Competing for Talents

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May 12, 2009

Abstract: Two organizations compete for high quality agents from a fixed population of heterogeneous qualities by designing how to distribute their resources among members according to their quality ranking. The peer effect induces both organizations to spend the bulk of their resources on higher ranks in an attempt to attract top talents that benefit the rest of their membership. Equilibrium is asymmetric, with the organization with a lower average quality offering steeper increases in resources per rank. High quality agents are present in both organizations, while low quality agents receive no resources from either organization and are segregated by quality into the two organizations. A stronger peer effect increases the competition for high quality agents, resulting in both organizations concentrating their resources on fewer ranks with steeper increases in resources per rank, and yields a greater equilibrium difference in average quality between the two organizations.

Acknowledgments: We thank Dirk Bergemann, Simon Board, Jeff Ely, Mike Peters, and seminar audience at University of British Columbia, Columbia University, Duke University, University of Michigan, University of Southern California, University of Toronto, University of Western Ontario and Yale University for helpful comments and suggestions.

1. Introduction

Consider an academic department trying to improve its standing by hiring a new faculty member. Several economic forces influence such a decision. First, if the potential appointee is of high quality, the presence of such a colleague in the department will make the department more attractive to other faculty members due to the peer effect and may therefore help the department's other recruiting efforts. Second, the new recruit can upset the department's existing hierarchical structure and bring about implications for the internal distribution of departmental resources. "Salary inversion" is often seen as a potential problem in academia (Lamb and Moates, 1999; Siegfried and Stock, 2004). More generally, conventional wisdom in personnel management emphasizes the importance of "internal relativity" in the reward structure of any organization. In other words, the decision to make a job offer cannot be viewed in isolation; instead the entire reward structure of the organization has to be taken into account. Third, in a thin market with relatively few employers, the recruitment efforts of one department will affect the availability of the talent pool for another department. Hiring decisions in one department therefore have implications for the sorting of talents across all departments that need to be considered in an analysis of strategic competition for talents.

In this paper we develop a model of the competition for talents which incorporates all these economic forces. While the concern for the quality of one's peers, or the "peer effect," is widely acknowledged in the education literature (e.g., Coleman et al., 1966; Summers and Wolfe, 1977; Lazear, 2001; Sacerdote, 2001), and modeled extensively in the literature on locational choice (De Bartolome, 1990; Epple and Romano, 1998), the implications for organization design and especially organization competition, have received little attention.[1] We take the first step with a stylized model to study organizational strategies to attract

[1] In De Bartolome (1990), two communities decide their public service output and tax rate by majority voting, while in Epple and Roman (1998), private schools choose admission and tuition policies to maximize profits in a competition equilibrium with free entry. Neither paper addresses the issue of strategic competition between organizations in a non-cooperative game. The existing economic literature on the competition for talents typically focuses on either the informational spillovers resulting from offers and counter-offers (Bernhardt and Scoones, 1993; Lazear, 1996), or the implications of raiding for firms' incentive to offer training (Moen and Rosen, 2004). Tranaes (2001) studies the impact of raiding opportunities on unemployment in a search environment.

talents in the presence of the peer effect, and analyze the resulting equilibrium pattern of sorting of talents.

Section 2 introduces a game between two organizations, A and B. Talents have one-dimensional types distributed uniformly, and a utility function linear in the average type of the organization they join (the peer effect) and the resource they receive in the organization. Each organization faces a fixed capacity constraint that allows it to accept half of an exogenously given talent pool, and a fixed total budget of resources that can be allocated among its ranks. There are three stages of the game. In the first stage of resource distribution, the two organizations each simultaneously choose a budget-balanced schedule that associates the rank of each agent by type with an amount of resource the agent receives. In the second stage of talent sorting, after observing the pair of resource distribution schedules, all agents simultaneously choose one organization to apply to. In the third and final stage of admissions, each organization admits a subset of its applicants no larger than the capacity. The payoff to each organization is zero if the capacity is not filled, and is given by the average type otherwise. The payoff to any agent that is not admitted by an organization is zero.

Finding a subgame perfect equilibrium in the above game of organizational competition is difficult, because the space of resource distribution schedules is large, and because little can be said in general about the continuation equilibrium given an arbitrary pair of resource distribution schedules. In section 3 we develop an indirect approach to characterize a candidate for subgame perfect equilibrium resource distribution schedules. This approach is based on the quantile-quantile plot of any given sorting of talents between the two organizations, defined as follows.[2] For each rank r, the quantile-quantile plot identifies the type that has this rank in organization B, and gives the fraction of higher types in organization A. Since the type distribution is uniform, the difference in the average types between A and B, referred to as "quality difference," has a one-to-one relation with the integral of the quantile-quantile plot. Thus, to characterize a continuation equilibrium for a given pair of resource distribution schedules, instead of using two type distribution

[2] The quantile-quantile plot, also known as QQ-plot, is widely used in graphical data analysis for comparing distributions (e.g., Wilk and Gnanadesikan, 1968; Chambers et al., 1983).

functions for the two organizations, we use the associated quantile-quantile plot. Further, given any resource distribution schedule of organization B and any quantile-quantile plot, we can construct a resource distribution schedule for A that yields the quantile-quantile plot as a continuation equilibrium. Since the payoff to an agent is a linear function of the amount of the resource it receives from the organization it joins and the mean type of the organization, the budget associated with such resource distribution schedule is a weighted integral of the quantile-quantile plot. This observation implies that minimizing the resource budget that A needs in order to achieve any given quality difference in a continuation equilibrium against a given resource distribution schedule of organization B, is a linear programming problem in the quantile-quantile plot, because both the objective function and the constraint are linear.

Our first key result, which we call the "Raiding Lemma," characterizes the minimum resource budget for A to achieve any quality difference in a continuation equilibrium, depending on B's resource distribution schedule. This minimum budget is achieved by targeting the ranks in B which have the lowest resource-to-rank ratio (suitably adjusted to incorporate the equilibrium quality difference). A critical implication of the Raiding Lemma is that the minimum resource budget for A to achieve any quality difference in a continuation equilibrium depends on B's resource schedule only through the amount of resource received by the ranks with the lowest resource-to-rank ratio. Thus, if B wants to maximize the resource budget needed for A to achieve any quality difference in a continuation equilibrium, it should offer a resource distribution schedule that is linear in rank. An immediate implication is that for any quality difference, there is a unique budget-balanced resource distribution schedule for B that maximizes the minimum budget required for A's resource distribution schedule to be consistent with the quality difference in a continuation equilibrium. We refer to the resulting budget requirement for A as the "resource budget function," which is a function of quality difference.

Despite the characterization in the Raiding Lemma, finding all subgame perfect equilibria is difficult because for an arbitrary pair of resource distribution schedules, there are typically multiple continuation equilibria. In section 4 we focus on subgame perfect equilibria with the largest quality difference between the two organizations by adopting the

“A-dominant” criterion that selects the continuation equilibrium with the largest difference in average types in favor of A for any pair of schedules. Under this selection criterion, the unique candidate for equilibrium quality difference is the largest value of the difference in average types for which the budget function is below the exogenous resource budget available to A, and the associated resource distribution schedule for B is the unique candidate for equilibrium schedule. As suggested in the Raiding Lemma, there are many best responses of organization A to the candidate schedule for B. We construct a subgame perfect equilibrium by identifying a unique schedule for A to which the candidate schedule for B is a best response. This equilibrium schedule for A is also linear for ranks above some critical point, thus giving B no incentive to deviate and target the ranks in A with the lowest resource-to-rank ratio.

Our analysis has sharp predictions for both equilibrium resource distribution schedules and equilibrium sorting of talents. In our equilibrium, which we show is unique under the selection criterion, the targets of competition are the top talents; only these types receive positive shares of resources from either organization. Furthermore, equilibrium resource distribution schedules are systematically different between the high quality organization A and the low quality organization B. This is true even if the two organizations have the same resource budget, so long as the peer effect is sufficiently strong. The organization that in equilibrium attracts a higher average quality of talents has a more egalitarian distribution of resources than the low quality organization, which is disadvantaged by the peer effect and must concentrate its resources on a smaller set of top talents. The equilibrium sorting of talents exhibits mixing of top talents, with a greater share of them going to the high quality organization, while segregation occurs for all types that receive no resources in equilibrium, with the better types going to the high quality organization.

The results in our paper offer potentially testable implications for organization resource distributions and the sorting of talents. Comparative statics analysis reveals that if the resource advantage of the dominant organization becomes greater, or if the peer effect becomes more important in the talents' utility function, the equilibrium quality difference between the two organizations is greater. The weaker organization finds it more difficult to compete against the dominant organization, and as a result must focus its resources on fewer ranks at the top. This leads to steeper resource distribution schedules for both organizations.

We complete the introduction by stressing that our contributions in this paper are both a descriptive analysis of the implications of the peer effect for organization competition and the equilibrium sorting pattern, and a methodological one based on the quantile-quantile plot approach. Since the quantile-quantile approach is quite involved, after presenting the model in section 2 but before introducing the quantile-quantile plot approach in section 3, we present a description of the subgame perfect equilibrium and some of its key properties, and a heuristic explanation of why there are no profitable deviations. We hope to use section 2.2 to inform the reader who is primarily interested in the equilibrium organization strategies and talent sorting pattern, and to illustrate why the quantile-quantile plot approach is necessary to establish the equilibrium.

2. The Model

There are two organizations, A and B. Each organization $i = A, B$ is endowed with a measure 1 of positions and a fixed resource budget Y_i to be allocated among its members. We assume that $Y_A \geq Y_B$. There is a continuum of agents, of measure 2. Agents differ with respect to a one-dimensional characteristic, called "type" and denoted by θ, which may be interpreted as ability or productivity. The distribution of θ is uniform on the interval $[0, 1]$.

We consider an extensive game with perfect information of three stages. In the first stage (resource distribution stage), each organization i, $i = A, B$, chooses a "resource distribution schedule," which is a function $S_i : [0, 1] \to \mathbb{R}_+$ stipulating how Y_i is allocated among i's members according to their rank by type: For each $r \in [0, 1]$, $S_i(r)$ denotes the amount of resources received by an agent of type θ when a fraction r of the organization's members are of type smaller than θ.[3] We assume that each S_i is weakly increasing, that is, organizations can only adopt "meritocratic" resource distribution schedules in which

[3] We implicitly assume that the resource distribution schedules cannot directly depend on the type of agents. This and other assumptions are briefly discussed in section 5.

members of higher ranks receive at least as much resources as lower ranks. The monotonicity of S_i ensures that it is differentiable almost everywhere; we use $S_i'(r)$ to denote the derivative at rank r when it exists, and we make the additional technical assumption that S_i' has a countable number of discontinuities. The monotonicity assumption ensures integrability, and we require that it satisfies the resource constraint, i.e., $\int_{r=0}^{r=1} S_i(r)\mathrm{d}r \leq Y_i$. In the second stage (application stage) of the game, after observing the pair of distribution schedules chosen by the two organizations, all agents simultaneously apply to join either A or B. In the third and final stage (admissions stage), after observing the pool of applicants, each organization chooses the lowest type that will be admitted subject to the capacity constraint.

The payoff to each organization is zero if the capacity is not filled, and is given by the average type otherwise. The payoff to any agent that is not admitted by an organization is 0. If admitted by organization i, $i = A, B$, the payoff to an agent of type θ is given by

$$V_i(\theta) = \alpha S_i(r_i(\theta)) + m_i, \tag{1}$$

where m_i is the average type of agents in organization i, $r_i(\theta)$ is the quantile rank of the agent in i, and α is a positive constant that represents the weight on the concern for the resource he receives relative to the concern for the average type (the peer effect). The payoff is zero if an agent does not join either organization.

We have made two simplifying assumptions in setting up the game of organization competition. First, the type distribution is uniform; second, resources and peer average type are perfect substitutes in the payoff function of the agents. Neither assumption is conceptually necessary for a model of organization competition. However they are critical to make our approach in the analysis successful; this will become clear in section 3 when we introduce the quantile-quantile plot.

2.1. Continuation equilibrium

We begin the analysis with a characterization of the continuation equilibrium for a given pair of resource distribution schedules. In the continuation game following (S_A, S_B), if the outcome is a pair of type distribution functions (H_A, H_B) for A and B, then for any type

θ the rank in organization i, $i = A, B$, is given by $r_i(\theta) = H_i(\theta)$, and the average type in organization i is $m_i = \int \theta \, \mathrm{d}H_i(\theta)$. The payoffs V_A and V_B to type θ from joining A and B are defined as in (1), with $H_i(\theta)$ replacing $r_i(\theta)$. Thus, a continuation equilibrium is characterized by (H_A, H_B) that satisfy two conditions: (i) $H_A(\theta) + H_B(\theta) = 2\theta$ for all $\theta \in [0, 1]$; and (ii) if H_i is strictly increasing at θ and $H_j(\theta) > 0$, then $V_i(\theta) \geq V_j(\theta)$, for $i, j = A, B$ and $i \neq j$. Condition (i) is necessary because each agent's outside option is zero and each organization must fill all positions in any continuation equilibrium in which all agents join one organization. If condition (ii) does not hold, then because H_i is strictly increasing at θ some type just above θ is applying to organization i in the continuation equilibrium, and such type could instead profitably deviate by successfully applying to j. It is immediate that for any (H_A, H_B) that satisfies the above two conditions, there is a continuation equilibrium in terms of application and admission strategies.

