Efficiency and Policy in the Presence of Indivisible Goods and Widespread Externalities*

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Abstract

We consider a benchmark model in which an indivisible good generates pollution. We introduce costly sharing to the model. *In the presence or absence of sharing*, we find the efficient pollution level and the efficient allocation. Except for a relatively small sharing cost parameter, the efficient allocation necessarily involves favouring one group over another but only the size and not the identity of the groups matter. In the absence of any re-distribution of income the outsiders may be worse off in the efficient outcome relative to the laissez-faire outcome. Since a market-based policy favours the upper income group while compensating outsiders through a redistribution of income (based on ownership), we compare a market-based outcome (that targets the efficient pollution level) to the laissez-faire outcome. We provide necessary and sufficient conditions for the Pareto domination of the laissez-faire outcome by the targeted market equilibrium outcome. Pareto domination occurs the lower the sharing cost, the less skewed the distribution of income is toward the poor, or the larger the middle class.

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1. Introduction

In order to develop effective public policy in the presence of negative externalities generated by indivisible goods, it is crucial to understand the implications of indivisibility for efficient environmental targets and Pareto improving environmental policy. Unlike the case of divisible goods, the efficient allocation of an indivisible good need not be common across individuals with common preferences. One group is necessarily favoured over a group of outsiders so that the efficient outcome need not Pareto dominate the laissez-faire outcome. We find the efficient pollution level along with an associated efficient allocation. We give necessary and sufficient conditions for the efficient outcome to *khs*-dominate the laissez-faire outcome after a redistribution of income based on ownership of the indivisible good.[1] We explore the market-based policy in which transferable permits are distributed. We consider the policy that targets the efficient pollution level. When the good is indivisible this targeted policy allocates the indivisible good efficiently. The targeted policy outcome Pareto dominates the laissez-faire outcome if and only if (denoted by iff) the efficient outcome khs-dominates the laissez-faire outcome.

We also explore the implications of transforming the indivisibility of the good. We assume that individuals can share a unit of the good through joint ownership. Joint ownership is associated with an inconvenience cost. As in the case of no sharing, for each non-negative sharing cost, we find the efficient pollution level and an associated efficient allocation and we consider a market-based policy that targets the efficient pollution level. When the sharing cost is low enough, all individuals receive a common allocation in the efficient outcome. Otherwise, in the efficient outcome, one group is favoured over a group of outsiders and the size and allocation of this group varies with the cost of sharing. As in the case of no sharing, we give necessary and sufficient conditions for the efficient outcome to *khs*-dominate the laissez-faire outcome. Unlike the case of no sharing, however,

[1] Outcome 1 $\mathcal{KHS}-$dominates outcome 2 if it is possible to reallocate income so that outcome 1, after reallocation, Pareto dominates outcome 2 but that the reverse is impossible. (See [3], [6], [7].) Outcome 1 *khs*-dominates outcome 2 if the transfers are based on ownership.

the targeted policy may allocate the goods inefficiently. We show that whenever the efficient outcome *khs*-dominates the laissez-faire outcome, this targeted policy outcome Pareto dominates the laissez-faire outcome even when the resulting market allocation is inefficient. We give necessary and sufficient conditions for the existence of either a sharing cost or a distribution of income for which this targeted policy outcome Pareto dominates the laissez-faire outcome. We find that Pareto domination occurs the lower the sharing cost, the less skewed the distribution of income toward the poor, or the larger the middle class.

[8] and [15] examine the use of transferable permits to reduce the consumption of a strictly indivisible good that generates a negative externality. These papers compare the welfare of the poor in the laissez-faire outcome to that in the equilibrium of a permit market that targets a given pollution level. In these papers, the given distribution of income is skewed toward lower income levels and consumers' income-independent common preferences are represented by a specific utility function. Given the functional forms and a given inefficient pollution target, their results indicate that the use of permits may make the poor worse off.[2] Though a permit scheme results in a transfer of money from the favoured rich to the poor outsiders, the transfer need not improve the outcome for the poor.[3] We extend their models in five directions. (1) We generalize the distribution of income and the utility function over consumption. (2) We introduce sharing by using a general parametrized family of sharing cost functions.[4] (3) We solve for the efficient pollution level and allocation in the presence or absence of sharing. (4) We show that the efficient outcome *khs*-dominates the laissez-faire outcome whenever the marginal income in the favoured group is large enough. (5) We give necessary and sufficient conditions for Pareto domination of the laissez-faire outcome by the targeted policy outcome and offer some policy recommendations.

We make the simplifying assumption that the variability of the negative ex-

[2] The efficient level of pollution in [8] and [15] is 0 so any positive target is inefficient.
[3] [5] assumes that an equal distribution of tradable permits ensures equitability.
[4] [2] adds a specific sharing function to the examples used in [8] and [15].

ternalities generated by ownership and use of our indivisible good is attributed solely to aggregate ownership.[5] We justify this assumption as follows. While the literature on negative externalities has primarily focused on reducing pollution by providing incentives for productive innovations or reduced usage,[6] [1] finds evidence that consumption is excessive globally and the rate of resource use is unsustainable. A reduction in the amount of materials used by an economy reduces the total amount of waste in the environment. (See [4].) Reducing this material throughput requires a change in attitude towards ownership. If a product's importance lay in the service that it provides but current ownership structures produce idle capacity then it may be feasible to re-organize the market so that one unit may service multiple users. Such a market is known as a Product-Service System. The establishment of the United Nations Environmental Programme's sustainable consumption programme has generated interest in such systems.[7]

The private automobile is a good candidate for a system change. [9] estimates that the car sits idle 95% of the time and may cost up to one third of individual net income. Car sharing not only reduces the total amount of resources used for mobility, it may also reduce use and thereby mitigate traffic congestion and lower noxious emissions.[8] We show that a restriction of the number of units consumed promotes sharing of the good and therefore may improve social welfare. Some other examples of indivisible goods that have idle capacity include lawn tractors, recreational property, washing machines, etc. [10] looks at the factors affecting the institutionalization of product service systems like car sharing.

We assume that joint ownership is feasible and that it comes at a cost. Although sharing makes sense for indivisible goods that sit idle for much of the time, attempts at sharing can be successful only if either the price of the indivisible good is high enough or the cost of sharing is low enough. For example, though early

[5] [8] and [15] make this assumption with respect to their indivisible good, the automobile.

[6] See [11] for multiple policies that mitigate the many externalities generated by automobiles.

[7] The UN site is http://www.unep.fr/scp/design/pss.htm

[8] [12] estimate that car sharing reduces mileage per person by 42% to 50% due to its low annual fee and high marginal cost based on time and distance driven.

attempts at organized car sharing were unsuccessful, recent cost saving technological innovations have contributed to the rapid growth in the number and size of car-sharing organizations over the last 2 decades (see See [13] and [14]). In 1990, Singapore instituted a policy aimed at reducing new car ownership by increasing the price of owning a car. A car sharing organization club was launched in 1997.[9]

Governments may be interested in using permit markets to reduce both the consumption of goods that generate negative externalities and the excess capacity associated with indivisible goods. It is therefore important to understand the ownership patterns and equity implications of such polices. If a government wants to promote an efficient use of resources, then the catalyst for shared use may be a permit system that increases the price of the indivisible good. If the cost of sharing is low, then individuals could enjoy the services of the indivisible good without the expense of sole ownership. If the cost of sharing is high, then outsiders may be compensated for their loss of services provided by the indivisible good. Our results suggest that additional policies aimed at reducing the inconvenience cost of sharing, increasing the wealth of the poorest individuals, or increasing the size of the middle class would improve the outcome of the permit system.

The structure of the paper is as follows: Section 2 describes the model. Section 3 finds the efficient pollution level and allocation of the good. Section 4 analyzes the equilibrium of a market-based permit system that targets the efficient pollution level. Section 5 summarizes the main results.

2. Model

Ownership of an indivisible good is tied to the generation of a widespread negative externality. Each individual is affected negatively as a result of the aggregate consumption of the indivisible good, but each individual's contribution to the aggregate is of measure zero. We assume that preferences over an indivisible good and a composite commodity are common across individuals and that there are no

[9] Singapore increased the price of owning a car by requiring owners to bid for a Certificate of Entitlement. (http://www.expatsingapore.com/once/cost.shtml)

income effects so that utility is quasilinear over the indivisible good and the composite commodity. Since preferences are common and there are no income effects, income rather than taste differentiates choices, so that consumers are denoted by their income level I. Income is distributed according to the continuously differentiable distribution function G over $[G^{-1}(0), G^{-1}(1)]$. We assume that I denotes income available to spend on the indivisible good so that it is feasible to spend all on the indivisible good. Let $\alpha(I)$ denote the indivisible good consumption of an individual with income I and let x parameterize the disutility of sharing the indivisible good. The utility function of an individual who faces aggregate consumption captured by α, and who consumes A units of the indivisible good and m other goods is

$$\phi(A) - \kappa(A, x) + m - \pi\left(\int_{G^{-1}(0)}^{G^{-1}(1)} \alpha(I) G'(I) \, dI\right)$$

where ϕ denotes the positive consumption utility over the indivisible good, $\kappa(A, x)$ denotes the negative consumption utility or inconvenience cost of sharing A units of the indivisible good when the sharing cost parameter is $x \geq 0$, and π denotes the disutility associated with the widespread externality generated by aggregate consumption of the indivisible good. Thus

$$\phi(A) - \kappa(A, x) \tag{2.1}$$

can be interpreted as the net consumption utility over the indivisible good and π is the cost of pollution generated by aggregate consumption.

