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CARTELS, PROFITS AND EXCESS CAPACITY*

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1. INTRODUCTION

Our purpose is to study a model of the behavior of a cartel. There are a number of reasons why this is an interesting exercise. Whenever they have not been outlawed, cartels have existed, and frequently flourished, in a wide variety of industries. Their modes of operation have been diverse, and their longevity far from uniform, but they cannot be dismissed as transitory phenomena.[2] From a theoretical point of view, the fact that the firms in an industry can collectively benefit from colluding rather than competing means that there is an incentive to form a cartel. There is always the problem that an entrant may upset a collusive arrangement, but in any industry with a barrier to entry, it is in the interests of all firms to reach a binding collusive agreement. Even if such an agreement is outlawed, there may be a collusive arrangement which is self-enforcing (as, for example, in the models of Shubik [1959, Ch. 10], Friedman [1971], Stigler [1964], Radner [1980], and Green and Porter [1984]). Finally, in order to evaluate the desirability of making cartels illegal, it is necessary to understand their behavior, and how it depends on the nature of the demand for output and the available technology.

We address two questions. What collusive agreement will a group of firms of possibly different sizes reach? What implications does the nature of the agreement have for the choice of size by each member? Regarding the latter, we formalize a simple idea: collusive firms will carry excess capacity to make their threats to act competitively more damaging, so that they obtain a more favorable agreement. Capacity is costly, but it is worth acquiring, even if not used in

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[2] For a discussion of the extent of cartelization, see Scherer [1980, Ch. 6]; for detailed accounts of several major cartels, see Stocking and Watkins [1946].

production, if it sufficiently improves a firm's bargaining position. Our model is very simple. There are two firms, with possibly different capacities. Up to its capacity, each firm has the same, constant average cost of production. We assume that there is a barrier to entry, so that the market is not "contestable".

First, we fix the capacities. The firms negotiate an agreement, which involves an output quota for each firm, and a price at which output is sold. The outcome of the negotiation is determined by the damage each firm can inflict on the other by undercutting the monopoly price. (We have examined elsewhere (in Osborne and Pitchik [1983]) the consequence, in this part of the model, of assuming that the firms threaten to expand output, rather than cut prices. The assumption of threatened price-wars seems more appropriate for many industries.)

We find (see Proposition 1) that the profit per unit of capacity of the small firm is always at least equal to that of the large one, and if the joint capacity of the firms exceeds the monopoly output, then the inequality is strict. The reason is that each firm, regardless of size, can equally well disrupt the collusive outcome — in this respect each firm has the same power. The large firm can inflict more damage, but the net effect favors the small firm (per unit of capacity). (Stigler's [1964] model yields the same conclusion if information is imperfect; our result derives solely from the threat-potential of each firm.) We also find (see Proposition 2) that the ratio of the unit profit of the small firm to that of the large firm is higher, the lower is demand relative to capacity. Thus, the model predicts that, if capacities are fixed while demand varies cyclically, this ratio will vary counter-cyclically.

If we want to compare the outcome in our model with one which is "competitive", there are two alternatives. In the standard "perfectly competitive" outcome, both firms sell at the same price, so that their unit profits are the same. The outcome of price competition (between firms with limited capacities) predicted by the Bertrand-Edgeworth model involves the same unit profits for both firms for a wide range of capacity pairs (essentially, unless the large firm has more than enough capacity to serve demand at the breakeven price; for the details, see Osborne and Pitchik [1986]). Thus, whenever the industry capacity is neither very small nor very large relative to demand, the unit profit is the same for both firms in either of the "competitive" outcomes, while the small firm fares better in the cartel. If the cartel outcome can be achieved by an implicit agreement, this provides a criterion to distinguish between competitive and collusive industries.

The capacities must ultimately be chosen by the firms. We incorporate this by assuming that capacities are chosen once-and-for-all by all firms before negotiation over price and output quotas[3] (so that the structure is similar to that of the model of Kreps and Scheinkman [1983]). The capacities are thus neither objects of

[3] Rather than assuming that the industry is starting from scratch, we could suppose that it is currently competitive, and that adjustments in capacity are being made, in anticipation of subsequent collusion. Or, the industry may currently be a monopoly, which is changing its capacity in response to an entrant with which it will collude.

negotiation, nor strategic variables for the firms during negotiation. The idea is that, given some inflexibility of capacity, it is to the advantage of a firm to choose its capacity before entering negotiations. Of course, if the cartel lasts for a long time, there is scope for subsequent adjustments of capacity. Even so, in the absence of perfect enforcement, agreement on a capacity-reduction may be much less likely than agreement on a price-hike: if a firm cheats on the former, its opponent is in a weak position, while any change in price is easily reversible.

We find (see Corollary 5) that if the cost of capacity is relatively low (less than half the monopoly profit margin) then the sum of the capacities chosen by the firms exceeds the sum of the negotiated output quotas — i.e. the choices of the firms result in excess capacity in the industry. The reason for this is straightforward. The more capacity a firm has, the more potent the threats it can make, and hence the larger its negotiated profit. If capacity is not too costly, it pays a firm to build more capacity, even if it is not all used in production (it is "used" to threaten the other firm and maintain the firm's negotiated profit). If the firms choose their capacities in the expectation of competing (as in Kreps and Scheinkman [1983]), rather than colluding, then there is no use for excess capacity, and the outcome always involves full utilization. The same is obviously true if there is a single firm in the industry.

