G(T, n_t^s);$$

where c_t^h = home production in period *t*, consumed directly by the representative worker;

G = home production function;

T = total time available; and

n_t^s = supply of labor in period *t*.

The function G is assumed to be increasing with T, decreasing with n_t^s, strictly concave, and continuously differentiable and satisfies the Inada condition for an interior solution of n_t^s.

The flow budget constraint of the representative worker is characterized as follows:

(7) $$c_t^n + \frac{M_t^n}{p_t} = n_t^s \frac{w_t}{p_t} + \frac{M_{t-1}^n + \Delta_t^n}{p_t}\text{; and}$$

(8) $$c_t^n \leq \frac{M_{t-1}^n + \Delta_t^n}{p_t}.$$

where c_t^n = consumption in period t;

M_t^n = money acquired in period t by the representative worker; and

Δ_t^n = money received in period t by the representative worker.

All demands in (6) must be nonnegative. The two types of consumption, c_t^h and c_t^n, are perfect substitutes.

The preferences of all individuals are represented by the following utility function:

(9) $$E\sum_{i=t}^{\infty} b^{i-t} U(c_{t+i});$$

where U = felicity function; and

β = subjective discount factor.

The function U is increasing, concave, and continuously differentiable. Furthermore, it satisfies the Inada conditions for an interior solution.

In addition, there is a government in the economy that manages the money supply. The monetary injections, $\Delta_t^1, \Delta_t^2,$ and Δ_t^n, introduced by this government are stochastic variables which can take positive (transfers) or negative (taxes) values. In either case, they are seen as lump sum by each recipient. The stochastic processes followed by the monetary injections vary depending on the simulations, and will be specified in sections 5 and 6.

We assume the equilibria in the economy to be monetary and competitive. In other words, the sequences of allocations and prices must be consistent with the following conditions: individuals take prices as given and behave according to rational expectations, workers maximize (9) subject to (6) and (7), entrepreneurs maximize (9) subject to (1) through (5), money is valued, and all markets clear. In addition, the equilibria studied in this paper are recursive in sense that prices are a function of the state variables of the economy. These state variables include present harvest, labor and materials invested in the previous period at stage 1 of production, total money holdings after the monetary injections, and all variables useful to forecast future monetary injections.

There are three markets in this economy: output, labor, and money. The respective market clearing conditions are given by the following equations:

(10) $$c_t^1 + c_t^2 + x_t^1 + x_t^2 = y_t;$$

(11) $$n_t^s = n_t^1 + n_t^2; \text{ and}$$

(12) $$M_t^1 + M_t^2 + M_t^n = M_{t-1}^1 + \Delta_t^1 + M_{t-1}^2 + \Delta_t^2 + M_{t-1}^n + \Delta_t^n.$$

3. OPTIMAL INDIVIDUAL BEHAVIOR

This section describes the optimal behavior of workers and entrepreneurs. When entrepreneurs behave optimally, their consumption, investment, and portfolio choices satisfy the standard Euler conditions. For assets held in positive amounts, a unit of output brings the same utility whether immediately consumed or sold for one of these assets and then the gross return consumed at maturity. For assets with a binding non-negativity constraint, a unit of output brings more utility to the entrepreneur if it is immediately consumed than sold for one of these assets. Algebraically, these Euler conditions are formalized with the following relations:

(13) $$U'\left(c_t^1\right) = b\ E\left[U'\left(c_{t+1}^2\right)\frac{p_t}{p_{t+1}}\right];$$

(14) $$U'\left(c_t^2\right) \geq bE\left[U'\left(c_{t+1}^1\right)\frac{p_t}{p_{t+1}}\right] \text{with equality if } M_t^2 > 0;$$

(15) $$U'\left(c_t^1\right) = b^2 E\left[U'\left(c_{t+2}^1\right)F_{x1t}\right];$$

(16) $$U'\left(c_t^1\right) = b^2 E\left[U'\left(c_{t+2}^1\right)F_{n1t}\frac{p_t}{w_t}\right];$$

(17) $$U'\left(c_t^2\right) = bE\left[U'\left(c_{t+1}^1\right)F_{x2t}\right];$$

(18) $$U'\left(c_t^2\right) = bE\left[U'\left(c_{t+1}^1\right)F_{n2t}\frac{p_t}{w_t}\right].$$

The Inada conditions assumed for U and F imply that consumption and investment in labor and materials at both stages of production are always positive, so that the demand for money at stage 1 of production is ensured to be positive. As a result, conditions (13) and (15) to (18) do not contemplate the possibility of corner solutions. However, condition (14) considers a possible corner solution because the demand for money at stage 2 of production may be zero.