We assume that $\phi'(A) > 0$, $\phi''(A) < 0$, $\phi(0) = 0$, $\phi(1) > 1$, and that π is increasing, convex and that $\pi(0) = 0$, $\pi'(0) + 1 < \phi'(0)$. We now discuss the disutility or cost of sharing, $\kappa(A, x) \geq 0$.[10] We introduce the following notation to make it easier to discuss the relationship between fractions and the family of sharing cost functions. Let $\lceil A \rceil$ denote the largest integer less than or equal to A

[10] These functions generalize $(\pi(z) = \sigma z, \sigma > 1)$ used in [8] (with $\phi(A) = 2A$) and used in [15] (with $\phi(A) = 2A^{1/2}$). There is no counterpart to $\kappa(A, x)$.

and let $\lfloor A \rfloor$ denote the smallest integer greater than or equal to A. We assume when $x > 0$, that for every fraction $A \in (\lceil A \rceil, \lfloor A \rfloor)$, (i) $\kappa(A,x) > 0$ so that the cost of sharing is incurred if the consumer chooses to share, (ii) $\kappa'_x(A,x) > 0$ for $x \in \Re_+$ (where $\Re_+$ denotes the non-negative real numbers), (iii) $-\infty < \kappa'_A(A,x) \leq 0$ so that the cost of sharing decreases as the quantity of sharing decreases, and (iv) $\kappa(A,x)$ is twice continuously differentiable with $k''_{AA} \geq 0$ and $k''_{Ax} < 0$. We assume that when $A \in \mathbb{N}_+$ (where $\mathbb{N}_+$ denotes the non-negative integers) then $\kappa(A,x) = 0$ for all $x \geq 0$, (i.e., $\phi(A) - \kappa(A,x) + m = \phi(A) + m$ for $A \in \mathbb{N}_+$) so that the consumer incurs no cost of sharing whenever the consumer chooses not to share. We assume that $\kappa(A,0) = \kappa'_A(A,0) = 0$ for all $A \in \Re_+$ and that $\lim_{A \uparrow \lfloor A \rfloor} \kappa'_A(A,x) = -x$. Thus, κ decreases in A from something positive to 0 for $A \in (\lceil A \rceil, \lfloor A \rfloor]$ and then jumps up to something positive again immediately to the right of $A = \lfloor A \rfloor$ so that there is a discontinuity from the right at each integer. Thus, κ is left-continuous for every A but is right-discontinuous at each integer.[11] Therefore $\phi(A) - \kappa(A,x) + m$ is continuous in A for $A \in \Re \diagdown \mathbb{N}_+$, but is discontinuous from the right for $A \in \mathbb{N}_+$.

Let $MRS_x(A,m)$ denote the marginal rate of substitution given the sharing cost parameter x. Given $x > 0$, for every commodity bundle (A,m),

$$MRS_x(A,m) = \phi'(A) - \kappa'_A(A,x) > \phi'(A) = MRS_0(A,m). \qquad (2.2)$$

For any fraction $A \in (\lceil A \rceil, \lfloor A \rfloor)$, indifference curves are decreasing and convex in A and $MRS_x(A,m)$ increases in x. In addition, $MRS_x(A,m)$ is continuous at any non-integer but is continuous only from the left at any integer.

Since the laissez-faire outcome (LFO) is not the focus of the paper, we formulate the model so that each consumer demands 1 unit of the indivisible good in the LFO. We assume that the measure of suppliers is greater than the measure of consumers, the supply of the indivisible good is perfectly elastic at $p = 1$, and $G^{-1}(0) \geq p = 1$. Since m is a composite commodity, its price is normalized to 1. Thus, the price ratio is 1. We assume that $\phi'(1) = 1$ so that

[11] One example of such a function κ is $\kappa(A,x) = x(\lfloor A \rfloor - A)$. A second example is $\kappa(A,x) = x(1/(A - \lceil A \rceil) - 1)$ if $\lceil A \rceil < A < \lfloor A \rfloor$, 0 otherwise.

$MRS_x(1,m) \geq \phi'(1) = 1$. Thus, if $x = 0$ (that is, the indivisible good is made divisible at no cost), then, independent of α, each consumer demands 1 unit of the indivisible good. Since 1 unit is preferred to any non-negative number when $x = 0$, then 1 unit is preferred to any other integer when sharing is not an option. If $x > 0$, then, since the cost of sharing is zero at an integer but positive everywhere else, each consumer still demands 1 unit at the price 1. Thus, in the LFO, $p = 1$, $\alpha(I) = 1$, $\int \alpha(I)\, G'(I)\, dI = 1$ and the utility of individual I is given by $\phi(1) + I - 1 - \pi(1)$.

3. Efficient Outcome

We find the optimal pollution level and allocation of the indivisible good (that is, the aggregate utility-maximizing pollution level and indivisible good allocation) under two different assumptions: no sharing and sharing (cost parameter $x \geq 0$). Since $p = 1$, the individual I who consumes A units spends $I - A$ on the composite good. Given A, α, and I, individual I's utility equals

$$\phi(A) - \kappa(A, x) + I - A - \pi\left(\int \alpha(I)\, dI\right) \tag{3.1}$$

Since aggregate income is constant and equal to

$$I^G = \int_{G^{-1}(0)}^{G^{-1}(1)} I G'(I) dI \tag{3.2}$$

the income allocation is irrelevant for any given efficient pollution and indivisible good allocation. In the efficient outcome (denoted by EO) each individual I who is allocated A units is allocated income $I - A$.

As shown in Theorems 3.1 and 3.4, there may be a favoured group and a group of outsiders in the EO. The size of the favoured group in the EO is fixed but the identity of its members is irrelevant. Corollaries 3.2 and 3.5 determine the conditions for which the EO Pareto dominates the LFO independent of the identity of the favoured group. When Pareto domination fails, the favoured group is better off and the outsiders are worse off than at the LFO. In this case, we use a variant of KHS-domination[12] for comparison purposes. Since preferences are common across consumers, the only equitable redistribution of income is based solely on ownership. When the favoured group in the EO consists of the highest-income group we call this EO the distinguished efficient outcome (denoted DEO). When the DEO KHS-dominates the LFO by redistributing income based on ownership of the indivisible unit, we say that the DEO *khs*-dominates the LFO. Corollaries 3.2 and 3.5 determine the conditions for which the DEO *khs*-dominates the LFO. We begin with the case in which sharing is not an option.

3.1. No sharing

In the case that sharing is not an option, individuals either buy 1 unit at the price 1 or buy 0 units so that $\kappa(A, x)$ disappears from the above utility. The aggregate pollution level is z when each consumer in a set of measure z buys 1 unit and each in a set of measure $1 - z$ buys 0 units. In this case, since there are no income effects, utility can be aggregated using (3.1) and (3.2) as $z(\phi(1) - 1) - \pi(z) + I^G$. We find the optimal pollution level (denoted by z_n^*) in the case of no-sharing.

In order to state Theorem 3.1, we let $\overline{z}$ denote the solution in z to

$$\phi(1) = \pi'(z) + 1 \tag{3.3}$$

Theorem 3.1. *If there is no sharing then*

$$z_n^* = \begin{cases} 1 & \textit{if} \quad \pi'(1) + 1 < \phi(1) \\ \overline{z} & \textit{if} \quad \pi'(0) + 1 \leq \phi(1) \leq \pi'(1) + 1 \\ 0 & \textit{if} \quad \phi(1) < \pi'(0) + 1 \end{cases} \tag{3.4}$$

The optimal allocation assigns 1 *to each in a group of size* z_n^* *and* 0 *to all else.*

Unless otherwise stated, all proofs omitted from the text are in the Appendix. Theorem 3.1 is intuitive. If $\overline{z} \in (0, 1)$, then when each in a set of measure $\overline{z}$ buys 1 unit of the indivisible good and each in a set of measure $1 - \overline{z}$ buys 0 units,

[12] See footnote 1.

the benefit of an additional consumer moving from 0 to 1 unit (in terms of the discrete jump in utility from 0 to $\phi(1)$) equals the cost (in terms of price and extra pollution) of an additional consumer moving from 0 to 1.

Let I_f denote the bottom income level of the highest-income group of size $f \in [0,1]$ so that $I_f = G^{-1}(1-f)$.

Corollary 3.2. *Suppose that there is no sharing. If $z_n^* = \overline{z} \in (0,1)$, then the EO Pareto dominates the LFO iff*

$$\pi(1) - \pi(\overline{z}) \geq \phi(1) - 1 \tag{3.5}$$

When (3.5) fails, the DEO khs-dominates the LFO iff

$$I_{\overline{z}} \geq \frac{(1-\overline{z})}{\overline{z}} \left[\phi(1) - 1 - (\pi(1) - \pi(\overline{z}))\right] + 1. \tag{3.6}$$

The intuition is as follows. If (3.5) holds, the benefit from the reduction in pollution is greater than the decrease in consumption utility suffered by the outsiders in the EO so that all are better off relative to the LFO. If (3.5) fails, the outsiders are worse off in the EO. A redistribution based on ownership can make the outsiders better off in the DEO relative to the LFO if the marginal favoured income is large enough (that is, if (3.6) holds). We note that, in [8] and in [15], the optimal pollution level is $z_n^* = 0$, so that the EO Pareto dominates the LFO.[13]

3.2. Costly Sharing: $x \geq 0$

In the case that there is costly sharing, individuals may buy any fraction of a unit but need not buy a common fraction so that we first find the optimal allocation of the indivisible good. For any given pollution target z, Proposition 3.3 states the optimal allocation of the indivisible good for any sharing cost parameter $x \geq 0$. While there are a continuum of feasible allocations, the optimal allocation takes only one form. The feasible optimal allocation of $z \in [0,1]$ units of pollution

[13] In these two papers $\phi(1) - 1 < \pi(1) - \pi(0)$.

allocates a common fraction $A \in [z,1]$ to each member of a favoured group of size z/A and 0 to each outsider. Given x, $p=1$, (3.1), and (3.2), the aggregate utility that results when z units of pollution are generated by allocating a common fraction $A \geq z$ at a per unit price of 1 to each member of a group of size z/A is

$$\frac{z\left(\phi\left(A\right)-\kappa\left(A,x\right)\right)}{A}-z+I^{G}-\pi\left(z\right). \tag{3.7}$$

Thus, given (z,x), the optimal common fraction, $A^*(z,x)$, maximizes the average of net consumption utility (that is, the average of (2.1) over $A \in [z,1]$.