We also find (see Proposition 6) that if the size and elasticity of demand at the monopoly price are fixed, while at every other price demand becomes more elastic, then excess capacity increases. This makes sense: if demand is more elastic, a price-cut is more potent (it is less damaging to its perpetrator, and more damaging to the other firm), so that the marginal benefit of an extra unit of capacity is greater. Hence, given the unit cost of capacity, the equilibrium sizes chosen are larger.

In a slightly different context, the idea that a firm will build excess capacity as a threat has been modeled before. Spence [1977] constructed an equilibrium in which an existing firm installs capacity which deters entry. Subsequently it has been pointed out that the credibility of the threat underlying this equilibrium depends on the nature of the demand function. Dixit [1980] shows that if the marginal revenue of each firm is decreasing in the output of the other firm, then the threat is not credible: if a firm actually enters, then it is not in the interest of the existing firm to carry out its threat. Bulow, Geanakoplos, and Klemperer [1985] show that if Dixit's assumption is violated (as it may be for "reasonable" demand functions) then Spence's threats may in fact be credible. (Spence [1979] and Fudenberg and Tirole [1983] study the credibility of entry-deterring capacity choices by an incumbent in multi-period models.) In our model, neither firm has the advantage of being an "incumbent": the capacities are chosen simultaneously, not sequentially. Under this assumption, a model like Spence's, in which there is competition (rather than collusion, as in our model) in the second stage, has no perfect equilibria in which the firms hold excess capacity. This is because capacity in excess of that used in production can have no effect on the competitive profits. In our model, the nature of the collusive agreement *is* affected by the threats the firms can carry out, and these depend on the capacities of the firms.

Benoît and Krishna [1987] have recently studied the issue of excess capacity in a multi-period model in which, as in ours, the firms are symmetric. Rather than using a model of negotiation, as we do, they look at equilibria in a repeated game, which may involve "implicit collusion". Their work is discussed further in the next section.

The (independent) work of Brander and Harris [1983] is also related to ours. The main difference concerns the modeling of the negotiation, given fixed capacities. Brander and Harris assume simply that the monopoly profit is divided in proportion to the capacities of the firms, while we derive the negotiated profit shares from a model which recognizes that these shares depend on the nature of the available threats, so that the leverage of heterogeneous firms at the bargaining table is not, in general, in proportion to capacities.

2. THE SOLUTION CONCEPT

The solution we use for the negotiation of the collusive agreement (given the capacity choices) has two stages. First, actions to be taken in the event of disagreement ("threats") are simultaneously announced; second, a compromise is agreed upon, based on the payoffs at these announced threats. The negotiated payoffs in the second stage are those given by Nash's [1950] bargaining solution, relative to disagreement payoffs equal to those obtained when the threats are carried out. In addition to Nash's original arguments to support this solution (elaborated upon by Binmore [1981]), it is shown by Binmore [1980] that the solution coincides with the limit (as the period between proposals goes to zero) of the unique perfect equilibrium of a very natural strategic bargaining model due to Rubinstein [1982]. Further, in our context, Nash's model prescribes splitting equally the excess of the monopoly profit over the payoffs when the threats are carried out, a rule which by itself has intuitive appeal.

We impose credibility on the threats by insisting that they are the actions which would actually be carried out in the event of disagreement — the Nash equilibrium strategies in the price-setting game, given the chosen capacities. We refer to the resulting solution as the *credible threats solution*.[4] In associating credibility with threats equal to the Nash equilibrium strategies in the price-setting game, we are assuming that it is not possible to make a commitment to the threat of setting a certain price. If full commitment is possible, then the appropriate solution is Nash's [1953] "variable threat" solution, in which threats are chosen simply to generate the best bargaining position, without regard to their

[4] The solution makes sense only if the Nash equilibrium strategies — or at least the Nash equilibrium payoffs — in the price-setting game are unique. For the game here, this is the case, as we show in Osborne and Pitchik [1986]. (The results of Kreps and Scheinkman [1983] are not quite sufficient, since they are obtained under an assumption on demand which is more restrictive than the one here.)

consequences if carried out. Since the assumption of no commitment to price-threats seems most appealing in the context of our model, we do not report in detail the results of applying the Nash variable threat solution. However, it turns out (perhaps surprisingly) that these results are very similar to those we obtain using the credible threats solution; we discuss this further in Sections 5 and 6 below.

We assume that payoff is transferable. However, we show later (see Proposition 3) that (for each capacity pair) the negotiated profits can be realized by each firm selling an output at most equal to its capacity, so that no monetary transfers are required. Thus the solution to the whole game has the following structure. First, capacities are simultaneously chosen; then output quotas are negotiated, demands for these quotas being backed by threats to deviate from the monopoly price; finally, outputs equal to the negotiated quotas are produced and sold at the monopoly price.