When workers behave optimally, their consumption, labor supply, and money demand must satisfy the following first order conditions:

$$U'\left(c_t^h\right) \geq b E\left[U'\left(c_{t+1}^h\right)\frac{p_t}{p_{t+1}}\right] \text{with equality if } c_t^n < \frac{M_{t-1}^n + \Delta_t^n}{p_t}\text{; and} \tag{19}$$

$$\frac{w_t}{p_t} = -\frac{\P G(T, n_t^s)}{\P n_t^s}. \tag{20}$$

Condition (19) is analogous to the entrepreneurs' Euler relations (12) and (13). Condition (20) establishes that the worker must obtain, at the margin, the same return by either supplying a unit of labor in the market or by employing it in home production.

In general, the optimal policy functions for workers and entrepreneurs depend on the individual state variables that characterize their wealth and liquidity. For entrepreneurs, the state variables are a_t^1 at stage 1, and x_{t-1}^1, n_{t-1}^1, and a_t^2 at stage 2. The unique state variable relevant to the workers is $\frac{M_{t-1}^n + \Delta_t^n}{p_t}$. In addition, the optimal policy functions depend on current prices and monetary injections, together with other variables useful in predicting future prices and monetary injections.

4. THE STEADY STATE

In the absence of monetary injections, the present model is deterministic. In this case, an equilibrium converges to a steady state where output, its allocation, and prices are constant. This steady state solves the deterministic counterparts of (13) to (20) together with (1) to (12) with the time subscripts omitted from all the variables.[2] This section characterizes the properties of this steady state important in order to understand the neighboring stochastic dynamics, which are studied in the next section.

In the steady state, workers do not hold money because its gross real return is unity (prices are constant) and their subjective discount factor is less than one, so that condition (18) holds with strict inequality. Likewise, money is demanded by entrepreneurs only to provide liquidity at stage 2 because (13) implies $c^1 > c^2$ and thus condition (14) must hold with strict inequality. Consequently, money is only demanded by entrepreneurs at stage 1 of production to provide liquidity at stage 2.

In the steady state, conditions (15) and (16) imply that the real return of circulating capital invested at stage 1 in either materials $\left(F_{x1}\right)$ or labor $\left(F_{n1}\dfrac{p}{w}\right)$ is ß^2 over two periods (or ß$^{-1}$ per period). Meanwhile, conditions (17) and (18), together with (13), imply that the real return of circulating capital invested at stage 2 in either materials $\left(F_{x2}\right)$ or labor $\left(F_{n2}\dfrac{p}{w}\right)$ is ß$^{-2}$ per period. Consequently, circulating capital invested in either stage of production dominates money

[2]The existence of a solution for this system of equations is easily proved using the standard techniques used to prove existence of a steady state in the Neoclassical growth model. Similarly, techniques analogous to those used in the analysis of the Neoclassical growth model can then be used to prove uniqueness and convergence. An explicit solution of the steady state and the deterministic dynamics around it exists if *U* is logarithmic and *F* is Cobb-Douglas.

in rate of return ($ß^{-2}$ and $ß^{-1}$ versus 1). Moreover, circulating capital invested at stage 2 earns a higher return per period than circulating capital invested at stage 1. Hence, the allocation of labor and materials, purchased with circulating capital, is inefficient: too much circulating capital is invested at stage 1 of production when its return is low and too little at stage 2 when its return is high. For this reason, monetary shocks that increase the liquidity available to entrepreneurs at stage 2 of production improve the allocation of labor and materials, hence increasing total factor productivity. Vice versa, monetary shocks that reduce liquidity at stage 2 worsen the allocation of labor and materials, resulting in reduced total factor productivity.

