## Corrections and updates for first printing of Osborne's "An Introduction to Game Theory" (Oxford University Press, 2004)

## 2023.3.12

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## Corrections

	Page,	Line	Correction
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4	The first letter of the text in Section 1.2 should be upper case.
6	Add a space after the period on the last line.
30	Two lines above Exercise 30.1, replace "pursing" with "pursuing".
31	The "A" in the caption of Figure 31.1 should be upright, not italic.
42	At the end of the first sentence of the paragraph starting on line 10 of
	Section 2.8.4 add "where $v_i$ is an increasing function".
49,1	Replace "by her other two actions" with "by her action of voting for
	her favorite candidate."
50	Replace the last sentence of the second bullet point of item <i>a</i> , on lines
	15–16, with "Thus player <i>i</i> is at least as well off naming $x_i^*$ as she is
	naming <i>x<sub>i</sub></i> ."
56	On the line below (56.2), replace " $c \ge 0$ " with " $c > 0$ ".
58, -6	Replace the first instance of "increases" with "increase".
59, -3	Replace " $c \ge 0$ " with " $c > 0$ ".
62, -9	Replace "firm's" with "firms'".
63	Replace the last two sentences of Exercise 63.1 with "Find the Nash
	equilibria of the game. Find an action profile $(x_1, \ldots, x_n)$ at which each
	firm's output is higher than it is at any Nash equilibrium."

- 67 In the third line of Exercise 67.1, insert "continuous" between "any" and "function".
- 69 In part *a* of Exercise 68.2, replace the first sentence with "Suppose that when the firms' prices are the same, each firm obtains a positive fraction of the demand when the common price is less than  $c_2$ , but firm 1 obtains all the demand when the common price is  $c_2$  or more."
- 74 Replace the second sentence of Exercise 74.1 with the following two sentences. "Show that if less than one third of the citizens' favorite positions are equal to the median favorite position (*m*), then the game has no Nash equilibrium. Argue as follows."
- 75 Exercise 75.2 should specify the players' payoffs when no citizen stands as a candidate and when several candidates tie for first place. Replace the fifth through eighth sentences with: "Winning confers the benefit b, and citizens care about the position of the winning candidate. Specifically, the payoff of a citizen with favorite position x who becomes a candidate is the expected value of  $-|x - x_i^*| + b/k - c$  (as  $x_i^*$  ranges over the positions of the winning candidates) if she ties with k - 1 other candidates for first place, and the expected value of  $-|x - x_i^*| - c$  if she is not one of the candidates tied for first place. (For any number z, |z| denotes the absolute value of z: |z| = z if z > 0 and |z| = -z if z < 0.) The payoff of a citizen with favorite position x who does not become a candidate is the expected value of  $-|x - x_i^*|$  if some citizen becomes a candidate, and K, with K < b - c, if no citizen becomes a candidate." The exercise should also include the assumption that less than one-third of the citizens' favorite positions are equal to the median favorite position.
- 76, 10–11 Replace "one prefers x to z and the other prefers z to x" with "both prefer z to x".
  - 78 In Figure 78.2, replace " $B_1(p_2)$ " with " $B_1(t_2)$ " and replace " $B_2(p_1)$ " with " $B_2(t_1)$ ".
  - 83 In the first line of the second paragraph, change "complete" to "perfect" (for consistency with other terminology).
  - 83 The first sentence of the item "Preferences" just below the middle of the page is hard to follow. A better version is: "Denote by  $b_i$  the bid of player *i* and by  $\overline{b}$  the highest bid submitted by a player other than *i*. If either (*a*)  $b_i > \overline{b}$  or (*b*)  $b_i = \overline{b}$  and the number of every other player who bids  $\overline{b}$  is greater than *i*, then player *i*'s payoff is  $v_i - \overline{b}$ ."