In a continuation equilibrium an agent will join organization i whenever he prefers organization i and his type is higher than the lowest type in that organization. The types lower than the maximum of the lowest types in the two organizations do not have a choice as they are only acceptable to one organization. The other types are free to choose which organization to join, and are the targets of the organization competition. Because of the linearity in the payoff functions (1), which organization these types choose depends on the comparison of the resources they get from A and B, adjusted by the same factor determined by the quality difference $m_A - m_B$. Thus, existence of a continuation equilibrium for any (S_A, S_B) can be established by a fixed point argument, as follows. For any difference $m_A - m_B$, there is a unique pair of type distribution functions (H_A, H_B) that satisfies condition (i) and (ii) above. To see this, note that regardless of choices of other types, the highest type $\theta = 1$ chooses A if $S_A(1) > S_B(1) - (m_A - m_B)/\alpha$, B if the opposite inequality holds strictly, and mixes if it is an equality. For any type $\theta < 1$, the choice between A and B depends on how the type is ranked in the two organizations, and thus only on the choices made by higher types. Assigning types from the top of the distribution yields a unique pair of distributions functions (H_A, H_B), which implies a new quality difference. A continuation equilibrium is a fixed point in this mapping.

Typically multiple continuation equilibria exist for a given pair of resource distribution schedules. For example, if the two distribution schedules are identical, there is a continuation equilibrium with zero quality difference. For a sufficiently large peer effect (sufficiently small α), there is another continuation equilibrium with perfect segregation, generating the maximum possible quality difference. Moreover the equilibrium with maximal quality difference is "stable" to small perturbations in distribution functions (H_A, H_B) while the equilibrium with zero quality difference is "unstable." In this paper we construct a subgame perfect equilibrium by adopting the criterion that selects the continuation equilibrium with the largest quality difference in favor of A for any pair of resource distribution schedules. We refer to this criterion as "A-dominant."

2.2. A heuristic description of equilibrium schedules

The unique subgame perfect equilibrium in our model is generally asymmetric, with a positive quality difference $m_A^* - m_B^*$. To make an agent indifferent between organization A and organization B, the resource that the agent receives from organization B must exceed what he receives from A by a "peer premium" P^*, given by $(m_A^* - m_B^*)/\alpha$. We find that the equilibrium resource schedules S_A^* and S_B^* have the following properties:

(i) There is a threshold rank r_i^*, $i = A, B$, such that agents below that rank in organization i receives no resource from the organization.

(ii) Adjusted for the peer premium, the top rank agent in each organization receives the same resources; i.e., $S_A^*(1) + P^* = S_B^*(1)$.

(iii) The resource-to-rank ratio in A adjusted by the peer premium, $(S_A^*(r) + P^*)/r$, is constant for all ranks above r_A^*.

(iv) $S_B^*(r)$ is linear above the threshold rank r_B^*.

Figure 1 illustrates the equilibrium schedules S_A^* and S_B^*, with S_A^* shifted up by the premium P^*. Note that $r_A^* < r_B^*$, so that organization A has more ranks receiving resources than B, but S_A^* is flatter than S_B^* above the corresponding threshold ranks.

The equilibrium sorting of types, represented by a pair of distribution functions (H_A^*, H_B^*), is as follows: all types $\theta < \frac{1}{2} r_B^*$ are forced to join organization B, even though they receive no resources from either organization and strictly prefer A to B; all types $\theta \in \left[\frac{1}{2} r_B^*, \frac{1}{2}(r_B^* + r_A^*)\right)$ choose A exclusively; types $\theta \geq \frac{1}{2}(r_B^* + r_A^*)$ mix between A and B in such a way that each θ is indifferent between rank $H_A^*(\theta)$ and rank $H_B^*(\theta)$, with

$$S_A^*(H_A^*(\theta)) + P^* = S_B^*(H_B^*(\theta)),$$

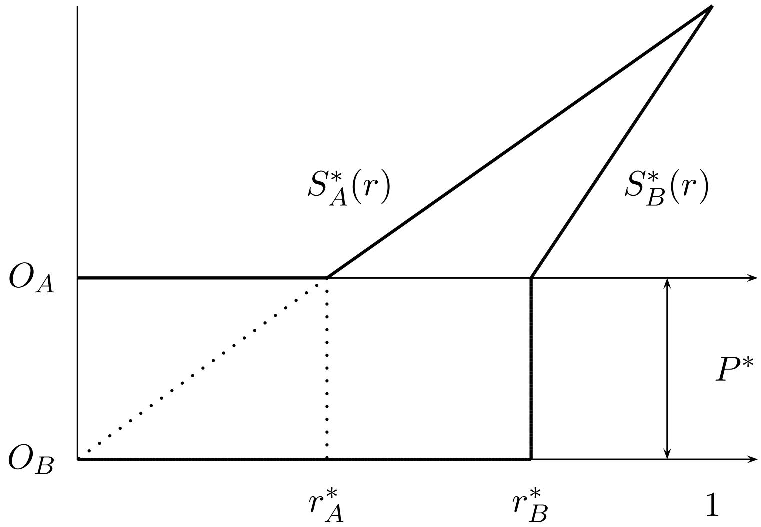

Figure 1

which, by linearity of the two schedules, implies that a constant fraction $(1-r_A^*)/(2-r_B^*-r_A^*) > \frac{1}{2}$ of each type joining A. These equilibrium distribution functions (H_A^*, H_B^*) allow us to calculate the mean type in each organization, with the equilibrium quality difference:

$$m_A^* - m_B^* = \frac{1}{2} - \frac{1}{2}(1+r_A^*)(1-r_B^*). \tag{2}$$

Under properties (i)-(iv) above and given the peer premium P^*, the equilibrium schedules are uniquely identified by three variables, r_A^*, r_B^* and $S_B^*(1)$, and determined by the two budget constraints

$$\begin{aligned} \frac{1}{2}(1-r_A^*)(S_B^*(1)-P^*) &= Y_A, \\ \frac{1}{2}(1-r_B^*)(S_B^*(1)+P^*) &= Y_B; \end{aligned} \tag{3}$$

and by property (iii), which is equivalent to

$$S_B^*(1) r_A^* = P^*. \tag{4}$$

Finally, the peer premium P^* has to be consistent with the equilibrium quality difference (2) derived from sorting.

In this subsection, we focus on providing a heuristic explanation for the properties of the equilibrium resource distribution schedules, leaving the formal construction of such an equilibrium to sections 3 and 4. Property (i) is an intuitive consequence of organization competition for talents with a fixed amount of resources. Because of the benefits that high quality agents confer to the rest of the members of an organization, organizations put the bulk of their resources to attract higher types who naturally occupy higher ranks in the organizations. The zero reservation value of agents implies that low types are captives to organization B and receive no resources in equilibrium. The low rank agents in organization A strictly prefers A to B due to the peer premium, and therefore there is no need to compensate them with organization resources. Property (ii) of the equilibrium schedules is also intuitive. If $S_A^*(1) + P^*$ is not equal to $S_B^*(1)$, then then one of the two organizations is wasting its resources on the top quality agent in the talent pool.

Property (iii) is key to our characterization of equilibrium schedules. It requires S_A^* to be linear in r for $r \geq r_A^*$, and the premium adjusted resource-to-rank ratio at $r = r_A^*$ to be equal to the same ratio at $r = 1$, which implies $S_B^*(1)r_A^* = P^*$. We claim that given S_A^*, for any deviation schedule S_B with the property that no ranks receiving any resources get less than P^* or more than $S_A^*(1) + P^*$, there is a continuation equilibrium with a quality difference equal to the equilibrium difference $m_A^* - m_B^*$. Under our selection criterion, this claim then establishes that B cannot benefit from any deviation from S_B^*. Further, if S_A^* is linear but does not satisfy property (iii), then there is a deviation schedule for B such that the sorting may produce a smaller quality difference. The formal argument is difficult because the space of possible deviation schedules is large. It will be presented in the proof of Proposition 3 after introducing the quantile-quantile plot approach. Here, we illustrate the logic of the argument through an example.

Given S_A^*, let $\tilde{S}_B(r)$ be a step function that "targets" $\tilde{r}_A$ (where $\tilde{r}_A \geq r_A^*$) by giving the same amount that rank $\tilde{r}_A$ receives from A, adjusted by the peer premium P^*, to ranks above some $\tilde{r}_B$ and no resources to all lower ranks. That is,

$$\tilde{S}_B(r) = \begin{cases} S_A^*(\tilde{r}_A) + P^* & \text{if } r \geq \tilde{r}_B = 1 - Y_B/(S_A^*(\tilde{r}_A) + P^*); \\ 0 & \text{otherwise} \end{cases}$$

where the expression for $\tilde{r}_B$ follows from the budget constraint. This is a simple and special deviation strategy, but the Lemma 4 in the subsequent section establishes that such targeted raiding of talents is optimal. With such a raiding strategy by B, types above some cutoff θ_1 all join A and occupy ranks above $\tilde{r}_A$, and so $2(1 - \theta_1) = 1 - \tilde{r}_A$;

types below θ_1 but above a lower cutoff θ_2 all join B to occupy ranks above $\tilde{r}_B$, and so $2(\theta_1 - \theta_2) = 1 - \tilde{r}_B$; types below θ_2 and above a third cutoff θ_3 all join A to occupy ranks below $\tilde{r}_A$, and so $2(\theta_2 - \theta_3) = \tilde{r}_A$; and finally, types below θ_3 all join B. The implied quality difference is

$$\tilde{m}_A - \tilde{m}_B = 2\tilde{m}_A - 1 = (\theta_1 + 1)(1 - \tilde{r}_A) + (\theta_2 + \theta_3)\tilde{r}_A - 1 = \frac{1}{2} - \tilde{r}_A(1 - \tilde{r}_B).$$

Using the expression for $\tilde{r}_B$, we have

$$\tilde{m}_A - \tilde{m}_B = \frac{1}{2} - \frac{Y_B r_A^* \tilde{r}_A (1 - r_A^*)}{P^* \tilde{r}_A (1 - r_A^*) + (S_B^*(1) r_A^* - P^*)(\tilde{r}_A - r_A^*)}.$$

For comparison, we rewrite the equilibrium quality difference (2) using the budget constraint for B in (3) as

$$m_A^* - m_B^* = \frac{1}{2} - \frac{Y_B r_A^* (1 + r_A^*)}{P^*(1 + r_A^*) + (S_B^*(1) r_A^* - P^*)}.$$

Thus, if S_A^* satisfies property (iii), $\tilde{m}_A - \tilde{m}_B$ is equal to $m_A^* - m_B^*$ for all $\tilde{r}_A$. If instead $S_B^*(1) r_A^* < P^*$, then the difference $\tilde{m}_A - \tilde{m}_B$ evaluated at $\tilde{r}_A = 1$ is strictly smaller than $m_A^* - m_B^*$, implying that organization B should target rank 1 which has the smallest resource-to-rank ratio in A. In the opposite case of $S_B^*(1) r_A^* > P^*$, the difference $\tilde{m}_A - \tilde{m}_B$ evaluated at $\tilde{r}_A = r_A^*$ is strictly smaller than $m_A^* - m_B^*$, which means that B can potentially benefit by targeting rank r_A^* because it has the smallest resource-to-rank ratio.

Property (iv) is similar to property (iii). Since organization B has to pay at least P^* to be competitive against A, the linearity of S_B^* above r_B^* in property (iv) implies that the resource-to-rank ratio in B from A's perspective, given by $(S_B^*(r) - P^*)/(r - r_B^*)$, is constant for all ranks above r_B^*. Using a similar argument as the above for property (iii), we can illustrate the claim that if S_B^* satisfies property (iv), there is a continuation equilibrium with a quality difference equal to the equilibrium difference $m_A^* - m_B^*$ for any deviation schedule of A with the property that ranks receiving resources get no more than $S_B^*(1) - P^*$. However, this claim does not imply that there is no deviation schedule S_A that, against S_B^*, produces a quality difference higher than $m_A^* - m_B^*$ as a fixed point of the mapping from quality difference to quality difference. Thus, under our selection criterion, we cannot conclude directly from the claim that S_A^* is a best response against S_B^*. To illustrate this point, suppose that $Y_A = Y_B = \frac{1}{2}$, and consider $\tilde{m}_A - \tilde{m}_B = 0$ and $\tilde{S}_A(r) = \tilde{S}_B(r) = r$. The schedule $\tilde{S}_B(r)$ satisfies property (iv) for $\tilde{m}_A - \tilde{m}_B = 0$ because the premium is zero, which implies that for any schedule S_A there is a continuation equilibrium with zero quality difference against $\tilde{S}_B$. However, for any α between $\frac{1}{2}$ and 1, if A chooses a flat schedule that gives $\frac{1}{2}$ to each rank including the lowest, there is a continuation equilibrium resulting in perfect segregation: if $m_A = \frac{3}{4}$ and $m_B = \frac{1}{4}$, the median type prefers rank 0 in A with a payoff of $\frac{1}{2}\alpha + \frac{3}{4}$, to rank 1 in B with a payoff of $\alpha + \frac{1}{4}$. In contrast, for the same values of α, there is no continuation equilibrium with quality difference equal to $\frac{1}{2}$ under the pair of schedules $\tilde{S}_A(r) = \tilde{S}_B(r) = r$, because even with $m_A = \frac{3}{4}$ and $m_B = \frac{1}{4}$, the median type strictly prefers rank 1 in B with a payoff of $\alpha + \frac{1}{4}$ to rank 0 in A with a payoff of $\frac{3}{4}$. This example illustrates the additional complication involved in characterizing A's best response to a fixed S_B due to the problem of multiple continuation equilibria in the presence of the peer effect. The conclusion that $S_A^*(r)$ is a best response against $S_B^*(r)$ when the latter satisfies property (iv) for the equilibrium quality difference $m_A^* - m_B^*$ is obtained by first using the quantile-quantile approach to characterize in Proposition 1 the minimum budget of resources needed to achieve any fixed quality difference, and then showing in Proposition 2 that against S_B^*, any greater quality difference than $m_A^* - m_B^*$ requires more resources than Y_A.