Similar properties characterize the steady state of economies with a growing money supply at a constant rate. In the present case, prices grow over time at the rate of growth of the money supply. Consequently, the gross return on money is equated to the inverse of the gross rate of growth of the money supply. Efficiency in the allocation of labor and materials is inversely related to the rate of growth of the money supply. An efficient allocation can be achieved following Friedman's (1969) optimum quantity of money rule, that is, with a steady decline of the money supply consistent with a gross return on money equal to $ß^{-1}$. In this case, the rates of return on money and circulating capital at both stages of production are equalized. In this paper, we study equilibria in which the government does not pursue the optimum quantity of money rule.

5. STOCHASTIC DYNAMICS

We now turn to the study of the economy's reaction to stochastic monetary injections. To this end, the model adopts the following specialized functional forms: the felicity function U is assumed to be logarithmic, the production function F is assumed to be Cobb-Douglas, and the home production function G is assumed to have a special form that yields an isoelastic labor supply:

(21) $$U(c_t) = \ln c_t ,$$

(22) $$y_t = \left(\bar{k}\right)^{a_k} \left(x_{t-2}^1\right)^{a_{x1}} \left(n_{t-2}^1\right)^{a_{n1}} \left(x_{t-1}^2\right)^{a_{x2}} \left(n_{t-1}^2\right)^{a_{n2}} \text{, and}$$

(23) $$G(T, n_t^s) = T - \frac{\left(n_t^s\right)^{1+\frac{1}{q}}}{1+\dfrac{1}{q}};$$

where a_i = income share of factor i $\left(i = \bar{k}, x^1, n^1, x^2, n^2\right)$, and

2 = elasticity of the labor supply with respect to the real wage.

The monetary injections are assumed to be sufficiently small so that the economy remains at an equilibrium around the steady state characterized in the previous section. In this equilibrium, money is a dominated asset in rate of return and thus is only demanded by workers because of the cash in advance constraint and to provide liquidity services to entrepreneurs at their second stage of production.

The effect of monetary injections depends crucially on whom money is handed to or taken away from. We assume that money is handed to (or taken from) entrepreneurs at their first stage of production. This assumption follows the large literature on liquidity effects where monetary injections are only received by entrepreneurs in the process of acquiring money. Specifically, monetary injections have the following form:

(24) $$\Delta_t^1 = g_t y_t p_t ;$$

where g_t is a stochastic variable that follows a Markov process. Notice that even if the monetary injection is proportional to the entrepreneurs' nominal wealth at their first stage of production, it is handed in a lump-sum manner to each one of them.

With these functional forms, we conjecture that the optimal policy functions for entrepreneurs have the following form:

(25) $$c_t^1 = \tilde{c}_1(\gamma_t)a_t^1,\ \ x_t^1 = \tilde{x}_1(\gamma_t)a_t^1,\ \ n_t^1 = \tilde{n}_1(\gamma_t)a_t^1 p_t / w_t,\ M_t^1 = \tilde{m}_1(\gamma_t)a_t^1 p_t,$$

$$c_t^2 = \tilde{c}_2(\gamma_t)a_t^2,\ x_t^2 = \tilde{x}_2(\gamma_t)a_t^2,\ n_t^2 = \tilde{n}_2(\gamma_t)a_t^2 p_t / w_t,\ \text{and}\ \ M_t^2 = 0\,.$$

These policy functions together with (2), (4), (24) and (25) imply:

(26) $$a_t^1 = y_t(1+\gamma_t^1)\text{, and}$$

(27) $$a_t^2 = \frac{M_{t-1}^1}{p_t} = \tilde{m}_1(\gamma_t^n)a_{t-1}^1\frac{p_{t-1}}{p_t}\,.$$

To prove our conjecture about the policy functions, we check that these functions satisfy the entrepreneurs' budget equations and the first order Euler conditions for utility maximization. This verification is done by directly inserting (26) into equations (1) to (5) and (13) to (18). After simplification using (29), (30), and the partial derivatives of (22), we obtain the following system of functional equations:

(28) $$\tilde{c}_1(\gamma_t) + \tilde{x}_1(\gamma_t) + \tilde{n}_1(\gamma_t) + \tilde{m}_1(\gamma_t) = 1\,,$$

(29) $$\tilde{c}_2(\gamma_t) + \tilde{x}_2(\gamma_t) + \tilde{n}_2(\gamma_t) = 1\,,$$

(30) $$\tilde{c}_1(\gamma_t)^{-1} = \beta\tilde{m}_1(\gamma_t)^{-1}E\left[\tilde{c}_2(\gamma_t)^{-1}\right]\cdot$$

(31) $$\tilde{c}_1(\gamma_t)^{-1} = \beta^2\alpha_{x1}\tilde{x}_1(\gamma_t)^{-1}E_t\left[\tilde{c}_1(\gamma_{t+2})^{-1}(1+\gamma_{t+2})^{-1}\right],$$

(32) $$\tilde{c}_1(\gamma_t)^{-1} = \beta^2 \alpha_{n1}\tilde{n}_1(\gamma_t)^{-1} E_t\left[\tilde{c}_1(\gamma_{t+2})^{-1}(1+\gamma_{t+2})^{-1}\right],$$

(33) $$\tilde{c}_2(\gamma_t)^{-1} = \beta\,\alpha_{x2}\tilde{x}_2(\gamma_t)^{-1} E_t\left[\tilde{c}_1(\gamma_{t+1})^{-1}(1+\gamma_{t+1})^{-1}\right],$$

(34) $$\tilde{c}_2(\gamma_t)^{-1} = \beta\,\alpha_{n2}\tilde{n}_2(\gamma_t)^{-1} E_t\left[\tilde{c}_1(\gamma_{t+1})^{-1}(1+\gamma_{t+1})^{-1}\right].$$

This system determines the functions: $\tilde{c}_1(\gamma_t),\ \tilde{c}_2(\gamma_t),\ \tilde{x}_1(\gamma_t),\ \tilde{x}_2(\gamma_t),\ \tilde{n}_1(\gamma_t),\ \tilde{n}_2(\gamma_t),$ and $\tilde{m}_1(\gamma_t)$. While, in general, there does not exist an explicit analytical solution, these functions can easily be computed with standard numerical algorithms.

To complete the description of an equilibrium, we use the market clearing conditions and the workers' optimal behavior. Let μ_t be the proportion of the money supply held by entrepreneurs at stage 2 of production after the monetary injection. The market clearing condition for money implies:

(35) $$a_t^2 = \mu_t \frac{M_t^s}{p_t} = \mu_t y_t\left[\tilde{m}_1(\gamma_t) + \tilde{n}_1(\gamma_t)\right] + \mu_t \tilde{n}_2(\gamma_t) a_t^2.$$

Rearranging,

(36) $$a_t^2 = \frac{\mu_t a_t^1\left[\tilde{m}_1(\gamma_t) + \tilde{n}_1(\gamma_t)\right]}{1 - \mu_t \tilde{n}_2(\gamma_t)}.$$

Clearing of the labor market implicitly determines n_t^s as a function of γ_t, a_t^1, and w_t / p_t:

(37) $$n_t^s = \left[\tilde{n}_1(\gamma_t) + \tilde{n}_2(\gamma_t)\right]\frac{p_t}{w_t}$$

This equation together with (20) implicitly determines w_t / p_t as a function of γ_t and a_t^1:

(38) $$\frac{w_t}{p_t} = \left[\tilde{n}_1(\gamma_t)a_t^1 + \tilde{n}_2(\gamma_t)a_t^2\right]^{\frac{1}{1+q}}$$

Since the representative worker is at a corner solution with respect to money holdings, the budget constraint of the representative worker (7) implies that c_t^n is also a function of γ_t, μ_t and a_t^1.