- 85–87 In the third line of the text on page 85, in the third line of Section 3.5.3 on page 86, and in the fifth line from the bottom of page 87, change "complete" to "perfect" (for consistency with other terminology).
  - 86 In Exercise 86.1, change the word "action" on lines 1, 2, 4, and 6 to "policy" (to avoid confusion with the actions in the strategic game).
  - 91 Delete the parenthetical claim on the last line of Exercise 91.1 (which is not correct). (Probably in each case the pair of actions given is the unique equilibrium in which neither player's action is weakly dominated.)
- 94 The fourth word of the caption of Figure 94.1 should be "shows".
- 110, 16 Replace "an" at the end of the line with "a".
- 110 Replace "left" with "right" on the second line of Exercise 110.1.
- 115, 1 Replace "a swimmer" with "any swimmer".
- 118 In line 2 of Exercise 118.2, replace " $k \le m$ " with " $2 \le k \le m$ ".
- 130 Change Proposition 130.1 to: "Every symmetric strategic game with vNM preferences in which each player's set of actions is finite has a symmetric mixed strategy Nash equilibrium."
- 130, -16 Delete "of the equilibrium".
- 136, 1 Add "by" after "generated".
- 143, 21 Replace F(z) with  $F_i(z)$ .
  - 145 Replace " $x_2$  and  $y_2$ " on the line below the display with " $x_1$  and  $y_1$ ".
- 145, -8 Delete " $a_1$ " at the end of the line.
  - 148 Line 7 of Section 4.12.2 is confusing. Replace it with "*v* defined by  $v(x) = \sqrt{u(x)}$  for all *x*, for which v(0) = 0, v(1) = 1, and v(5) = 2, is not consistent with such a".
  - 150 In each of the games in Exercise 150.1, replace the actions *C* and *D* with *Q* and *F*.
  - 185 The continuity assumption in Exercise 185.2*b* is not stated correctly. Replace it with "Assume also that the players' preferences are continuous: if a player prefers *P* to its complement (the remainder of the cake), then there is a subset *P*' of *P* not equal to *P* such that the player prefers *P*' to its complement."
- 187, 2 Replace "in" with "is".
- 189 Replace first " $q_1^*$ " with " $q_2^*$ " on line 6 of paragraph starting "We conclude".
- 195, 10–11 Delete the clause "so it needs to pay each legislator more than  $V_Y/\mu$ ", which is not correct and not needed.
  - 197 At the start of the penultimate line of the item *Terminal histories*, replace  $y^1 + \cdots + y^T < k_2$  with  $y^1 + \cdots + y^{T-1} < k_2$ .

- 202–203 The term "equilibrium path" is used without explanation. It is synonymous with "equilibrium outcome". (That is, the equilibrium path is the terminal history generated by the equilibrium strategies.)
  - 209 In Figure 209.2 the pair of payoffs to the action pair (B, R) should be (2, 1) (not (2, 0)).
- 215, -12 Insert "the negative of" before "the average distance".
- 215, -11 Replace the first "of" with "to".
- 216, -4 The word "that" should be "than".
- 220, 8 Replace "*x* beats *y* beats x" with "*x* beats *y* beats *z*".
- 221 Replace "Exercise 220.1" in Exercise 221.2 with "Exercise 220.1b".
- 227, 11 Delete the string ", rejects all offers x < 0,".
- 228 Change the penultimate sentence of Exercise 228.1 to "Specify each game precisely and find the outcomes of subgame perfect equilibria in which no player's vote in either period is weakly dominated."
- 240 Change "a" to "an" on the second line of Example 240.2 and delete "with f(0) = 0" from the third line.
- 241 Modify the definition of a coalitional game with transferable payoff: A coalitional game has **transferable payoff** if there is a collection of payoff functions, one representing each player's preferences, and, for each coalition *S*, a number v(S), such that for every action of *S* the sum of the payoffs of the members of *S* is v(S) and for every *S*-allocation  $x_S$ of v(S), *S* has an action that yields  $x_S$ .
- 241 Change "2,..., m" to "2,..., m + 1" on the line above the displayed equation.

254–255 The argument in the last paragraph of page 254 and the last two points in (255.1) are not correct. An owner sells her horse if and only if her index is at most  $k^*$  and a nonowner buys a horse if and only if her index is at most  $k^*$ . (A player whose valuation is  $p^*$  trades only if her index is at most  $k^*$ .) The last paragraph on page 254 should be replaced by the following paragraph.

Finally, I argue that the set of owners who sell their horses is  $\{1, \ldots, k^*\}$  and the set of nonowners who buy a horse is  $\{1, \ldots, k^*\}$ . If  $\max\{\sigma_{k^*}, \beta_{k^*+1}\} < p^* < \min\{\beta_{k^*}, \sigma_{k^*+1}\}$ , this result follows immediately from the paragraph before Exercise 254.1. If  $p^* = \max\{\sigma_{k^*}, \beta_{k^*+1}\}$  or  $p^* = \min\{\beta_{k^*}, \sigma_{k^*+1}\}$  then it follows from that paragraph combined with the fact that every trade involves both an owner and a nonowner. (For example, if  $p^* = \sigma_{k^*} > \beta_{k^*+1}$ , then  $\beta_{k^*} > p^*$ , so that the set of nonowners who buy a horse is  $\{1, \ldots, k^*\}$ , and  $\sigma_{k^*-1} < p^* < \sigma_{k^*+1}$ , so that owners with index at most  $k^* - 1$  sell their horses and owner with index at least  $k^* + 1$  do not sell their horses. Consequently owner  $k^*$  must sell her horse.)