3. The Quantile-Quantile Plot Approach

Definition 1. Given a pair of type distribution functions (H_A, H_B), the associated quantile-quantile plot $t : [0,1] \to [0,1]$, is given by

$$t(r) \equiv 1 - H_A\left(\inf\{\theta : H_B(\theta) = r\}\right). \tag{5}$$

The above definition associates to each pair of type distribution a unique non-increasing function on the unit interval. The variable $t(r)$ is the fraction of agents in organization A of type higher than the type with rank r in organization B.[4] For example, if the distribution

[4] The quantile-quantile plot is usually defined as $1 - t(r)$. We use the non-standard definition for convenience of notation.

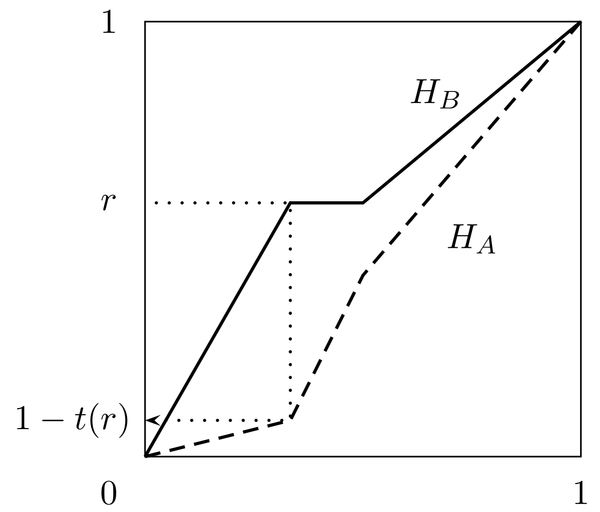

Figure 2

of talents is perfectly segregated with the higher types exclusively in organization A, then $t(r) = 1$ for all r; and if there is even mixing so that the distribution of types is identical across the two organizations, then $t(r) = 1 - r$ for every r. The infimum operator in the definition (5) is a convention to handle the case where H_B is flat over some interval, that is, when there is local segregation with all types in the interval going to organization A. See Figure 2 for a graphical illustration of the quantile-quantile plot.

Conversely, each non-increasing function $t : [0, 1] \to [0, 1]$ identifies a unique pair of type distribution functions (H_A, H_B), with $H_A(\theta) = 2\theta - H_B(\theta)$ for all θ, where H_B is given by

$$H_B(\theta) = \begin{cases} 0 & \text{if } 2\theta \leq 1 - t(0), \\ \sup\{r : 2\theta \geq r + 1 - t(r)\} & \text{if } 1 - t(0) < 2\theta < 2 - t(1), \\ 1 & \text{if } 2\theta \geq 2 - t(1). \end{cases} \quad (6)$$

For any type θ around which H_B is increasing, which means that types around θ are not joining organization A exclusively, $H_B(\theta)$ is uniquely defined by r such that $r+1-t(r) = 2\theta$; if there is no solution to the equation, then $H_B(\theta)$ is either 0 or 1. For any type θ such that H_B is flat just above θ, then $H_B(\theta)$ is given by 1 minus the rank of the highest type that does not exclusively join A.

Thus, there is a one-to-one mapping from a pair of type distribution functions to a non-increasing function on the unit interval. The convenience of working with quantile-quantile plot is made explicit by the following lemma, where we show that, for any pair of type distribution functions, the difference in average type between the two organizations only depends on the integral of the associated quantile-quantile plot.[5]

Lemma 1. *Let* (H_A, H_B) *be a pair of type distribution functions such that* $H_A(\theta) + H_B(\theta) = 2\theta$, *and* t *the associated quantile-quantile plot. Then*

$$m_A - m_B = -\frac{1}{2} + \int_0^1 t(r)\ \mathrm{d}r.$$

Proof. Using the definition of $t(r)$ and a change of variable $\theta = \inf\{\theta' : H_B(\theta') = r\}$, we can write

$$-\frac{1}{2} + \int_0^1 t(r)\ \mathrm{d}r = \frac{1}{2} - \int_{\underline{\theta}_B}^{\overline{\theta}_B} H_A(\theta)\ \mathrm{d}H_B(\theta),$$

where $\underline{\theta}_B = \sup\{\theta : H_B(\theta) = 0\}$ and $\overline{\theta}_B = \inf\{\theta : H_B(\theta) = 1\}$. Since $H_A(\theta) = 2\theta - H_B(\theta)$, the right-hand-side of the above equation is equal to

$$\frac{1}{2} - 2m_B + \int_{\underline{\theta}_B}^{\overline{\theta}_B} H_B(\theta)\ \mathrm{d}H_B(\theta).$$

The claim then follows immediately from the fact that $m_A + m_B = 1$. *Q.E.D.*

By Lemma 1, since the difference in average types is the integral of the quantile-quantile plot minus $\frac{1}{2}$, we will also refer to the integral

$$T = \int_0^1 t(r)\ \mathrm{d}r$$

as the "quality difference." Since the agents' payoff function is linear, the function

$$P(T) = \frac{1}{\alpha}\left(T - \frac{1}{2}\right)$$

can be interpreted as the peer premium of A over B, in that any agent would be just indifferent between the two if the agent receives from B a resource greater than what he receives from A by that premium.

[5] The definition of quantile-quantile plot does not rely on the assumption that θ is distributed uniformly on $[0, 1]$. However, the representation of a pair of type distribution functions through its associated quantile-quantile plot is not generally useful because the difference in average type cannot be written as an integral of the quantile-quantile plot.

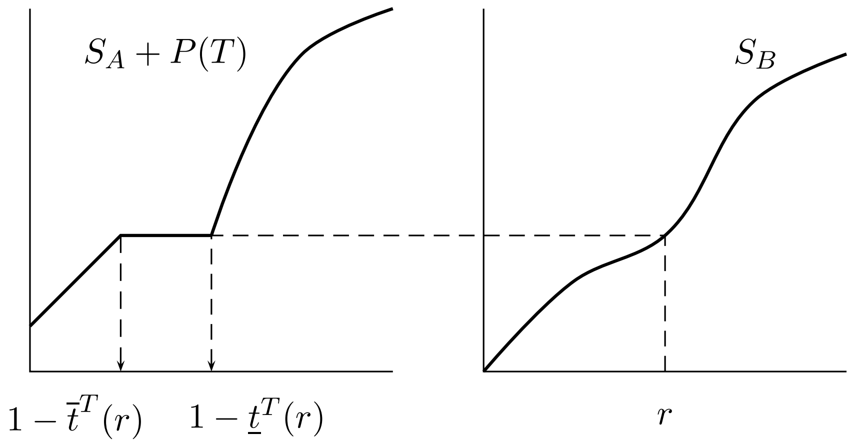

Figure 3

Now we provide a characterization of a continuation equilibrium for given S_A and S_B in terms of a fixed point in the quality difference T. For any quality difference $T \in [0,1]$, let $\underline{t}^T$ and $\bar{t}^T$ be the quantile-quantile plots defined as

$$\underline{t}^T(r) = \begin{cases} 1 & \text{if } S_A(0) + P(T) > S_B(r), \\ 1 - \sup\{\tilde{r} \in [0,1] : S_A(\tilde{r}) + P(T) \leq S_B(r)\} & \text{otherwise;} \end{cases}$$

and

$$\bar{t}^T(r) = \begin{cases} 0 & \text{if } S_A(1) + P(T) < S_B(r), \\ 1 - \inf\{\tilde{r} \in [0,1] : S_A(\tilde{r}) + P(T) \geq S_B(r)\} & \text{otherwise.} \end{cases}$$

The functions $\underline{t}^T(r)$ and $\bar{t}^T(r)$, are the lower bound and the upper bound on the quantile-quantile plot consistent with a continuation equilibrium with quality difference T. The agent who has rank r in B must have rank at most $1 - \underline{t}^T(r)$ in A or he would prefer to switch; he must also have rank at least $1 - \bar{t}^T(r)$ in A or otherwise some agent from A would want to switch. See Figure 3. It is immediate that given (S_A, S_B), a pair of type distribution functions (H_A, H_B) with $H_A(\theta) + H_B(\theta) = 2\theta$, is a continuation equilibrium if and only if the associated quantile-quantile plot t satisfies $\underline{t}^T(r) \leq t^T(r) \leq \bar{t}^T(r)$ for all $r \in [0,1]$, for $T = \int_0^1 t(r)\ \mathrm{d}r$. Thus, a continuation equilibrium for any (S_A, S_B) is given by a quality difference $T \in [0,1]$ that is a fixed point of the correspondence

$$D(T) = \left[\int_0^1 \underline{t}^T(r)\ \mathrm{d}r, \int_0^1 \bar{t}^T(r)\ \mathrm{d}r \right]. \tag{7}$$

Existence of a fixed point follows from an application of Tarski's fixed point theorem.

3.1. The Raiding Lemma

In this subsection, we solve the problem of minimizing the amount of resources organization A needs to have a continuation equilibrium with a fixed quality difference $T \geq \frac{1}{2}$ against any fixed resource distribution schedule S_B for organization B. The solution provides a characterization of the cheapest way for A to raid organization B for its talents, a result we refer to as the "Raiding Lemma." The resulting minimum resource budget for A is denoted as $C(T; S_B)$. The approach we take is to work with quantile-quantile plot to construct a continuation equilibrium consistent with quality difference T against S_B, which transforms the problem of finding the cheapest S_A into the problem of finding the corresponding quantile-quantile plot.

First, suppose that $S_B(T) \leq P(T)$. By the definition of $\underline{t}^T$, we have $\underline{t}^T(r) = 1$ for all r such that $S_B(r) < P(T)$, and thus there is no continuation equilibrium with quality difference T regardless of the resource distribution schedule S_A. Moreover, even if A gives no resources to all of its ranks, there is a continuation equilibrium with quality difference strictly larger than T. In this case, we write $C(T; S_B) = 0$. Second, suppose $S_B(T) \geq P(T)$. Then, for any quantile-quantile plot t, with $\int_0^1 t(r)\ \mathrm{d}r = T$ and $t(r) = 1$ for any r such that $S_B(r) < P(T)$, let S_A^t be the pointwise smallest resource distribution schedule that satisfies

$$S_A^t(1 - t(r)) \geq \max\{S_B(r) - P(T), 0\} \ \text{ for all } r \in [0,1]. \tag{8}$$

By construction, given (S_A^t, S_B) we have $\underline{t}^T(r) \leq t(r) \leq \bar{t}^T(r)$ for all $r \in [0,1]$. It follows that T is a fixed point of the mapping (7) and hence there exists a continuation equilibrium with quality difference T for the pair of schedules (S_A^t, S_B).

By construction, S_A^t is the resource distribution schedule with the minimum integral for which there is a continuation equilibrium with quantile-quantile plot t. It follows that

$$\begin{aligned} &C(T; S_B) = \min_t \int_0^1 S_A^t(r)\ \mathrm{d}r \\ &\text{subject to } \int_0^1 t(r)\mathrm{d}r = T; \ \text{ and } t(r) = 1 \text{ if } S_B(r) < P(T). \end{aligned} \tag{9}$$

After a change of variable $\tilde{r} = t^{-1}(1-r)$ and integration by parts, we have

$$\int_0^1 S_A^t(r)\ \mathrm{d}r = -\int_{t^{-1}(1)}^{t^{-1}(0)} \Delta(\tilde{r})t'(\tilde{r})\ \mathrm{d}\tilde{r} = \int_{t^{-1}(1)}^{t^{-1}(0)} t(\tilde{r})\Delta'(\tilde{r})\ \mathrm{d}\tilde{r} - \Delta(t^{-1}(1)),$$

where for notational convenience we have defined

$$\Delta(\tilde{r}) = \max\{S_B(\tilde{r}) - P(T), 0\}$$

as the premium-adjusted resource distribution schedule of B. We can then rewrite the minimization problem (9) as

$$\begin{aligned} &\min_t \int_{t^{-1}(1)}^{t^{-1}(0)} t(r)\Delta'(r)\ \mathrm{d}r - \Delta(t^{-1}(1)) \\ &\text{s.t.} \int_0^1 t(r)\mathrm{d}r = T, \end{aligned} \tag{10}$$

where we have dropped the second constraint of (9) since it will be satisfied by any solution to (10).

In the minimization problem (10), both the objective function and the constraint are linear in the choice variable t. This feature is used extensively to characterize the solution of (10). The next lemma, proved in the appendix, establishes two properties of the solution. First, there is always a solution to (10) which takes the form in t of a step function. In particular, if the quality-adjusted schedule Δ of B is locally concave in the neighborhood of some rank r, then a quantile-quantile plot that is flat around r does better than any decreasing plot; and if Δ is locally convex around r, then a t that is a step function with two values in the neighborhood of r does better. Second, due to the linear nature of (10), there exists a step function t that solves (10) with at most one value strictly between 0 and 1. This simple characterization is why we have chosen to deal with the quantile-quantile plot instead of a pair of type distribution functions directly. The linear programming nature of the minimization problem is a result of the two key assumptions: the type distribution is uniform, and the agents' payoff function is linear. Without either assumption our quantile-quantile plot approach would not be analytically advantageous.