When $A^*(z,x) \in (z,1)$, $A^*(z,x)$ will satisfy first order conditions. Otherwise, $A^*(z,x)=z$ or $A^*(z,x)=1$. We introduce the following notation to demarcate the two boundaries. We implicitly define Z_0 as a function of x by[14]

$$\left(\frac{\phi\left(Z_0\right)-\kappa\left(Z_0,x\right)}{Z_0}\right)-\left(\phi'\left(Z_0\right)-\kappa'_A\left(Z_0,x\right)\right)=0 \tag{3.8}$$

(so that $Z_0(x)$ equates the average net consumption utility to the marginal net consumption utility and therefore the average net consumption utility is maximized at $Z_0(x)$ over all A); we let X_0 denote the inverse of Z_0 so that X_0 is a function of $z>0$; we define x_1 to be the solution in x to

$$\phi\left(1\right)=\phi'\left(1\right)-\kappa'_A\left(1,x\right) \tag{3.9}$$

(so that $Z_0(x_1)=1$ or $X_0(1)=x_1$); and we let

$$\mu_G\left(S\right)=\int_{I\in S}G'\left(I\right)dI \tag{3.10}$$

denote the measure of a set S with respect to the distribution G. We can now state Proposition 3.3 succinctly.

Proposition 3.3. *Given $z \geq 0$ and $x \geq 0$, the optimal allocation (denoted by $\alpha_{z,x}(I)$) of the indivisible good among the population satisfies*

$$\alpha_{z,x}\left(I\right)=\begin{cases} A^*\left(z,x\right) & \text{if } \; I\in S \text{ for some } \mu_G\left(S\right)=z/A^*\left(z,x\right) \\ 0 & \text{if } \; I\notin S \end{cases} \tag{3.11}$$

[14] For $x=0$, (3.8) is $\phi(Z_0)/Z_0-\phi'(Z_0)=0$. By L'Hopital's rule, $\lim_{Z_0\downarrow 0}\phi(Z_0)/Z_0=\phi'(0)$. Since $\phi(0)=0$, ϕ is concave, $\phi(z)>\phi'(z)z$ for $z>0$, $Z_0(0)=0$.

where the common fraction given to each in the favoured group is

$$A^*(z,x) = \begin{cases} z & \textit{if} \quad 0 \le x \le X_0(z) \\ Z_0(x) & \textit{if} \quad X_0(z) \le x \le x_1 \\ 1 & \textit{if} \quad x \ge x_1 \end{cases} \tag{3.12}$$

Proposition 3.3 is intuitive. If x is small enough, the good is essentially divisible so that z units are allocated homogeneously across all individuals. As x increases, the homogenous allocation becomes too costly relative to the allocation in which each in a favoured group receives $Z_0(x) \in (z,1)$ and each outsider receives 0. The gain in reduced sharing costs (newly formed favoured members share less than previously and the newly formed outsiders do not share at all) outweighs the loss in consumption utility (the discrete drop in consumption to 0 for the new outsiders outweighs the continuous increase in consumption for the favoured members). If x is so large that the good is essentially indivisible, each in a favoured group of size z receives 1. Any further increase in x has no effect on the allocation.

Theorem 3.4 characterizes the optimal pollution level as a function of $x \ge 0$.[15] Given $x \ge 0$ and $A^*(z,x)$, the optimal pollution level equates marginal social benefit to marginal social cost. At any pollution level $z \ge 0$, the marginal social cost (denoted by $MC(z)$) of another unit of pollution is

$$MC(z) = \pi'(z) + 1 \tag{3.13}$$

The marginal social benefit (denoted by $MB(z,x)$) of another unit of pollution is defined in a piecewise fashion as follows. If there are no outsiders, then

$$MB(z,x) = \phi'(z) - \kappa'_A(z,x) \tag{3.14}$$

If each in a favoured group of size $z/Z_0(x)$ consumes $Z_0(x) \in (0,1]$ while all else consume 0, then the marginal social benefit of another unit of pollution is the

[15] If we allow sharing with $x=0$ in [8] and [15] then (using $A=z$ in (3.7) since $x<x_s$), in [8], aggregate utility $2z-(1+\sigma)z+I_G$ is maximized at $z=0$ since $\sigma>1$; in [15], $2z^{\frac{1}{2}}-(1+\sigma)z+I_G$ is maximized at $z=1/(1+\sigma)^2$.

average net consumption utility

$$MB(z,x) = \frac{\phi(Z_0(x)) - \kappa(Z_0(x),x)}{Z_0(x)} \tag{3.15}$$

since an increase in z increases the size of the favoured group and each new member consumes $Z_0(x)$ rather than 0. Lastly, if each in a favoured group of size z consumes 1 while all else consume 0, then the marginal social benefit of another unit of pollution is the average net consumption utility

$$MB(z,x) = \phi(1) - \kappa(1,x) \tag{3.16}$$

since an increase in z increases the size of the favoured group and each new member consumes 1 rather than 0.

A candidate for optimal pollution satisfies $MB(z,x) = MC(z)$. Using (3.13) and (3.14), we define $\overrightarrow{Z}(x)$ implicitly by

$$\phi'\left(\overrightarrow{Z}\right) - \kappa'_A\left(\overrightarrow{Z},x\right) = \pi'\left(\overrightarrow{Z}\right) + 1. \tag{3.17}$$

$\overrightarrow{Z}(x)$ is a candidate optimal pollution level only if $0 \leq x \leq X_0\left(\overrightarrow{Z}(x)\right)$ and $0 \leq \overrightarrow{Z}(x) \leq 1$. Using (3.8), (3.13) and (3.15), we define $\overleftarrow{Z}(x)$ implicitly by

$$\phi'(Z_0(x)) - \kappa'_A(Z_0(x),x) = \frac{\phi(Z_0(x))}{Z_0(x)} - \frac{\kappa(Z_0(x),x)}{Z_0(x)} = \pi'\left(\overleftarrow{Z}\right) + 1. \tag{3.18}$$

$\overleftarrow{Z}(x)$ is a candidate optimal pollution level only if $X_0\left(\overleftarrow{Z}(x)\right) \leq x \leq x_1$ and $0 \leq \overleftarrow{Z}(x) \leq 1$. Using (3.13) and (3.16) we see that $\overline{z}$ as defined in (3.3) is a candidate optimal pollution level only if $x_1 \leq x$ and $0 \leq \overline{z} \leq 1$.

Since the first order conditions for the optimal pollution level (denoted by $Z_c^*(x)$) vary, we introduce the following notation to demarcate the boundaries. The Implicit Function Theorem can be used to obtain that, as x increases, $\overrightarrow{Z}$ increases, $\overleftarrow{Z}$ decreases, and $Z_0(x)$ increases. If $Z_c^*(x) = \overrightarrow{Z}(x)$, then this option ends when $\overrightarrow{Z}(x) = \overleftarrow{Z}(x)$ at x_s which solves

$$\phi'(Z_0(x)) - \kappa'_A(Z_0(x),x) = \pi'(Z_0(x)) + 1, \tag{3.19}$$

or when $\overrightarrow{Z}(x) = 1$ at x_2 where x_2 solves

$$\phi'(1) - \kappa'_A(1, x) = \pi'(1) + 1 \tag{3.20}$$

If $Z_c^*(x) = \overleftarrow{Z}(x)$, then each of the favoured members receive $Z_0(x)$ so that this option ends when either $\overleftarrow{Z}(x) = \overline{z}$ and $Z_0(x) = 1$ at x_1 (defined by (3.9)), or when $\overleftarrow{Z}(x) = 0$ at x_3 which solves

$$\pi'(0) + 1 = \phi'(Z_0(x)) - \kappa'_A(Z_0(x), x) \tag{3.21}$$

Theorem 3.4. *If $x \geq 0$ then the optimal level of pollution is*

$$Z_c^*(x) = \begin{cases} \overrightarrow{Z}(x) & \textit{if} \quad 0 \leq x \leq \min\{x_s, x_2\} < x_1 \\ \overleftarrow{Z}(x) & \textit{if} \quad x_s \leq x \leq \min\{x_1, x_3\} < x_2 \\ z_n^* & \textit{if} \quad \textit{otherwise} \end{cases}$$

where z_n^* satisfies (3.4) and the optimal allocation $A_c^*(x) = A^*(Z_c^*(x), x)$.