Two alternative approaches to modeling collusive outcomes deserve attention. One is the use of a noncooperative solution to a repeated game, as, for example, in Friedman [1971], Green and Porter [1984] and Benoît and Krishna [1987]. A noncooperative equilibrium in which each firm earns more each period than it does in a single-period Nash equilibrium can be interpreted as "implicitly collusive". A problem is that there is typically a very large set of outcomes supported by (even subgame perfect) Nash equilibria of such games. Indeed, consider the following model. In period 1, the firms choose capacities; in period 2 and in each subsequent period they choose prices. Fix a pair of capacities in period 1. The "Folk Theorem" tells us that, with no discounting, every individually rational split of the monopoly profit (given the total capacity) can be supported as the average payoff in a subgame perfect equilibrium of the subgame beginning in period 2. In particular, for each capacity pair, the split given by our solutions can be supported. Thus the outcome in our game (including the chosen capacity pair) can be supported as a subgame perfect equilibrium of the whole game. That is, our outcome can be sustained by self-enforcing strategies. The noncooperative approach requires some extra assumption to generate a determinate outcome; we argue that the assumption we use is particularly appealing. (Rather than adding assumptions to obtain a unique equilibrium, Benoît and Krishna [1987] show that all equilibria share certain properties. In particular, they show that, in the many-period game described above with discounting, all perfect equilibria except one involve excess capacity. This is of interset, although the size and comparative static properties of the excess capacity are not determinate.) The framework of a repeated game is particularly useful when considering the stability of an "implicitly collusive" agreement. Our main interest is in characteristics of the explicit agreement reached by heterogeneous firms within a cartel; in its present form, the theory of repeated games is not well-suited to address this issue.

A second alternative is to model bargaining as an extensive game. Rubinstein's [1982] model is not sufficient for our purposes, since it involves a given outcome in the event of disagreement; if one allows for "variable threats" within his framework, it is possible that, depending on the degree of commitment to threats allowed, the outcome coincides with the solution we adopt here.

3. THE ECONOMIC STRUCTURE

There are two firms. Firm i has capacity k_i; when analyzing the outcome for fixed capacities, we assume that $k_1 \geqq k_2 > 0$. Each firm can produce the same good at the same, constant unit cost $c \geqq 0$ up to its capacity. Let p be the excess of price over unit cost and let $S = [-c, \infty)$. We frequently refer, somewhat loosely, to an element of S as a "price". For each price p, let $d(p)$ be the aggregate demand for the output of the firms (given the prices of all other goods). We assume that

(1) there exists $p_0 > 0$ such that $d(p) = 0$ if $p \geqq p_0$ and $d(p) > 0$ if $-c \leqq p < p_0$, and d is smooth and decreasing on $(-c, p_0)$.

For each price p, let $\pi(p) = pd(p)$. Given (1), π attains a maximum on S. To save on notation we choose the units in which price is measured so that the maximizer is 1, and the units for quantity so that the maximum is also 1. We assume that[5]

(2) π is strictly concave on $[0, p_0]$.

We now define the profits of the firms if they noncooperatively choose prices p_1 and p_2. Suppose they set different prices, say $p_i < p_j$. Depending on the capacity of firm i, there may be some demand left over for firm j. Precisely how much remains depends on the preferences of the consumers and the way the available quantity is rationed, not just on the aggregate demand function d. We assume that there is a large number of identical consumers with preferences which do not have any "income effect".[6] It is natural to assume that the rationing scheme is chosen by firm i. If firm i is concerned solely with its own payoff, this assumption does not generate a determinate outcome, since i's payoff is independent of the scheme chosen.

However, in the bargaining model we use, it is to the advantage of a firm to choose a threat which reduces the payoff of firm j as much as possible. A rationing scheme which does this, independently of the action of firm j, is the following: each consumer is allowed to buy the same fraction of k_i (rather, for example, than some consumers being allowed to buy as much as they want, while others

[5] Except for our analysis in Section 6, it is sufficient to assume that π is strictly concave on $[0, 1]$ and decreasing on $[1, p_0]$.

[6] Precisely, for each quantity of the good which the firms produce, the marginal rate of substitution between that good and any other good is independent of the quantities of the other goods. We also need to assume that, given income and the prices of all other goods, a consumer's demand for all other goods is positive for every price of the good produced by the firms.

are not allowed to buy anything). Given that this is a dominant strategy, a firm will always adopt it when issuing a threat, and we can focus on the choice of prices. Under these assumptions,[7] the demand facing firm j when $p_i < p_j$ is $d(p_j) - k_i$. If $p_i = p_j = p$, we assume that demand is allocated in proportion to capacities[8] (if these are large enough to serve that demand). Thus the profit of firm i at any price pair (p_i, p_j) is

$$(3) \quad h_i(p_i, p_j) = \begin{cases} L_i(p_i) \equiv p_i \min(k_i, d(p_i)) & \text{if} \quad p_i < p_j \\ \phi_i(p) \equiv p \min(k_i, k_i d(p)/k) & \text{if} \quad p_i = p_j = p \\ M_i(p_i) \equiv p_i \min(k_i, \max(0, d(p_i) - k_j)) & \text{if} \quad p_i > p_j, \end{cases}$$

where $k = k_1 + k_2$. Examples of the functions L_i, ϕ_i, and M_i are shown in Figure 1. For each pair (k_1, k_2) of capacities, let $H(k_1, k_2)$ be the game in which the (pure) strategy set of each firm is S (the set of prices), and the payoff function of firm i $(=1, 2)$ is h_i.