Lemma 2. *There exists a solution t to (10) which assumes at most one value strictly between 0 and 1.*

Now we use the above lemma to provide an explicit characterization of the solution to (10) and a value for $C(T;S_B)$. The lemma implies that we can restrict the search for a solution to (10) to two cases. In the first case, consider a quantile-quantile plot t that has just one positive value. In this case t is entirely characterized by its only discontinuity point, say $\hat{r}$, because the constraint $\int_0^1 t(r)\ \mathrm{d}r = T$ and the assumption of $t(r) = 0$ for $r > \hat{r}$, imply that $t(r) = T/\hat{r}$ for all $r \le \hat{r}$. Also note that $\hat{r} \ge T$ must hold in this case. In the second case, consider a quantile-quantile plot t that has one value strictly between 0 and 1, with two discontinuity points defined as $r^1 = \sup\{r : t(r) = 1\}$ and $r^0 = \sup\{r :\ t(r) > 0\}$. Using the constraint $\int_0^1 t(r)\ \mathrm{d}r = T$, we have $t(r) = (T - r^1)/(r^0 - r^1)$ for $r \in (r^1, r^0)$. Note that $r^1 \le T \le r^0$ in this case. Thus,

$$C(T;S_B) = \min\left\{\min_{\hat{r}\ge T} \frac{T}{\hat{r}}\Delta(\hat{r}),\ \min_{T\ge r^1\ge 0;\ 1\ge r^0\ge T} \Delta(r^1) + \frac{T-r^1}{r^0-r^1}\left(\Delta(r^0)-\Delta(r^1)\right)\right\}. \quad (11)$$

Using the above characterization for $C(T;S_B)$ it is possible to obtain a solution to problem (10) depending on Δ. To do so, let $\underline{\Delta}$ be the largest convex function with $\underline{\Delta}(0) = 0$ which is pointwise smaller than Δ. In other words, $\underline{\Delta}$ is obtained as the lower contour of the convex hull of the function Δ and the origin:

$$\underline{\Delta}(r) = \min\{y :\ (r,y) \in co(\{(\tilde{r},\tilde{y}) : 0 \le \tilde{r} \le 1; \tilde{y} \ge \Delta(\tilde{x})\} \cup (0,0))\}.$$

The next lemma provides a simple characterization of the discontinuity points of a solution to (10) which depends only on the functions Δ and $\underline{\Delta}$. In particular, it states that if $\Delta(T) = \underline{\Delta}(T)$, then there is a solution to (10) with only one discontinuity point at exactly T. The solution is a step function equal to 1 for $r \le T$ and equal to 0 for $r > T$. When $\Delta(T) > \underline{\Delta}(T)$ instead, there is a solution t to (10) such that t has two discontinuity points $r^1 < T$ and $r^0 > T$. The two discontinuity points are the largest $r < T$ and the smallest $r > T$ at which the function Δ coincides with the function $\underline{\Delta}$. The solution t equals 1 up to r^1 and becomes 0 at r^0.

Lemma 3. (The Raiding Lemma) *Let $Q = \{r : \Delta(r) = \underline{\Delta}(r)\} \cup \{0,1\}$, and denote as $\overline{Q}$ the closure of Q. (i) If $T \in \overline{Q}$, then the following quantile-quantile plot with one discontinuity point at $\hat{r} = T$ solves (10):*

$$t(r) = \begin{cases} 1 & \text{if } r \le T; \\ 0 & \text{otherwise.} \end{cases}$$

(ii) If $T \notin \overline{Q}$, the following quantile-quantile plot with two discontinuity points at $r^1 = \sup\{r \in \overline{Q} : \ r < T\}$ and $r^0 = \inf\{r \in \overline{Q} : \ r > T\}$ solves (10):

$$t(r) = \begin{cases} 1 & \text{if } r < r^1; \\ (T - r^1)/(r^0 - r^1) & \text{if } r^1 \leq r \leq r^0; \\ 0 & \text{if } r > r^0. \end{cases}$$

In both cases, the value of the objective function is $C(T; S_B) = \underline{\Delta}(T)$.

Proof. First we show that the objective function assumes the value $\underline{\Delta}(T)$ at the claimed solution in both cases. In the first case, at the claimed solution, the value of the objective function is $\Delta(T)$ which is equal to $\underline{\Delta}(T)$ because $T \in \overline{Q}$. In the second case, when $T \notin \overline{Q}$, at the claimed solution the value of the objective function is given by

$$\frac{r^0 - T}{r^0 - r^1}\Delta(r^1) + \frac{T - r^1}{r^0 - r^1}\Delta(r^0) = \frac{r^0 - T}{r^0 - r^1}\underline{\Delta}(r^1) + \frac{T - r^1}{r^0 - r^1}\underline{\Delta}(r^0) = \underline{\Delta}(T),$$

where the second equality holds because $\underline{\Delta}$ is linear between r^1 and r^0.

Next we show that (11) is never smaller than $\underline{\Delta}(T)$. First, for all $r \geq T$,

$$\underline{\Delta}(T) \leq \frac{r - T}{r}\underline{\Delta}(0) + \frac{T}{r}\underline{\Delta}(r) \leq \frac{T}{r}\Delta(r),$$

where the first inequality follows from the fact that $\underline{\Delta}$ is convex and the second from $\underline{\Delta}(r) \leq \Delta(r)$ for all r. Similarly, for all $r^1 \leq T$ and $r^0 \geq T$, we have

$$\underline{\Delta}(T) \leq \frac{r^0 - T}{r^0 - r^1}\underline{\Delta}(r^1) + \frac{T - r^1}{r^0 - r^1}\underline{\Delta}(r^0) \leq \frac{r^0 - T}{r^0 - r^1}\Delta(r^1) + \frac{T - r^1}{r^0 - r^1}\Delta(r^0).$$

Thus the claimed solution minimizes (11). *Q.E.D.*

Figure 4 illustrates the second case of the above lemma. The quality difference to be achieved for organization A is T. The discontinuity points of the quantile-quantile plot in the statement of the lemma, r^0 and r^1, are identified in the diagram. For any other quantile-quantile plot with discontinuity points $\tilde{r}^0$ and $\tilde{r}^1$, the resource requirement for the plot to be a continuation equilibrium is greater. This is because from (11) the resource requirement is equal to a weighted average of the quality-adjusted resource schedule of B at $\tilde{r}^1$ and $\tilde{r}^0$, with the weights such that the average of $\tilde{r}^1$ and $\tilde{r}^0$ equals T. Similarly, for

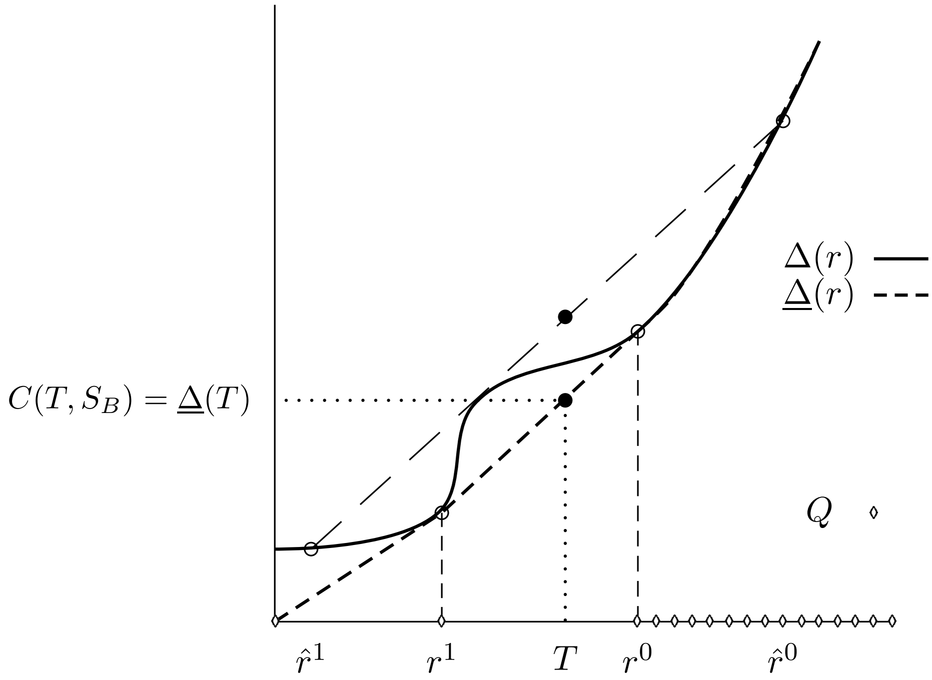

Figure 4

any quantile-quantile plot with a single discontinuity point, say $\hat{r}$, the resource requirement for the plot to be a continuation equilibrium is also greater, because it is an average of zero and the quality-adjusted resource schedule of B at $\hat{r}$, with the weights such that average of zero and $\hat{r}$ equals T.

Given the solution $t(r)$ to (10) characterized by the Raiding Lemma, we can recover the resource distribution schedule $S_A^t(r)$ through (8). In the first case of the lemma, (8) implies that $S_A^t(r) = \Delta(T)$ for all r. Thus S_A^t is flat and is such that the lowest rank in A is just indifferent between staying in A at rank 0 and switching to B for rank T. The resulting continuation equilibrium exhibits segregation with types between $\frac{1}{2}T$ and $\frac{1}{2}(1+T)$ joining A and every other type joining B. The interpretation is that A achieves the quality difference T by targeting a single rank $\hat{r} = T$ in organization B, and giving to all ranks in A the minimum amount of resources needed to make them indifferent between joining A and joining B at rank T. This solves the resource minimization problem for A because rank T in B has the lowest resource-to-rank ratio. In the second case of the

Raiding Lemma, we have

$$S_A^t(r) = \begin{cases} \Delta(r^1) & \text{if } r \le 1-(T-r^1)/(r^0-r^1); \\ \Delta(r^0) & \text{if } r > 1-(T-r^1)/(r^0-r^1). \end{cases} \tag{12}$$

That is, S_A^t is a step function with two levels, such that the lowest rank in A is indifferent between staying and joining B at rank r^1, and the lowest rank receiving the higher level of resources in A is indifferent between staying and joining B at rank r^0. In the resulting continuation equilibrium, types between $\frac{1}{2}r^1$ and $\frac{1}{2}(r^1+(r^0-T)/(r^0-r^1))$, and types between $\frac{1}{2}(r^0+(r^0-T)/(r^0-r^1))$ and $\frac{1}{2}(1+r^0)$ join organization A, and all other types join B. Organization A achieves quality difference T by targeting two ranks in B, r^1 and r^0, and giving the minimum resources to its own ranks between 0 and $(r^0-T)/(r^0-r^1)$ so that they are indifferent between staying and joining B at rank r^1, and to all higher ranks the minimum resources so that they are indifferent between staying and joining B at rank r^0. This solves the resource minimization problem for A because ranks r^1 and r^0 in B have the lowest resource-to-rank ratios.

3.2. The resource budget function

First, we characterize the resource distribution schedule S_B that maximizes the minimum amount of resources needed by organization A to have a continuation equilibrium with quality difference T. Formally, for each $T \ge \frac{1}{2}$, we study the maximization problem

$$\begin{aligned} E(T) \equiv & \max_{S_B} C(T; S_B) \\ & \text{subject to } \int_0^1 S_B(r)\, \mathrm{d}r \le Y_B. \end{aligned} \tag{13}$$

The Raiding Lemma suggests that organization B would be wasting its resources if it chooses a schedule S_B such that $\Delta(r) > \underline{\Delta}(r)$ for some r. Our next result shows that, as a result, there is a solution S_B to (13) such that $S_B(r)$ is 0 for all r below some critical rank $\tilde{r}$, equals $P(T)$ at $\tilde{r}$, and has a constant slope between $\tilde{r}$ and 1.

Lemma 4. *Given any resource distribution schedule S_B that satisfies the budget constraint, there exists another resource distribution schedule $\tilde{S}_B$ given by*

$$\tilde{S}_B(r) = \begin{cases} 0 & \text{if } r < \tilde{r}, \\ P(T) + \beta(r-\tilde{r}) & \text{if } r \ge \tilde{r}; \end{cases} \tag{14}$$

for some $\tilde{r} \in [0,1]$ and $\beta \ge 0$, such that $\int_0^1 \tilde{S}_B(r)\, \mathrm{d}r = Y_B$ and $C(T; \tilde{S}_B) \ge C(T; S_B)$.