Theorem 3.4 is intuitive. If the sharing cost parameter x is small enough, the optimal allocation is uniform. As x increases, $Z_c^*(x) = \overrightarrow{Z}(x)$ increases because the increased sharing cost and pollution are offset by the increased benefit from the decrease in sharing and the increase in aggregate consumption of the indivisible good. The optimal pollution increases until either $\overrightarrow{Z}(x) = 1$ or $\overrightarrow{Z}(x) = \overleftarrow{Z}(x)$ when the benefit of the increase equals the cost of the increase (Since ϕ is concave and π is convex, eventually the benefit must be less than the cost). When $Z_c^*(x) = \overleftarrow{Z}(x)$, the allocation awards $Z_0(x)$ to each in a favoured group of size $\overleftarrow{Z}(x)/Z_0(x)$. As x increases, $Z_c^*(x) = \overleftarrow{Z}(x)$ decreases, the common consumption level, $Z_0(x)$, increases, and the optimal size of the favoured group decreases. This makes sense because the increased sharing cost and decreased aggregate consumption of the indivisible good that is associated with an increased x are offset by both the decrease in pollution and the decrease in sharing. If x is so large that no sharing occurs, the optimal pollution level is determined by Theorem 3.1.

Corollary 3.5 is the costly sharing analogue of Corollary 3.2 when $Z_c^*(x) \neq z_n^*$. If $Z_c^*(x) = z_n^*$, then there is no sharing so that Corollary 3.2 applies.

Corollary 3.5. *Suppose that $x \geq 0$. If $Z_c^*(x) \neq z_n^*$, then the EO Pareto dominates the LFO iff $Z_c^*(x) = \overrightarrow{Z}(x)$* **or** *$Z_c^*(x) = \overleftarrow{Z}(x)$ and*

$$\pi(1) - \pi\left(\overleftarrow{Z}(x)\right) \geq \phi(1) - 1 \tag{3.22}$$

When $Z_c^*(x) = \overleftarrow{Z}(x)$ and (3.22) fails, the DEO khs-dominates the LFO iff

$$I_{\frac{\overleftarrow{Z}(x)}{Z_0(x)}} \geq \frac{\left(1 - \frac{\overleftarrow{Z}(x)}{Z_0(x)}\right)}{\frac{\overleftarrow{Z}(x)}{Z_0(x)}} \left[\phi(1) - 1 - \left(\pi(1) - \pi\left(\overleftarrow{Z}(x)\right)\right)\right] + Z_0(x). \tag{3.23}$$

In the case that $Z_c^*(x) = \overleftarrow{Z}(x)$ and each in the favoured group receives $Z_0(x)$, the intuition is as follows. When (3.22) holds, the benefit from the reduction in pollution is greater than the decrease in consumption utility suffered by the outsiders in the EO so that everyone is better off relative to the LFO. When (3.22) fails, then the outsiders are worse off in the EO. A redistribution based on ownership in the DEO can make the outsiders better off than in the LFO only if the lowest income of the favoured group in the DEO is large enough.

4. Policy Instruments

We now find conditions under which the outcome of the market-based permit policy (denoted by $\langle g, h \rangle$) is efficient.[16] Let policy $\langle g, h \rangle$ denote a market for coupons that is established through a uniform allocation of g coupons and a requirement that h coupons be submitted per unit of the indivisible good. The coupon price adjusts until supply equals demand in the coupon market. The policy outcome is efficient iff the pollution level satisfies Theorem 3.4 and the allocation of the indivisible good satisfies Proposition 3.3. As in the DEO, the policy may result in a favoured group and a group of outsiders. The trade in permits redistributes income away from those those in the favoured group to the outsiders. We first consider the case in which there is no sharing.

[16] The efficient outcome can also be reached if the government imposes a suitable tax on ownership which it then distributes among the outsiders.

Theorem 4.1. *If there is no sharing and $z_n^* = \overline{z} \in (0,1)$, then the market equilibrium outcome of the policy $< g, h >$ is efficient iff*

$$\frac{g}{h} = \overline{z} \tag{4.1}$$

Theorem 4.1 is intuitive. In the absence of sharing, indivisibility implies that, in the policy equilibrium, either all individuals are indifferent between 1 and 0 units of the indivisible good or all prefer 1 unit to 0 and those with high enough income obtain the good. Thus, under policy $\langle g, h \rangle$, the fraction g/h determines the size of the group whose members receive 1 unit. The allocation is optimal whenever g/h equals the optimal pollution level.

Corollary 4.2. *If there is no sharing and $z_n^* = \overline{z} \in (0,1)$, then the policy $\langle g, h \rangle$ for which (4.1) holds Pareto dominates the LFO iff (3.6) holds.*

Thus, if there is no sharing, then, by Corollaries 3.2 and 4.2, the equilibrium of the targeted policy $\langle g, h \rangle$ Pareto dominates the LFO iff the DEO *khs*-dominates the LFO. Corollary 4.2 is intuitive. In the case of no sharing, in both the DEO and the targeted policy equilibrium, each favoured group member receives 1 unit and each outsider receives 0. Since trade in permits results in a transfer from the favoured group to the outsiders, the outsiders are better off under $\langle g, h \rangle$ relative to the DEO. Thus, if the DEO Pareto dominates the LFO, then so does the targeted policy equilibrium. If instead, the outsiders in the DEO are worse off relative to the LFO, then it is possible to compensate the outsiders after a redistribution of income whenever the DEO *khs*-dominates the LF0. In this case, the targeted policy equilibrium also transfers an amount that compensates the outsiders. If the DEO does not *khs*-dominate the LFO, then it is impossible to make the outsiders better off relative to the LFO through a transfer of income so that the targeted policy equilibrium cannot Pareto dominate the LFO.

When sharing is possible and the cost parameter is $x \geq 0$, the analogues of Theorem 4.1 and Corollary 4.2 are Theorems 4.3 and 4.5 and Corollaries 4.4 and 4.6. In the presence of sharing, the policy equilibrium allocation may be inefficient even if the pollution level is efficient.

Theorem 4.3. *If $x \geq 0$ and $Z_c^*(x) \in (0,1)$ then the pollution level in the equilibrium of policy instrument $\langle g, h \rangle$ is efficient iff*

$$\frac{g}{h} = Z_c^*(x) \tag{4.2}$$

and the allocation is efficient iff

$$I_{\frac{Z_c^*(x)}{A_c^*(x)}} \geq \pi'(Z_c^*(x))(A_c^*(x) - Z_c^*(x)) + A_c^*(x) \tag{4.3}$$

An efficient policy equilibrium Pareto dominates the LF0.

Corollary 4.4. *Suppose $x \geq 0$ and $Z_c^*(x) \in (0,1)$, then the policy $\langle g, h \rangle$ (for which (4.2) holds) Pareto dominates the LFO* ***unless***

$$\phi(1) - 1 > \pi(1) - \pi(Z_c^*(x)) \tag{4.4}$$

AND either *$Z_c^*(x) = \overleftarrow{Z}(x)$ and (3.23) fails* **or** *$Z_c^*(x) = \overline{z}$ and (3.6) fails.*

Thus, whenever the DEO khs-dominates the LFO and the pollution target is optimal, the policy outcome Pareto dominates the LFO even if the market allocation is inefficient. As seen below, the policy outcome may Pareto dominate the LFO even if the DEO does not khs-dominate the LFO.

Theorem 4.5 gives necessary and sufficient conditions for the existence of a targeted policy equilibrium that does not Pareto dominate the LFO. For any given income distribution, G, with support $[\underline{I}, \overline{I}]$, we introduce a set, F, of parametrized families of distributions that are derived from G. Each family of distributions $\{G^t\}_{t\in[o,\infty)}$ is parametrized by t for any fixed original G. If $t_1 < t_2$, then G^{t_1} first order stochastically dominates G^{t_2} so that, as t increases, G^t puts more of its weight on something that is getting closer to the lower limit of support $\underline{I}(t)$ which converges to $\lim_{t\uparrow\infty} \underline{I}(t) \geq 1$ (denoted by $\underline{I}^0$).

Theorem 4.5. *Let ϕ, κ, π, and G be given for which $Z_c^*(x) \in (0,1)$ for all $x \geq 0$. (i) If $x > x_s$, then for every family $\{G^t\}_{t\in[o,\infty)} \in F$, derived from G, there exists*

$\widehat{t} < \infty$ for which $t > \widehat{t}$ implies the targeted policy equilibrium does not Pareto dominate the LFO iff

$$g\underline{\rho}^0 < \phi(1) - 1 - \left(\pi(1) - \pi\left(\frac{g}{h}\right)\right) \tag{4.5}$$

where $\underline{\rho}^0$ solves

$$\frac{\phi\left(\frac{\underline{I}^0 + g\rho}{1+h\rho}\right) - \kappa\left(\frac{\underline{I}^0 + g\rho}{1+h\rho}, x\right)}{\frac{\underline{I}^0 + g\rho}{1+h\rho}} = 1 + h\rho \tag{4.6}$$

and $g/h = Z_c^*(x)$. (ii) There exists $\widehat{x} \geq x_1$ for which $x > \widehat{x}$ implies the targeted policy equilibrium does not Pareto dominate the LFO iff (3.6) fails.

We note that in (i) of Theorem 4.5 $x > x_s$ is fixed and one can transform any given distribution into a family of distributions in F in a variety of ways. For example one can decrease the upper limit of the support, decrease the lower limit of support or skew the distribution towards the poor with no change in support. Whether or not any particular family of distributions will lead to a failure of Pareto domination depends on the poorest individual of the limiting distribution in the family relative to ϕ, κ, π, and x. In (ii) of Theorem 4.5 G is fixed and one can vary $x > x_1$. Whether any given x leads to a failure of Pareto domination depends on the marginal individual given G relative to ϕ, κ, π, and x.

