An Introduction to Game Theory

Please send comments to
Martin J. Osborne
Department of Economics
150 St. George Street
University of Toronto
Toronto, Canada M5S 3G7
email: martin.osborne@utoronto.ca

This version: 2002/7/25

Note: The categorization of exercises according to the availability of their solutions is tentative. No solutions are currently publicly available.
Note: The Index currently covers only Chapters 1 and 2 (and a few other bits and pieces).
Note: Bad line breaks and page breaks have not yet been corrected.

An Introduction to Game Theory

Martin J. Osborne

New York Oxford OXFORD UNIVERSITY PRESS 2002

OXFORD UNIVERSITY PRESS
Oxford New York
Athens Auckland Bangkok Bombay
Calcutta Cape Town Dar es Salaam Deli
Florence Hong Kong Istanbul Karachi
Kuala Lumpur Madras Madrid Melbourne
Mexico City Nairobi Paris Singapore
Taipei Tokyo Toronto

and associated companies in
Berlin Ibadan

Published by Oxford University Press, Inc.,
198 Madison Avenue, New York, New York 10016

Oxford is a registered trademark of Oxford University Press

This book was typeset by the author, who is greatly indebted to
Donald Knuth (TEX), Leslie Lamport (LATEX), Diego Puga (mathpazo),
Christian Schenk (MiKTEX), Ed Sznyter (ppctr), Timothy van Zandt (PSTricks),
and others, for generously making superlative software freely available.
The main font is 10pt Palatino.

Library of Congress Cataloging-in-Publication Data
Osborne, Martin John.
An Introduction to Game Theory / Martin J. Osborne
p. cm.
Includes bibliographical references and index.
ISBN ?????
1. Game Theory. 2. Economics, Mathematical. I. II. Title.
HB144.O???? 2002

Contents

Preface

GAME-THEORETIC REASONING pervades economic theory and is used widely in other social and behavioral sciences. This book presents the main ideas of game theory and shows how they can be used to understand economic, social, political, and biological phenomena. It assumes no knowledge of economics, political science, or any other social or behavioral science. It emphasizes the ideas behind the theory rather than their mathematical expression, and assumes no specific mathematical knowledge beyond that typically taught in US and Canadian high schools. (Chapter 17 reviews the mathematical concepts used in the book.) In particular, calculus is not used, except in the appendix of Chapter 9 (Section 9.8). Nevertheless, all concepts are defined precisely, and logical reasoning is used throughout. My aim is to explain the main ideas of game theory as simply as possible while maintaining complete precision; the more comfortable you are with tight logical analysis, the easier you will find the arguments.