Proof. Let $\Delta_{S_B}(r) = \max\{S_B(r) - P(T), 0\}$. First, since $C(T; S_B)$ only depends on $\Delta_{S_B}(r)$, we have $C(T; S_B) \geq C(T; \tilde{S}_B)$ for any $\tilde{S}_B$ such that $\tilde{S}_B(r) = 0$ whenever $\tilde{S}_B(r) < P(T)$. Second, by the Raiding Lemma, $C(T; S_B) = \underline{\Delta}_{S_B}(T)$ for any resource distribution schedule S_B, implying that $C(T; S_B) = C(T; \tilde{S}_B)$ for any $\tilde{S}_B$ such that $\Delta_{\tilde{S}_B} = \underline{\Delta}_{S_B}$. Thus for any S_B, there is a resource distribution schedule $\tilde{S}_B$ which is convex whenever positive and $\Delta_{\tilde{S}_B}(0) = 0$ such that $C(T; \tilde{S}_B) \geq C(T; S_B)$. Finally, for any S_B that is convex whenever positive and $\Delta_{S_B}(0) = 0$, there is an $\tilde{S}_B$ which is linear when positive such that $C(T; \tilde{S}_B) \geq C(T; S_B)$. The lemma then immediately follows from the resource constraint because binding the constraint increases the resource requirement for A. *Q.E.D.*

By Lemma 4, we can restrict to resource distribution schedules of the form (14) when characterizing a solution to (13). In other words, solving (13) boils down to finding the point of discontinuity $\tilde{r}$ of the S_B schedule. When S_B has a discontinuity at $\tilde{r}$, the resource constraint for B requires the slope of $S_B(r)$ to be

$$\beta = \frac{2}{(1-\tilde{r})^2}[Y_B - P(T)(1-\tilde{r})].$$

Given any such schedule, the quality-adjusted resource distribution schedule of B is 0 for all $r < \tilde{r}$ and equal to $\beta(r - \tilde{r})$ for all $r \geq \tilde{r}$. It then follows from the Raiding Lemma that $C(T; S_B) = \beta(T - \tilde{r})$ for all $\tilde{r} \leq T$, and $C(T; S_B) = 0$ otherwise. Thus, the value of any solution to (13), $E(T)$, is given by

$$E(T) = \max_{\tilde{r}\in[0,T]} \frac{2(T-\tilde{r})}{(1-\tilde{r})^2}\left[Y_B - P(T)(1-\tilde{r})\right]. \tag{15}$$

It is straightforward to show that the solution to the maximization problem (15) is given by a discontinuity point $\tilde{r} = r(T)$, defined as

$$r(T) = \begin{cases} T & \text{if } Y_B \leq P(T)(1-T); \\ 1 - 2Y_B(1-T)/[Y_B + P(T)(1-T)] & \text{otherwise.} \end{cases} \tag{16}$$

Thus, if organization B's resource budget is not enough to cover the peer premium $P(T)$ for all ranks above T, then B is not competitive at all against A at quality difference T.[6]

6 In this case, there is a continuation equilibrium with a quality difference at least as large as T even if A pays nothing to all its ranks. The solution to (15) is irrelevant.

Provided that this is not the case, $r(T)$ is increasing in T, with $r\left(\frac{1}{2}\right) = 0$ and $r(1) = 1$. Thus, the optimal way for B to deter A from achieving a greater quality difference in A's favor is to concentrate more of its resources to reward its higher-rank members.

Substituting $r(T)$ from equation (16) into (15), we obtain an explicit form for the resource budget function:

$$E(T) = \begin{cases} 0 & \text{if } Y_B \leq P(T)(1-T); \\ [Y_B - P(T)(1-T)]^2/[2Y_B(1-T)] & \text{otherwise.} \end{cases} \tag{17}$$

The following proposition characterizes the properties of this resource budget function.

Proposition 1. *The resource budget function $E(T)$ satisfies $\lim_{T\to 1} E(1) = \infty$ and $E\left(\frac{1}{2}\right) = Y_B$. Moreover, (i) if $\alpha Y_B > \frac{1}{2}$, then $E'(T) > 0$ for all $T \geq \frac{1}{2}$; (ii) if $\alpha Y_B \in \left[\frac{1}{16}, \frac{1}{2}\right]$, then there exists a $\hat{T}$ such that $E'(T) < 0$ for $T \in \left(\frac{1}{2}, \hat{T}\right)$ and $E'(T) > 0$ for $T \in (\hat{T}, 1)$; and (iii) if $\alpha Y_B < \frac{1}{16}$, then there exist T_- and T_+ such that $E'(T) < 0$ for $T \in \left(\frac{1}{2}, T_-\right)$, $E(T) = 0$ for $T \in [T_-, T_+]$ and $E'(T) > 0$ for $T \in (T_+, 1)$.*

Proof. Substituting $T = \frac{1}{2}$ and $T = 1$ into the resource budget function (17), we immediately have $E\left(\frac{1}{2}\right) = Y_B$, and $\lim_{T\to 1} E(T) = \infty$.

Next, when $E(T) > 0$, its derivative is positive if and only if

$$\alpha Y_B + (1-T)\left(3T - \frac{5}{2}\right) > 0.$$

The above holds for all $T \geq \frac{1}{2}$ if $\alpha Y_B > \frac{1}{2}$, thus establishing (i).

When $\alpha Y_B \leq \frac{1}{2}$, there exists a unique $\hat{T} \in \left[\frac{1}{2}, 1\right]$ such that $E'(T) > 0$ for $T > \hat{T}$ while the opposite holds for $T < \hat{T}$. Finally, from (17) we have that $E(T) = 0$ when $Y_B \leq P(T)(1-T)$. The quadratic equation $Y_B = P(T)(1-T)$ has two real roots T_- and T_+ in $\left[\frac{1}{2}, 1\right]$ when $\alpha Y_B \leq \frac{1}{16}$, and no real root otherwise. Claims (ii) and (iii) follow immediately. *Q.E.D.*

See Figure 5 for the three different cases of $E(T)$. An increase in the target quality difference T has two opposite effects on the resource budget $E(T)$ required for organization A to achieve the target in a continuation equilibrium. On one hand, to achieve a greater quality difference T organization A must be competitive with more ranks in B and this

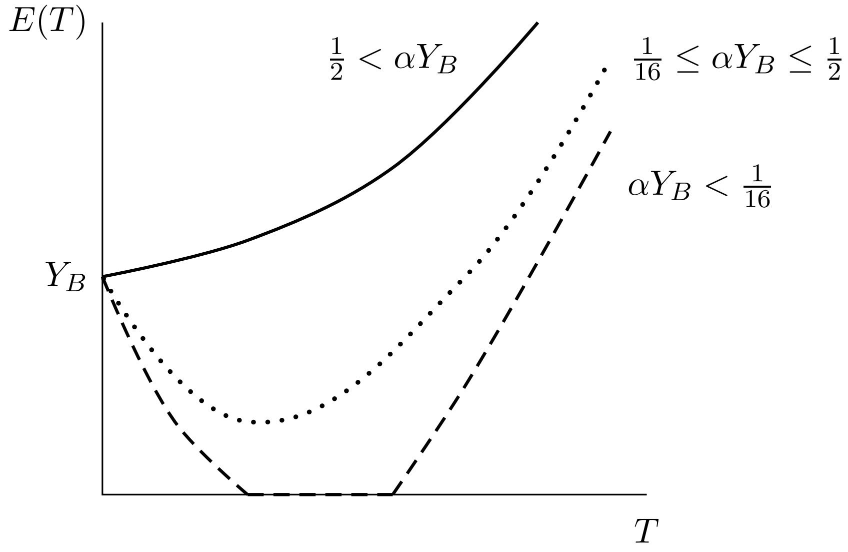

Figure 5

requires a larger budget. On the other hand, a greater T also increases the peer premium that A enjoys over B and this reduces the resource requirement. The first effect dominates when the peer effect is relatively small, which happens when either α or Y_B is large. This explains why $E(T)$ is monotonically increasing in T when αY_B is greater than $\frac{1}{2}$. In contrast, the peer effect is strong and $E(T)$ may decrease when αY_B is small. Indeed, the resource requirement to achieve some intermediate values of T can be zero because organization B's resource budget Y_B is not even sufficient to cover the peer premium $P(T)$ for all its ranks above T. Note that for $T = \frac{1}{2}$, the quality difference is zero, and by adopting a linear resource distribution schedule for all its ranks, B can make sure that A needs at least the same amount of resources. Finally, for sufficiently large T, the resource budget function must be increasing. This is because by concentrating its resources on a few top ranks organization B can make it increasingly costly for A to achieve large quality differences. Indeed, it is impossible for A to induce the perfect segregation of types in the sorting stage regardless of the resource advantage it has, because B could give all its limited resource to an arbitrarily small number of its top ranks.

4. The A-Dominant Equilibrium

To study the game in which the two organizations compete by choosing resource distribution schedules we must specify which continuation equilibrium is played in the sorting stage given a pair of schedules. For the remainder of the paper we assume that organization A is dominant in that the continuation equilibrium with the largest quality difference T is played in the sorting stage. This is a natural selection criterion since we have assumed that $Y_A \geq Y_B$. Given any pair of resource distribution schedules, the largest quality difference T corresponds to the largest fixed point of the mapping

$$\overline{D}(T) = \int_0^1 \bar{t}^T(r)\,\mathrm{d}r, \tag{18}$$

and the equilibrium quantile-quantile plot is given by $\bar{t}^T$.[7] We will refer to the continuation equilibrium with the largest T as the A-dominant continuation equilibrium, the selection criterion as the A-dominant criterion, and the resulting subgame perfect equilibrium as the A-dominant equilibrium.

An important implication of the A-dominant criterion is that the resource budget function $E(T)$, characterized in Proposition 1, can be used directly to establish a lower bound on the equilibrium quality difference in the game of organization competition. Note that since $Y_A \geq Y_B$, the A-dominant criterion implies that the lower bound is at least $\frac{1}{2}$. This is because, for any resource distribution schedule S_B, when $S_A = S_B$, there is a continuation equilibrium where the type distributions in the two organizations are identical. Given the budget function $E(T)$, define

$$T^* = \max\left\{T \in \left[\frac{1}{2}, 1\right] :\ E(T) = Y_A\right\}. \tag{19}$$

For any $Y_A \geq Y_B$, the quality difference T^* is well defined. Moreover, by the characterization in Proposition 1, we have $T^* = \frac{1}{2}$ if and only if $Y_A = Y_B \geq 1/(2\alpha)$. For all other values of Y_A and Y_B such that $Y_A \geq Y_B$, we have $T^* > \frac{1}{2}$. Finally, given T^* we can define the associated resource distribution schedule S_B^* that achieves $E(T^*)$, as follows. Let $r_B^* = r(T^*)$ in equation (16) and define

$$S_B^*(r) = \begin{cases} 0 & \text{if } r < r_B^*; \\ P(T^*) + 2[Y_B - P(T^*)(1 - r_B^*)](r - r_B^*)/(1 - r_B^*)^2 & \text{if } r \geq r_B^*. \end{cases} \tag{20}$$

[7] Fixed points of D which are non-extremal may be unstable in the sense that small perturbations in the quality difference T can cause agents to switch organizations in such a way that moves T further away from the initial fixed point. On the other hand, the A-dominant equilibrium is always stable.

The following proposition verifies that T^* is both the lower and the upper bound of the equilibrium quality difference.[8] It follows that T^* is the unique candidate for the equilibrium quality difference. Since S_B^* is the only schedule that achieves $E(T^*)$ by Lemma 4, it is also the only candidate for equilibrium resource distribution schedule for B.

PROPOSITION 2. *In any A-dominant equilibrium of the organization competition game, the quality difference is T^* given by (19). Further, B's resource distribution schedule is S_B^* given by (20).*

PROOF. First, T^* is a lower bound on the equilibrium quality difference. This is because $C(T^*; S_B) \leq E(T^*) = Y_A$ for any S_B, and hence there is a resource distribution schedule S_A that satisfies A's resource constraint and such that under (S_A, S_B) there is a continuation equilibrium with quality difference $T \geq T^*$.

Second, T^* is also an upper bound on the equilibrium quality difference. From the Raiding Lemma, we have $C(T; S_B^*) = S_B^*(T) - P(T)$. Since S_B^* is linear, $C(T; S_B^*)$ is also linear in T. By the definition of T^*, we have $C(T^*; S_B^*) = Y_A$, so that $C(T; S_B^*) > Y_A$ for all $T > T^*$ if and only if $C(T; S_B^*)$ has a positive slope in T. Since S_B^* solves the maximization problem (13), by the envelope theorem, the derivative of $C(T; S_B^*)$ with respect to T at T^* is equal to $E'(T)$ at $T = T^*$, which is positive by (19). *Q.E.D.*

That T^* is a lower bound on the equilibrium quality difference follows directly from the definition of T^* and the A-dominant criterion. Further, when organization B chooses resource distribution schedule S_B^*, in order to achieve the quality difference T^* organization A must expend all its available resources. However, this may fail to guarantee that a larger quality difference is out of reach for A, because a larger T increases the peer premium and frees some resources for A. The above proposition establishes that given S_B^*, this peer effect is dominated by the additional resource requirement for obtaining a greater quality difference than T^*. This is because, by the Raiding Lemma, the minimum resource budget

[8] Alternatively we can define a zero-sum "resource distribution game" in which the two organizations simultaneously choose their resource distribution schedules. For any (S_A, S_B), the payoff to organization A is the quality difference $\overline{T}(S_A, S_B)$ in the A-dominant continuation equilibrium. Then T^* corresponds to the minmax value, or $\min_{S_B} \max_{S_A} \overline{T}(S_A, S_B)$, and S_B^* corresponds to the solution.

$C(T; S_B^*)$ required for A to achieve any quality difference T is linear in T, with a slope equal to $E'(T)$ at $T = T^*$ by the envelope theorem. Even though the resource budget function $E(T)$ may be decreasing, from Figure 5, it is necessarily increasing at $T = T^*$. Since $C(T^*; S_B^*) = E(T^*) = Y_A$, the result that $C(T; S_B^*)$ is a positively-sloped linear function in T implies that by choosing S_B^*, organization B ensures that the minimum resource budget required to achieve any quality difference greater than T^* is strictly greater than Y_A, and hence T^* is an upper bound on the quality difference in any A-dominant equilibrium.