Theorem 4.5 gives conditions under which one can find a distribution or a sharing cost for which Pareto domination fails. The permit price, $\underline{\rho}^0$, defined by (4.6), is the price that leaves the limiting poorest individual indifferent between trading all permits or keeping all permits. Given a distribution G, there exists a stochastically dominated transformation of G for which Pareto domination fails iff the value of the permit endowment at $\underline{\rho}^0$ is not enough to compensate an outsider for their loss in consumption despite the reduction in pollution. In particular, this allows for Pareto domination even in the absence of *khs*-domination and in the presence of an inefficient allocation since both can occur when (4.5) fails. Given a distribution G, there exists $x > x_1$ for which Pareto domination of the LFO by the targeted policy equilibrium fails iff *khs* domination by the DEO fails.

Corollary 4.6. *Given ϕ, κ, π, $x > x_s$ and G, there exists a family $\{G^t\}_{t\in[1,\infty)} \in F$, for which there exists $\widehat{t} < \infty$ such that $t > \widehat{t}$ implies Pareto domination fails iff $\underline{\rho}^0$ satisfies 4.5 when $\underline{I}^0 = 1$.*

If $x > x_s$, Corollary 4.6 tells us if there exists a stochastically dominated transformation of G for which Pareto domination fails. Theorem 4.5 states when there exists such a transformation in any given family in F. Relative to ϕ, κ and π, if either the marginal individual is poor enough or the lower bound of the support is low enough, then either increasing the cost of sharing with G fixed,[17] or transforming the distribution by decreasing the income of the richest individual,[18] will eventually lead to the absence of Pareto domination. However, even if the poorest individual has the lowest possible income (which equals 1), Pareto domination may result no matter the upper income. This occurs when $\underline{\rho}^0$ is large enough to compensate the outsiders.[19] Note also that, if the distribution of income is relatively skewed enough toward lower income levels, then Pareto domination may fail even if the highest income level is infinity.[20] Lastly, if the gain from the decrease in pollution is higher than the loss from the decrease in consumption then Pareto domination occurs regardless of the income distribution.[21]

5. Conclusion

When an indivisible good generates negative externalities, the efficient aggregate consumption necessarily allocates this good asymmetrically across consumers.

[17] $4 < \widehat{x} < 5$ if $G(I)$ is uniform on $[1, 1.1]$, $\phi(A) = 2A^{1/2}$, $\kappa(A,x) = x(\lfloor A \rfloor - A)$, $\pi(z) = z^2$.

[18] $3/2 < \widehat{t} < 7/3$, $x_s < x = 0.95 < x_1$, $\overleftarrow{Z}(0.95) \simeq 0.503$, $\underline{\rho}^0 \simeq 0.09$ satisfies (4.5) if $G^t(I)$ uniform on $[1, 1 + 1/(t+1)]$, $\phi(A) = 2A^{\frac{1}{2}}$, $\kappa(A,x) = x(1/A - 1)$, $\pi(z) = 0.974\,339\,5111 z^{1.1}$.

[19] $\widehat{t} = \infty$, $x = 0.89$, $\overleftarrow{Z}(0.89) \simeq 0.507$, $g\underline{\rho}^0 \simeq 0.513\,3$ violates (4.5) if $G^t(I)$ uniform on $[1, 1/(t+1) + 1]$, $\phi(A) = 2A^{1/2}$, $\kappa(A,x) = x(\lfloor A \rfloor - A)$, $\pi(z) = z^2$.

[20] $\widehat{t} = 16k$, $\overline{z} = 0.5$, $x = 1.119\,160\,55 > x_1, \overline{z} = 0.5$ if $\phi(A) = 2A^{\frac{1}{2}}$, $\kappa(A,x) = x(1-A)$ and $\pi(z) = 0.974\,339\,511\,1 z^{1.1}$, $G^t(I) = 1 - k/t(I-1)$ on $[k/t + 1, \infty)$, for $k > 1/16$. If $t > \widehat{t}$, Pareto domination fails though income increases to ∞. This family of distribution functions includes that used in [8] and [15].

[21] If $\phi(A) = 2A^{\frac{1}{2}}$, $\kappa(A,x) = x(1-A)$, $\pi(z) = (4/3)z^3$, then $Z_c^*(x_s) \simeq 0.53$ so that (4.4) is violated for all $x \geq 0$. Outsiders are better off under the targeted market policy regardless of G.

Given the presence of insiders and outsiders, the efficient outcome may not Pareto dominate the laissez-faire outcome. We give necessary and sufficient conditions for the efficient outcome to Pareto dominate the laissez-faire outcome after a suitable transfer of income from those in the favoured group to the outsiders. These necessary and sufficient conditions coincide with those that guarantee Pareto domination of the laissez-faire outcome by the permit policy that targets the efficient pollution level. The targeted policy increases the full price of the indivisible good via the permit price by enacting a 'tax on ownership' that compensates the outsiders for their loss in consumption in the presence of Pareto domination.

Our model provides a simple setting in which to analyze the policy implications of negative externalities in the presence of indivisibilities. We introduce sharing as a way to mitigate the inefficiencies arising from indivisibility. Intuitively, when the cost of sharing is low, indivisibilities are transformed into divisibilities so that both the efficient outcome and the targeted policy equilibrium Pareto dominate the laissez-faire outcome. As the cost of sharing increases to infinity, the efficient outcome and the targeted policy equilibrium converge nonmonotonically to the case of no sharing.

In the presence of sharing, Pareto domination of the laissez-faire outcome may occur even when the targeted policy equilibrium does not result in the efficient allocation. Pareto domination occurs the lower the sharing cost or the higher the income of the marginal consumer. Our results suggest that the efficiency properties of the targeted policy would improve if combined with policies aimed at reducing the inconvenience cost of sharing, increasing the wealth of the poorest individuals, and increasing the size of the middle class.

A. Appendix

Proof. (THEOREM 3.1): π is convex and increasing in $z \in [0, 1]$. ■
Proof. (COROLLARY 3.2): Replace (3.18) with (3.3), (3.22) with (3.5), (3.23) with (3.6), $\overleftarrow{Z}$ with $\overline{z}$ and Z_0 with 1 in the proof of Corollary 3.5. ■

Proof. (PROPOSITION 3.3): Given the pollution level z there is a measure z of indivisible goods. A feasible allocation $\alpha : \left[G^{-1}(0), G^{-1}(1)\right] \longrightarrow [0,1]$ satisfies $\int_{[G^{-1}(0),G^{-1}(1)]} \alpha(I)\, G'(I)\, dI = z$. Since $\phi(0) = \kappa(0,x) = 0$, aggregate welfare (associated with α and z) is

$$I^G - \pi(z) + \int_{S_\alpha} \left(\phi(\alpha(I)) - \alpha(I) - \kappa(\alpha(I), x)\right) G'(I)\, dI$$

where $\alpha(I) = 0$ iff $I \notin S_\alpha$. The optimal allocation $\alpha_z^*(I)$ solves problem (A.1):

$$\max_{\alpha:[G^{-1}(0),G^{-1}(1)]\longrightarrow[0,1]} \int_{S_\alpha} H(\alpha(I), x)\, G'(I)\, dI \text{ s.t.} \tag{A.1}$$

$$\int_{S_\alpha} \alpha(I)\, G'(I)\, dI = z$$

where $H(\alpha(I), x) = \phi(\alpha(I)) - \alpha(I) - \kappa(\alpha(I), x)$. If α^* solves (A.1) then

$$\frac{d}{d\varepsilon}\left(\int_{S_{\alpha^*}} H(\alpha^*(I) + \varepsilon\eta(I), x)\, G'(I)\, dI\right)|_{\varepsilon=0} = 0$$

for every $\varepsilon \geq 0$ and every $\eta : \left[G^{-1}(0), G^{-1}(1)\right] \longrightarrow [0,1]$ for which

$$\alpha^* + \varepsilon\eta : \left[G^{-1}(0), G^{-1}(1)\right] \longrightarrow [0,1] \text{ and } \int_{[G^{-1}(0),G^{-1}(1)]} \eta(I)\, G'(I)\, dI = 0, \tag{A.2}$$

That is, α^* is optimal only if

$$\int_{S_\alpha} \eta(I)\, H_A'(\alpha^*(I), x)\, G'(I)\, dI = 0 \text{ for all feasible } \eta \tag{A.3}$$

Since $H_A'(A,x)$ is decreasing in A and since (A.3) must hold for any function η that satisfies (A.2), it is immediate that $\alpha^*(I)$ must be constant on S_{α^*}.

The feasible set of optimal allocation functions (denoted by $OA(z)$) is therefore the set of allocation functions $\alpha : \left[G^{-1}(0), G^{-1}(1)\right] \longrightarrow [0,1]$ such that, there exists $A \in [z,1]$ and a set S_α for which $\mu_G(S) = z/A$ and

$$\alpha(I) = \begin{cases} A & \text{if} \quad I \in S_\alpha \\ 0 & \text{if} \quad I \notin S_\alpha \end{cases}$$

If the pollution level equals z and we vary the fraction $A \in [z, 1]$ that is allocated to a favoured group of size z/A we can find the optimal level of A by maximizing the aggregate utility function (3.7) with respect to A. That is, the solution to (A.1) is $\alpha \in OA(z)$ whose constant value maximizes average net consumption utility

$$AVG(A) \equiv \frac{\phi(A) - \kappa(A, x)}{A} \tag{A.4}$$

over $A \in [z, 1]$. Since $AVG(A)$ increases in A for $A \leq Z_0(x)$ (defined by 3.8) and decreases for $A \geq Z_0(x)$, $\alpha_{z,x}(I)$ satisfies (3.11) and $A^*(z, x)$ satisfies (3.12).