The last two points in (255.1) should be replaced by the following point.

every owner with index at most  $k^*$  sells her horse, and every other owner retains her horse; every nonowner with index at most  $k^*$  buys a horse, and no other nonowner buys a horse.

The word "four" should be deleted on the last line of page 255.

- 274, 3 Change "got" to "go".
- 276,1–2 Change the first two occurrences of "player 2" to "player 1".
- 286, -8 Delete the word "posterior".
  - 287 In the first line of Section 9.4.2 replace "its cost" with "firm 2's cost" and in the third line replace "it cost" with "its cost".
- 288, 8 Replace second occurrence of  $q_2(P(q_1 + q_2) c_L)$  with  $q_2(P(q_1 + q_2) c_H)$ .
- In lines 3, 11, and 13 of Exercise 288.1, replace  $\ell$  and h with L and H.
- 289 In the description of the states above the figure, replace " $0 \le v_i \le \overline{v}$ " with " $\underline{v} \le v_i \le \overline{v}$ ".
- 290, -5 Replace "equations" with "inequalities".
  - 291 Replace "a decreasing" with "an increasing" on line 16 and "increases" with "decreases" on line 17.
  - 295 The claim in the last sentence on the page is too strong: the appendix contains only suggestive arguments, not a proof.
  - 296 Replace "the highest of the other players' bids" on line 5 with "the highest of the other players' valuations".

- 296 Change the last sentence of the first paragraph of the subsection "Comparing equilibria ..." to "I now argue that the same is true, under the assumptions of this section, when the players are uncertain of each other's valuations."
- 303 In the description of the beliefs in the middle of the page, replace the  $\pi$  near the start of the second line with  $\Pr(G \mid g)$ , the  $1 \pi$  near the end of the second line with  $\Pr(I \mid g)$ , the  $\pi$  near the middle of the fifth line with  $\Pr(G \mid b)$ , and the expression involving  $1 \pi$  near the start of the sixth line with  $\Pr(I \mid b)(1 q)^k q^{n-k-1}$ .
- 307 In part *c* of Exercise 307.1, replace "one of the player's actions" with "an action of one of the players". In part *d*, replace "second" with "first".
- 308–309 To deduce the solution of the differential equation near the bottom of page 308, the initial condition  $\beta(\underline{v}) = \underline{v}$  is needed. Given that this initial condition is needed to find the equilibrium bidding function, the part of Exercise 309.2 asking for a proof that the equilibrium bidding function satisfies the condition should be removed. See the website for the book for a version of Section 9.8.1 that corrects these two points, treats more carefully the boundary cases in which  $v = \underline{v}$  and  $v = \overline{v}$ , and explains the argument more clearly.
  - 310 Two lines below (310.1), replace  $Pr{X < v}$  with Pr(X < v). On the following line, delete "= 0".
- 314, 11 Replace "had" with "has".
- 314 At the end of the next to last item in Definition 314.1, add "such that for every history h in any given member of the partition, the set A(h) of actions available is the same".
- 319, -2 Change the weak inequality to a strict inequality.
- 320–321 Replace  $0 \le \varepsilon < \frac{1}{4}$  by  $0 < \varepsilon < \frac{1}{4}$  on the bottom line of the page, on the seventh line from the bottom, and on the last line of the caption of Figure 321.1.
  - 321 In the bottom row of the right-hand table in the bottom panel of Figure 321.1, interchange the entries in the columns headed *XY* and *YX*, so that  $1/(2 4\varepsilon)$  is in the column headed *XY* and 0 is in the column headed *YX*.
- 324, 14–15 Replace "histories in that set" with "actions available at that set".
  - 327 Replace the 7th line of Example 327.3 with "Pr((High, Raise) according to  $\beta) = \frac{1}{2}p_H$  and Pr((Low, Raise) according to  $\beta) = \frac{1}{2}p_L$ , so by (327.1), the".
  - 330 In the 7th line of Example 330.1, replace "the history is *Acquiesce*" with "the history is *Unready*".