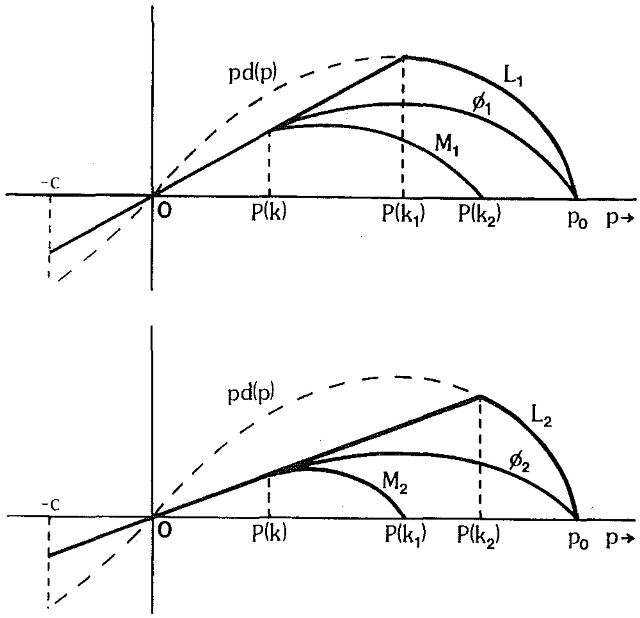

FIGURE 1
THE FUNCTIONS L_i, ϕ_i, AND M_i DEFINED IN (3). $L_i(p)$ (RESP. $\phi_i(p)$, $M_i(p)$) IS THE PROFIT OF FIRM i WHEN IT SETS THE PRICE p AND FIRM j SETS A HIGHER (RESP. THE SAME, A LOWER) PRICE

[7] Which are the same as those adopted by Kreps and Scheinkman [1983].

[8] If one adopts a rule in this case which is more favorable to the small firm (for example, the demand is split equally), then the negotiated profit of the small firm is higher, and that of the large firm is lower.

By colluding, the firms can obtain the monopoly profit. It is easy to check that under our assumptions this is achieved by both firms selling at the same price[9]. Let $P: [0, \infty) \to S$ be the inverse demand function defined by $P(q) = d^{-1}(q)$ if $0 < q \leqq d(-c)$, $P(0) = p_0$ (see (1)), and $P(q) = -c$ otherwise. Then, if the capacity of the industry is k, the monopoly profit is $Z(k) \equiv \max_q \{qP(q) : q \leqq k\}$. Given our normalization, and our assumptions on demand, we have

$$Z(k) = \begin{cases} kP(k) & \text{if} \quad 0 \leqq k \leqq 1 \\ 1 & \text{if} \quad 1 \leqq k. \end{cases} \tag{4}$$

4. THE MODEL OF NEGOTIATION

Assume that $(p_1, p_2) \in S \times S$ is a pair of threats. Then the negotiated payoff of firm i $(=1, 2)$ is

$$\begin{aligned} v_i(p_i, p_j) &= h_i(p_i, p_j) + [Z(k) - h_i(p_i, p_j) - h_j(p_j, p_i)]/2 \\ &= [Z(k) + h_i(p_i, p_j) - h_j(p_j, p_i)]/2. \end{aligned}$$

Thus the excess of the monopoly profit over the sum of the threat payoffs is split equally between the firms; a large firm is powerful only because it can issue more damaging threats. Using (3) we have

$$v_i(p_i, p_j) = \begin{cases} [Z(k) + L_i(p_i) - M_j(p_j)]/2 & \text{if} \quad p_i < p_j \\ [Z(k) + \phi_i(p) - \phi_j(p)]/2 & \text{if} \quad p_i = p_j = p \\ [Z(k) + M_i(p_i) - L_j(p_j)]/2 & \text{if} \quad p_i > p_j. \end{cases}$$

For each pair (k_1, k_2) of capacities, let $V(k_1, k_2)$ be the (constant-sum) game in which the (pure) strategy set of each firm is S (the set of prices), and the payoff function of firm i $(=1, 2)$ is v_i.

In our solution (the "credible threats solution"), each firm chooses its threat so as to maximize its disagreement payoff, given the threat of the other firm. Thus the pair of *equilibrium threats* is the (unique) pair of equilibrium strategies in the capacity-constrained price-setting game $H(k_1, k_2)$, and the *negotiated profits* are the payoffs corresponding to this strategy pair in $V(k_1, k_2)$. We denote the negotiated profit of i by $v_i^*(k_1, k_2)$.

5. THE EQUILIBRIUM THREATS AND NEGOTIATED PROFITS FOR FIXED CAPACITIES

The equilibrium threats have intuitively appealing properties. We refer to

[9] If $p_i < p_j$ then (under our assumptions on demand) firm j sells at most $[1 - k_i/d(p_i)]d(p_j)$ units. (The maximum is achieved when i rations its supply by allowing a fraction of the consumers to buy all they want.) The sum of the firms' profits is consequently at most $\max\{p_i d(p_i), p_j d(p_j)\}$. (Note that if the consumers' preferences differ, it might not be optimal for the firms to set $p_i = p_j$.)

the output of a monopolist without any capacity constraint as the "unconstrained monopoly output". If the industry capacity is less than this output, then the monopoly price is $P(k)$, and since both firms are producing at capacity, neither can improve its bargaining position by threatening to cut price (this only reduces its own payoff, and does not affect that of its opponent). That is, the "threat" of each firm is to set the price $P(k)$, and the negotiated profit of firm i is just $k_iP(k)$. On the other hand, if there is a lot of overcapacity in the industry —if the capacity of each firm exceeds the demand at the breakeven price— then the equilibrium threat of each firm is to set the breakeven price. In this case, the extra capacity of the large firm has no effect on its bargaining position, and the negotiated profit of each firm is the same; thus the profit per unit of capacity of the small firm exceeds that of the large one.