Note that the $\lim_{x \uparrow x_1} A^*(z, x) = 1$, $X_0(1) = x_1$ so that $X_0^{-1}(x_1) = 1$, when $x = x_1$ and so, $X_0^{-1}(x)$ tends to 1 as x increases to x_1. That is, when $x = x_1$ each member of a favoured group of size z receives 1 unit of the indivisible good. ■

Proof. (THEOREM 3.4): By (3.17), (3.18), Theorem 3.1 and Proposition 3.3,

$$Z_c^*(x) = \begin{cases} \overrightarrow{Z}(x) & \text{if } 0 \leq x \leq X_0\left(\overrightarrow{Z}(x)\right) < x_1, \overrightarrow{Z}(x) \in [0, 1] \\ \overleftarrow{Z}(x) & \text{if } X_0\left(\overleftarrow{Z}(x)\right) \leq x \leq x_1, \overleftarrow{Z}(x) \in [\max\{0, \overline{z}\}, 1], Z_0(x) \in \left[\overleftarrow{Z}(x), 1\right] \\ z_n^* & \text{otherwise} \end{cases}$$

$\phi'(0) > \pi'(0) + 1$ and (3.17) imply $\overrightarrow{Z}(0) > 0$. Since, as x increases, $\overrightarrow{Z}(x)$ and $Z_0(x)$ increase and $\overleftarrow{Z}(x)$ decreases, we obtain, by (3.3), (3.8), (3.9), (3.17), (3.18), (3.19), (3.20), (3.21) that (i) $\overrightarrow{Z}(x_2) = Z_0(x_1) = 1$, (ii) $\overrightarrow{Z}(x_s) = \overleftarrow{Z}(x_s) = Z_0(x_s)$, (iii) either $0 < x_s < x_1 < x_2$ or $0 < x_2 < x_1 < x_s$ (iv) $0 \leq x < x_s < x_1 < x_2$ or $0 \leq x < x_2 < x_1 < x_s$ implies $Z_0(x) < \overrightarrow{Z}(x) < \overleftarrow{Z}(x)$, $\overrightarrow{Z}(x) < 1$ and $x < X_0\left(\overrightarrow{Z}(x)\right) < X_0\left(\overleftarrow{Z}(x)\right) < X_0\left(\overleftarrow{Z}(x_2)\right) = X_0(Z_0(x_1)) = x_1$ implies $Z_c^*(x) = \overrightarrow{Z}(x)$ (v) $0 < x_2 < x < x_1 < x_s$ implies $Z_0(x) < 1 < \overrightarrow{Z}(x) < \overleftarrow{Z}(x)$ and $\pi'(1) + 1 < \phi(1)$ implies $Z_c^*(x) = 1$, (vi) $0 < x_s \leq x < x_1 < x_2$ and $x < x_3$ imply $\max\{0, \overline{z}\} < \overleftarrow{Z}(x) < \overrightarrow{Z}(x) < Z_0(x) < 1$ and $X_0\left(\overleftarrow{Z}(x)\right) \leq X_0(Z_0(x)) = x < x_1$ implies $Z_c^*(x) = \overleftarrow{Z}(x)$, (vii) If $x_s < x_1 < \min\{x_2, x_3\}$ and $x_1 < x$, then $\overleftarrow{Z}(x) < \overline{z}$, $\pi'(0) + 1 < \phi(1) < \pi'(1) + 1$ and **either** $x_2 < x$ and $\overrightarrow{Z}(x) > 1$ **or** $x < x_2$ and $\overrightarrow{Z}(x) < 1$ and $X_0\left(\overrightarrow{Z}(x)\right) < X_0(1) = X_0(Z_0(x_1)) < x$, **so that** $Z_c^*(x) = \overline{z}$ (viii) If $x_s < x_1 < x_2$ and $x_s < x_3 < x_1$ and $x_3 < x$, then

$\overleftarrow{Z}(x) < 0$, $\phi(1) < \pi'(0) + 1$ and **either** $x_2 < x$ and $\overrightarrow{Z}(x) > 1$ **or** $x_1 < x < x_2$ **or** $x_1 < x_2 < x$ and $\overrightarrow{Z}(x) < 1$ and $X_0\left(\overrightarrow{Z}(x)\right) < X_0(1) = X_0(Z_0(x_1)) < x$, **so that** $Z_c^*(x) = 0$. ■

Proof. (COROLLARY 3.5): If $Z_c^*(x) = \overrightarrow{Z}(x) \in (0,1)$, then all are better off at the EO than at the LFO since each receives the average which, by optimality, improves welfare. If $Z_c^*(x) = \overleftarrow{Z}(x) \in [\max\{\overline{z}, 0\}, 1]$, then each member of a favoured group of size $\overleftarrow{Z}(x)/Z_0(x)$ receives $Z_0(x)$ and the EO Pareto dominates the LFO iff (3.22) holds (the outsiders are better off). If $Z_c^*(x) = \overleftarrow{Z}(x)$ and (3.22) fails then the DEO *khs*-dominates the LFO iff there exists M for which

$$
\begin{aligned}
\phi(Z_0(x)) - \kappa(Z_0(x), x) - (\phi(1) - 1) - Z_0(x) + \pi(1) - \pi\left(\overleftarrow{Z}(x)\right) &\geq M \quad (1)\\
\left(\frac{1 - \frac{\overleftarrow{Z}(x)}{Z_0(x)}}{\frac{\overleftarrow{Z}(x)}{Z_0(x)}}\right)\left(\phi(1) - 1 - \left(\pi(1) - \pi\left(\overleftarrow{Z}(x)\right)\right)\right) &= M \quad (2)\\
I_{\frac{\overleftarrow{Z}(x)}{Z_0(x)}} - Z_0(x) &\geq M \quad (3)
\end{aligned}
$$

where line (1) indicates that insiders are better off; line (2), outsiders no worse off; line (3) insiders have adequate income. Lines (2) and (3) are satisfied whenever (3.23) holds as required. It remains to show that line (2) implies line (1). When line (2) holds, line (1) is satisfied iff

$$
\frac{\overleftarrow{Z}(x)}{Z_0(x)}\left(\phi(Z_0(x)) - \kappa(Z_0(x), x) - Z_0(x)\right) \geq \left(\phi(1) - 1 - \left(\pi(1) - \pi\left(\overleftarrow{Z}(x)\right)\right)\right) \tag{A.5}
$$

By (3.18), (3.3) and the fundamental theorem of calculus, (A.5) holds iff

$$
\overleftarrow{Z}(x)\,\pi'\left(\overleftarrow{Z}(x)\right) + \int_{\overleftarrow{Z}(x)}^{1} \pi'(z)\,dz \geq \pi'(\overline{z})
$$

which holds since $\overleftarrow{Z}(x) \geq \overline{z}$ and π is convex. ■

Proof. (THEOREM 4.1): Since $z_n^* = \overline{z} \in (0,1)$, by Theorem 3.1, each individual in a group of size $\overline{z}$ is allocated 1 unit and each outsider is allocated 0 units in the efficient allocation. Since policy instrument $\langle g, h\rangle$ results in pollution level g/h, efficiency requires that (4.1) holds. We now suppose that (4.1) holds and prove that the equilibrium allocation is efficient. If $\rho(x)$ is the market price of permits and h permits are required to buy 1 unit, then increasing consumption from 0 to 1 unit increases expenditure by $h\rho(x)+1$ and increases net consumption utility by $\phi(1)-\phi(0)=\phi(1)$. By (3.3), if $\rho(x)$ satisfies

$$h\rho(x)=\phi(1)-1=\pi'(\overline{z}) \tag{A.6}$$

then each individual is indifferent between buying 1 unit of the good and buying 0 units of the good. In this case, the equilibrium permit price is $\rho_{H,n}=\pi'(\overline{z})/h$ and 1 unit is allocated to each in a group of size $\overline{z}$ provided

$$I_{\overline{z}}+g\rho_{H,n}\geq h\rho_{H,n}+1 \text{ for } I\in S$$

or equivalently, by (A.6) and $g/h=\overline{z}$,

$$I_{\overline{z}}\geq(1-\overline{z})(\phi(1)-1)+1 \tag{A.7}$$

If inequality (A.7) fails, then the price of permits must decrease below that of $\pi'(\overline{z})/h$. The price decreases until $\rho_{L,n}=(I_{\overline{z}}-1)/(h-g)$ where $I_{\overline{z}}+g\rho=1+h\rho$ and aggregate demand equals $\overline{z}$. The failure of (A.7) implies $\rho_{L,n}\in\left(0,\rho_{H,n}\right)$. ■