The price level p_t follows from the equilibrium in the money market and the equation relating the effects of injections in the money supply:

(39) $$M_t^s = \left[\tilde{m}_1(\gamma_t)a_t^1 + \tilde{n}_1(\gamma_t)a_t^1 + \tilde{n}_2(\gamma_t)a_t^2\right]p_t\text{, and}$$

(40) $$M_t^s = M_{t-1}^s + \gamma_t y_t p_t\ .$$

The previous two paragraphs describe how all real endogenous variables at time *t* are functions of γ_t, μ_t and a_t^1. First, the variable a_t^1 is equal to predetermined output y_t. Second, the proportion μ_t is determined by the initial money holdings and the monetary shock:

(41) $$\mu_t = \frac{M_{t-1}^1}{M_{t-1}^1 + M_{t-1}^n + \gamma_t y_t p_t} = \frac{\tilde{m}_1(\gamma_t)y_{t-1}}{\tilde{m}_1(\gamma_t)y_{t-1} + \tilde{n}_1(\gamma_{t-1})y_{t-1} + \tilde{n}_2(\gamma_{t-1})a_{t-1}^2 + \gamma_t y_t p_t}\ .$$

Finally, the monetary shock γ_t follows an exogenous Markov process. Hence, the equilibrium amounts of inputs are a function of y_t, μ_t, and γ_t. Given the production function (22), present output depends on the previous two period inputs, which in turn depend on y_{t-1}, y_{t-2}, γ_{t-1}, γ_{t-2}, μ_{t-1}, and μ_{t-2}. Consequently, the law of motion of output is a difference equation where y_t depends on y_{t-1}, y_{t-2}, γ_{t-1}, γ_{t-2}, μ_{t-1}, and μ_{t-2}.

6. NUMERICAL EVALUATION

This section evaluates the stochastic dynamics of the model around the steady state. The parameters used are the following: $\beta = 1.02^{-1}$, $\theta = 1$, and $\alpha_i = 0.2$ for all i. The rate of time preference (β) is chosen so as to be consistent with an annual steady state rate of return on capital of 8 percent. Although the wage elasticity of labor supply (2) adopts a higher value in our parametrization than estimated by microeconomic analyses of labour, we opt for an intermediate value which is lower than values typically used in the real business cycle literature. The shares of output accruing to the various factors of production (α_i) chosen in the present analysis are consistent with a scenario in which 40 percent accrues to materials, one third of the remaining 60 percent to physical capital and the balance to labor.

Furthermore, we adopt a simple stochastic environment in which it is assumed that the shocks to the money supply are identically and independently distributed and are exogenous to the agents' decision process. In particular, we posit that there exist two possible states of the world, normal and abnormal, where each occurs with probability 0.9 and 0.1, respectively. Each state is characterized by a different rate of change in the monetary supply: in the normal state, the stock of money increases by 0.01 and in the abnormal state, it increases by 0.02.

Figures 1 to 6 display the impulse response functions generated by a simulation in which the model economy experiences one period of the abnormal state after a long sequence of normal periods. Monetary injections are distributed to entrepreneurs in the first stage of production. The first stage entrepreneurs, with no monetary holdings at the beginning of the period, are in the process of acquiring money. The money they receive from the monetary injection is not spent until the next period. As a result of the monetary injection, factors of production are shifted from household production and second stage production to first stage production. Indeed, we observe from Figures 5 and 6 that both labor and materials used by the first stage entrepreneurs increase in the period coincident with the increase in the money stock. These increases overwhelm the declines of labor and materials used by the second stage entrepreneurs. Hence, the demand for labour experiences a net increase, which is reflected by an associated increase in the real wage rate. The increase in the demand for labor in stage one production is reflected by the increase in the real wage in Figure 4. Comparing the movement in the real wage to the movement in total product in Figure3, we find that real wages are quite procyclical except for the period immediately after the shock. Figure 3 also maps the response of consumption to the monetary injection. In fact, movements in total consumption follows movements in total product very closely, except in the period of the monetary injection. This gap is due to the fact that output brought to the market in the injection period is unresponsive to the injection itself, unlike consumption, as it has been determined in the two previous periods. Although the injection period real wage has increased, the initial drop in total consumption is attributed to the entrepreneurs' reduced purchasing power in response to the increase in prices as depicted in Figure 2.