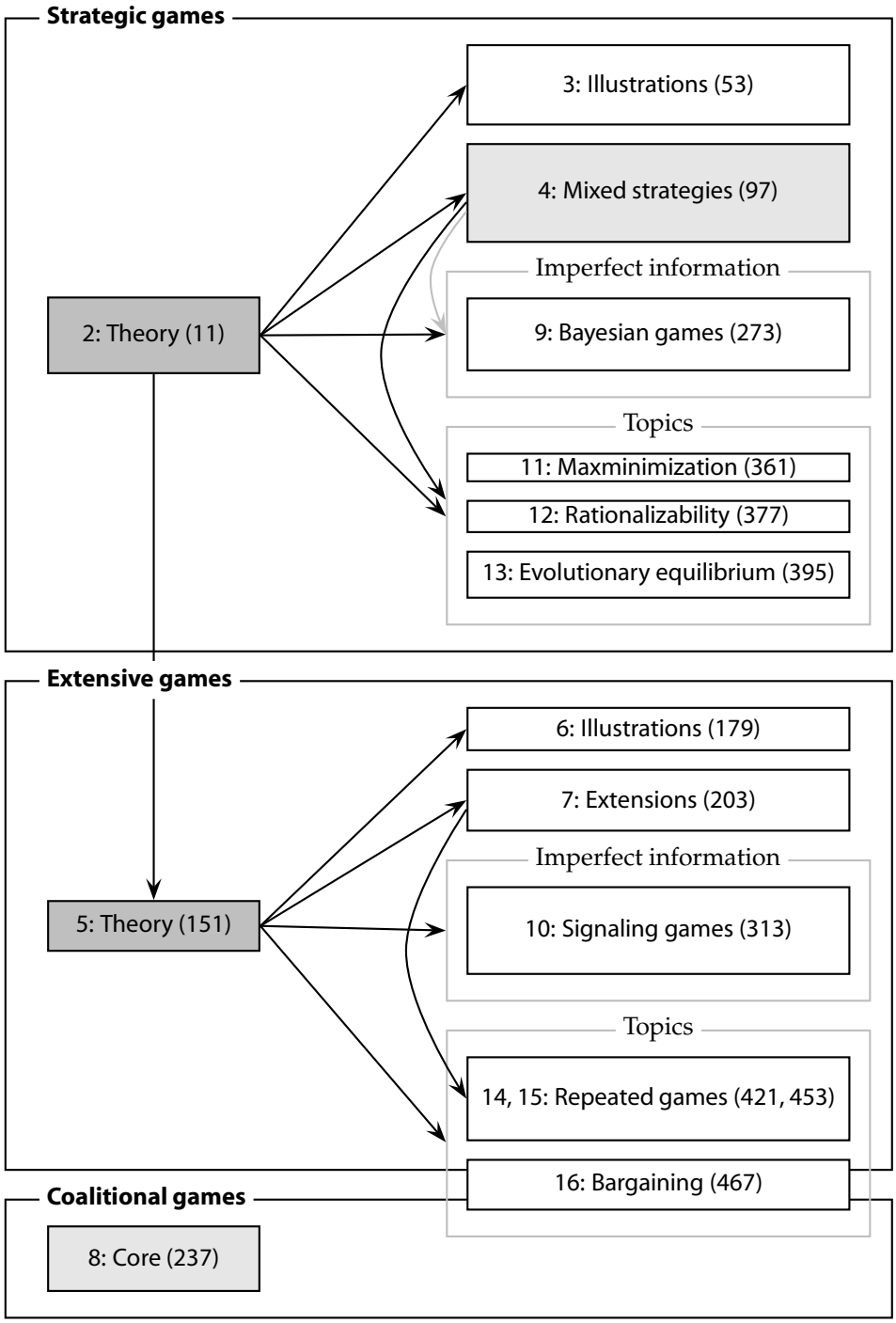
The only way to appreciate the theory is to see it in action, or better still to put it into action. So the book includes a wide variety of illustrations from the social and behavioral sciences, and over 280 exercises.

The structure of the book is illustrated in the figure on the next page. The gray boxes indicate core chapters (the darker gray, the more important). A black arrow from Chapter i to Chapter j means that Chapter j depends on Chapter i. The gray arrow from Chapter 4 to Chapter 9 means that the latter depends weakly on the former; for all but Section 9.7 only an understanding of expected payoffs (Section 4.1.3) is required, not a knowledge of mixed strategy Nash equilibrium. (Two chapters are not included in this figure: Chapter 1 reviews the theory of a single rational decision-maker, and Chapter 17 reviews the mathematical concepts used in the book.)

Each topic is presented with the aid of "Examples", which highlight theoretical points, and "Illustrations", which demonstrate how the theory may be used to understand social, economic, political, and biological phenomena. The "Illustrations" introduce no new theoretical points, and any or all of them may be skipped without loss of continuity. The "Illustrations" for the key models of strategic and extensive games are grouped in separate chapters (3 and 6).

The limited dependencies between chapters mean that several routes may be taken through the book.