4.1. Equilibrium construction

Given the result in Proposition 2 that S_B^* specified in (20) is the unique candidate equilibrium resource distribution schedule for organization B, a necessary condition for an A-dominant equilibrium is a resource distribution schedule S_A that is a best response to S_B^*. The Raiding Lemma suggests that there are many such schedules. To see this, note that in equation (11), since S_B^* as given in (20) is linear with a discontinuous jump at r_B^*, we have $\Delta(r) = 0$ for all $r \leq r_B^*$, and increases linearly with r for $r \geq r_B^*$. It follows that for any r^1 and r^0 satisfying $r_B^* \leq r^1 \leq T^* \leq r^0$, the quantile-quantile plot

$$t(r) = \begin{cases} 1 & \text{if } r < r^1; \\ (T^* - r^1)/(r^0 - r^1) & \text{if } r^1 \leq r \leq r^0; \\ 0 & \text{if } r > r^0; \end{cases}$$

has an integral equal to T^*, and thus solves (10) for $T = T^*$.[9] The associated resource distribution schedule S_A^t for A is a step function given by equation (12), with $T = T^*$. Since $\Delta(r)$ is linear above r_B^*, by the Raiding Lemma the integral of S_A^t is

$$C(T^*; S_B^*) = S_B^*(T^*) - P(T^*) = Y_A, \tag{21}$$

where the last equality follows from the definition of T^*. Thus, the schedule S_A^t satisfies the budget constraint of A, and is therefore a best response to S_B^*.

[9] In the proof of the Raiding Lemma, under the schedule S_B^*, we have $T^* \in \overline{Q}$ and thus the solution t is given by case (i) of the lemma, with a single discontinuity point of t at T^*. However, because S_B^* is linear above r_B^*, the same value of the objective function in (10) is also achieved by case (ii) of the lemma, with any $r^1 \neq r^0$ satisfying $r_B^* \leq r^1 \leq T^* \leq r^0$.

In fact, since S_B^* has the property that the resource-to-rank ratio, adjusted for the peer premium, is constant for all ranks above r_B^*, any resource distribution schedule that exhausts the resource constraint and satisfies $S_A(1) \leq S_B^*(1) - P(T^*)$ so that the highest rank in A is not overpaid, is a best response to S_B^*. To construct an A-dominant equilibrium, in the next proposition we identify a particular resource distribution schedule S_A^* against which the unique candidate schedule S_B^* is a best response. As hinted by the logic of the Raiding Lemma, this S_A^* resembles S_B^* in that it is zero up to some rank r_A^* and then linear for ranks higher than r_A^*, with $S_A^*(1) = S_B^*(1) - P(T^*)$, but unlike S_B^*, it is continuous at r_A^*.

Proposition 3. *Let $r_A^* = P(T^*)/S_B^*(1)$ and S_A^* be defined by*

$$S_A^*(r) = \begin{cases} 0 & \text{if } r < r_A^*; \\ (r - r_A^*)S_B^*(1) & \text{if } r \geq r_A^*. \end{cases}$$

The strategy profile (S_A^*, S_B^*) forms an A-dominant equilibrium.

Proof. By definition we have

$$\int_0^1 S_A^*(r)\ \mathrm{d}r = \frac{(S_B^*(1) - P(T^*))^2}{2S_B^*(1)}.$$

Using the linearity of S_B^*, we can rewrite equation (21) as

$$Y_A = (S_B^*(1) - P(T^*))\frac{T^* - r_B^*}{1 - r_B^*}. \tag{22}$$

Using the above relation and the equation (16) for $r_B^* = r(T^*)$, we can directly verify that

$$\int_0^1 S_A^*(r)\ \mathrm{d}r = Y_A.$$

To show that S_A^* is a best response to S_B^*, it suffices to establish that the integral of $\bar{t}^{T^*}$ is equal to T^*. We have

$$\bar{t}^{T^*} = \begin{cases} 1 & \text{for } r \leq r_B^*; \\ 1 - S_A^{*-1}(S_B^*(r) - P(T^*)) & \text{for } r \in (r_B^*, 1]. \end{cases}$$

With a change of variable $\tilde{r} = S_A^{*-1}(S_B^*(r) - P(T^*))$ and integration by parts, we obtain

$$\int_0^1 \bar{t}^{T^*}\ \mathrm{d}r = 1 - \frac{1}{S_B^{*\prime}}\left(S_A^*(1) - \int_{r_A^*}^1 S_A^*(\tilde{r})\ \mathrm{d}\tilde{r}\right) = 1 - \frac{1}{S_B^{*\prime}}\left(S_A^*(1) - Y_A\right),$$

which is equal to T^*, because $S_A^*(1) = S_B^*(1) - P(T^*)$ and $Y_A = S_B^*(T^*) - P(T^*)$.

Under the A-dominant criterion, to prove that S_B^* is a best response to S_A^*, it suffices to verify that, given S_A^*, for any S_B there is a continuation equilibrium with quality difference $T \geq T^*$. From the definition of $\bar{t}^{T^*}$, we have

$$\int_0^1 \bar{t}^{T^*}(r)\ \mathrm{d}r = 1 - \int_{r^0}^{r^1} S_A^{*-1}(S_B(r) - P(T^*))\ \mathrm{d}r,$$

where r^0 is the lowest rank in B that receives more resources than $P(T^*)$, and r^1 is the highest rank that receives less resources than $S_A^*(1) + P(T^*)$. By the construction of S_A^*,

$$S_A^{*-1}(S_B(r) - P(T^*)) = \frac{S_B(r)}{S_B^*(1)}.$$

Using the above expression and the resource constraint for B, we have

$$\int_0^1 \bar{t}^{T^*}(r)\ \mathrm{d}r \geq 1 - \frac{Y_B}{S_B^*(1)}.$$

Using equation (20) for the schedule S_B^* and eliminating $(1 - r_B^*)$ through equation (16), we can verify that

$$S_B^*(1) = \frac{2Y_B}{1 - r_B^*} - P(T^*) = \frac{Y_B}{1 - T^*}. \tag{23}$$

Therefore, the mapping (18) has one fixed point greater than or equal to T^*. *Q.E.D.*

The construction of S_A^* is already illustrated in Figure 1, where we have used P^* for $P(T^*)$. Since $r_A^* = P(T^*)/S_B^*(1)$, the critical rank r_A^* is the intersection between the line connecting the origin and the end point of S_B^* and the horizontal line representing the peer premium $P(T^*)$ in Figure 1. As a result, from organization B's point of view, A's premium-adjusted schedule, $S_A^*(r) + P(T^*)$, has the same resource-to-rank ratio at every rank greater than or equal to r_A^*. As in the Raiding Lemma, each rank that receives a positive amount of resource from S_A^* is equally costly for B to raid, making it impossible for B to reduce the quality difference by changing S_B^*. It is remarkable that the critical rank r_A^*, given by $P(T^*)/S_B^*(1)$, which uniquely makes A's quality-adjusted schedule have the same resource-to-rank ratio for all ranks receiving resources, coincides with the rank uniquely determined by the budget constraint $\int_0^1 S_A^*(r)\ \mathrm{d}r = Y_A$. This coincidence is a consequence of the linearity of S_B^*, as made clear by the proof of the proposition. In a non-linear model it may still be possible to identify a candidate equilibrium strategy for organization B, however, if such candidate is non-linear, there is no reason to expect that the resource schedule for A as constructed in the proposition with a constant resource-to-rank ratio would meet its budget constraint.

4.2. Comparative statics

Since the equilibrium resource distribution schedules constructed in Proposition 3 are piece-wise linear, they can be described through the following intercepts and slopes. For organization A, the critical type r_A^* such that the ranks below receive no resources satisfies

$$r_A^* = \frac{P(T^*)}{S_B^*(1)} = \frac{P(T^*)(1-T^*)}{Y_B} \tag{24}$$

where the second equality follows from equation (23). For organization B, from equation (16) with $T = T^*$, the critical type r_B^* such that the ranks below receive no resources satisfies

$$1 - r_B^* = \frac{2Y_B(1-T^*)}{Y_B + P(T^*)(1-T^*)}. \tag{25}$$

Using equation (23), we can write the slope of S_A^* for ranks above r_A^* as

$$S_A^{*\prime} = S_B^*(1) = \frac{Y_B}{1-T^*}. \tag{26}$$

For organization B, the slope of S_B^* for ranks above r_B^* satisfies

$$S_B^{*\prime} = \frac{S_B^*(1) - P(T^*)}{1-r_B^*} = \frac{Y_A}{T^* - r_B^*} = \frac{1-r_A^*}{1-r_B^*}\,\frac{Y_B}{1-T^*}, \tag{27}$$

where the second equality follows from the linearity of S_B^* (equation 22), and the third equality follows from (23).

When $Y_A = Y_B \geq 1/(2\alpha)$, the unique candidate quality difference T^* given in Proposition 2 is equal to $\frac{1}{2}$, and the unique candidate schedule S_B^* for B has $r_B^* = 0$. Since the peer premium $P(T^*)$ is 0, the equilibrium schedule S_A^* for A has $r_A^* = 0$, and the two equilibrium schedules coincide. Thus the equilibrium given in Proposition 3 is symmetric, even though by adopting the selection criterion that picks out the continuation equilibrium with the largest quality difference for any competing resource distribution schedules, we have given the dominant organization A the greatest advantage allowed by the peer effect, because the peer effect is too weak.

As long as the peer effect is sufficiently strong, regardless of whether the two organizations have the same or different resource budgets, the equilibrium constructed in Proposition 3 is asymmetric. As already mentioned in section 2.2, there are two features of asymmetry between S_A^* and S_B^* constructed in Proposition 3 that are immediate from Figure 1. First, $r_A^* < r_B^*$, so that more ranks receive positive resources in organization A than in B. Second, $S_A^{*'} < S_B^{*'}$, so that S_A^* is flatter than S_B^* for ranks that receive positive resources in respective organizations. Our model therefore suggests that the organization resources are less concentrated at the top ranks in the dominant organization than in the weaker organization. This is perhaps not surprising, as the weaker organization must compensate for the peer premium $P(T^*)$ that results from the weaker peer effect compared to the dominant organization. The asymmetry in the resource distribution schedules between the two organizations in equilibrium is not due to the disparity in the resource budget. Even if $Y_B = Y_A$ and thus organization B faces no resource disadvantage, if the peer effect is strong relative to the resource budget, that is, if $\alpha Y_B < \frac{1}{2}$, under our selection criterion there is an asymmetric equilibrium in which the dominant organization enjoys a peer premium and employs a resource distribution schedule that is less concentrated in the top ranks.

The mixing pattern between the two organizations in the type space can be determined as follows. Given the equilibrium schedules (S_A^*, S_B^*), we can first derive the implied quantile-quantile plot t^{T^*}.[10] Using the definition of $\bar{t}^{T^*}$, we have

$$\bar{t}^{T^*} = \begin{cases} 1 & \text{if } r \leq r_B^*; \\ (1-r)(1-r_A^*)/(1-r_B^*) & \text{if } r > r_B^*. \end{cases}$$

[10] Since S_A^* and S_B^* are strictly increasing above the corresponding critical ranks of r_A^* and r_B^*, the upper bound and the lower bound, $\bar{t}^{T^*}$ and $\underline{t}^{T^*}$ coincide, except at a common discontinuity point r_B^*. They have the same integral equal to T^*. However, this does not imply that there is no other fixed point $T \in \left[\frac{1}{2}, T^*\right)$ of the mapping (7). That is, there may be a continuation equilibrium with a quality difference smaller than T^*.

Using equation (6), we can then recover the underlying distribution of types across the two organizations, as already mentioned in section 2.2. Our model thus predicts both segregation and mixing in the A-dominant equilibrium. Low and intermediate types receive no resources in either organization, and thus they segregate completely, with the low types left to join B and the intermediate types going to A and receiving the peer premium. In contrast, high types receive the same resources from A and B, adjusted for the quality difference, and they are mixed between A and B in a constant proportion determined by the slopes of the two equilibrium resource distribution schedules. The dominant organization A gets a greater share of the high types than B because S_A^* is flatter than S_B^*: starting from type $\frac{1}{2}(r_B^* + r_A^*)$ that is indifferent between joining A at rank r_A^* and joining B at rank r_B^*, a flatter schedule S_A^* in A means that the increase in the amount of resource received by each higher rank is smaller in A than in B, and thus the rank must grow faster with type in A than in B to maintain the indifference of each higher type between the two organizations. As a result, the dominant organization gets a greater share of the types that in equilibrium it competes for against the weaker organization.