Meanwhile, the diversion of productive resources to the first stage of production leads to a drop in the resources used in this period's second stage production. It is precisely this shift in the allocation of resources which leads to a productive inefficiency since the entrepreneurs with the best opportunities (those at the second stage of production) are adversely affected by the monetary injections. Consequently, since second stage production is reduced by the diversion of resources from the second to the first stage entrepreneurs, the total output of the model economy experiences a drop in the period immediately following the monetary injection. Nonetheless, since the first stage production process increases the use of labor and materials, the second period after the monetary injection experiences a large increase in production[3].

Important in the dynamic effects of the monetary injections is the relationship between the money supply and prices. Figure 2 shows that the increase in inflation is smaller than the increase in the money supply. The immediate result is that real money balances increase. However, this increase is far from equally distributed. The entrepreneurs at the first stage of production receive all the additional money balances, while entrepreneurs at the second stage of production and workers see the value of their money balances erode with the price hike. This redistribution of liquidity is the source of the redistribution of resources after the injection.

As a result of the monetary injection the relative size of the two sectors of production is altered. The sector managed by the entrepreneurs that received the injection becomes large relative to the other sector. This imbalance persists for a few periods because the entrepreneurs with more output have also more resources to invest. Over time, the two sectors rebalance themselves because in periods with relatively low output the intertemporal price of goods (the real interest rate) is relatively high encouraging growth for the small sector. The oscillatory pattern of the long term dynamics of the model is a consequence of the mechanical two period length of each project. With longer lasting projects the dynamics will also last longer. Moreover, if projects were of different length, the jagged oscillatory pattern would likely

[3]Production in the present model is final output, while in the national accounts GDP is a measure of value added.

disappear.

7. CONCLUSION

In environments where credit contracts are difficult to implement and where productive processes take time to mature, entrepreneurs must find a different avenue to facilitate their transactions. The model presented in this paper has illustrated how money can fill this role. Indeed, money can provide an important role in improving the timing of entrepreneurs' transactions in periods where cash flows are restricted by projects not yet completed.

The model we have built combines tractability and a role of money in financing some of the input in production without the "split family" device of Lucas (1980). As a result, a monetary injection in the model generates long lasting effects without having to assume ad-hoc costs of adjusting money balances. The source of these persistent effects is not only the wealth effects present in the seminal contributions of Grossman and Weiss (1983) and Rotemberg (1984), but also the shifts of productive resources across the various projects in the economy. In the model, these long lasting effects induce an oscillatory pattern in output and other real variables similar to those of Grossman and Weiss (1983) and Rotemberg (1984). However, this unrealistic feature could be remedied in more complicated models with different projects lasting different lengths of time.