Proof. (COROLLARY 4.2): By the proof of Theorem 4.1, the favoured individuals are at least as well off as the outsiders (modulo income). The outsiders are no worse off relative to the LFO if

$$\pi(1)-\pi(\overline{z})+g\rho\geq\phi(1)-1 \tag{A.8}$$

If (3.5) holds then (3.6) holds since, by assumption, $G^{-1}(0)\geq 1$, and, in addition, (A.8) is satisfied since $\rho>0$. If (3.5) fails and (A.7) holds, then $\rho^*=\rho_{H,n}$ and both (3.6) and (A.8) hold since the convexity of π, and $\pi(0)=0$ imply that $\pi(1)-\pi(\overline{z})\geq(1-\overline{z})\pi'(\overline{z})$ which implies that (i) by (3.3), the right side of (A.7) is larger than that of (3.6), and (ii), by (3.3), (A.6), and $g/h=\overline{z}$, if $\rho=\rho_{H,n}$, then (A.8) holds. If (3.5) and (A.7) fail, the equilibrium permit price is $\rho^*=\rho_{L,n}$. If $\rho^*=\rho_{L,n}$ then (3.6) and (A.8) are equivalent since $\overline{z}=g/h$. ■

Proof. (THEOREM 4.3): Since policy $< g, h >$ results in pollution level g/h, Theorem 3.4 implies that efficiency in pollution results if and only if (4.2) holds. We now suppose that (4.2) holds and prove that the equilibrium allocation is efficient iff (4.3) holds. If $\rho(x)$ is the market price of permits and h permits are required to buy 1 unit, then the price ratio is $h\rho(x) + 1$. By Theorem 3.4,

$$MAR\left(A_c^*(x), x\right) = \pi'\left(Z^*(x)\right) + 1 \tag{A.9}$$

where we denote $MRS\left(A_c^*(x), I - A_c^*(x)\right)$ by $MAR\left(A_c^*(x), x\right)$ since, by (2.2), MRS is independent of income. Let $\rho_e(x)$ denote the solution to

$$h\rho + 1 = \pi'\left(Z^*(x)\right) + 1 \tag{A.10}$$

By construction, $MAR\left(A_c^*(x), x\right) = h\rho_e(x) + 1$ so that, at $\rho_e(x)$, individuals prefer consuming $A_c^*(x)$ to any other $A \in (0, 1]$. We now show that $\rho_e(x)$ is the equilibrium price $\rho^*(x)$ iff (4.3) holds.

$\rho^*(x) = \rho_e(x)$ iff $A_c^*(x)$ and $\rho_e(x)$ satisfy the budget constraint

$$I_{\frac{Z_c^*(x)}{A_c^*(x)}} \geq \left(h\rho_e(x) + 1\right) A_c^*(x) - g\rho_e(x) \tag{A.11}$$

and $A_c^*(x)$ is at least as good as 0. If $0 \leq x \leq \min\{x_1, x_2\} < x_s$, then $A_c^*(x) = \overrightarrow{Z}(x)$ and (3.8), (3.17), (3.19) imply $AVG\left(\overrightarrow{Z}(x), x\right) \geq MAR\left(\overrightarrow{Z}(x), x\right)$ which is equivalent, by (A.9) and (A.10), to

$$\phi\left(\overrightarrow{Z}(x)\right) - \kappa\left(\overrightarrow{Z}(x), x\right) + I + g\rho_e(x) - \left(1 + h\rho_e(x)\right)\overrightarrow{Z}(x) \geq I + g\rho_e(x)$$

so that all prefer $\overrightarrow{Z}(x)$ to 0. Each individual is able to buy $\overrightarrow{Z}(x)$ at $\rho_e(x)$ since (4.2), $A_c^*(x) = \overrightarrow{Z}(x)$, $G^-(0) \geq 1$ and $I \geq 1 \geq \overrightarrow{Z}(x)$ imply (4.3) and (A.11). If $Z_c^*(x) = \overleftarrow{Z}(x)$, $A_c^*(x) = Z_0(x)$, then (3.8), (3.18) and (A.10) imply that all are indifferent between $Z_0(x)$ and 0. By (4.2) and (A.10), the budget constraint (A.11) is satisfied by $Z_0(x)$ and $\rho_e(x)$, iff (4.3) holds. If $Z_c^*(x) = \overline{z}$, $A_c^*(x) = 1$, $x_s < x_1$, $\min\{x_1, x_3\} \leq x$, then (3.3) and (A.10) imply that all are indifferent between 1 and 0. Each individual $I \geq I_{\overline{z}}$ is able to buy 1 at $\rho_e(x)$ iff (4.3) holds since (4.2) implies that (4.3) and (A.11) are equivalent.

If (4.3) does not hold, then demand is less than supply of permits at $\rho_e(x)$ so that $\rho^*(x) < \rho_e(x)$ and any individual $I \geq I_{Z_c^*(x)/A_c^*(x)}$ demands more than $A_c^*(x)$ so that the allocation is inefficient. Lastly, since π is convex and increasing, whenever (4.3) holds, $\rho^*(x) = \rho_e(x)$ and (3.3), (3.18) and (A.10) imply that

$$g\rho_e(x) \geq (\phi(1) - 1) - (\pi(1) - \pi(Z_c^*(x)))$$

so that outsiders are better off relative to the LFO. ■

Proof. (COROLLARY 4.4): If (4.4) is violated then the reduction in pollution cost is greater than the loss in utility from decreased consumption so that Pareto domination is achieved by the policy equilibrium. $Z_c^*(x) \in (0,1)$ and convexity of π imply that $\pi(1) - \pi(Z_c^*(x)) > (\phi(1) - 1)(1 - Z_c^*(x))$ so that whenever (4.3) holds, the DEO khs-dominates the LF0 and by Theorem 4.3, the market outcome Pareto dominates the LFO. We now assume that (4.4) holds, (4.3) fails, and that either $Z_c^*(x) = \overline{z}$ and (3.6) holds or $Z_c^*(x) = \overleftarrow{Z}(x)$ and (3.23) holds.

Suppose that $Z_c^*(x) = g/h$. Since (4.3) fails, demand for permits is less than the supply at $\rho_e(x)$. In this case, the equilibrium price of permits, $\rho^*(x) < \rho_e(x)$, and the resulting allocation is inefficient. If $\rho < \rho_e(x)$, then $MAR(A_c^*(x), x) > 1+\rho$, and there are three possible market equilibrium systems of equations: system $(A, E, 0)$, system $(1, E, 0)$ or system $(E, 0)$ (obtained by replacing G^t with G in the system equations shown in the proof of Theorem 4.5). In system $(A, E, 0)$, for each ρ, there are two marginal incomes $\widehat{I}_a(\rho) < \widehat{I}_b(\rho)$. Those with high income $I > \widehat{I}_b(\rho)$ demand $A(\rho) = (\widehat{I}_b(\rho) + g\rho)/(I+\rho) > A_c^*(x)$; those with moderate income $\widehat{I}_b(\rho) > I > \widehat{I}_a(\rho)$ where $(\widehat{I}_a(\rho) + g\rho)/(I+\rho) < A_c^*(x)$ demand $(I + g\rho)/(I+\rho)$ (i.e., spend all income on the indivisible good); those with low income $I < \widehat{I}_a(\rho)$ demand 0. In system $(1, E, 0)$ those with high income $I > \widehat{I}_b(\rho)$ demand $1 = \left(\widehat{I}_b(\rho) + g\rho\right)/(1 + h\rho) \geq A_c^*(x)$; those with moderate income $\widehat{I}_b(\rho) > I > \widehat{I}_a(\rho)$ demand $(I+g\rho)/(I+\rho)$ and $(\widehat{I}_a(\rho)+g\rho)/(I+\rho) < A_c^*(x)$; those with low income $I < \widehat{I}_a(\rho)$ demand 0. In system $(E, 0)$ those with high income $I > \widehat{I}_a(\rho)$ demand $(I + g\rho)/(I+\rho)$, where $(\widehat{I}_a(\rho) + g\rho)/(I+\rho) < A_c^*(x) < (\overline{I} + g\rho)/(I+\rho) < 1$; those with income $I < \widehat{I}_a(\rho)$ demand 0. In each of these systems, individual $\widehat{I}_a$, the poorest individual to spend all income on the indivisible good, is indifferent between spending all and spending 0 on this good, expenditure on the indivisible good increases in I, and, in equilibrium, supply of permits equals demand.

Let ρ_0 denote the price that makes individual $I_{Z_c^*/A_c^*(x)}$ indifferent between spending all and spending 0 on the indivisible good. If $\rho^* = \rho_0$ then since $\widehat{I}_a(\rho_0) = I_{Z_c^*/A_c^*(x)}$ we obtain that expenditure by $I > I_{Z_c^*/A_c^*(x)}$ is at least as much as that by $I_{Z_c^*/A_c^*(x)}$ which is at least as much as the right side of (3.6) if $Z_c^*(x) = \overline{z}$ (respectively (3.23) if $Z_c^*(x) = \overleftarrow{Z}(x)$). Thus, if $\rho^* = \rho_0$, then each outsider receives strictly more than required to make them better off than at the LFO. If instead, $\rho^* > \rho_0$ then $g\rho^* > g\rho_0$ so that again, each outsider is better off than at the LFO. Lastly, if $\rho^* < \rho_0$, then (i) each individual $I > I_{Z_c^*/A_c^*(x)}$ spends at least as much as does $I_{Z_c^*/A_c^*(x)}$ since expenditure increases in income and (ii) we argue below that $I_{Z_c^*/A_c^*(x)}$ spends all income on the indivisible good so that the outsiders are better off than at the LFO. The argument for (ii) is as follows. Since the value of holding 0 when $\rho^* < \rho_0$ is below that of holding 0 when the market price equals ρ_0 we obtain that $\widehat{I}_a(\rho^*) \leq I_{\overleftarrow{Z}(x)/Z_0(x)}$. If the quantity bought by $I_{\overleftarrow{Z}(x)/Z_0(x)}$ is greater than $Z_0(x)$ then the demand for permits is greater than supply. Thus, $\widehat{I}_a(\rho^*) \leq I_{\overleftarrow{Z}(x)/Z_0(x)} < \widehat{I}_b(\rho^*)$ so that $I_{Z_c^*/A_c^*(x)}$ spends all income on the indivisible good so that, as argued above, the outsiders are better off than at the LFO. ■

Proof. (THEOREM 4.5): (i) First suppose that $x_s < x < x_1$. We show below that, as t increases, the market equilibrium price decreases to its asymptote $\underline{\rho}^0$. When $x_s \leq x \leq x_1$ if $\rho^* = \rho_e$, then Pareto domination follows from Theorem 4.3. If demand is less than supply at ρ_e, the equilibrium solves system $(A, E, 0)$, system $(1, E, 0)$, or system $(E, 0)$. System $(1, E, 0)$ is identical to system $(A, E, 0)$ except that $A = 1$ in lines (1), (3) and (7) and the equality in line (1) becomes weakly greater. System $(E, 0)$ is identical to system $(A, E, 0)$ except that $\widehat{I}_b = \overline{I}(t)$ in lines (3), (5), (6), and (7), and the equality in line (1) becomes weakly greater.