When industry capacity is neither very small nor very large, the equilibrium threats are mixed strategies.[10] As industry capacity varies in this middle range, the threats vary continuously from one extreme to the other, and the negotiated unit profits also vary continuously. Thus, unless the industry capacity is very small, the unit profit of the small firm exceeds that of the large one (see Proposition 1 below).

The equilibrium threats and negotiated profits can be specified explicitly; the details are given in the Appendix. The negotiated profits have a number of natural properties. The negotiated profit of firm i $(=1, 2)$ is continuous in (k_1, k_2), nondecreasing in k_i, and nonincreasing in k_j (an increase in capacity cannot make a firm worse off; an increase in the capacity of its opponent cannot make it better off). Also, for a wide range of capacity pairs, the negotiated profit of firm i $(=1, 2)$ is concave in k_i (see Lemma 7 in the Appendix). That is, the marginal benefit of an increase in capacity is decreasing. Our result on the negotiated unit profits of the two firms is as follows.

PROPOSITION 1. *If* $k>1$ *and* $k_1>k_2$ *then* $v_1^*(k_1, k_2)/v_2^*(k_1, k_2)<k_1/k_2$; *otherwise* $v_1^*(k_1, k_2)/v_2^*(k_1, k_2)=k_1/k_2$.

PROOF. We use the formulas for the negotiated profits given in Table 2 of the Appendix. For the regions in which the equilibrium threats are pure, the argument is quite simple. If $k\leqq 1$ (region Ia), $k_2\geqq d(0)$ (region II), or $k_1=k_2$ the result is immediate. If $k>1$ then $Z(k)$ (the monopoly profit) is 1 (see (4)) so that $v_1^*(k_1, k_2)+v_2^*(k_1, k_2)=1$. Hence it is enough to show that $v_1^*(k_1, k_2)<k_1/k$. In region Ib we have $k>1$, so $kP(k)<1$ and hence

$$v_1^*(k_1, k_2) = [1+(k_1-k_2)P(k)]/2 < [1+(k_1-k_2)/k]/2 = k_1/k.$$

Finally consider region III. Define $f(k_1, k_2)=[1+(k_1-k_2)B(k_2)/k_1]/2$. We shall first argue that $v_1^*(k_1, k_2)\leqq f(k_1, k_2)$ for all (k_1, k_2) in this region. In region

[10] Note that the negotiated outcome is still pure — only the threats involve randomization. Note also that one can interpret a mixed strategy in this context as a strategy of threatening to hold "sales" at various "reduced" prices over a period of time (see, e.g., Varian [1980]).

IIIa, we have equality. Further, given (A.2), v_1^* is constant in k_1 in region IIIb and so, since it is continuous in k_1, we have $v_1^*(k_1, k_2) < f(k_1, k_2)$ there.

It remains to show that $f(k_1, k_2) < k_1/k$ if (k_1, k_2) is in region III. Rearranging this inequality, we need to show that $k_1(1-B(k_2)) - k_2B(k_2) > 0$. Since $B(k_2) < 1$ and $k_1 > d(b(k_2)) - k_2 = B(k_2)/b(k_2)$ in region III, we have $k_1(1-B(k_2)) - k_2B(k_2) > B(k_2)[1-B(k_2) - k_2b(k_2)]/b(k_2) = B(k_2)[1-b(k_2)d(b(k_2))]/b(k_2) > 0$ (the last inequality since $pd(p) < 1$ if $p < 1$, and $b(k_2) < 1$). This completes the proof. □

We can also show that the larger the joint capacity of the firms relative to demand the lower the unit profit of the large firm relative to that of the small one.

PROPOSITION 2. *Fix* $0 < \alpha \leq 1$. *Then* $v_1^*(k_1, \alpha k_1)/v_2^*(k_1, \alpha k_1)$ *is constant in* k_1 *(equal to* $1/\alpha$*) if* k $(=(1+\alpha)k_1) < 1$, *decreasing in* k_1 *if* $1 < k < (1+\alpha)d(0)/\alpha$, *and constant (equal to* 1*) if* $(1+\alpha)d(0)/\alpha < k$.

PROOF. For regions Ia and II the result is immediate from Table 2 of the Appendix. In the remaining cases, let $J^*(k_1, k_2)$ be such that $v_1^*(k_1, k_2) = (1 + J^*(k_1, k_2))/2$, so that $v_2^*(k_1, k_2) = (1 - J^*(k_1, k_2))/2$. Then it is enough to show that $J^*(k_1, \alpha k_1)$ is decreasing in k_1. In region Ib we have $J^*(k_1, \alpha k_1) = (1-\alpha)k_1 \cdot P((1+\alpha)k_1)$, which is decreasing in k_1 by (2). In region IIIa we have $J^*(k_1, \alpha k_1) = (1-\alpha)B(\alpha k_1)$, which is decreasing in k_1, since B is decreasing. Finally, in region IIIb the negotiated profits are independent of k_1 (see (A.2)), while the profit of firm 1 is decreasing, and that of firm 2 increasing in k_2. This completes the proof. □

Finally, the following result allows us to interpret our model as one of negotiation over output quotas. We show that there is a feasible output level $x_i \leq k_i$ for firm i $(=1, 2)$ such that the negotiated profit of i is precisely the profit earned when the output x_i is sold at the monopoly price. (That is, the negotiated profits can be realized without any transfers). For each value of k, let $m(k)$ be the monopoly price (so that $m(k) = P(k)$ if $k \leq 1$, and $m(k) = 1$ otherwise).