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Figure 1

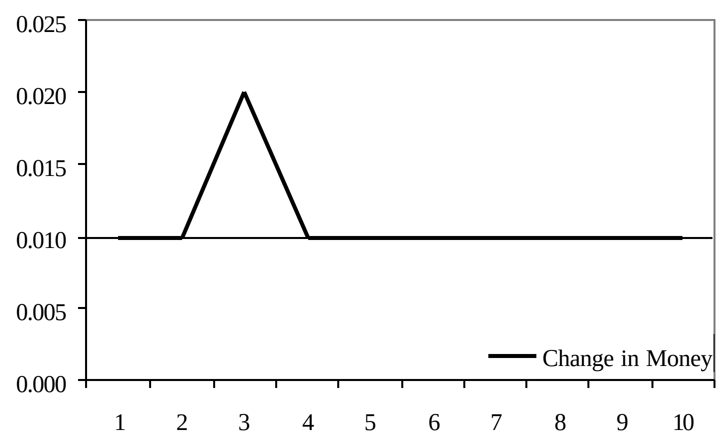


Figure 2

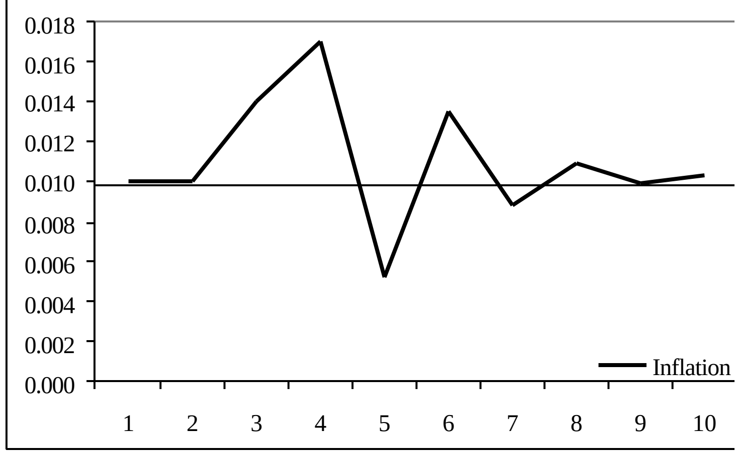
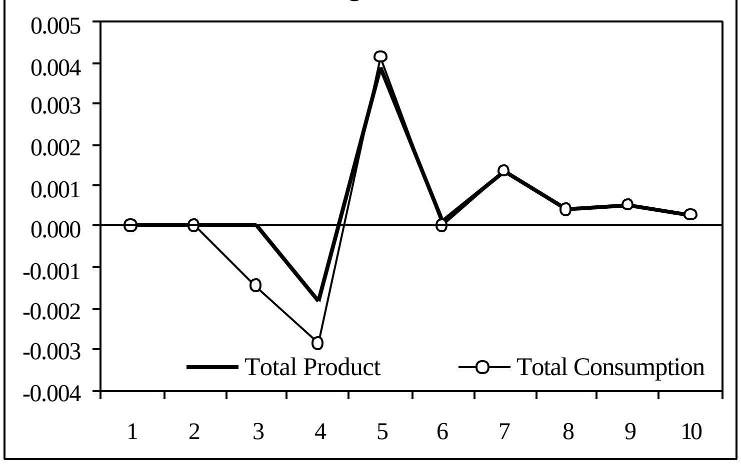


Figure 3



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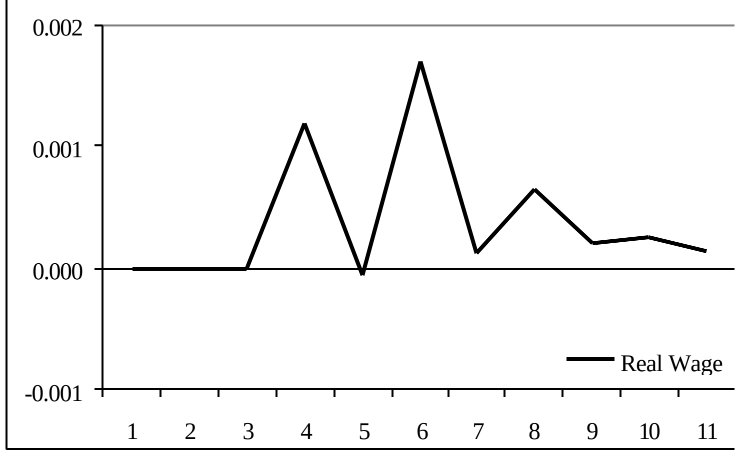

Figure 5

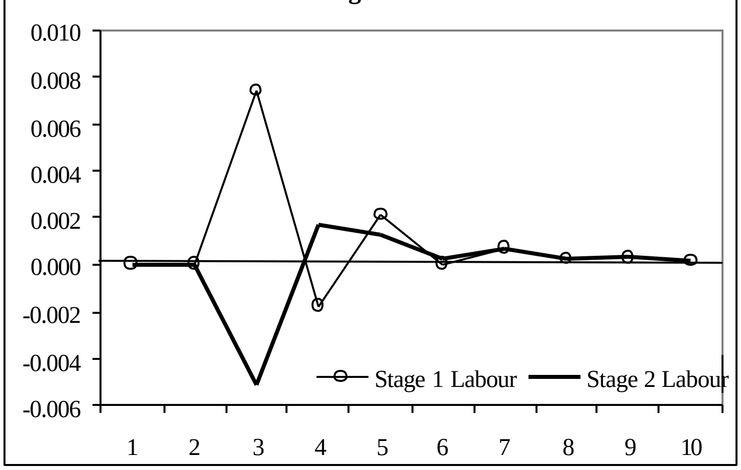

Figure 6