System $(A,E,0)$ follows.

$$\begin{aligned} MAR\,(A,x) &= 1+h\rho \quad (1) \\ AVG\left(\frac{\widehat{I}_a+g\rho}{1+h\rho},x\right) &= 1+h\rho \quad (2) \\ \int_{\widehat{I}_a}^{\widehat{I}_b}\left(\frac{I+g\rho}{1+h\rho}\right)G^{t\prime}(I)\,dI + A\left(1-G^{t}\left(\widehat{I}_b\right)\right) &= \frac{g}{h} \quad (3) \\ \frac{g}{h} &= \overleftarrow{Z}(x) \quad (4) \\ 1 &\geq \frac{\widehat{I}_b+g\rho}{1+h\rho} > \frac{\widehat{I}_a+g\rho}{1+h\rho} \quad (5) \\ 1 &\leq \underline{I} < \widehat{I}_a < \widehat{I}_b \leq \overline{I} \quad (6) \\ \frac{\widehat{I}_b+g\rho}{1+h\rho} &= A \quad (7) \end{aligned}$$

Suppose system $(A,E,0)$ is solved by $\left(\rho^{**}(t), \widehat{I}_a^*(\rho^{**}(t),t), \widehat{I}_b^*(\rho^{**}(t),t)\right)$ for $t_\tau > t_\sigma$. Since a change in t has no effect on lines (1) (2) and (7), $\widehat{I}_i^*(\rho^{**}(t_\sigma),t_\sigma) = \widehat{I}_i^*(\rho^{**}(t_\sigma),t_\tau)$ for $i=a,b$, so that since G^{t_σ} first order stochastically dominates G^{t_τ}, we obtain that the left-hand side of line (3) above decreases while the right-hand side is unaffected when $\rho=\rho^{**}(t_\sigma)$ and t increases to t_τ so that $\rho^{**}(t_\tau) < \rho^{**}(t_\sigma)$. Since $(I+g\rho)/(1+h\rho)$ increases in I and decreases in ρ and since the equilibrium price decreases as t increases from t_σ to t_τ, equality in line (3) ensures that $\widehat{I}_a^*$ decreases as t increases from t_σ to t_τ. Analogous arguments show that the equilibrium price and $\widehat{I}_a^*$ decrease as t increases for system $(1,E,0)$ and system $(E,0)$. Since $\underline{I}(t) < \widehat{I}_a^* < \overline{I}(t)$ and the supply of permits must equal demand, $lim_{t\to\infty}\rho^{**}(t) = \underline{\rho}^0$ (as t increases to ∞, G^t shifts weight closer to $\underline{I}(t)$ which decreases to $\underline{I}^0$). Thus, if $x_s < x < x_1$ and 4.5 fails then $\widehat{t}=\infty$, but if 4.5 holds, then, $\widehat{t}<\infty$, and the outsiders are worse off for all $t>\widehat{t}$. Analogous reasoning shows that the results hold if $x_1 \leq x$.

(ii) Given G and $x \geq x_1$, we now show that, as x increases, the market equilibrium price decreases to its asymptote $(I_{\overline{z}}-1)/(h-g)$ where $\overline{z}=g/h$. When $x \geq x_1$, if $\rho^*=\rho_e$, then Pareto domination follows from Theorem 4.3. If demand is less than supply at ρ_e, the equilibrium solves system $(E, 0)$ or system $(1, E, 0)$ (with G and $\overline{z}$ replacing G^t and $\overleftarrow{Z}(x)$ respectively). Suppose that $x_1 < x_\sigma < x_\tau$. Let $\widehat{I}_a^*(\rho, x)$ denote the solution to the equality in line (2) in the system $(E, 0)$ or $(1, E, 0)$ for any given (ρ, x). Let $\widehat{I}_b^*(\rho)$ denote the solution to the equality in line (7) in the system $(E, 0)$ or $(1, E, 0)$ for any given ρ. If $\rho = \rho^*(x_\sigma)$, and x increases from x_σ to x_τ then since AVG decreases in x we obtain that

$$AVG\left(\frac{\widehat{I}_a + g\rho^*(x_\sigma)}{1 + h\rho^*(x_\sigma)}, x_\tau\right) < AVG\left(\frac{\widehat{I}_a + g\rho^*(x_\sigma)}{1 + h\rho^*(x_\sigma)}, x_\sigma\right)$$

for any $\widehat{I}_a$ and since AVG increases in A in the feasible range, we obtain that

$$\widehat{I}_a^*(\rho^*(x_a), x_\tau) > \widehat{I}_a^*(\rho^*(x_a), x_a) \tag{A.12}$$

Let $S\left(\widehat{I}_a, \widehat{I}_b, \rho\right)$ denote the left side of the equality in line (3) of the system $(E, 0)$ or $(1, E, 0)$. Since (A.12) holds and $\widehat{I}_b$ depends directly only on ρ, we obtain that

$$S\left(\widehat{I}_a(\rho^*(x_a), x_\tau), \widehat{I}_b(\rho^*(x_a)), \rho^*(x_a)\right) < \overline{z}$$

so that the demand is less than supply at $\rho = \rho^*(x_a)$, when $x = x_\tau > x_\sigma$. Thus $\rho^*(x_\tau) < \rho^*(x_\sigma)$. So, we have shown that when $x > x_1$, the equilibrium price of permits decreases in the system $(E, 0)$ or $(1, E, 0)$.

Suppose we are in system $(E, 0)$. In this case, $\widehat{I}_b = \overline{I}$. The equality in line (2) of the system $(E, 0)$ implies that when ρ decreases, $\widehat{I}_a$ must increase since $(I + g\rho)/(1 + h\rho)$ decreases in ρ and $S\left(\widehat{I}_a, \overline{I}, \rho\right)$ decreases in $\widehat{I}_a$. As ρ^* decreases, $\left(\overline{I} + g\rho\right)/(1 + h\rho)$ increases until it equals 1 and we switch to system $(1, E, 0)$.

Now suppose we are in system $(1, E, 0)$. In this case, as x increases, $\rho^*(x)$ decreases and so $\widehat{I}_b^*(\rho^*(x))$ decreases. In this case, $S\left(\widehat{I}_a, \widehat{I}_b(\rho^*(x)), \rho^*(x)\right)$ must increase for any given $\widehat{I}_a$ so that, if $x_\tau > x_\sigma > x_1$, then

$$S\left(\widehat{I}_a(\rho^*(x_\sigma), x_\sigma), \widehat{I}_b(\rho^*(x_\tau)), \rho^*(x_\tau)\right) > \overline{z}$$

so that

$$\widehat{I}_a(\rho^*(x_\tau), x_\tau) > \widehat{I}_a(\rho^*(x_\sigma), x_\sigma)$$

Thus, $\widehat{I}_a^*(\rho^*(x), x)$ increases in x and $\widehat{I}_b^*(\rho^*(x))$ decreases in x. Since $I_{\overline{z}}$ solves $S(I_{\overline{z}}, I_{\overline{z}}, \rho_n) = \overline{z}$, and since $S\left(\widehat{I}_a(\rho^*(x), x), \widehat{I}_b(\rho^*(x)), \rho_e(x)\right) = \overline{z}$, where $\rho_e(x)$ satisfies (A.10), in the limit, as x increases, $\widehat{I}_a^*(\rho^*(x), x)$ asymptotes toward $I_{\overline{z}}$ from below, and $\widehat{I}_b^*(\rho^*(x), x)$ asymptotes toward $I_{\overline{z}}$ from above. In addition, since $\rho^*(x) < \rho_e(x)$ is decreasing in x, in the limit, $\rho^*(x)$ decreases to the solution, $\underline{\rho}$, to line (7) of system $(1, E, 0)$ when $I_b = I_{\overline{z}}$. Thus, if $g\underline{\rho} \geq \phi(1) - 1 - (\pi(1) - \pi(\overline{z}))$ where $g/h = \overline{z}$, or equivalently, if (3.6) holds, then, no matter what the value of $x \geq x_1$ in the regime $(1, E, 0)$, the outsiders are better off, but if (3.6) fails then, there exists $\widehat{x} < \infty$ for which $x > \widehat{x}$ implies the outsiders are worse off. ■

Proof. (COROLLARY 4.6): The proof follows directly from Theorem **4.5**. ■

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