For comparative statics analysis of the equilibrium constructed in Proposition 3, first consider a decrease in the concern for the peer effect, as represented by an increase in α.[11] (i) Examining the resource budget function (17) shows that a rise in α reduces the peer premium $P(T)$ and hence shifts up $E(T)$. Since equilibrium T^* is defined by $E(T^*) = Y_A$, this means that equilibrium quality difference between the two organizations falls as agents put less weight on the peer effect. (ii) Since $r(T)$ is increasing in T and decreasing in α in equation (16), a rise in α lowers r_B^*. For organization A, using the budget function (17) to express the condition $E(T^*) = Y_A$ and substituting the result in (24), we get

$$(1 - r_A^*)^2 = 2(1 - T^*)\,\frac{Y_A}{Y_B}.$$

Hence a fall in T^* also implies a fall in r_A^*. Thus, a fall in the concern for the peer effect causes both the dominant organization and the weaker organization to give positive resources to more ranks. (iii) As α increases, since r_A^* decreases, the resource budget

[11] We assume that the equilibrium is asymmetric, i.e., it is not the case that $Y_A = Y_B = Y$ and $\alpha Y \geq \frac{1}{2}$. Otherwise, an increase in α has no effects on the equilibrium.

constraint for A implies that $S_A^*(1)$ decreases and S_A^* becomes flatter. Moreover, S_B^* becomes flatter as well, because $S_B^*(1) - P(T^*)$ is equal to $S_A^*(1)$ and thus decreases, while r_B^* also decreases. Thus, the disparity in resources between the higher and lower ranks in both organizations becomes smaller as α increases. (iv) Finally, since a smaller peer effect reduces both r_B^* and r_A^*, and since the advantage of the dominant organization derives from the higher quality of its members, a reduction in the importance of the peer effect increases the set of types that are present in both organizations. Moreover, using (24) and (25), and noting that Y_A equals $E(T^*)$, which is given by (17), we have

$$\frac{1 - r_A^*}{1 - r_B^*} = \frac{1 + r_A^*}{1 - r_A^*} \frac{Y_A}{Y_B}. \tag{28}$$

The right-hand-side of the above relation is decreasing in α because r_A^* decreases with α. Thus, when the peer effect becomes less important, the dominant organization gets a smaller share of these top talents.

Next, consider an increase in Y_B, the resource budget of the weaker organization.[12] Using equation (17) and the definition of T^*, we have

$$Y_A = \frac{[Y_B - P(T^*)(1 - T^*)]^2}{2Y_B(1 - T^*)} = \left[1 - \frac{P(T^*)(1 - T^*)}{Y_B}\right]^2 \frac{Y_B}{2(1 - T^*)}. \tag{29}$$

(i) The first part of the above equation implies that T^* decreases as Y_B increases. Thus, an increase in Y_B reduces the equilibrium quality difference T^*. (ii) From the second part of equation (29), since T^* decreases, both $Y_B/(1 - T^*)$ and $P(T^*)(1 - T^*)/Y_B$ decrease as Y_B increases. It then follows from equation (16) that the critical rank r_B^* in B decreases, and from equation (24) that the critical rank r_A^* in A also decreases. Thus, as the resource budget of the weaker organization increases, it is able to compete for more ranks, and in equilibrium both organizations end up providing positive resources to more types. (iii) From (26), S_A^* becomes flatter because $Y_B/(1 - T^*)$ decreases as Y_B increases. Moreover, for organization B, from equation (25) we have

$$T^* - r_B^* = (1 - T^*) \frac{1 - r_A^*}{1 + r_A^*}, \tag{30}$$

[12] We assume $Y_B < Y_A$ to be consistent with the selection criterion.

which increases because r_A^* decreases and T^* decreases. From (27), $S_B^{*'}$ decreases. Thus, as the resource disadvantage of B decreases, both organizations in equilibrium adopt resource distribution schedules that are less concentrated at the top. (iv) From (28) the fraction $(1 - r_A^*)/(1 - r_B^*)$ decreases because r_A^* decreases as Y_B increases. This means that as organization B reduces the resource disadvantage, it attracts a larger fraction of high types in equilibrium. Thus, effects of an increase in Y_B are qualitatively similar to an increase in α. In both cases, the peer effect becomes weaker, and the equilibrium becomes more symmetric between the two organizations.

Finally, consider the effects of an increase in the resource budget of the dominant organization. (i) An increase in Y_A raises T^* because the budget function is upward sloping at T^*. (ii) Since $r(T)$ is increasing in T from equation (16), organization B responds by raising the critical rank r_B^* below which it devotes no resources. Thus, the effects of an increase in Y_A on T^* and r_B^* are opposite of the effects of an increase in Y_B. However, the effect on r_A^* is generally non-monotone. By equation (24), an increase in Y_A raises the critical rank r_A^* if and only if $T^* < \frac{3}{4}$. This is due to two opposing effects: the increase in r_B^* induces A to devote more resources to the top ranks to stay competitive with B, but with a greater resource budget, this may not lead to a reduction in the range of ranks that receive positive resources. The first effect dominates when the competition is tougher because T^* is relatively small. (iii) By equation (26), an increase in Y_A always makes the schedule S_A^* steeper for ranks above r_A^* because T^* increases. To study the effect of an increase in Y_A on the slope of S_B^*, we distinguish two cases. If $T^* \geq \frac{3}{4}$, then $S_B^{*'}$ increases by equation (27), because both r_A^* and $1 - r_B^*$ decrease with T^*. In the second case, with $T^* < \frac{3}{4}$, from equation (30) $T^* - r_B^*$ decreases because r_A^* increases, and thus by (27), $S_B^{*'}$ again increases with Y_A. Thus, in both cases, under competition from an organization with a greater resource budget, B adopts a resource distribution schedule that is more skewed toward top ranks. (iv) Finally, to study the effect of an increase in Y_A on the mixing of high types between A and B, we again distinguish two cases. In the first case, with $T^* \geq \frac{3}{4}$, the fraction $(1 - r_A^*)/(1 - r_B^*)$ increases, because r_A^* decreases while r_B^* increases as Y_A increases. In the second case, with $T^* < \frac{3}{4}$, from equation (28), the fraction $(1 - r_A^*)/(1 - r_B^*)$ again increases, because r_A^* increases with Y_A. Thus, in either case, as organization A gains a greater advantage in resource budget, it also attracts a larger fraction of high types in equilibrium.

4.3. Equilibrium uniqueness

By Proposition 2, S_B^* is the unique candidate equilibrium schedule for organization B. As demonstrated in section 4.1, there are many best responses of A to S_B^*, including S_A^*. In particular, any resource distribution schedule that satisfies $S_A(1) \leq S_B^*(1) - P(T^*)$ is a best response. Our first step is to establish that the condition for A not to overpay its top rank is also necessary for S_A to be an equilibrium resource distribution schedule.

Lemma 5. *If S_A is an A-dominant equilibrium resource distribution schedule for A, then $S_A(r) \leq S_B^*(1) - P(T^*)$ for all $r < 1$.*

Proof. Suppose that there is a rank $\underline{r} < 1$ such that $S_A(r) > S_B^*(1) + P(T^*)$ for all $r \in (\underline{r}, 1]$. Consider a modified resource distribution schedule $\tilde{S}_A$ that coincides with $S_A(r)$ except $\tilde{S}_A(r) = S_A(\underline{r})$ for all $r \in (\underline{r}, 1]$. Then, for any $r, \tilde{r} \in [0, 1]$, we have $\tilde{S}_A(r) \geq S_B^*(\tilde{r}) - P(T^*)$ whenever $S_A(r) \geq S_B^*(\tilde{r}) - P(T^*)$. By the definition of $\bar{t}^{T^*}$, T^* is still a fixed point of the mapping $\overline{D}(T)$ under $(\tilde{S}_A, S_B^*)$. Since by construction $\int_0^1 \tilde{S}_A(r)\, \mathrm{d}r < \int_0^1 S_A(r)$, there exists some other resource distribution schedule of A which satisfies the budget constraint and, against S_B^*, yields a fixed point $\tilde{T}$ of $\overline{D}(T)$ that is strictly greater than T^*. Thus S_A is not a best response to S_B^*. *Q.E.D.*

The next step in establishing the uniqueness of (S_A^*, S_B^*) is to show that any equilibrium schedule S_A must be linear when positive. Otherwise, if $S_A(r)$ is locally convex around some rank $\hat{r}$ with $S_A(\hat{r}) > 0$, then organization B can modify S_B^* by identifying a rank $\tilde{r}$ that competes with $\hat{r}$, i.e., $\tilde{r}$ satisfying $S_B^*(\tilde{r}) = S_A(\hat{r}) + P(T^*)$, and flattening S_B^* around $\tilde{r}$ in a way that still satisfies the resource budget. Analogous to the argument used in establishing Lemma 2, this modification lowers the mapping $\overline{D}(T)$ at T^*, and thus T^* is no longer a continuation equilibrium quality difference.[13] However, in contrast to

[13] While Lemma 2 looks at the solution in quantile-quantile plot to (10) that minimizes the budget required for A to achieve a given quality difference T, the next lemma modifies the resource distribution schedule to reduce the integral of the quantile-quantile plot while satisfying the budget constraint for B. A step function solution in t in Lemma 2 corresponds to a step function in schedule S_B.

Lemma 2, under the A-dominant criterion, we also need to show that the modification does not create a larger fixed point than T^*. This additional step is accomplished by making the modification around $\tilde{r}$ sufficiently small. Similarly, if $S_A(r)$ is locally concave at $\hat{r}$, B can identify a rank $\tilde{r}$ that competes with $\hat{r}$ and replace S_B^* with a step function of two values in a sufficiently small neighborhood of $\tilde{r}$. The proof of the following lemma is in the appendix.

LEMMA 6. *If S_A is an A-dominant equilibrium resource distribution schedule for A, then S_A is a linear function in the range of r where $S_A(r)$ is positive.*

The equilibrium resource distribution schedule S_A^* constructed in Proposition 3 is linear above rank r_A^*, which is uniquely identified by connecting the starting point and the end point of S_B^*. As shown in the proof of the proposition, it satisfies the resource budget constraint of A. By the above two lemmas, any candidate equilibrium schedule for A must not overpay its top rank, and is linear whenever it is positively valued. Since a candidate equilibrium schedule for A must satisfy the budget constraint, the uniqueness of S_A^* as an equilibrium schedule for A follows immediately once we show that if S_A is an A-dominant equilibrium resource distribution schedule for A, then (i) $S_B^*(1) \leq S_A(1) + P(T^*)$ so that B is not overpaying its top rank, and (ii) S_A is continuous at the critical rank $\hat{r}$ defined as $\inf\{r \in [0,1] : S_A(r) > 0\}$ so that there is no jump at the lowest rank that receives a positive amount of resources. For the first property, if $S_A(1) < S_B^*(1) - P(T^*)$, then B can modify S_B^* by reducing the amount of resources for ranks around the very top rank 1, and redistributing the resources to ranks just below r such that $S_B^*(r) - P(T^*) = S_A(1)$. With a similar argument as in the proof of Lemma 6, we can show that for a sufficiently small modification B can decrease the largest fixed point of the mapping $\overline{D}(T)$. For the second property, if S_A is discontinuous at $\hat{r}$, then B can modify S_B^* by reducing the amount of resources for ranks just above r_B^*, and redistributing the resources to ranks around the very top rank 1. This modification reduces the the mapping $\overline{D}(T)$ for all $T \geq T^*$. Thus, if either of the two properties is not satisfied, S_B^* is not a best response for B. The following proposition immediately follows.

PROPOSITION 4. *In the unique A-dominant equilibrium, the resource distribution schedules for A and B are S_A^* and S_B^*.*

As remarked after Proposition 3, the equilibrium resource distribution schedule S_A^*, as well as S_B^*, has the property that the premium-adjusted resource-to-rank ratio is constant for ranks that receive positive resources. As demonstrated in the proof of Lemma 6, the linear feature of our model, embodied in the uniform type distribution and in the linear payoff function for the agents, plays a critical role in the uniqueness result. Proposition 4 strengthens the result from Proposition 2 that the equilibrium quality difference is unique. Thus, the resource distributions in the two organizations and the sorting of types described in section 4.2 are necessary implications of the A-dominant equilibrium.

5. Discussion

For a fixed pair of increasing resource distribution schedules, the payoff specification in (1) captures the trade-off facing the agent between relative rank and average type when choosing between the two organizations. In Damiano, Li and Suen (forthcoming), we study a more general model of this trade-off with exogenously fixed concern for relative ranking, with different focuses on competitive equilibrium implementation and welfare implications. In terms of the model presented here, the benchmark model of Damiano, Li and Suen assumes that the resource distribution schedules are fixed at $S_A(r) = S_B(r) = r$ with $Y_A = Y_B = \frac{1}{2}$. The concern of an agent for relative ranking in the organization he joins, referred to as "pecking order effect," may be motivated by self-esteem (Frank, 1985) or competition for mates or resources (Cole, Mailath and Postlewaite, 1992; Postlewaite, 1998). The equilibrium pattern of mixing and segregation characterized in Damiano, Li and Suen is of an "overlapping interval" structure, where the very talented are captives in the high quality organization and the least talented are left to the low quality organization, while the intermediate talents are present in both. In the present model instead, agents do not directly care about their relative ranking in the organization. The concern for relative ranking is generated endogenously because the organizations choose how to distribute resources according to ranks. The equilibrium sorting pattern, given in Proposition 3, is qualitatively different from an overlapping interval structure, with top talents mixing between the two organizations and lower types perfectly segregation. Thus, mixing occurs for the intermediate types and high types are captives when the trade-off between the peer effect and the pecking order effect does not respond to organizational choices as in Damiano, Li and Suen (forthcoming), while under competition between organization, the high types are the only ones receiving resources and mix between organizations.

Organizations in our model have a fixed capacity of half of the talent pool and must fill all positions. In particular, an organization cannot try to improve its average talent by rejecting low types even though the capacity is not filled. We have made this assumption in order to circumvent the issue of size effect, and focus on implications of sorting of talents. We may justify the assumption of fixed capacity if the peer effect enters the preferences of talents in the form of total output (measured by the sum of individual types) as opposed to the average type, and the objective of the organization is to maximize the total output. Since all agents contribute positively to the total output, in this alternative model all positions will be filled.