PROPOSITION 3. $v_i^*(k_1, k_2) \leq k_i m(k)$ *for* $i = 1, 2$, *for all* (k_1, k_2).

PROOF. In region Ia we have $v_i^*(k_1, k_2) = k_iP(k)$ and $m(k) = P(k)$, so the result follows. In the remaining regions we have $k > 1$, so that $m(k) = 1$. Given this, and Proposition 1, it is enough to show that $v_2^*(k_1, k_2) \leq k_2$. Now, since v_1^* is nondecreasing in k_1 and $v_1^*(k_1, k_2) + v_2^*(k_1, k_2) = 1$ in regions Ib, II, and III, v_2^* is decreasing in k_1 there. Hence if $k_2 \leq 1/2$ then $v_2^*(k_1, k_2) \leq v_2^*(1-k_2, k_2) = k_2$, while if $k_2 > 1/2$ then $v_2^*(k_1, k_2) \leq v_2^*(k_2, k_2) = 1/2 < k_2$, completing the proof. □

Propositions 1 through 3 remain true when the credible threats solution is replaced by Nash's variable threat solution (i.e. when it is assumed that complete commitment to threats is possible). In fact, when the equilibrium threats under our solution are pure, they (and hence the negotiated profits) are the same in Nash's solution. When they are mixed, they take the same general form in

Nash's solution, and the negotiated profits coincide for some range of capacity pairs. Thus it appears that our results are somewhat robust to the precise form of bargaining solution used.

6. THE CAPACITY CHOICES OF THE FIRMS

We now allow each firm to choose its capacity before entering negotiations. The choices are made simultaneously, and for each pair (k_1, k_2) the (negotiated) profits are those described above. We assume that the unit cost of capacity is the same for both firms, equal to u. We are interested in the Nash equilibrium of the game $W^*(u)$ in which the strategic variable of each firm is its capacity, and the payoff $w_i^*(k_i, k_j)$ of firm i is the negotiated profit corresponding to (k_i, k_j) minus the cost of k_i. Since we have defined the negotiated profit $v_i^*(k_1, k_2)$ only when $k_1 \geqq k_2$, the appropriate definition of w_i^* is

$$w_i^*(k_i, k_j) = \begin{cases} v_2^*(k_j, k_i) - uk_i & \text{if} \quad k_i < k_j \\ v_1^*(k_i, k_j) - uk_i & \text{if} \quad k_i \geqq k_j. \end{cases}$$

We show in the following results that for each value of $u > 0$ the game $W^*(u)$ has a unique pure-strategy equilibrium; at this equilibrium there is excess capacity if and only if $u > 1/2$ (i.e. the unit cost of capacity exceeds half of the excess of the monopoly price over unit cost); and the more costly capacity, the smaller the excess capacity. The line of argument is quite simple. If (k_1, k_2) is an equilibrium of $W^*(u)$ then $\partial w_i^*(k_i, k_j)/\partial k_i = 0$ for $i = 1, 2$. By examining the derivatives, it is easy to show that this can happen only if $k_i = k_j$. Finally, we can characterize the values of k^* such that $\partial w_i^*(k^*/2, k^*/2)/\partial k_i = 0$ and show that $w_i^*(x, k^*/2) \leqq w_i^*(k^*/2, k^*/2)$ for all $x \geqq 0$, so that $(k^*/2, k^*/2)$ is in fact an equilibrium. (The functions b and B which appear in the statement of the next result are defined in the Appendix.)

PROPOSITION 4. *For each $u > 0$ the game $W^*(u)$ has a unique pure-strategy equilibrium. The equilibrium strategy pair $(k_i^*(u), k_i^*(u))$ is characterized as follows. For each $0 < x < 2d(0)$ let*

$$f(x) = \begin{cases} 2P(x) + xP'(x) & \text{if} \quad 0 < x \leqq 1 \\ P(x) & \text{if} \quad 1 < x \leqq \bar{x} \\ 2B(x/2)/x & \text{if} \quad \bar{x} < x < 2d(0) \end{cases} \tag{5}$$

(see Figure 2), where $\bar{x}$ is such that $\bar{x} = d(b(\bar{x}/2))$. Then for each $0 < u < P(0)$ there is a unique point $k^*(u) < 2d(0)$ such that $f(k^*(u)) = 2u$, and we have $k_i^*(u) = k^*(u)/2$ for $i = 1, 2$.