- Add a period to the end of the caption of Figure 331.2.
- 331 Replace Exercise 331.2 (which is incorrect) with the following exercise. EXERCISE 331.2 (Weak sequential equilibrium and Nash equilibrium in subgames) Consider the variant of the game in Figure 331.1 shown in Figure 332.1, in which the challenger's initial move is broken into two steps. Show that this game has a weak sequential equilibrium in which the players' actions in the subgame following the history *In* do not constitute a Nash equilibrium of the subgame.
- 332–333 Replace the last word on page 332 and the first word on page 333 with "a weak", and replace the penultimate word of the sentence with "strong".
- 333, -5 Replace "choose" with "chooses".
- 334, -6 Replace "and weak challenger" with "and a weak challenger".
- 335 Replace the second and third sentences of Exercise 335.1 with the following sentences (which reduce the demands of the exercise). "For what values of these payoffs, if any, does the game have a weak sequential equilibrium in which a strong challenger chooses *Ready* and a weak one chooses *Unready* (a separating equilibrium)? For what values of these payoffs, if any, does the game have a weak sequential equilibrium in which both types of challenger choose *Unready* (a pooling equilibrium)?"
- 335, -8 Insert "when it is quiet" after "obtaining the food".
- 335, -4 Replace r < (1-S)/(1-V) with r < (1-S)/(1-(1-p)V).
- 336 In Figure 336.1, the payoff of the parent after the history (*Not hungry, Squawk, Keep*) should be 1 + rV(1 t), not 1 + rV.
- 344 Replace each of the seven occurrences of the string t b with t + b.
- 347 In the third display, replace the second *b* with 4*b*.
- 348 The point 1 on the horizontal axis should be 15mm further to the right.
- 350, -9 Interchange "receiver" and "sender".
- 353 Replace the first occurrence of  $y_0 + 4b$  with  $y_0 + 3b$  in the fourth line of the display.
- 381, -10 Replace  $X_i^{t-1}$  with  $X_j^{t-1}$ .
  - 388 On the third line of Exercise 388.1*c* replace  $w_3$  with  $w_3/2$ .
- 389, 11 The outcomes that survive are (T, L) and (T, C) (not (T, L) and (T, R)).
- 391, 10 Replace "*B* are *R*" with "*B* and *R*".
- 394, -5 Delete the word "by".
- 406, 9 Change  $\gamma > 0$  to  $0 < \gamma < 1$ .
- 411 In the displayed statement, replace "if  $u(a^*, a^*) > u(b, b)$  for every action *b*" with "if  $u(a^*, a^*) > u(b, b)$  for every action  $b \neq a^{*''}$ .

- 415 Add a period to the end of the caption of Figure 415.1.
- 423, -1 Replace "history" with "terminal history".
- 424, -14 Replace "initial history" with "empty history".
- 424, -11 Replace "history" with "nonterminal history".
- 424, -7 Replace "history" with "nonterminal history".

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426 Replace (a^1, \ldots, a^T) with (a^1, \ldots, a^t) in the second line of Section 14.6.
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- 426, -4 Replace "initial history" with "empty history".
- 429 Replace t + k + 1 with t + k two lines above the penultimate display and one line above the last display.
- 438, 6 Replace "player's" with "players'".
- 439 In Exercise 439.1, add the assumption that the number of periods is at least 3, and in part (*b*) change the condition on  $\epsilon$  to  $2 < \epsilon < 4$ .
- 442, -3 Replace "initial history" with "empty history".
- 443, 3 Replace "initial history" with "empty history".
- 444, 8 Replace the phrase "in a subgame following a history" with "in the whole game and in a subgame following a history".
- 447 Move the phrase "where *j* is the other player" from the line below the display to the end of the following (parenthetical) sentence, and replace  $b^t$  near the end of the line below the display with  $b_i^t$ .
- 448, -8 Delete the first (or the second) "more".
- 452, -5 Replace "with probability  $\frac{1}{2}$ " with "with the same probability".
- 457, -2 Change  $k \ell$  to  $k \ell + 1$ .
- 469, -12 Change  $\delta_1^2 x_2$  to  $\delta_2^2 x_2$ .
  - 487 The sentence starting after the sentence containing the first display should be "Then the maximizer of  $(v'_1 d'_1)(v'_2 d'_2)$  over U' is  $(\alpha_1 v_1^* + \beta_1, \alpha_2 v_2^* + \beta_2)$ , where  $(v_1^*, v_2^*)$  is the maximizer of  $(\alpha_1 v_1 + \beta_1 d'_1)(\alpha_2 v_2 + \beta_2 d'_2)$  over U." In the following display, the second occurrence of  $d'_1$  should be  $d'_2$  and the second occurrence of  $d_1$  should be  $d'_2$ .
  - 498, 8 Delete one of the commas following *x*.
- 501, 10 Change  $p_n x_n$  to  $p_i x_i$ .
- 505, 10 Change "all prime numbers" to "all prime numbers greater than two".
- 505, 11 Change "if a number is prime" to "if a number is prime and greater than two".

## Updates

Dhillon and Lockwood (2003) is now

Dhillon, Amrita, and Ben Lockwood (2004), "When are plurality rule

voting games dominance-solvable?", *Games and Economic Behavior* **46**, 55–75.