- At a minimum, you should study Chapters 2 (Nash Equilibrium: Theory) and 5 (Extensive Games with Perfect Information: Theory).
- Optionally you may sample some sections of Chapters 3 (Nash Equilibrium: Illustrations) and 6 (Extensive Games with Perfect Information: Illustrations).



The structure of the book. Each chapter is represented by a box whose area is proportional to the length of the chapter; the number in parentheses is the page on which the chapter begins. Boxes shaded gray correspond to core chapters; dark gray chapters are more central that light gray ones. An arrow from Chapter i to Chapter j means that Chapter i is a prerequisite for Chapter j. The gray arrow from Chapter 4 to Chapter 9 means that the latter depends only weakly on the former.

- You may add to this plan any combination of Chapters 4 (Mixed Strategy Equilibrium), 9 (Bayesian Games, except Section 9.7, which requires Chapter 4), 7 (Extensive Games with Perfect Information: Extensions and Discussion), 8 (Coalitional Games and the Core), and 16 (Bargaining).
- If you read Chapter 4 (Mixed Strategy Equilibrium) then you may in addition study any combination of the remaining chapters covering strategic games, and if you study Chapter 7 (Extensive Games with Perfect Information: Extensions and Discussion) then you are ready to tackle Chapters 14 and 15 (Repeated Games).

All the material is intended to be accessible to undergraduate students. A one-semester course for third or fourth year North American economics majors (who have been exposed to a few of the main ideas in first and second year courses) could cover up to about half the material in the book in moderate detail.

Osborne and Rubsintein (1994), a graduate text, offers a more advanced treatment of the field. With few exceptions, the two books use the same notation and terminology. The exceptions are noted on the website for this book (see the end of the preface).

Examples modeling economic, political, and biological phenomena

The main examples that involve economic, political, or biological phenomena are listed below. (Examples used to make a primarily game-theoretic point are omitted.)

Economic phenomena

Accident law: Section 3.6
Adverse selection: Exercise 282.4
Auctions: Section 3.5 (perfect information), Example 142.1 (all-pay), Section 9.6 (imperfect information), Section 9.8 (imperfect information)
Bertrand's model of oligopoly: Section 3.2, Exercise 134.2, Exercise 143.2, Exercise 190.1, Exercise 213.1, Exercise 392.4 (dominance solvability), Exercise 457.1 (repeated game), Exercise 461.1, Exercise 461.2, Section 15.4 (repeated game with imperfectly observable actions)
Chain-store game: Example 230.1
Collective decision-making: Section 2.9.4
Common property: Section 3.1.5
Competition in product characteristics: Exercise 74.2
Cournot's model of oligopoly: Section 3.1, Exercise 134.1, Section 9.4 (imperfect information), Exercise 388.3 (rationalizability)
Entry into an industry by a financially-constrained challenger: Exercise 173.1
Entry into a monopolized industry: Section 7.2
Exit from a declining industry: Section 7.5

Expert diagnosis: Section 4.6
Firm–union bargaining: Exercise 175.1, Exercise 225.1, Exercise 490.2
Holdup game: Section 6.1.2
Market games: Exercise 209.3, Example 244.1, Section 8.4, Section 8.5, Section 16.2
Matching: Example 240.3, Section 8.7
Ownership and the distribution of wealth: Example 238.2, Example 243.1, Section 8.3
Price competition between sellers: Exercise 126.1, Exercise 210.1
Provision of a public good: Exercise 31.1, Section 2.8.4, Exercise 43.1, Exercise 130.3, Section 9.5, Exercise 388.2 (rationalizability)
Reporting a crime (private provision of a public good): Section 4.8
"Rotten kid theorem": Exercise 175.2
Signaling ability with education: Section 10.7
Signaling quality with conspicuous expenditure: Section 10.6
Stackelberg's model of duopoly: Section 6.2
Strategic information transmission: Section 10.8
Timing product release: Exercise 78.1