PROOF. First we claim that in a pure-strategy equilibrium of $W^*(u)$ we have $k_1^*(u) = k_2^*(u)$. To show this, note that if $k_i^*(u) > 0$ for $i = 1, 2$, and w_i^* is differ-

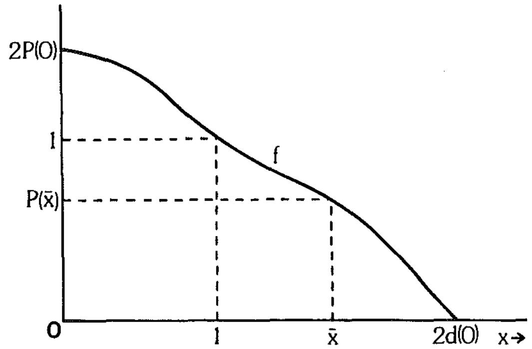

FIGURE 2

THE FUNCTION f DEFINED IN (5). WHEN THE UNIT COST OF CAPACITY IS u, EACH FIRM CHOOSES THE CAPACITY k^* SUCH THAT $f(k^*)=2u$

entiable at $(k_i^*(u), k_j^*(u))$, then we need $\partial w_i^*(k_i^*(u), k_j^*(u))/\partial k_i=0$ for $i=1, 2$. Now, w_i^* is differentiable except on the boundary between regions IIIa and IIIb. But the right-hand derivative of w_2^* exceeds the left-hand derivative in the exceptional case, so that an equilibrium cannot lie there. By calculating the derivatives in each region for the remaining values of (k_i, k_j) it is easy to show that $\partial w_i^*(k_i^*(u), k_j^*(u))/\partial k_i=0$ for $i=1, 2$ only if $k_i^*(u)=k_j^*(u)<d(0)$. It is also easy to check that if $k_i^*(u)>0$ then $k_j^*(u)>0$, so that the only possible equilibrium in which $k_i^*(u)=0$ for some i is $(k_i^*(u), k_j^*(u))=(0, 0)$.

We now argue that for each $u>0$ there is a unique number $k<2d(0)$ such that first-order conditions $\partial w_i^*/\partial k_i=0$ for $i=1, 2$ are satisfied. A calculation shows that $\partial w_i^*(k/2, k/2)/\partial k_i=0$ for $i=1, 2$ if and only if $f(k)=2u$ (where f is defined in (5)). To show that for each $0<u<P(0)$ this equation has a unique solution, note that f is continuous, $f(0)=2P(0)$, and $f(2d(0))=0$. Thus it is enough to show that f is decreasing. On $(1, 2d(0))$ this follows from the fact that P and B are decreasing. On $(0, 1)$ we have $f(k)=P(k)+\Pi'(k)$, where $\Pi(k)\equiv kP(k)$. But (2) implies that Π is concave. Hence $f'(k)<0$. Thus k^* is decreasing in u for $0<u<P(0)$. If $u\geqq P(0)$ there is no $k>0$ such that $f(k)=2u$. In this case it is easy to check that $(k_1, k_2)=(0, 0)$ is an equilibrium.

It remains to show that for $i=1, 2$ we have $w_i^*(x, k^*(u)/2)\leqq w_i^*(k^*(u)/2, k^*(u)/2)$ for all $x\geqq 0$ (i.e. $(k^*(u)/2, k^*(u)/2)$ is actually an equilibrium). If $x\geqq k^*(u)/2$ this follows from the concavity of v_1^* in k_1 (see Lemma 7 in the Appendix). If $x<k^*(u)/2$ then since v_2^* is concave in k_2 except possibly in regions Ib and IIIb, we need to consider only what happens if $(k^*(u)/2, x)$ is in these regions. The remainder of the argument involves a straightforward but tedious examination of the several cases; the details are omitted. □

A consequence of this result is the following.

COROLLARY 5. *At the unique equilibrium the capacities chosen by the firms are decreasing in the unit cost of capacity* (u); *there is excess capacity if* *and only if this unit cost is less than* $1/2$.

PROOF. It is enough to note that $f(1)=1$ (see (5)), and f is decreasing, so that by the Proposition, $k^*(u)<1$ if and only if $u<1/2$. □

This result is quite striking because the critical value of u is independent of the shape of the demand function. It depends, of course, on our normalization (the excess of the monopoly price over unit cost is 1), but is otherwise insensitive to changes in the shape of the demand function.

Finally, we analyze the effect of a change in the elasticity of demand on the equilibrium capacity choices of the firms. Fix the point on the demand function where the elasticity is one, and make the function more elastic at every other price. Then the normalization is the same in both cases, and demand increases at every price below the monopoly price. Hence, $P(x)$ increases for each $x>1$ and $B(x)$ increases for each $x>0$. Thus if $u<1/2$ then, from Proposition 4, the chosen capacities increase (since $k^*(u)$ then exceeds 1). In fact, it is clear that it is enough to assume that the demand increases at every price below 1, while at 1 both demand and the slope of the demand function are fixed; what happens at prices in excess of 1 is irrelevant. Thus we have the following.

PROPOSITION 6. *Suppose the point on the demand function where the price elasticity of demand is unity is fixed, while demand increases at all lower prices. Then if originally there is excess capacity, its size increases.*

Given the close relation between the properties of the negotiated profits under the credible threats solution and under Nash's variable threat solution (remarked upon at the end of Section 5), it is not surprising that the equilibria of $W^*(u)$ under the two solutions coincide. (We omit the argument establishing this.) Once again, it seems that our results are robust to variations in the bargaining solution used.