We have restricted organization strategies to meritocratic resource distribution schedules. This is a natural assumption given how we model the sorting of talents after organizations choose their schedules. Non-meritocratic resource distribution schedules would create incentives for talented agents to "dispose of" their talent. Another assumption we have made about organization strategies is that resource distributions do not depend on type directly. This is a reasonable assumption in the presence of the resource constraint; a resource distribution schedule that depends directly on type might exceed the resources available or leave some resources unused depending on the distribution of types that join the organization. Moreover, at the equilibrium quality difference and against the equilibrium resource distribution schedule of the rival organization, each organization cannot improve its quality by deviating to a resource distribution schedule that depends on type as well as on rank. This is because any continuation equilibrium after such a deviation can be replicated by a deviating schedule that depends on type only. Our equilibrium is thus robust to deviations allowed by a richer strategy space.

Our main results of linear resource distribution schedules rely on the assumption of uniform type distribution. This assumption implies that the impact on the quality difference of an exchange of one interval of types for another interval between the two organizations depends only on the difference in the average types of the two intervals.

Together with the assumption that the payoff functions of agents is linear, this property allows us to transform the problem of finding the optimal response in resource distribution schedules to a linear programming problem in quantile-quantile plots, and characterize the solution in the Raiding Lemma. We leave the question of whether the method we develop in this paper is applicable to more general type distributions to future research.

Appendix

Proof of Lemma 2. We prove the lemma by establishing that, for any quantile-quantile plot t with $\int_0^1 t(r)\ \mathrm{d}r = T$, there exists another quantile-quantile plot $\tilde{t}$ with $\int_0^1 \tilde{t}(r)\ \mathrm{d}r = T$ which assumes at most one value strictly between 0 and 1 and satisfies

$$\int_{\tilde{t}^{-1}(1)}^{\tilde{t}^{-1}(0)} \tilde{t}(r)\Delta'(r)\ \mathrm{d}r - \Delta(\tilde{t}^{-1}(1)) \leq \int_{t^{-1}(1)}^{t^{-1}(0)} t(r)\Delta'(r)\ \mathrm{d}r - \Delta(t^{-1}(1)). \tag{A.1}$$

We first prove that $\tilde{t}$ assumes a countable number of values, and then show that it assumes at most one value strictly between 0 and 1.

Suppose t is strictly decreasing and Δ' is monotone on some open interval (r_-, r_+). Let $r^0 \in (r_-, r_+)$ solve

$$(r_+ - r_-)t(r^0) = \int_{r_-}^{r_+} t(r)\ \mathrm{d}r,$$

and let $\hat{r} \in (r_-, r_+)$ solve

$$(\hat{r} - r_-)t(r_-) + (r_+ - \hat{r})t(r_+) = \int_{r_-}^{r_+} t(r)\ \mathrm{d}r.$$

We construct a new quantile-quantile plot $\tilde{t}$ that is identical to t outside the interval (r_-, r_+), and such that $\tilde{t}(r) = t(r^0)$ for all $r \in (r_-, r_+)$ if Δ' is decreasing, and $\tilde{t}(r) = t(r_-)$ for all $r \in (r_-, \hat{r}]$ and $\tilde{t}(r) = t(r_+)$ for all $r \in (\hat{r}, r_+)$ if Δ' is increasing. By construction $\tilde{t}$ is a decreasing function and $\int_0^1 \tilde{t}(r)\ \mathrm{d}r = \int_0^1 t(r)\ \mathrm{d}r$, and further

$$\int_0^1 S_A^{\tilde{t}}(r)\ \mathrm{d}r - \int_0^1 S_A^t(r)\ \mathrm{d}r = \int_{r_-}^{r_+} \left(\tilde{t}(r) - t(r)\right) \Delta'(r)\ \mathrm{d}r.$$

If Δ' is decreasing on (r_-, r_+), then

$$\int_{r_-}^{r_+} \big(\tilde{t}(r) - t(r)\big)\, \Delta'(r)\ \mathrm{d}r = \int_{r_-}^{r^0} \big(t(r^0) - t(r)\big)\, \Delta'(r)\ \mathrm{d}r + \int_{r^0}^{r_+} \big(t(r^0) - t(r)\big)\, \Delta'(r)\ \mathrm{d}r$$
$$\leq \Delta'(r^0) \int_{r_-}^{r^0} \big(\tilde{t}(r^0) - t(r)\big)\ \mathrm{d}r + \Delta'(r^0) \int_{r^0}^{r_+} \big(\tilde{t}(r^0) - t(r)\big)\ \mathrm{d}r,$$

which is equal to 0. If otherwise Δ' is increasing, then

$$\int_{r_-}^{r_+} \big(\tilde{t}(r) - t(r)\big)\, \Delta'(r)\ \mathrm{d}r = \int_{r_-}^{\hat{r}} \left(t(r_-) - t(r)\right) \Delta'(r)\ \mathrm{d}r + \int_{\hat{r}}^{r_+} \left(t(r_+) - t(r)\right) \Delta'(r)\ \mathrm{d}r$$
$$\leq \Delta'(\hat{r}) \int_{r_-}^{\hat{r}} \left(t(r_+) - t(r)\right)\ \mathrm{d}r + \Delta'(\hat{r}) \int_{\hat{r}}^{r_+} \left(t(r_-) - t(r)\right)\ \mathrm{d}r,$$

which is equal to 0. The above argument implies that, for any t, there is a $\tilde{t}$ which assumes at most two values on any interval on where Δ' is monotone and t is strictly decreasing, it is identical to t oustide any such intervals, and (A.1) holds. The assumptions that t is monotone and Δ' has a countable number of discontinuities imply that there are countably many intervals $I \subset [0, 1]$ such that: (i) t is continuous and strictly decreasing on I; (ii) Δ' is monotone on I; and (iii) there is no open interval $I' \supset I$ that satisfies (i) and (ii). Further, t assumes a countable number of different values outside the union of all such intervals. This concludes the proof of the claim.

We can now restrict attention to quantile-quantile plots t which assume a countable number of values. Suppose there are two consecutive intervals I^j and I^{j+1}, such that t assumes value t^j on I^j and value t^{j+1} on I^{j+1}, for some $1 > t^j > t^{j+1} > 0$. Consider a new quantile-quantile plot $\tilde{t}_\epsilon$ defined as follows:

$$\tilde{t}_\epsilon(r) = \begin{cases} t^j + \epsilon/(r_+^j - r_-^j) & \text{if } r \in I^j; \\ t^{j+1} - \epsilon/(r_+^{j+1} - r_-^{j+1}) & \text{if } r \in I^{j+1}; \\ t(r) & \text{otherwise.} \end{cases}$$

For ϵ small, $\tilde{t}_\epsilon$ is a decreasing function. Moreover, $\int_0^1 \tilde{t}_\epsilon(r)\ \mathrm{d}r = \int_0^1 t(r)\ \mathrm{d}r$ and

$$\int_0^1 S_A^{\tilde{t}_\epsilon}(r)\ \mathrm{d}r - \int_0^1 S_A^t(r)\ \mathrm{d}r = \frac{\epsilon}{r_+^j - r_-^j} \int_{r_-^j}^{r_+^j} \Delta'(r)\ \mathrm{d}r - \frac{\epsilon}{r_+^{j+1} - r_-^{j+1}} \int_{r_-^{j+1}}^{r_+^{j+1}} \Delta'(r)\ \mathrm{d}r.$$

Since $\int_0^1 S_A^{\tilde{t}_\epsilon}(r)\ \mathrm{d}r - \int_0^1 S_A^t(r)\ \mathrm{d}r$ is linear in ϵ, there is some ϵ for which $\tilde{t}$ assumes one fewer value than t and does at least as well as t for (10).

Proof of Lemma 6. First, suppose that $S_A(\hat{r}) > 0$ at some $\hat{r}$, and is locally strictly increasing, and convex. Define a linear function $\underline{S}_A$ such that $\underline{S}_A(\hat{r}) = S_A(\hat{r})$ and $\underline{S}_A(r) < S_A(r)$ for all $r \neq \hat{r}$ in a neighborhood of $\hat{r}$. By Lemma 5, we can define a rank $\tilde{r}$ in B such that $S_B^*(\tilde{r}) = S_A(\hat{r}) + P(T^*)$. For each ϵ, we construct the following resource distribution schedule S_B^ϵ for B that coincides with S_B^* except when $r \in (\tilde{r}-\epsilon, \tilde{r}+\epsilon)$, in which case $S_B^\epsilon(r)$ is equal to $S_B^*(\tilde{r})$. By construction, for all ϵ, the schedule S_B^ϵ respects the resource constraint. For any T, the quantile-quantile plot $\bar{t}^T$ under S_A and S_B^ϵ is identical to that under S_A and S_B^* for all r outside the interval $(\tilde{r}-\epsilon, \tilde{r}+\epsilon)$, and is otherwise a constant equal to the value of the quantile-quantile plot under S_A and S_B^* at $\tilde{r}$, which is $1 - S_A^{-1}(S_B^*(\tilde{r}) - P(T))$. It follows that the change in the value of $\overline{D}(T)$, when B switches from S_B^* to S_B^ϵ, is given by

$$\int_{\tilde{r}-\epsilon}^{\tilde{r}+\epsilon} S_A^{-1}(S_B^*(r) - P(T))\ \mathrm{d}r - \int_{\tilde{r}-\epsilon}^{\tilde{r}+\epsilon} S_A^{-1}(S_B^*(\tilde{r}) - P(T))\ \mathrm{d}r.$$

At $T = T^*$, the second term of the above expression equals $2\epsilon\hat{r}$. For the first term, by definition of $\underline{S}_A$ we have

$$S_A^{-1}(S_B^*(r) - P(T^*)) \leq \underline{S}_A^{-1}(S_B^*(r) - P(T^*)),$$

for all $r \in (\tilde{r}-\epsilon, \tilde{r}+\epsilon)$, with strict inequality for all r different from $\tilde{r}$. Since both $\underline{S}_A$ and S_B^* are linear in the corresponding intervals,

$$\underline{S}_A^{-1}(S_B^*(r) - P(T^*)) = \hat{r} + (r - \tilde{r})\ \frac{S_B^{*\prime}}{\underline{S}_A'}$$

for all $r \in (\tilde{r}-\epsilon, \tilde{r}+\epsilon)$. Thus, the first term in the expression for the change in the value of $\overline{D}(T)$ is strictly smaller than $2\epsilon\hat{r}$. This implies that at $T = T^*$, for all small and positive ϵ, the value of $\overline{D}(T)$ decreases when B switches from S_B^* to S_B^ϵ. It follows that T^* is no longer a fixed point of $\overline{D}(T)$.

The mapping $\overline{D}(T)$ as a function of T under S_A and S_B^ϵ has the following properties when ϵ goes to 0: (i) $\overline{D}(T)$ converges uniformly to the mapping $\overline{D}(T)$ under S_A and S_B^*;

and (ii) the derivative of $\overline{D}(T)$ with respect to T converges uniformly to the derivative of $\overline{D}(T)$ under S_A and S_B^*. Using property (ii), the fact that the derivative of $\overline{D}(T)$ under S_A and S_B^* is strictly smaller than 1 at T^* and continuous, we can establish that there exists a small and positive γ, such that the derivative of $\overline{D}(T)$ under S_A and S_B^ϵ is less than 1 for any $T \in (T^*, T^* + \gamma)$ and for all sufficiently small ϵ. It follows that $\overline{D}(T)$ under S_A and S_B^ϵ is strictly less than T for $T \in (T^*, T^* + \gamma)$ for sufficiently small ϵ. Since by property (i) for all sufficiently small ϵ, the mapping $\overline{D}(T)$ under S_A and S_B^ϵ has no fixed point greater than $T^* + \gamma$, the largest fixed point is strictly below T^*. Thus, S_B^* is not a best response to S_A.

Next, suppose that for some $\hat{r}$ such that $S_A(\hat{r}) > 0$, the schedule $S_A(r)$ is locally convex but $S_A(r)$ is constant just below $\hat{r}$. Then, for any small and positive η, construct the modification of S_B^* as above, but set $S_B^\epsilon(\tilde{r}) = S_A(\hat{r}) + P(T^*) + \eta$. Following the same argument as above, we can establish that this modification does strictly better for B than S_B^* for all η. As η goes to 0, the modified S_B^ϵ also satisfies B's resource constraint. Thus, S_B^* is not a best response to S_A.

Finally, suppose that for some $\hat{r}$ such that $S_A(\hat{r}) > 0$, the schedule $S_A(r)$ is locally concave. Define $\tilde{r}$ as in the first case of local convexity. Consider a modified schedule S_B^ϵ which is identical to S_B^* outside the interval $(\tilde{r} - \epsilon, \tilde{r} + \epsilon)$, and is equal to $S_B^*(\tilde{r} - \epsilon)$ for all $r \in (\tilde{r} - \epsilon, \tilde{r}]$ and to $S_B^*(\tilde{r} + \epsilon)$ for all $r \in (\tilde{r}, \tilde{r} + \epsilon)$. Following a similar argument as in the case of local convexity, we can show that for sufficiently small ϵ, the largest fixed point of the mapping $\overline{D}(T)$ under S_A and S_B^ϵ is strictly smaller than T^*. Thus, S_B^* is also not a best response to S_A in this case.

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