Political phenomena

Agenda control: Section 6.1.3 (perfect information), Section 10.9 (imperfect information)
Allocating resources in election campaigns: Exercise 139.2
Approval voting: Exercise 47.3
Buying votes in a legislature: Section 6.3
Cohesion of governing coalitions in legislatures: Exercise 226.2
Collective decision-making: Section 2.9.4
Committee decision-making: Section 7.4
Electoral competition between citizen-candidates: Exercise 73.2
Electoral competition between policy-motivated candidates: Exercise 73.1
Hotelling's model of electoral competition: Section 3.3, Exercise 194.3, Exercise 194.4, Section 7.3 (strategic voters), Exercise 388.1 (rationalizability), Exercise 456.2 (repeated game)
Juries: Section 9.7
Lobbying as an auction: Exercise 88.3
Majority game: Example 239.1, Exercise 243.2
Vote trading: Exercise 245.2
Voter participation: Exercise 32.2, Exercise 116.1
Voting: Section 2.9.3 (strategic game), Section 8.6 (coalitional game), Exercise 307.1 (swing voter's curse), Exercise 392.3 (dominance solvability)
Voting by alternating veto: Exercise 172.1

Biological phenomena

Evolution of sex ratio: Section 13.6
Hawk–Dove: Example 400.2, Example 406.2, Section 13.3.2
Hermaphroditic fish: Exercise 16.1
Nesting behavior of wasps: Section 13.5
Reciprocal altruism: Box on page 447
Sibling competition: Section 13.4
Signaling hunger (Sir Philip Sydney game): Exercise 335.2
War of attrition: Section 3.4

Personal pronouns

The English language lacks a third person singular pronoun widely interpreted to be sex-neutral. In particular, many experiments have shown that "he" in not neutral[1], a finding consistent with the observation that whereas people may say "when an airplane pilot is working, he needs to concentrate", they do not usually say "when a flight attendant is working, he needs to concentrate" or "when a secretary is working, he needs to concentrate". To quote the *American Heritage Dictionary* (third edition, page 831), "Thus *he* is not really a gender-neutral pronoun; rather it refers to a male who is to be taken as the representative member of the group referred to by its antecedent. The traditional usage, then, is not simply a grammatical convention; it also suggests a particular pattern of thought." Like many writers, I regard as unacceptable the bias implicit in the use of "he" for individuals of unspecified sex. The *New Oxford Dictionary of English* states the case clearly: "[the use of *he* to refer to a person of unspecified sex] has become . . . a hallmark of old-fashioned language or sexism in language." Writers have become sensitive to this issue in the last fifty years, but the lack of a sex-neutral pronoun "has been felt since at least as far back as Middle English" (*Webster's Dictionary of English Usage*, Merriam-Webster Inc., 1989, 499). A common solution has been to use "they",[2] a usage that the *New Oxford Dictionary of English* endorses (and employs). In some contexts this usage sounds natural, but in others it does not; it can also create ambiguity when the pronoun follows references to more than one person. I choose a different solution: I use "she" exclusively. Obviously this usage, like that of "he", is not sex-neutral, but it may help to counterbalance the widespread use of "he", and seems unlikely to do any harm.

[1] See, for example, Janice Moulton, George M. Robinson, and Cherin Elias, "Sex bias in language use: 'neutral' pronouns that aren't", *American Psychologist* **33** (1978), 1032–1036; Janet Shibley Hyde, "Children's understanding of sexist language", *Development Psychology* **20** (1984), 697–706; and John Gastil, "Generic pronouns and sexist language: the oxymoronic character of masculine generics", *Sex Roles* **23** (1990), 629–643.

[2] For a discussion of the history of the use of "they", see Ann Bodine, "Androcentrism in prescriptive grammar: singular 'they', sex-indefinite 'he', and 'he or she' ", *Language in Society* **4** (1975), 129–146.