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APPENDIX: THE EQUILIBRIUM THREATS AND OUTCOME OF NEGOTIATION FOR FIXED CAPACITIE

Here we describe the equilibrium threats and negotiated profits for each pair of capacities. For each $0 \leqq x \leqq d(0)$, suppose that

$$b(x) \text{ maximizes } p(d(p)-x) \text{ over } p \in S,$$

and let $B(x)=b(x)(d(b(x))-x)$. If firm i sets the price $p_i<b(k_i)$, and $b(k_i)>P(k)$, then the best price for firm j to charge out of all those in excess of p_i is $b(k_i)$, independently of p_i (i.e. $b(k_i)$ maximizes M_j in this case). The nature of the equilibrium threats depends on the relation between $b(k_2)$, $P(k)$, and $P(k_1)$; the various

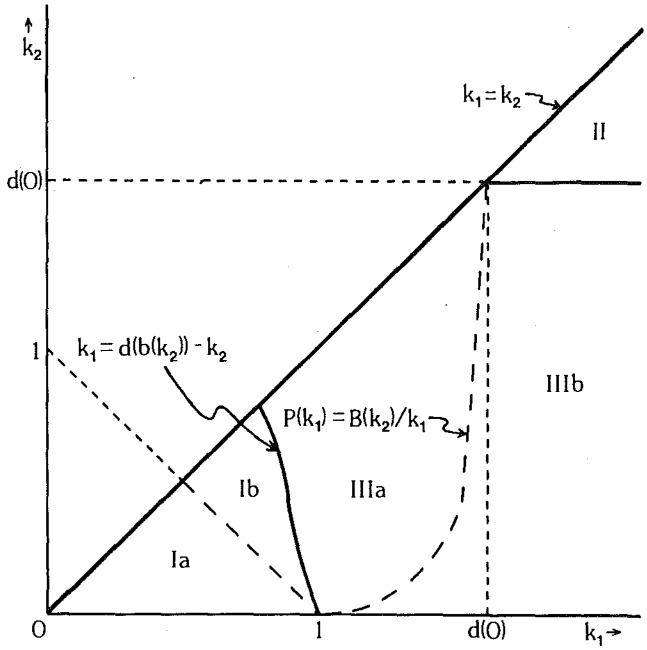

FIGURE 3

THE CHARACTER OF THE SOLUTION AS A FUNCTION OF (k_1, k_2). IN REGIONS I AND II THE EQUILIBRIUM THREATS ARE PURE STRATEGIES; IN REGION III THEY ARE MIXED

regions are illustrated in Figure 3. Region III is the one in which the equilibrium threats are mixed strategies. In this case these strategies F_i both have the form

$$F_i(x) = \begin{cases} 0 & \text{if} \quad x \leqq a \\ G_i(x) & \text{if} \quad a \leqq x < b(k_2) \\ 1 & \text{if} \quad b(k_2) \leqq x, \end{cases} \tag{A.1}$$

where G_i: $[a, b(k_2)] \to [0, 1]$ is continuous, $G_i(a)=0$ $(i=1, 2)$, $G_1(b(k_2)) \leqq 1$, and $G_2(b(k_2))=1$ (so that F_2 is continuous, while F_1 may have an atom at $b(k_2)$). The precise forms of G_i (which follow from the results of Osborne and Pitchik [1986]) are specified in Table 1, together with the pure strategy equilibrium

TABLE 1

Region	Equilibrium Threats
I	Pure: $(P(k), P(k))$
II	Pure: $(0, 0)$
III	Mixed, of type (A.1), with $G_i^*(p) = \dfrac{L_j(p) - L_j(a^*)}{L_j(p) - M_j(p)}$, where $a^* < b(k_2)$ is such that $L_1(a^*) = M_1(b(k_2))$.

threats for regions I and II. Since $L_1(x)=\pi(x)$ in IIIb, it follows that

(A.2) in region IIIb, a^* is independent of k_1.

Expressions for the negotiated profits are given in Table 2. In the cases of pure strategy equilibrium, these can be calculated directly from the equilibrium strategies. For region III, note that 1's payoff $v_1(p, F_2^*)$ is constant, equal to its equilibrium payoff, for p in $[a^*, b(k_2)]$, and so, since F_2^* is continuous, 1's negotiated profit is just $v_1(b(k_2), F_2^*)$. This can be calculated explicitly (given the form of F_2^*); since the monopoly profit is 1 in region III, the sum of the negotiated profits is 1, so the negotiated profit of firm 2 follows immediately.

It is easy to check that the negotiated profits have the continuity properties claimed in the text. A tedious differentiation also shows that the negotiated profit of i is nondecreasing in k_i and nonincreasing k_j; we also have the following.

LEMMA 7. *v_1^* is concave in k_1. v_2^* is concave in k_2 on the union of regions Ia, II, and IIIa.*

TABLE 2

Region	Negotiated Profits[a]
Ia	$(k_1P(k), k_2(P(k))$
Ib	$[1+(k_1-k_2)P(k)]/2$
II	$1/2$
IIIa	$[1+(k_1-k_2)B(k_2)/k_1]/2$
IIIb	$[1+B(k_2)-k_2a^*]/2$

[a] For all regions except Ia, only the negotiated profit of firm 1 is given; in these cases, negotiated profit of firm $2=1-$(negotiated profit of firm 1).

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