References

The "Notes" section at the end of each chapter attempts to assign credit for the ideas discussed. Several cases present difficulties. In some cases, ideas evolved over a long period of time, with contributions by many people, making their origins hard to summarize in a sentence or two. In a few cases, my research has led to a conclusion about the origins of an idea different from the standard one. In all cases, I cite the relevant papers without regard to their difficulty.

Over the years, I have taken exercises from many sources. I have attempted to remember the origins of the ones I use in this book, and give credit appropriately.

Conventions, numbering, and nomenclature

I use the term "dollar" for a unit of money because it is probably recognizable as such to a majority of readers of this book, even if they pay their bills in rupees, yuan, or pa'anga.

In formal definitions, the terms being defined are set in **boldface**. Terms set in *italics* are informal definitions.

Definitions, propositions, examples, and exercises are numbered according to the page on which they appear. If the first such object on page n is an exercise, for example, it is called Exercise n.1; if the next object on that page is a definition, it is called Definition n.2. For example, the definition of a strategic game with ordinal preferences on page 11 is Definition 11.1. This scheme allows numbered items to found rapidly, and also facilitates precise index entries.

Symbol/term	Meaning
⍰	Exercise, with solution on website
⁉	Hard exercise, with solution on website
?	Exercise, with solution available only to instructors (see website for conditions)
⁉	Hard exercise, with solution available only to instructors (see website for conditions)
▶	Definition
■	Result
◆	Example: a game that illustrates a game-theoretic point
Illustration	A game, or family of games, that shows how the theory can illuminate observed phenomena

Acknowledgements

I owe a huge debt to Ariel Rubinstein. I have learned, and continue to learn, vastly from him about game theory. His influence on this book will be clear to anyone

familiar with our jointly-authored book *A course in game theory*. Had we not written that book and our previous book *Bargaining and markets*, I doubt that I would have embarked on this project.

I was privileged as a graduate student to learn game theory from Robert Aumann, Sergiu Hart, Mordecai Kurz, Al Roth, and Robert Wilson at Stanford University.

Discussions over the years with Jean-Pierre Benoît, Haruo Imai, Vijay Krishna, Michael Peters, and Carolyn Pitchik have improved my understanding of many game-theoretic topics.

Many people have generously commented on all or parts of drafts of the book. I am particularly grateful to Gian Luigi Albano, Jeffrey Banks, Nikolaos Benos, Ted Bergstrom, Tilman Börgers, Randy Calvert, Vu Cao, In-Koo Cho, Rachel Croson, Eddie Dekel, Marina De Vos, Laurie Duke, Patrick Elias, Mukesh Eswaran, Xinhua Gu, Costas Halatsis, Joe Harrington, Hiroyuki Kawakatsu, Lewis Kornhauser, Jack Leach, Simon Link, Bart Lipman, Kin Chung Lo, Massimo Marinacci, Peter McCabe, Massimo Morelli, Nathan Nunn, Barry O'Neill, Robin G. Osborne, Marco Ottaviani, Carolyn Pitchik, Marie Rekkas, Bob Rosenthal, Al Roth, Matthew Shum, Giora Slutzki, Michael Smart, Nick Vriend, Charles A. Wilson, and Luís Zemborain.

I thank also the anonymous reviewers consulted by Oxford University Press and several other presses; the suggestions in their reviews greatly improved the book.

The book has its origins in a course I taught at Columbia University in the early 1980s. My experience in that course, and in courses at McMaster University and the University of Toronto, brought the book to its current form. The Kyoto Institute of Economic Research at Kyoto University and the School of Economics at the Australian National University provided me with splendid environments in which to work on the book during visits in 1999 and 2001.

I maintain a website for the book. The current URL is
`http://www.economics.utoronto.ca/osborne/igt/`

MARTIN J. OSBORNE
`martin.osborne@utoronto.ca`
`http://www.economics.utoronto.ca/osborne/`
Department of Economics, University of Toronto
150 St. George Street, Toronto, Canada, M5S 3G7
July 2002